

2026 EMMI workshop on light-ion collisions

Cumulants of Mean Transverse Momentum and Elliptic Flow

in the Hydrodynamic Model of Heavy-Ion Collisions

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Based on : Phys. Rev. C, 114(2026) 3, 034910. arXiv:2605.26737

21 September 2026, Heidelberg



The standard mixed correlator $\rho([p_T], v_2^2)$

later denoted $\rho_{1,1}$ in our notation.

INTEGRATED CORRELATOR

Božek, Phys. Rev. C 93 (2016) 044908 (arXiv:1601.04513)

$$\rho([p_T], v_2^2) = \frac{\text{Cov}([p_T], v_2^2)}{\sqrt{\text{Var}([p_T])} \sqrt{\text{Var}(v_2^2)}}$$

A three-particle correlator: the numerator correlates v_2^2 with $[p_T]$; the denominator uses the dynamical (self-correlation-removed) variance of $[p_T]$.

Introduced by Božek (2016)

The standard tool for studying collective-flow correlations between mean transverse momentum and harmonic flow. Measurements available across small and large collision systems.

p_T -DIFFERENTIAL GENERALIZATIONS

Parida, Samanta & Ollitrault, Phys. Lett. B 868 (2025) 139729 (arXiv:2506.18690)

$$v_{02}(p_T) = \frac{\text{Cov}(n(p_T), v_2^2)}{\langle n(p_T) \rangle \langle v_2^2 \rangle}$$

differential via the normalised p_T -spectrum $n(p_T)$.

Samanta & Božek, Phys. Rev. C 109 (2024) 064910 (arXiv:2308.11565)

$$\rho([p_T], V_2 V_2^*(p_T)) = \frac{\text{Cov}([p_T], V_2 V_2^*(p_T))}{\sqrt{\text{Var}([p_T])} \sqrt{\text{Var}(V_2 V_2^*(p_T))}}$$

differential via complex flow vector $V_2(p_T)$.

No experimental measurement available yet.

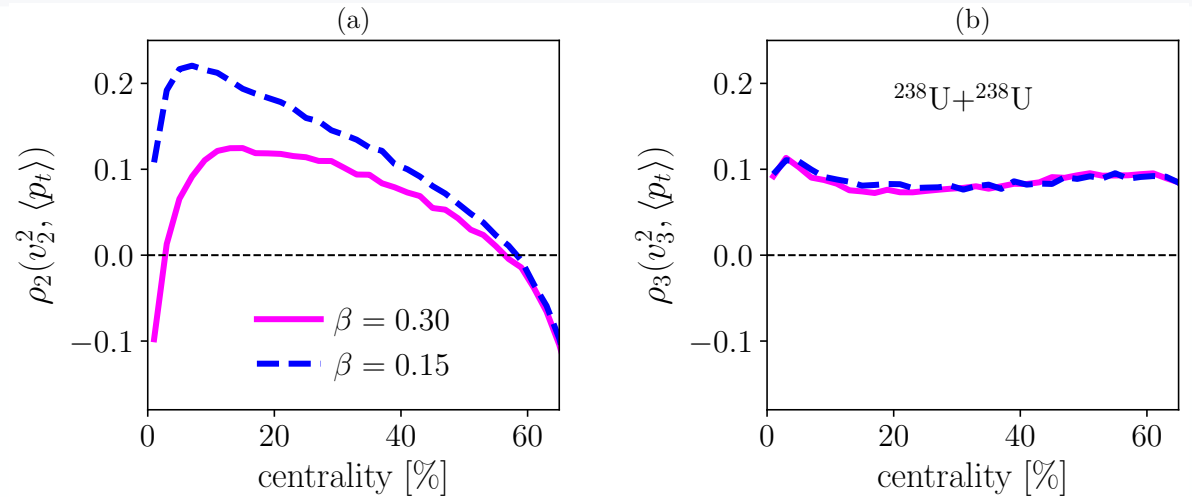
Why does $\rho([p_T], v_2^2)$ matter?



Nuclear shape

Great sensitivity to the quadrupole deformation of the colliding nuclei — used to constrain β .

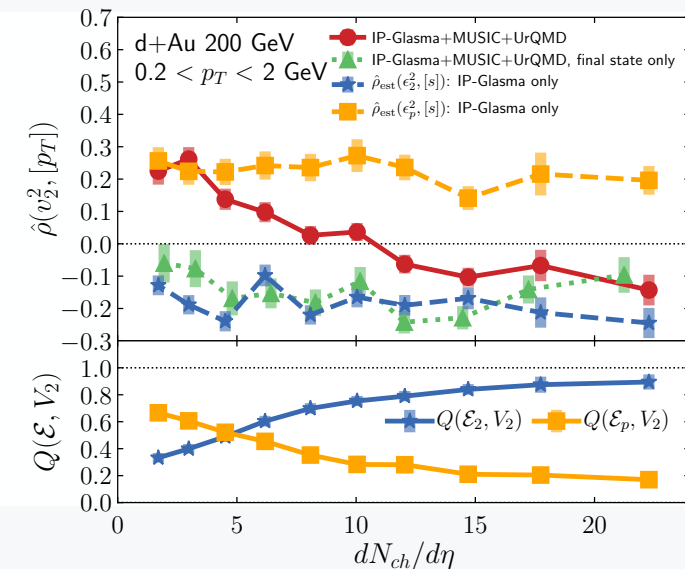
Giaccalone, Phys. Rev. C 102 (2020) 024901 (arXiv:2004.14463)



Initial-state momentum anisotropies

A hybrid IP-Glasma+MUSIC+UrQMD calculation predicts a sign change with multiplicity in p+Au, d+Au, p+Pb — absent without initial-state momentum anisotropies.

Giaccalone, Schenke & Shen Phys. Rev. Lett. 125 (2020) 192301 (arXiv:2006.15721)



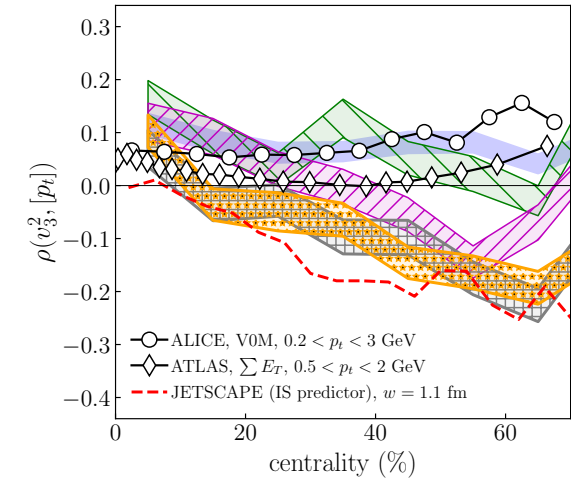
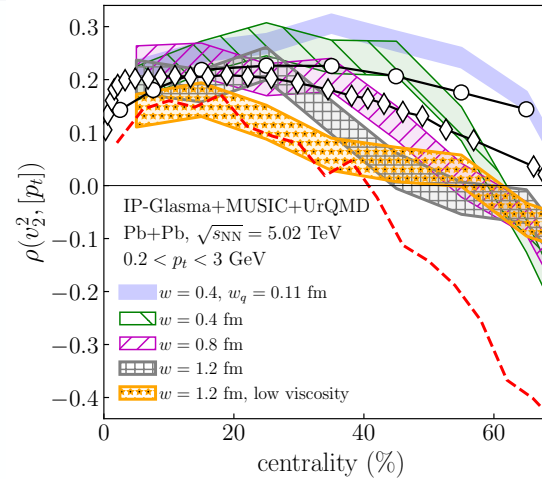
Why does $\rho([p_T], v_2^2)$ matter?



Nucleon substructure

Distinct sensitivity to nucleon size in off-central collisions; existing data favor a transverse length scale of about 0.5 fm.

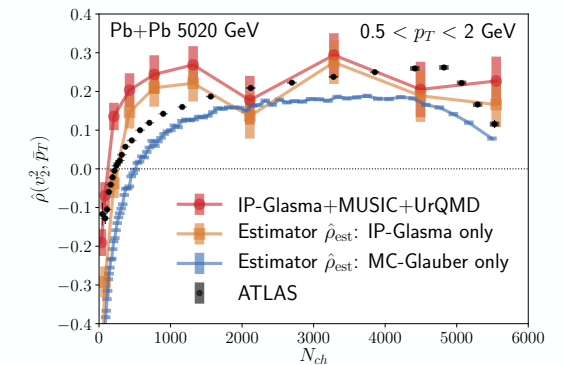
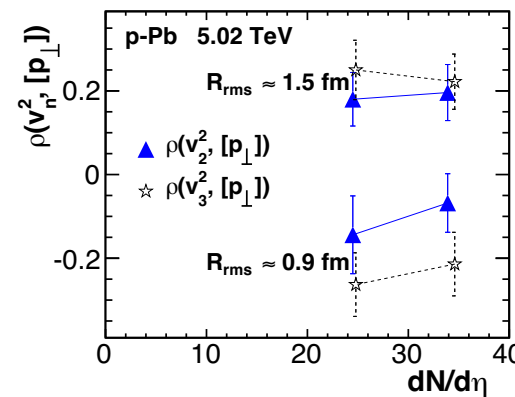
Giacalone, Schenke & Shen, Phys. Rev. Lett. 128 (2022) 042301 (arXiv:2111.02908)



Discriminates the initial state model

Sensitive to the mechanism of initial energy deposition in p+Pb ([arXiv:1601.04512](#)), and to MC-Glauber vs. IP-Glasma differences in Pb+Pb at 5.02 TeV.

Schenke, Shen & Teaney, Phys. Rev. C 102 (2020) 034905 (arXiv:2004.00690)



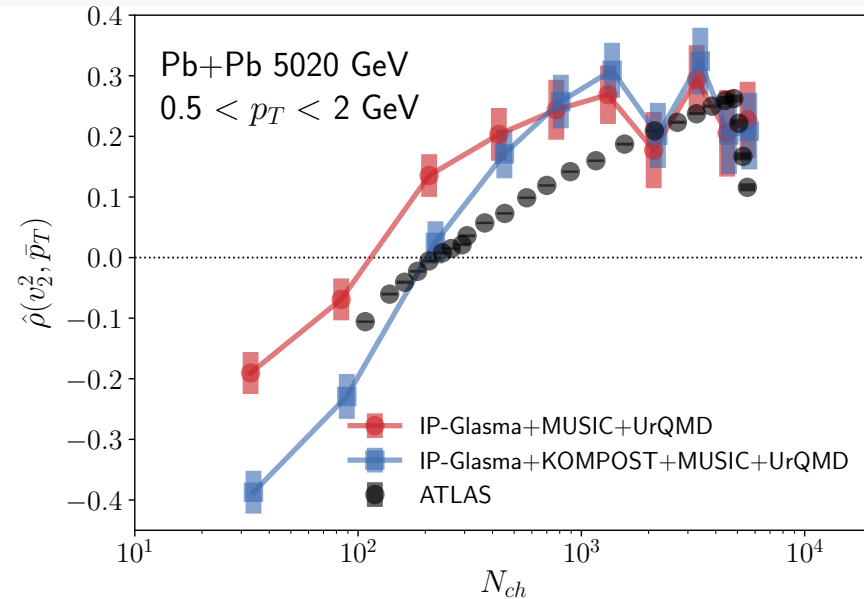
Why does $\rho([p_T], v_2^2)$ matter?



Effect of the pre-equilibrium stage

The pre-equilibrium contribution to $\rho([p_T], v_2^2)$ is larger for smaller multiplicities, where the subsequent hydrodynamic evolution is shorter and has less time to erase the imprint of the earliest stage.

Schenke, Shen & Teaney, Phys. Rev. C 102 (2020) 034905 (arXiv:2004.00690)



Taken together

$\rho([p_T], v_2^2)$ is sensitive to nuclear shape, initial-state momentum anisotropies, nucleon substructure, the initial-state model itself, and the pre-equilibrium stage — making it one of the richest observables in heavy-ion collisions.

A natural question

$\rho([p_T], v_2^2)$ is useful — and it's the **first-order** cumulant.

What do the higher orders tell us — and how are they related?

1

Do higher-order cumulants carry independent information, or are they fixed by the lowest order?

2

Can they be predicted from simple initial-state estimators — or do they require full hydrodynamic simulations?

3

What new experimental handles could they give us — e.g. on non-flow effects?

→ This motivates the systematic study of higher-order $[p_T] - v_2^2$ cumulants presented here.

Event observables and the correlation coefficient

Parida & Božek, Phys. Rev. C (arXiv:2605.26737)

Nielsen et. al., Eur. Phys. J. C 85 (2025) 1016 (arXiv:2504.03044)

Event-by-event quantities

$$[p_T]$$

mean transverse
momentum per
particle of an event

$$V_n = v_n e^{-in\psi_n}$$

complex n -th order flow
vector; magnitude v_n ,
event-plane angle ψ_n .

$$\delta p_T, \delta v_n^2$$

deviations from the
ensemble (event)
average

Moment generating function

$$G[t, s, s^*] = \sum_{i,k,l=0}^{\infty} \langle [p_T]^i (V_n)^k (V_n^*)^l \rangle \frac{t^i s^k s^{*l}}{i!k!l!}$$

Cumulant generating function

$H[t, s, s^*] = \ln(G[t, s, s^*])$ generates all cumulants

$\langle [p_T]^i (V_n)^k (V_n^*)^l \rangle_c$ by differentiation.

STANDARD CORRELATOR : FIRST ORDER CUMULANT

$$\langle [p_T] V_n V_n^* \rangle_c = \langle \delta p_T \delta v_n^2 \rangle$$

Covariance of $[p_T]$ and v_n^2

Cumulants beyond covariance

$$\langle [p_T]^2 V_n V_n^* \rangle_c = \langle (\delta p_T)^2 \delta v_n^2 \rangle$$

$$\langle [p_T]^3 V_n V_n^* \rangle_c = \langle (\delta p_T)^3 \delta v_n^2 \rangle - 3 \langle (\delta p_T)^2 \rangle \langle \delta p_T \delta v_n^2 \rangle$$

$$\langle [p_T]^3 V_n^2 V_n^{*2} \rangle_c = \langle (\delta p_T) (\delta v_n^2)^2 \rangle - 2 \langle v_n^2 \rangle \langle \delta p_T \delta v_n^2 \rangle$$

built order-by-order from δp_T and δv_n^2 via the generating-function expansion.

Two families of scaled cumulants

Removing overall magnitudes of $[p_T]$ and v_n^2 by normalisation.



Higher order in $[p_T]$

$$\rho_{k,1}([p_T], v_n^2) = \frac{\langle [p_T]^k v_n^2 \rangle_c}{\sigma_{p_T}^k \langle v_n^2 \rangle}$$

$\rho_{k,1}$ for $k = 1, 2, 3$ — normalized by $\sigma_{p_T}^k$, the standard-deviation power that matches the correlation coefficient dimensionless.

Probes of p_T fluctuations correlated with v_n^2 .



Higher order in v_n^2

$$\rho_{1,k}([p_T], v_n^2) = \frac{\langle [p_T] V_n^k V_n^{*k} \rangle_c}{\sigma_{p_T} c_n\{2k\}}$$

$\rho_{1,k}$ for $k = 1, 2, 3$ — normalized by the 2k-particle

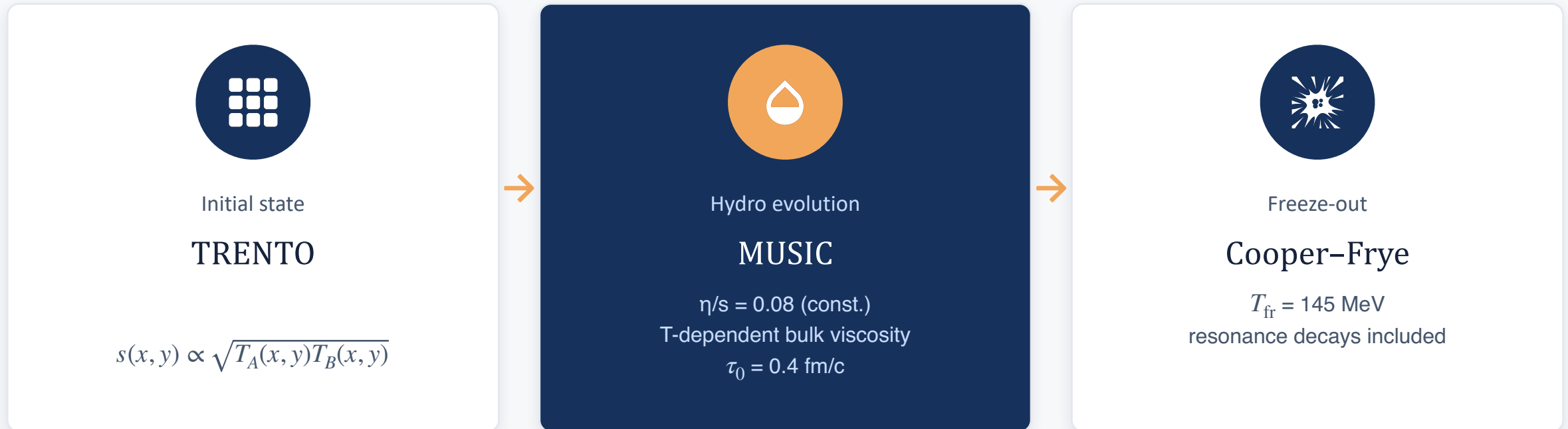
flow cumulant $c_n\{2k\} = \langle V_n^k V_n^{*k} \rangle$

$$c_n\{2\} = \langle v_n^2 \rangle$$

$$c_n\{4\} = \langle v_n^4 \rangle - 2\langle v_n^2 \rangle^2$$

$$c_n\{6\} = \langle v_n^6 \rangle + 12\langle v_n^2 \rangle^3 - 9\langle v_n^2 \rangle \langle v_n^4 \rangle$$

Relativistic viscous hydrodynamic model



SYSTEM

Pb+Pb at $\sqrt{s_{NN}} = 5.02$ TeV

EQUATION OF STATE

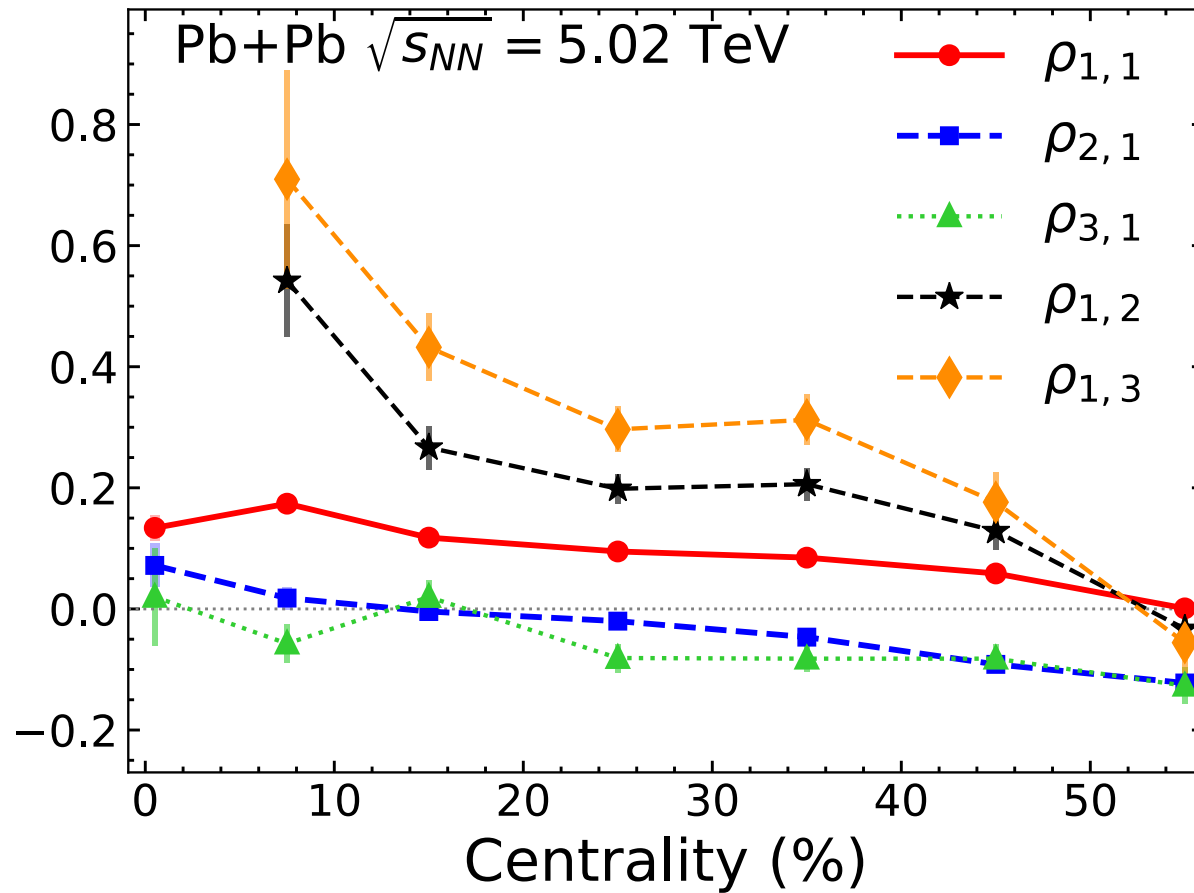
lattice QCD ($\mu_B = 0$), matched to hadron
resonance gas

MULTIPLICITY CORRECTION

$$\delta\hat{O} = \delta O - \frac{\text{Cov}(O, N_{\text{ch}})}{\sigma_{N_{\text{ch}}}^2} \delta N_{\text{ch}}$$

removes
multiplicity-fluctuation bias

Scaled cumulants vs. centrality



$$\rho_{1,1}([p_T], v_2^2) > 0$$

the standard, well-measured correlator;
positive across centrality, largest in mid-
central collisions.

$$\rho_{2,1} \text{ and } \rho_{3,1}$$

cumulants of 2nd/3rd order in $[p_T]$ are small
and turn negative in peripheral collisions.

$$\rho_{1,2} \text{ and } \rho_{1,3}$$

cumulants of 2nd/3rd order in v_2^2 are the
largest in magnitude, and share the sign of
 $\rho_{1,1}$.

Relation between the cumulant of different order in v_2^2

STEP 1 — Gaussian model of elliptic-flow distribution

In the Gaussian model, the flow vector V_2 fluctuates as a 2D Gaussian of width σ_v in the complex plane about it's mean $\langle V_2 \rangle = v_0$. The magnitude $v_2 = |V_2|$ then follows a Bessel-Gaussian distribution.

$$p(v_2^2) = \exp\left(-\frac{v_2^2 + v_0^2}{2\sigma_v^2}\right) I_0\left(\frac{v_0\sqrt{v_2^2}}{\sigma_v^2}\right)$$

Voloshin et. al. , Phys. Lett. B 659 (2008) 537-541 (arXiv:0708.0800)

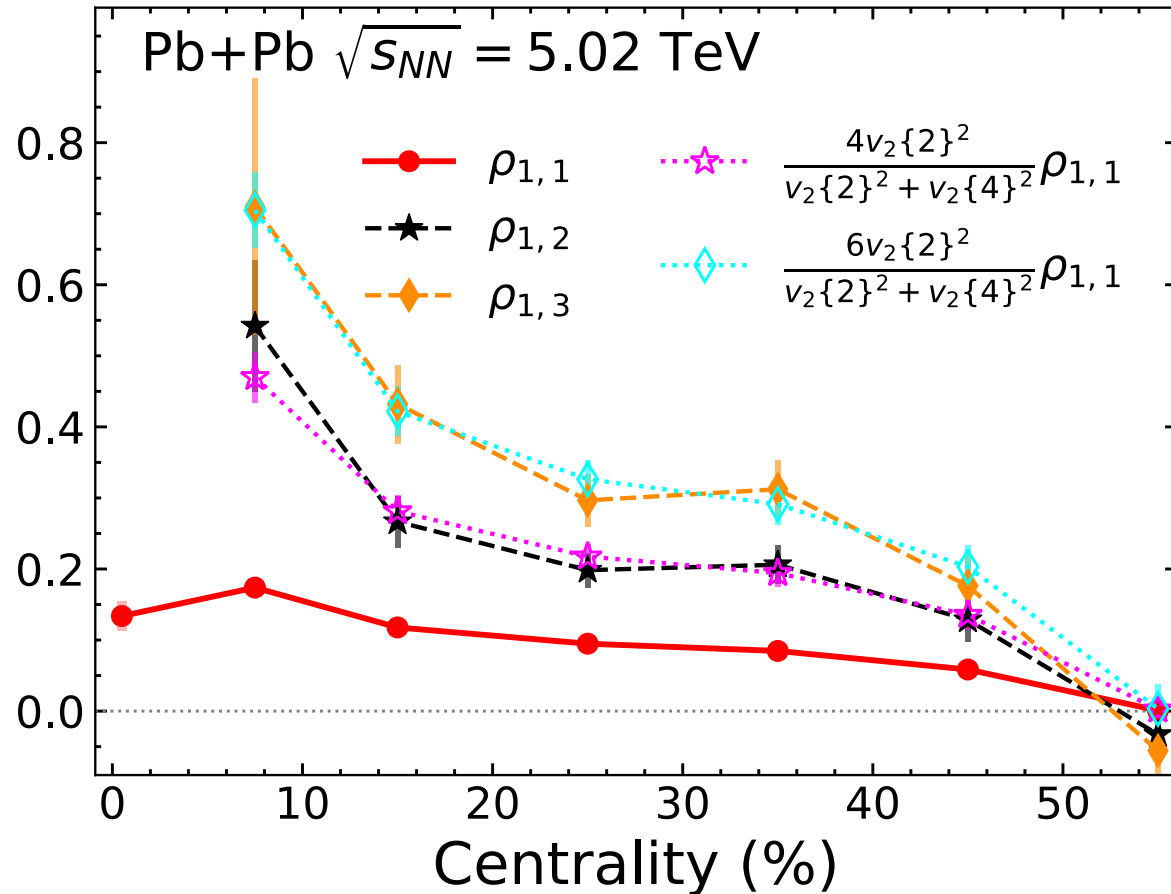
STEP 2 — correlate with $[p_T]$

Add a Gaussian in $[p_T]$ coupled to v_2^2 with a single strength parameter κ :

$$P([p_T], v_2^2) = A \exp\left(-\frac{([p_T] - \langle p_T \rangle)^2}{2\sigma_p^2} + \frac{\kappa([p_T] - \langle p_T \rangle)(v_2^2 - v_0^2 - 2\sigma_v^2)}{\sigma_v\sigma_p}\right) p(v_2^2)$$

Analytically calculate the scaled cumulants $\rho_{1,1}, \rho_{1,2}, \rho_{1,3}$ in leading order in κ - and, because they share the single parameter κ , this fixed ratios between them.

The predicted scaling relations hold in hydro

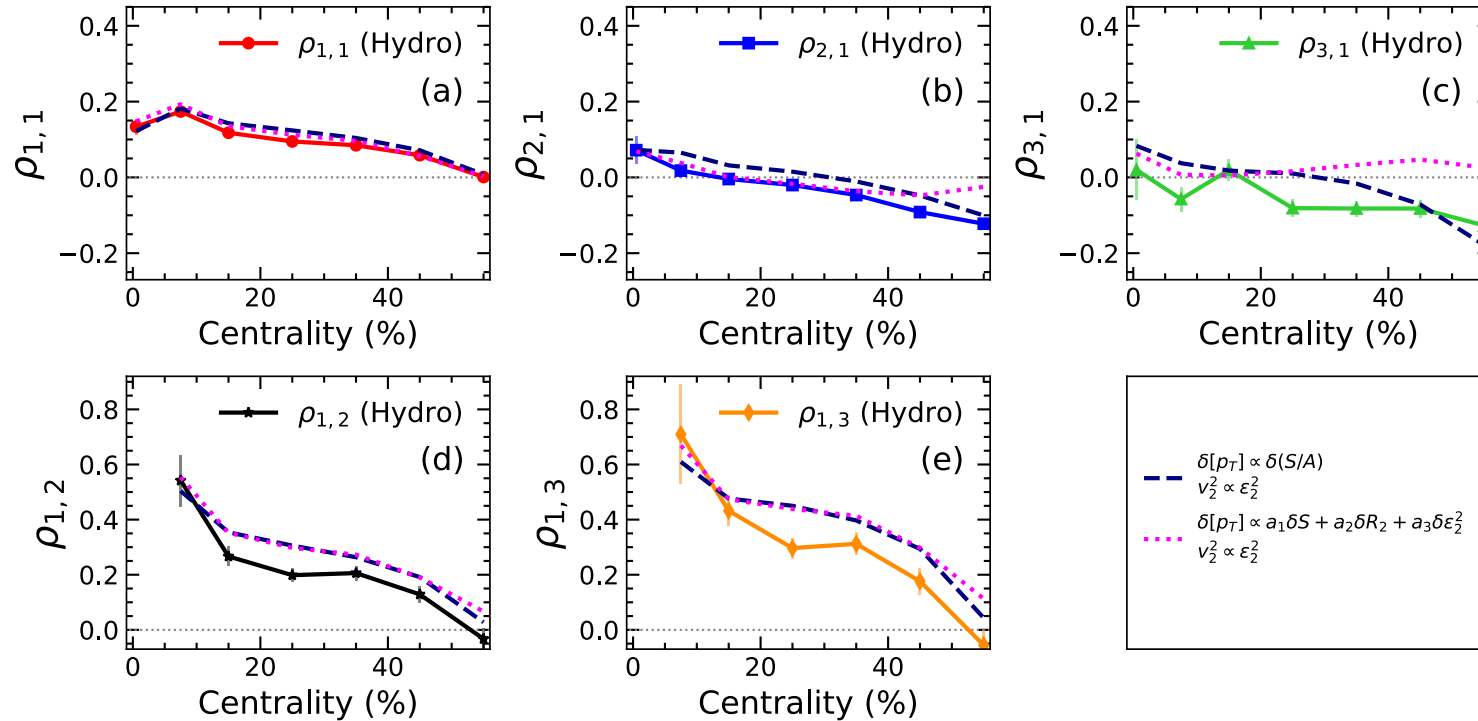


LEADING-ORDER SCALING

$$\rho_{1,2} \simeq \frac{4\langle v_2^2 \rangle}{(\langle v_2^2 \rangle + v_2\{4\}^2)}\rho_{1,1} \quad \rho_{1,3} \simeq \frac{3}{2}\rho_{1,2}$$

- Dotted lines = predictions from $\rho_{1,1}$ alone, using the scaling relations derived from the Bessel–Gaussian ansatz.
- Agreement is close across the full centrality range — the second- and third-order v_2^2 cumulants are quantitatively tied to the lowest-order correlator.
- By contrast, cumulants of higher order in $[p_T]$ ($\rho_{2,1}$ and $\rho_{3,1}$) are NOT captured by this simple ansatz — they require independent skewness/kurtosis information in the $P([p_T], v_2^2)$.

Can initial-state moments predict the cumulants?



PREDICTOR 1 — simplest estimator

$$\delta[p_T] \propto \delta\left(\frac{S}{A}\right), \quad v_2^2 \propto \epsilon_2^2$$

*no fit parameters
Based on entropy S ,
transverse elliptic area A ,
eccentricity ϵ_2^2 .*

PREDICTOR 2 — linear fit

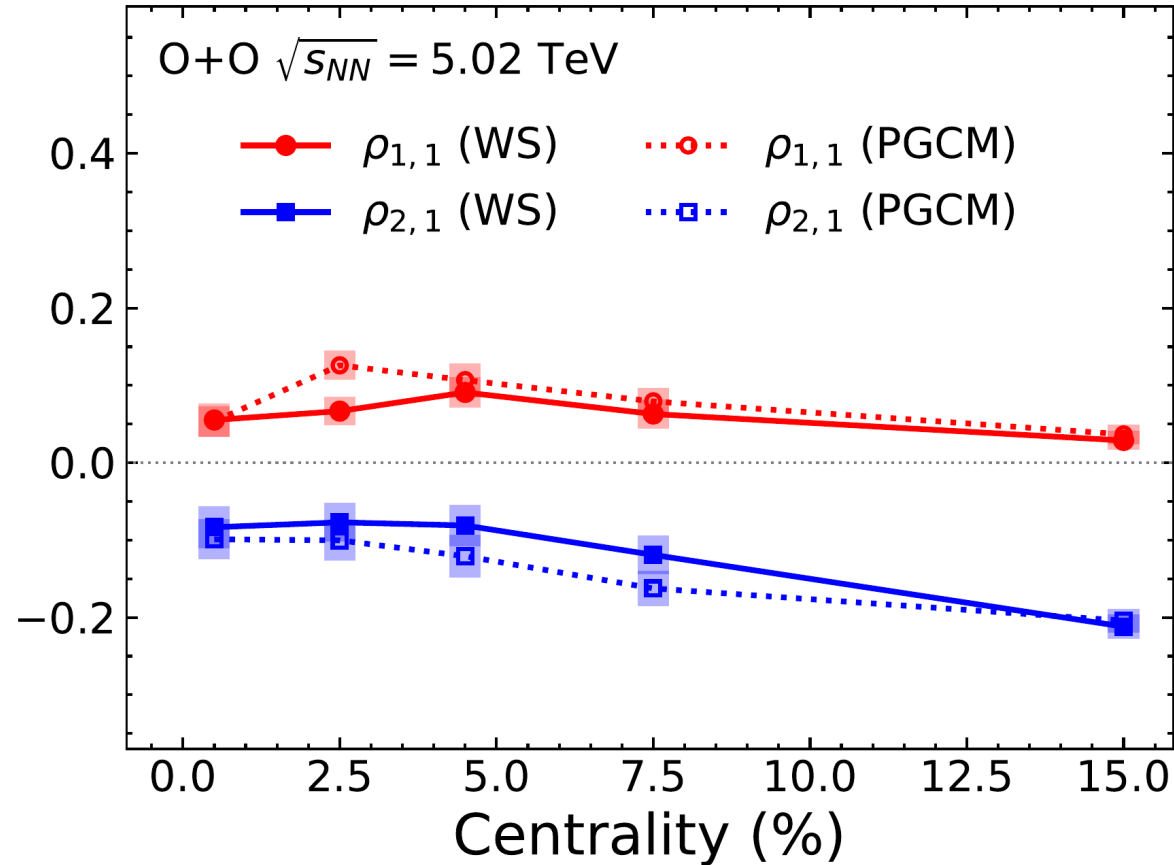
$$\delta[p_T] \propto a_1 \delta S + a_2 \delta R_2 + a_3 \delta \epsilon_2^2$$

fit per centrality bin to maximize correlation with the hydro result.

Reproduces $\rho_{1,1}, \rho_{1,2}, \rho_{1,3}, \rho_{2,1}$ qualitatively — but not $\rho_{3,1}$, whose centrality dependence is too steep.

O+O: higher-order cumulants

The same framework applied to a light system, where the collision geometry is set by the structure of the ^{16}O nucleus.



TWO CONFIGURATIONS OF THE ^{16}O NUCLEUS

Hydrodynamic predictions for O+O at $\sqrt{s_{NN}} = 5.02$ TeV, using a Woods–Saxon density (WS) and an ab-initio PGCM density that carries the genuine correlations and clustering of ^{16}O .

Measurements could be compared to model result of higher order cumulants to test hydrodynamic behaviour in such small system.

Very small difference between WS and PGCM configuration results in central O+O collisions in the higher-order cumulant $\rho_{2,1}$. Can we get structure information with higher order cumulant $\rho_{2,1}, \rho_{3,1}$? More investigation required with higher statistics.

Summary

1 Higher-order cumulants computed

First hydrodynamic calculation of scaled cumulants $\rho_{k,1}$ and $\rho_{1,k}$ between $[p_T]$ and v_2^2 , up to $k = 3$, for Pb+Pb at 5.02 TeV.

2 Simple analytic relations derived

A Bessel–Gaussian ansatz for the joint $[p_T] - v_2^2$ distribution predicts $\rho_{1,2} \simeq \frac{4\langle v_2^2 \rangle}{(\langle v_2^2 \rangle + v_2\{4\}^2)} \rho_{1,1}$ and $\rho_{1,3} \simeq \frac{3}{2} \rho_{1,2}$

— confirmed by the hydro results.

3 Initial-state predictors work — partially

Simple estimators from the initial entropy profile reproduce most cumulants qualitatively, but not $\rho_{3,1}$; full hydro is still needed for quantitative precision.

4 An experimental target

The derived relations, and the second-order flow cumulants $\rho_{2,1}$ (req. 4 particle correlation), $\rho_{1,2}$ (req. 5 particle correlation) are directly testable — offering a new handle on non-flow effects and the collective origin of $[p_T] - v_2^2$ correlations.

Thank you

Questions and discussion welcome

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Backup

