

A quantum fluid of 10 atoms

Uri Sharell, EMMI Workshop: Light-Ion Collisions 24/09/2026

Work in preparation with Tilman Enss, Aleksas Mazeliauskas, Toshali Mitra and Manuel Mützel

Part of ISOQUANT ABC Project with groups of Selim Jochim and Silvia Masciocchi



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Collaborators



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Part of larger collaborative project with groups
of Selim Jochim and Silvia Masciocchi

1. What is a fluid ?



On the rheology of cats

M.A. Fardin^{1, 2, 3, *}

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CNRS UMR 5672, 46 Allée d'Italie, 69364 Lyon cedex 07, France.*

²*The Academy of Bradylogists.*

³*Member of the Extended McKinley Family (EMF).*

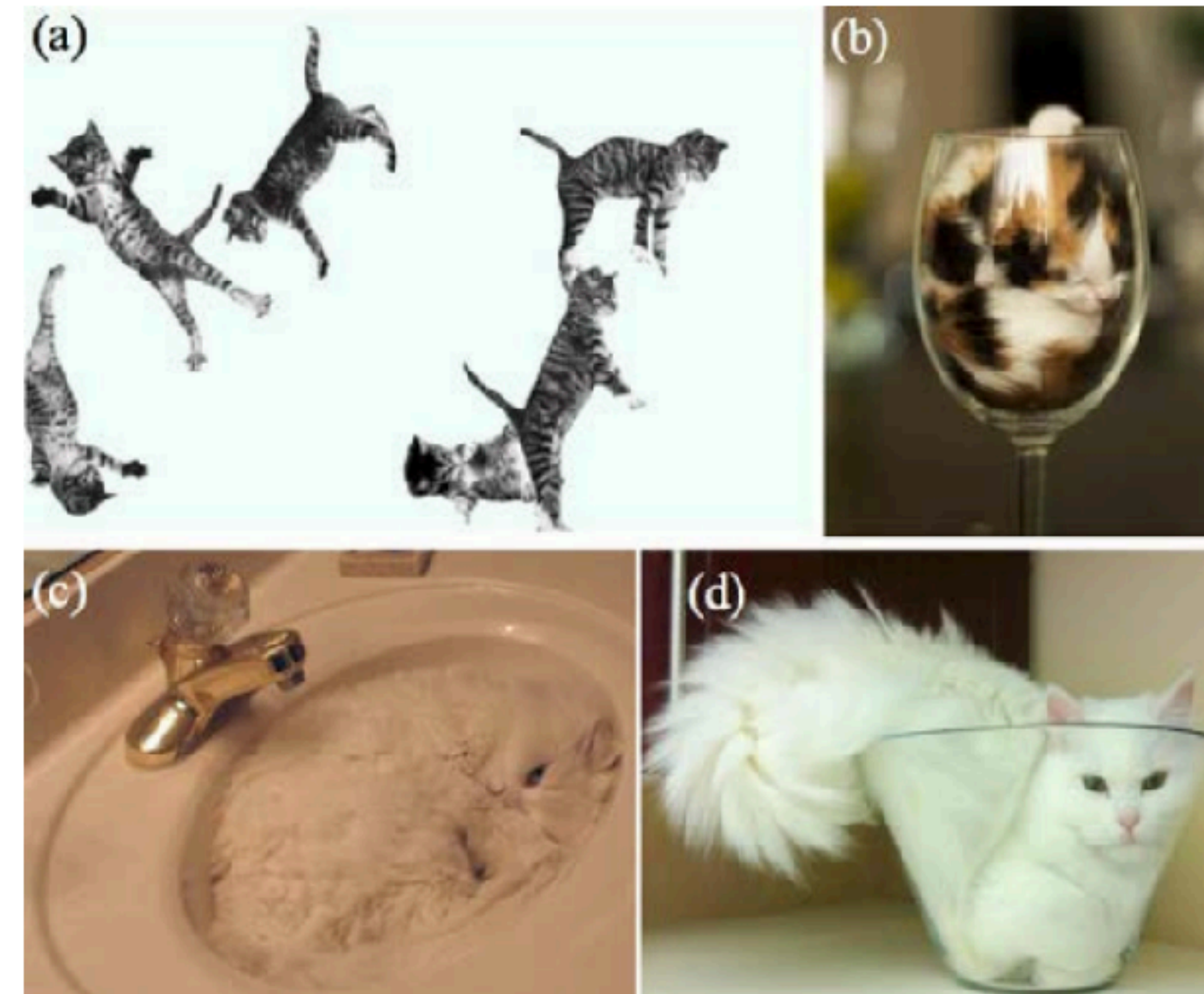
(Dated: July 9, 2014)

IgNobel Prize 2017

In this letter I highlight some of the recent developments around the rheology of *Felis catus*, with potential applications for other species of the felidae family. In the linear rheology regime many factors can enter the determination of the characteristic time of cats: from surface effects to yield stress. In the nonlinear rheology regime flow instabilities can emerge. Nonetheless, the flow rate, which is the usual dimensional control parameter, can be hard to compute because cats are active rheological materials.

παντα ρει! Everything flows! This famous aphorism used to characterize Heraclitus' thought is also the motto of *rheology*. "Everything flows and nothing abides; everything gives way and nothing stays fixed." a recipe for insubordination actually from Simplicius and Plato. Everything flows? Well, it depends on the definition of a *flow*; if sufficiently general, there is no doubt that there are no exceptions to the rule! What is a flow? What is a fluid? As pointed out from the start by Reiner, the essential value of rheology is to recognize that states of matter are a matter of time(s). The first time, is a *time of observation T*. What is true today may not be true tomorrow. Time over time, one day 49, the next 50.

Historically, the popular distinction between states of matter has been made based on qualitative differences in bulk properties. Solid is the state in which matter maintains a fixed volume and shape; liquid is the state



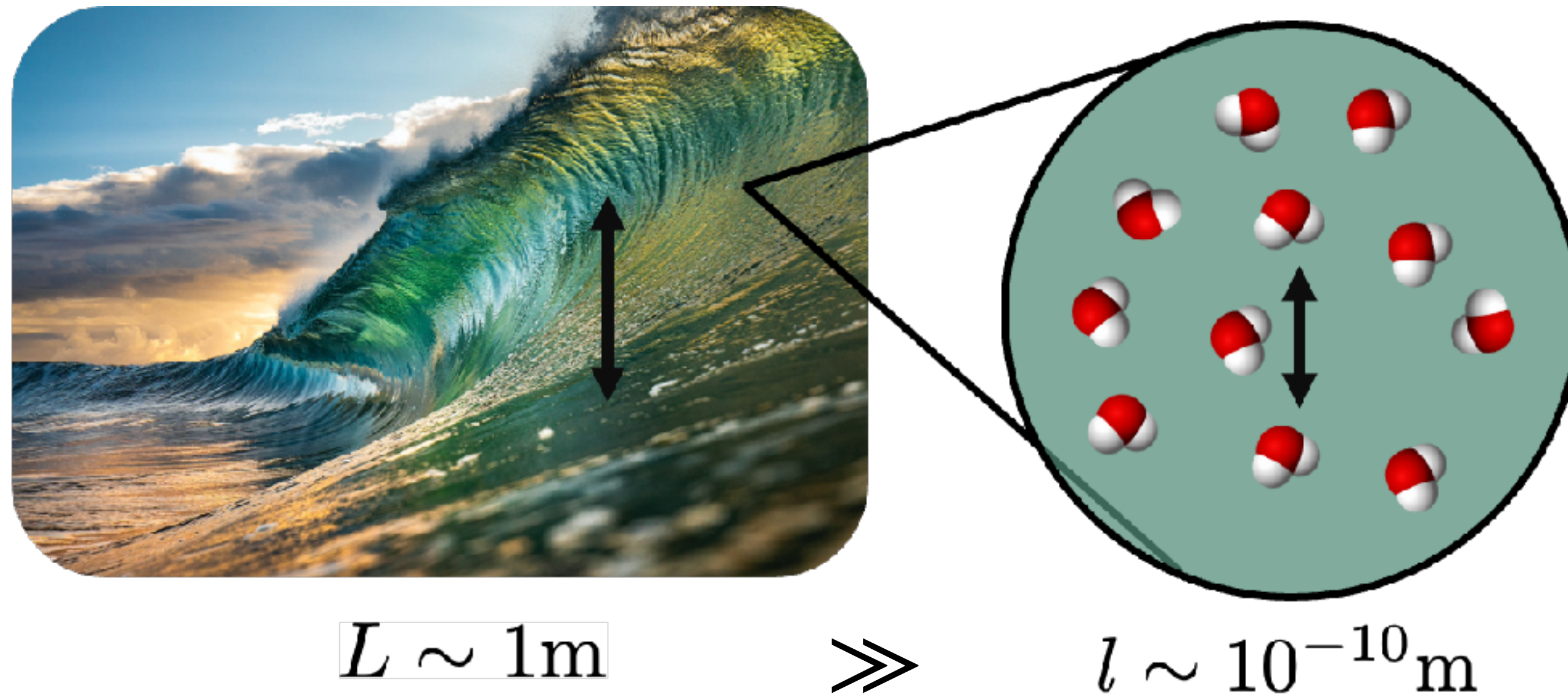
Elliptic flow as a measure of collectivity

- What is a fluid? “Something that obeys fluid dynamics”

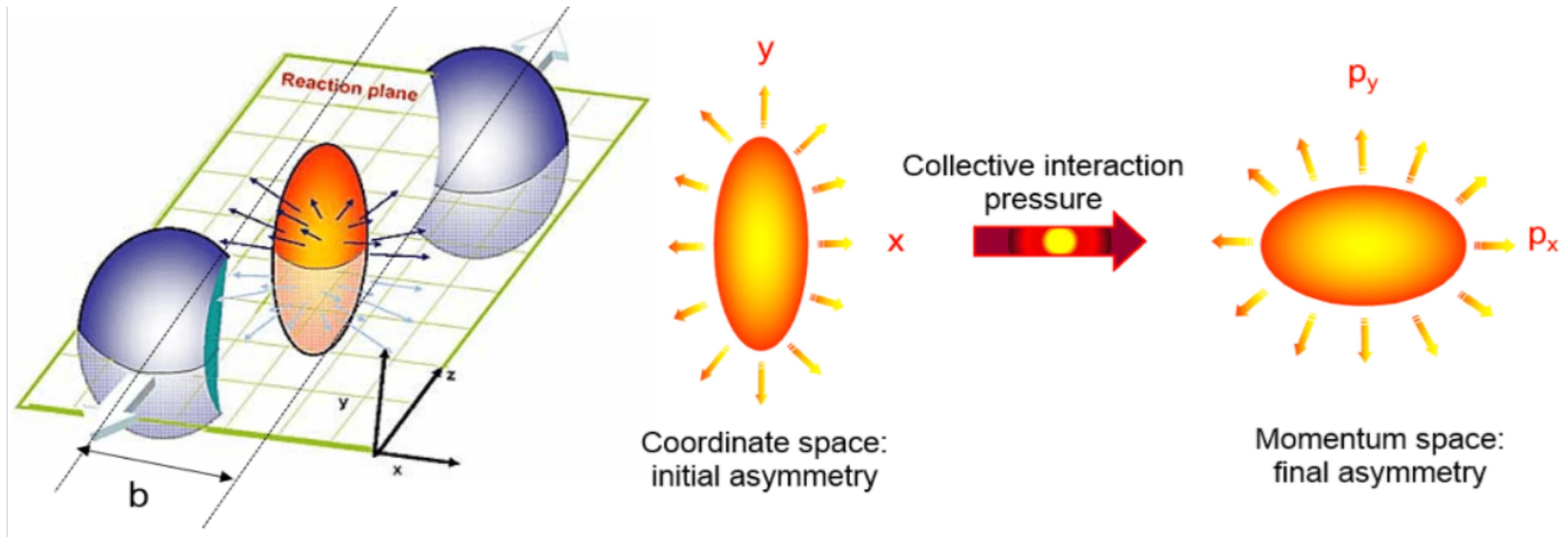
$$\frac{\partial \rho}{\partial t} + \nabla \cdot j = 0$$

$$\frac{\partial u}{\partial t} + u \cdot \nabla u = -\frac{\nabla P}{\rho} + \text{Viscous corrections}$$

$$(\nabla_\mu T^{\mu\nu} = 0)$$

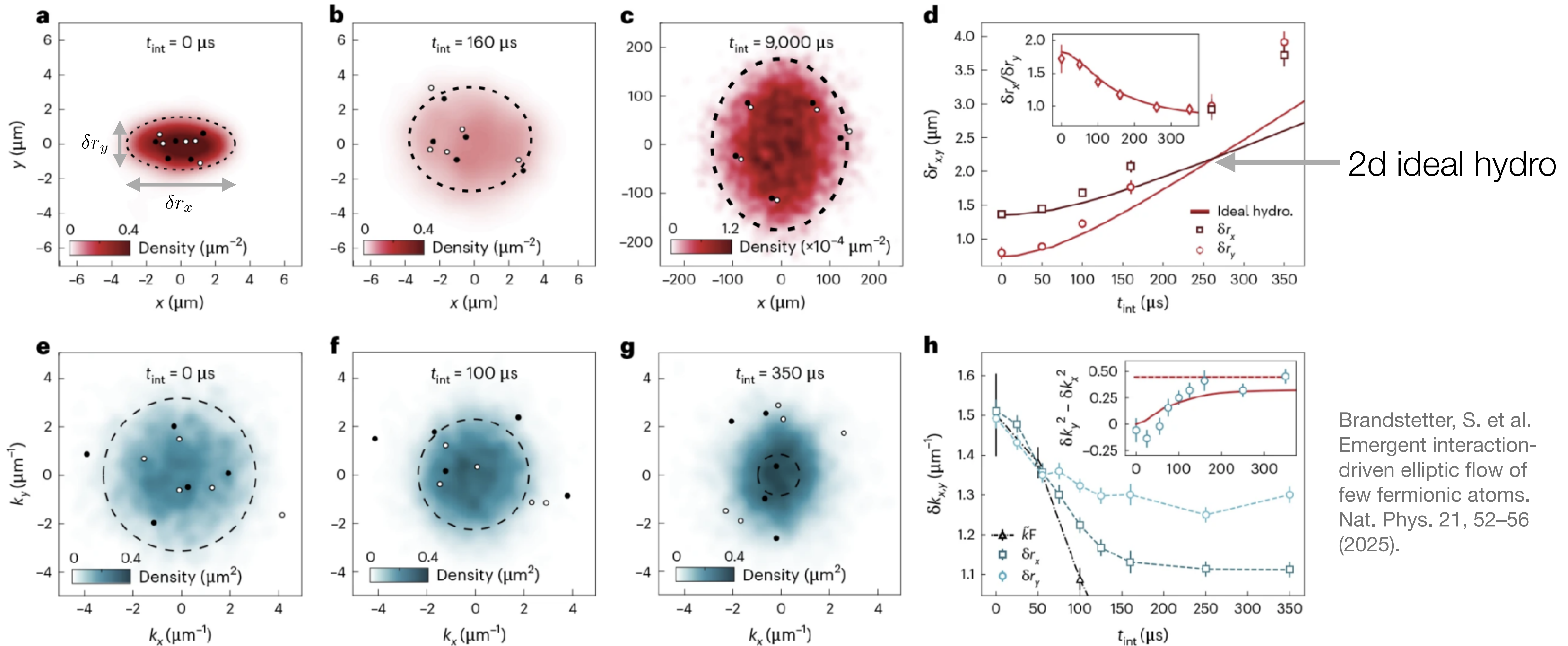


Heavy ions, elliptic flow



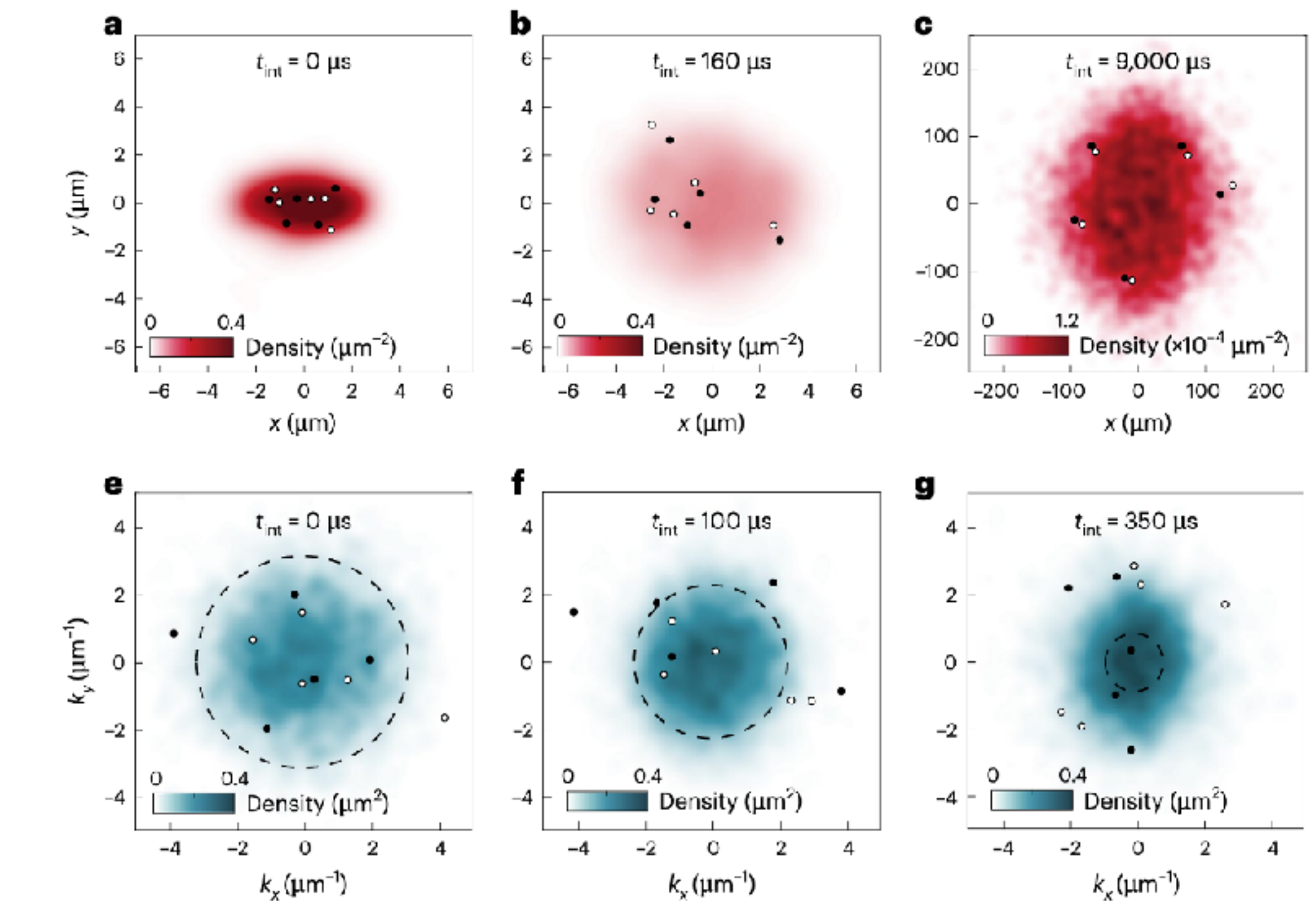
Nouicer, R. New state of nuclear matter: Nearly perfect fluid of quarks and gluons in heavy-ion collisions at RHIC energies. *The European Physical Journal Plus* 131, 70 (2016).

Elliptic flow in 10 atoms

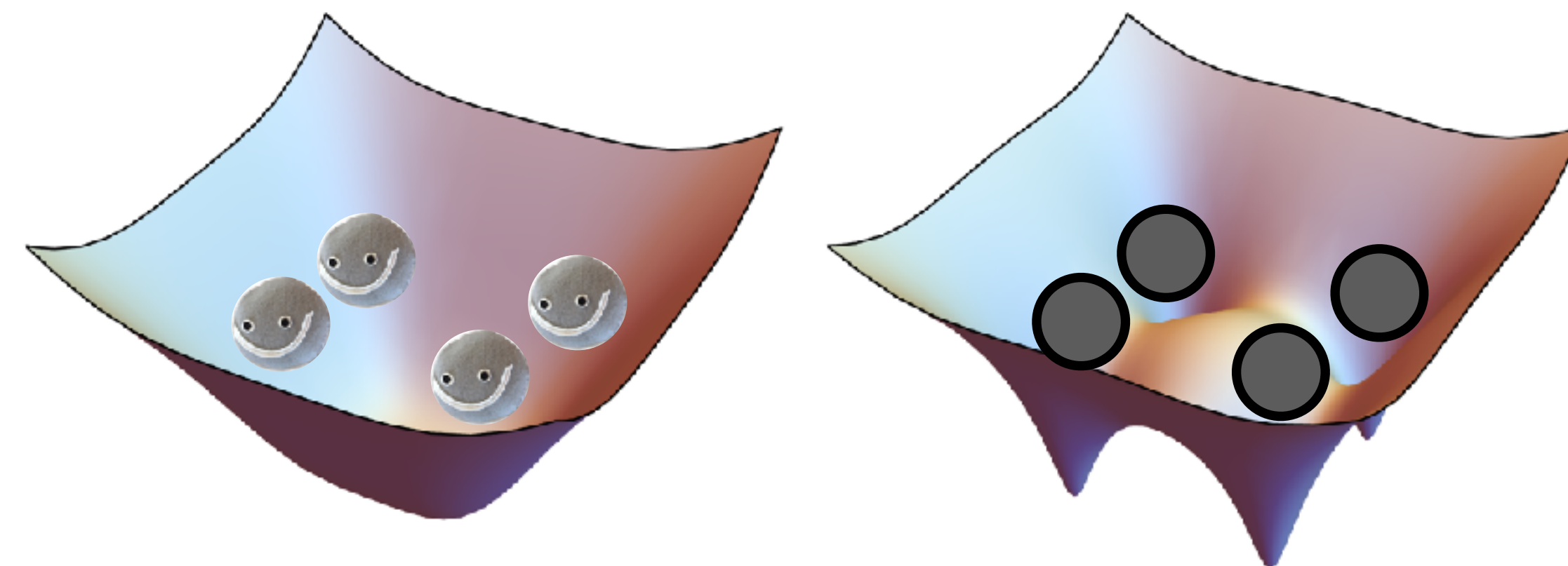


2 immediate questions

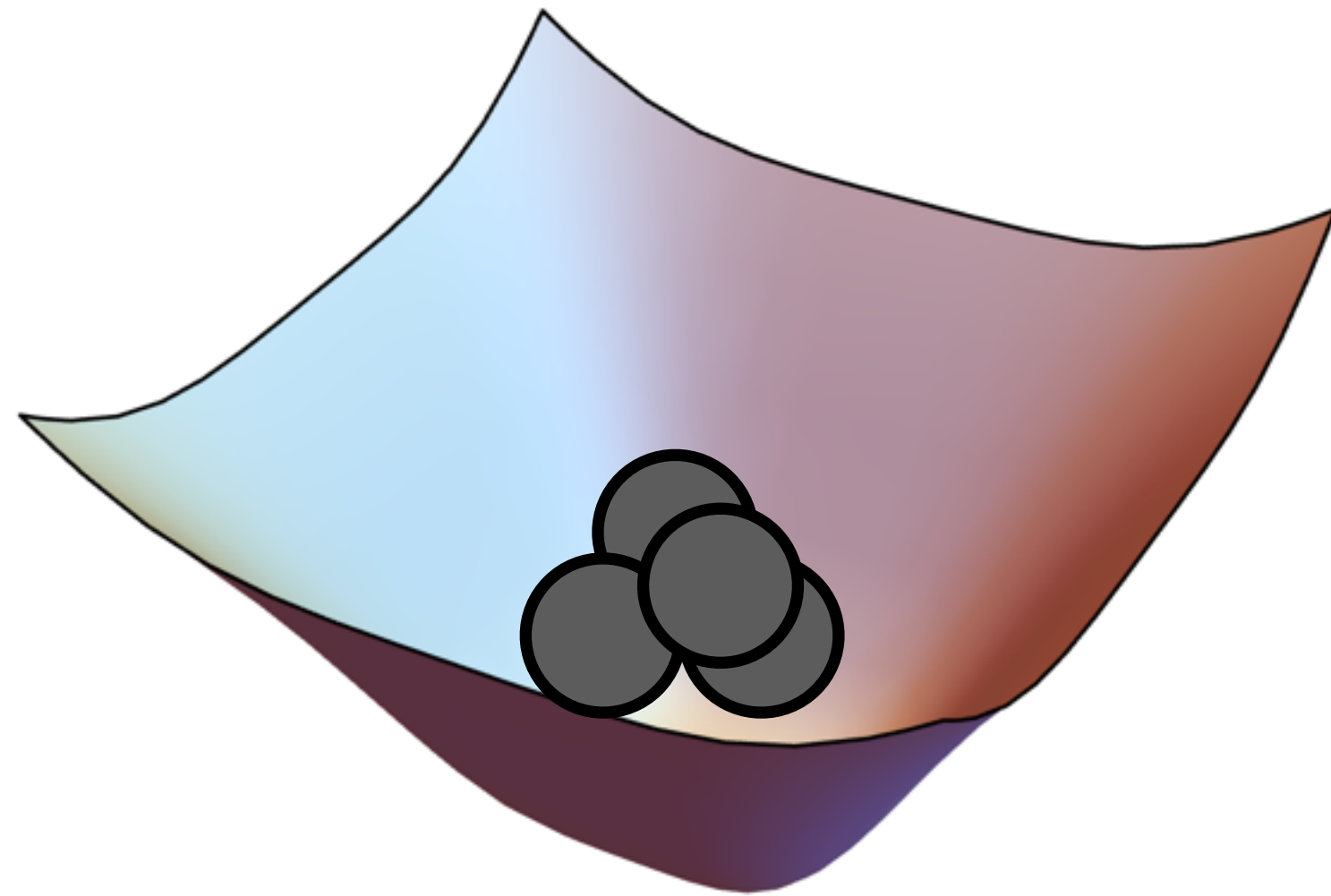
- Can a microscopic quantum theory predict this **quantitatively**?
- Is this **hydrodynamics** ? What transport coefficients are required by data ?



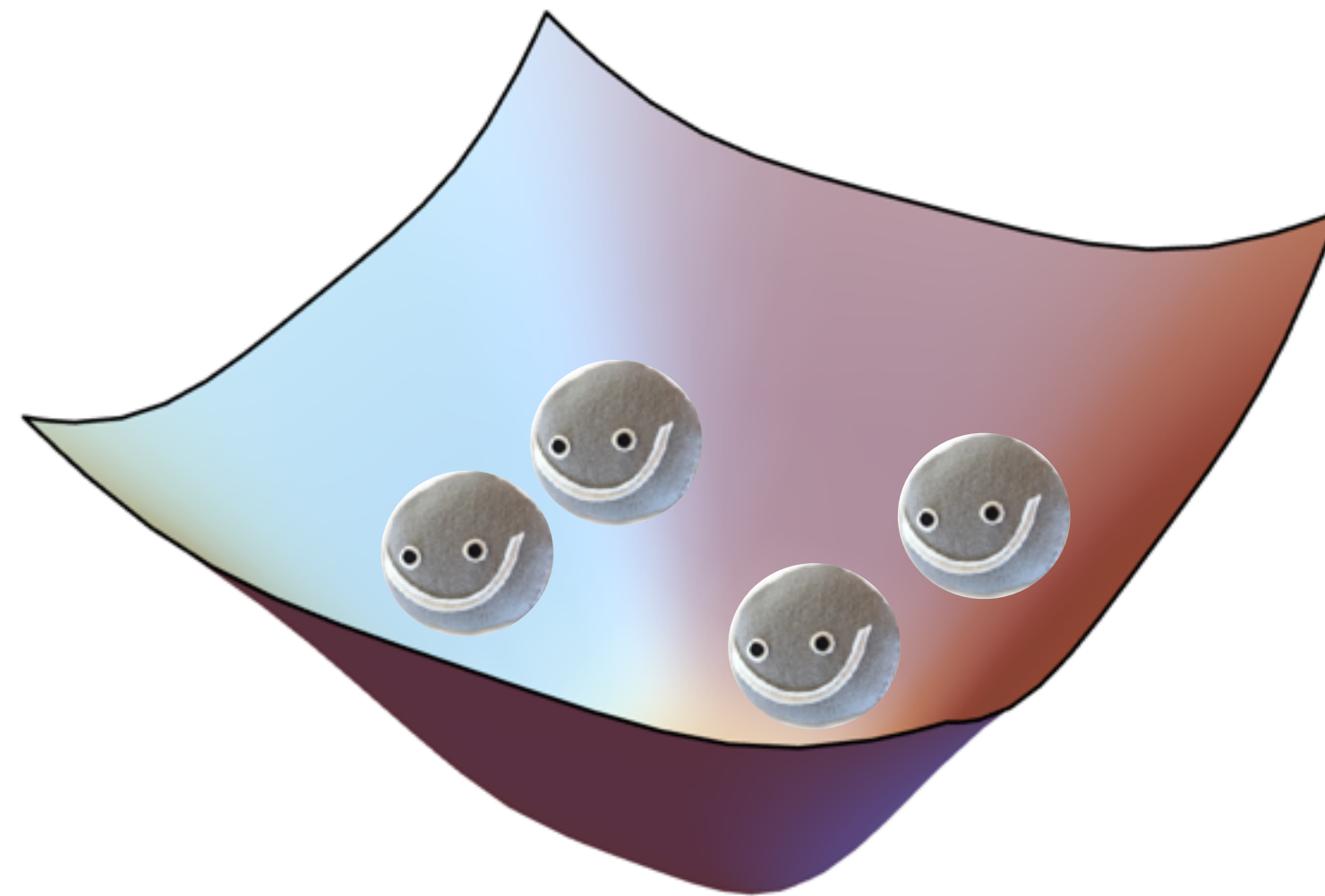
2. Density Functional Theory (DFT)



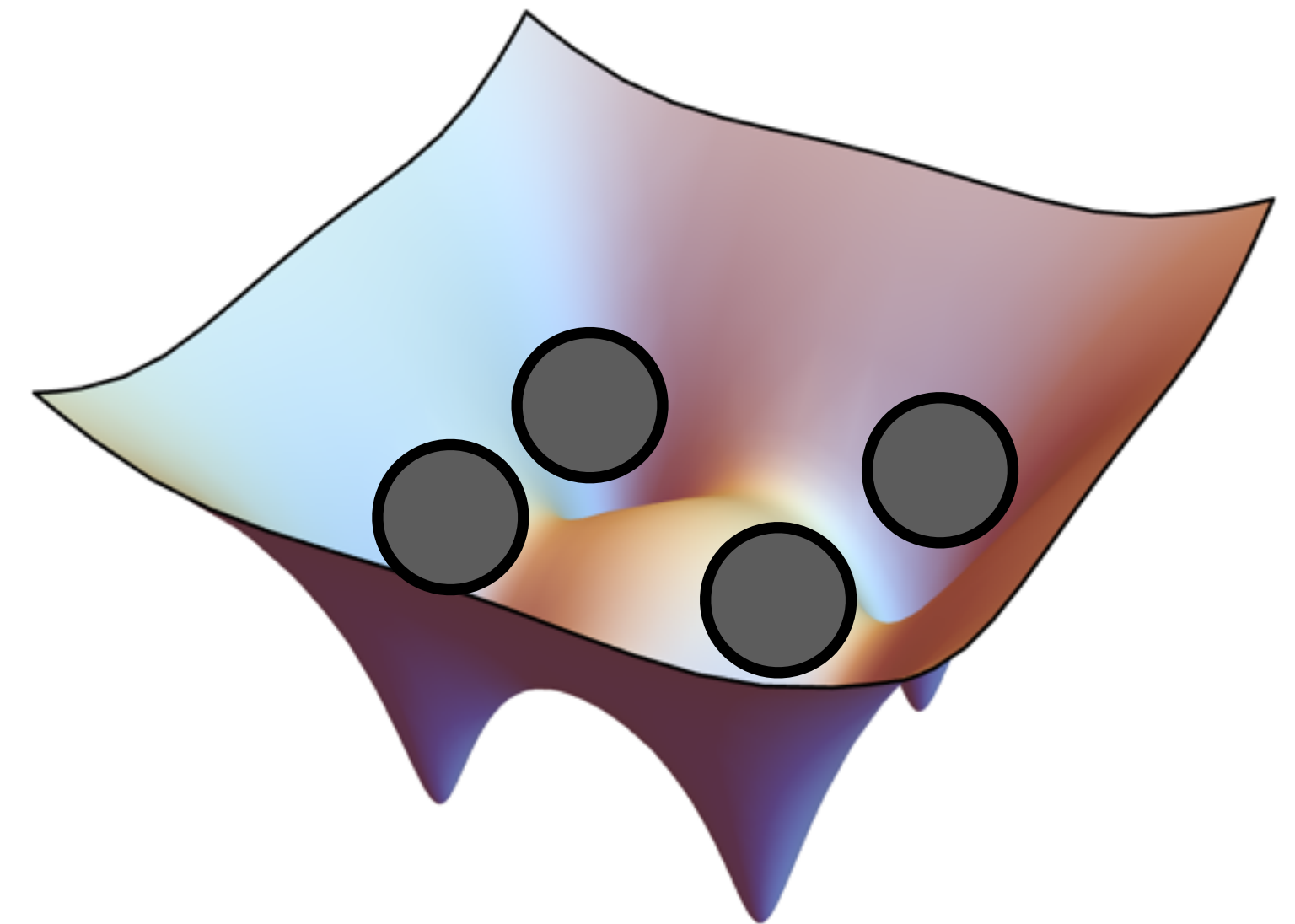
The idea



Noninteracting



Interacting



Noninteracting
+ aux. potential
= same density

The (extended) Superfluid local density approximation (SLDAE)

- We use SLDAE, assumes energy functional is built from **local** densities

Diagram illustrating the components of the SLDAE energy functional:

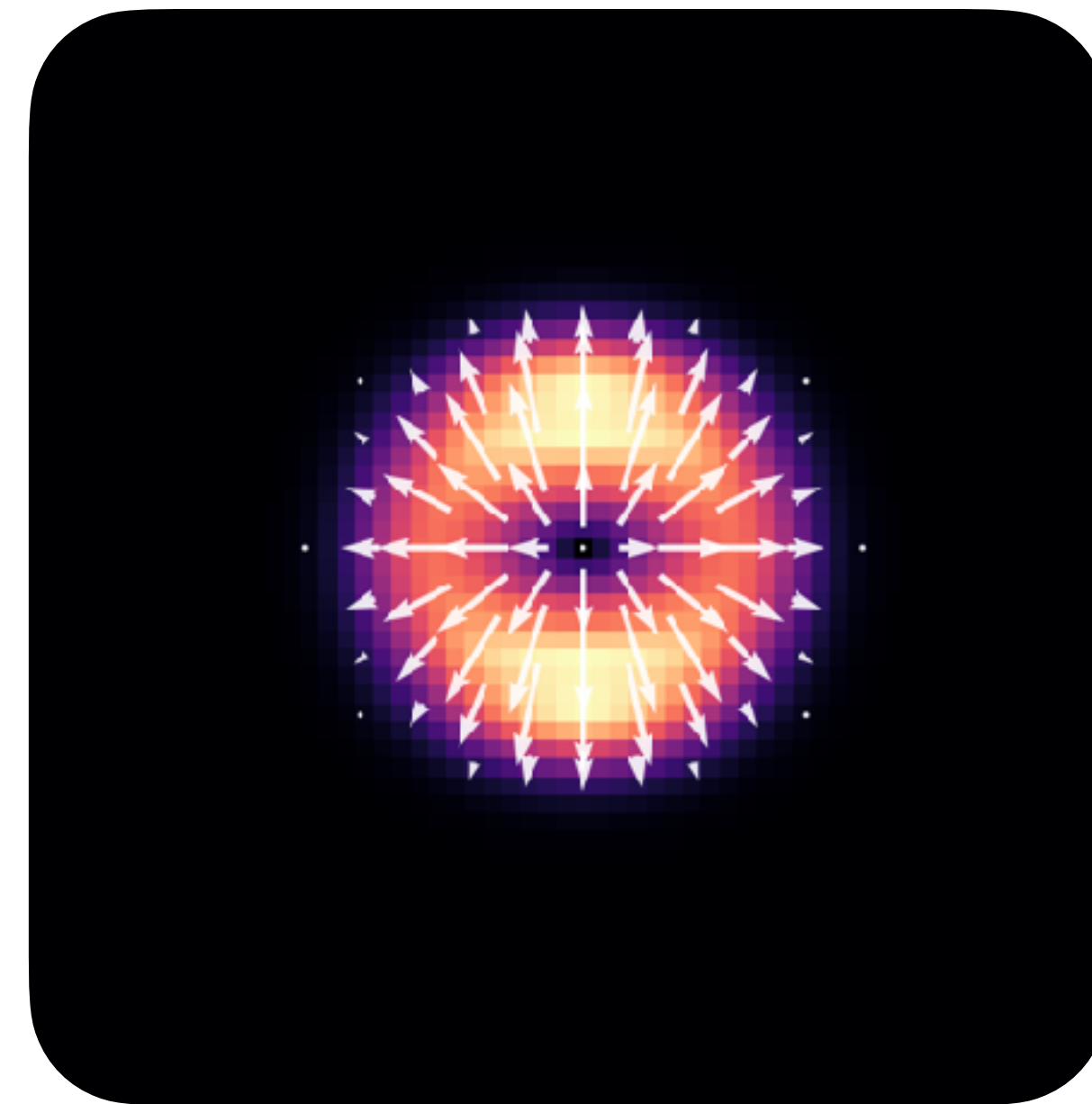
- $\tau_\sigma \sim \langle \psi_\sigma^\dagger \nabla^2 \psi_\sigma \rangle$ Kinetic energy
- $n_\sigma \sim \langle \psi_\sigma^\dagger \psi_\sigma \rangle$ Density
- $\nu \sim \langle \psi_\uparrow \psi_\downarrow \rangle$ Pairing
- $j_\sigma \sim \langle \psi_\sigma^\dagger \nabla \psi_\sigma \rangle$ Currents

$$\mathcal{E}_{\text{sldaе}} = A_\lambda \frac{\tau}{2} + \frac{3}{5} B_\lambda \varepsilon_F n + g_\lambda |\nu|^2 - (1 - A_\lambda) \frac{j^2}{2n}$$

[Wlazłowski, Magierski, Forbes, Bulgac, arXiv.2602.08982]

$$\lambda = |k_F a|$$

3. What does DFT predict ?



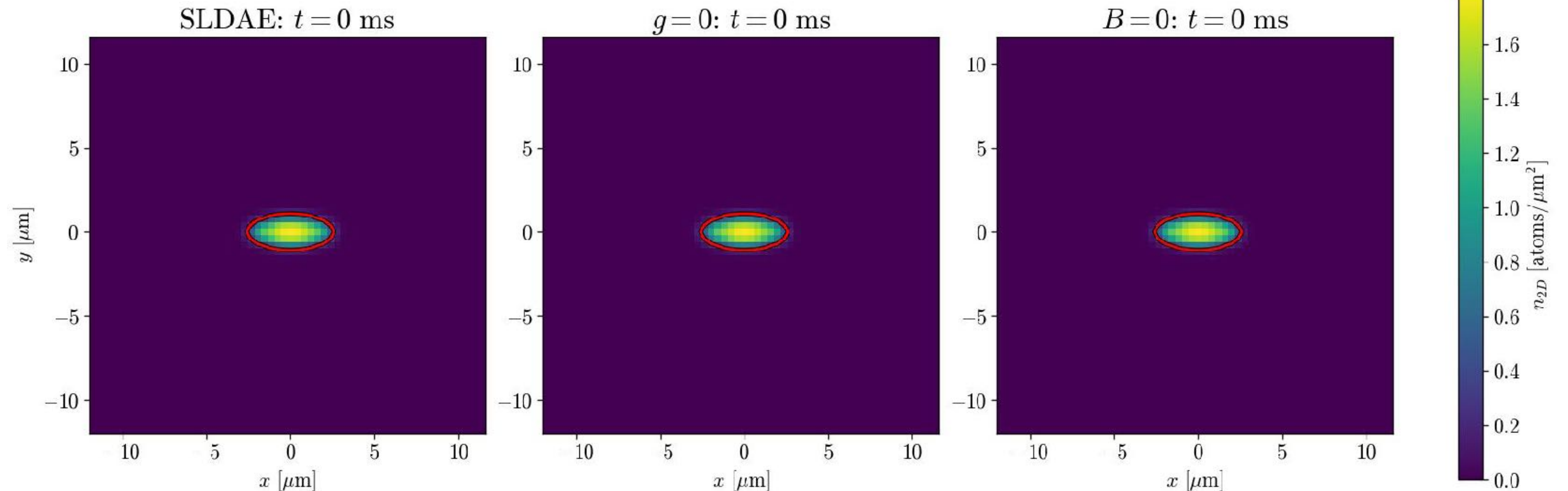
Elliptic flow

$$\mathcal{E}_{\text{sldaе}} = A_\lambda \frac{\tau}{2} + \frac{3}{5} B_\lambda \varepsilon_F n + g_\lambda |\nu|^2 - (1 - A_\lambda) \frac{j^2}{2n}$$

Both

Only Hartree-like term

Only pairing



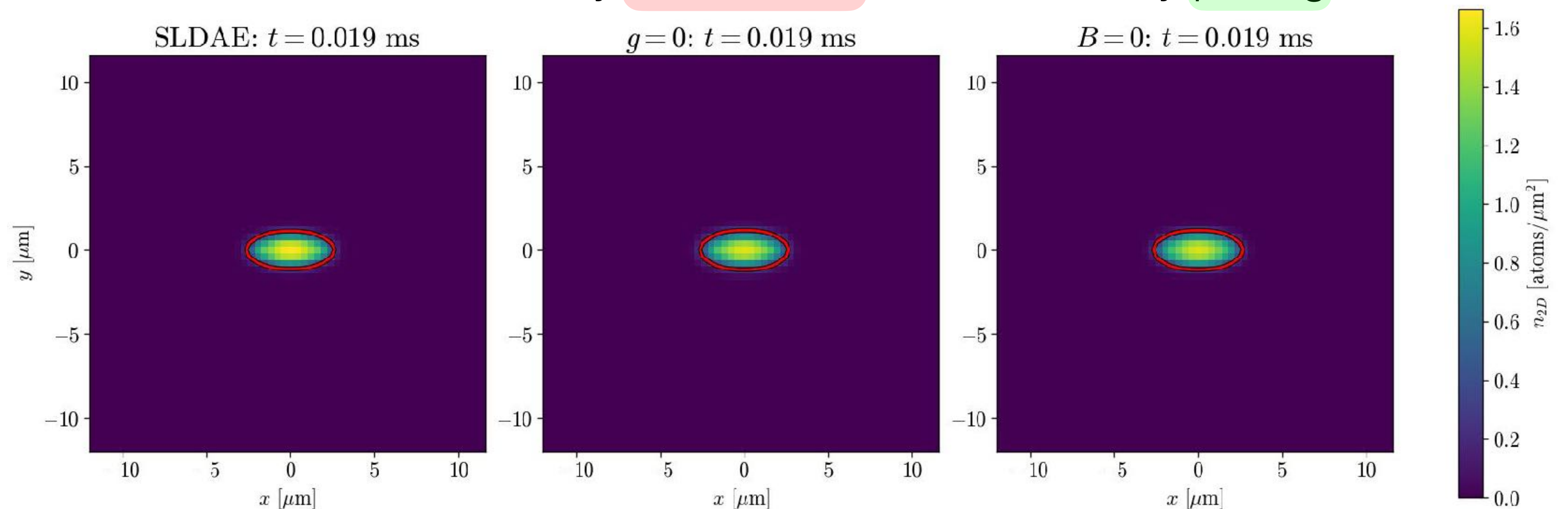
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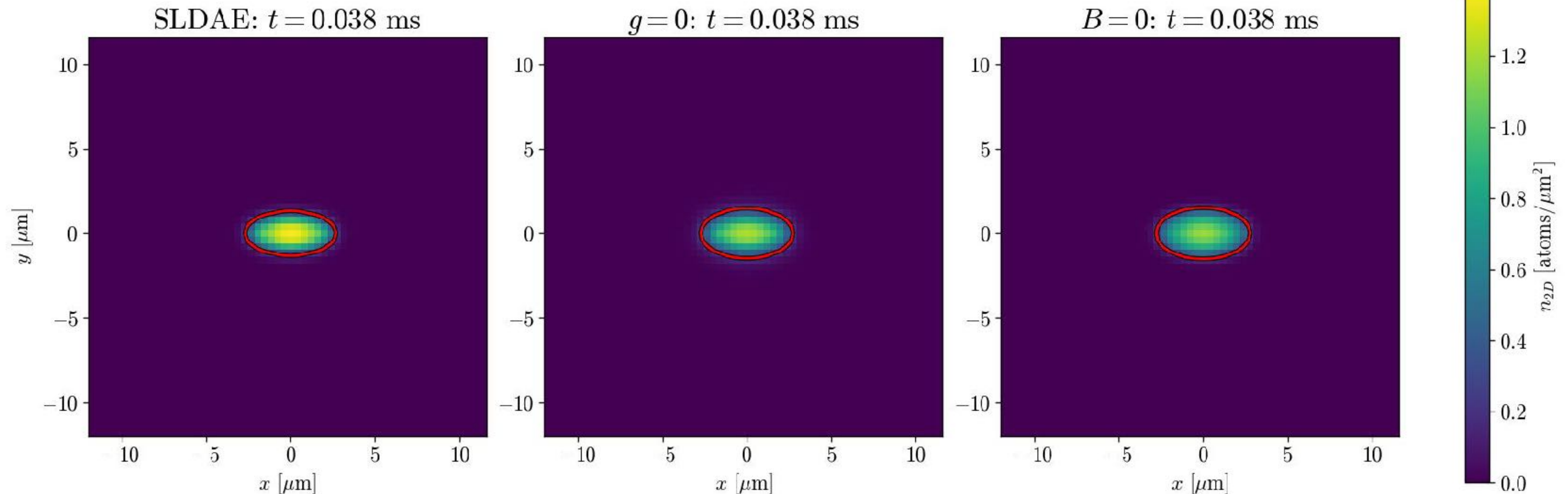
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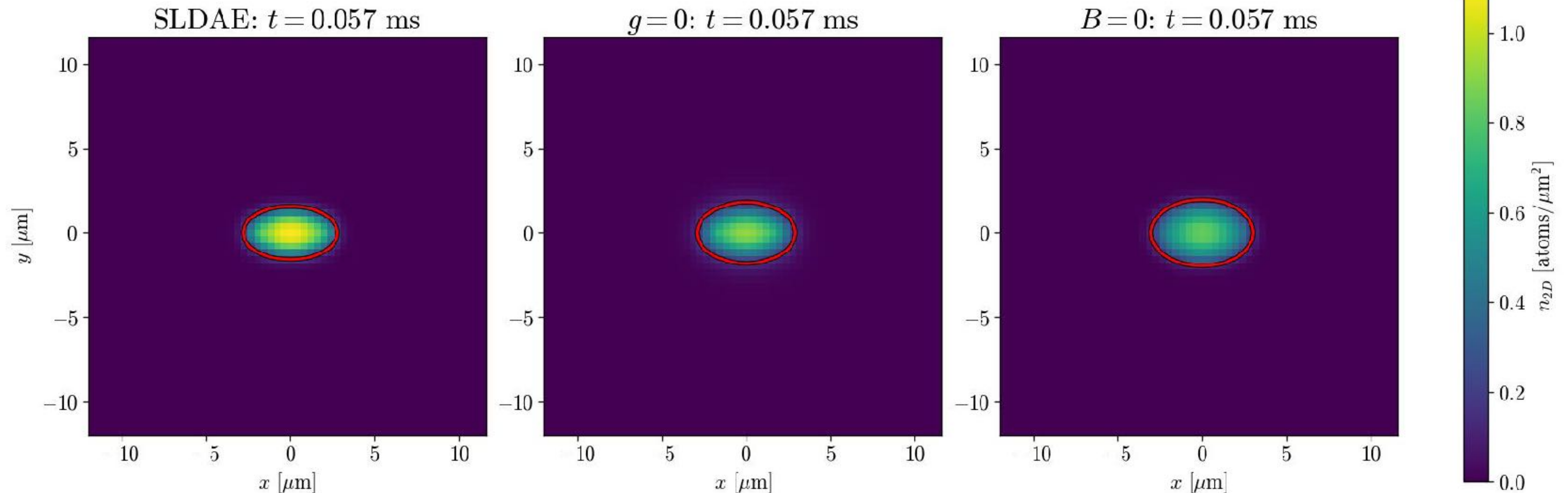
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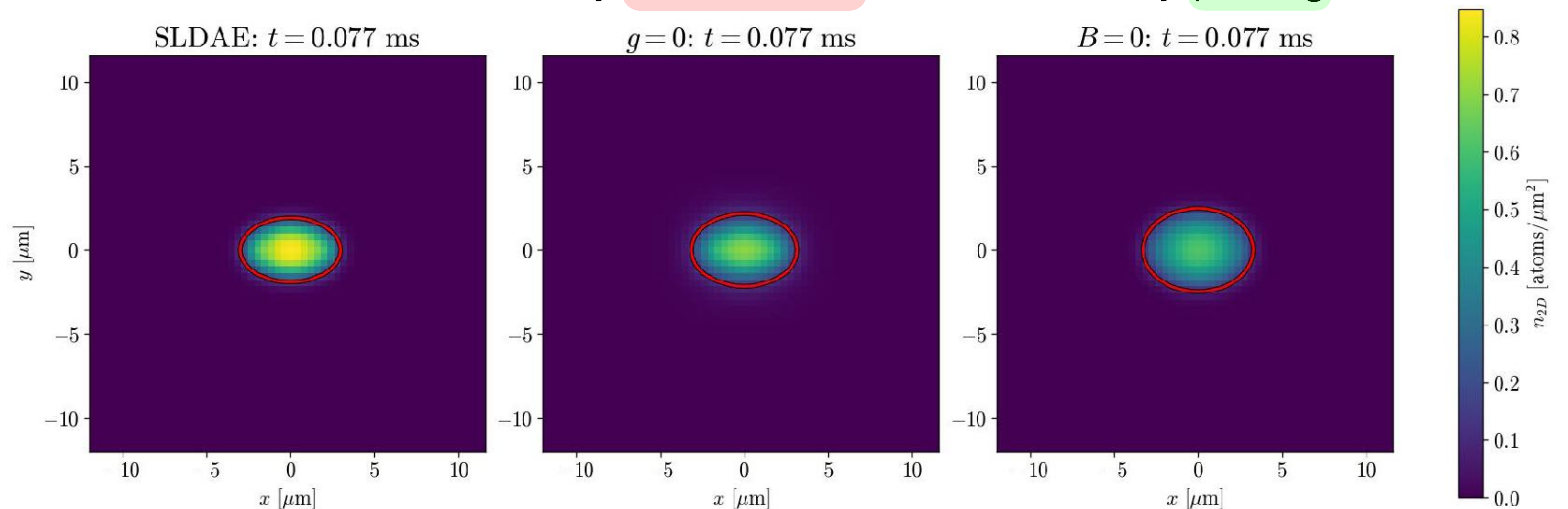
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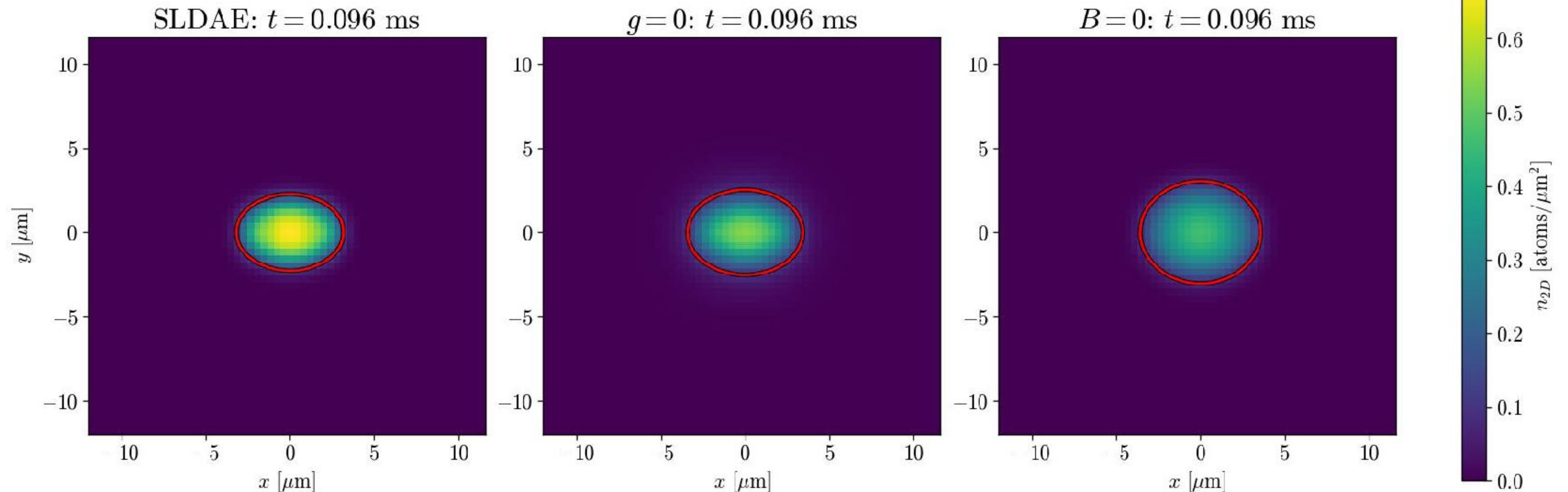
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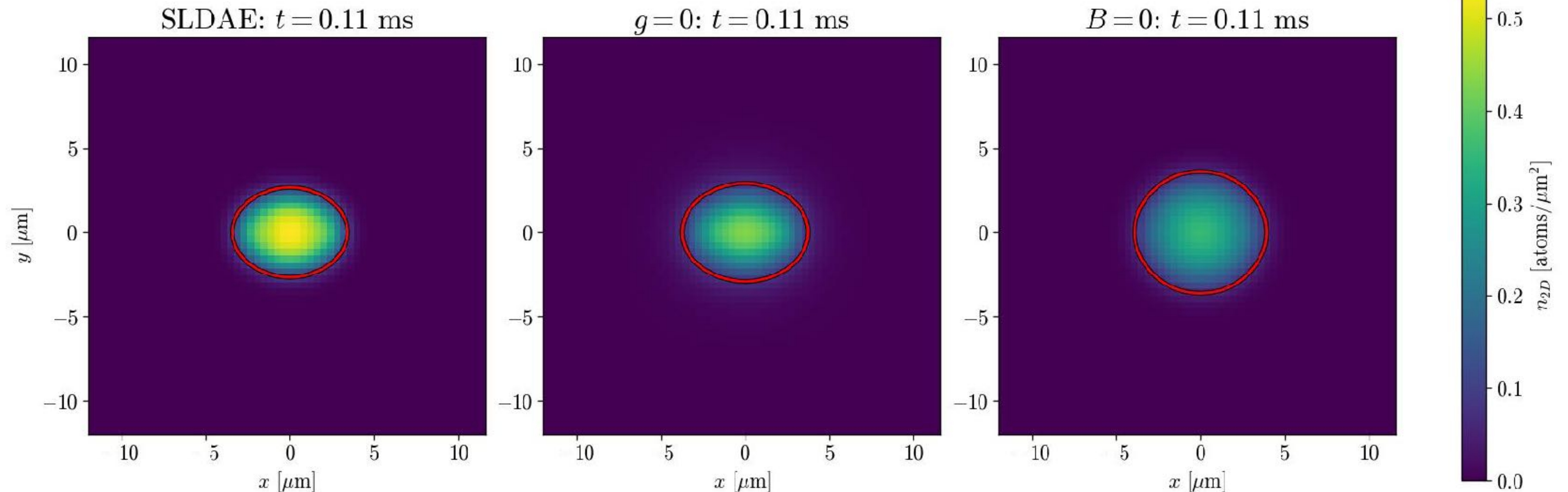
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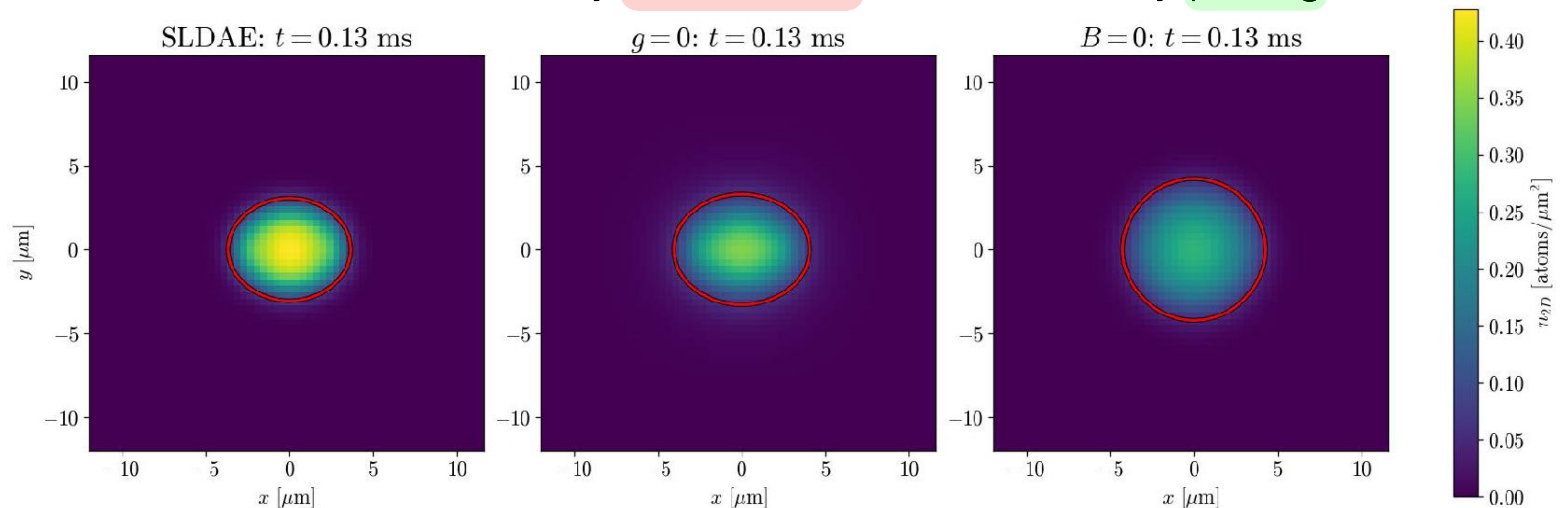
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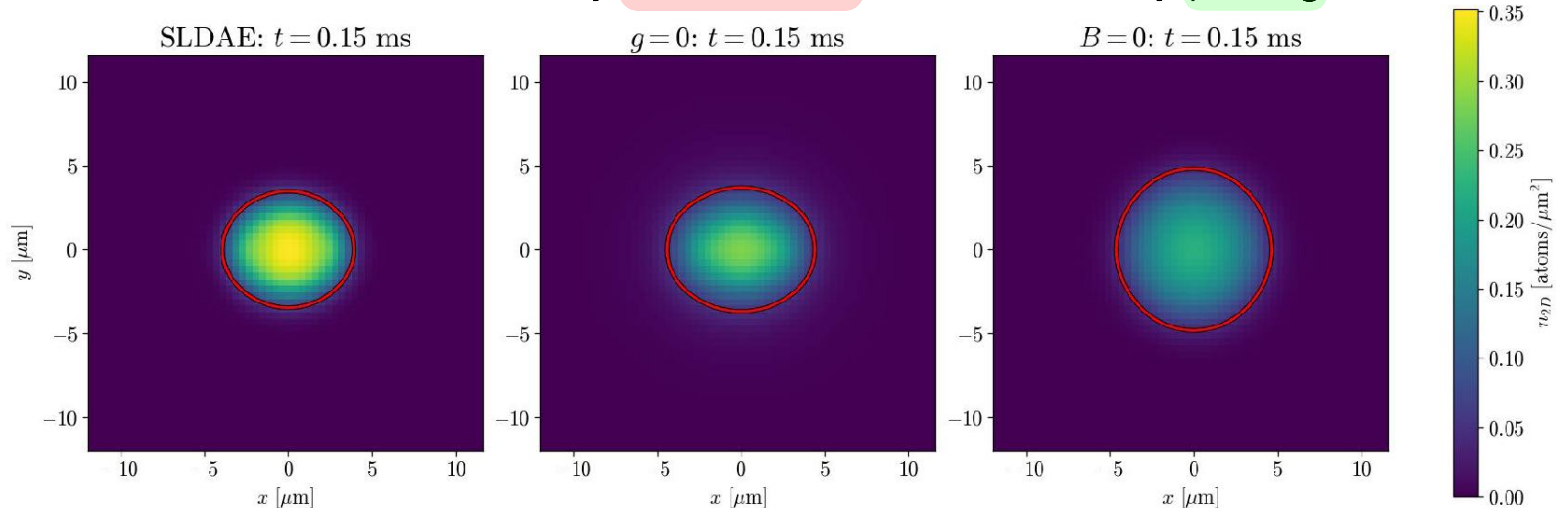
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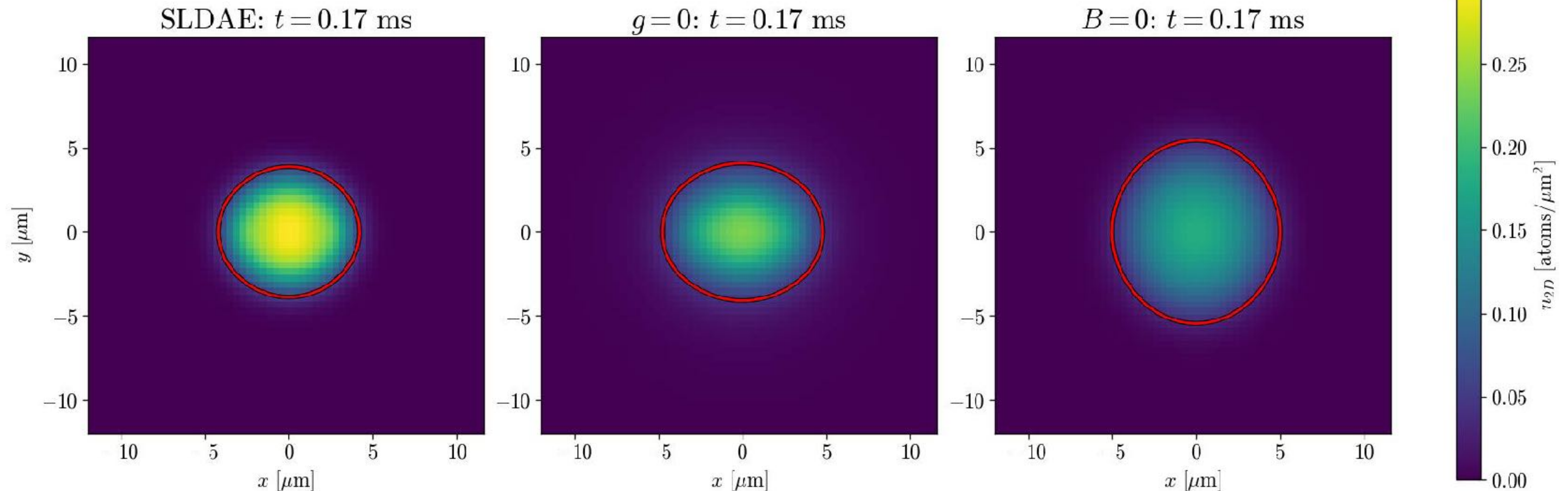
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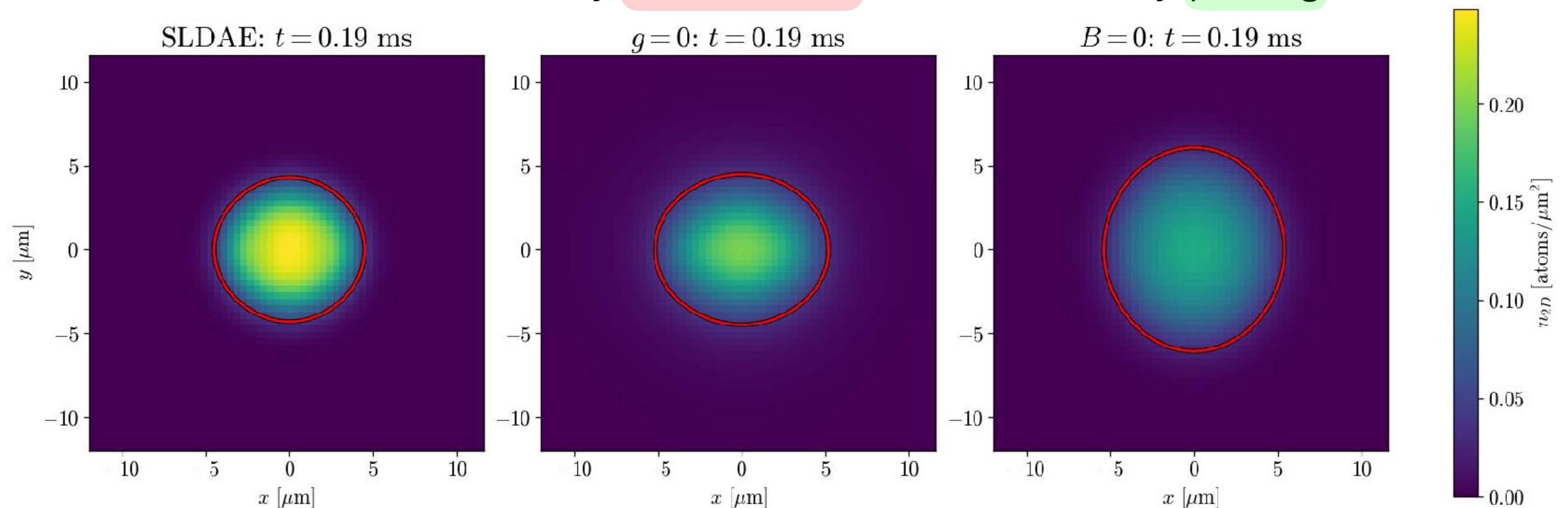
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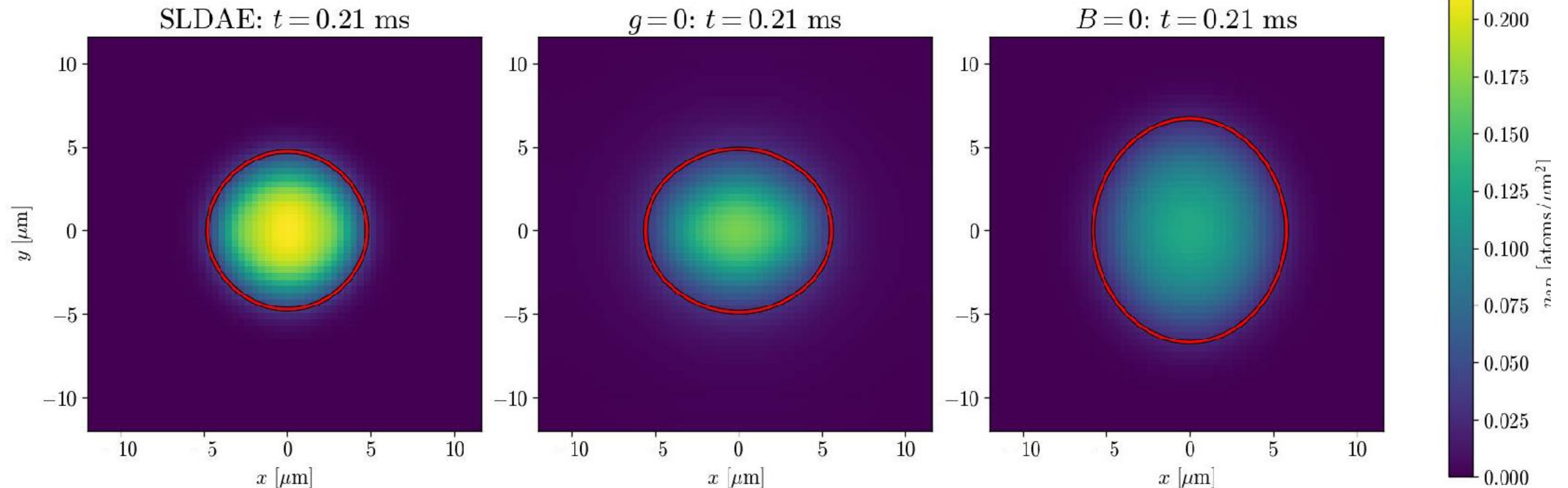
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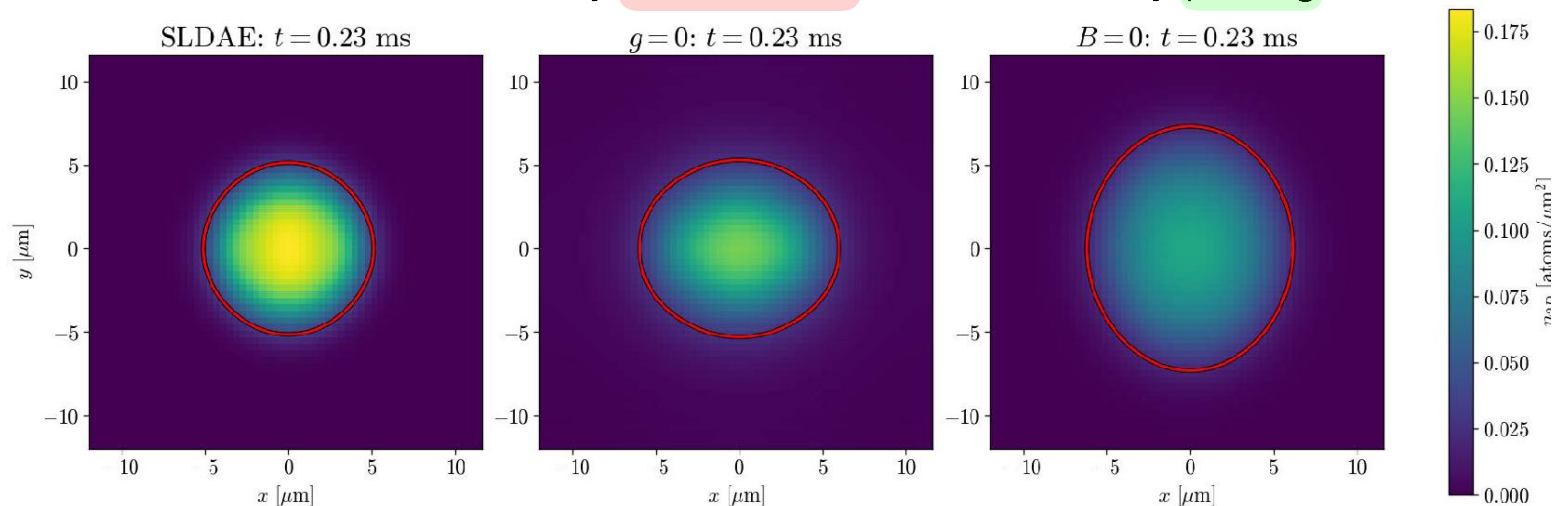
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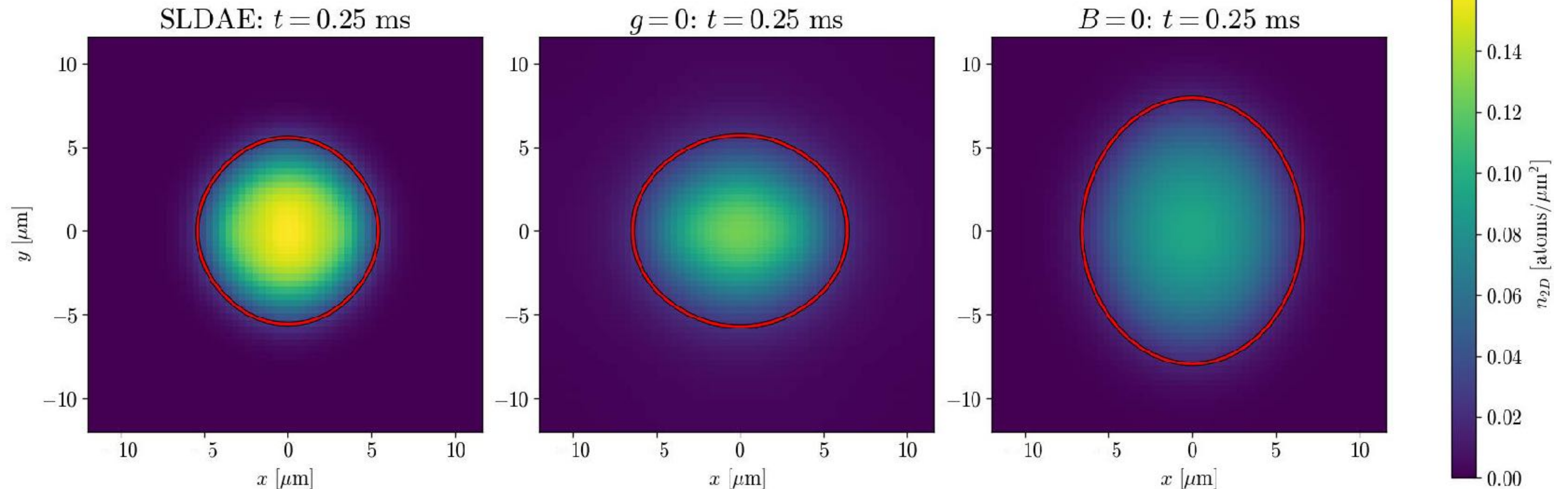
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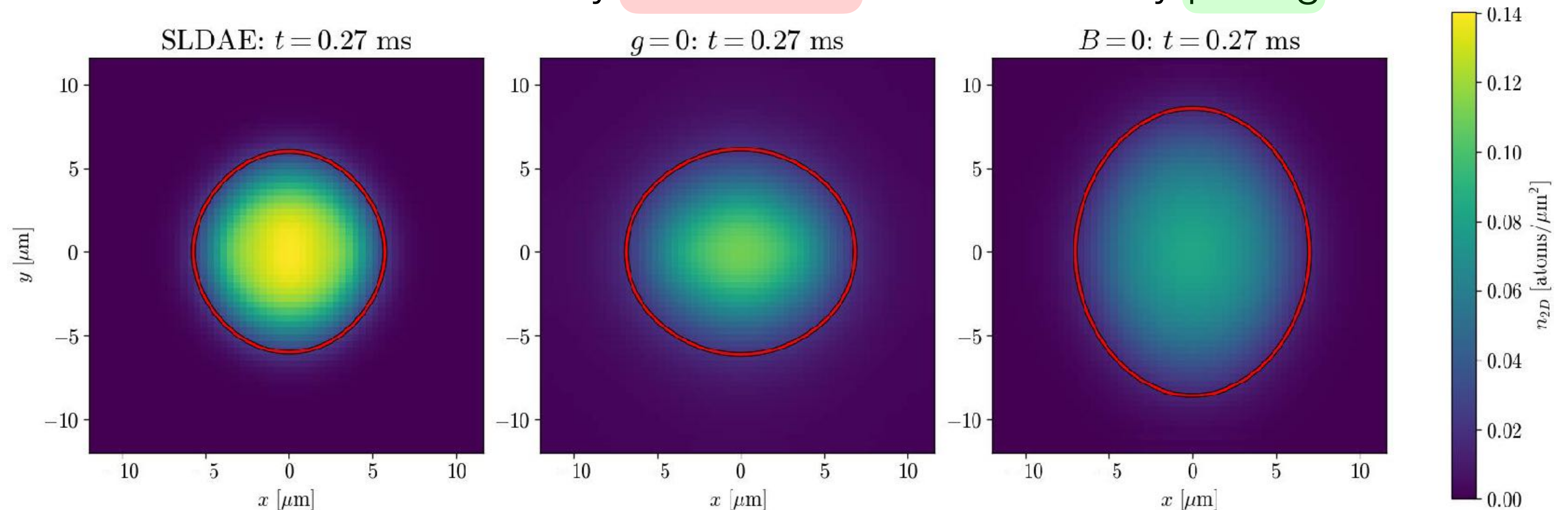
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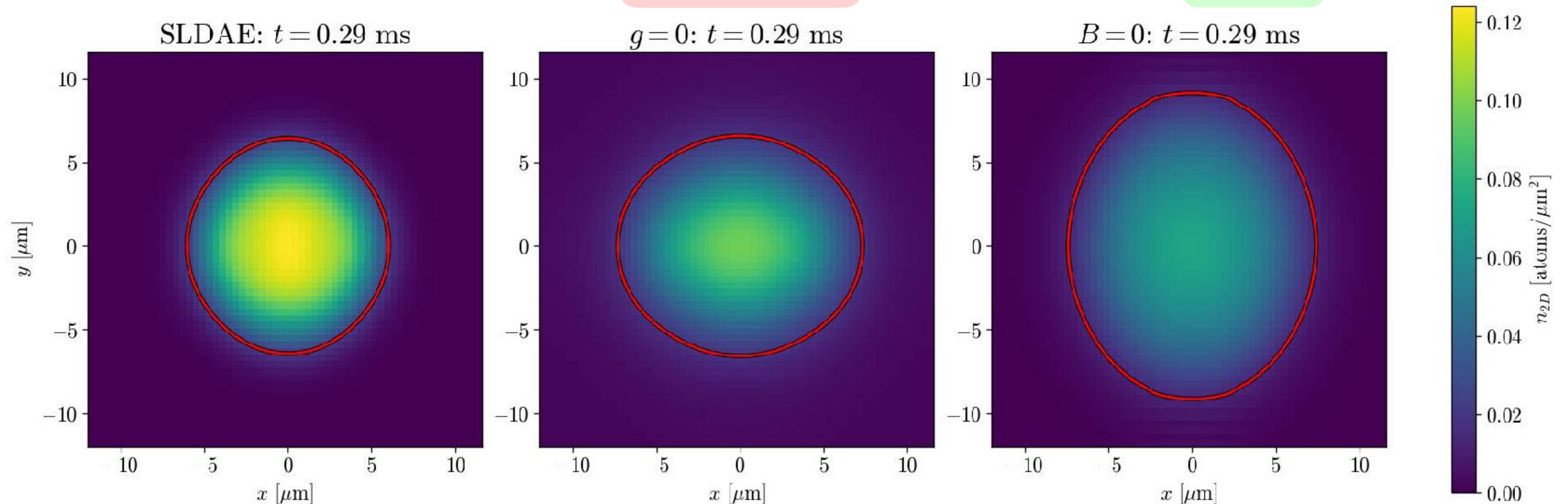
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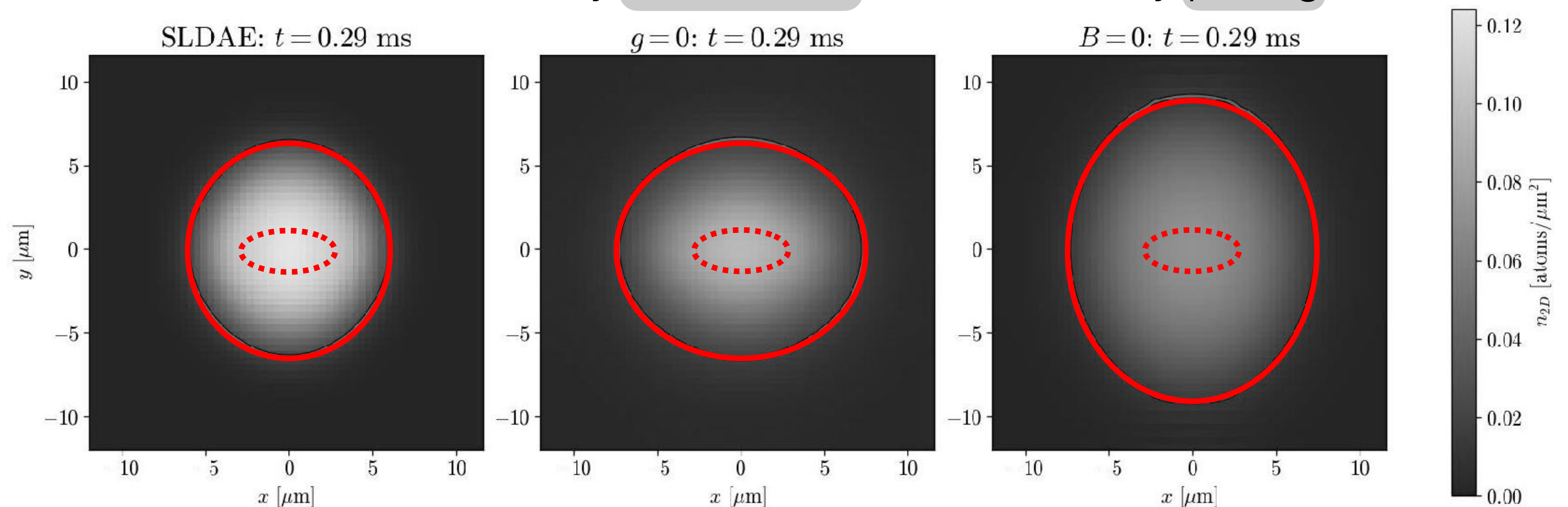
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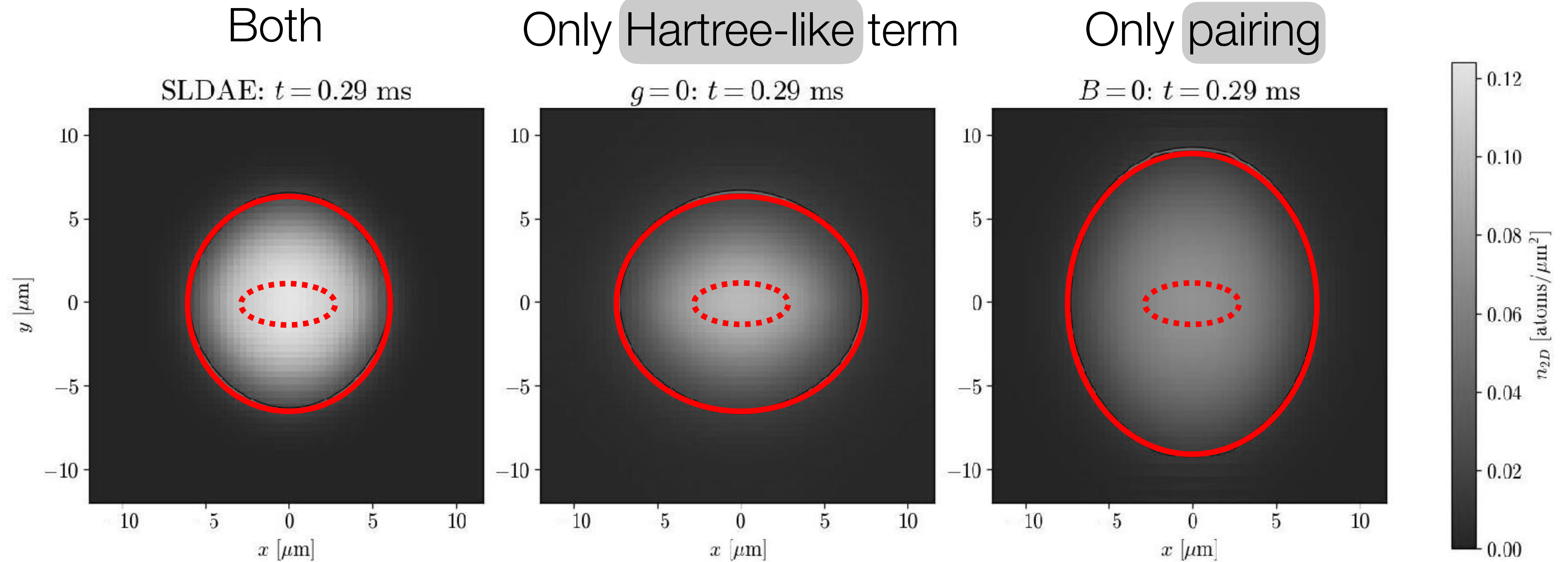
Only Hartree-like term

Only pairing

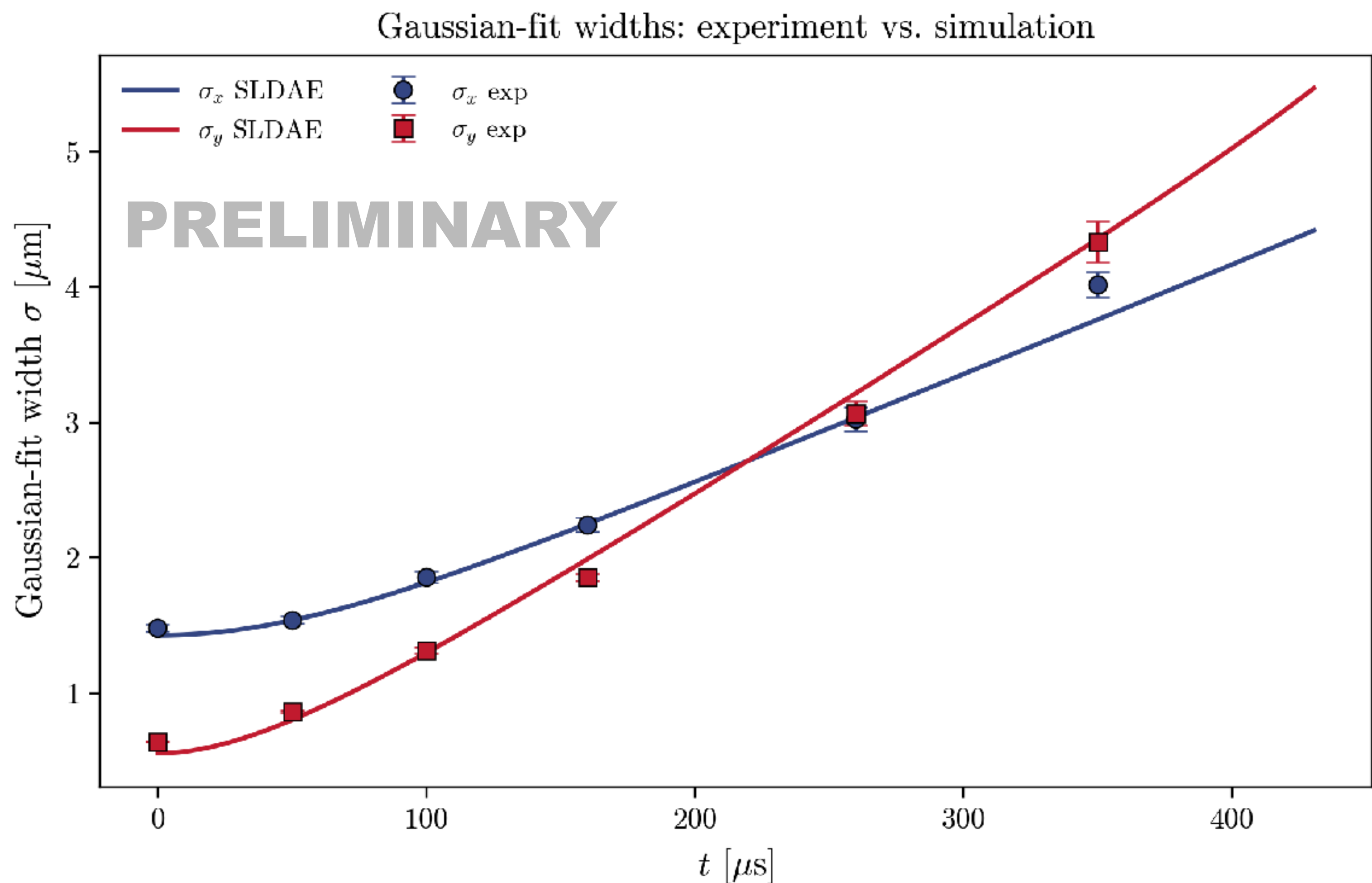


Elliptic flow

Pairing is essential for elliptic flow!



Comparison to experiments



Published experimental data from:

Brandstetter, S. et al.
Emergent interaction-driven elliptic flow of few fermionic atoms. Nat. Phys. 21, 52–56 (2025).

Comparing to experimental profiles

Thanks to

Guillermo
Sanchez Paladines

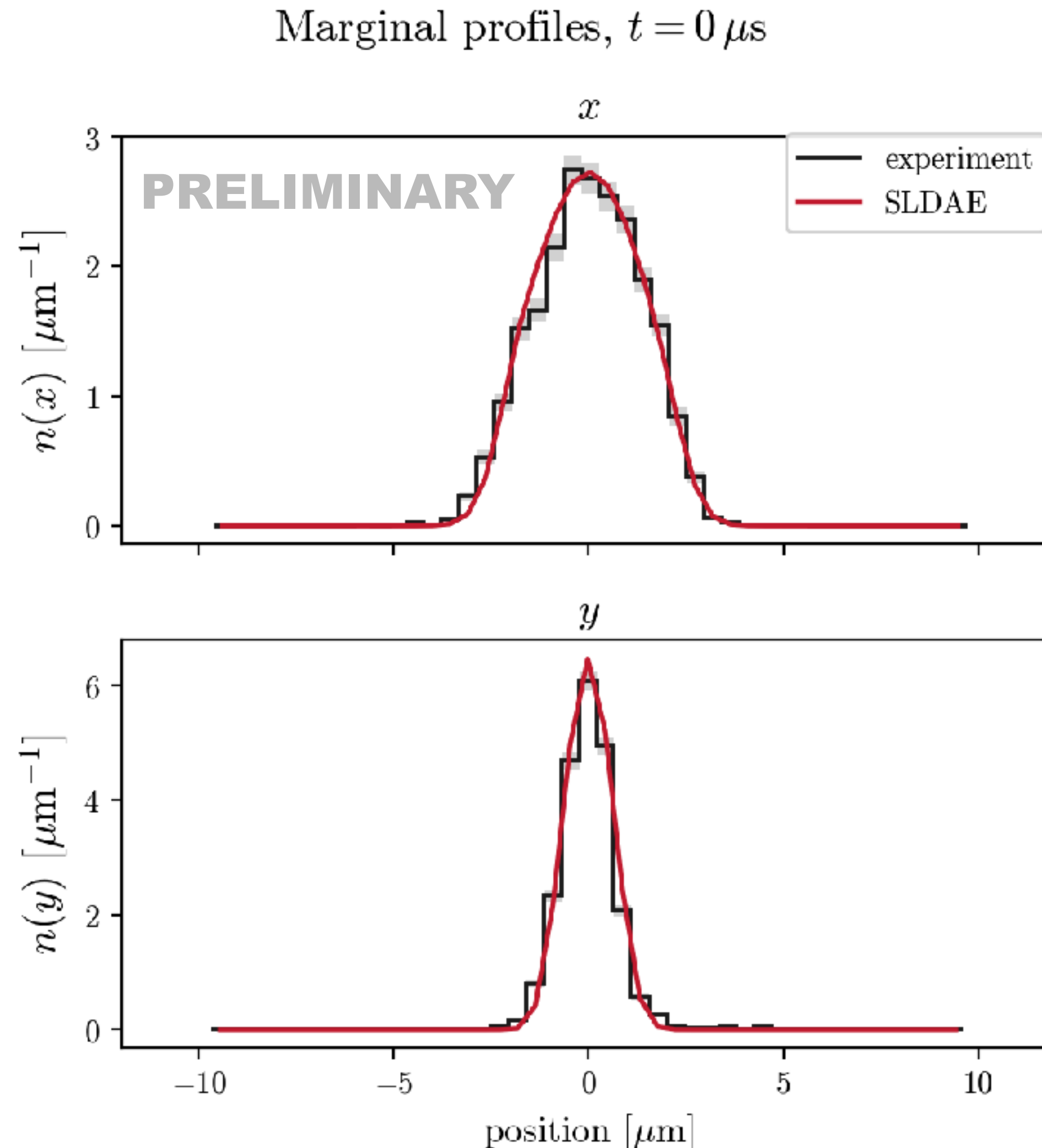


and

Carl Heintze



for help interpreting the
published experimental data



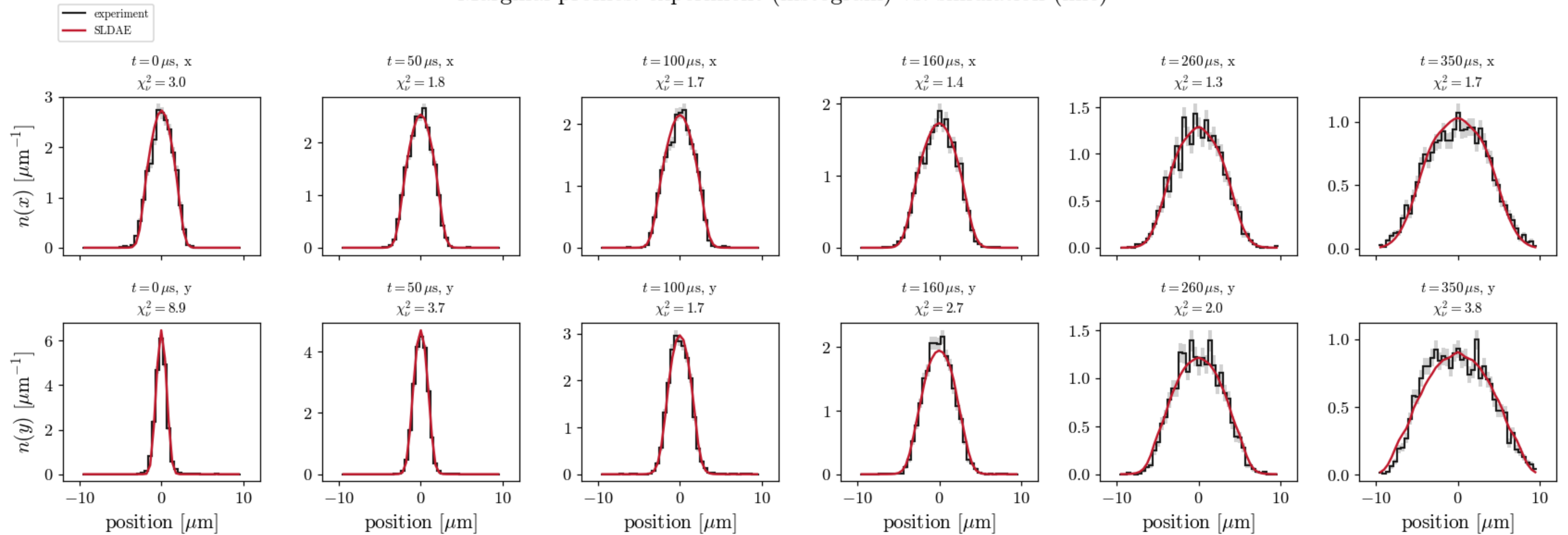
Published experimental data
from:

Brandstetter, S. et al.
Emergent interaction-driven
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(2025).

Profile summary

PRELIMINARY

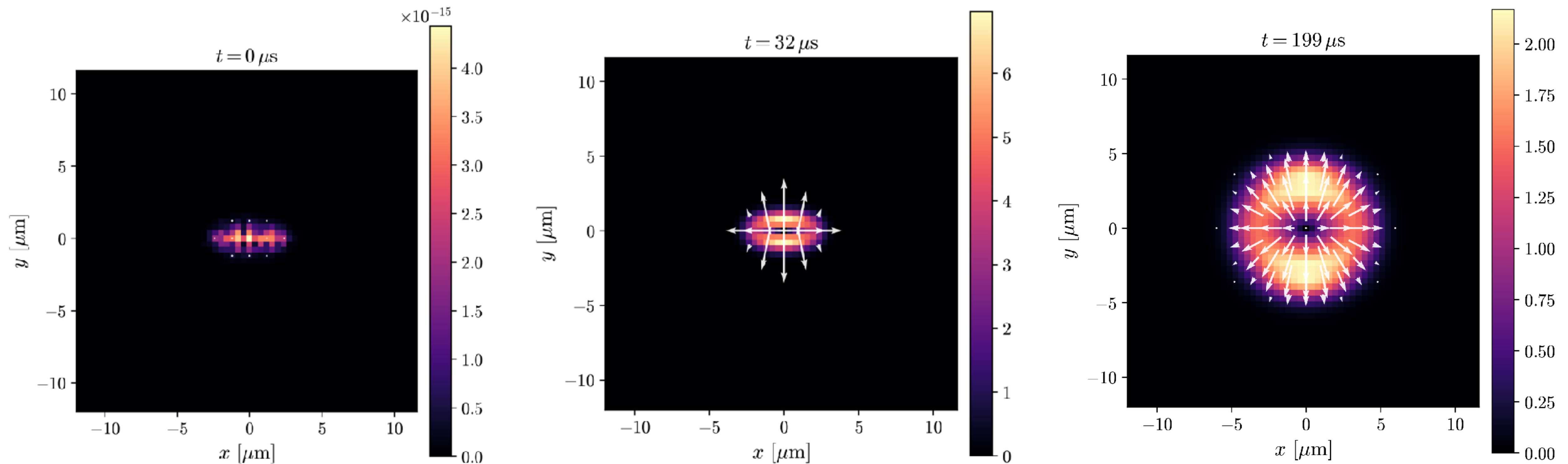
Marginal profiles: experiment (histogram) vs. simulation (line)



Time

Momentum space – current density

SLDAE: column current $|\mathbf{J}_{2D}|$ [atoms $\mu\text{m}^{-1}\text{ms}^{-1}$], arrows = flow direction



4. Hydrodynamics



Quantum pressure

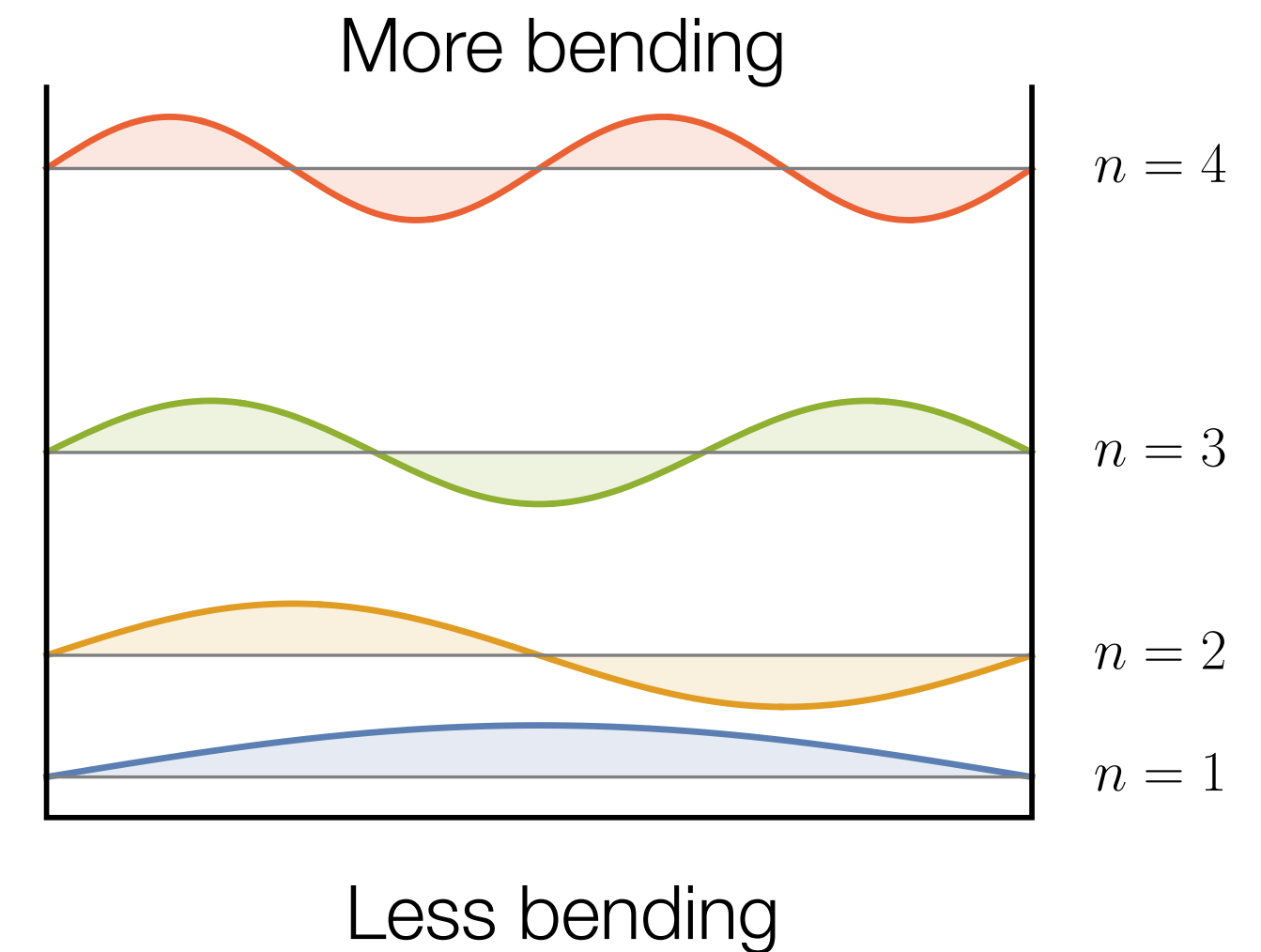
$$\psi = \sqrt{n} e^{i\theta} \quad \nabla \theta \propto v$$

$$\langle T \rangle = \frac{\hbar^2}{2m} \int |\nabla \psi|^2 = \int \frac{1}{2} m n v^2 + \frac{\hbar^2}{2m} \int |\nabla \sqrt{n}|^2$$

Phase winding: Flow

Amplitude bending:
Quantum pressure

$$n (\partial_t v_i + v_j \partial_j v_i) = - \partial_i P - n \partial_i V + \frac{\lambda_W}{2} n \partial_i \left(\frac{\nabla^2 \sqrt{n}}{\sqrt{n}} \right) + \partial_j (\eta \sigma_{ij})$$



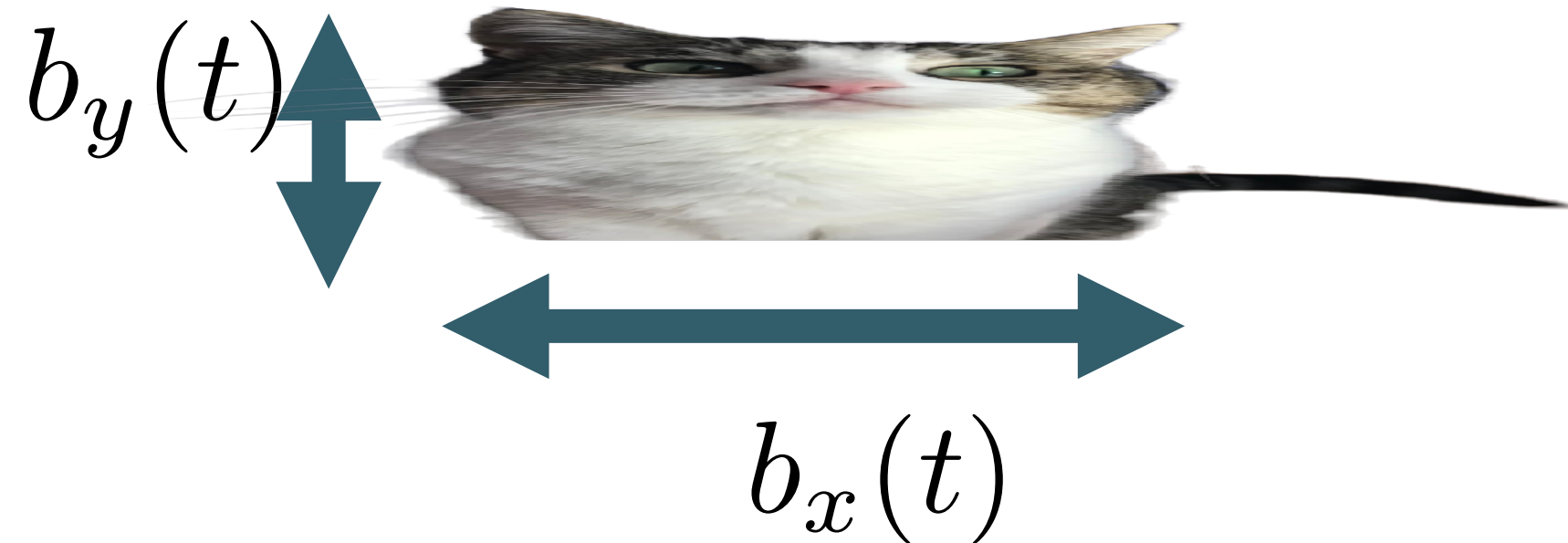
Shear viscosity

[Heyen, Flörchinger, Giacalone, arXiv:2408.06104]

Scaling Ansatz

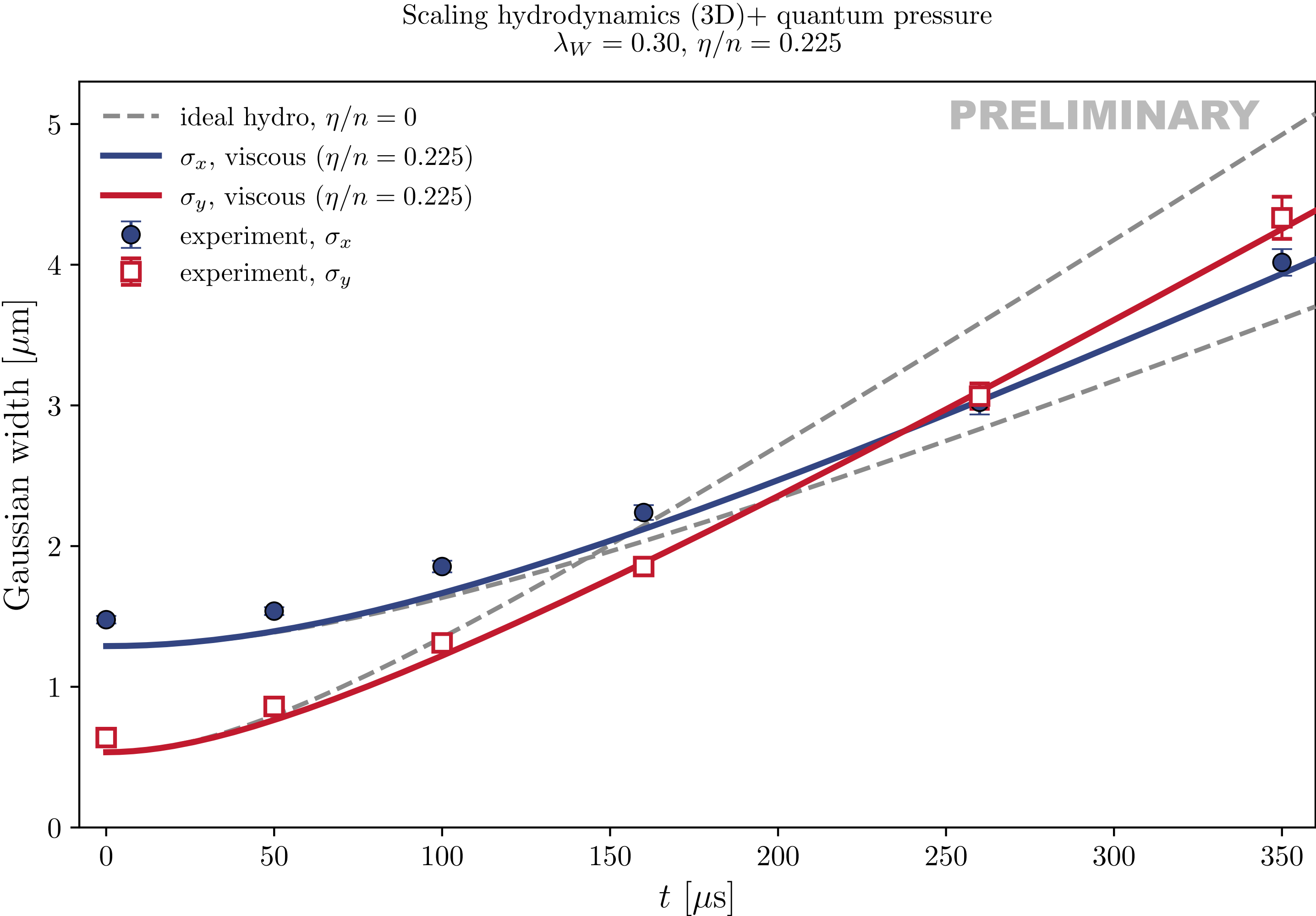
$$n(\vec{x}, t) = \frac{n_0(x_i/b_i)}{b_x b_y b_z}$$

Substitute and solve for $b_i(t)$

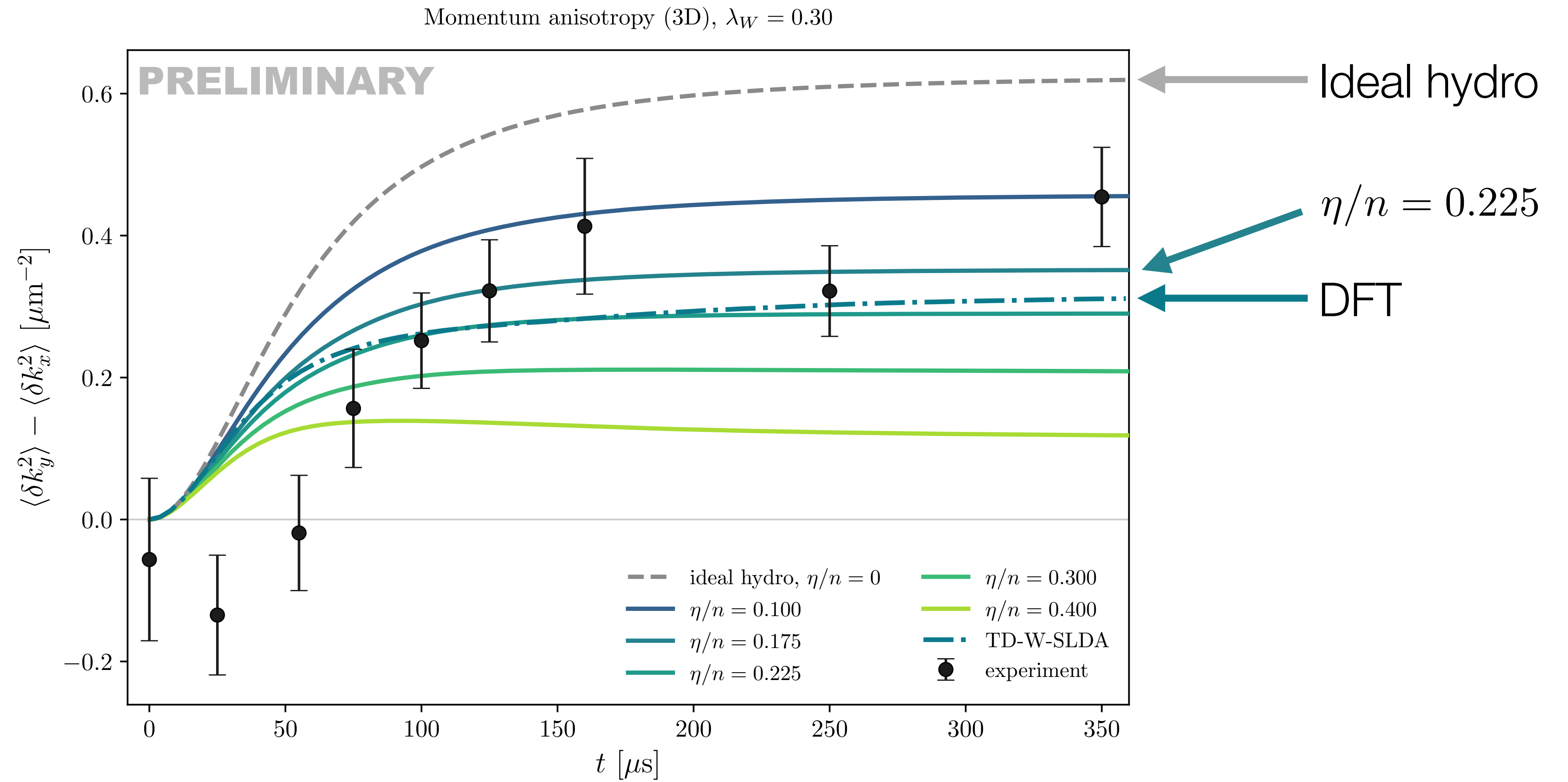


Betsy, born 2011

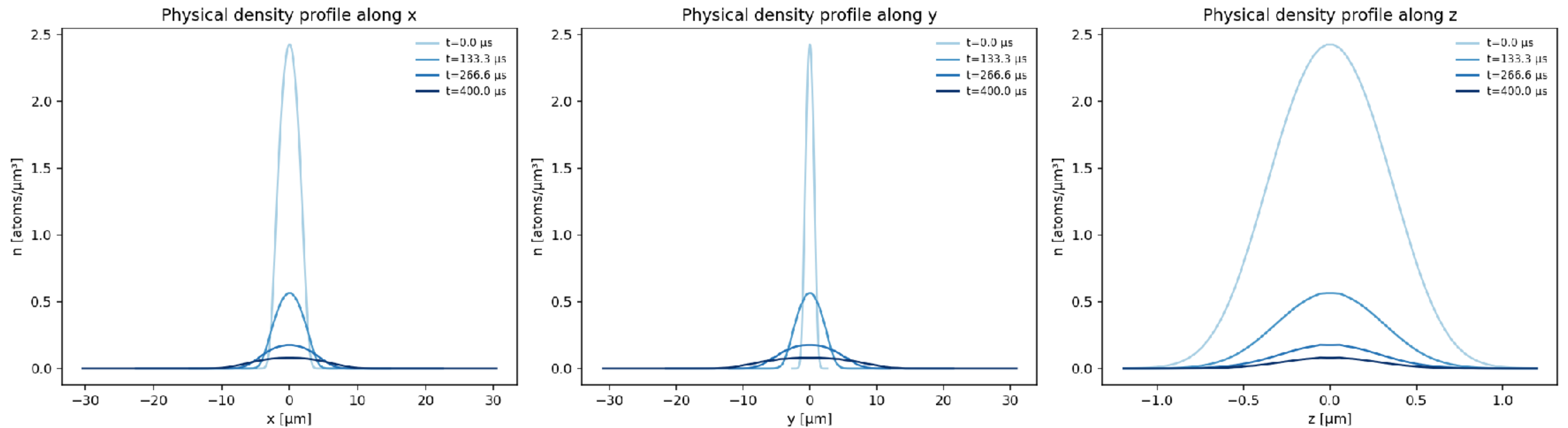
Euler + viscosity + quantum pressure



Momentum anisotropy

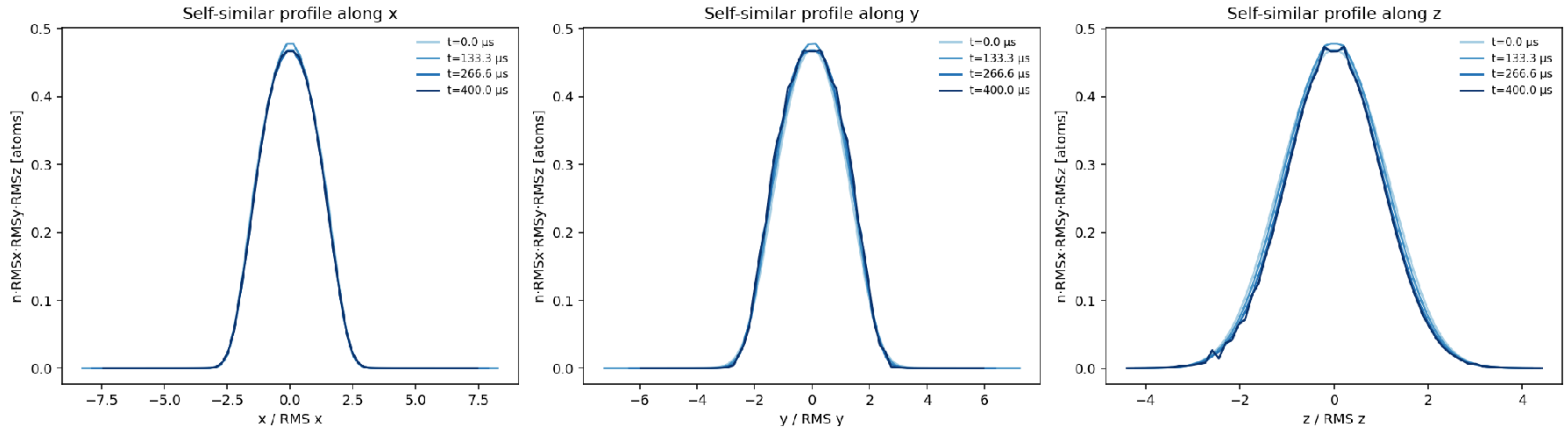


Full viscous anisotropic hydro simulations



Code from [Bluhm and Schäfer, arXiv:1512.00862]

Self similarity



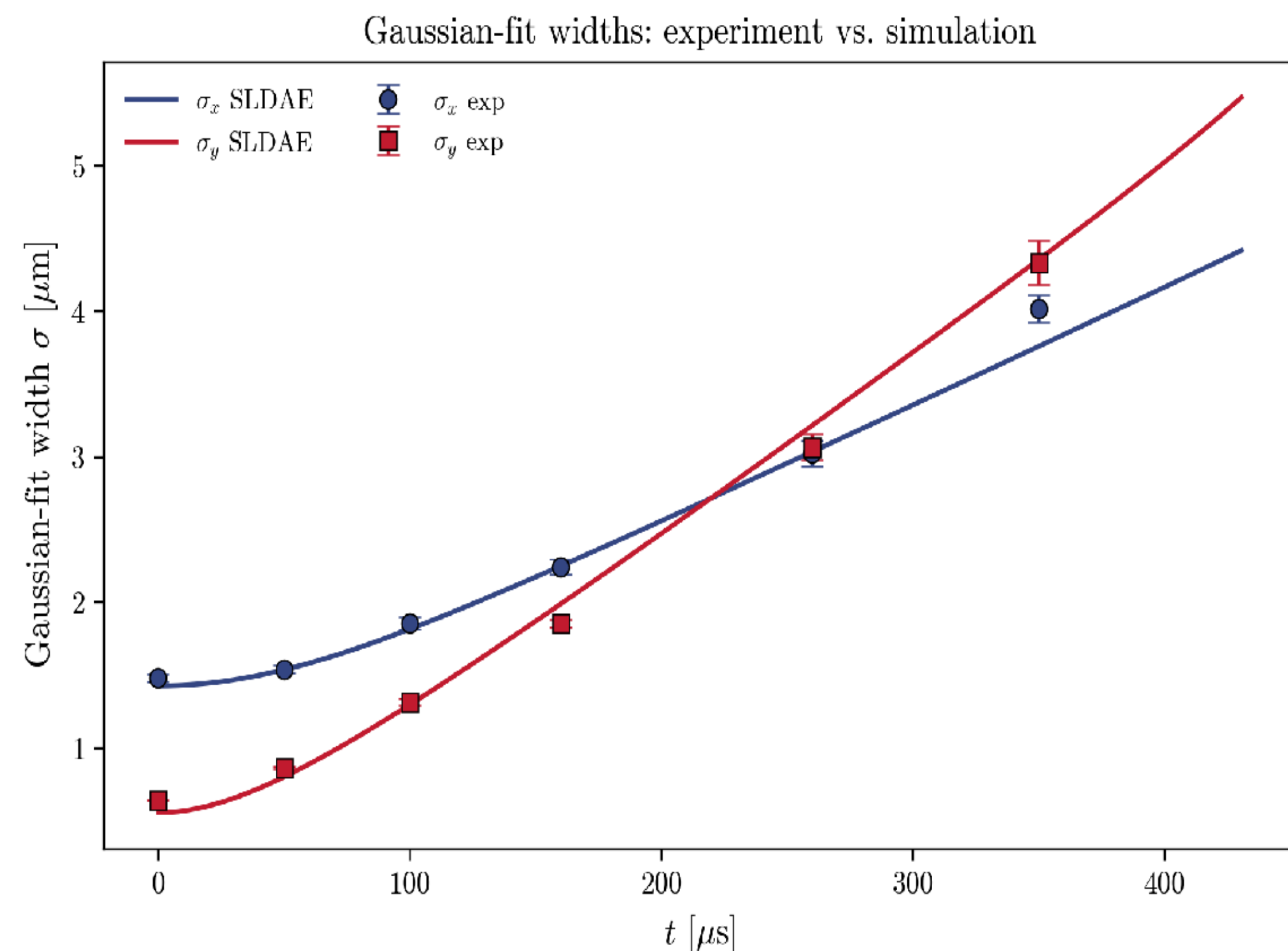
Justifies scaling Ansatz !

$$n(\vec{x}, t) = \frac{n_0(x_i/b_i)}{b_x b_y b_z}$$



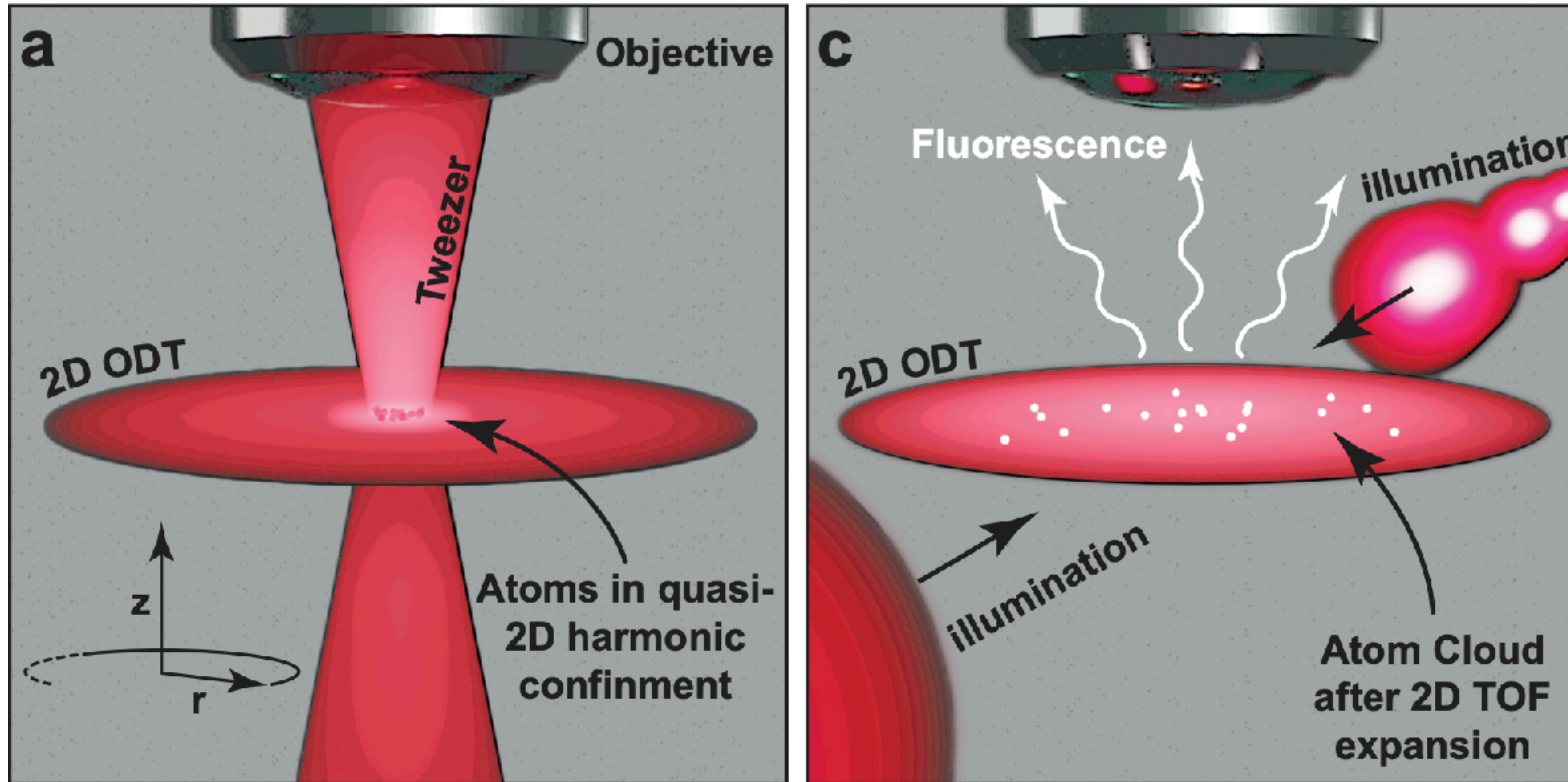
Summary and outlook

- DFT is a microscopic technique to solve many-body quantum problems
- Reproduces measured elliptic flow of 10 atoms (only inputs: scattering length + trap frequencies). Extendable to any atom number N .
- Pairing responsible for collectivity in this framework.
- Viscous scaling hydrodynamics with nonzero quantum pressure describes density evolution.
- Scaling Ansatz justified through full anisotropic hydro simulation.
- Thanks! Questions?



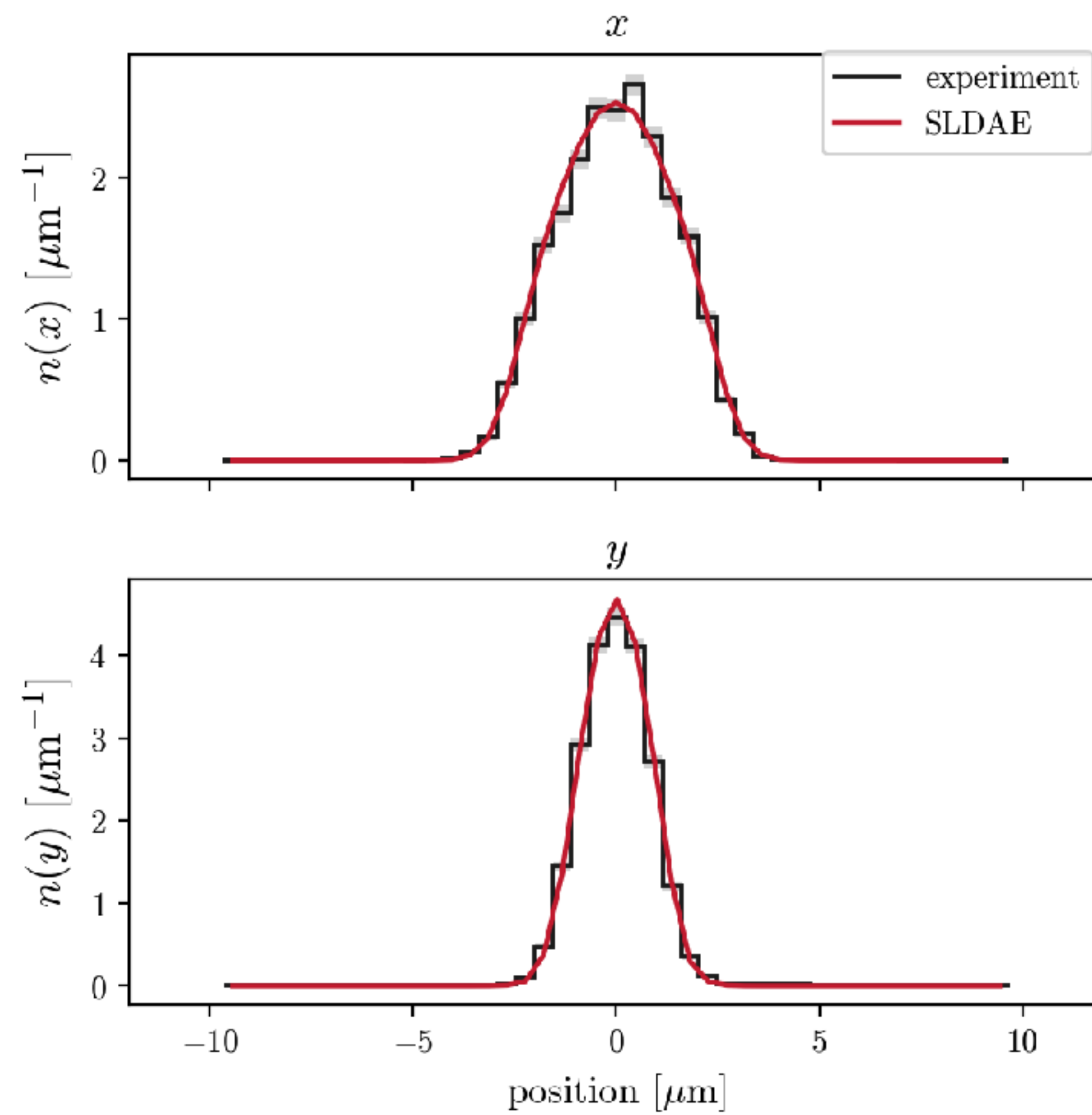
Email: sharell@thphys.uni-heidelberg.de

Backup slides

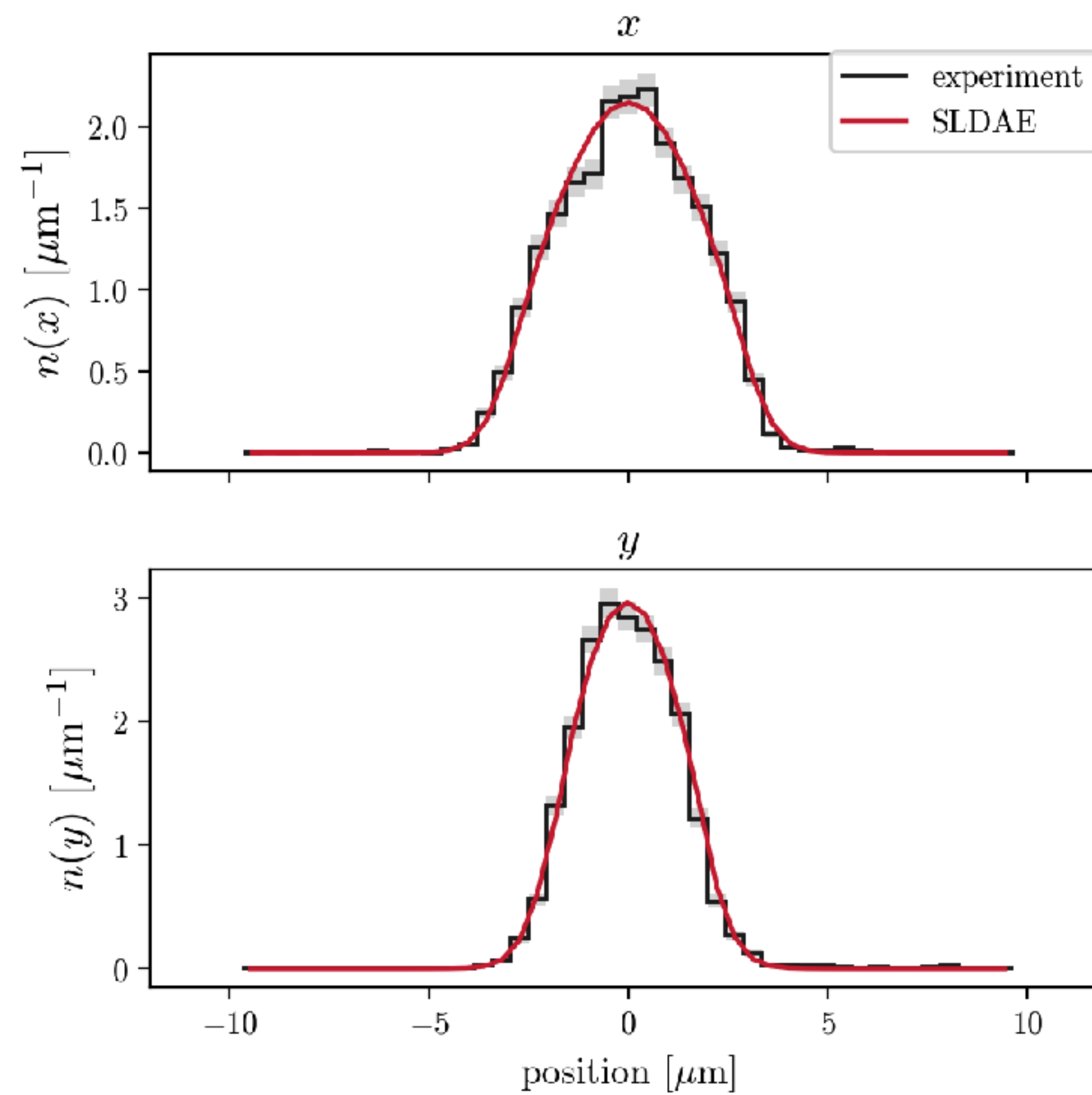


Holten, Bayha, Subramanian, Brandstetter, Heintze, Lunt, Preiss, and Jochim, arXiv:2109.11511

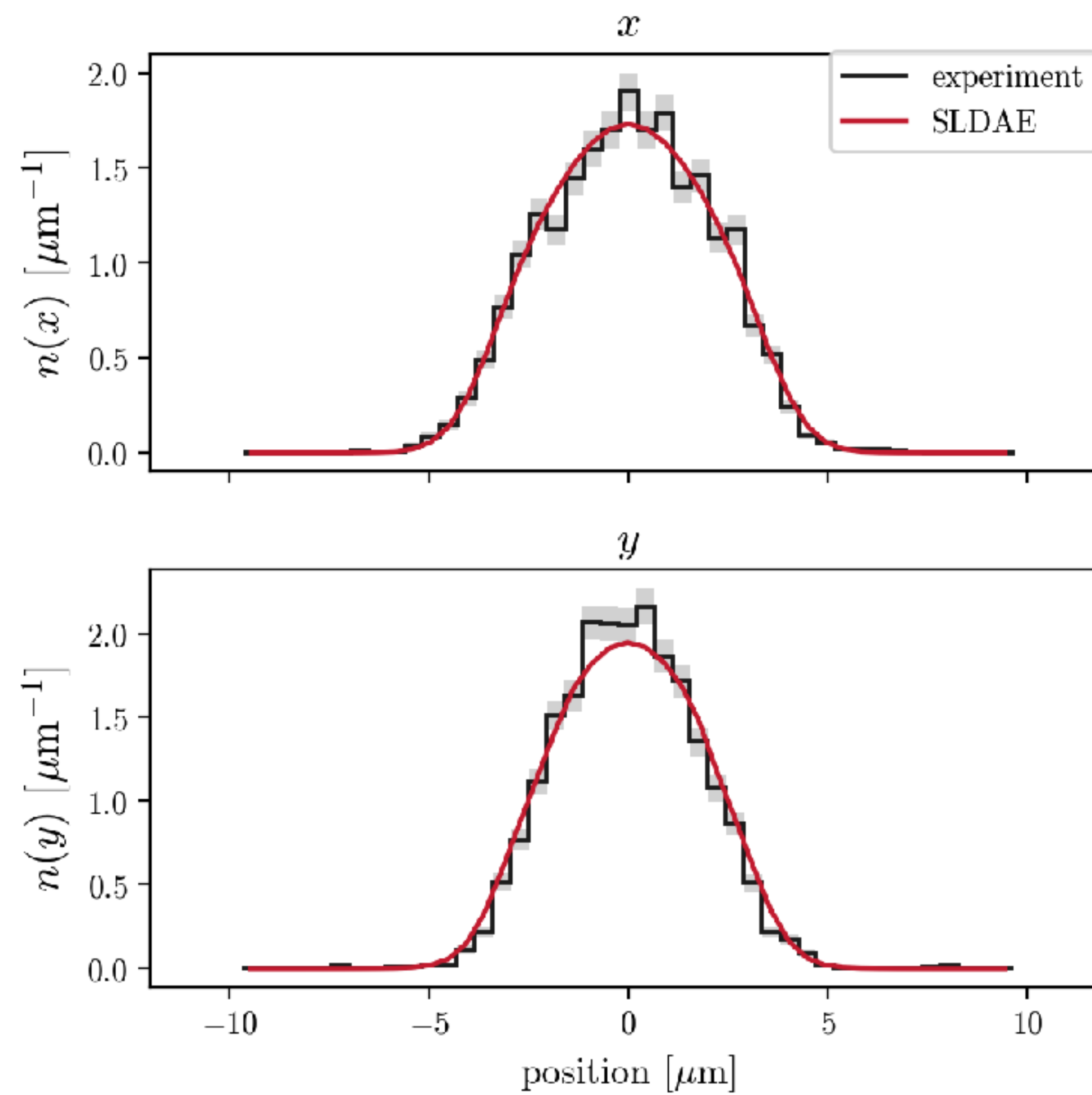
Marginal profiles, $t = 50 \mu s$



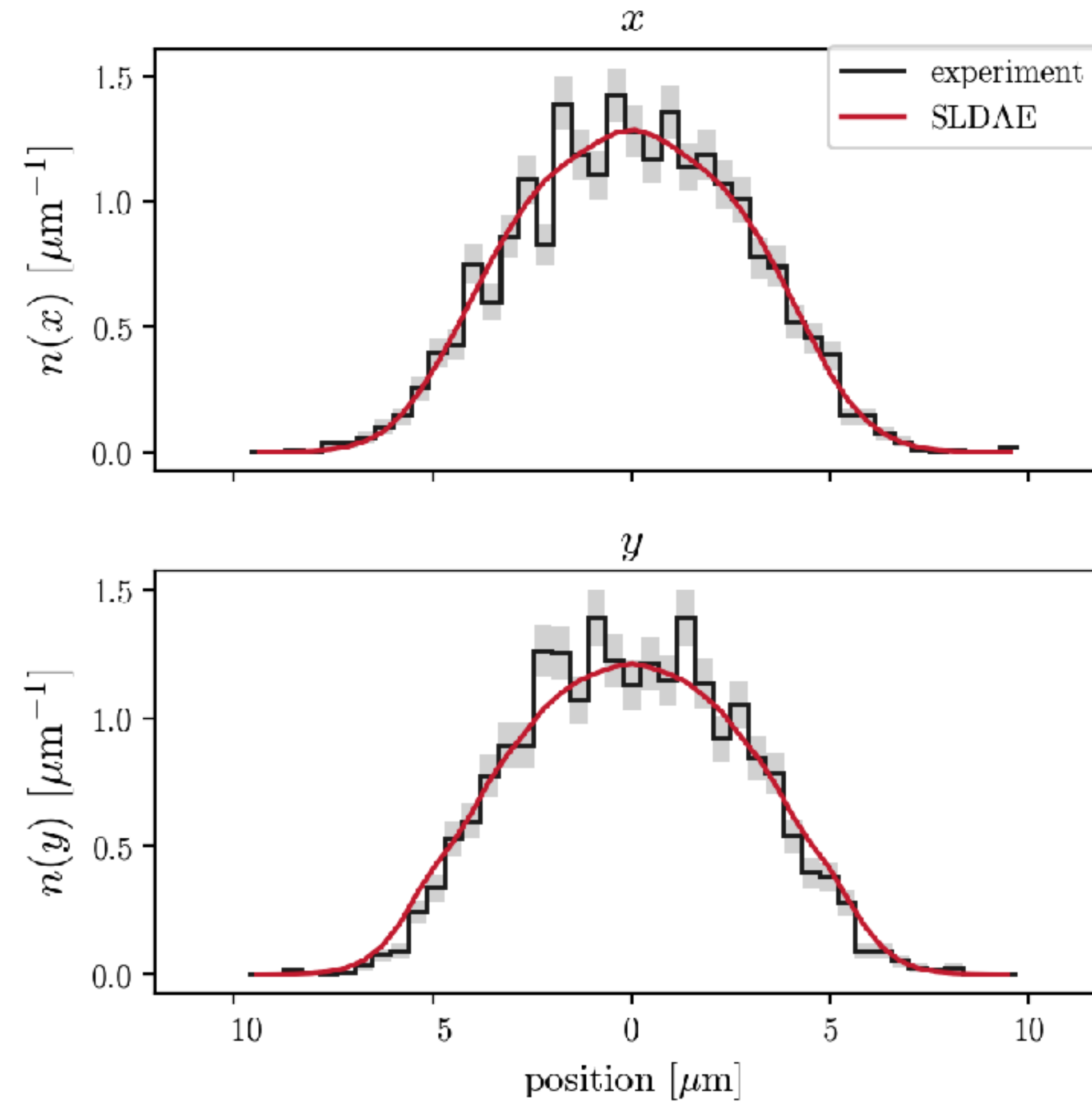
Marginal profiles, $t = 100 \mu s$



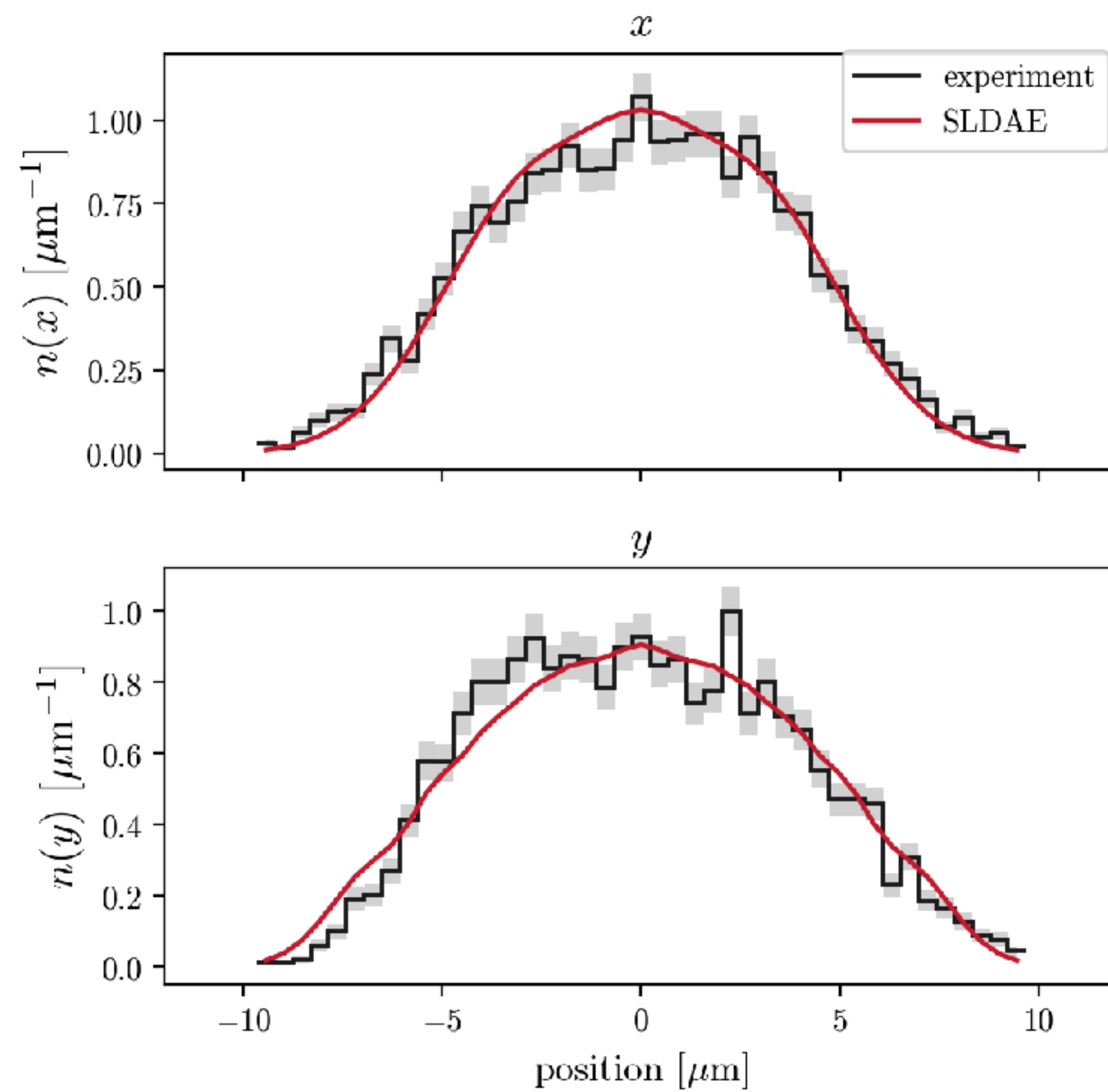
Marginal profiles, $t = 160 \mu s$



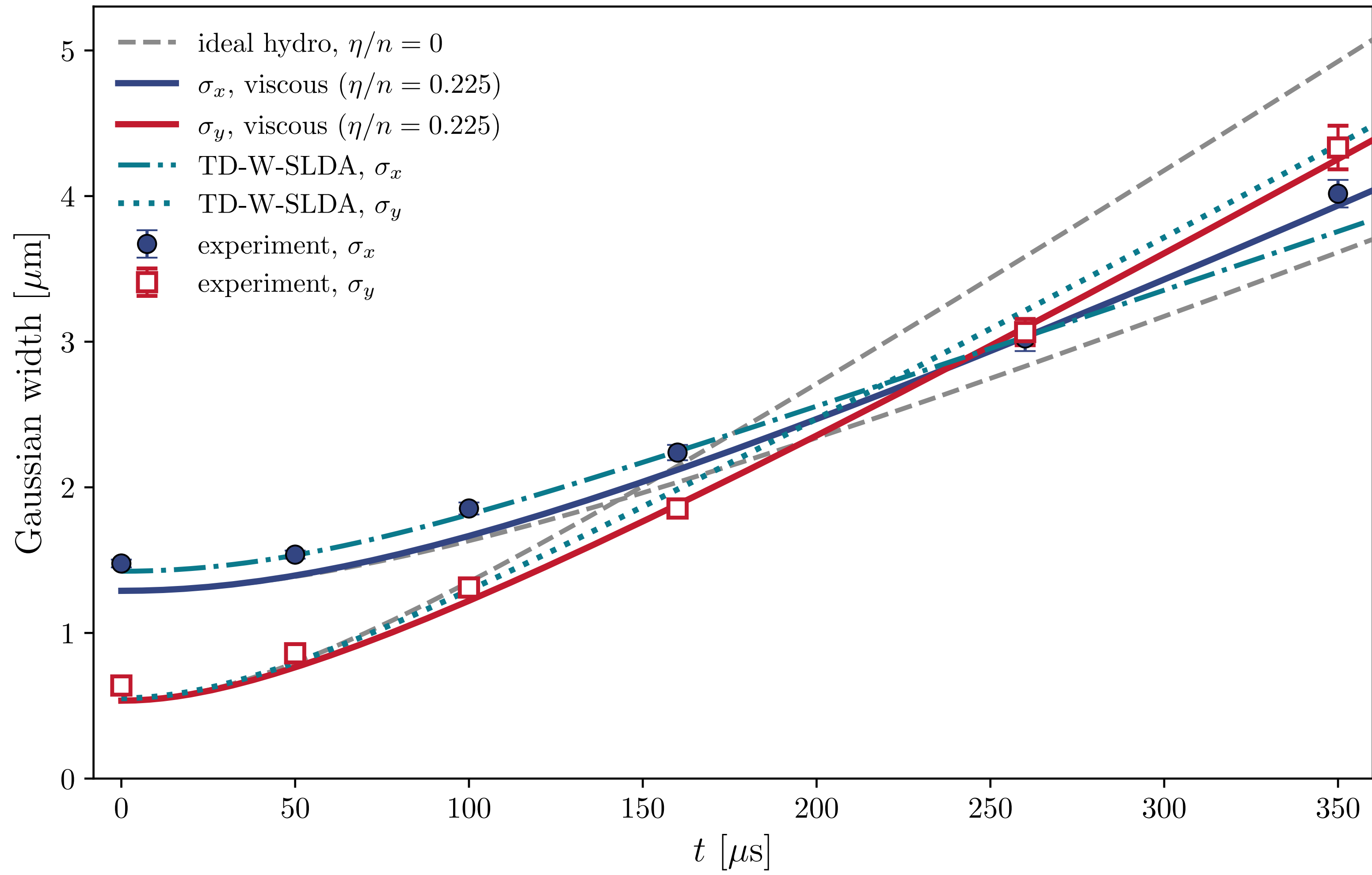
Marginal profiles, $t = 260 \mu s$



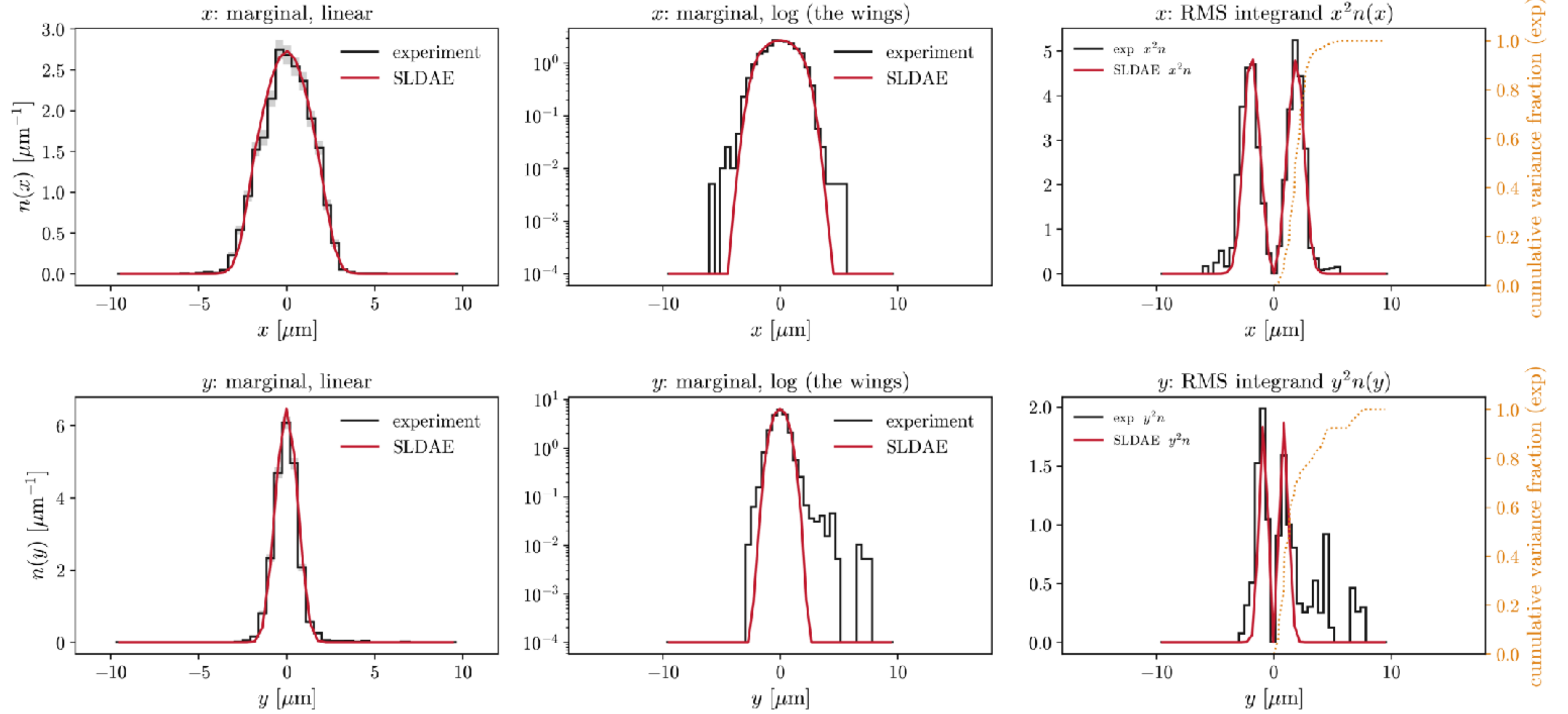
Marginal profiles, $t = 350 \mu\text{s}$

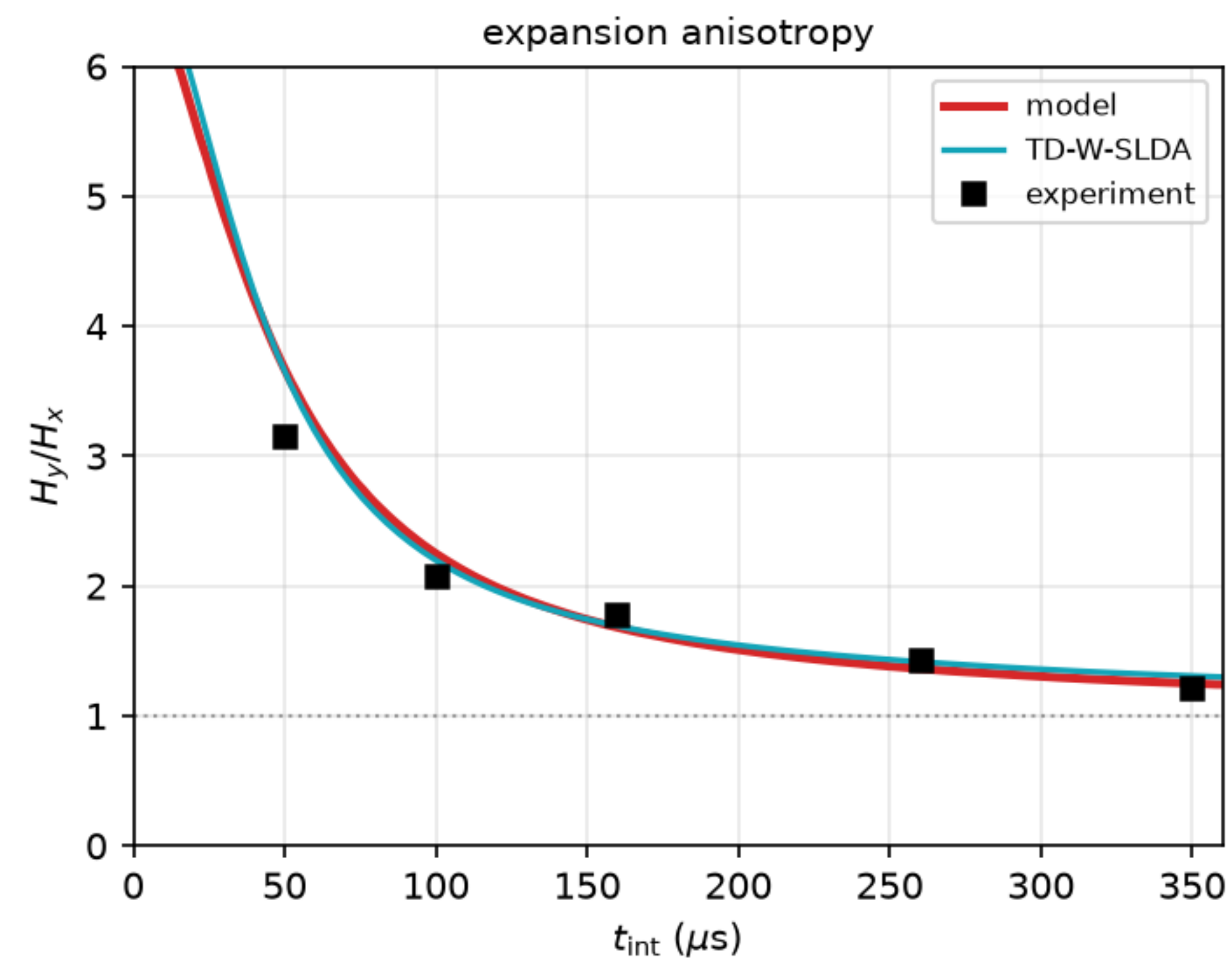
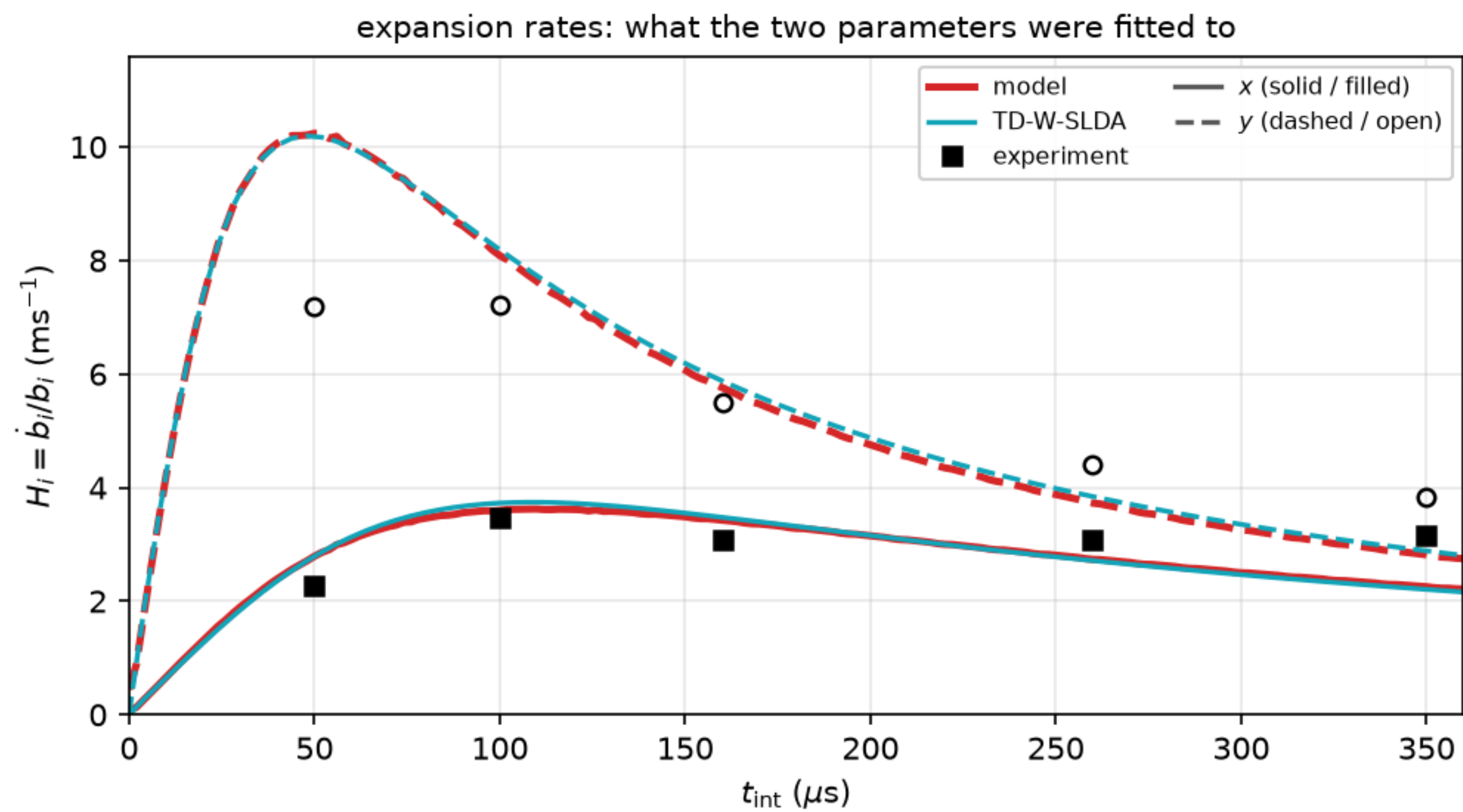


Scaling hydrodynamics (3D) + quantum pressure
 $\lambda_W = 0.30, \eta/n = 0.225$



$t = 0 \mu\text{s}$: core, wings, and what drives the RMS





How to find the auxiliary potential

- Exact diagonalisation: $\min_{\Psi} H[\Psi] = E_0 = H[\Psi_0]$
- Intractable for $N > 6$ $\Psi = \Psi(x_1, x_2, \dots, x_N), x_i \in \mathbb{R}^d$
- Hohenberg-Kohn Theorem: Existence of $\min_n \mathcal{E}[n] = E_0 = \mathcal{E}[n_0(x)]$

Mapping to noninteracting system via full energy functional

$$\mathcal{E}_{\text{ext}} = \int n(\mathbf{r}) V_{\text{ext}}(\mathbf{r}) d^3\mathbf{r} \quad \longrightarrow \quad v_{\text{aux}} = \frac{\delta \mathcal{E}_{\text{full}}}{\delta n}$$

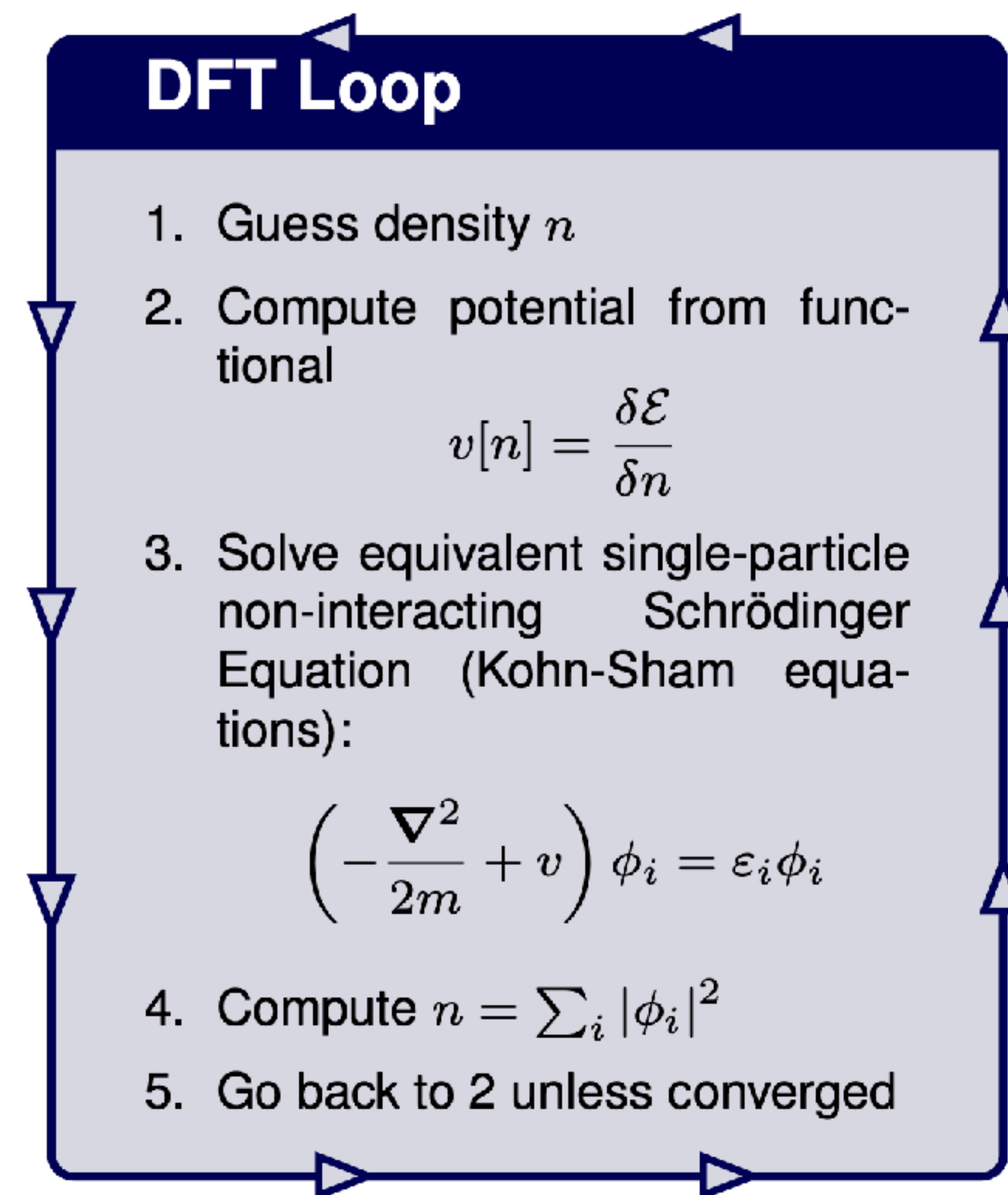
The self-consistent loop

Slater determinant

$$\Phi = \frac{1}{\sqrt{N!}} \det(\phi_i(x_j))$$

Single particle problem

$$\mathcal{H}\phi_i = \left(-\frac{\nabla^2}{2m} + V \right) \phi_i = \varepsilon_i \phi_i$$



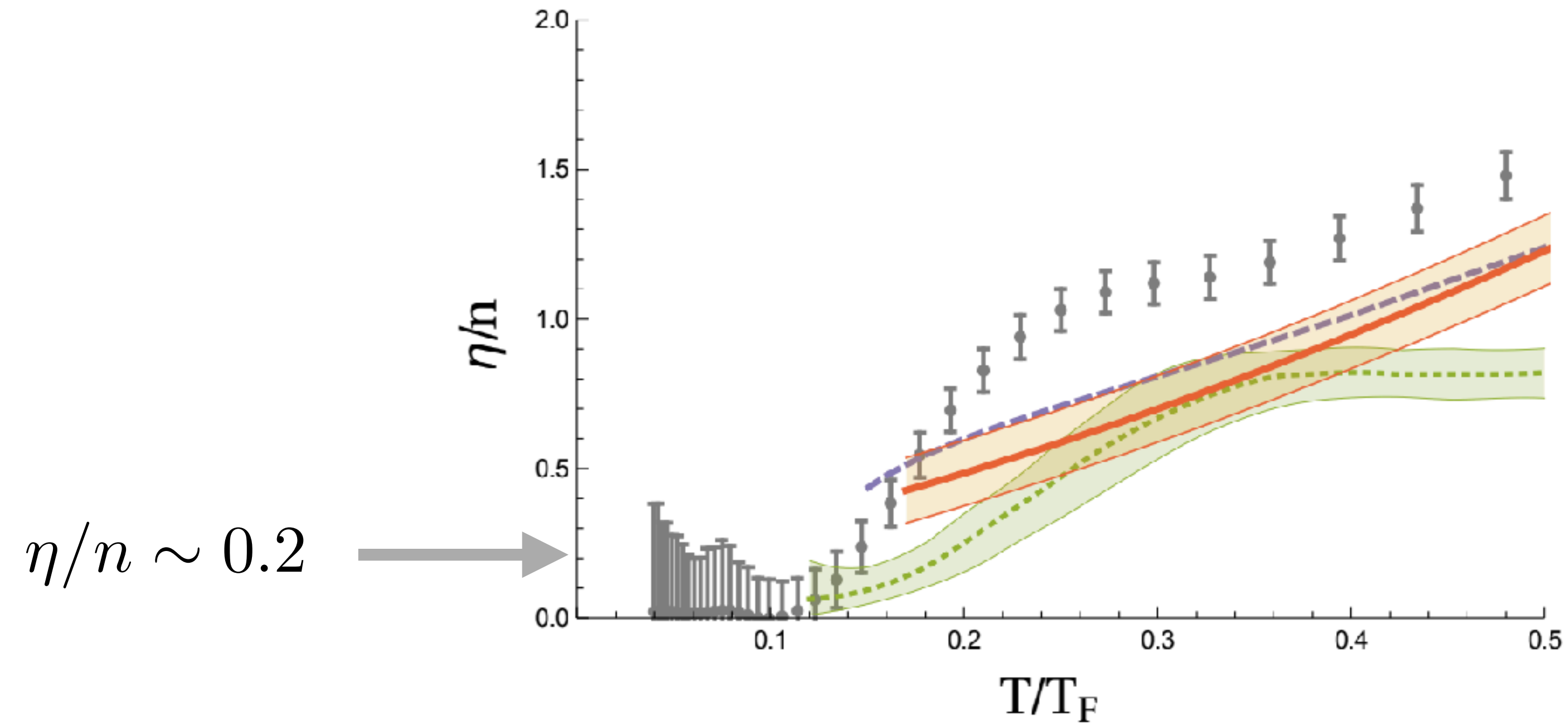


FIG. 4: Same as Fig. 3, zooming in on the low temperature regime. Our analysis (red band) is compared to the results (gray points) obtained in [1], the T -matrix calculation (dashed line) of Enss et al. [29], and the lattice calculation (green band) of Wlazlowski et al. [30].