

Anisotropic Flows and Universal Collective Response in Ion Collisions in Relativistic Boltzmann Transport

Vincenzo Nigara

in collaboration with

Maria Lucia Sambataro, Vincenzo Minissale, Salvatore Plumari, Vincenzo Greco

September 24th, 2026

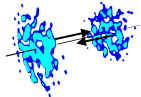
EMMI Workshop Light Ion Collisions 2026

Kirchhoff-Institut für Physik | University of Heidelberg



Standard Model of ultra-Relativistic Heavy-Ion Collisions

initial state
CGC, Glasma



**pre-equilibrium
dynamics**
kinetic theory



$\tau_{\text{coll}} \ll 1\text{fm}$

$\tau_{\text{Hydro}} \sim 1\text{fm}$

expansion of the QGP
viscous hydro/kinetic theory



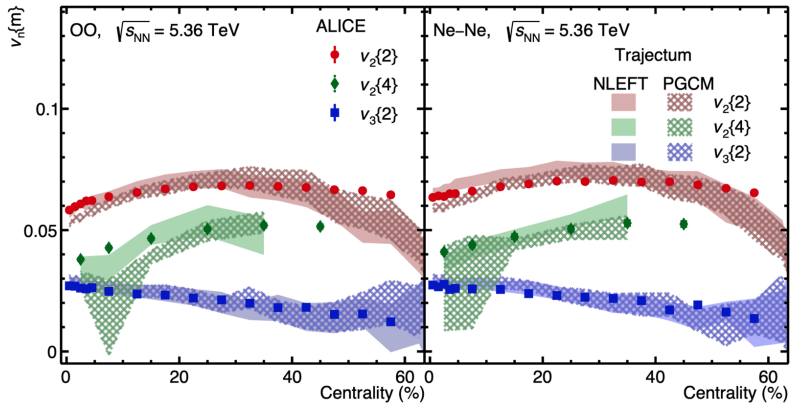
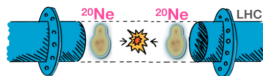
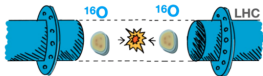
$\tau_{\text{freeze-out}} \sim 10\text{ fm}$

Hadron gas
Transport model



Adapted from: S. Schlichting, Quantifying the applicability of hydrodynamics in small systems (CERN Light Ion Workshop)

Hydrodynamic description of Light ions

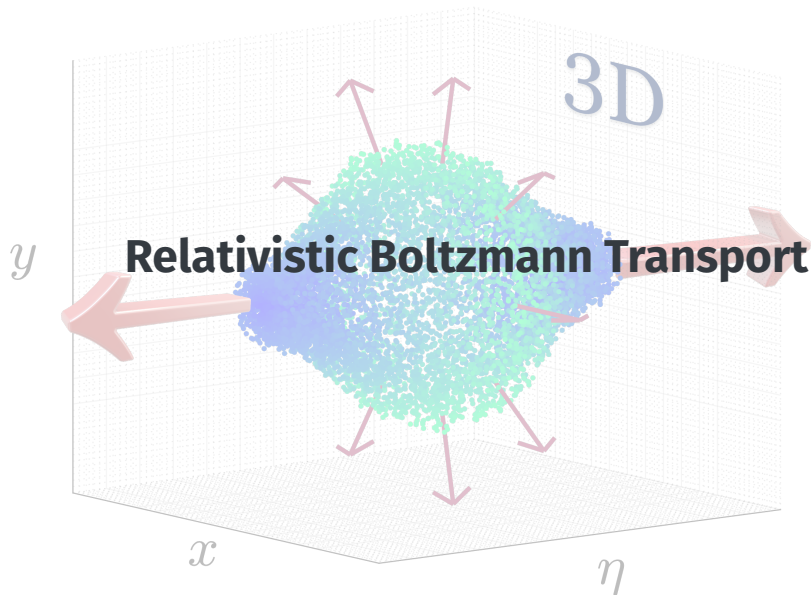


My open questions

#1 Are the imprints of initial conditions truly model-independent?

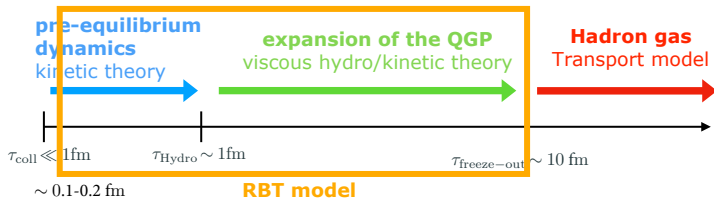
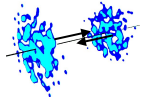
My open questions

- #1 Are the imprints of initial conditions truly model-independent?
- #2 Can we find signatures of out-of-equilibrium physics?



Standard Model of ultra Relativistic Heavy-Ion Collisions

initial state
CGC, Glasma



Adapted from: S. Schlichting, Quantifying the applicability of hydrodynamics in small systems (CERN Light Ion Workshop)

Relativistic Boltzmann Equation

Evolution equation for the one-body distribution function: $f(x, p) = \frac{dN}{d^3x d^3p / (2\pi)^3}$

$$\begin{cases} p^\mu \partial_\mu f(x, p) = C_{2 \leftrightarrow 2} [f(x, p)]_{\mathbf{p}} \\ f(\tau_0; \mathbf{x}, p) = f_0(\mathbf{x}, p) \end{cases}$$

Relativistic Boltzmann Equation

Evolution equation for the one-body distribution function: $f(x, p) = \frac{dN}{d^3x d^3p / (2\pi)^3}$

$$\begin{cases} p^\mu \partial_\mu f(x, p) = C_{2\leftrightarrow 2} [f(x, p)]_{\mathbf{p}} \\ f(\tau_0; \mathbf{x}, p) = f_0(\mathbf{x}, p) \end{cases}$$

Collision integral

$$\begin{aligned} C_{2\leftrightarrow 2} [f(x, p)]_{\mathbf{p}} &= \frac{1}{2} \int \frac{d^3k}{(2\pi)^3 k^0} \frac{d^3p'}{(2\pi)^3 p^0} \frac{d^3k'}{(2\pi)^3 k'^0} [f(x, p')f(x, k') - f(x, p)f(x, k)] W(p', k'|p, k) = \\ &= \frac{1}{2} \int \frac{d^3k}{(2\pi)^3 k^0} \frac{d^3p'}{(2\pi)^3 p^0} \frac{d^3k'}{(2\pi)^3 k'^0} [f(x, p')f(x, k') - f(x, p)f(x, k)] |\mathcal{M}|^2 \delta^{(4)}(p + k - p' - k'). \end{aligned}$$

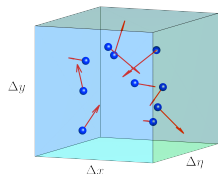
with $|\mathcal{M}|^2 = 16\pi s^2 \frac{d\sigma}{dt}$, s and t Mandelstam variables.

Relativistic Boltzmann Transport

We solve the Relativistic Boltzmann Equation in 3+1D
discretised time

$$\begin{cases} p^\mu \partial_\mu f(x, p) = C_{2 \leftrightarrow 2} [f(x, p)]_p \\ f(\tau_0; \mathbf{x}, p) = f_0(\mathbf{x}, p) \end{cases}$$

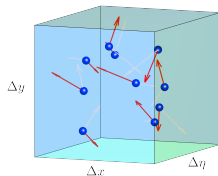
At $\tau = \tau_0$:



Initialisation

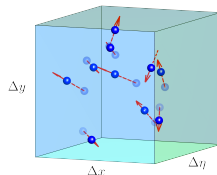
$f_0(x, p)$ sampled by 10^7 - 10^8
test particles

Repeat for every time step:



Collisions

Stochastic implementation of
the full $C_{2 \leftrightarrow 2} [f]_p$ cell by cell



Propagation

Test particles flow from one
collision to the next for Δt

Z. Xu, C. Greiner PRC 71 (2005); G. Ferini et al. PLB 670 (2009); S. Plumari et al. PRC 86 (2012)

Collision integral

- Test particles in a cell collide with probability $P_{22} \propto \sigma_{22}(x)$
Not real collisions, but a numerical implementation of C_{22}
- Fix cross section $\sigma_{22}(x)$ via the Chapman-Enskog formula

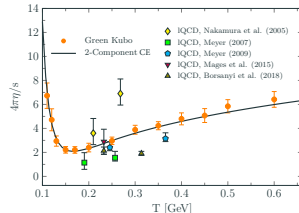
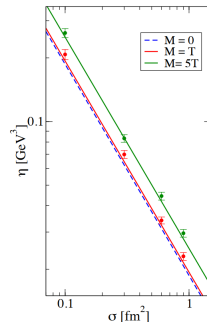
$$\eta = f(m/T) \frac{T}{\sigma_{22}} \Rightarrow \sigma_{22}(x) = f(m/T) \frac{T(x)}{\eta/s s(x)}$$

Limiting value for QGP $\eta/s \gtrsim 1/4\pi$

Interpolate from strong to weak coupling regime

$\eta/s \rightarrow 0$ ideal hydro

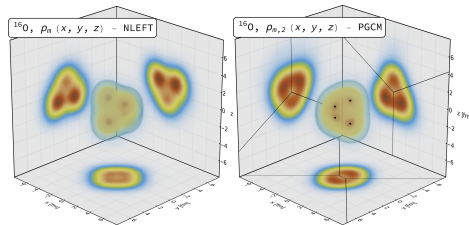
$\eta/s \rightarrow \infty$ free streaming



S. Plumari *et al.* PRC 86 (2012), G. Parisi, VN *et al.* PRD 113 (2026)

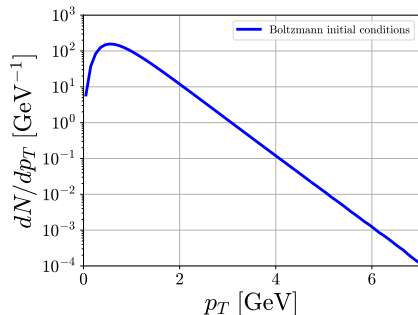
Preliminary results on light ions

- Initial spatial distribution. T_RENTO i.c. as particle density profiles
 - NLEFT
 - PGCM clustered

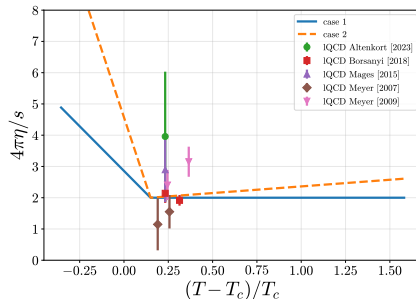


Giocalone et al. PRL (2025)

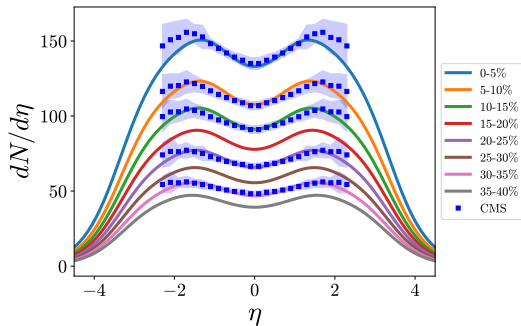
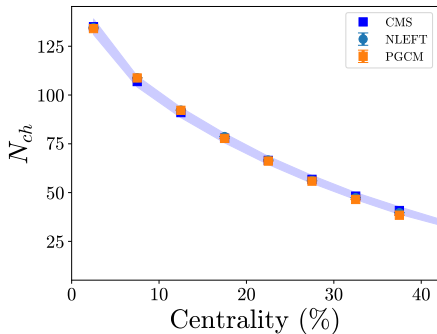
- Initial spatial distribution. T_RENTo i.c. as particle density profiles
 - NLEFT
 - PGCM clustered
- Initial momentum distribution: Boltzmann-Jüttner with $m = 0.5$ GeV ($\approx$ IQCD EoS at high T)



- Initial spatial distribution. T_RENTo i.c. as particle density profiles
 - NLEFT
 - PGCM clustered
- Initial momentum distribution: Boltzmann-Jüttner with $m = 0.5$ GeV ($\approx$ IQCD EoS at high T)
- Interaction regime:
 - η/s constant + freeze-out
 - η/s linear + steeper freeze-out

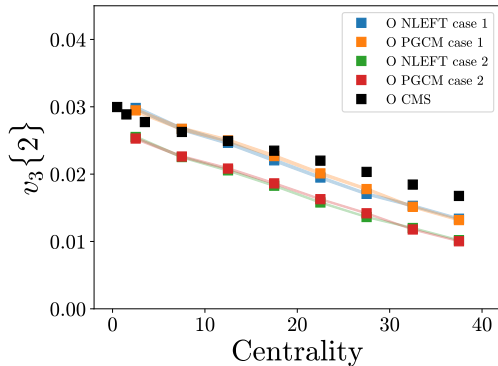
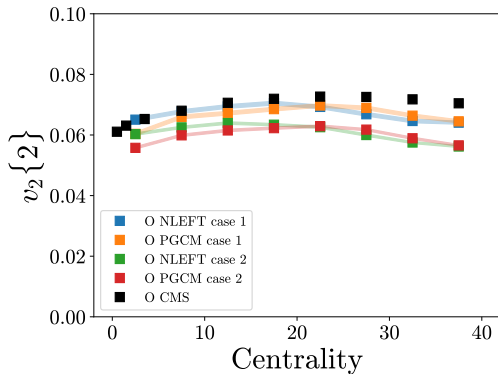


$dN/d\eta$ and multiplicities (^{16}O)



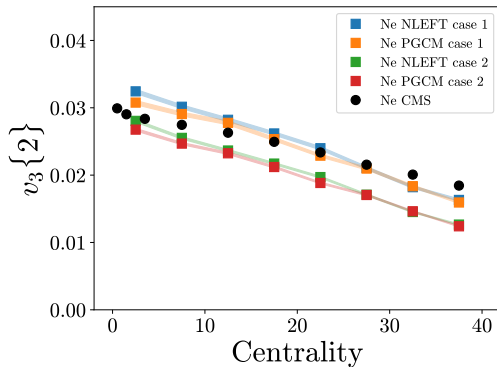
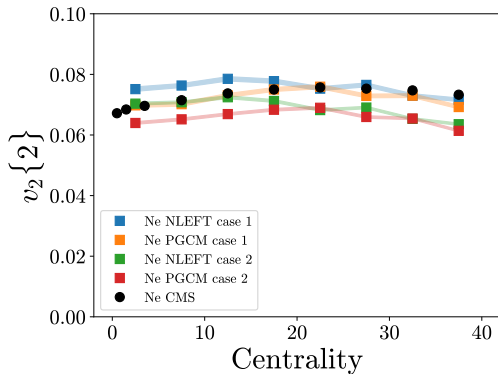
Little sensitivity to nuclear structure and interaction regime.
The dependence is set by the initial $dN/d\eta_s$.

v_n vs centrality ¹⁶O



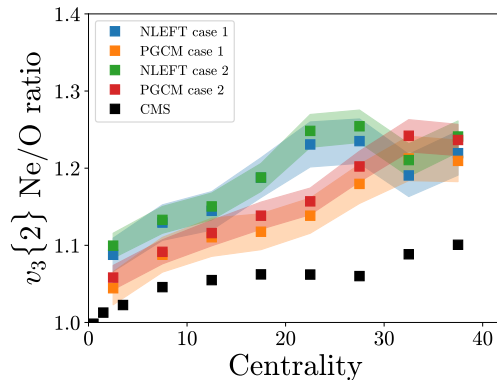
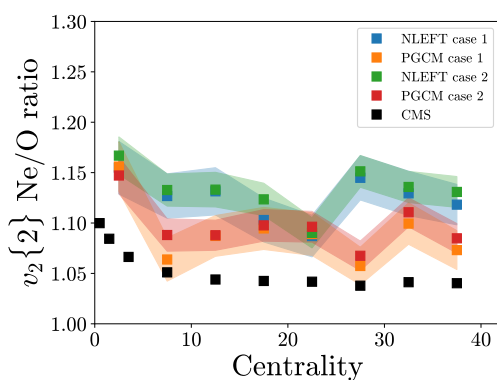
Qualitative agreement with experimental data
Sensitivity to η/s ; little dependence on initial conditions for v_3 .

v_n vs centrality ^{20}Ne



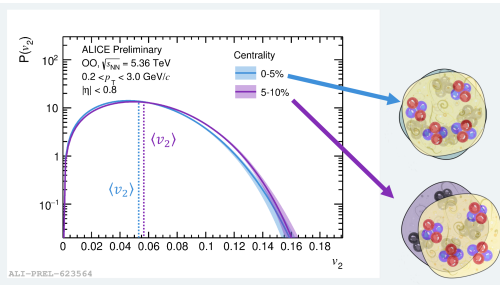
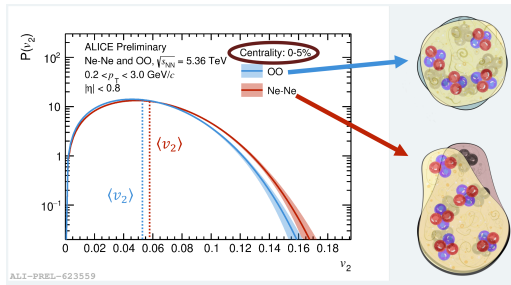
Qualitative agreement with experimental data.
Sensitivity to η/s and initial conditions.

v_n ratios vs centrality

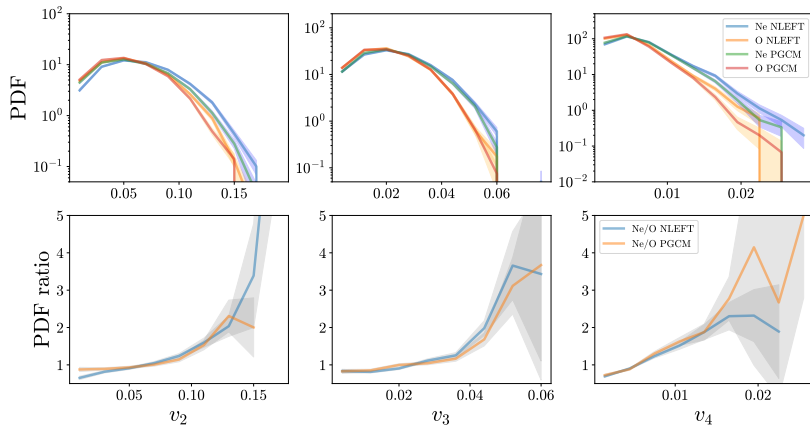


The sensitivity to η/s disappears — strong imprints of the initial conditions

Already preliminary results from ALICE:



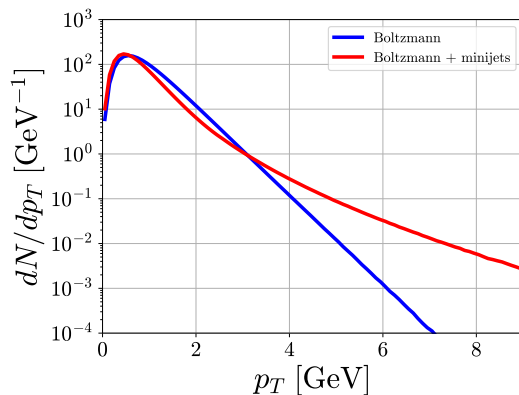
Probability of v_n : 0-5% centrality



Sensitivity to geometry and initial conditions, especially from the tails.
Can the functional shape and the ratios carry extra information?

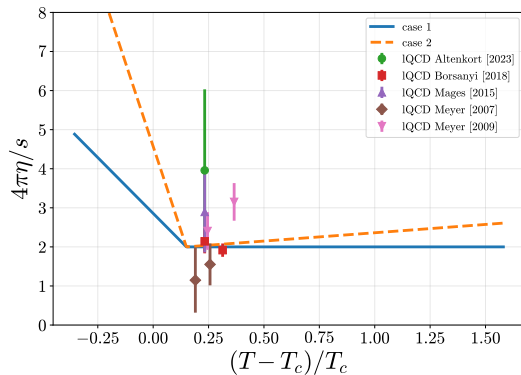
Looking for out-of-equilibrium effects

- Change the initial momentum distribution:
 - $p_T < 2.5$ GeV: Boltzmann-Jüttner distribution
 - $p_T > 2.5$ GeV: minijet tail (MC-EKRT)



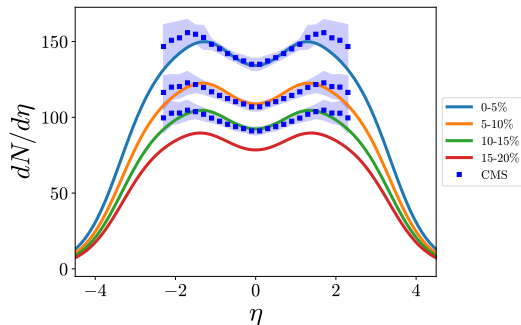
Looking for out-of-equilibrium effects

- Change the initial momentum distribution:
 - $p_T < 2.5$ GeV: Boltzmann-Jüttner distribution
 - $p_T > 2.5$ GeV: minijet tail (MC-EKRT)
- $\eta/s =$ case 1

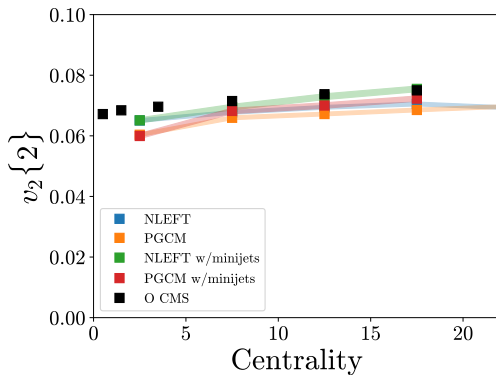
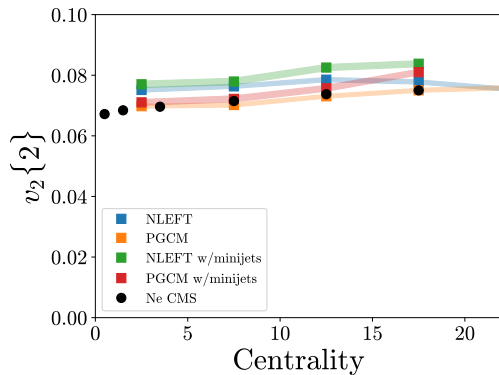


Looking for out-of-equilibrium effects

- Change the initial momentum distribution:
 - $p_T < 2.5$ GeV: Boltzmann-Jüttner distribution
 - $p_T > 2.5$ GeV: minijet tail (MC-EKRT)
- $\eta/s = \text{case 1}$
- Still reproduces the system multiplicity.

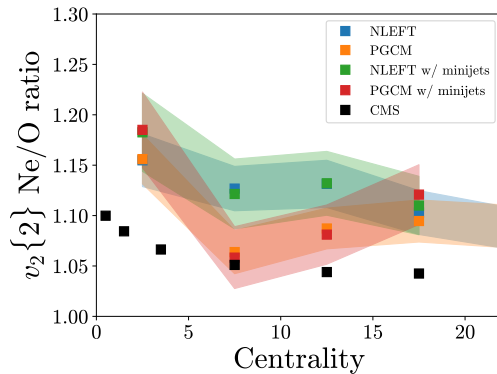


Out-of-equilibrium effects on integrated v_2



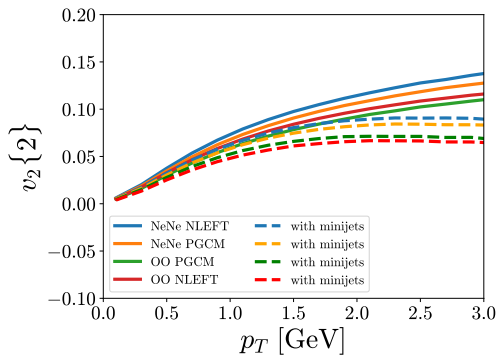
Weak effect on the integrated v_2

Out-of-equilibrium effects on integrated v_2

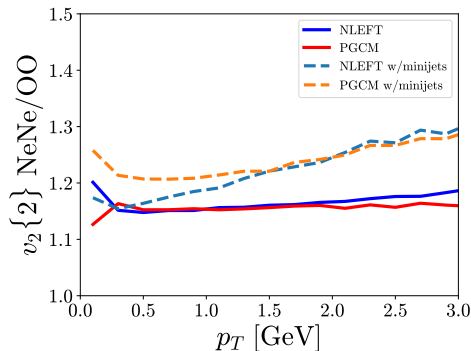
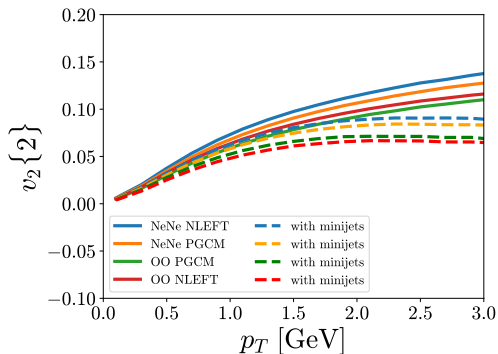


Weak effect on the integrated v_2 and on their ratios

Out-of-equilibrium effects on partonic $v_2(p_T)$ (0-5%)



Out-of-equilibrium effects on partonic $v_2(p_T)$ (0-5%)



Imprint on the $v_n(p_T)$ ratio?

Preliminary Answers

#1 Are the imprints of initial conditions truly model-independent?

Qualitative trend of observables such as v_n ratios vs centrality appears robust

#2 Can we find signatures of out-of-equilibrium physics?

Ratio of $v_2(p_T)$ may be affected by out-of-equilibrium effects

Outlook

- Effect of full lQCD EoS (Quasi-Particle Model)
- Impact of particlisation (Catania model: coal.+fragm.) → full predictions
- 3D initial conditions and rapidity dependence of observables

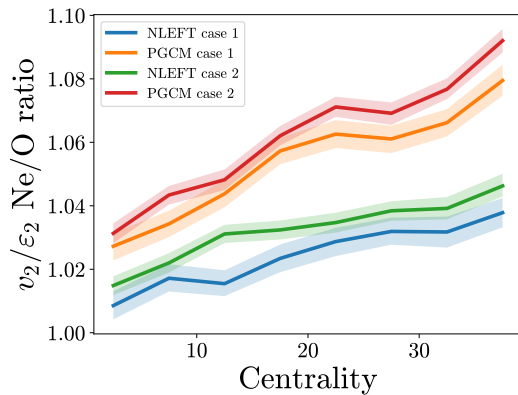
Thank you for your attention.

Backup slides

T_RENTo parameters

- reduced-thickness = 0
- nucleon-min-dist=0.5
- nucleon-width = 0.82
- fluctuation=0.7
- constit-number=5

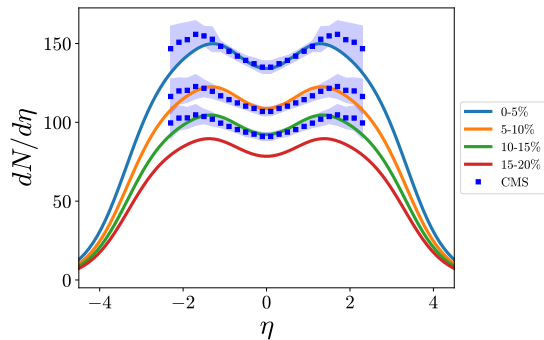
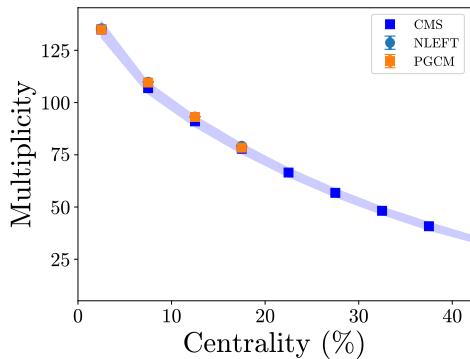
Universality of response functions?



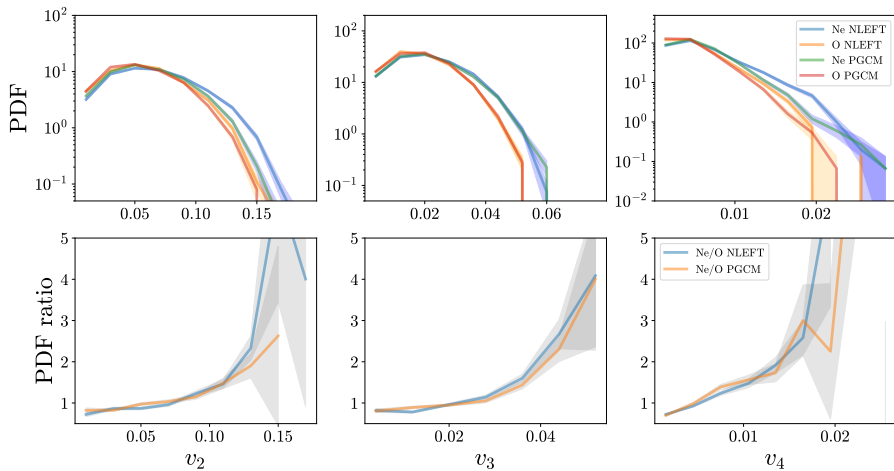
Non-universal response ratio.

Collective evolution is needed to bring out %-level effects.

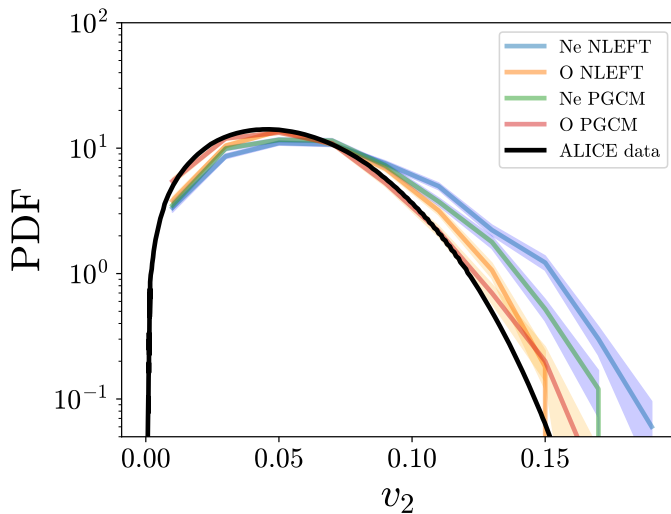
Multiplicity with minijets



Probability of v_n : 15-20% centrality



Probability: comparison with ALICE preliminary results



OO $v_2(p_T)$ with quark number scaling

