

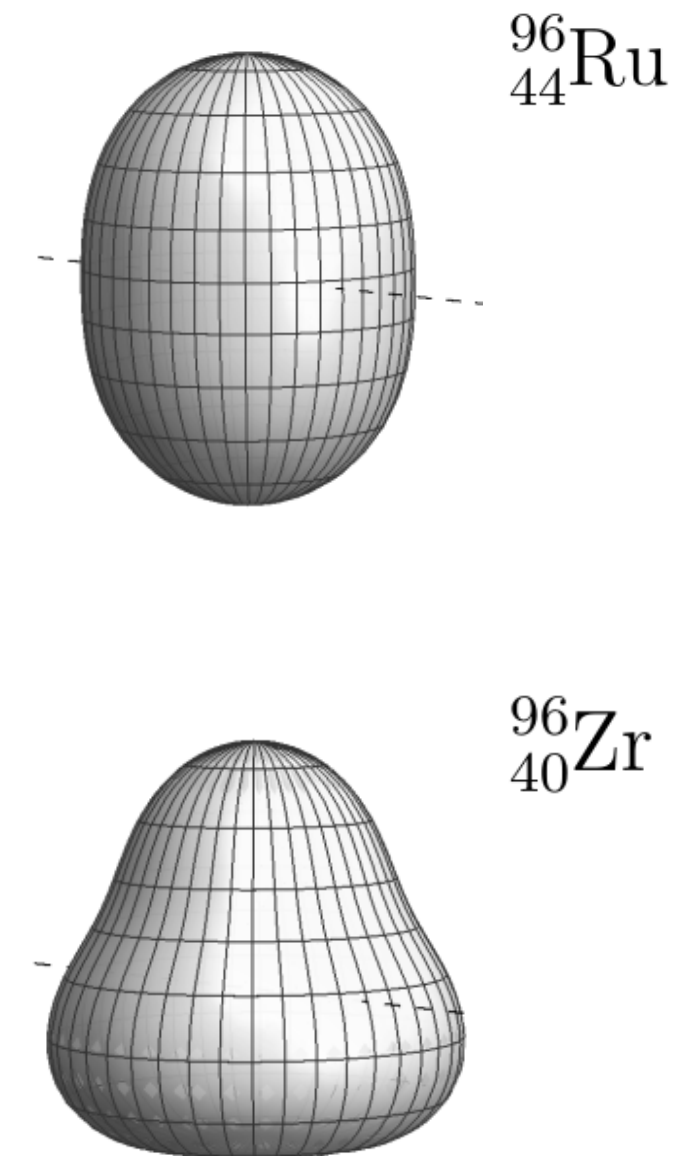
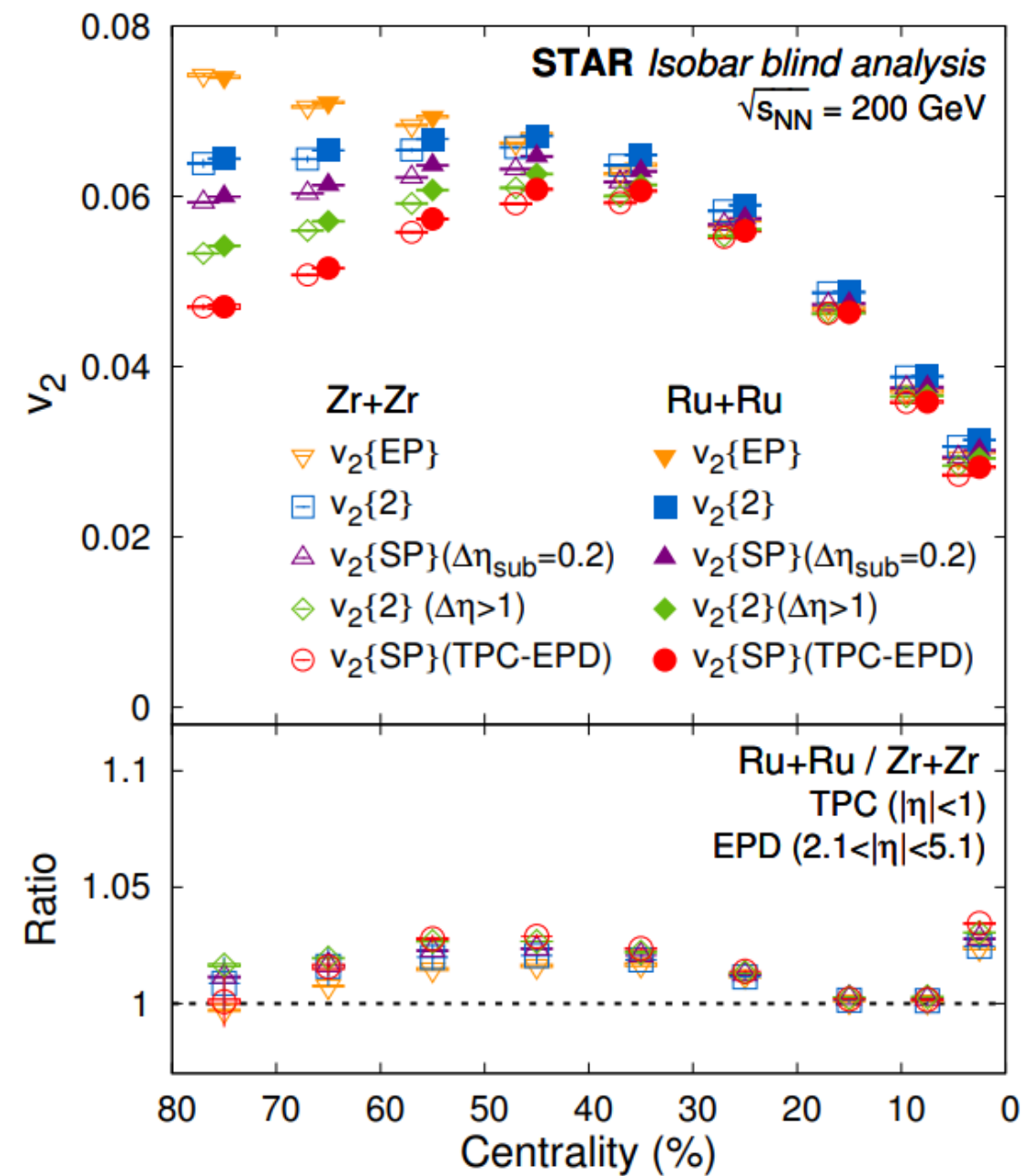
Operator-Level Nuclear Structure Constraints from Isobar Collisions

EMMI Workshop: Light-ion collisions 2026
September 23, 2026

João Paulo Picchetti, Matthew Luzum



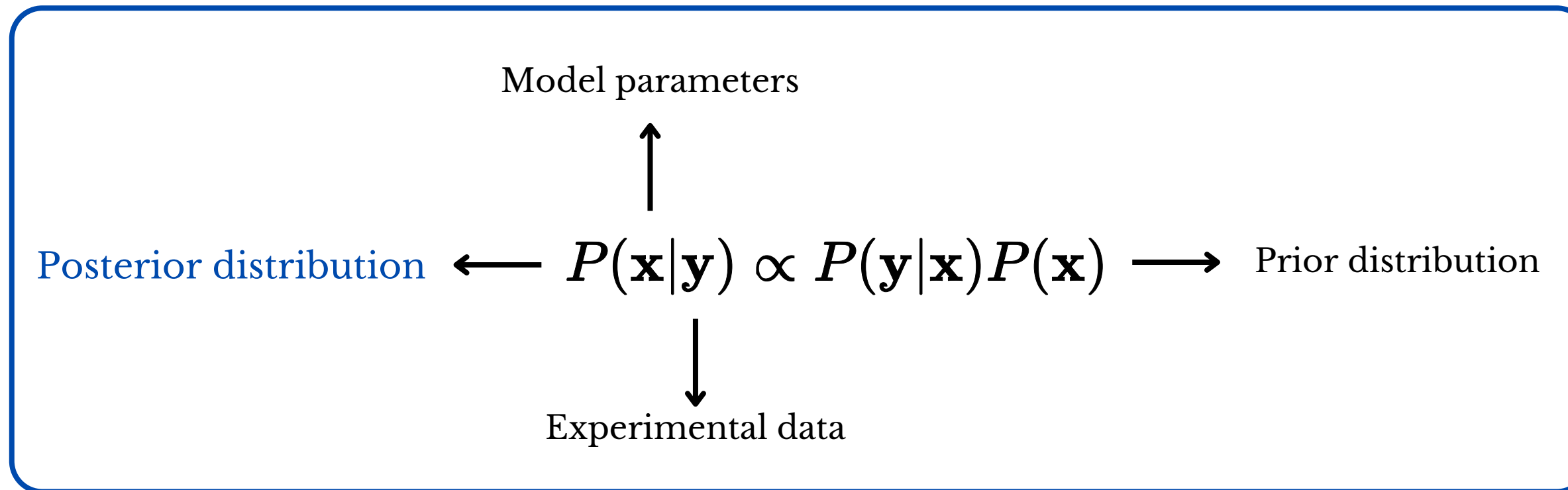
Isobar collisions: a microscope for nuclear structure



STAR Collaboration - Phys. Rev. C, 105, 1, 014901 (2022)

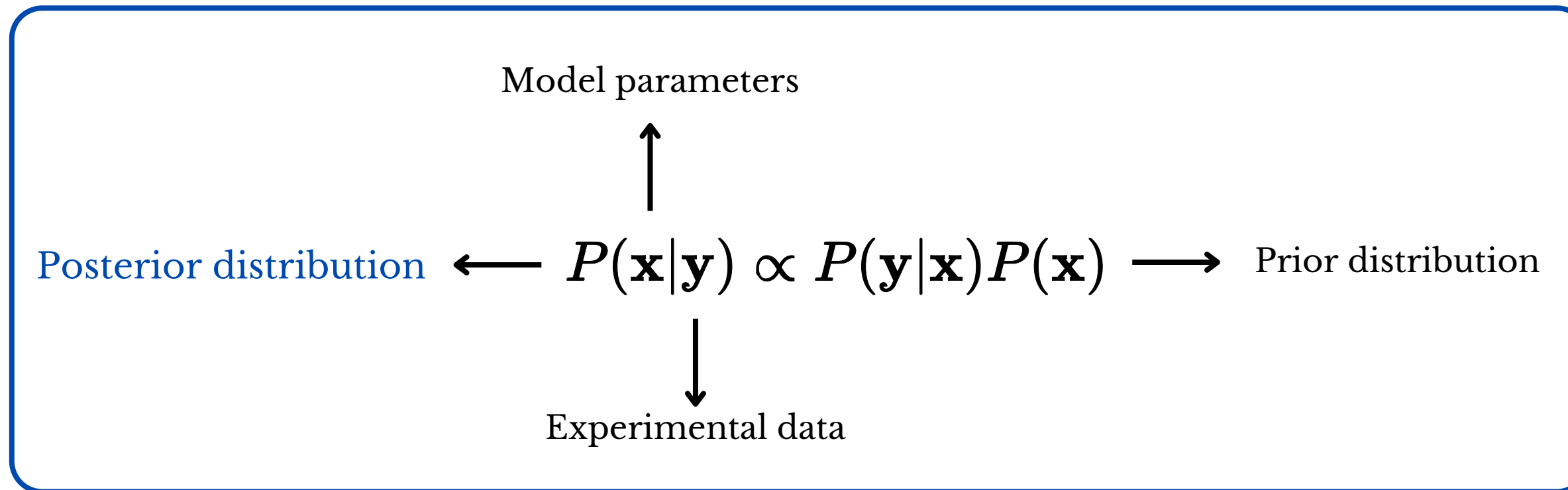
Bayesian parameter estimation

Information update after model-to-data comparison



Bayesian parameter estimation

Information update after model-to-data comparison



$$\frac{v_n^{\text{Ru}}}{v_n^{\text{Zr}}} = \frac{\kappa_n \varepsilon_n^{\text{Ru}}}{\kappa_n \varepsilon_n^{\text{Zr}}} = \frac{\varepsilon_n^{\text{Ru}}}{\varepsilon_n^{\text{Zr}}}$$

Initial state!

Long-range correlations: angular deformations

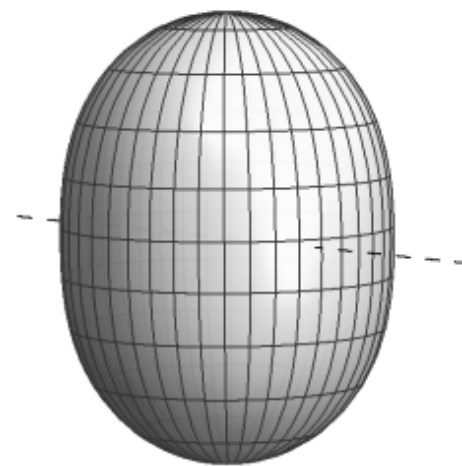
$$\rho^{(1)}(\mathbf{r}) = \frac{\rho_0}{1 + e^{[r-R(\theta,\phi)]/a}}$$

$$R(\theta, \phi) = R[1 + \beta_2(\cos \gamma Y_{2,0}(\theta, \phi) + \sin \gamma Y_{2,2}(\theta, \phi)) + \beta_3 Y_{3,0}(\theta, \phi)]$$

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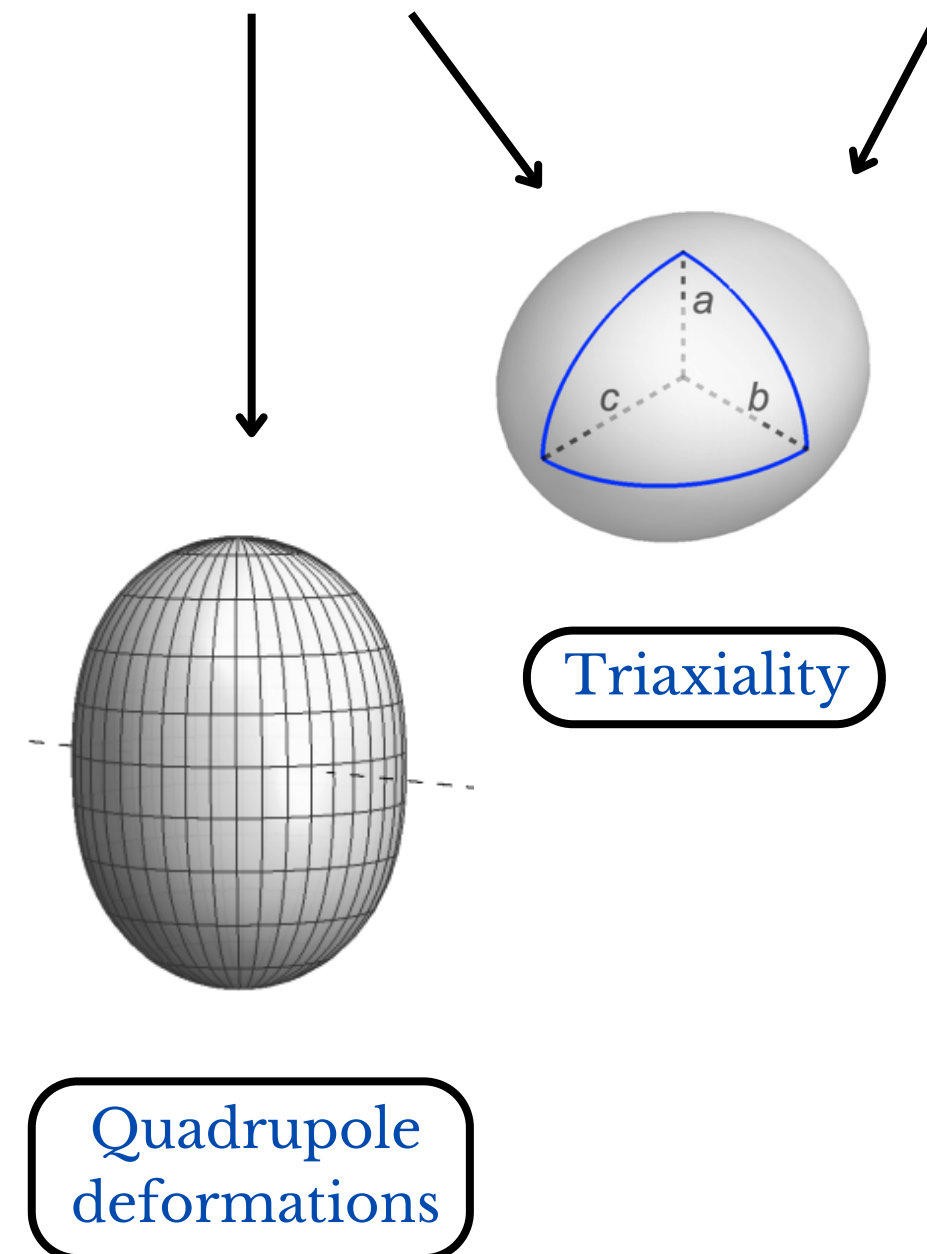


Quadrupole
deformations

Long-range correlations: angular deformations

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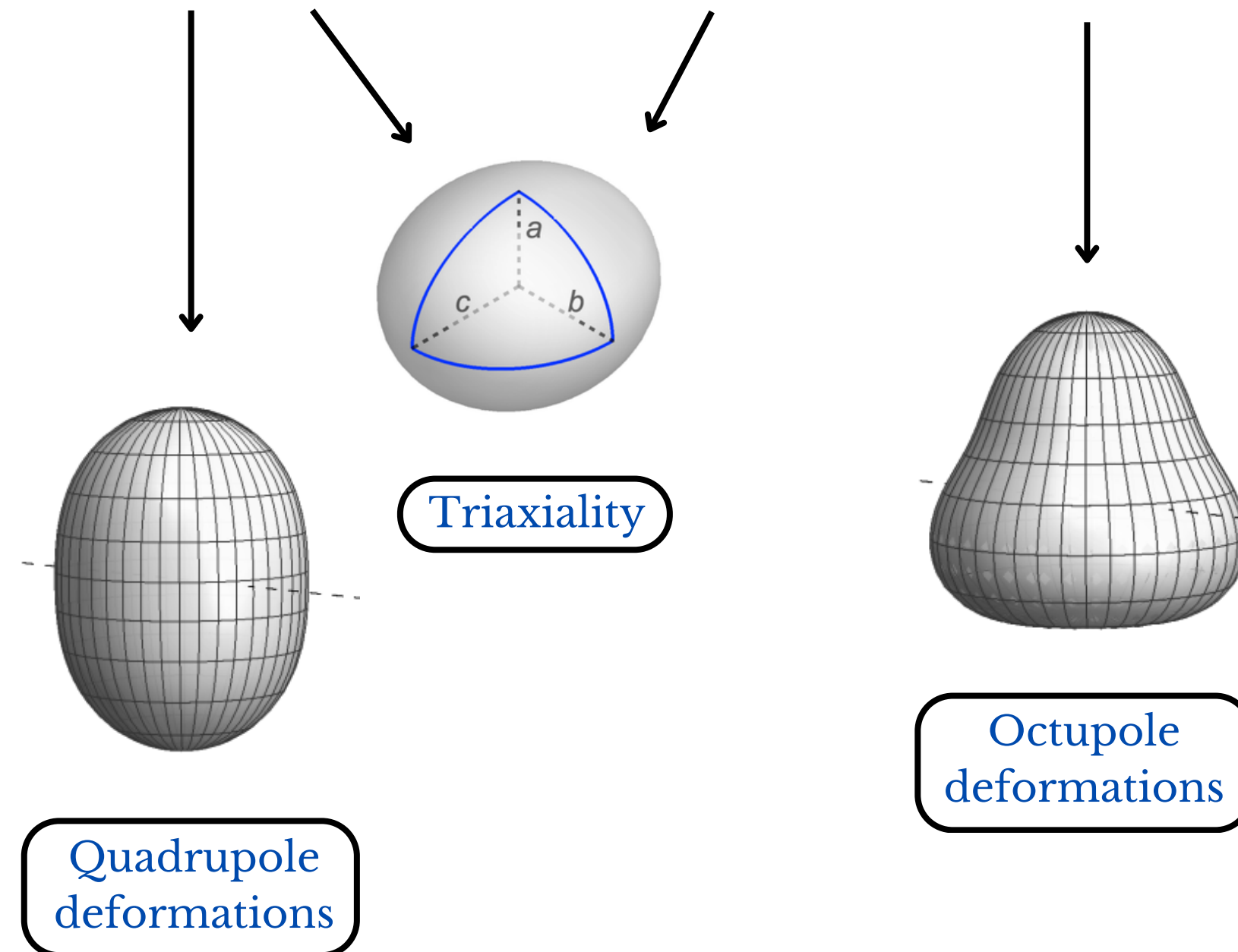
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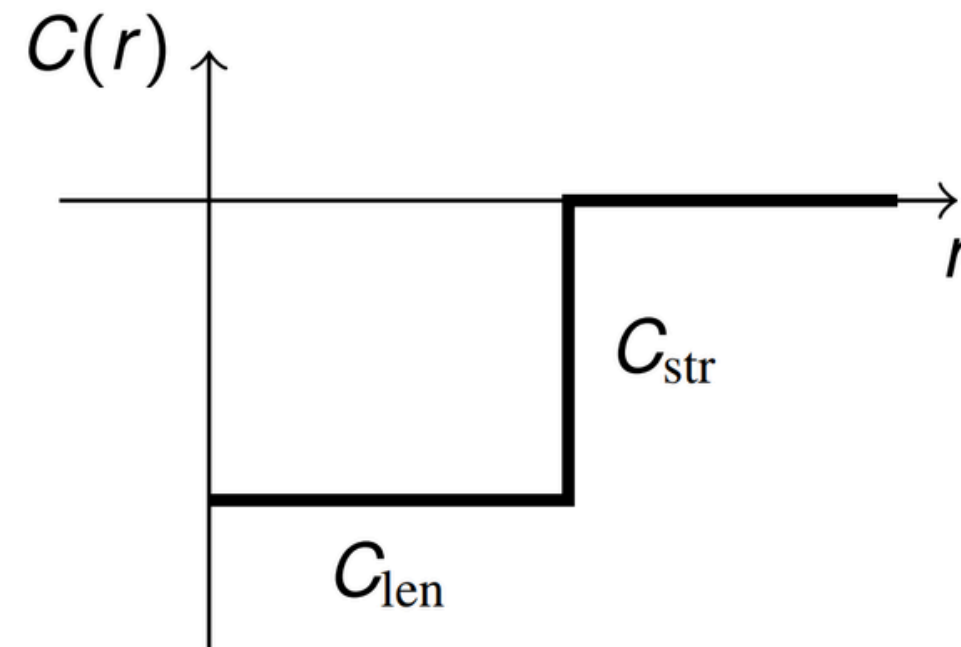
Short-range NN correlations: the step function

$$\rho^{(2)}(\mathbf{r}_1, \mathbf{r}_2) = \rho^{(1)}(\mathbf{r}_1)\rho^{(1)}(\mathbf{r}_2)[1 + C(\mathbf{r}_1, \mathbf{r}_2)]$$

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$$C(r) = \begin{cases} C_{\text{str}}, & r \leq C_{\text{len}} \\ 0, & r > C_{\text{len}} \end{cases}$$

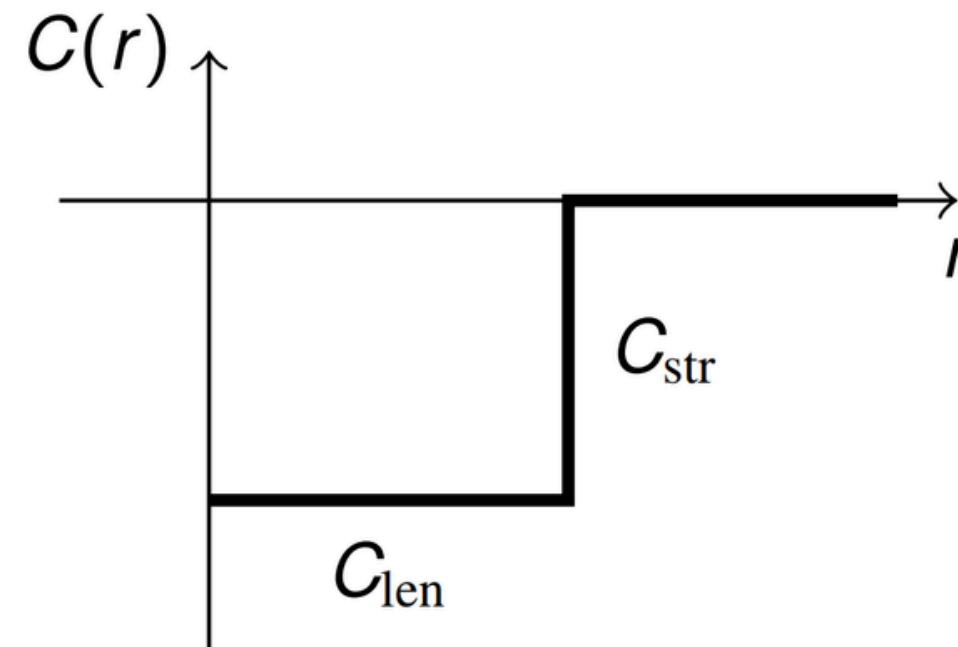


M. Luzum, J. Y. Ollitaut, M. Hippert - Eur. Phys. J. A, 59, 5, 110 (2023)

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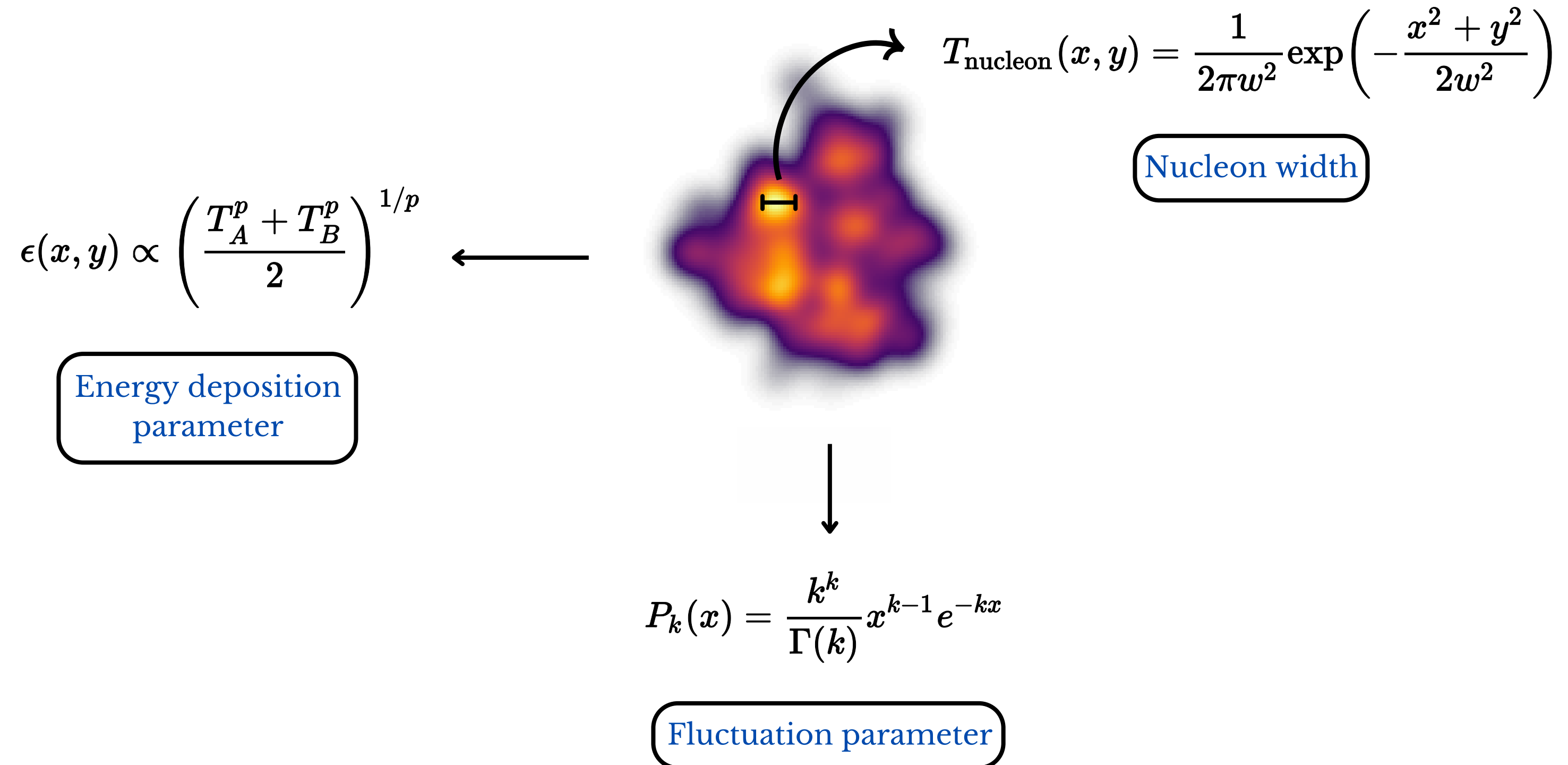


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Correlation volume $\longrightarrow C_{\text{vol}} = \int C(r) d^3 r = \frac{4\pi}{3} C_{\text{str}} C_{\text{len}}^3$

The T_{RENT}o model

Nucleus-nucleus collision: superposition of proton-proton collisions

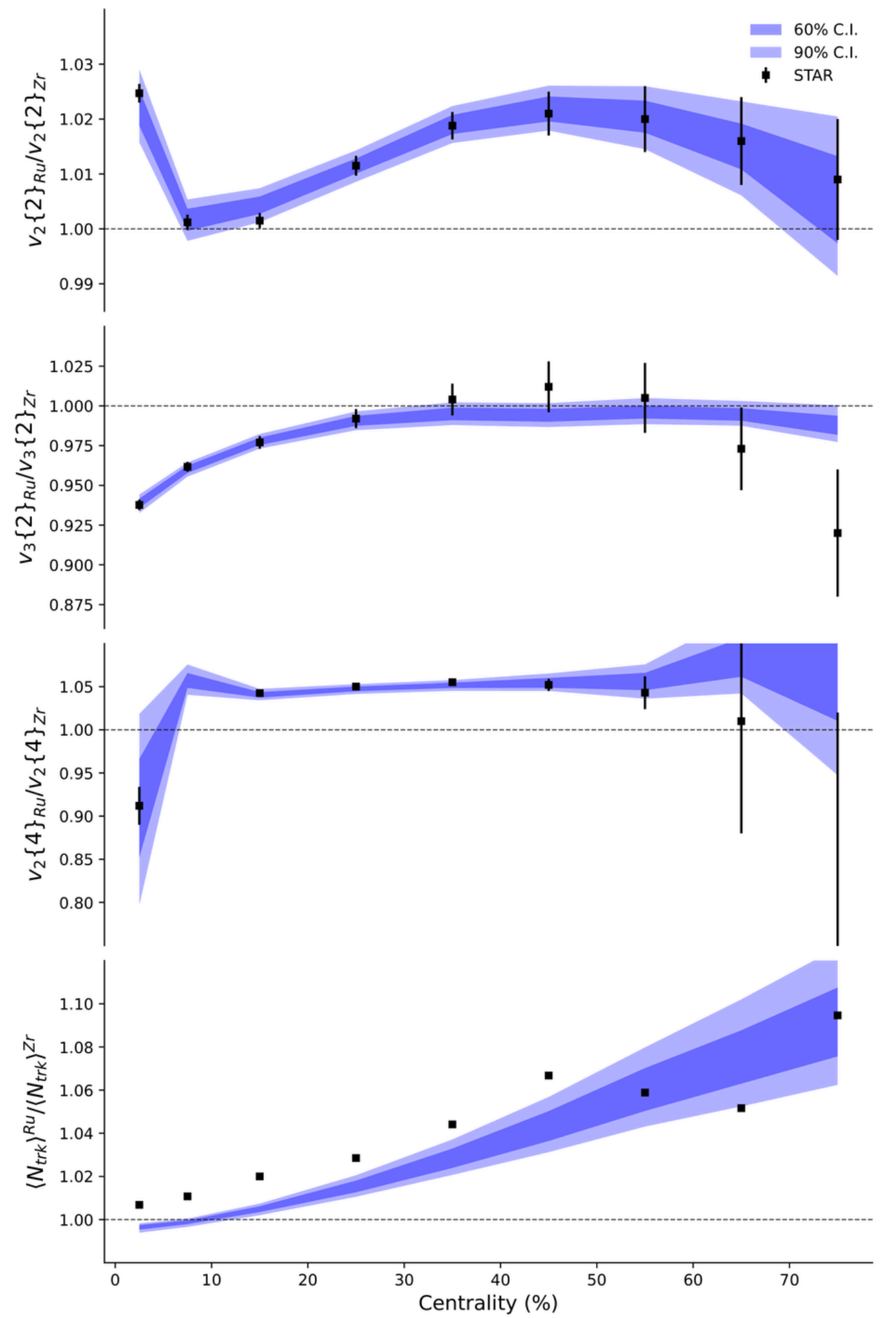
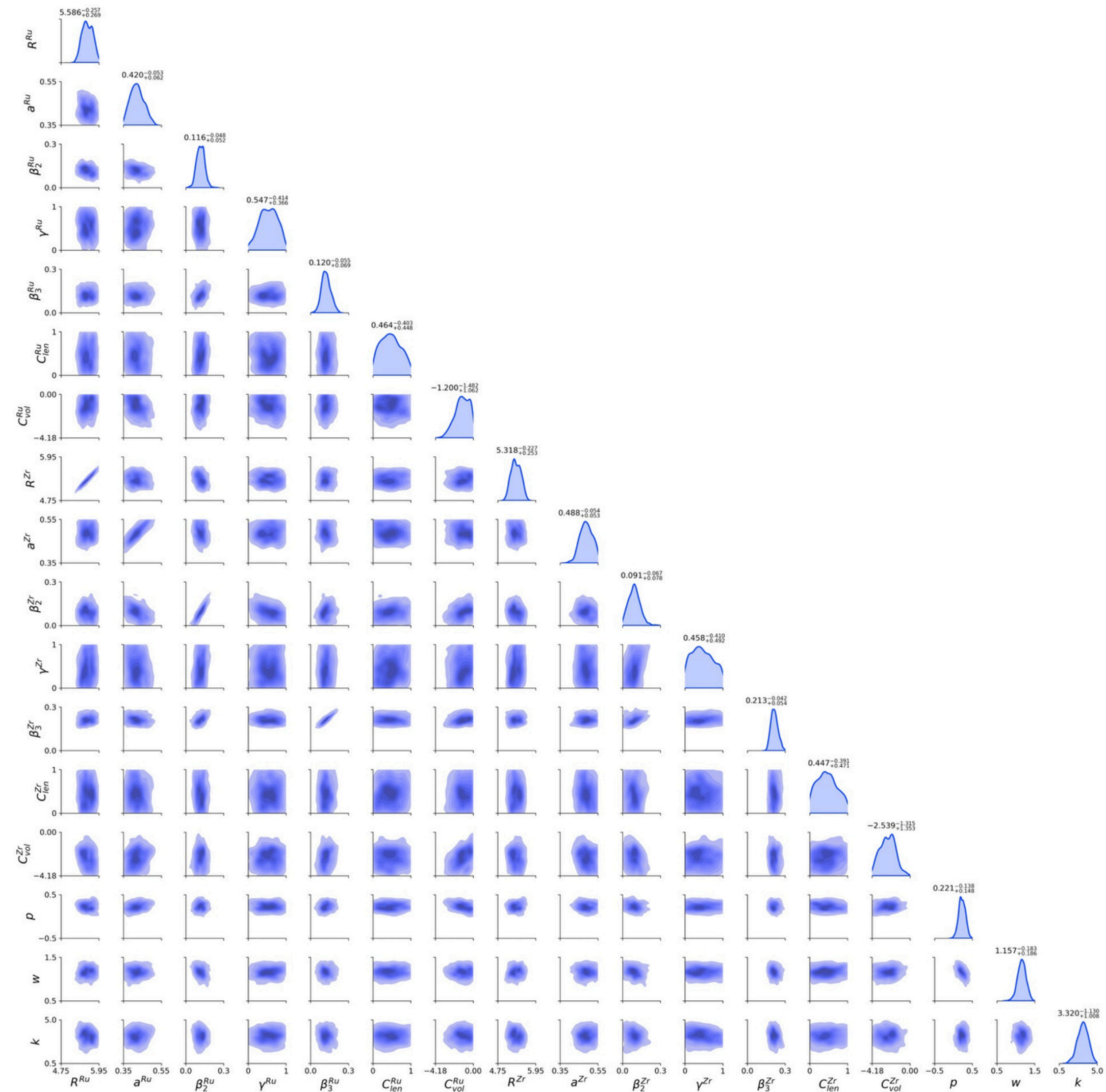


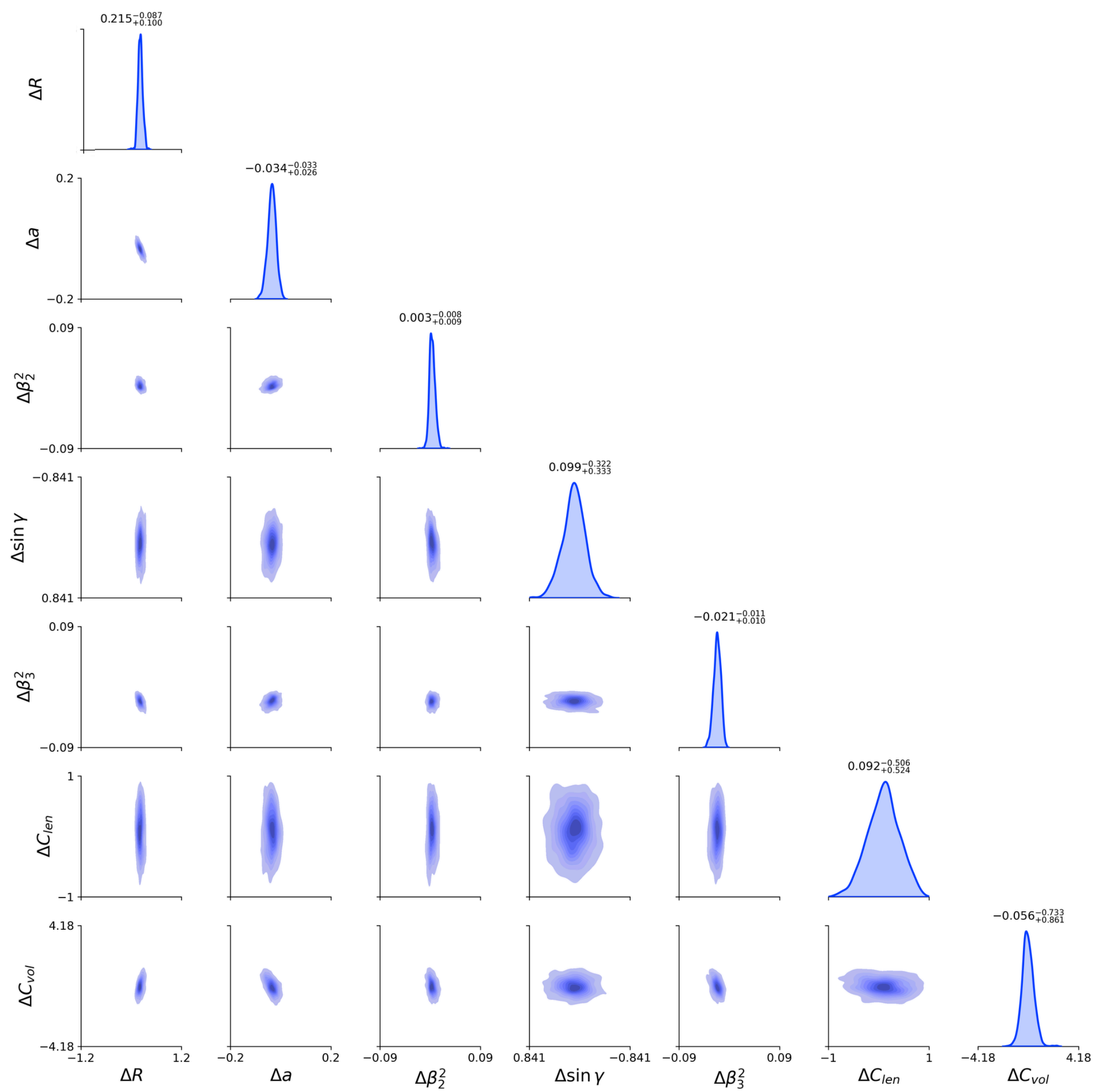
Analysis design

300 design points for emulator training

Parameter	Symbol	Prior range
Nuclear radius	$R^{\text{Ru,Zr}}$	[4.75,5.95] fm
Nuclear diffuseness	$a^{\text{Ru,Zr}}$	[0.35,0.55] fm
Quadrupole deformation	$\beta_2^{\text{Ru,Zr}}$	[0,0.3]
Triaxial deformation	$\gamma^{\text{Ru,Zr}}$	[0,1] rad
Octupole deformation	$\beta_3^{\text{Ru,Zr}}$	[0,0.3]
Correlation length	$C_{\text{len}}^{\text{Ru,Zr}}$	[0,1] fm
Correlation volume	$C_{\text{vol}}^{\text{Ru,Zr}}$	[-4.18,0] fm ³
Generalized mean	p	[-0.5,0.5]
Nucleon width	w	[0.5,1.5] fm
Fluctuation	k	[1.5,5]

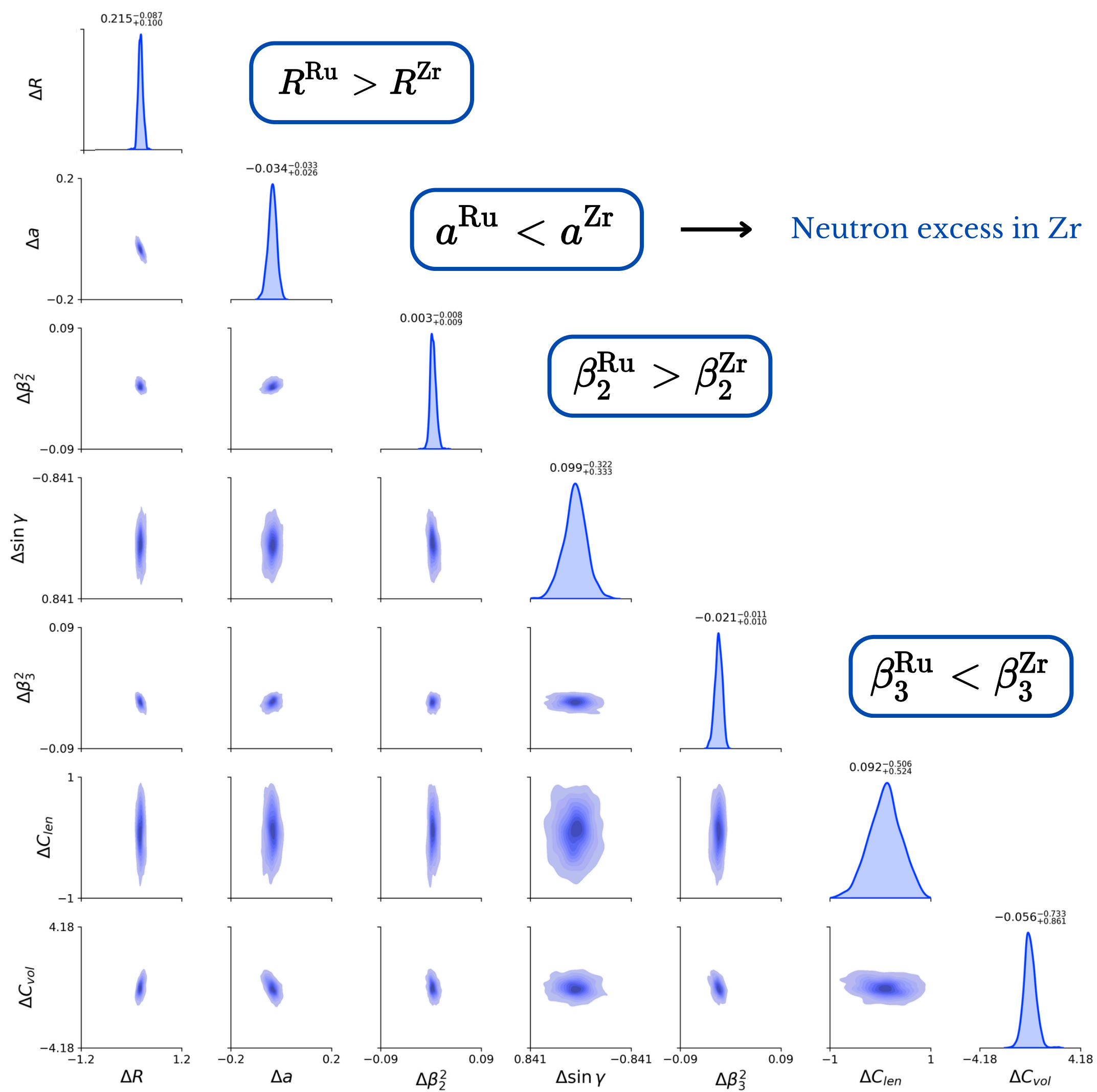
Observables	Predictors	Centrality classes
$v_n\{2\}^{\text{Ru}}/v_n\{2\}^{\text{Zr}}, n = 2, 3$	$\varepsilon_n\{2\}^{\text{Ru}}/\varepsilon_n\{2\}^{\text{Zr}}, n = 2, 3$	0 - 5 %, 5 - 10 %, ... , 70 - 80 %
$v_n\{4\}^{\text{Ru}}/v_n\{4\}^{\text{Zr}}, n = 2$	$\varepsilon_n\{4\}^{\text{Ru}}/\varepsilon_n\{4\}^{\text{Zr}}, n = 2$	0 - 5 %, 5 - 10 %, ... , 70 - 80 %
$\langle N_{\text{trk}} \rangle^{\text{Ru}}/\langle N_{\text{trk}} \rangle^{\text{Zr}}$	$\langle S \rangle^{\text{Ru}}/\langle S \rangle^{\text{Zr}}$	0 - 5 %, 5 - 10 %, ... , 70 - 80 %





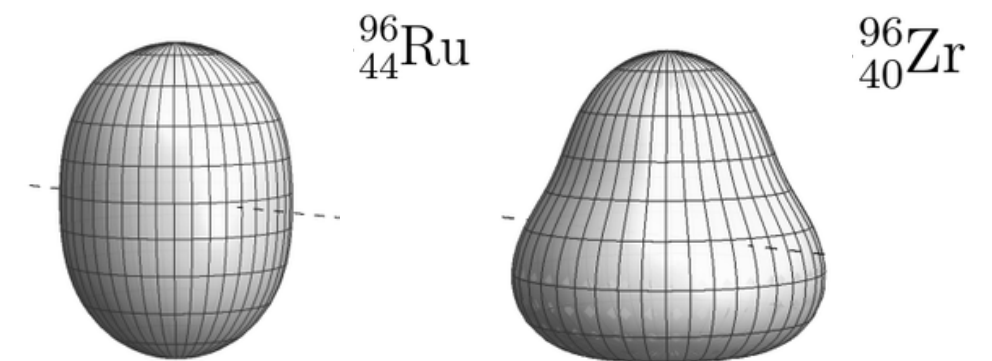
$$\Delta x_i = x_i^{\text{Ru}} - x_i^{\text{Zr}}$$

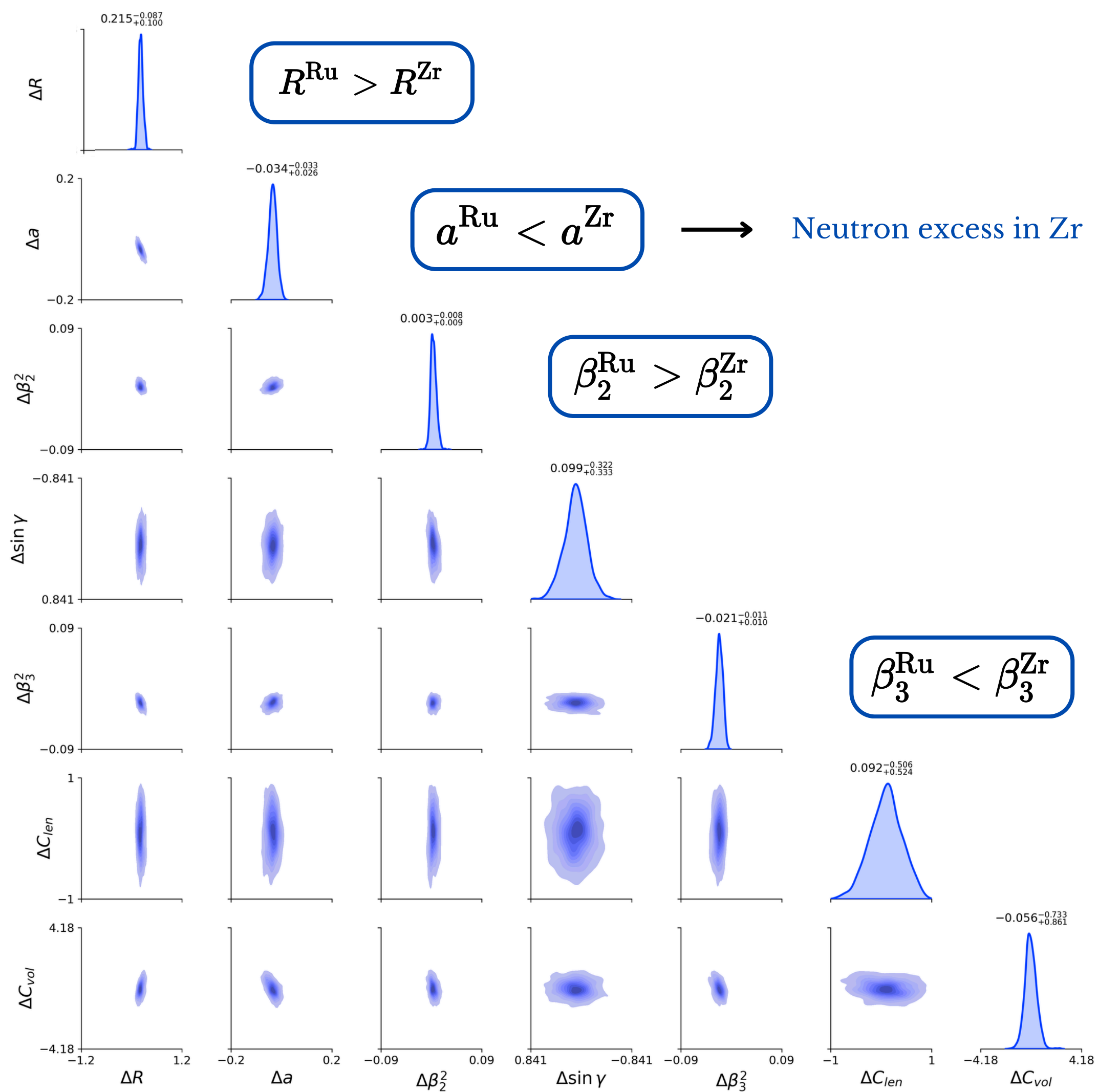
$$\Delta x_i^2 = (x_i^{\text{Ru}})^2 - (x_i^{\text{Zr}})^2$$



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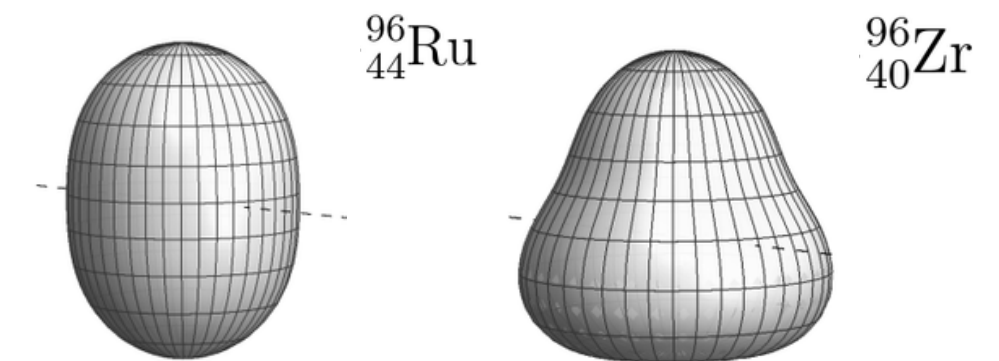
$$\Delta x_i^2 = (x_i^{\text{Ru}})^2 - (x_i^{\text{Zr}})^2$$

$$\frac{\mathcal{O}^{\text{Ru}}}{\mathcal{O}^{\text{Zr}}} \approx 1 + c_1 \Delta \beta_2^2 + c_2 \Delta \beta_3^2 + c_3 \Delta R + c_4 \Delta a$$

Jia, Jiangyong - Phys. Rev. C 105, 1 (2022)

Zhang, Chunjian and Jia, Jiangyong - Phys. Rev. Lett. 128, 2 (2022)

Jia, Jiangyong and Zhang, Chunjian - Phys. Rev. C 107, 2 (2023)



From parameters to nucleon correlations

$$s(\vec{x}) = \langle s(\vec{x}) \rangle + \delta s(\vec{x}) \quad , \quad \langle \delta s(\vec{x}) \rangle = 0 \quad , \quad s(\vec{x}) \propto \frac{t(\vec{x}) + t'(\vec{x})}{2}$$

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$$\langle \varepsilon_n^2 \rangle = \frac{1}{2A} \frac{1}{\langle \hat{R}_n \rangle^2} [\langle \hat{R}_{2n} \rangle + (A-1) \langle \hat{\mathcal{E}}_n \rangle]$$

average over ultra-central events



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$$\hat{R}_n(\mathbf{r}) = r_{\perp}^n \longrightarrow \text{One-body operator}$$

$$\langle \hat{R}_n \rangle = \int \rho^{(1)}(\mathbf{r}) \hat{R}_n(\mathbf{r}) d^3 r$$

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$$\hat{\mathcal{E}}_n(\mathbf{r}_1, \mathbf{r}_2) = r_{1\perp}^n r_{2\perp}^n e^{in(\phi_1 - \phi_2)} \longrightarrow \text{Two-body operator}$$

$$\langle \hat{\mathcal{E}}_n \rangle = \int \rho^{(2)}(\mathbf{r}_1, \mathbf{r}_2) \hat{\mathcal{E}}_n(\mathbf{r}_1, \mathbf{r}_2) d^3 r_1 d^3 r_2$$

Bridging high- and low-energy results

S. Bofos, B. Bally, T. Duguet, M. Frosini - Phys. Lett. B, 879, 140649 (2026):

$$\mathcal{B}_2^2(\text{HE}) = \frac{4\pi}{3} \frac{\langle \hat{\mathcal{E}}_2 \rangle}{\langle \hat{R}_2 \rangle^2}$$

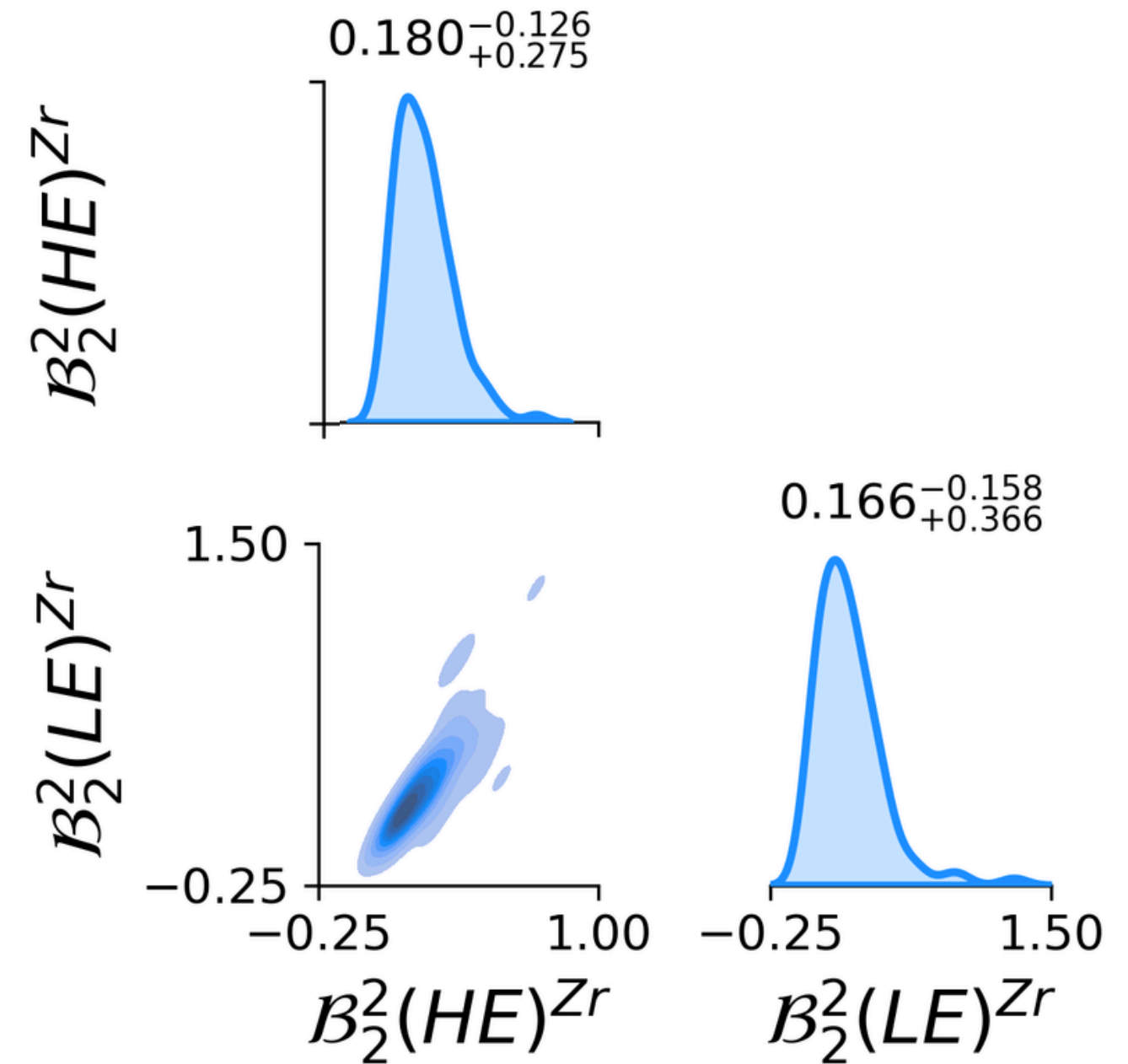
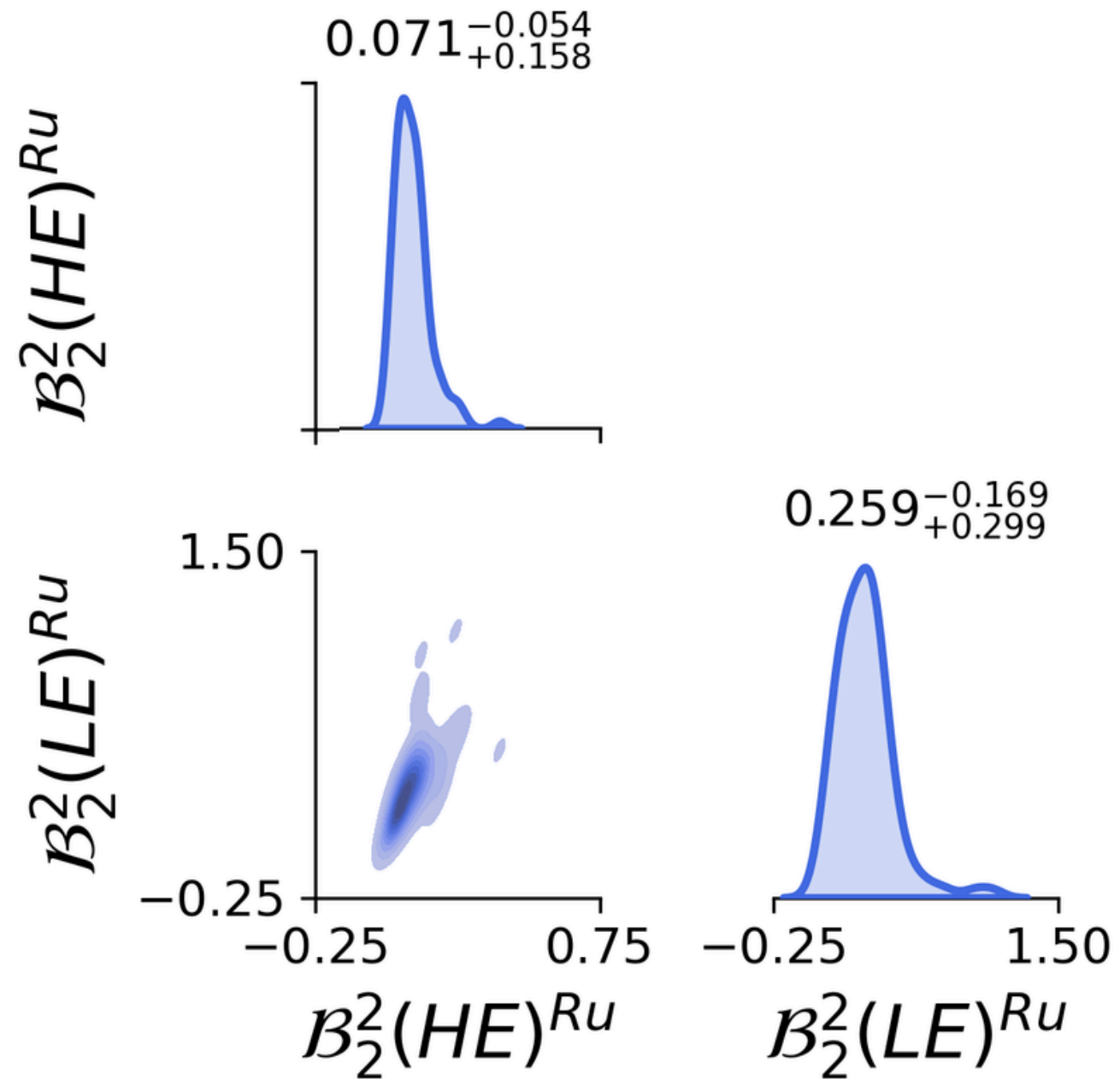
$$\mathcal{B}_2^2(\text{LE}) = \frac{16\pi^2}{9Z^2 e^2 R^4} P_2$$

$$P_2 = \langle Q_{20}^2 \rangle \longrightarrow \text{Kumar shape invariant of order 2}$$

K. Kumar - Phys. Rev. Lett. 28, 249 (1972)

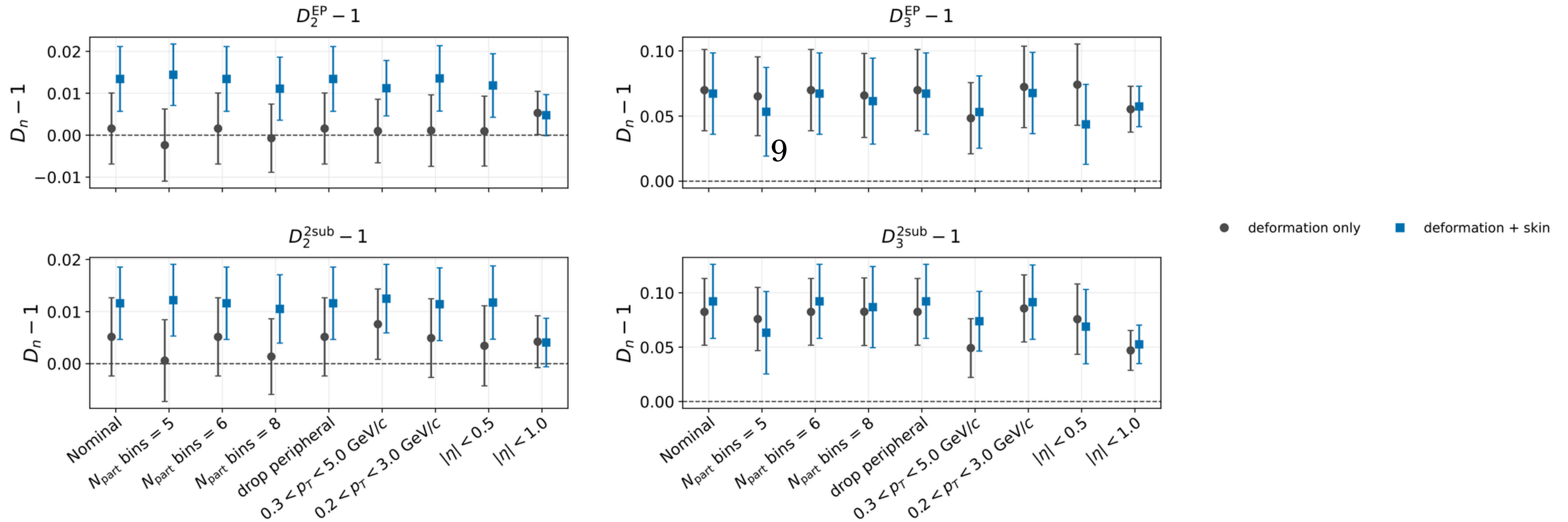
$$\langle Q_{20} \rangle = \int r^2 Y_{2,0}(\theta, \phi) \rho(r, \theta, \phi) d^3 r \quad (\text{Intrinsic quadrupole moment})$$

Distributions from posterior samples



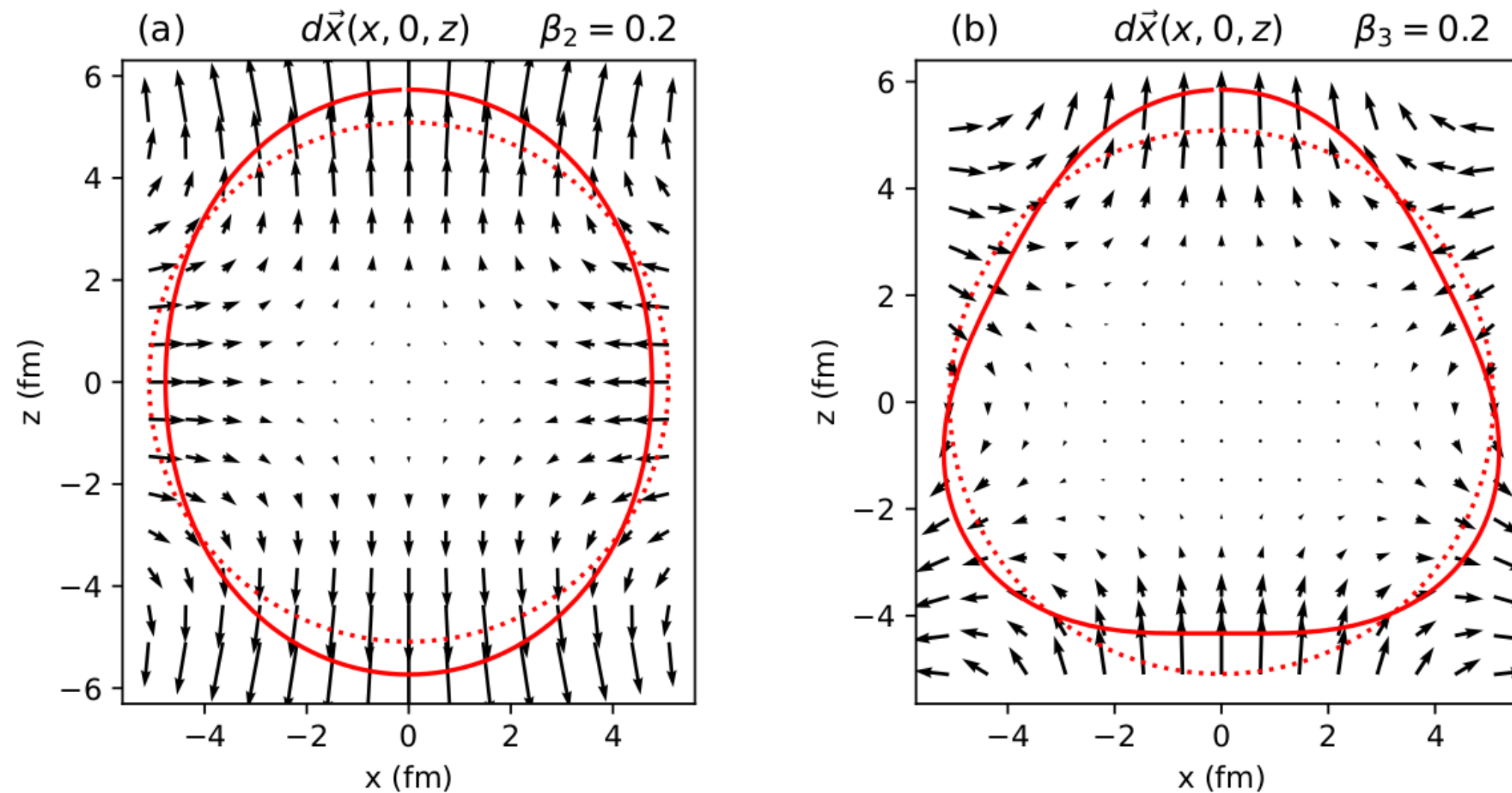
The importance of an analysis with a full model

$$D_n = \frac{v_n^{\text{Ru}} / v_n^{\text{Zr}}}{\varepsilon_n^{\text{Ru}} / \varepsilon_n^{\text{Zr}}}$$



Assembling the nuclei: Isobar Sampler

Start with a spherical nuclear configuration and **shift nucleons** to introduce the desired angular deformations and/or short range correlations

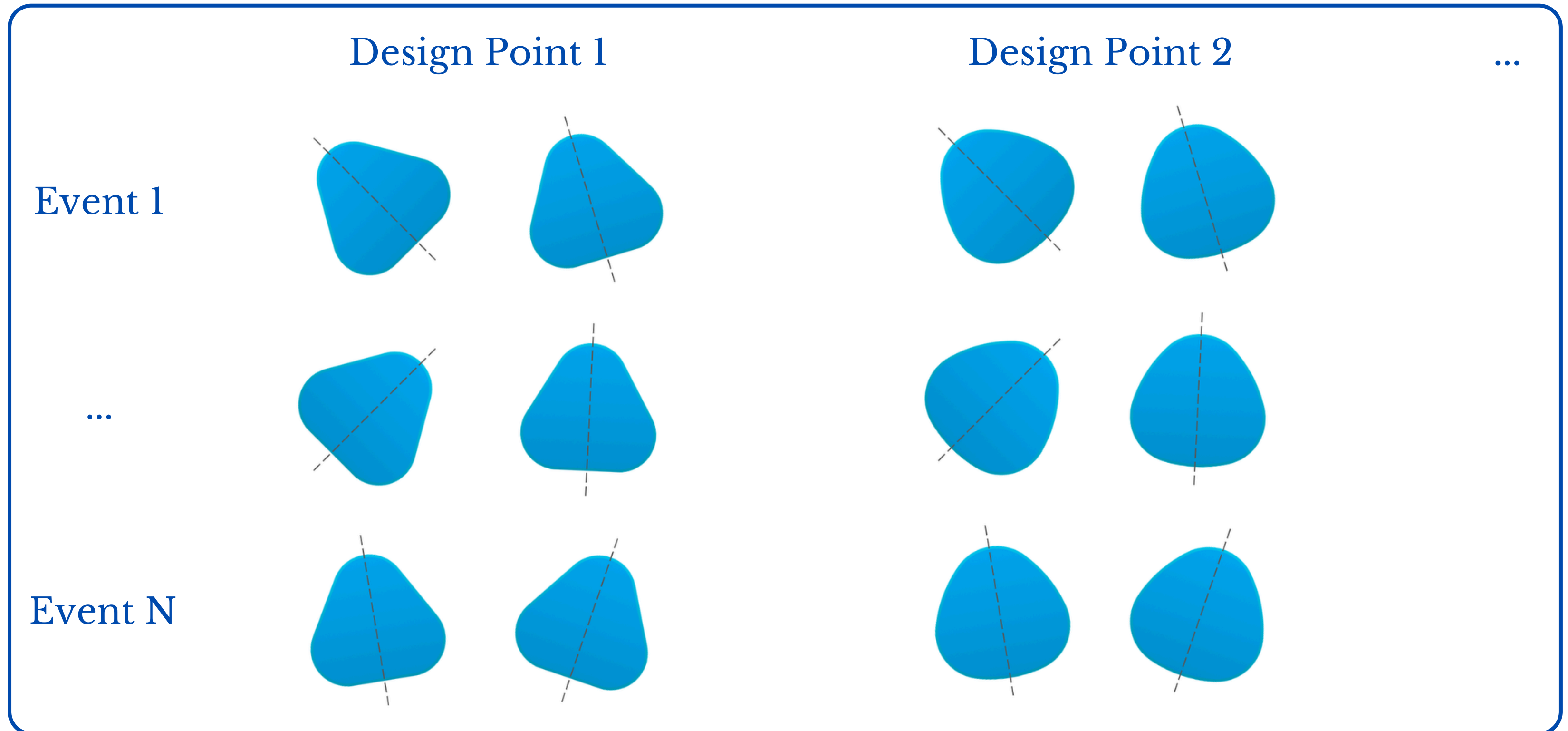


M. Luzum, J. Y. Ollitrault, M. Hippert - Eur. Phys. J. A, 59, 5, 110 (2023)

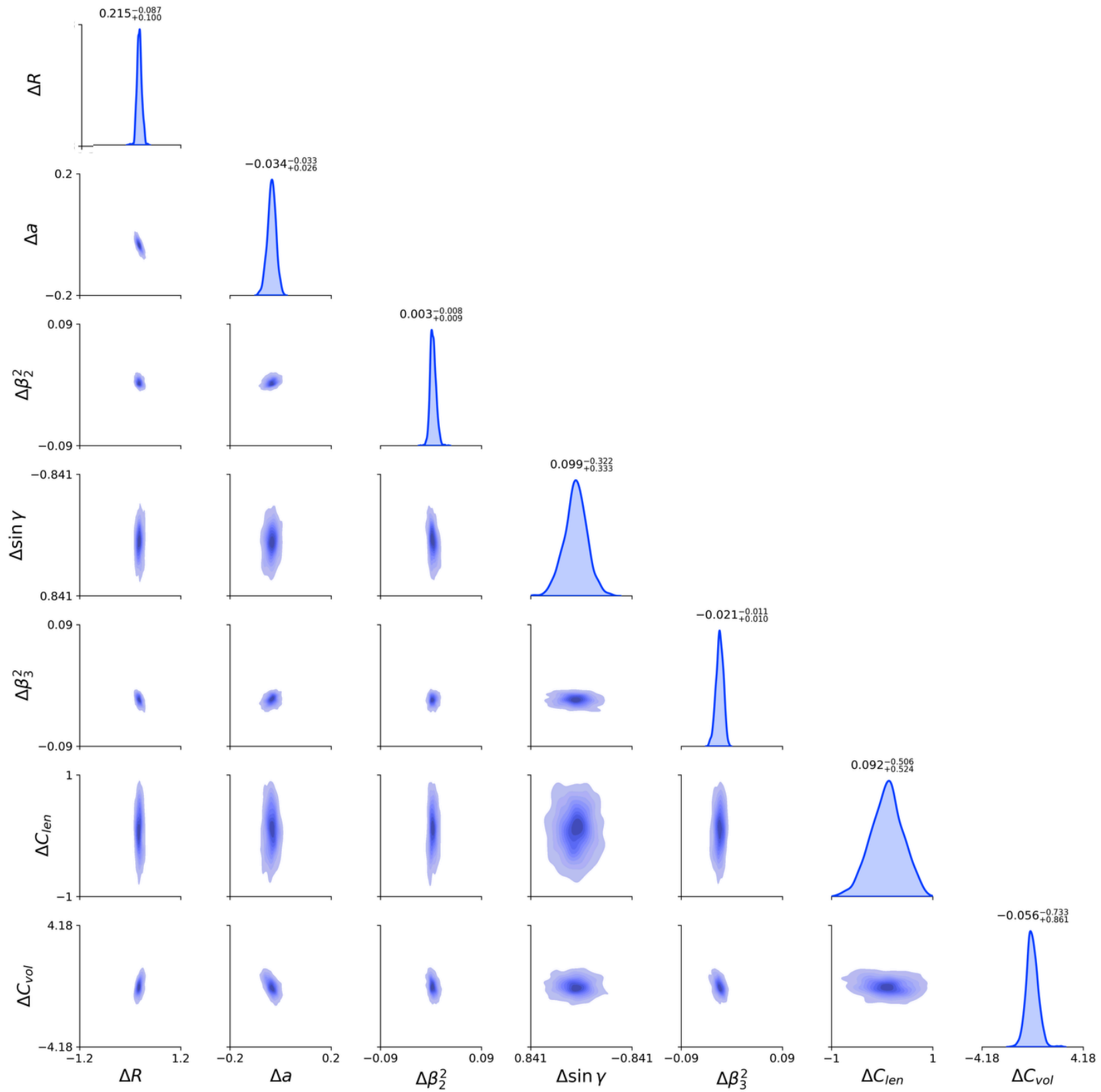
Highly correlated sets of nucleons' positions!

Synchronizing other fluctuations

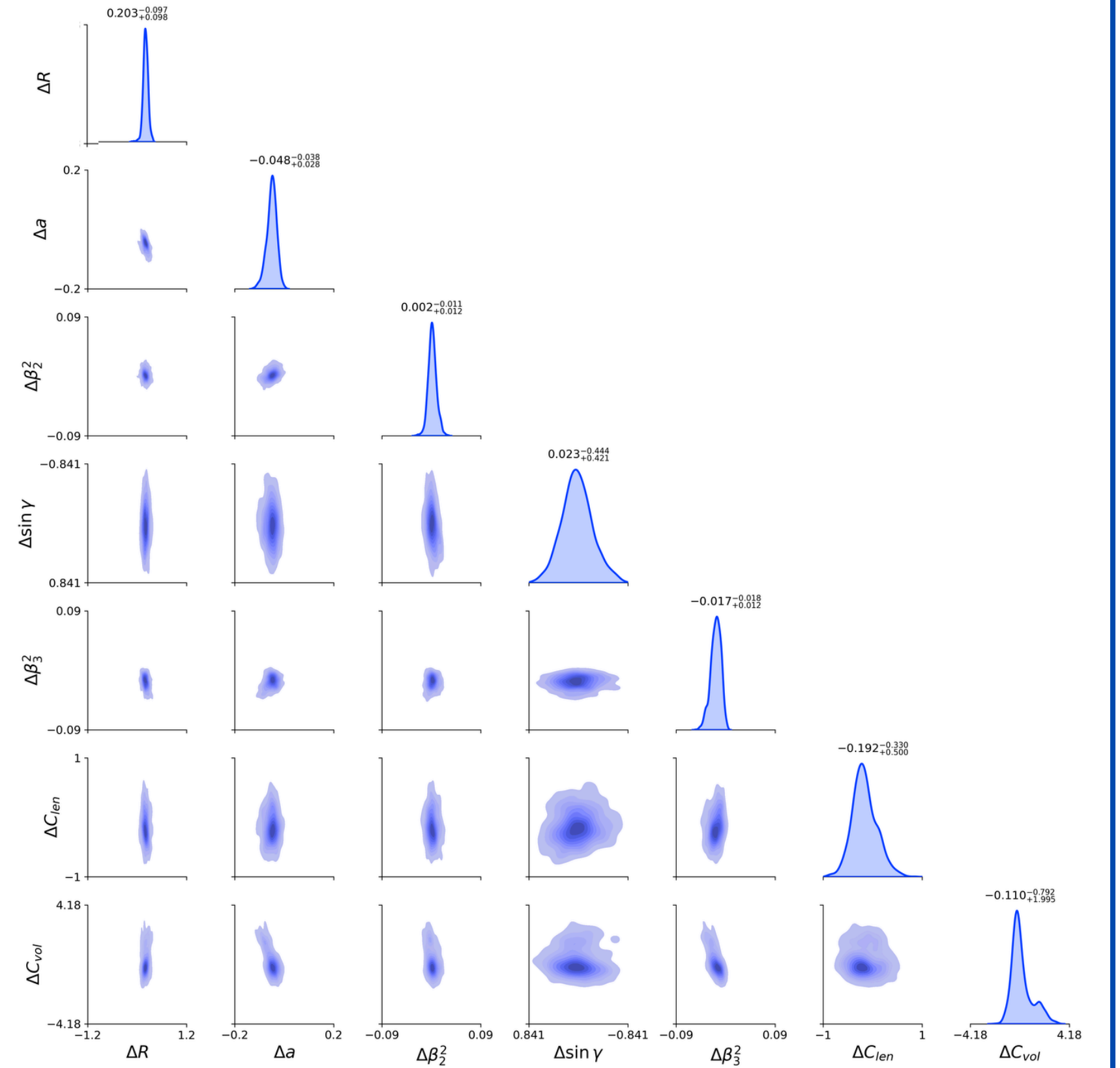
Same (scaled) impact parameter, Euler angles and gamma weights in each event



10^5 events



2500 events



Conclusions

Isobar collisions: a “microscope” for nuclear structure

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Toy Bayesian analysis with ratios of observables: precise constraints on isobars' relative properties

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Strong correlation between high- and low-energy pair of analogous quantities

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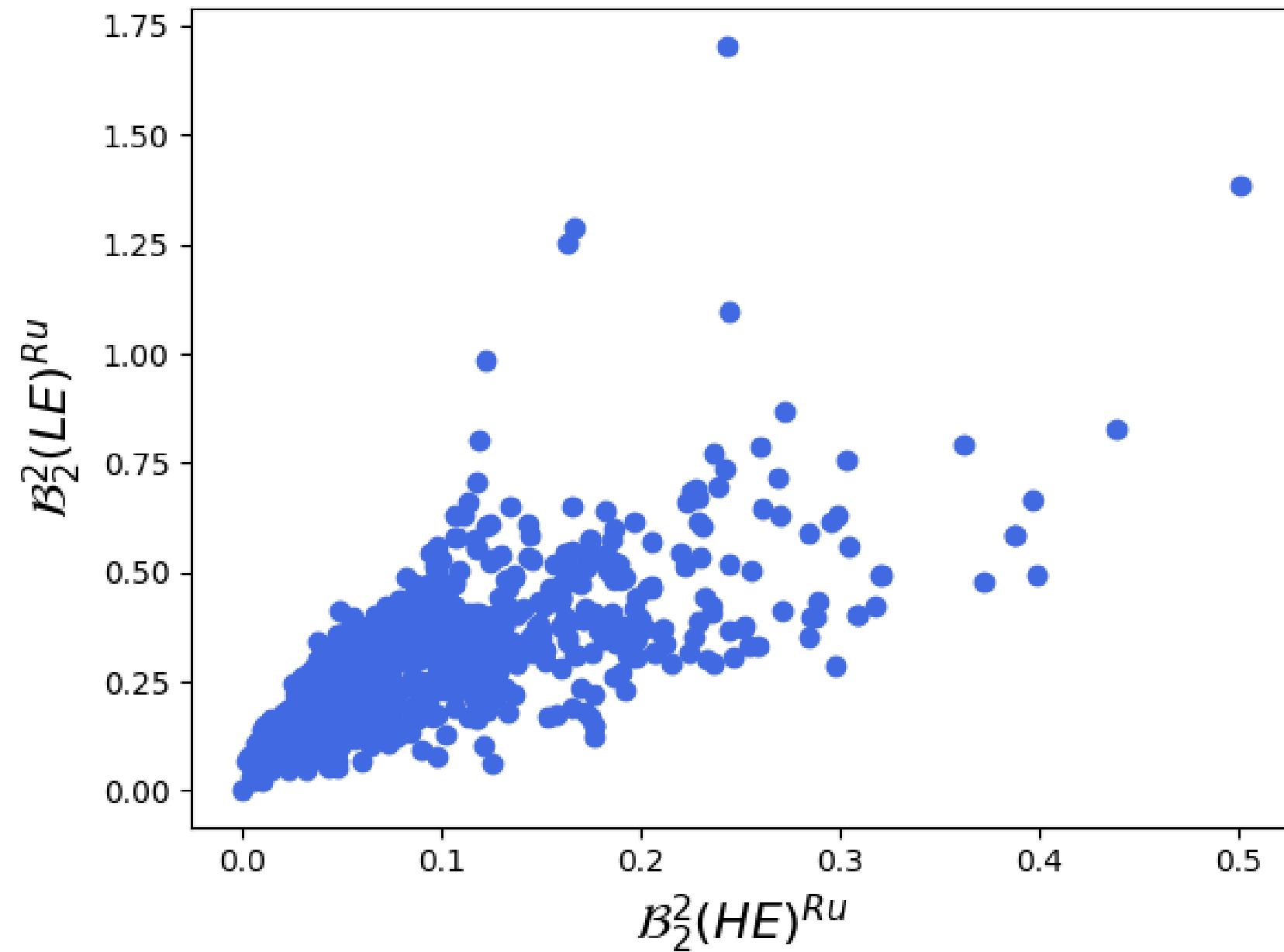
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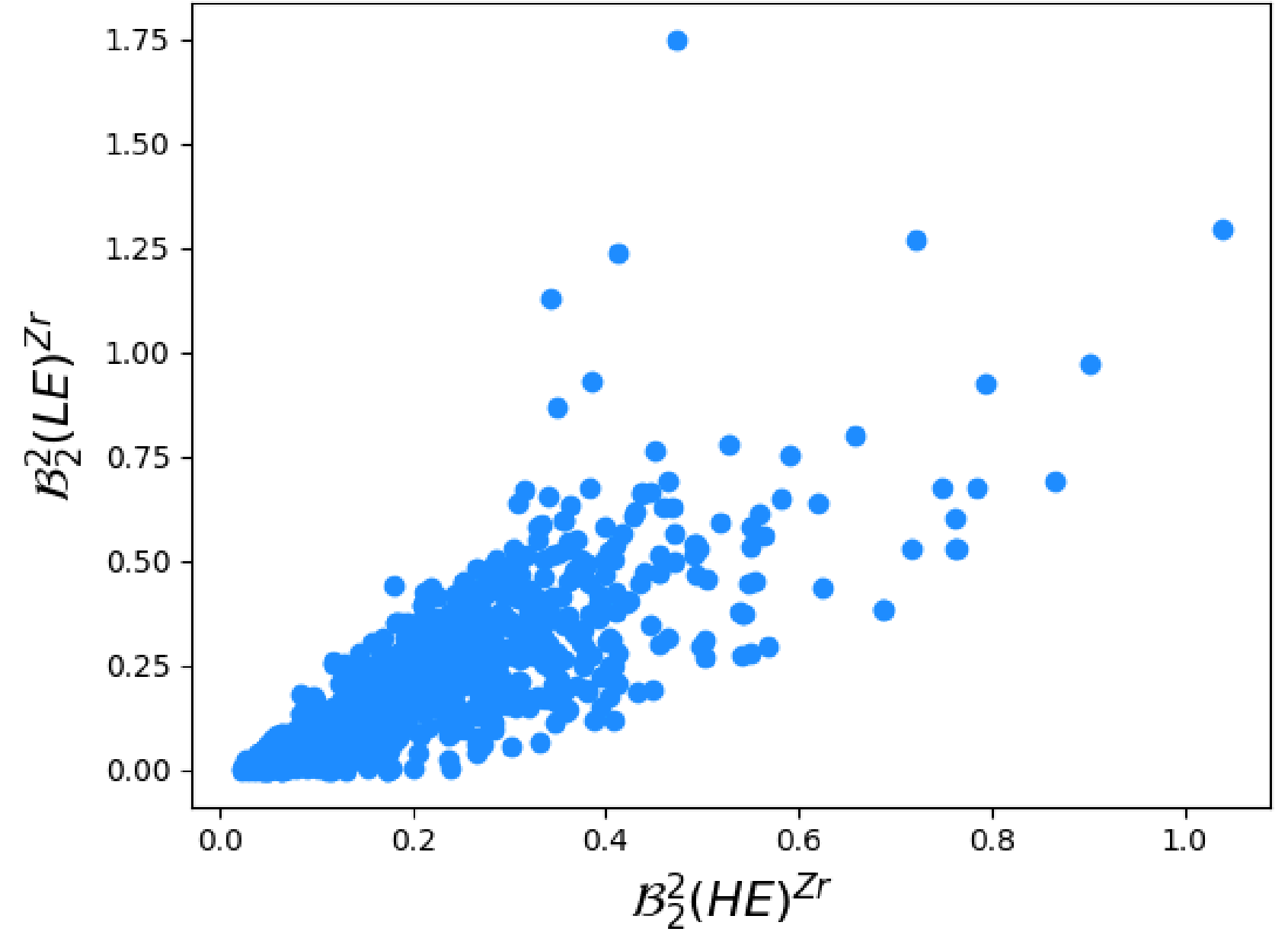
Thank you!

Backup

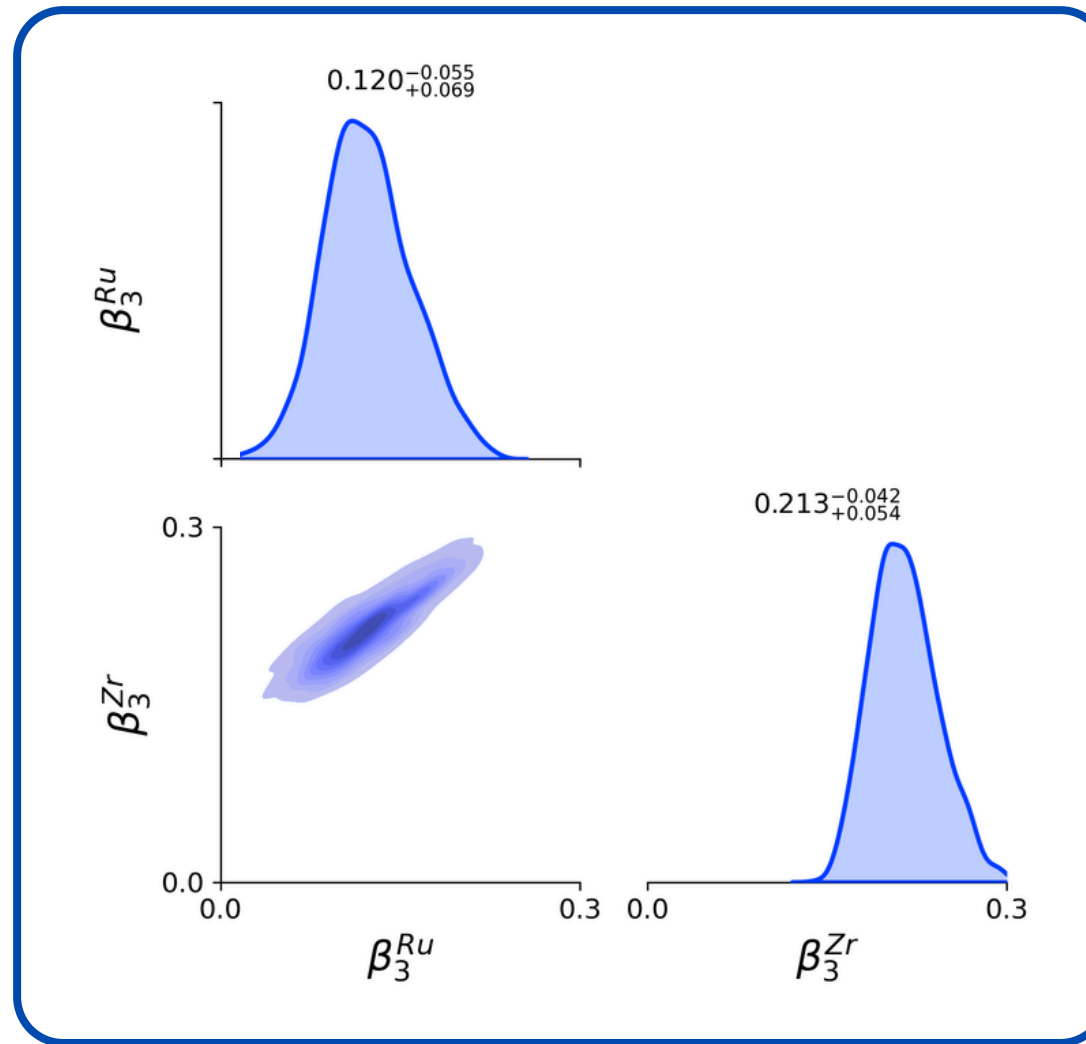
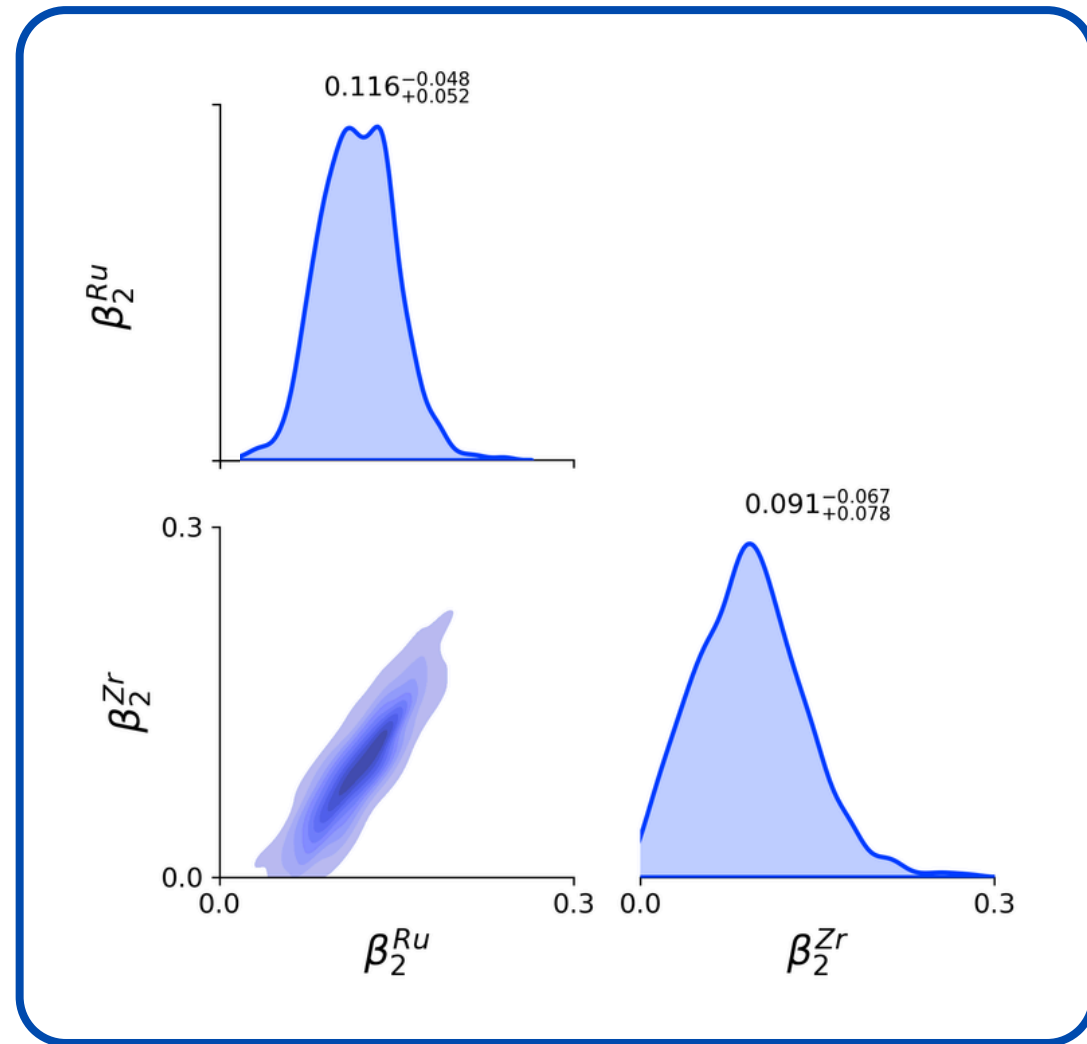
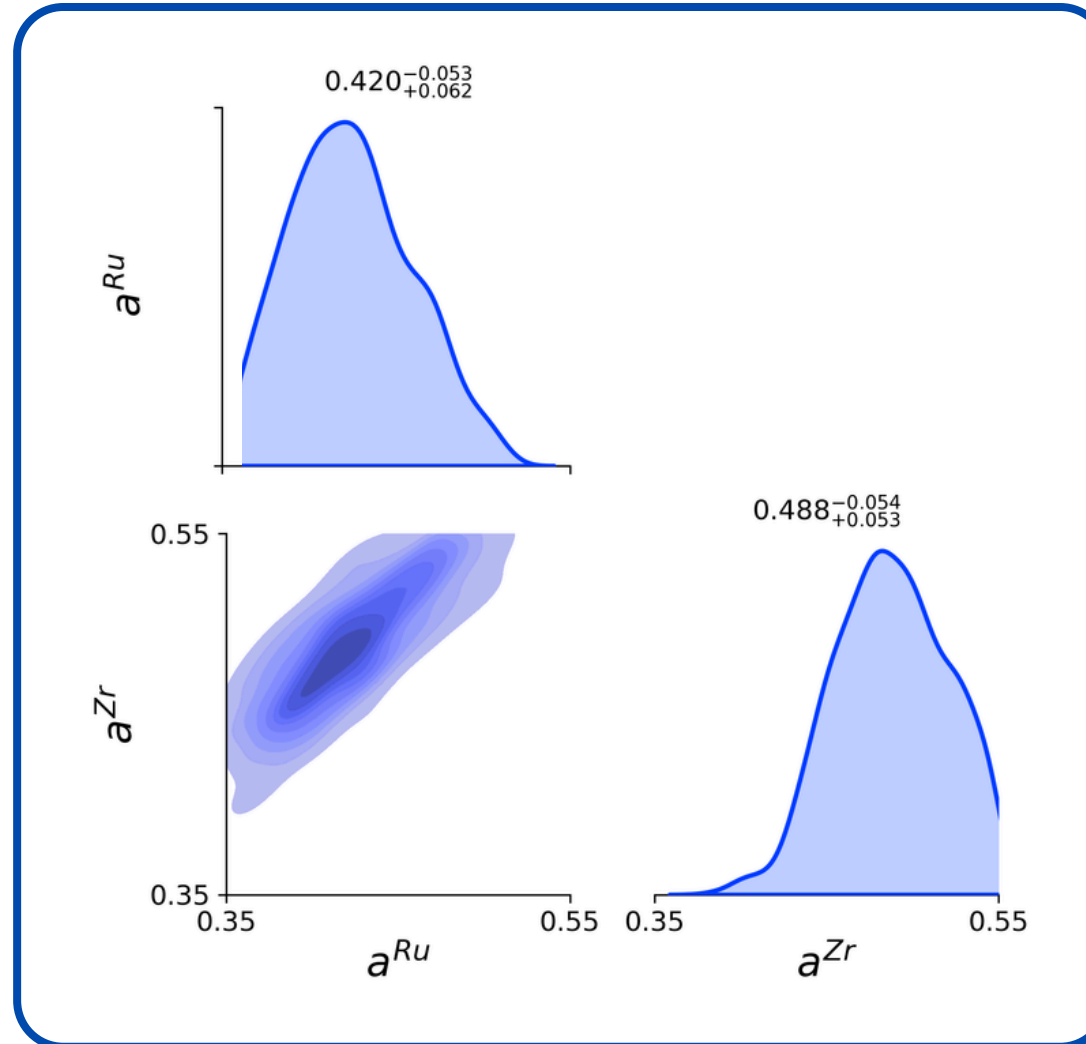
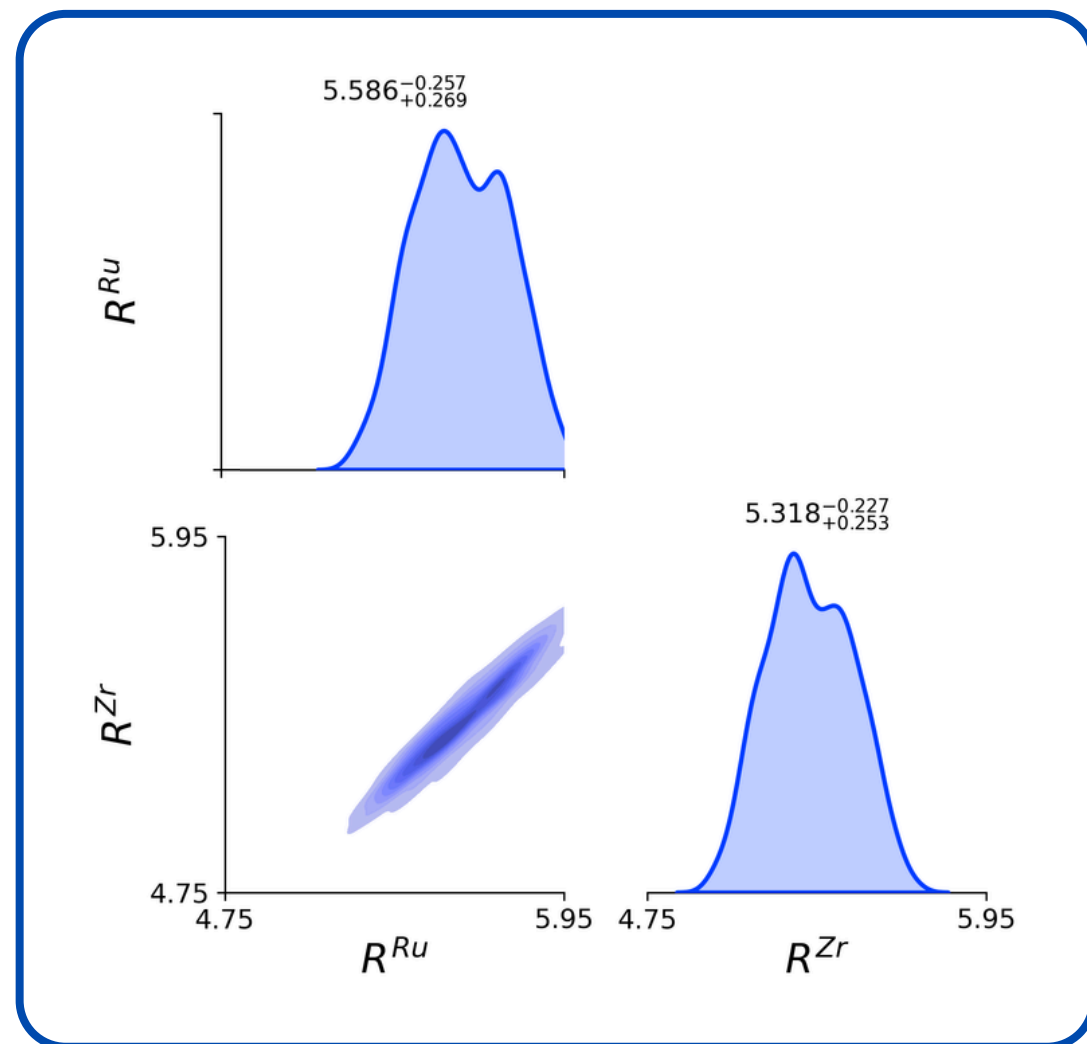
Quantifying the correlation



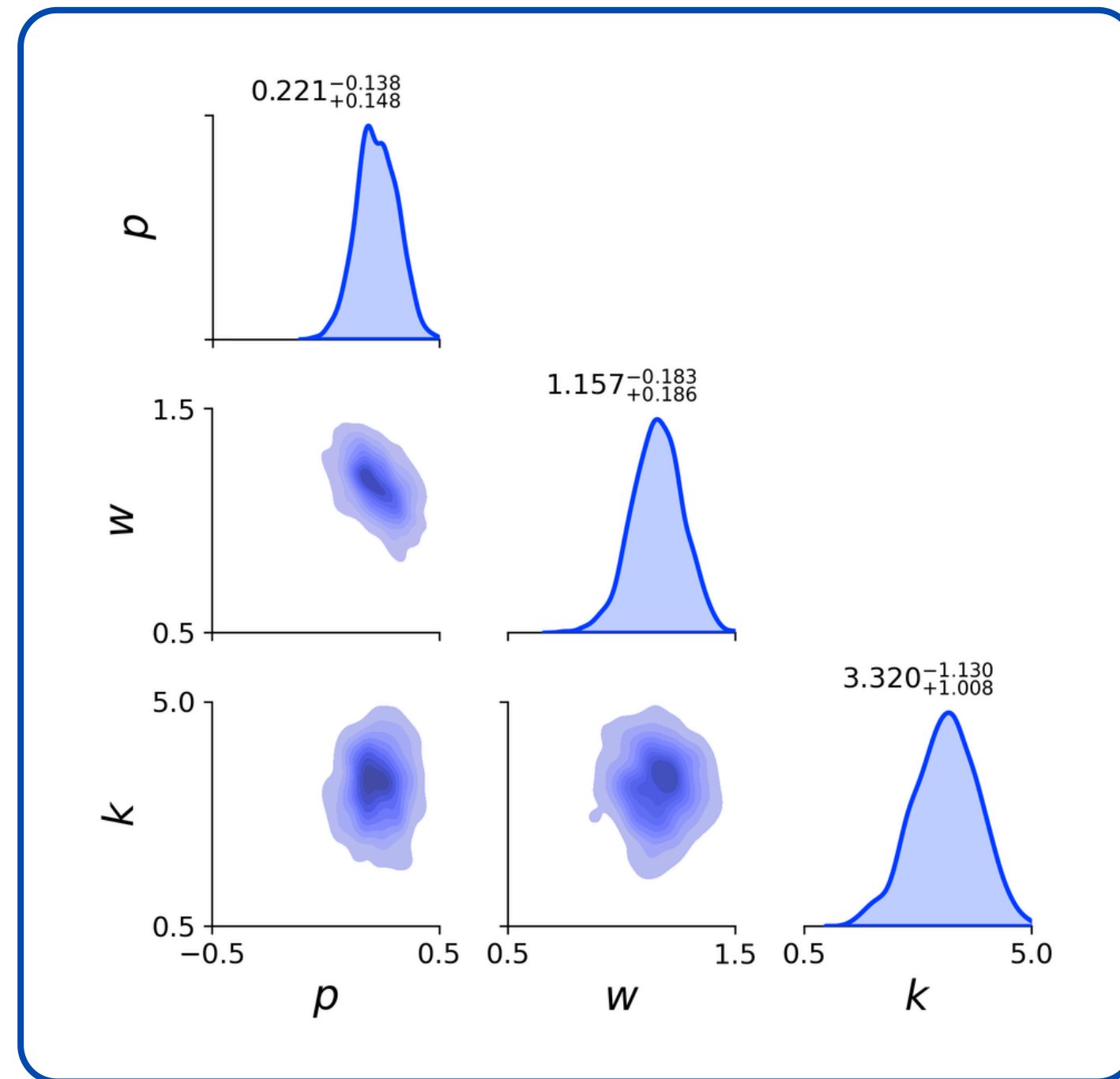
$$r = 0.698$$



$$r = 0.791$$

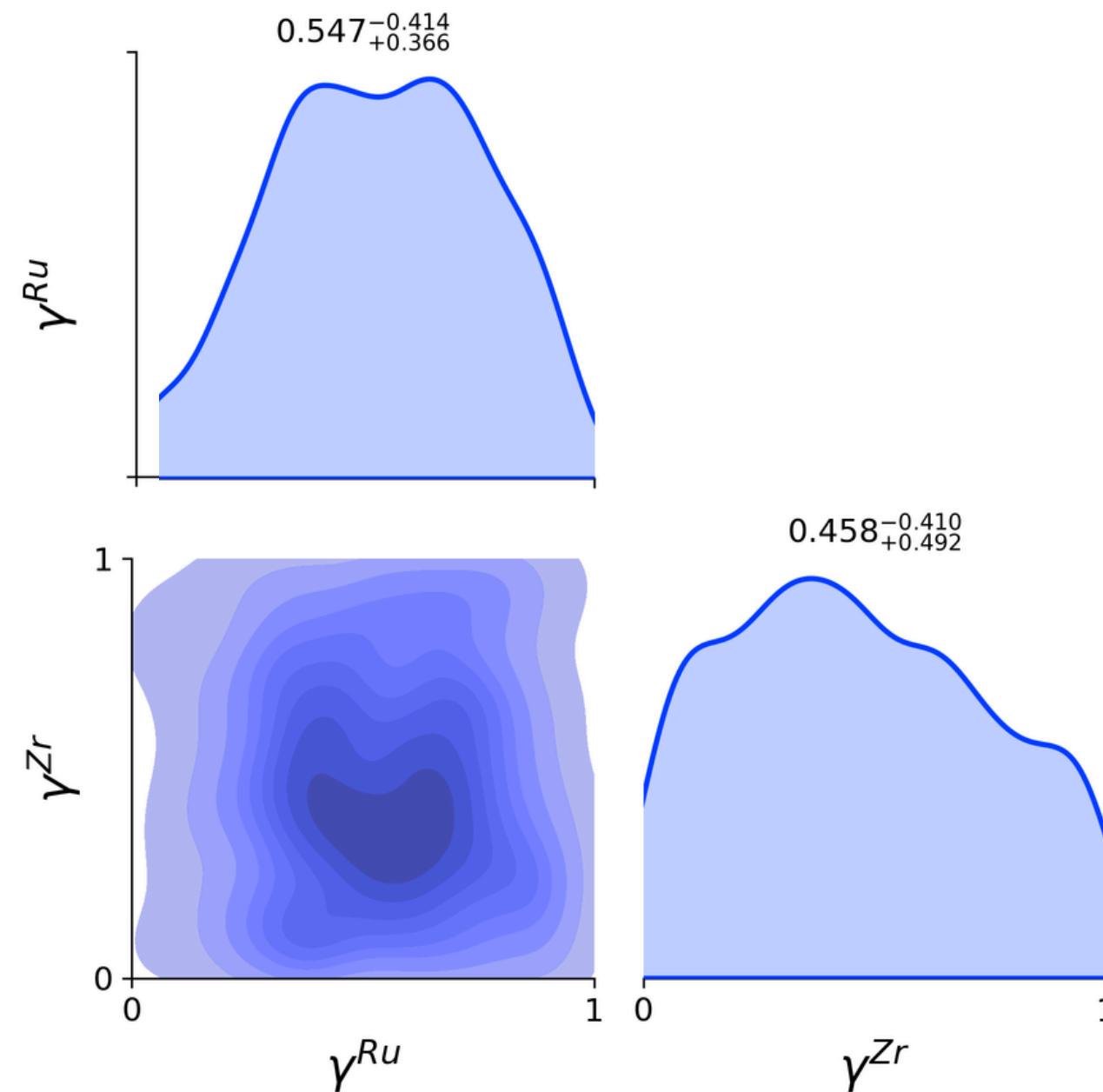


Posterior for initial state parameters

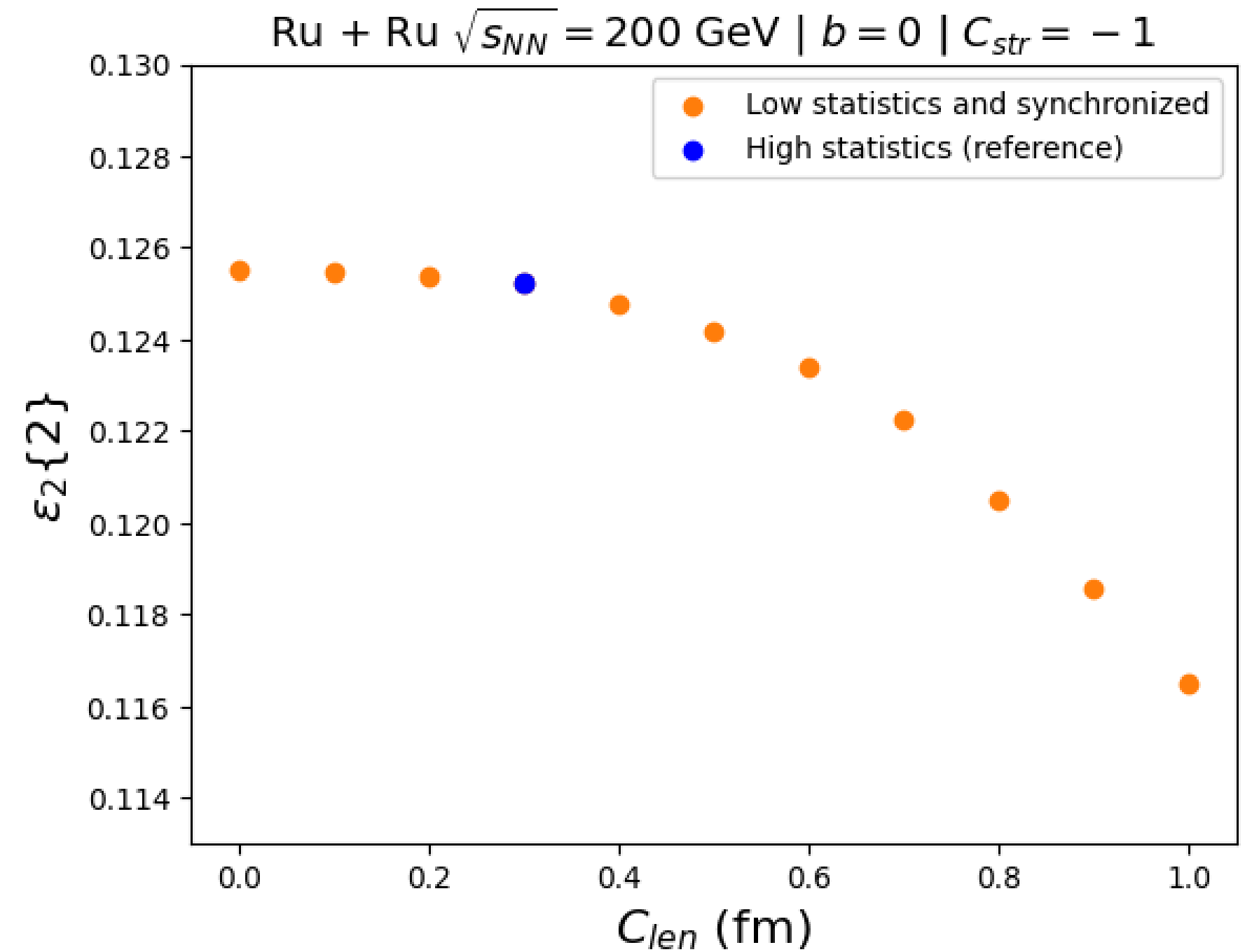
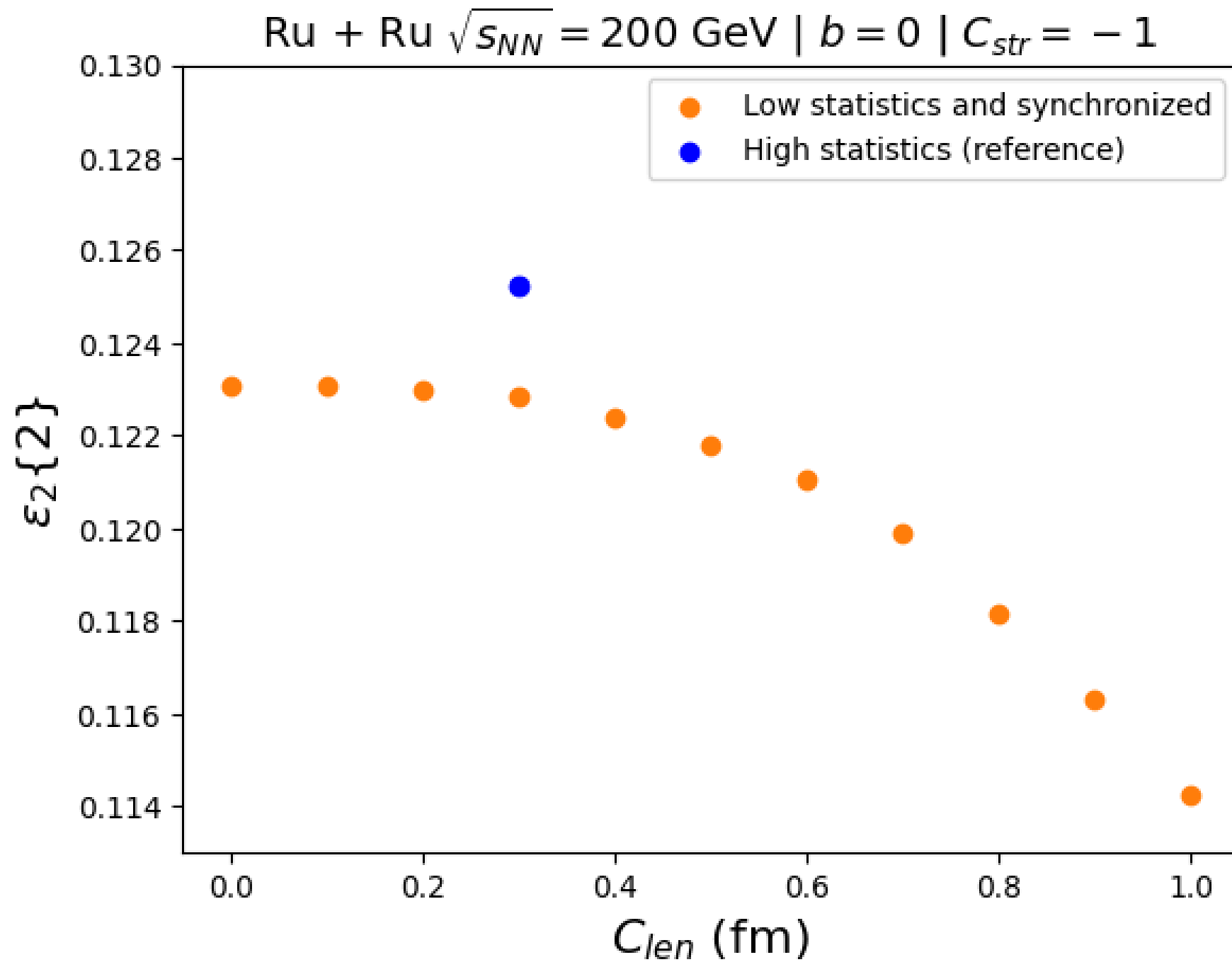


Posterior for the triaxiality parameter

$$R(\theta, \phi) = R[1 + \beta_2(\cos \gamma Y_{2,0}(\theta, \phi) + \sin \gamma Y_{2,2}(\theta, \phi)) + \beta_3 Y_{3,0}(\theta, \phi)]$$



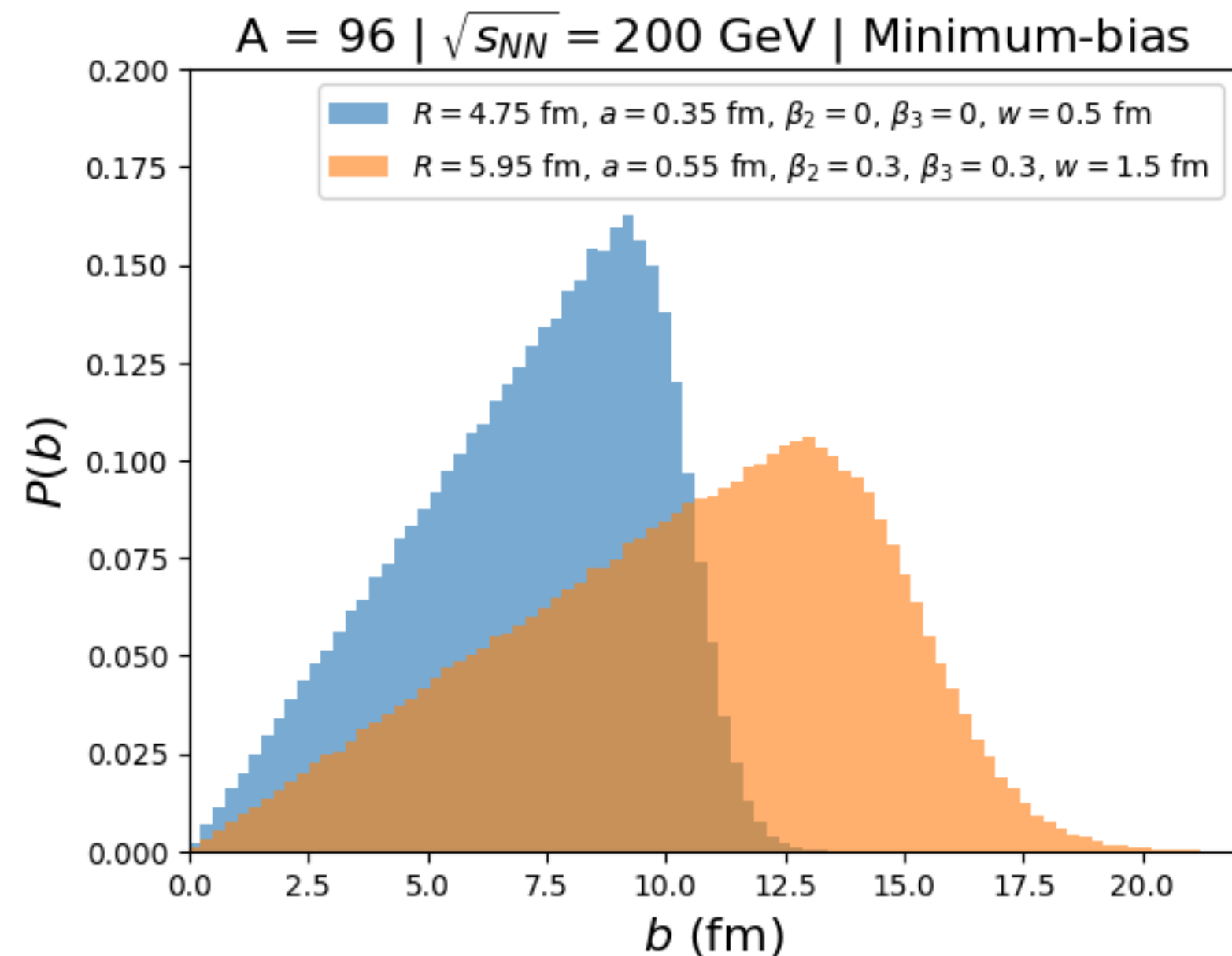
High-statistics reference point: 1D example



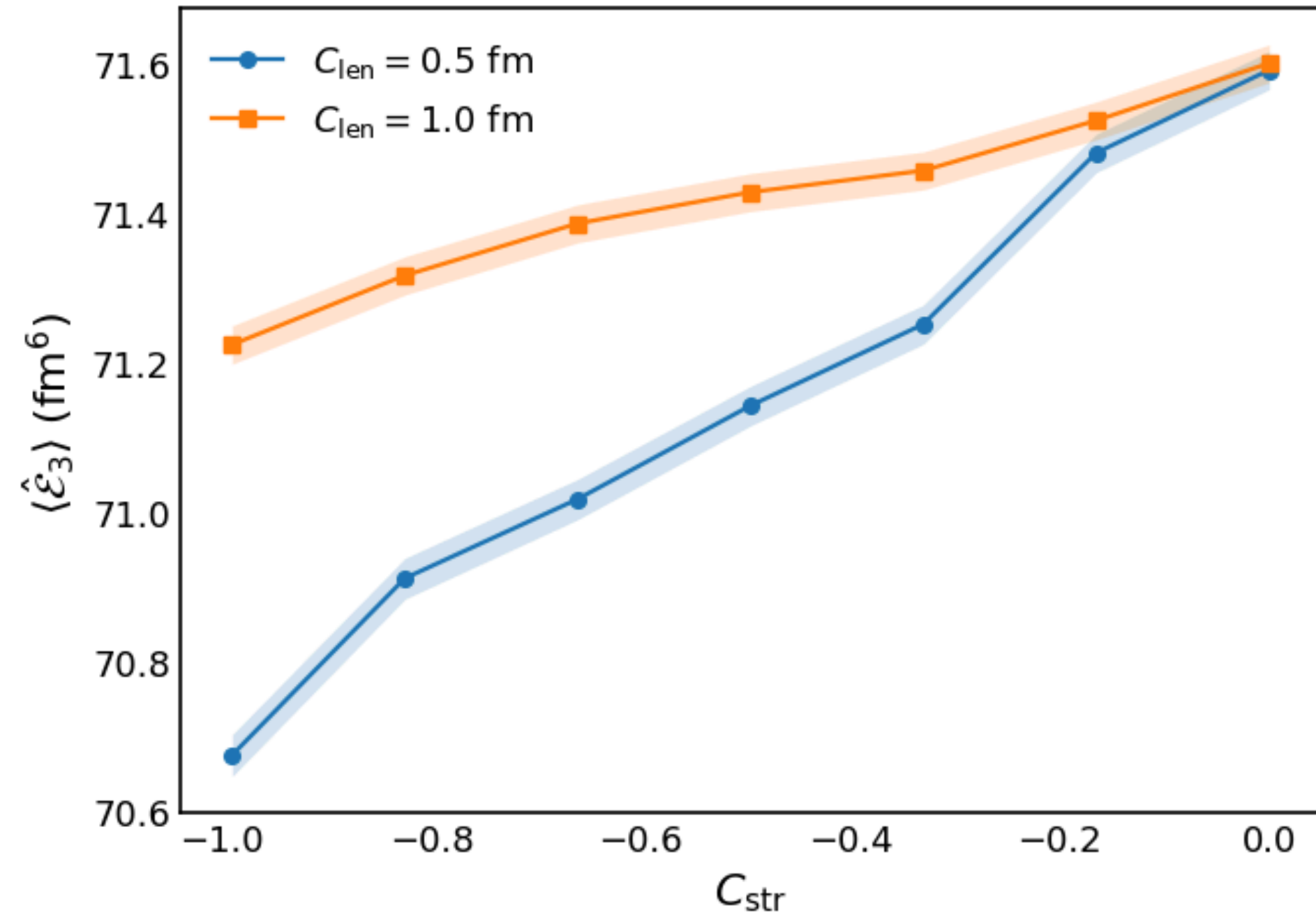
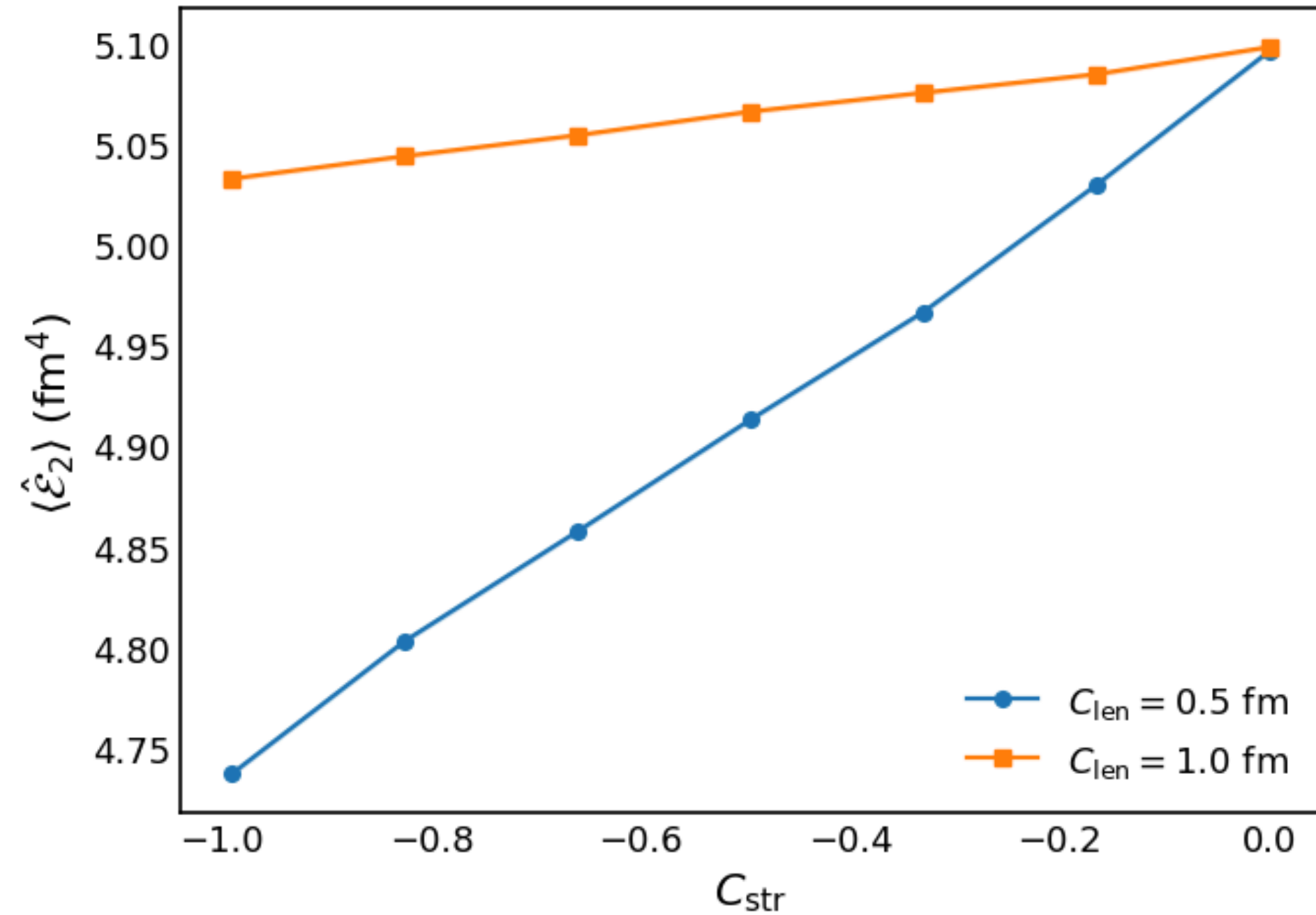
Choosing b_{max} for each design point

Construct an effective radius and expand up to second order in the deformations:

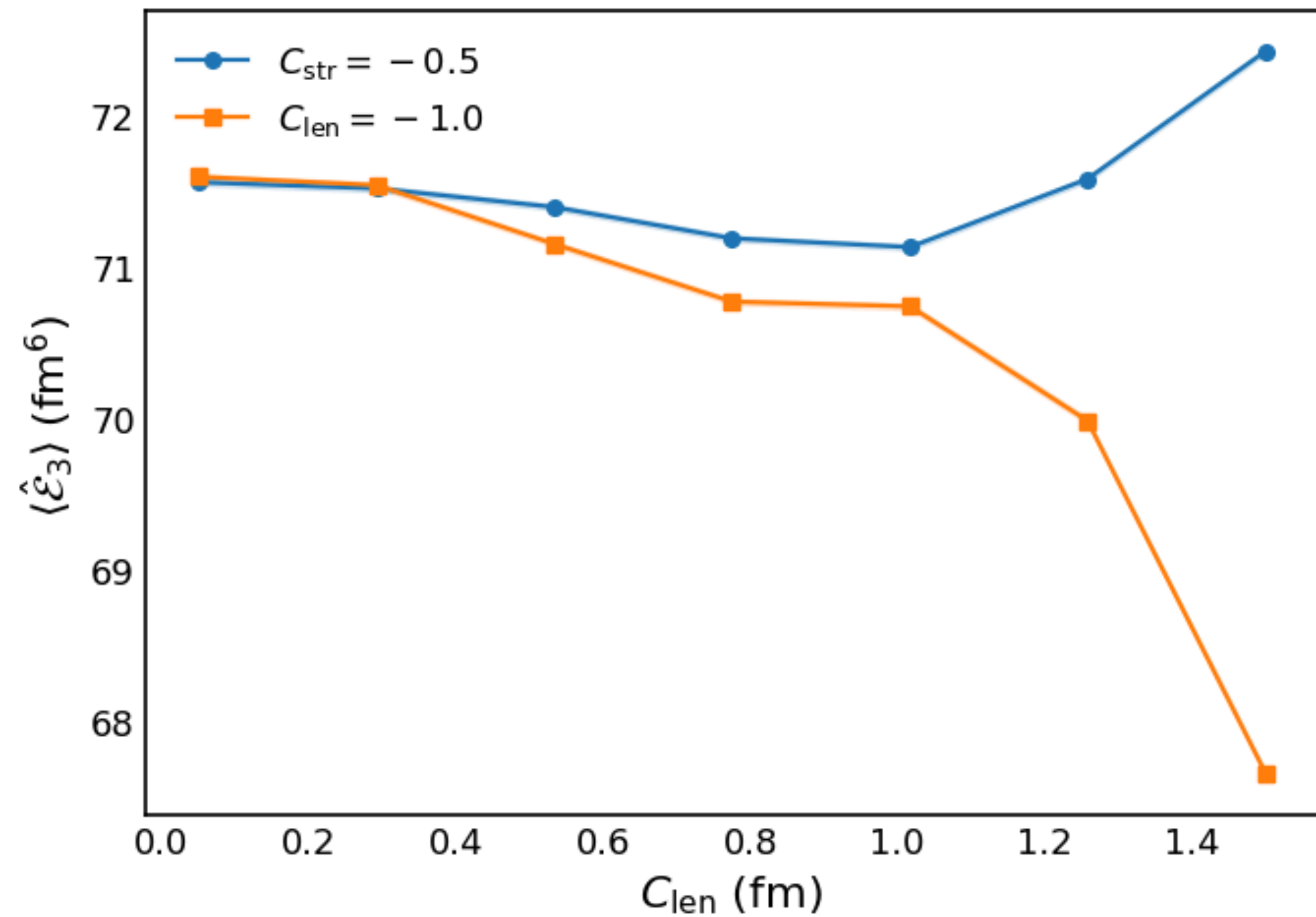
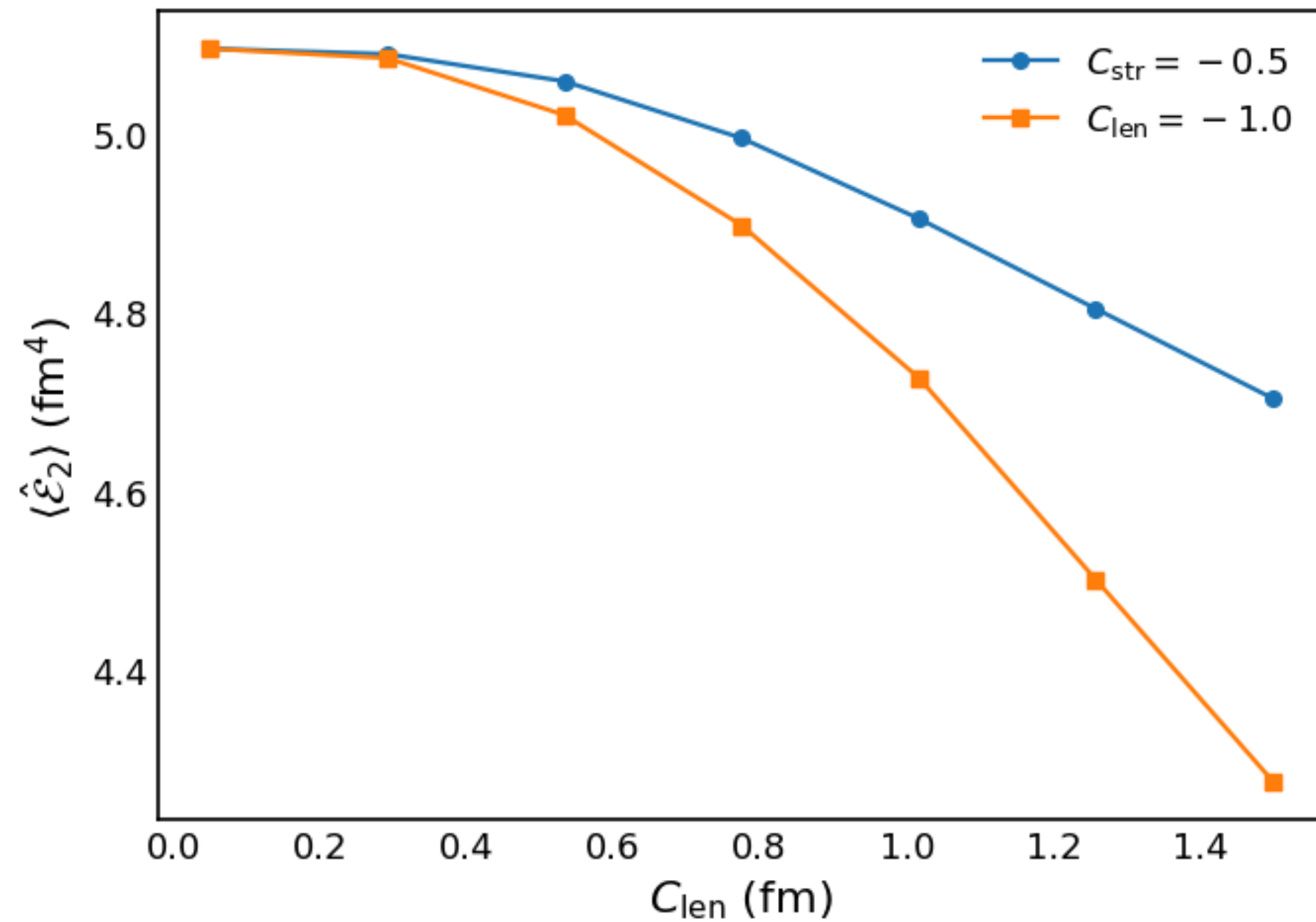
$$b_{\text{max}} = 1.789 \sqrt{R^2 \left[1 + \frac{7}{4\pi} (\beta_2^2 + \beta_3^2) \right] + \frac{7\pi^2}{3} a^2 + w + 0.5} \quad \text{fm}$$



Dependence on SRC parameters



Dependence on SRC parameters



Rigid-rotor with Gaussian density

$$\rho^{(1)}(r, \theta) = \frac{1}{(2\pi)^{3/2} R^3} e^{-\frac{r^2}{2R(\theta)^2}}$$

$$R(\theta) = R[1 + \beta_2 Y_{2,0}(\theta) + \beta_3 Y_{3,0}(\theta)]$$

$$\langle \varepsilon_2^2 \rangle = \frac{1}{A} + \frac{3}{4\pi} \beta_2^2 \quad \langle \varepsilon_3^2 \rangle = \frac{16}{3\pi A} + \frac{2048}{245\pi^3} \beta_3^2$$

One-body term: $\sim \frac{1}{A}$

Two-body/collective term: $\sim \beta_n^2$

Introducing the angular deformations

For deformed nuclei, the nuclear surface is not constant:

$$R \rightarrow R \left(1 + \sum_{l,m} \beta_{l,m} Y_l^m(\theta, \phi) \right)$$

We define a t-parameterized distribution as:

$$\rho(\vec{x}, t) = \rho \left(r - t \sum_{l,m} R \beta_{l,m} Y_l^m \right) \quad (1)$$

such that $t = 0$ represents a spherical distribution and $t = 1$ represents the desired deformed distribution

The shift in position of particles with respect to parameter t is described by a vector field $d\vec{x} = \vec{v}dt$

Positing a solution of the form of a gradient:

$$\vec{v} = \nabla \Phi$$

and inserting in the continuity equation:

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \vec{v}) = 0$$

we obtain an equation for the scalar field:

$$\frac{\partial \rho}{\partial t} + \nabla \rho \cdot \nabla \Phi + \rho \nabla^2 \Phi = 0$$

Decomposing the scalar field into multipole components:

$$\Phi(r, \theta, \phi) = \sum_{l,m} R \beta_{l,m} f_{l,m}(r) Y_l^m(\theta, \phi)$$

Inserting in the differential equation and using Equation (1):

$$f_{l,m}'' + f_{l,m}' \left(\frac{2}{r} + \frac{\rho'}{\rho} \right) - \frac{l(l+1)}{r^2} f_{l,m} - \frac{\rho'}{\rho} = 0$$

The radial part in fact doesn't depend on m

The final shift is given by:

$$\begin{aligned} d\vec{x} &= t \nabla \Phi = t R \sum_{l,m} \beta_{l,m} \nabla f_{l,m}(r) Y_l^m(\theta, \phi) = \\ &= \left[t R \sum_{l,m} \beta_{l,m} f_{l,m}' Y_l^m \right] \hat{r} + \left[t \frac{R}{r} \sum_{l,m} \beta_{l,m} f_{l,m} \frac{\partial}{\partial \theta} Y_l^m \right] \hat{\theta} + \left[t \frac{R}{r \sin \theta} \sum_{l,m} \beta_{l,m} f_{l,m} \frac{\partial}{\partial \phi} Y_l^m \right] \hat{\phi} \end{aligned}$$

Introducing the short range correlations

Pairs of nucleons with relative position $r = |\vec{x}_2 - \vec{x}_1|$ are shifted radially from their centers to a new separation $\tilde{r}(r)$

Conservation of the number of pairs must be satisfied:

$$\int_0^r = dr' r'^2 = \int_0^{\tilde{r}} dr' r'^2 [1 + C(\vec{x}_1, \vec{x}_2)]$$

Symmetric shifts $\longrightarrow d\vec{x}_i = \sum_{i \neq j} \frac{1}{2} (\tilde{r}_{ij} - r_{ij}) \hat{r}_{ij}$

The one-body and two-body densities are probabilities, and must integrate to unit. Sum rule:

$$\int \rho(\vec{x}_1) \rho(\vec{x}_2) C(\vec{x}_1, \vec{x}_2) d^3 x_1 d^3 x_2$$

Add a constant offset to ensure the sum rule is satisfied $\longrightarrow C(\vec{r}) = C_{\text{short}}(\vec{r}) + C_{\infty}$ where:

$$C_{\infty} \approx C_{\text{vol}} \int d^3 x \rho^2(\vec{x})$$

$$C_{\text{vol}} = \int d^3 r C_{\text{short}}(\vec{r})$$

In the case of the step correlation function, the correlation volume is:

$$C_{\text{vol}} = \frac{4\pi}{3} C_{\text{str}} C_{\text{len}}^3$$

and the new separations as a function of the original separations are given by:

$$\tilde{r}(r) = \begin{cases} r(1 + C_{\text{str}} + C_{\infty})^{-1/3}, & r \leq r_{\text{sw}} \\ r \left(\frac{1 - C_{\text{str}} C_{\text{len}}^3 / r^3}{1 + C_{\infty}} \right)^{1/3}, & r < r_{\text{sw}} \end{cases}$$

where:

$$r_{\text{sw}} = C_{\text{len}} (1 + C_{\text{str}} + C_{\infty})^{1/3}$$