

Event-by-event fluctuations of elliptic flow in O+O collisions

Renata Krupczak

Based on: arXiv:2606.22558

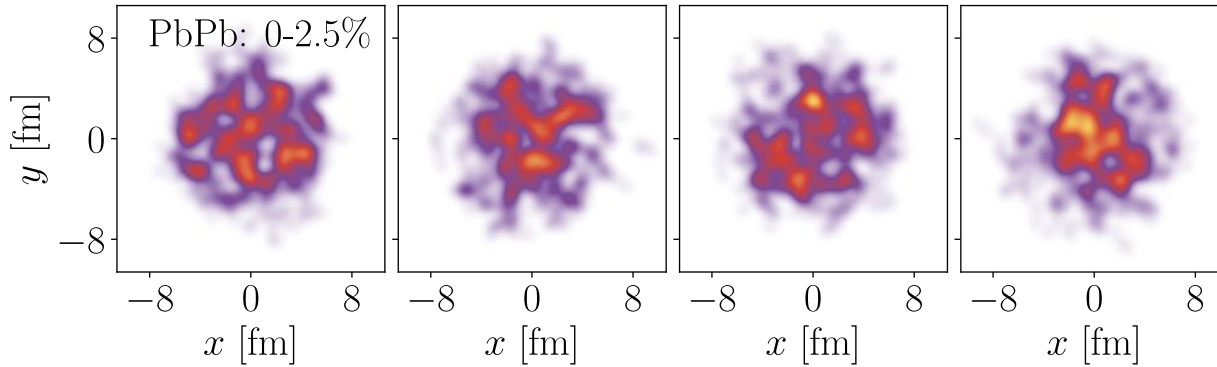
In collaboration with Nicolas Borghini and Hendrik Roch

Event-by-event fluctuations

Initial energy profiles in the transverse plane for heavy versus light ions

Based on MCGlauber

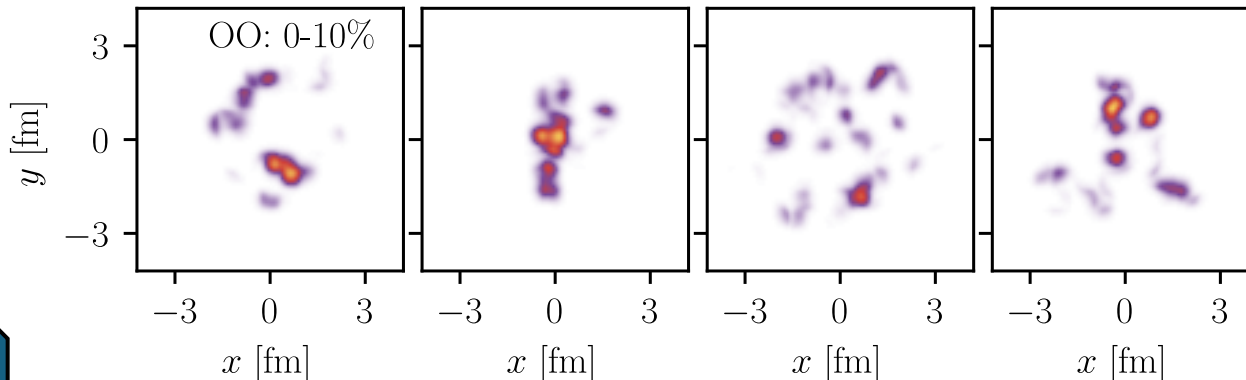
R. Krupczak et al., EPJ C 85 (2025), arXiv:2508.05336



Pb+Pb: 208 nucleons per nucleus
System size ~ 16 fm

Based on McDipper

R. Krupczak et al., arXiv:2606.22558



O+O: 16 nucleons per nucleus
System size ~ 6 fm

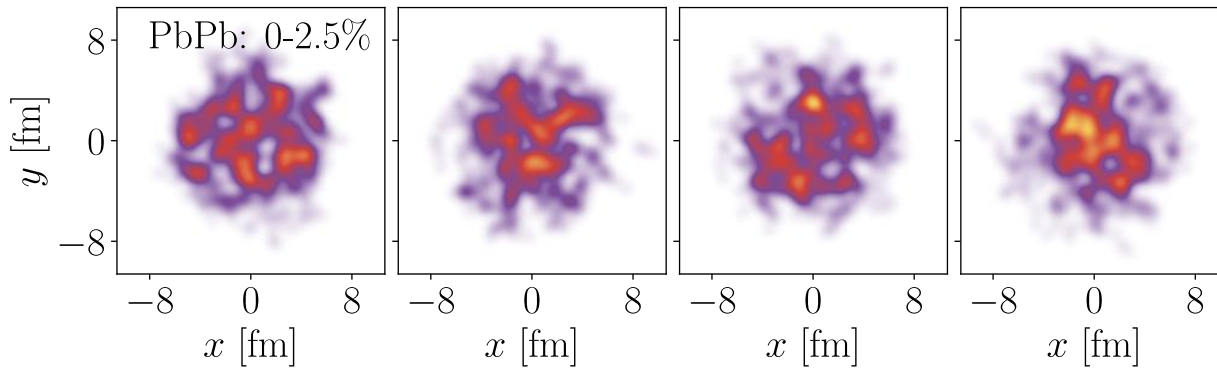
These fluctuations are relevant for describing precision final observables. Our goal here is **quantify the effect of these fluctuations.**

Event-by-event fluctuations

Initial energy profiles in the transverse plane for heavy versus light ions

Based on MCGlauber

R. Krupczak et al., EPJ C 85 (2025), arXiv:2508.05336

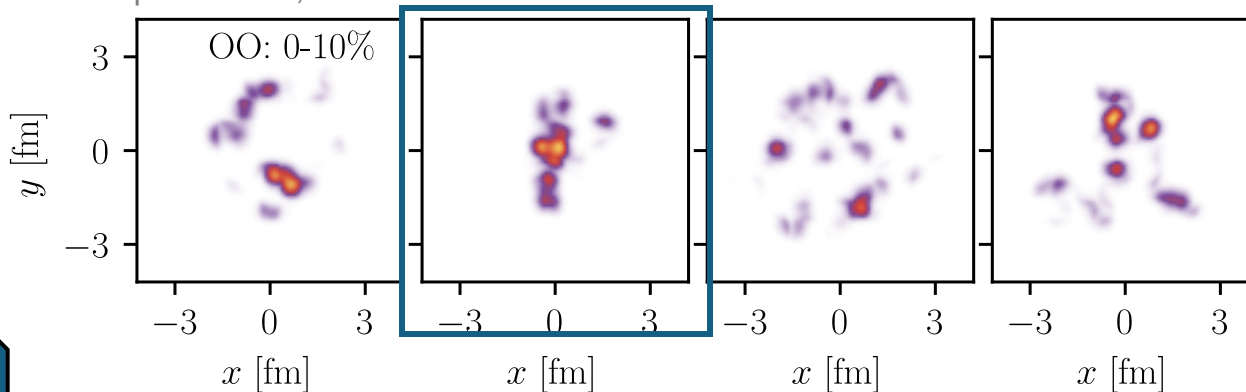


Pb+Pb: 208 nucleons per nucleus
System size ~ 16 fm

Oxygen collisions are not a scaled down version of lead collisions

Based on McDipper

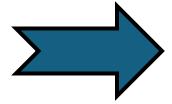
R. Krupczak et al., arXiv:2606.22558



O+O: 16 nucleons per nucleus
System size ~ 6 fm

Centrality is not linked to the usual geometry from the impact parameter

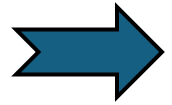
How do we quantify the effect of initial fluctuations on the observables?



Mode decomposition!

N. Borghini et al., PRC 107 (2023), arXiv: 2209.01176

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Mode decomposition!

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Each initial profile is decomposed into an average state plus a linear combination of fluctuation modes.

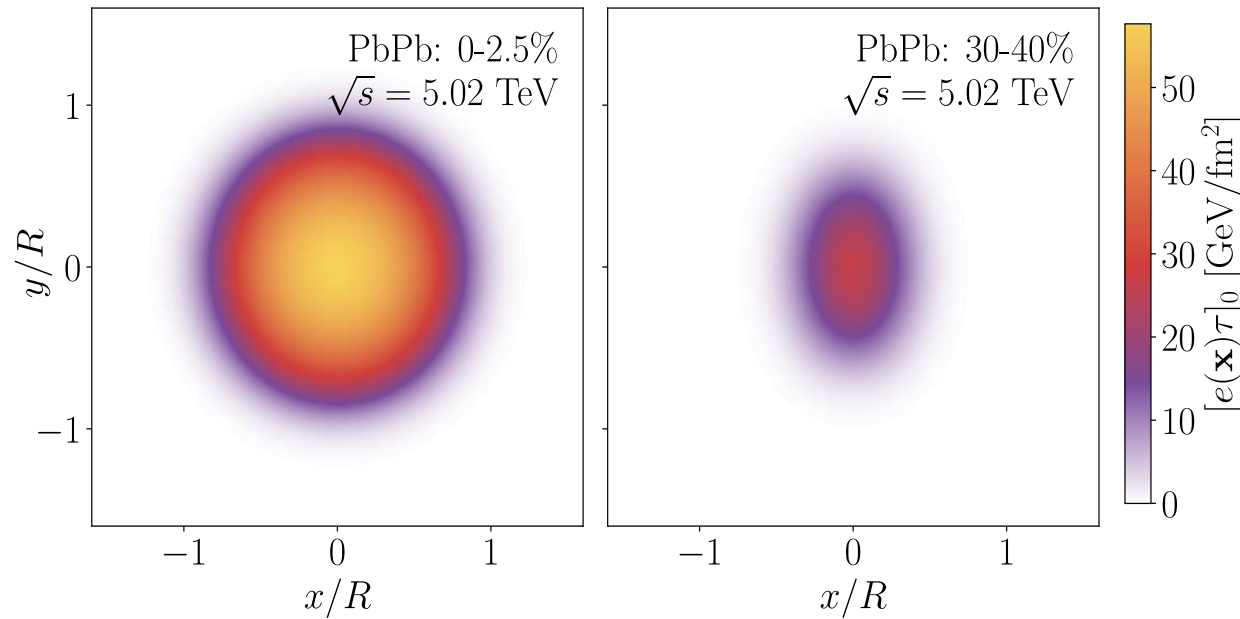
$$\Phi^{(i)}(x, y, \eta_s) = \bar{\Psi}(x, y, \eta_s) + \sum_l c_l^{(i)} \Psi_l(x, y, \eta_s)$$

Energy profile Average state $\downarrow$ Expansion coefficients $\searrow$ Fluctuation modes

Average State

$$\bar{\Psi}(x, y, \eta_s) \equiv \frac{1}{N_{\text{ev}}} \sum_{i=1}^{N_{\text{ev}}} \Phi^{(i)}(x, y, \eta_s)$$

For **large systems**, with a radius of 6.62 fm:



Characteristic **elliptical shape** for non-central collisions, due to the fixed direction of the impact parameter

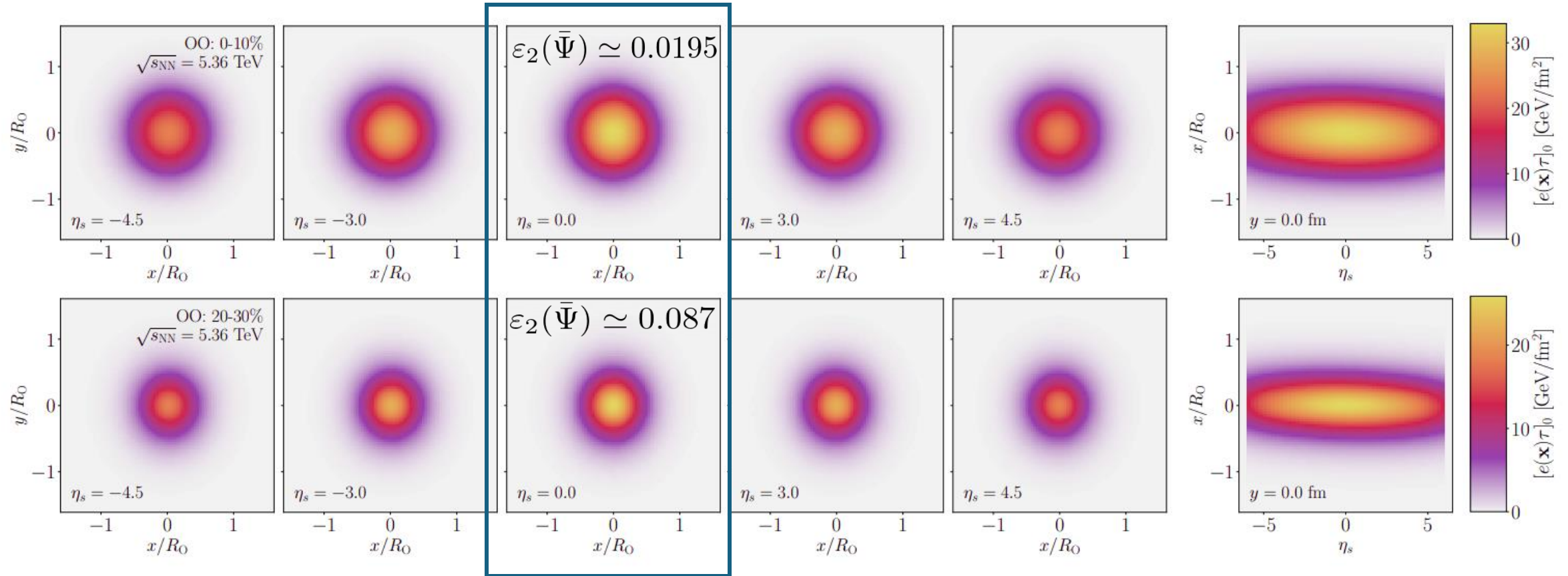
$$\varepsilon_2(\bar{\Psi}) \simeq 0.0174 \quad \varepsilon_2(\bar{\Psi}) \simeq 0.3214$$

R. Krupczak et al., EPJ C 85 (2025), arXiv:2508.05336

Average State

No clear elliptical shape, unlike large systems

The main contribution to the elliptical eccentricity is not produced by the overall impact parameter geometry



$R_O = 2.608$ fm

Initial profiles produced with McDipper

Fluctuation modes

$$\Phi^{(i)}(x, y, \eta_s) = \bar{\Psi}(x, y, \eta_s) + \sum_l c_l^{(i)} \Psi_l(x, y, \eta_s)$$

Fluctuation modes

Fluctuation modes

$$\Phi^{(i)}(x, y, \eta_s) = \bar{\Psi}(x, y, \eta_s) + \sum_l c_l^{(i)} \Psi_l(x, y, \eta_s)$$

Fluctuation modes



Correlation matrix between fluctuations: $\rho \equiv \frac{1}{N_{\text{ev}}} \sum_i \Phi^{(i)} \Phi^{(i)\text{T}} - \bar{\Psi} \bar{\Psi}^{\text{T}}$

Diagonalize this matrix:

$$\rho \tilde{\Psi}_l = \lambda_l \tilde{\Psi}_l$$

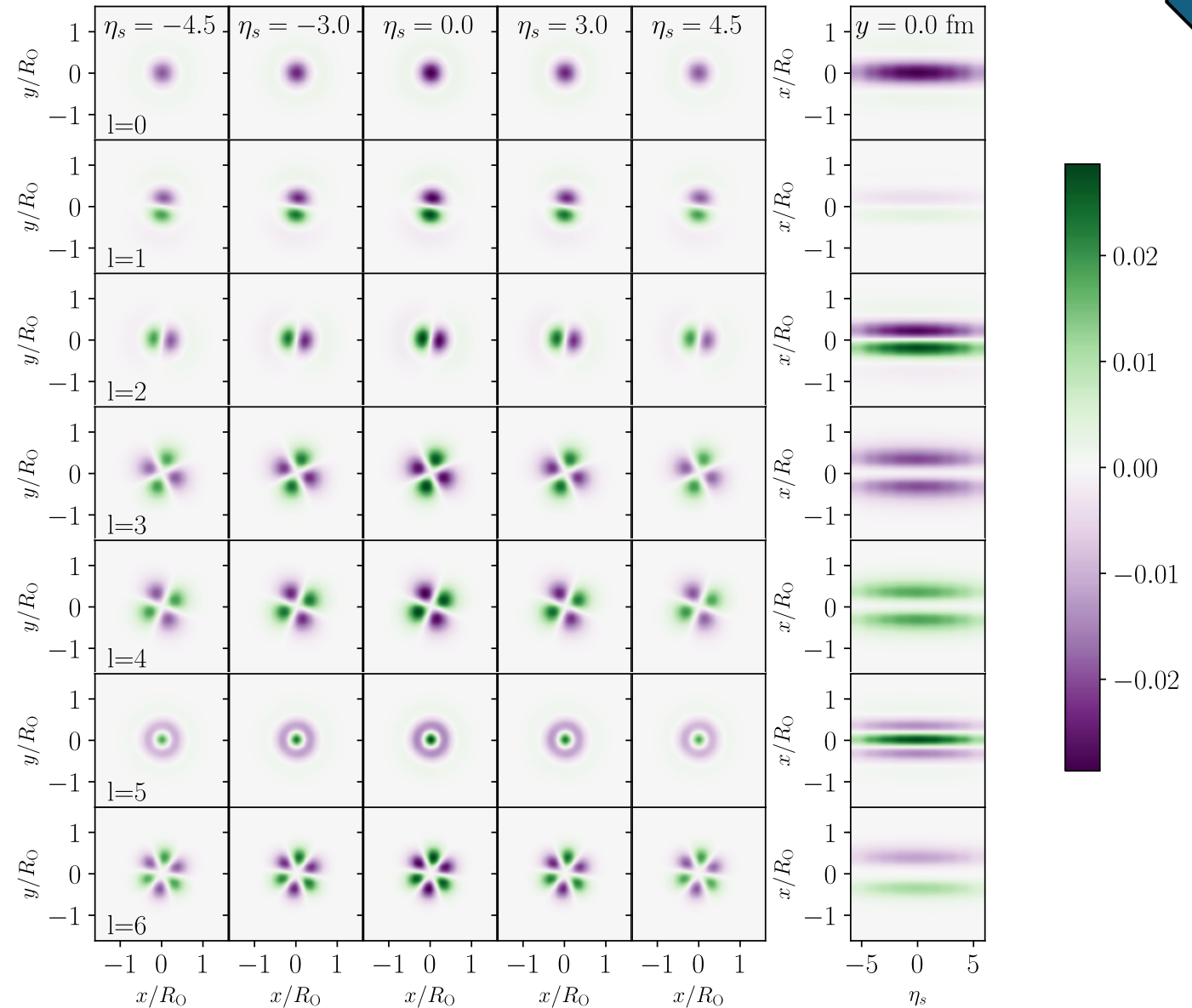
Fluctuation modes $\Psi_l = \sqrt{\lambda_l} \tilde{\Psi}_l$

Modes in O+O collisions

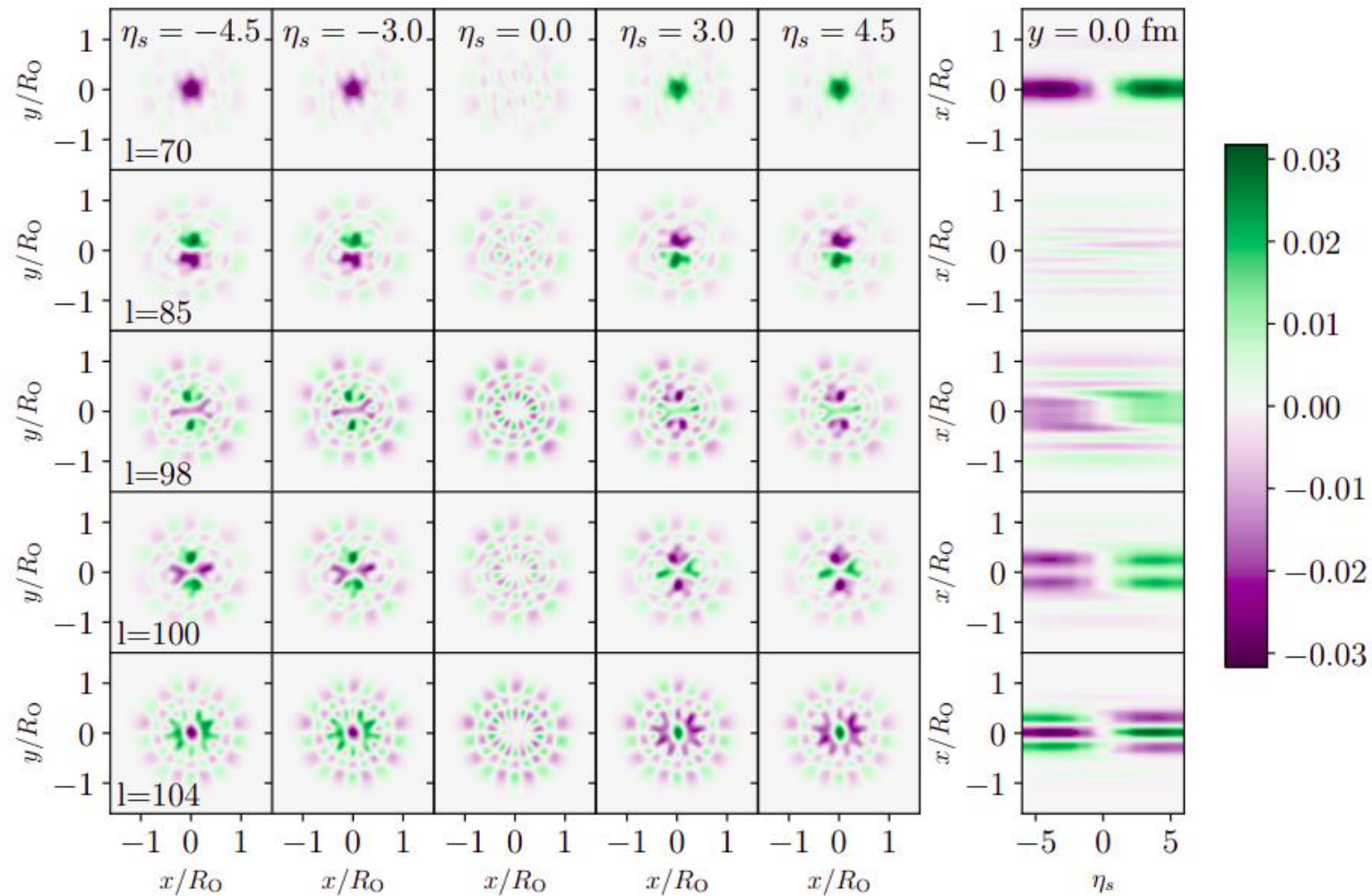
Modes show the generic structures observed in large systems: monopole, dipole, quadrupole, ...

The modes do not exhibit longitudinal tilt. This overall geometric feature is captured in the average state.

First 7 modes shown for 0-10% centrality. The modes are ordered by importance.



Odd modes in O+O collisions



Odd modes with longitudinal dependence.

Important for studying longitudinal decorrelations.

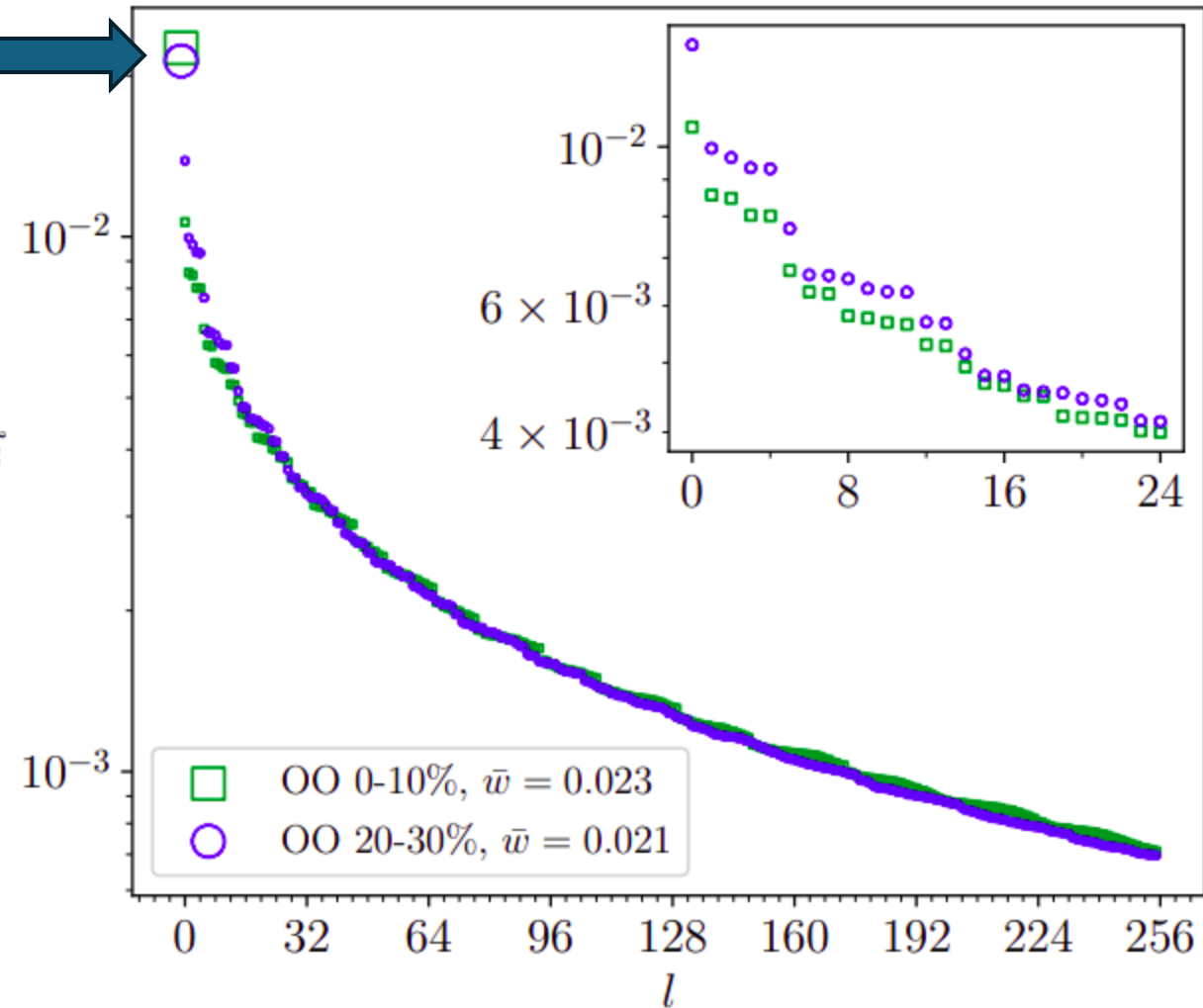
First 5 odd modes for 0-10% centrality bin

Weights and the modes contribution

$$\bar{w} \equiv \frac{\|\bar{\Psi}\|}{\sum_l \|\Psi_l\| + \|\bar{\Psi}\|}$$

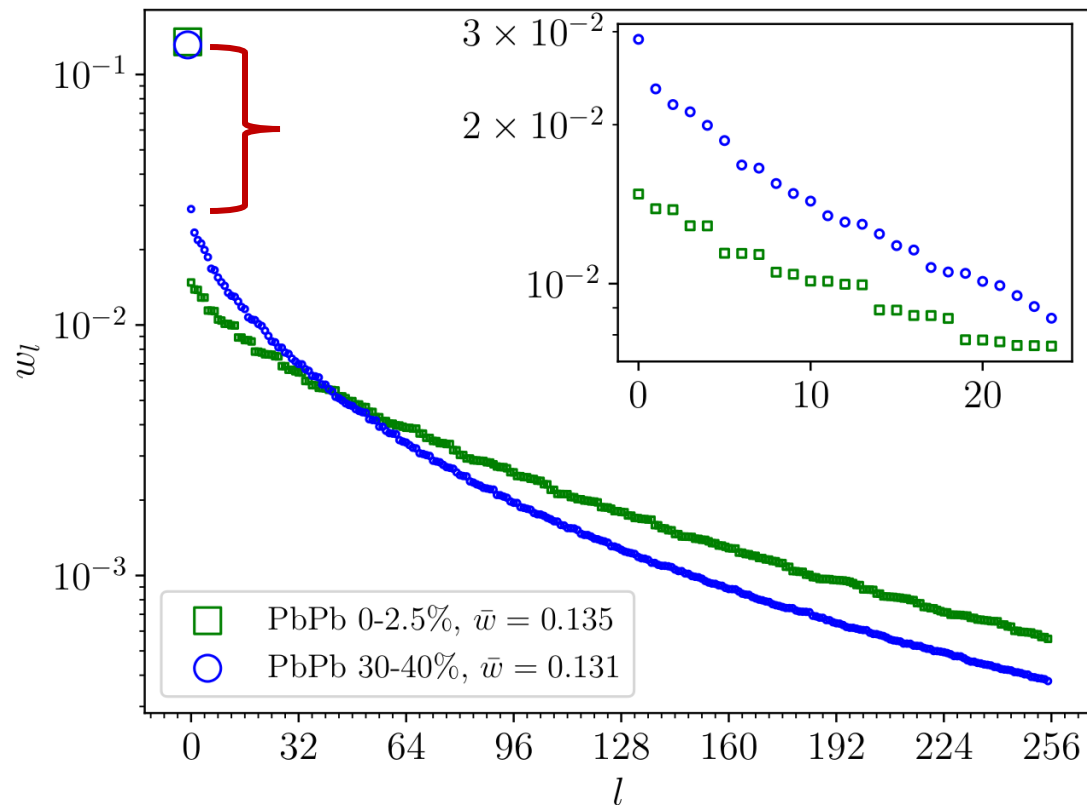
$$w_l \equiv \frac{\|\Psi_l\|}{\sum_l \|\Psi_l\| + \|\bar{\Psi}\|}$$

The average state contributes ~2%
The first modes contribute ~1%

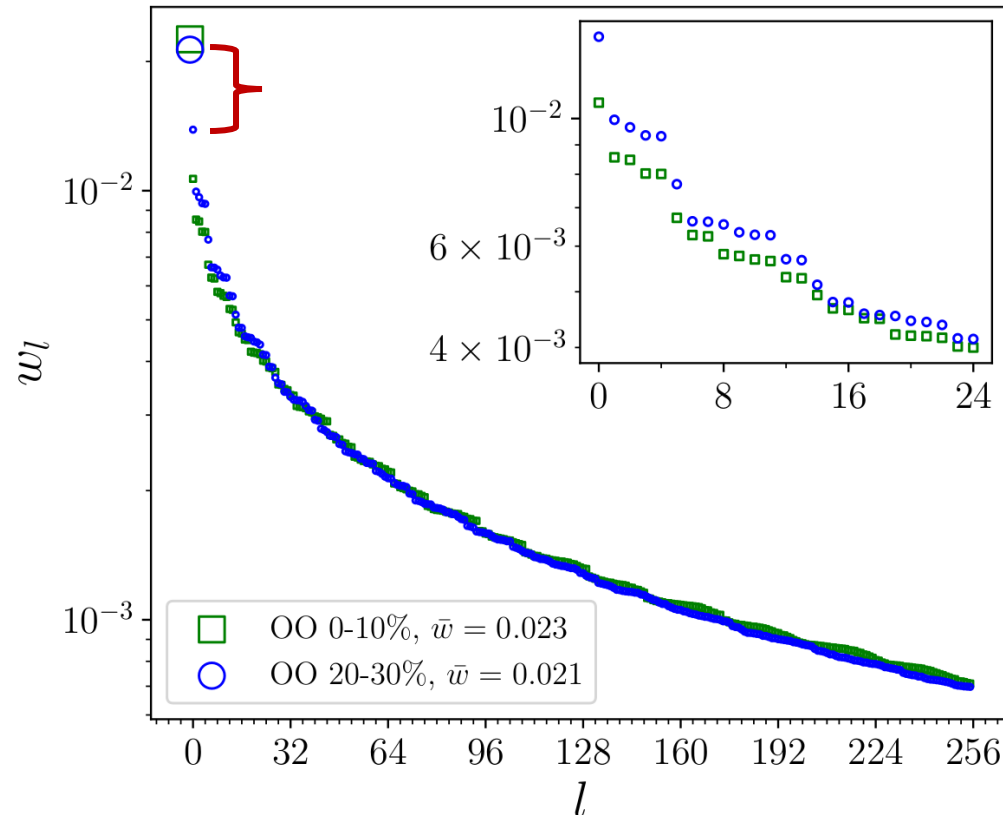


Weights in Pb+Pb and O+O

The average state contributes $\sim 13\%$
The first modes contribute $\sim 3\%$



The average state contributes $\sim 2\%$
The first modes contribute $\sim 1\%$

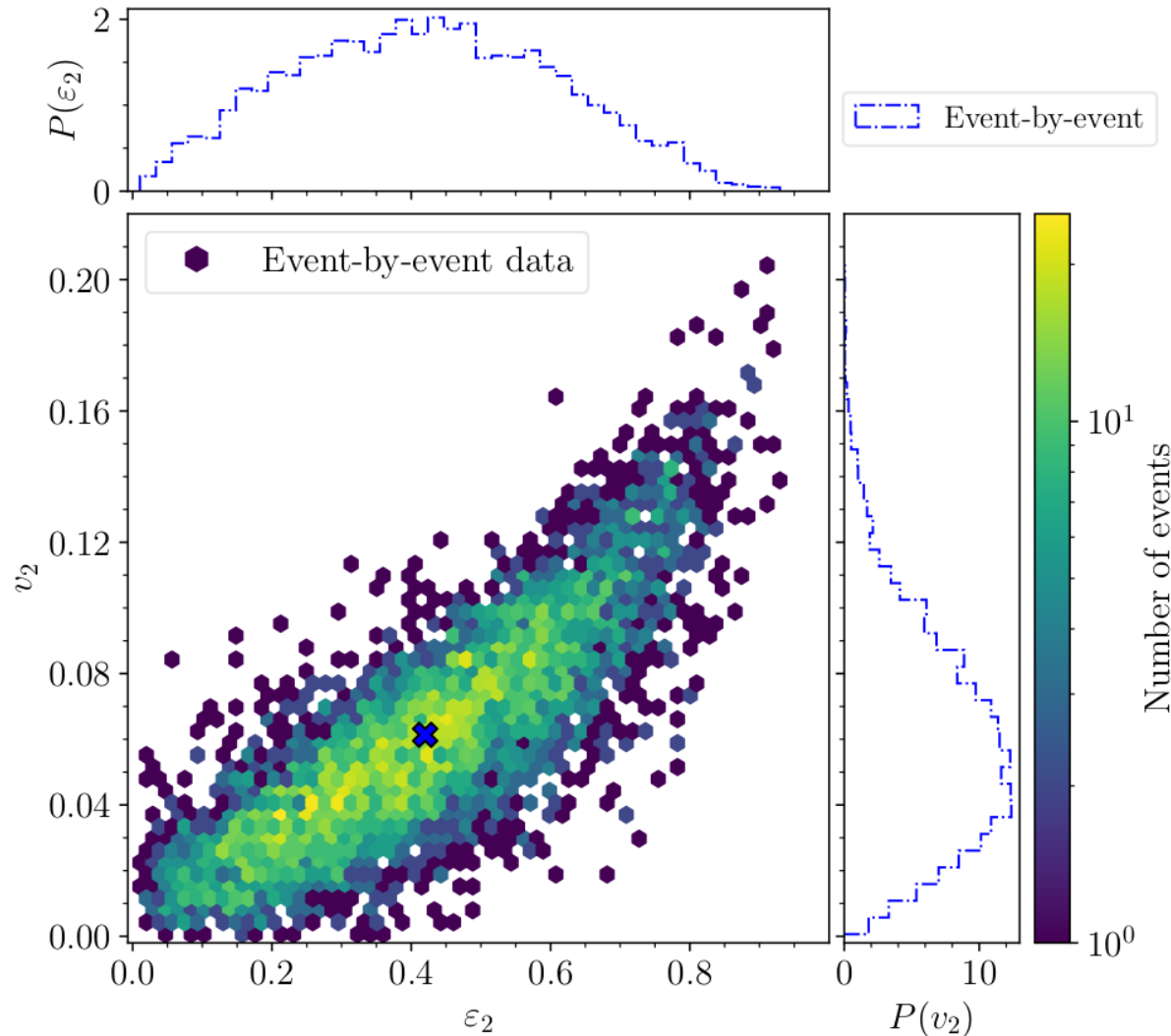


Fluctuations contribute more than the geometric shape associated with the impact parameter.
There is no difference across centralities for intermediate-size systems

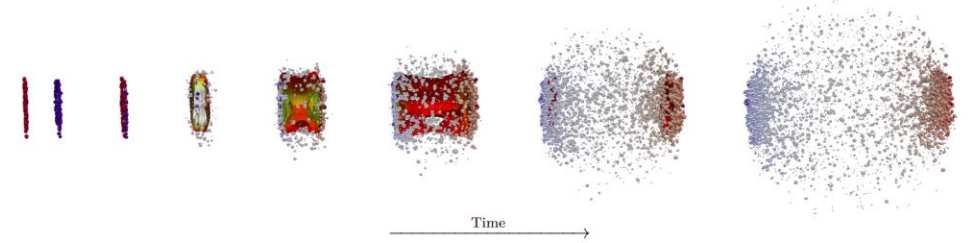
Effect of fluctuations in observables

Effect of fluctuations in observables

1. Event-by-event



Evolution with MUSIC (full 3D) + iSS



5000 events in 0-10% centrality bin.

The blue cross is the average of all the points.

$$\varepsilon_2 e^{2i\Phi_2} = - \frac{\int r^2 e^{2i\theta} e(r, \theta, \eta_s) r dr d\theta}{\int r^2 e(r, \theta, \eta_s) r dr d\theta}$$

$$v_2 e^{2i\Psi_2} \equiv \frac{\int e^{2i\phi} \frac{dN_{ch}}{p_T dp_T d\phi d\eta} p_T dp_T d\phi}{\int \frac{dN_{ch}}{p_T dp_T d\phi d\eta} p_T dp_T d\phi}$$

Effect of fluctuations in observables

1. Event-by-event

2. Linear Model

$$O_{\alpha} \left(\bar{\Psi} + \sum_l c_l \Psi_l \right) \simeq \bar{O}_{\alpha} + \sum_l c_l L_{\alpha,l}$$

Sample the expansion coefficients from a Gaussian distribution, using only 10 modes.

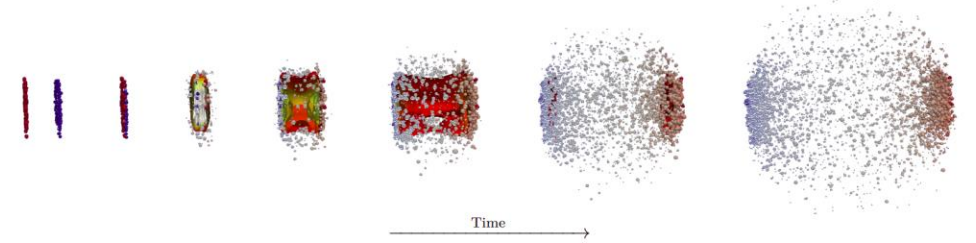
3. Analytical Gaussian fluctuation approximation (G.f.a.)

$$p(\{O_{\alpha_k}\}) = \frac{\exp \left[-\frac{1}{2} (O_{\alpha_i} - \bar{O}_{\alpha_i}) (\Sigma^{-1})_{ij} (O_{\alpha_j} - \bar{O}_{\alpha_j}) \right]}{(2\pi)^{d/2} \sqrt{\det \Sigma}}$$

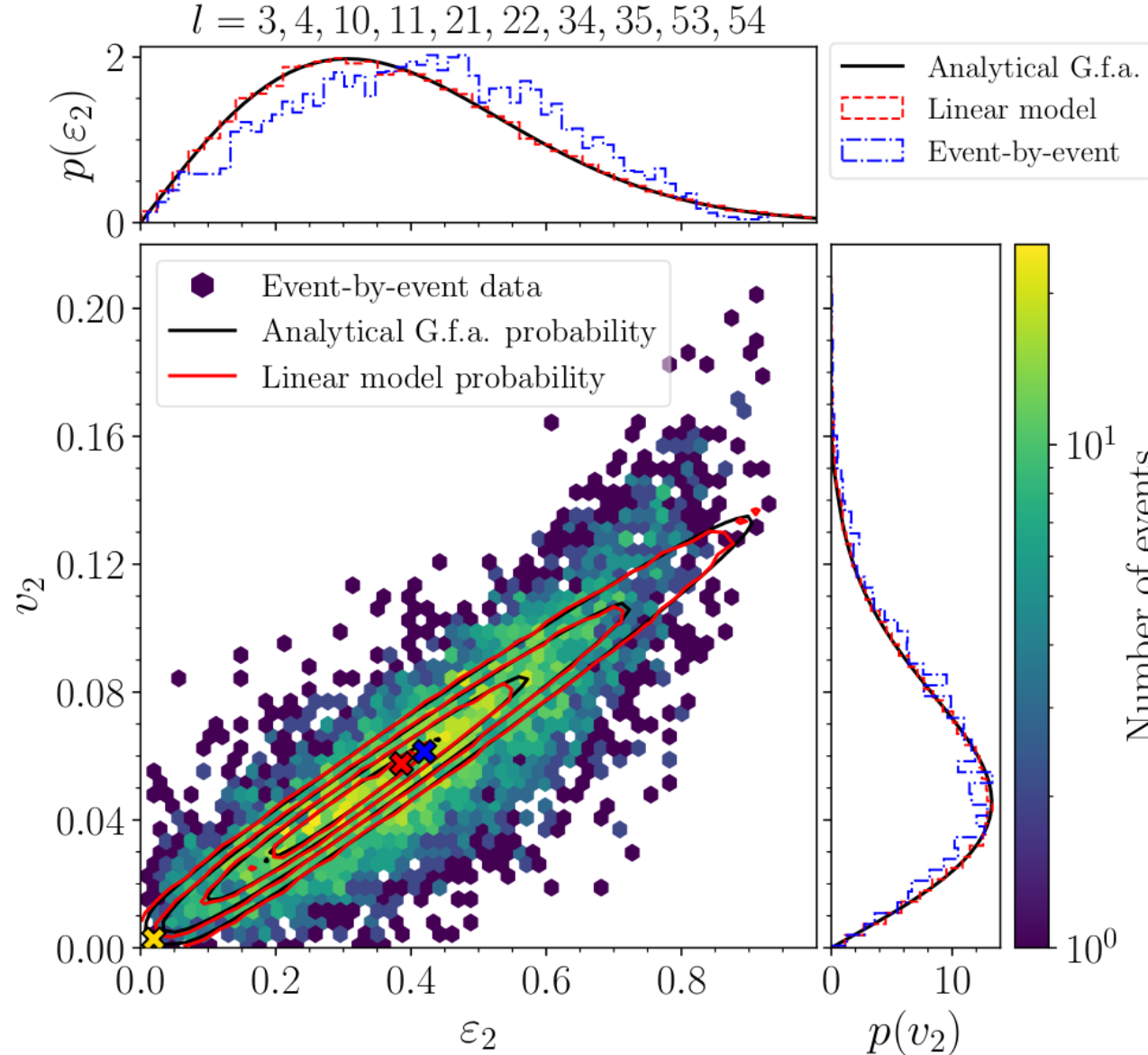
Using only 10 modes.

Covariance matrix $(\Sigma)_{ij} = \sum_l L_{\alpha_i,l} L_{\alpha_j,l}$

Evolution with MUSIC (full 3D) + iSS



Effect of fluctuations in observables



Gaussian assumption for the expansion coefficients.

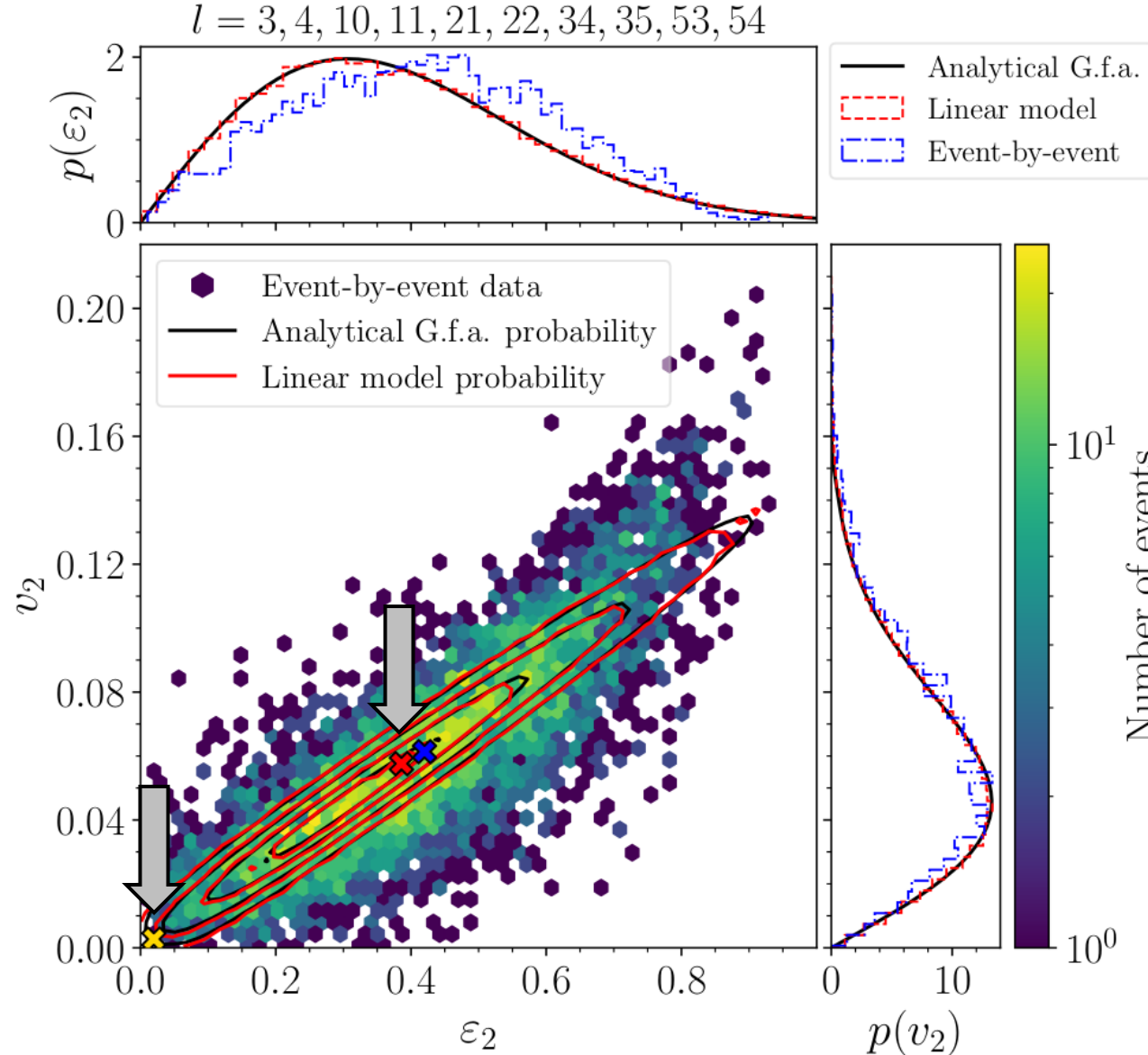
Elliptic flow is linear in the initial elliptical eccentricity:

$$v_2 = \kappa_{2,2}\varepsilon_2$$

However, the event-by-event data favors a cubic contribution:

$$v_2 = \kappa_{2,2}\varepsilon_2 + \kappa_{2,222}\varepsilon_2^3$$

Effect of fluctuations in observables



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The value for the average state (yellow cross) has nothing to do with the average of event-by-event data (blue cross). However, including only 10 selected modes produces the red cross.

Conclusions

Elliptic flow in O+O collisions has a larger contribution from event-by-event fluctuations than from the overall geometry. The system is more similar to small systems than to large systems in this respect.

Centrality definition in O+O collisions **is not directly linked to the impact parameter geometric shape.**

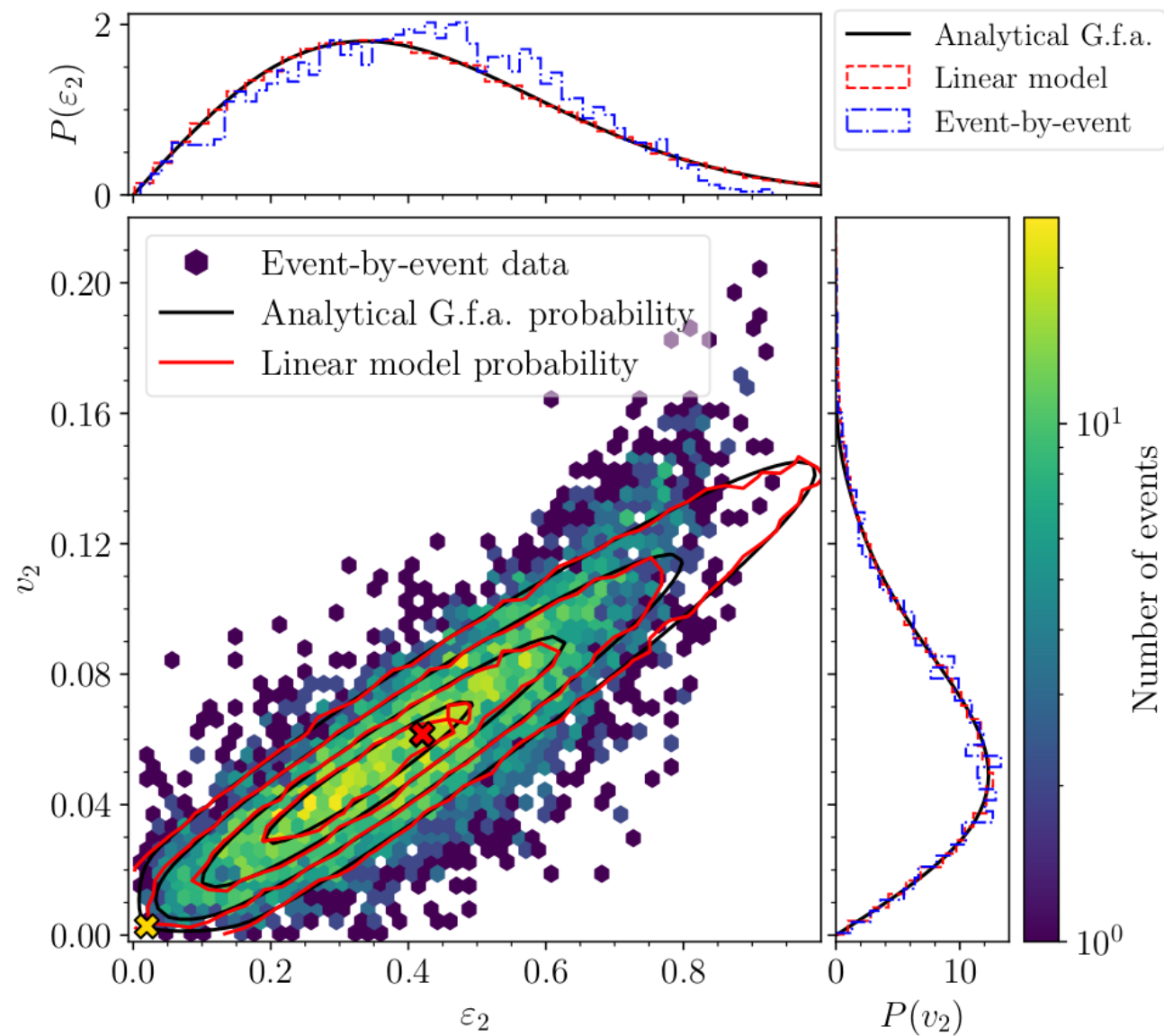
Our mode decomposition was performed in 3D for the first time.

We compute the joint probability distribution of the elliptic flow coefficient.

Our model can describe the overall behavior of the elliptical eccentricity and elliptic flow using only a small set of 10 selected modes. Including more modes improves the description.

Thank you for your attention!

Observables with the first 128 modes



Theory: Computation of observables

$$O_{\alpha}(\Phi^{(i)}) = O_{\alpha}(\bar{\Psi}) + \sum_l L_{\alpha,l} c_l^{(i)} + \frac{1}{2} \sum_{l,l'} Q_{\alpha,ll'} c_l^{(i)} c_{l'}^{(i)} + \mathcal{O}(c_l^3)$$

Taylor expansion around the average state in orders of the expansion coefficients

Linear response

$$L_{\alpha,l} = \left. \frac{\partial O_{\alpha}}{\partial c_l} \right|_{\bar{\Psi}}$$

Quadratic response

$$Q_{\alpha,ll'} = \left. \frac{\partial^2 O_{\alpha}}{\partial c_l \partial c_{l'}} \right|_{\bar{\Psi}}$$

But for a numerical implementation, the modes are plugged into the average state

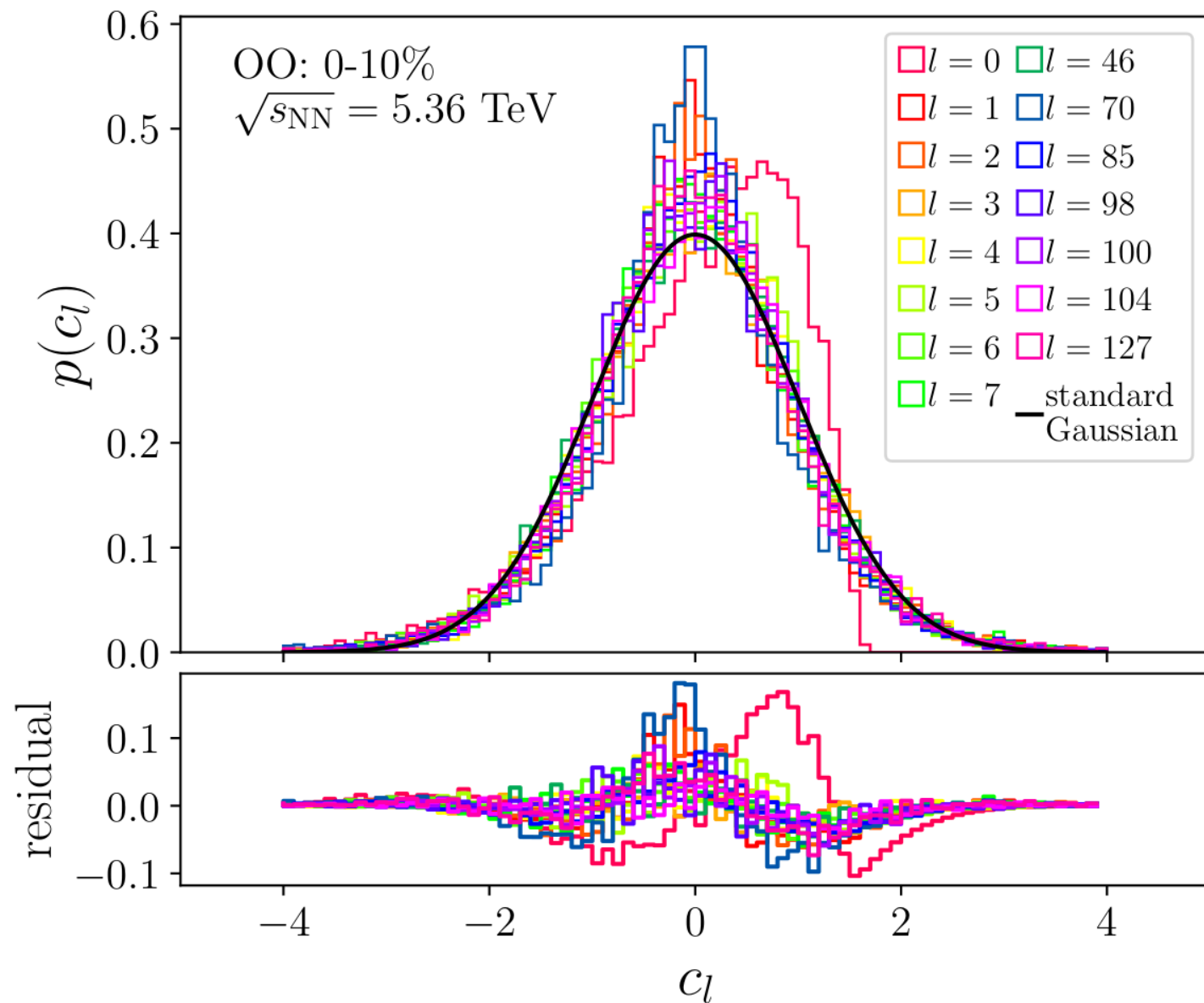
$$\Psi_l^{\pm} = \bar{\Psi} \pm \xi \Psi_l$$

Small coefficient. Here: 0.5

$$L_{\alpha,l} = \frac{O_{\alpha}(\Psi_l^{+}) - O_{\alpha}(\Psi_l^{-})}{2\xi}$$

$$Q_{\alpha,ll} = \frac{O_{\alpha}(\Psi_l^{+}) + O_{\alpha}(\Psi_l^{-}) - 2O_{\alpha}(\bar{\Psi})}{\xi^2}$$

Gaussian assumption for the expansion coefficients



Centrality definition

Estimator equation

G. Giacalone et al., PRL 123 (2019), arXiv:1908.02866

$$\left\langle \frac{dN_{\text{ch}}}{d\eta} \right\rangle = C(\eta/s) \frac{4}{3} \frac{N_{\text{ch}}}{S} C_{\infty}^{3/4} \left(4\pi \frac{\eta}{s} \right)^{1/3} \left(\frac{\pi^2}{30} \nu_{\text{eff}} \right)^{1/3} \int d^2\mathbf{x} [\tau\epsilon]_0^{2/3}$$

↓
Extra constant to be
calibrated to the specific
hydrodynamical model used
(here: 1.13)

↓
Initial energy density from
McDipper (calibrated with
 $K_g = 3.066$)

Longitudinal decorrelation

$$r_2(\eta_{s1}, \eta_{s2}) = \frac{\langle \varepsilon_{2,c}(-\eta_{s1})\varepsilon_{2,c}(\eta_{s2}) + \varepsilon_{2,s}(-\eta_{s1})\varepsilon_{2,s}(\eta_{s2}) \rangle}{\langle \varepsilon_{2,c}(\eta_{s1})\varepsilon_{2,c}(\eta_{s2}) + \varepsilon_{2,s}(\eta_{s1})\varepsilon_{2,s}(\eta_{s2}) \rangle}$$



Odd modes

