

Microscopic nuclear structure calculations for $^{76}\text{Se}/^{76}\text{Ge}$ isobar collisions

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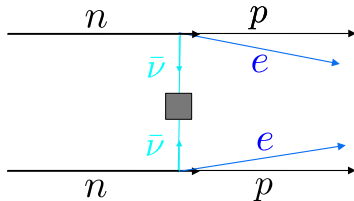
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Motivation for $^{76}\text{Se}/^{76}\text{Ge}$: double beta decay

Neutrinoless double beta decay

$$0\nu\beta\beta: 2n \rightarrow 2p + 2e^-$$



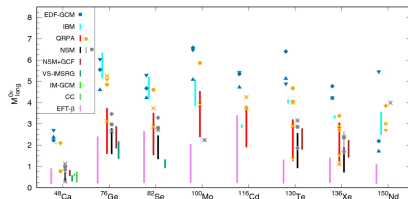
Beyond the Standard model!

- ▶ Dirac or Majorana particle?
- ▶ Absolute neutrino masses
- ▶ Matter-antimatter asymmetry

Half-life of $0\nu\beta\beta$

$$T_{1/2}^{-1} = G_{0\nu} \cdot |\langle m_{\beta\beta} \rangle|^2 \cdot |M_{0\nu}|^2$$

- ▶ Atomic and kinematics
- ▶ Majorana neutrino mass
- ▶ Nuclear matrix element are poorly known



JJ Gómez-Cadenas *et al.* Riv. Nuovo Cim. 46,619 (2023)

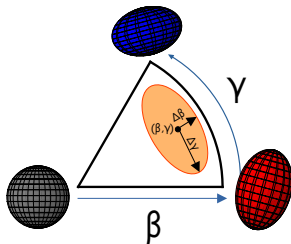
Predictions of $0\nu\beta\beta$ need precise knowledge of **nuclear structure!**

- ▶ Deformation of initial and final nuclei

Characterization of the deformation

Deformation parameters (β, γ)

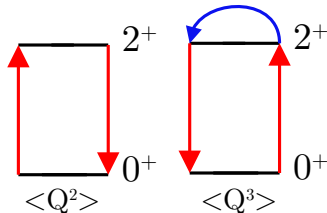
- ▶ Intrinsic frame
- ▶ β : magnitude of deformation
 - $\beta = 0$: Spherical
- ▶ γ : type of deformation
 - $\gamma = 0^\circ$: **Prolate**
 - $\gamma = 60^\circ$: **Oblate**
 - $0^\circ < \gamma < 60^\circ$: **Triaxial**



Shape invariants $\langle Q^n \rangle$: coupled quadrupole operators $Q_2 = r^2 Y_{20}$

- ▶ Laboratory frame
- ▶ $\langle Q^2 \rangle = \langle [Q_2 \times Q_2]_0 \rangle \rightarrow \beta^2$
- ▶ $\langle Q^3 \rangle = \langle [[Q_2 \times Q_2]_2 \times Q_2]_0 \rangle \rightarrow \gamma$
- ▶ All $\langle Q^n \rangle$ up to $\langle Q^6 \rangle$ are needed for fluctuations of (β, γ)

K. Kumar, PRL 28, 249 (1972)
D. Cline, Nuclear Structure 313–326 (1985)
A. Poves, et al. PRC 101, 054307 (2020)



Nuclear shell model

Schrödinger equation

$$\mathcal{H}|\Psi\rangle = E|\Psi\rangle$$

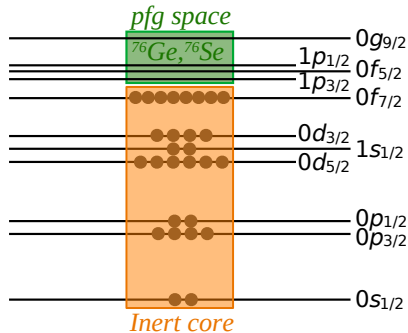
Nuclear shell model (NSM):

- ▶ Slater determinant basis $\{\Phi_i\}$
- ▶ Configuration mixing:
 $|\Psi_{\text{NSM}}\rangle = \sum_i C_i |\Phi_i\rangle$

Caurier E., et al. Rev. Mod. Phys. **77**, 427 (2005)

Effective $\mathcal{H}$ in **valence space**

- ▶ Nucleon-nucleon scattering
- ▶ Fit to selected nuclei near closed shells



Phenomenological interactions:

- ▶ JJ4BB¹, JUN45², GCN2850³, RGprolate⁴

¹ A. F. Lisetskiy, et al., PRC 70, 044314 (2004)

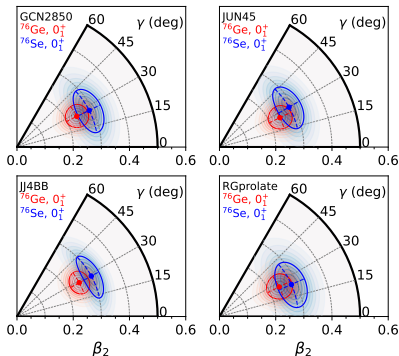
² M. Honma, et al., PRC 80, 064323 (2009)

³ E. Caurier, et al., PRL 100, 052503 (2008)

⁴ J. Menéndez, et al., PRC 80, 048501 (2009)

The shapes of $^{76}\text{Ge}/^{76}\text{Se}$

Shape invariants of $^{76}\text{Ge}/^{76}\text{Se}$
for different interactions:



D. Castillo, D. Frycz, B. Benavente & J. Menéndez
Phys. Lett. B 875, 140306 (2026)

Averaged (β, γ) for interactions:

- ▶ $(\beta, \gamma) \simeq (0.24 \pm 0.01, 27 \pm 2^\circ)$
- ▶ $(\beta, \gamma) \simeq (0.29 \pm 0.01, 26 \pm 3^\circ)$

General characteristics

- ▶ Mass distribution ($e_p = e_n$)
- ▶ ^{76}Se is **more deformed**
- ▶ ^{76}Ge and ^{76}Se are almost **maximally triaxial** $\gamma \approx 30^\circ$

We use JJ4BB for this study
(largest β difference)

- ▶ $^{76}\text{Ge} (\beta, \gamma) = (0.25, 29^\circ)$
- ▶ $^{76}\text{Se} (\beta, \gamma) = (0.30, 29^\circ)$

Experimental estimates ($E2$):

- ▶ $^{76}\text{Ge} (\beta, \gamma) \approx (0.26, 27^\circ)$
- ▶ $^{76}\text{Se} (\beta, \gamma) \approx (0.29, 24^\circ)$

J. Henderson et al. PRC 99, 054313 (2019)

A.D.Ayangeakaa et al. PRL 123, 102501 (2019)

Relativistic Ion collisions (RIC)

Shapes of nuclei have large impact in RIC analysis

- Final particle anisotropy:

$$\frac{dN}{d\phi} \propto 1 + 2 \sum_n v_n \cos[n(\phi - \phi_n)]$$

- Hydrodynamics: $\langle v_n^2 \rangle \propto \langle \varepsilon_n^2 \rangle$

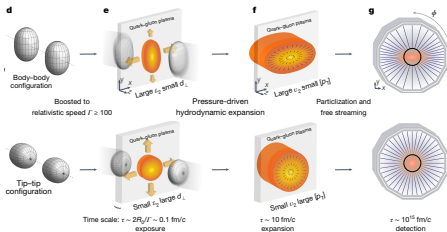
T. Duguet, et. al., PRL 135, 182301 (2025)

Connection to nuclear shape

- For a Gaussian density $\langle \varepsilon_2^2 \rangle = \frac{1}{A} + \frac{3}{4\pi} \beta_2^2$

S. Bofos, et. al., PLB 879, 140649 (2026)

- Shape invariants $\langle Q^2 \rangle \rightarrow \langle \beta_2^2 \rangle$



STAR Collaboration, Nature 67, 635 (2024)

- We predict final observable ratios using nuclear density informed by these nuclear structure calculations

Woods-Saxon density

Shape invariants from JJ4BB

- ▶ ^{76}Ge $(\beta_2, \gamma) = (0.25, 29^\circ)$
- ▶ ^{76}Se $(\beta_2, \gamma) = (0.30, 29^\circ)$

Root-mean radius from JJ4BB:

- ▶ ^{76}Ge $r_{rms} = 4.07$ fm
- ▶ ^{76}Se $r_{rms} = 4.08$ fm

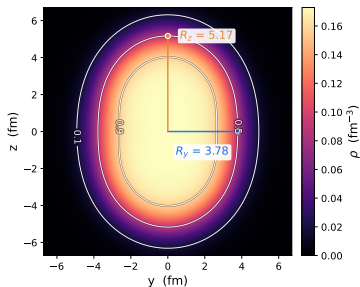
Matching Woods-Saxon and
shape invariants

- ▶ We fix $a^{WS} = 0.52$ fm
- ▶ Parameters to adjust:
 $(R_0^{WS}, \beta_2^{WS}, \gamma^{WS})$

B. Bally, et al. Eur. Phys. J. A 59, 58 (2023)

Woods-Saxon parametrization:

$$\rho^{WS}(\mathbf{r}) = \frac{\rho_0}{1 + e\left(\frac{r - R^{WS}}{a^{WS}}\right)},$$
$$R^{WS} = R_0^{WS} \left[1 + \beta_2^{WS} \cos \gamma^{WS} Y_{20} + \beta_2^{WS} \sin \gamma^{WS} \frac{Y_{22} + Y_{2-2}}{\sqrt{2}} \right]$$



Woods-Saxon density of $^{76}\text{Se}/\text{Ge}$

Woods-Saxon integrals:

$$\langle O \rangle_{WS} = \int d^3r \rho^{WS}(\mathbf{r}) O$$

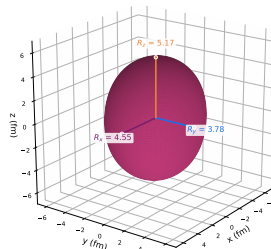
$$\sqrt{\langle r^2 \rangle}_{WS} \simeq r_{rms}; \beta_{2\mu} \propto Q_{2\mu} = r^2 Y_{2\mu}$$

$$\langle \beta_2 \rangle_{WS} = \sqrt{\langle \beta_{20}^2 \rangle_{WS} + 2 \langle \beta_{22}^2 \rangle_{WS}} \simeq \beta_2$$

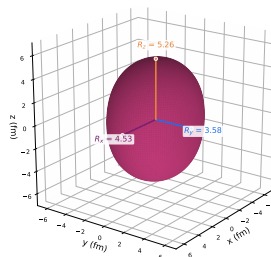
$$\langle \gamma \rangle_{WS} = \text{atan} \left(\frac{\sqrt{2} \langle Q_{22} \rangle_{WS}}{\langle Q_{20} \rangle_{WS}} \right) \simeq \gamma$$

^{76}Ge	r_{rms}/R^{WS}	β_2/β_2^{WS}	γ/γ^{WS}
NSM	4.07 fm	0.25	29°
WS	4.53 fm	0.28	33°
^{76}Se	r_{rms}/R^{WS}	β_2/β_2^{WS}	γ/γ^{WS}
NSM	4.08 fm	0.30	29°
WS	4.50 fm	0.35	34°

^{76}Ge - $\rho_0/2$ surface and principal radii
 $\beta = 0.283$, $\gamma = 33.5^\circ$, $R_0 = 4.53$ fm, $a = 0.52$ fm



^{76}Se - $\rho_0/2$ surface and principal radii
 $\beta = 0.345$, $\gamma = 34.5^\circ$, $R_0 = 4.5$ fm, $a = 0.52$ fm



Simulations of $^{76}\text{Se}/\text{Ge}$ collisions

► **SMASH**+fluidization

J. Weil et al., PRC 94, 054905 (2016)

► Midrapidity $\eta \leq 0.5$

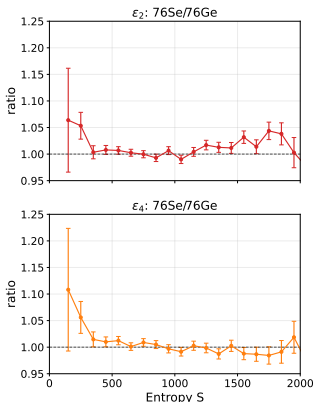
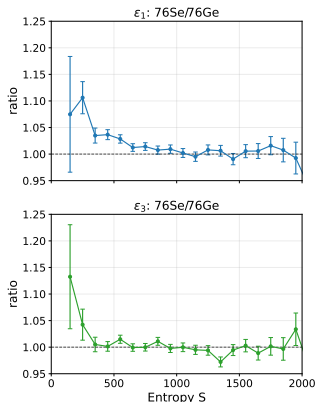
► 100.000 events

D. Frycz, A. Reina et al. in preparation

► $\varepsilon_1, \varepsilon_3, \varepsilon_4$ ratios are compatible with unity at 1σ for $S > 500$

► ε_2 ratio: 5 % enhancement in most central collisions

► $\langle \varepsilon_2^2 \rangle = \frac{1}{A} + \frac{3}{4\pi} \beta_2^2 \rightarrow 13\%$ estimate



Conclusions

Motivation for $^{76}\text{Se}/^{76}\text{Ge}$:

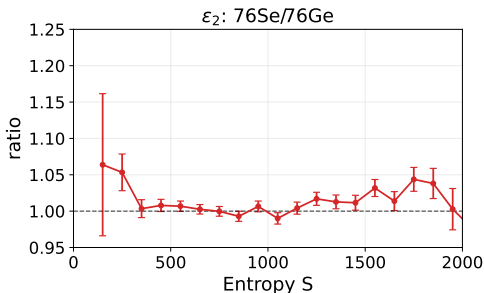
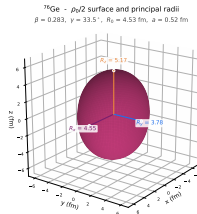
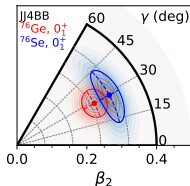
- ▶ $0\nu\beta\beta$ candidate
- ▶ Knowledge on $^{76}\text{Ge}/\text{Se}$ deformation

Shape invariants

- ▶ Characterization of the **deformation**
- ▶ Connection to **Woods-Saxon density**

Relativistic ion collisions:

- ▶ SMASH simulations
- ▶ Deformation difference induces a **5% ε_2 enhancement**



THANK YOU!!!

Feel free to ask any questions

Predictions of $^{76}\text{Se}/^{76}\text{Ge}$ collisions from nuclear structure calculations

Ángel Reina Ramírez^{1,2}, V.K. Magas^{1,2}, D. Frycz^{1,2}, J. Menéndez^{1,2}, J.M. Torres-Rincon^{1,2}, G. Giacalone³
¹*Departament de Física Quàntica i Astrofísica, Universitat de Barcelona, Martí i Franquès 1, 08028 Barcelona, Spain*
²*Institut de Ciències del Cosmos, Universitat de Barcelona, Martí i Franquès 1, 08028 Barcelona, Spain*
³*Theoretical Physics Department, CERN, 1211 Geneva 23, Switzerland*
(Dated: September 22, 2026)

Nuclear shapes: the Kumar-Cline lens

Dorian Frycz,^{1,2,3,*} Alfredo Poves,^{4,†} and Javier Menéndez^{1,2,‡}
¹*Departament de Física Quàntica i Astrofísica, Universitat de Barcelona, 08028 Barcelona, Spain*
²*Institut de Ciències del Cosmos, Universitat de Barcelona, 08028 Barcelona, Spain*
³*Facultat de Física, Universitat de Barcelona, c. Martí i Franquès, 1, 08028 Barcelona, Spain*
⁴*Departamento de Física Teórica and IFT-UAM/CSIC, Universidad Autónoma de Madrid, 28049 Madrid, Spain*
(Dated: September 21, 2026)

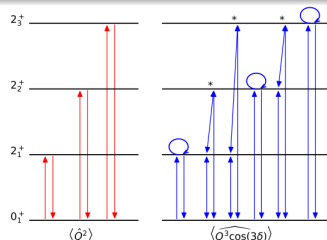
Shape invariants: Intermediate-state expansion

Intermediate-state expansion:

- ▶ (Infinite) expansion of $M_{if} = \langle i || Q || f \rangle \leftarrow Q_s \text{ or } B(E2)$
- ▶ Harder convergence for higher $\langle Q^n \rangle$

$$\langle Q^2 \rangle_i \sim \sum M_{it} M_{ti} \rightarrow \beta$$

$$\langle Q^3 \rangle_i \sim \sum M_{iu} M_{ut} M_{ti} \rightarrow \gamma$$



J. Henderson Phys Rev C 102, 054306 (2020)

Sum-rule method:

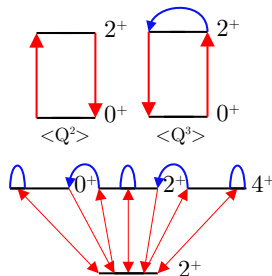
- ▶ Apply the $\hat{Q}_{20}$ operator to a state:

$$\hat{Q}_{20}|0^+\rangle = \text{SR}(2^+)^{1/2}|2^+\rangle$$

$$\langle Q^2 \rangle \sim \text{SR}(2^+), \langle Q^3 \rangle \sim \langle 2^+ | Q | 2^+ \rangle$$
- ▶ Apply $\hat{Q}_{20}$ twice $\rightarrow \langle Q^4 \rangle, \langle Q^6 \rangle$

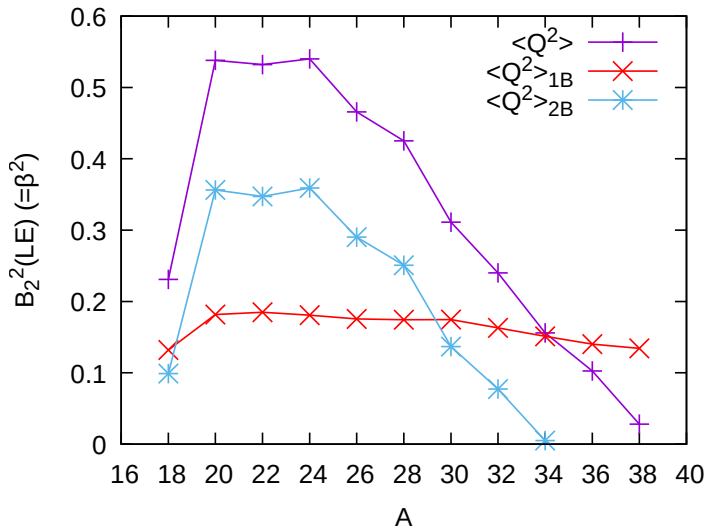
Poves, A., Nowacki, F. & Alhassid, Y. Phys. Rev. C 101, 054307 (2020)

- ▶ **NEW!:** Extension for $J \neq 0$

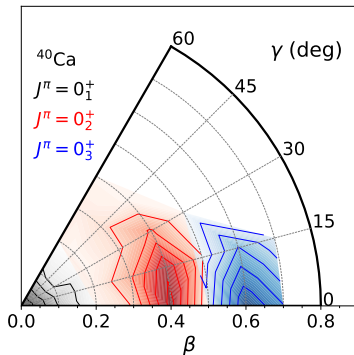
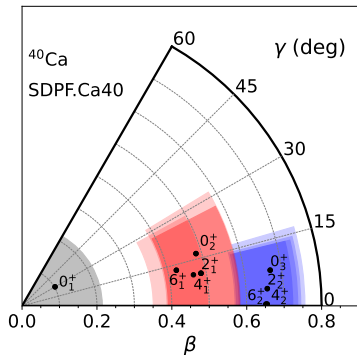


DF, A. Poves, J. Menéndez (in preparation)

2-body contribution of $\langle Q^2 \rangle$



Intrinsic vs laboratory frame connection



Shape invariants:

- ▶ Odd nuclei
- ▶ Nuclear vibrations
- ▶ Multipole deformations

Double-beta decay:

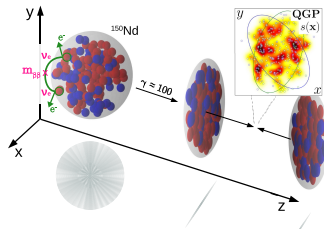
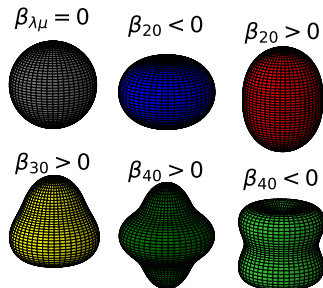
- ▶ Connection to seniority
- ▶ Extension to neutrinoless $\beta\beta$

Relativistic ion collisions:

- ▶ Shape invariants and observables
- ▶ Neutrinoless $\beta\beta$

Y. Li, et al., Phys. Rev. Lett. 135, 022301 (2025)

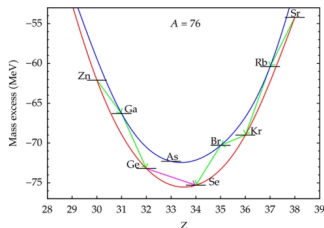
- ▶ Two-body correlations
- ▶ Neutron skin



Motivation for $^{76}\text{Se}/^{76}\text{Ge}$: double beta decay

Neutrino full double beta decay

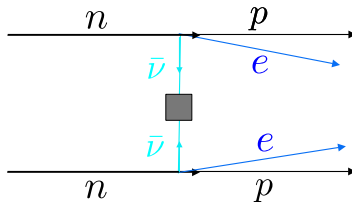
$$2\nu\beta\beta: 2n \rightarrow 2p + 2e^- + 2\bar{\nu}$$



Giuliani, et al. Adv. High Energy Phys. 2012, 857016, 2012.

Neutrinoless double beta decay

$$0\nu\beta\beta: 2n \rightarrow 2p + 2e^-$$



Standard model process

- ▶ When single β is forbidden
- ▶ Very slow but measured
- ▶ $^{76}\text{Ge} \rightarrow ^{76}\text{Se} + 2e^- + 2\bar{\nu}$

Beyond the Standard model!

- ▶ Dirac or Majorana particle?
- ▶ Absolute neutrino masses
- ▶ Matter-antimatter asymmetry

Predictions of $0\nu\beta\beta$ need precise knowledge of **nuclear structure!**

- ▶ Deformation of initial and final nuclei

Deformation and shape coexistence

Deformation is everywhere:

- ▶ Magic nuclei are spherical
- ▶ **Deformation** is enhanced mid-shell (most nuclei)

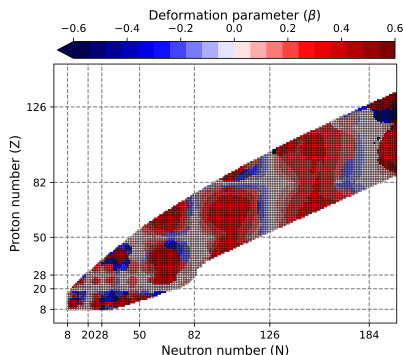
Quadrupole deformation:

- ▶ Most prevalent shape
- ▶ **Prolate**: elongated sphere
- ▶ **Oblate**: flattened sphere

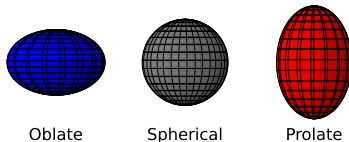
Shape coexistence:

- ▶ **Different shapes** coexist within the **same nucleus** at different energies
- ▶ Present in most nuclei

P. Garret, et al., Prog. Part. Nucl. Phys. 124, 103931 (2022)

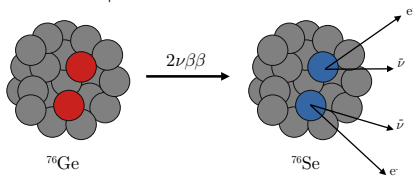
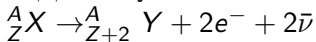


P. Möller, et al. Atom. Data Nucl. Data Tabl.
109-110 (2016)



Double-beta decays

$2\nu\beta\beta$ decays:

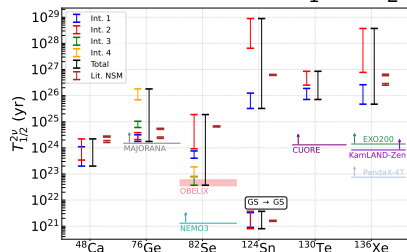


Nuclear matrix elements:

$$M_{GT}^2 = \langle \Psi_f | \hat{O}_{GT} | \Psi_i \rangle^2 \sim T_{1/2}^{-1}$$

related to half-lives ($T_{1/2}$)

Half-life predictions for $0_1^+ \rightarrow 0_2^+$:

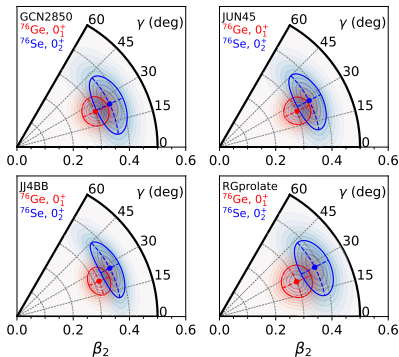


D. Castillo, D. Frycz, B. Benavente & J. Menéndez
[arXiv:2507.21868](https://arxiv.org/abs/2507.21868)

- ▶ Large $T_{1/2}$ uncertainties
 → interactions
- ▶ Structural differences
 → deformation?

Deformation impact on $2\nu\beta\beta$

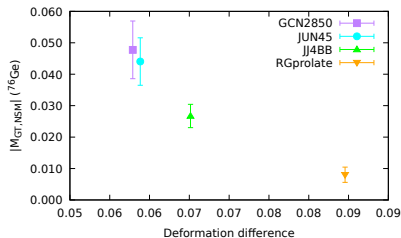
Shape invariants for different interactions:



D. Castillo, D. Frycz, B. Benavente & J. Menéndez
[arXiv:2507.21868](https://arxiv.org/abs/2507.21868)

Impact of deformation on $2\nu\beta\beta$:

- ▶ Deformation differences of $|\Psi_f\rangle$ and $|\Psi_i\rangle$ **reduce** M_{GT}
- ▶ δ : polar plane distance
- ▶ Correlation: M_{GT} with δ
- ▶ Large impact of triaxiality γ



Shape invariants

Deformation is characterized in the **intrinsic frame**: (β, γ)

However, measurements are done in **lab frame**: $(B(E2), Q_s)$

Conversion from lab to intrinsic frame:

$$Q_s(J) \sim Q_{0,s}, \quad J \geq 1/2$$

$$B(E2, J_i \rightarrow J_f) \sim Q_{0,t}^2$$

→ Perfect axial rotor assumption: $Q_{0,s} \simeq Q_{0,t}$

Shape invariants $\langle Q^n \rangle$: model independent

Couple Q_2 (tensor) to obtain a scalar:

- ▶ $\langle Q^2 \rangle = \langle [Q_2 \times Q_2]_0 \rangle \rightarrow \beta$
- ▶ $\langle Q^3 \rangle = \langle [[Q_2 \times Q_2]_2 \times Q_2]_0 \rangle \rightarrow \gamma$
- ▶ All $\langle Q^n \rangle$ up to $\langle Q^6 \rangle$ are needed for fluctuations of (β, γ)

K. Kumar, Phys. Rev. Lett. 28, 249 (1972)

D. Cline, Nuclear Structure 313–326 (1985)

