

Nuclear lattice effective field theory

Dean Lee (for Shihang Shen)
Facility for Rare Isotope Beams
Michigan State University
Nuclear Lattice EFT Collaboration

EMMI Workshop: Light Ion Collisions 2026
Kirchhoff-Institut für Physik, Heidelberg
September 23, 2026



Outline

Lattice effective field theory

Auxiliary field Monte Carlo

Wavefunction matching

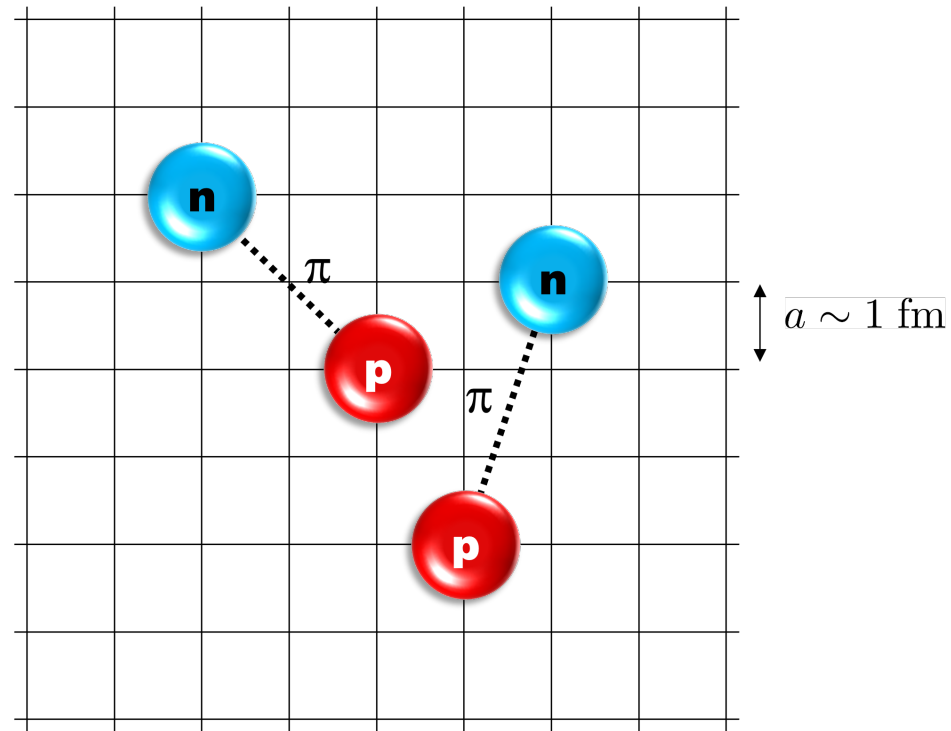
Pinhole algorithm

Mirror approximation

Multimodal superfluidity

Summary and outlook

Lattice effective field theory

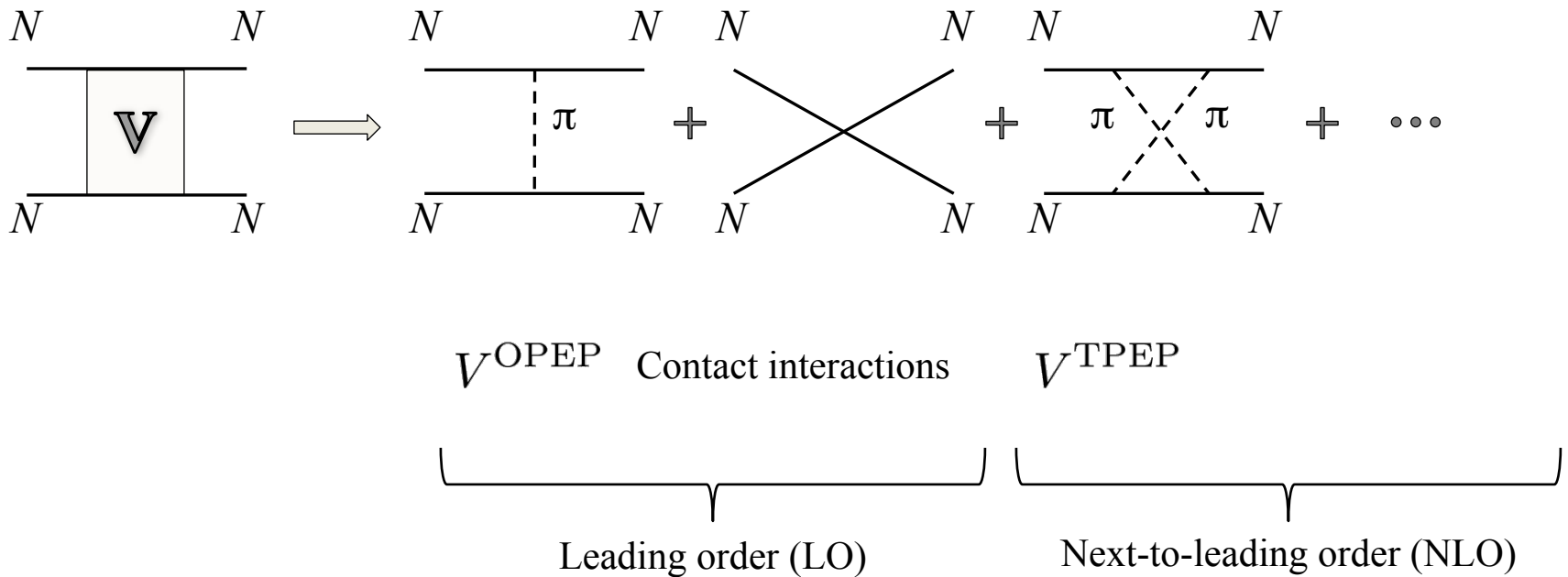


D.L., Annu. Rev. Nucl. Part. Sci. 75, 109 (2025)

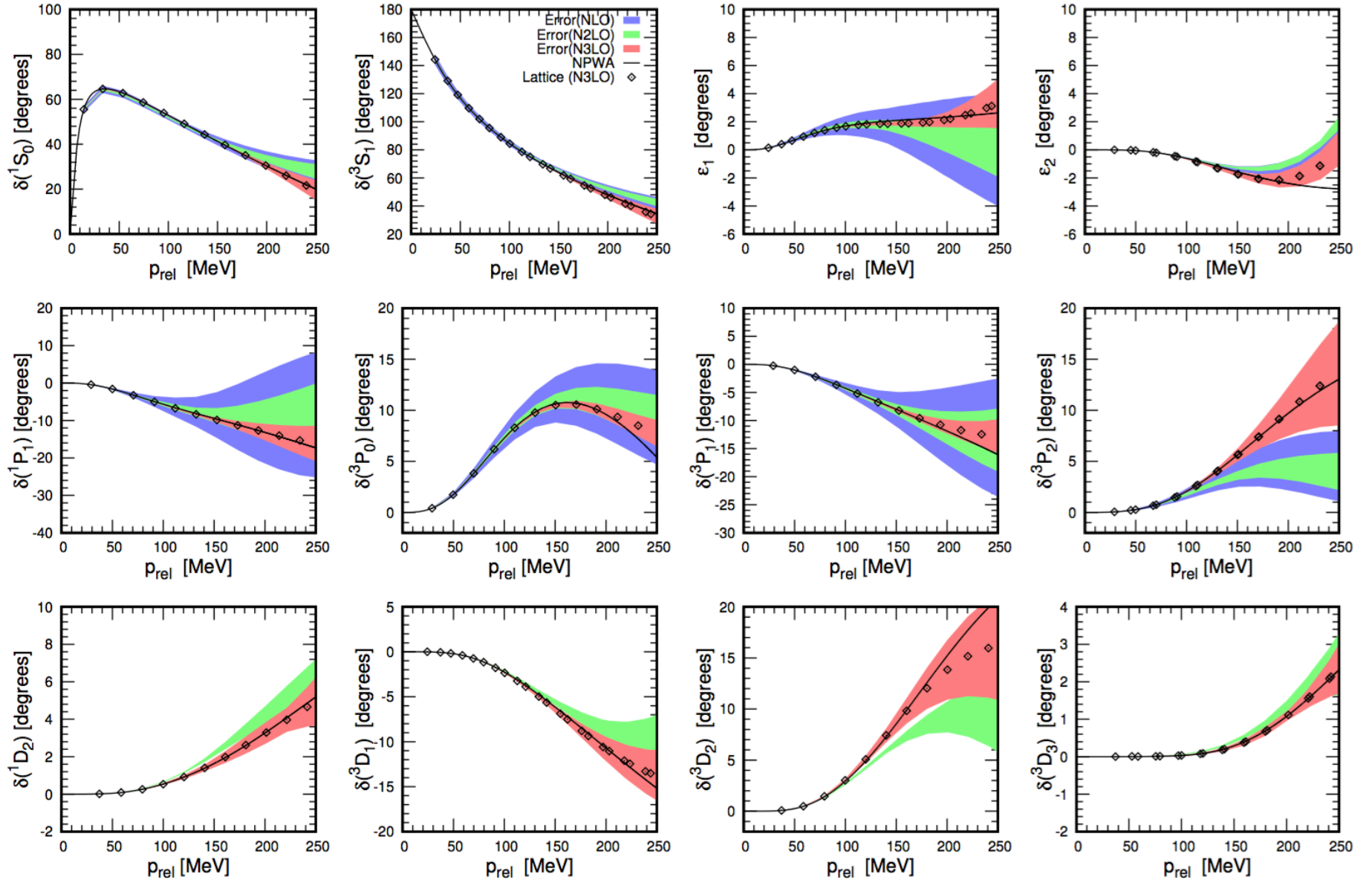
Lähde, Meißner, Nuclear Lattice Effective Field Theory (2019), Springer

Chiral effective field theory

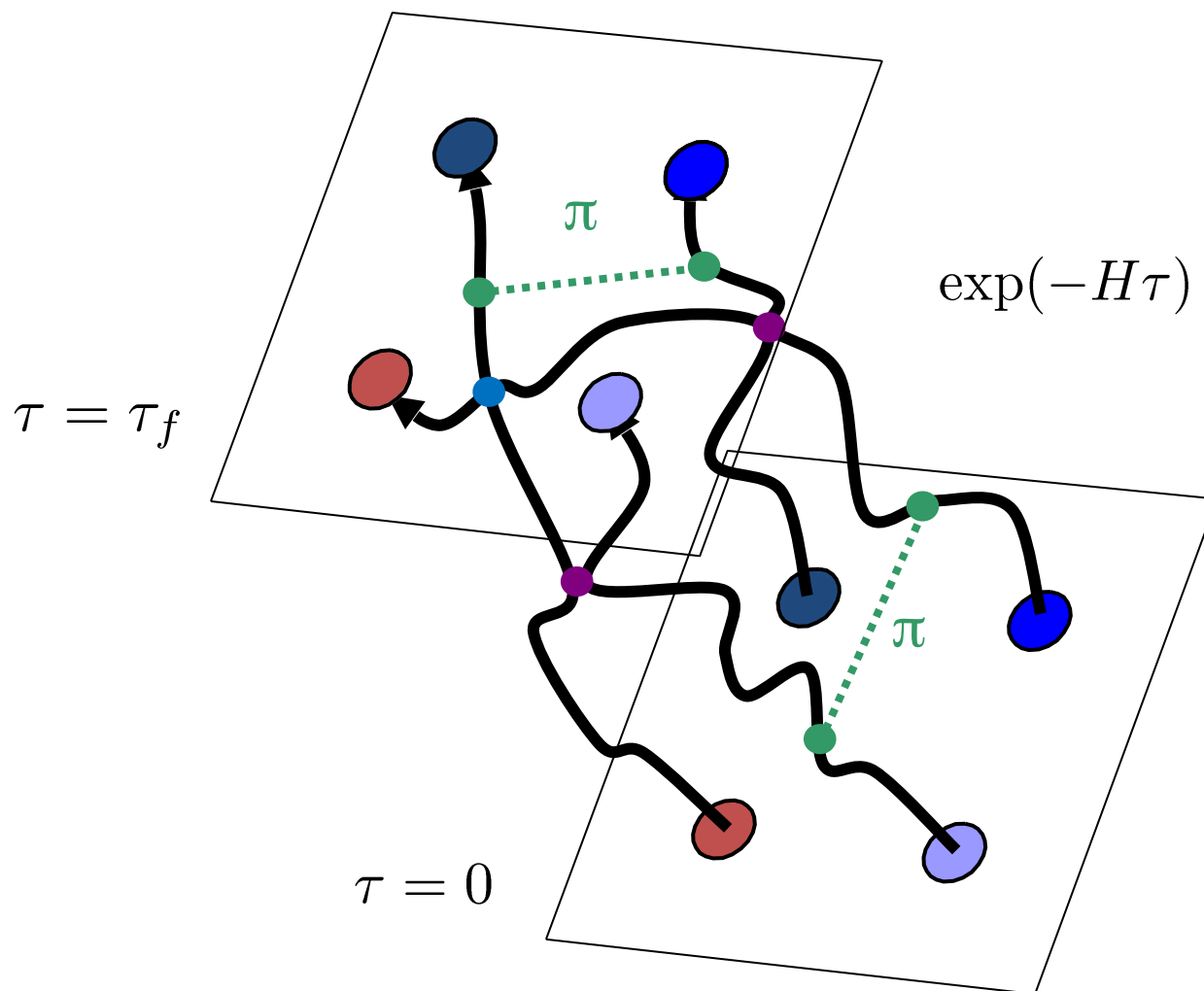
Construct the effective potential order by order



$$a = 1.315 \text{ fm}$$



Euclidean time projection



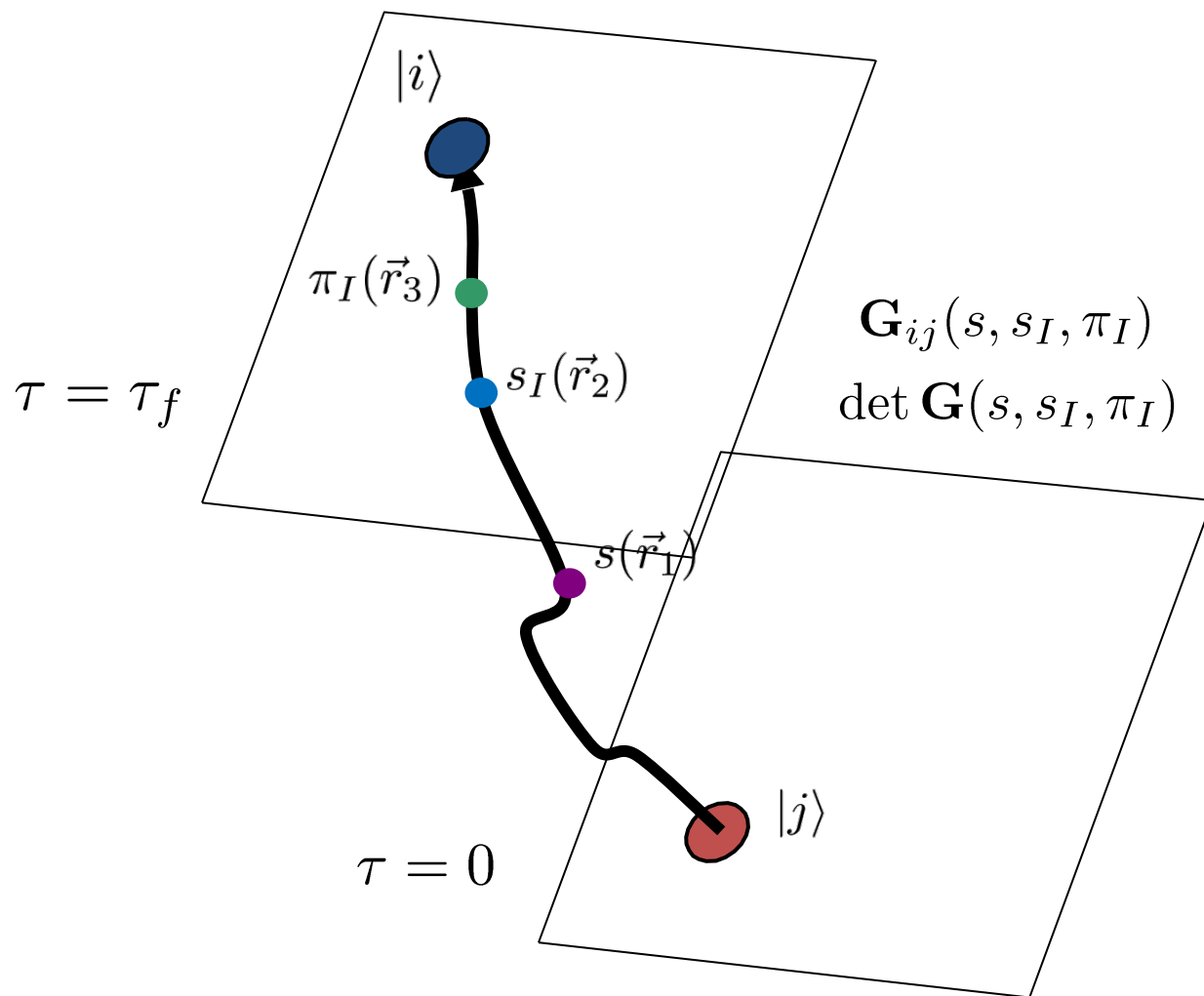
Auxiliary field method

We can write exponentials of the interaction using a Gaussian integral identity

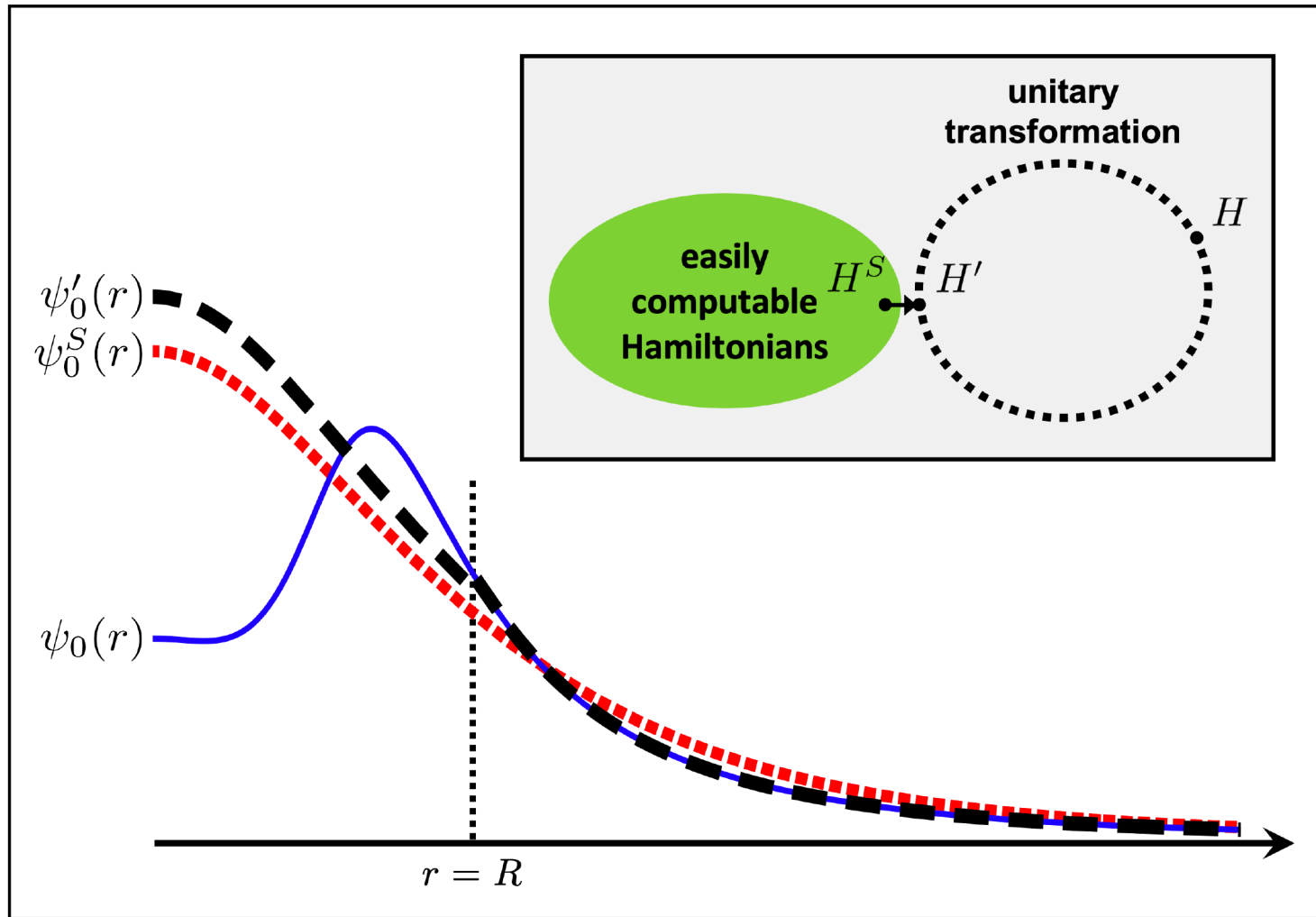
$$\exp \left[-\frac{C}{2} (N^\dagger N)^2 \right] \quad \text{X} \quad (N^\dagger N)^2$$

$$= \sqrt{\frac{1}{2\pi}} \int_{-\infty}^{\infty} ds \exp \left[-\frac{1}{2} s^2 + \sqrt{-C} s (N^\dagger N) \right] \quad \text{>} \quad s N^\dagger N$$

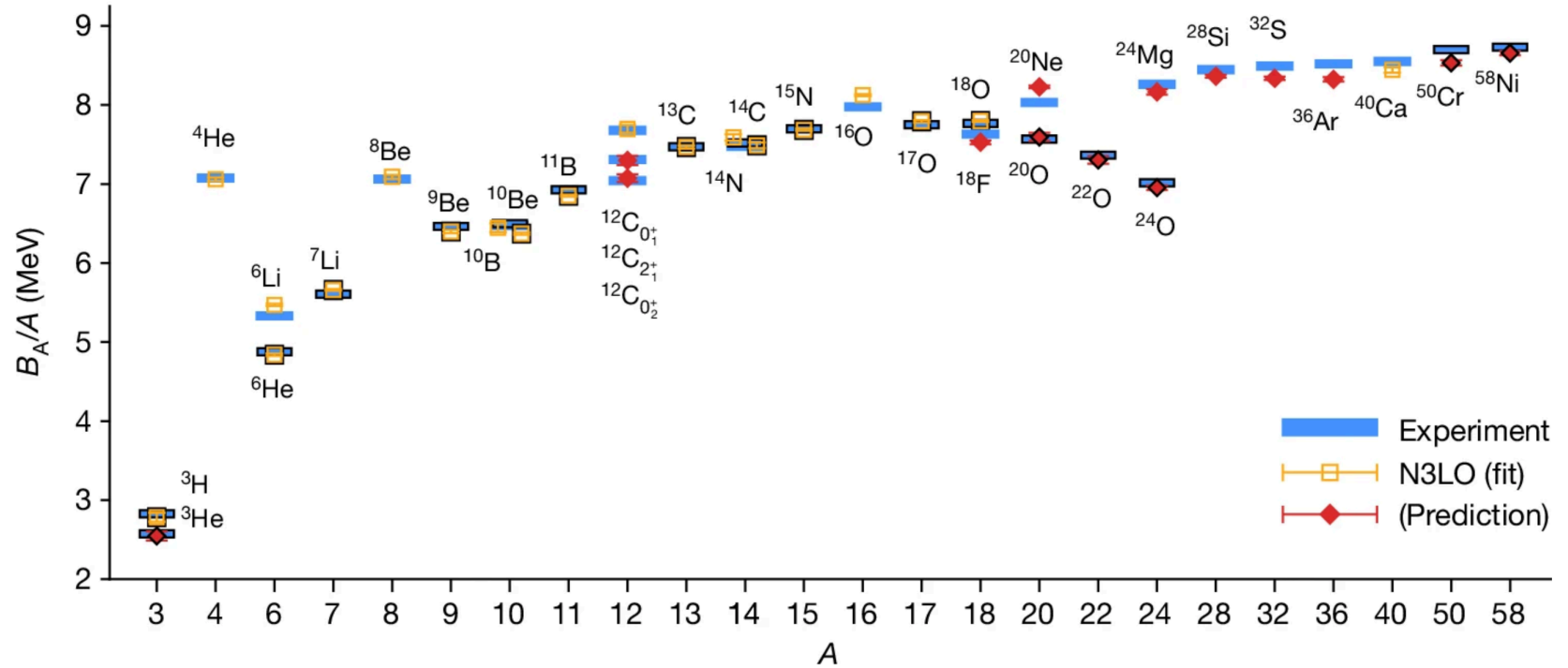
We remove the interaction between nucleons and replace it with the interactions of each nucleon with a background field.



Wavefunction matching

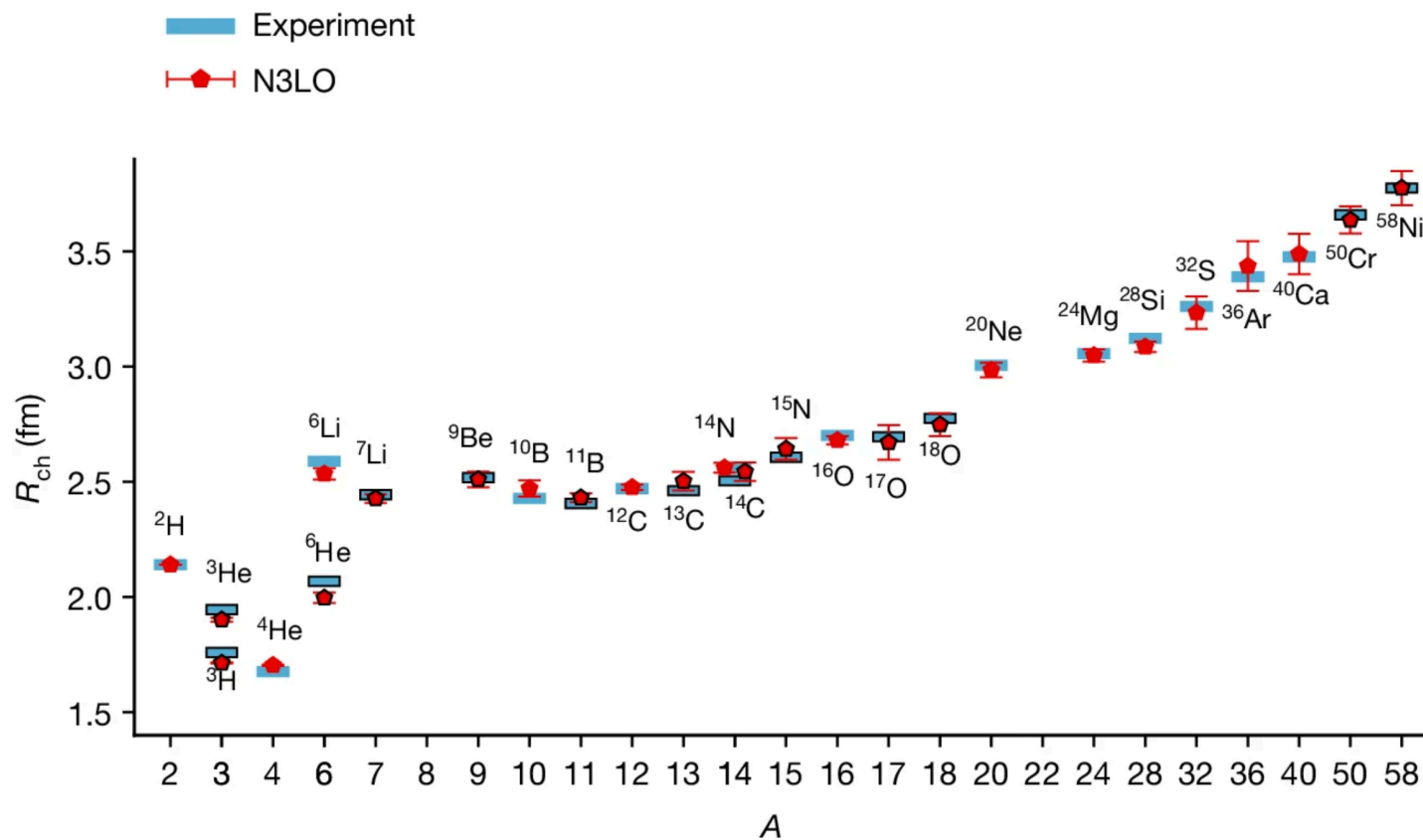


Binding energies



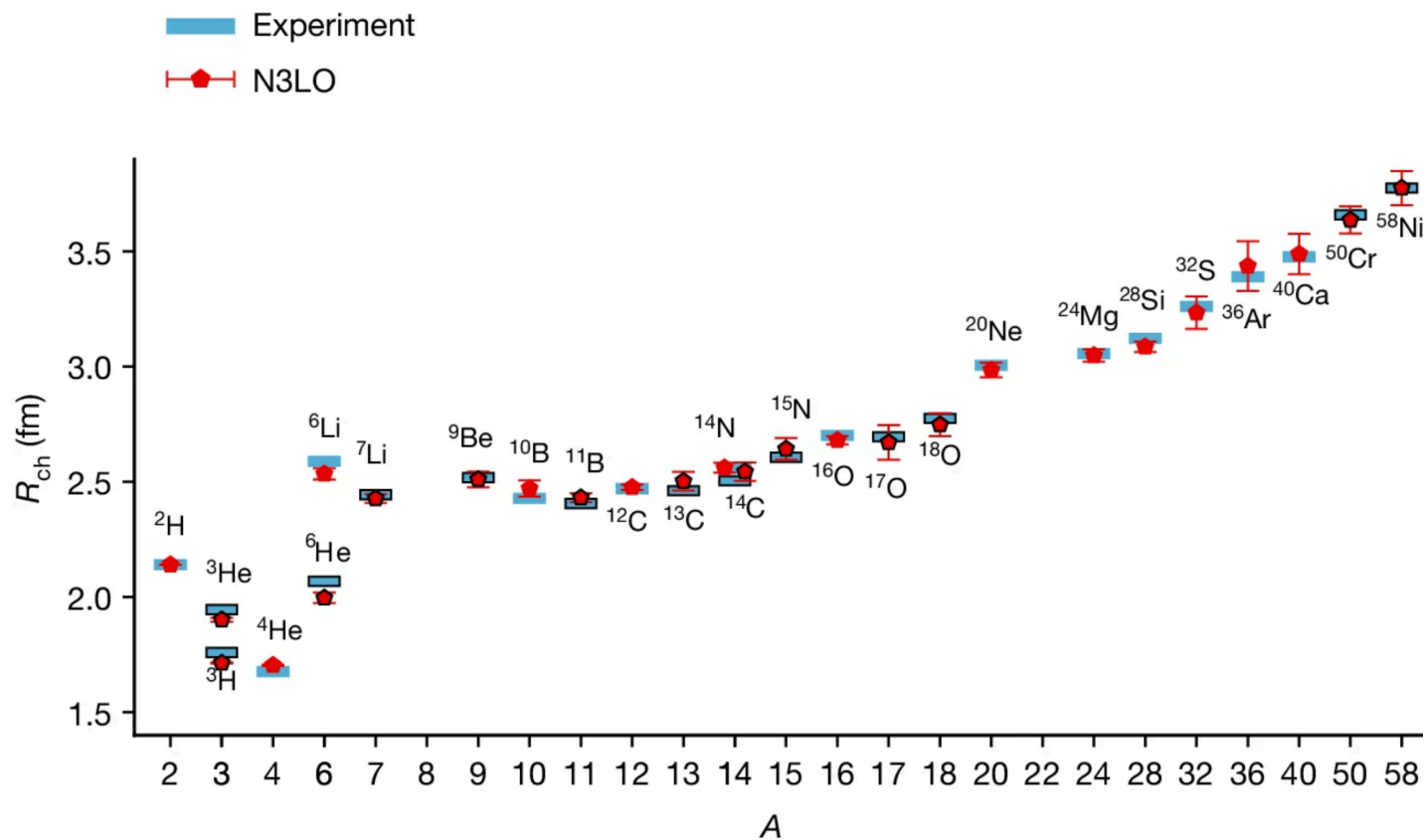
Elhatisari, Bovermann, Ma, Epelbaum, Frame, Hildenbrand, Krebs, Lähde, D.L., Li, Lu, M. Kim, Y. Kim, Meißner, Rupak, Shen, Song, Stellin, Nature 630, 59 (2024)

Charge radii



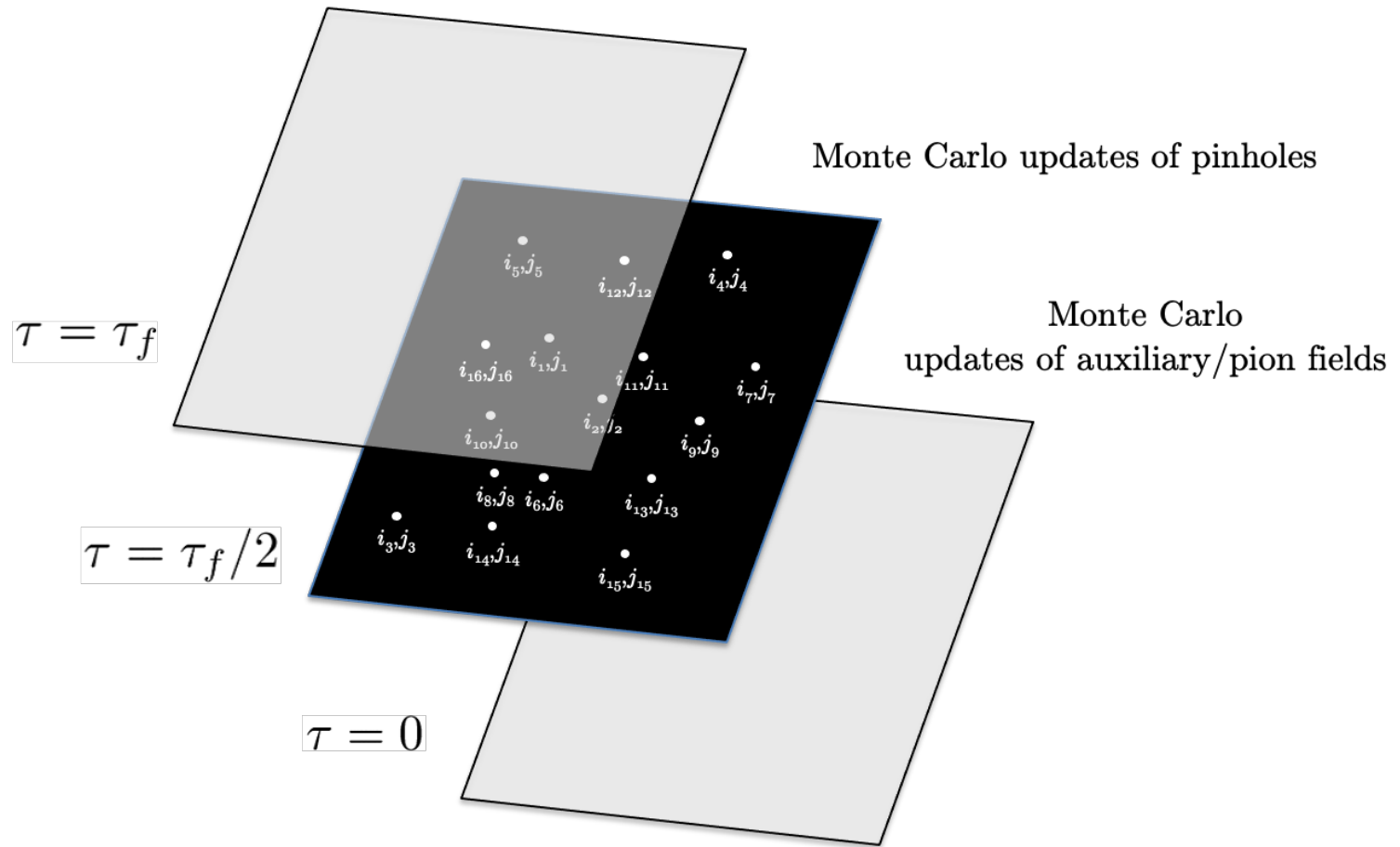
Elhatisari, Bovermann, Ma, Epelbaum, Frame, Hildenbrand, Krebs, Lähde, D.L., Li, Lu, M. Kim, Y. Kim, Meißner, Rupak, Shen, Song, Stellin, Nature 630, 59 (2024)

Charge radii

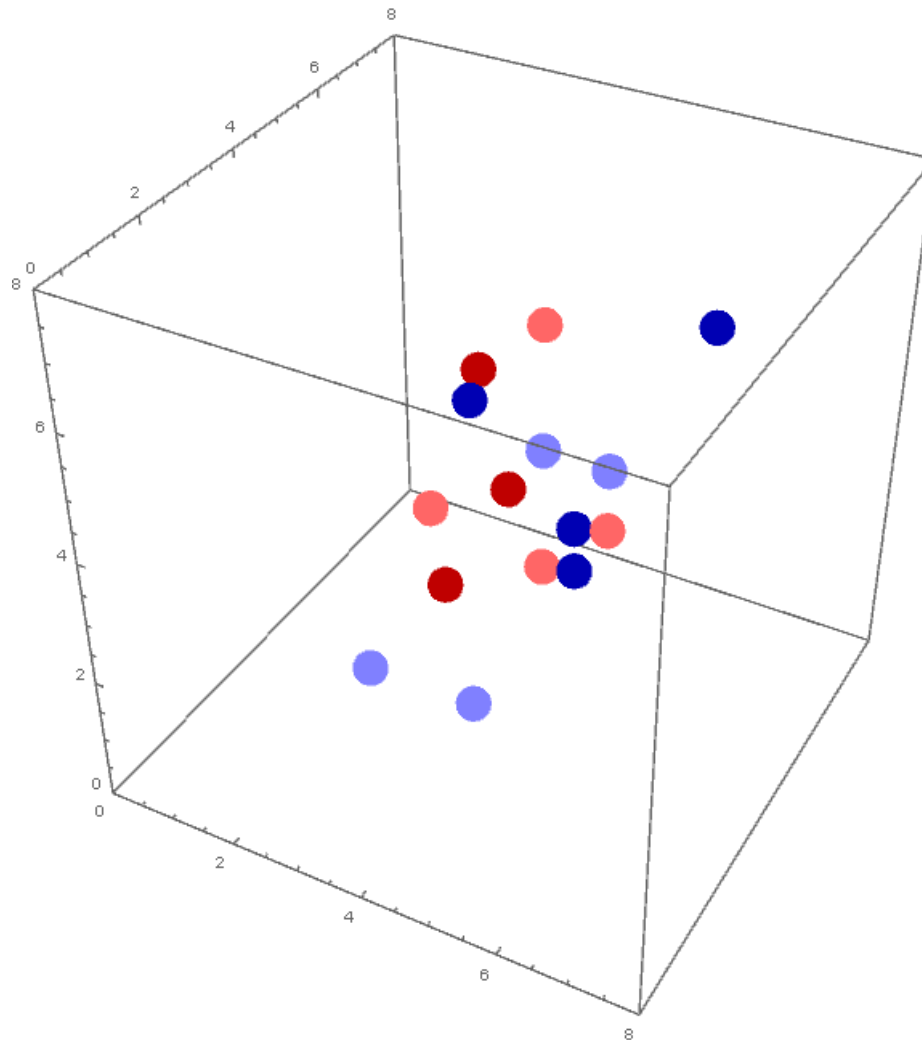


Elhatisari, Bovermann, Ma, Epelbaum, Frame, Hildenbrand, Krebs, Lähde, D.L., Li, Lu, M. Kim, Y. Kim, Meißner, Rupak, Shen, Song, Stellin, Nature 630, 59 (2024)

Pinhole algorithm

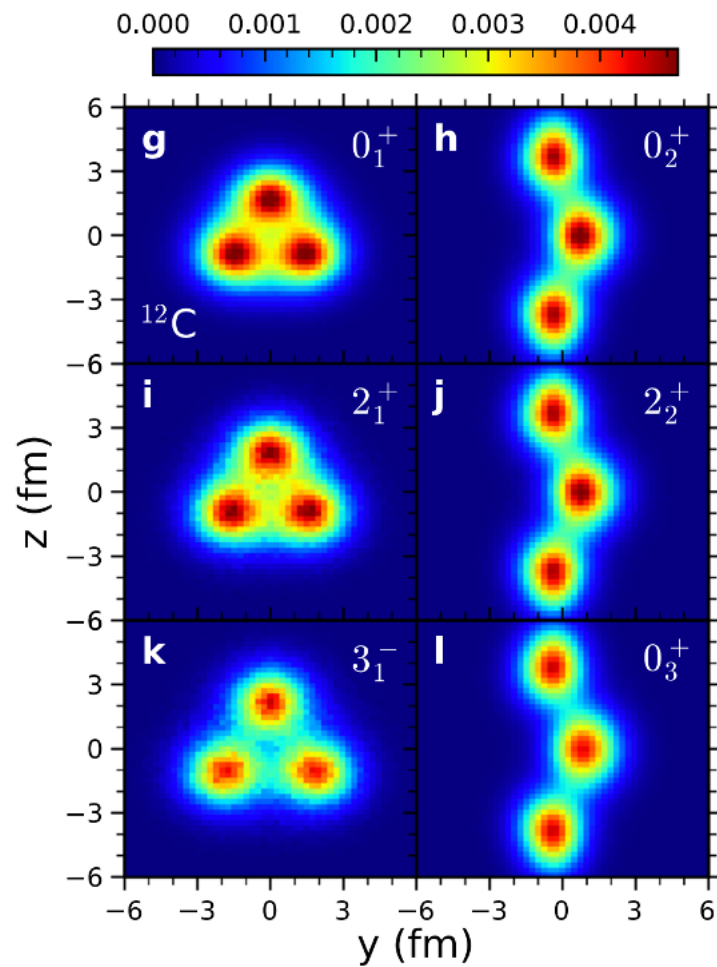


Pinhole configurations for ^{16}O



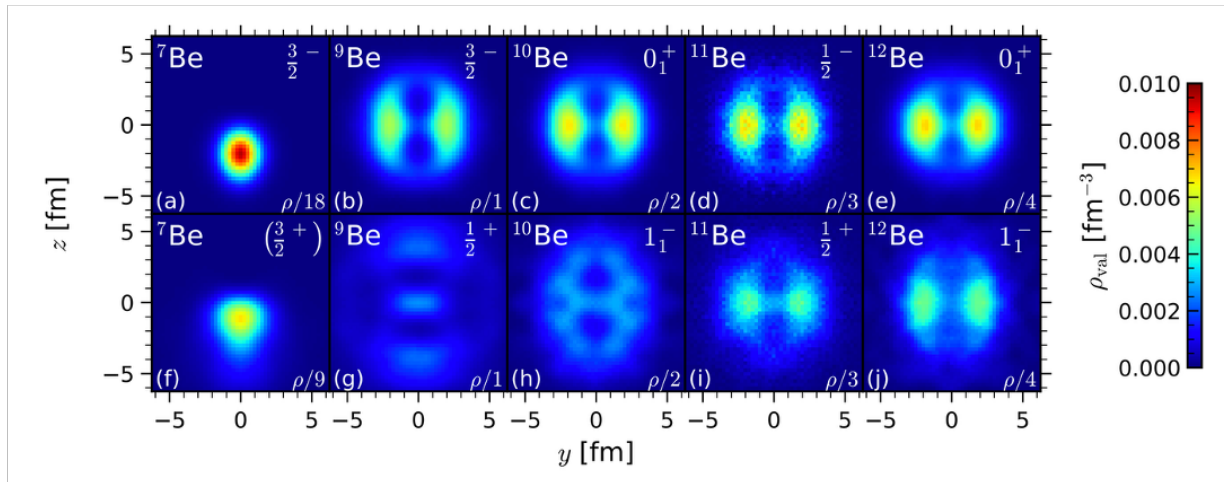
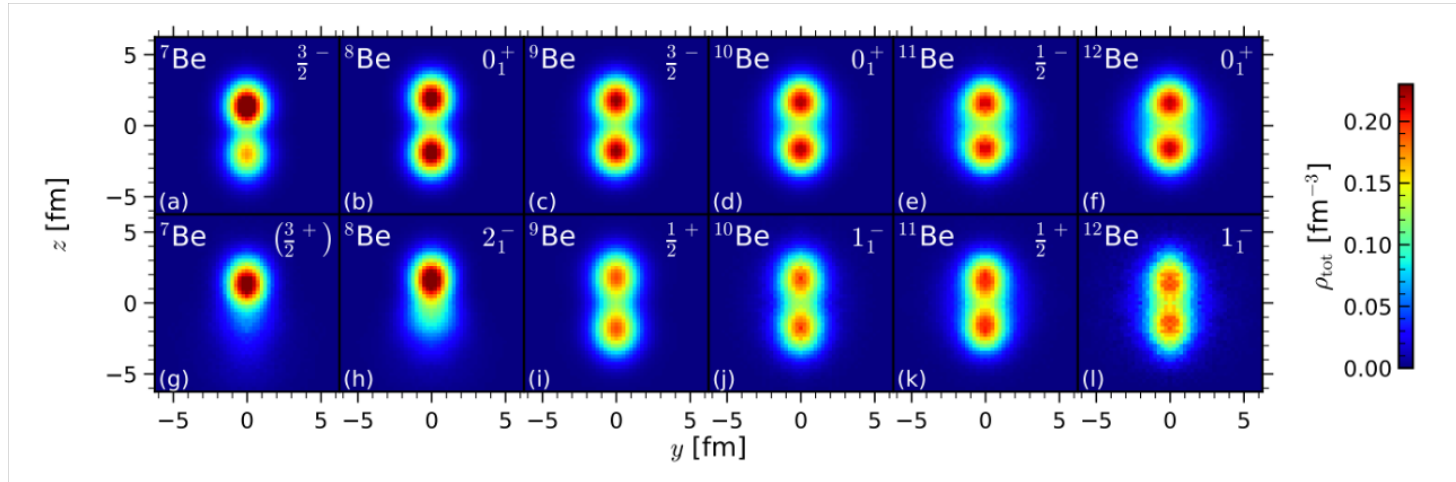
Summerfield, Lu, Plumberg, D.L., Noronha-Hostler, Timmins, Phys. Rev. C 104 L041901 (2021)

Emergent geometry of ^{12}C



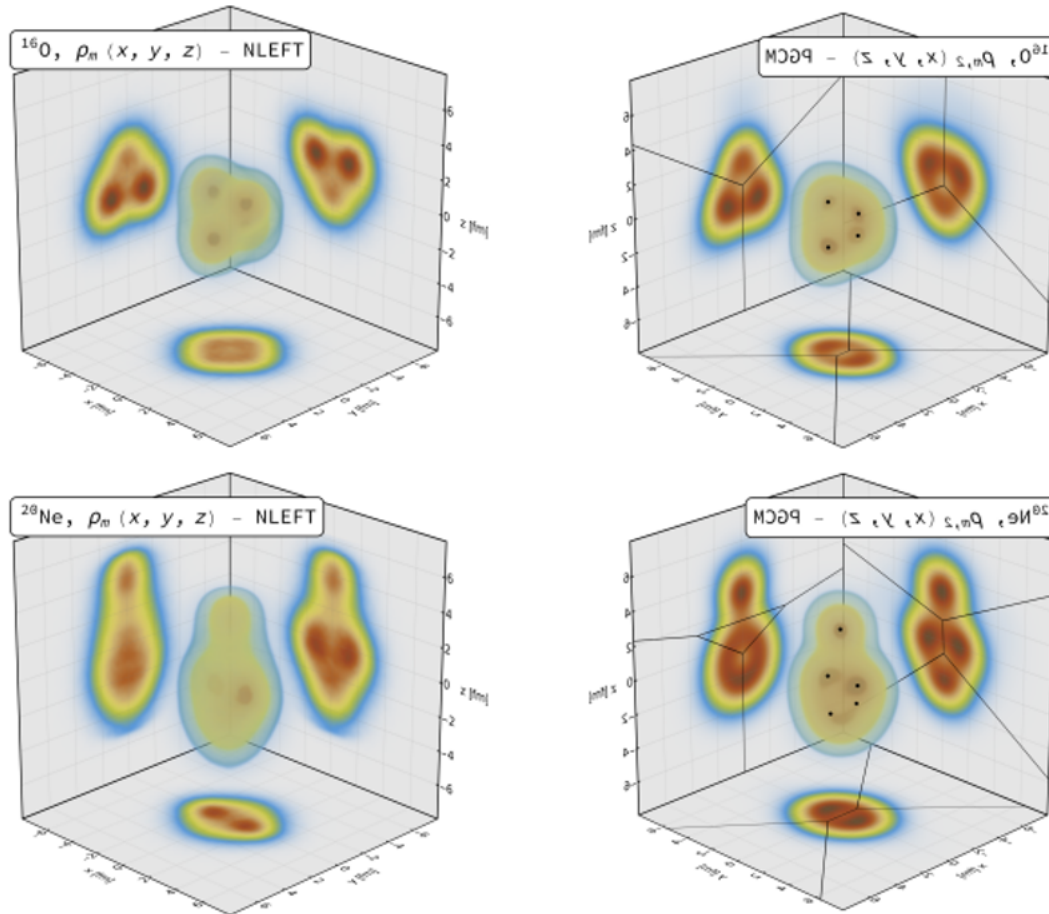
Shen, Elhatisari, Lähde, D.L., Lu, Meißner, Nature Commun. 14, 2777 (2023)

Beryllium isotopes



Shen, Elhatisari, D.L., Meißner, Ren, PRL 134, 162503 (2025)

Structure of ^{16}O and ^{20}Ne



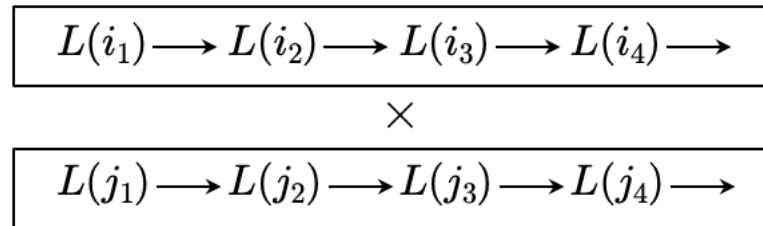
Giacalone et al., Phys. Rev. Lett. 135, 012302 (2025)

Giacalone et al., Phys. Rev. Lett. 134, 082301 (2025)

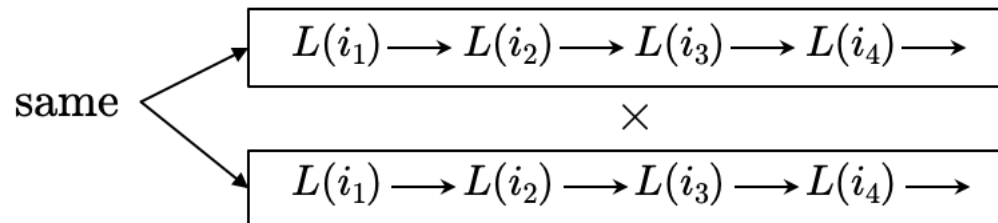
Mirror approximation

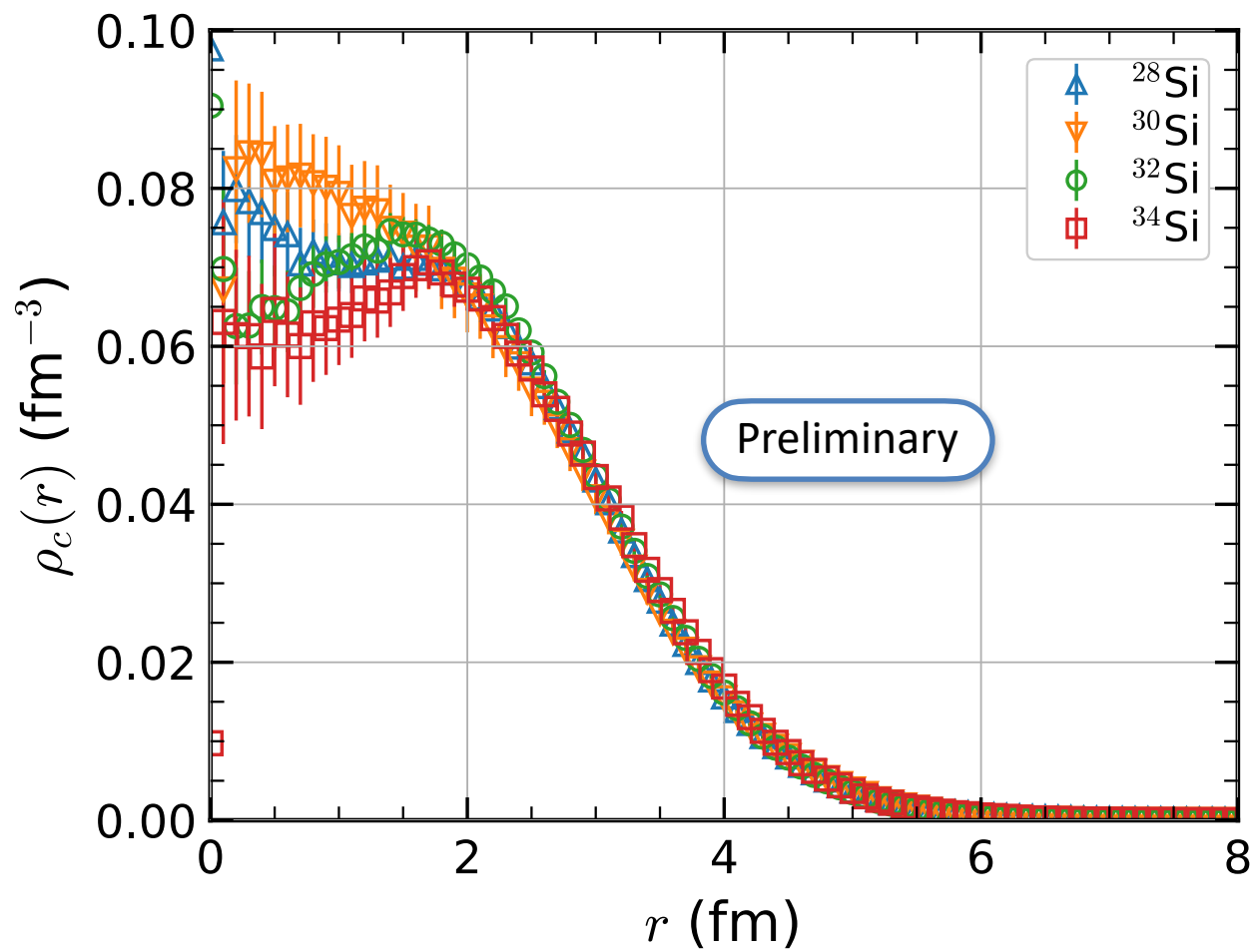
Symmetric product of two Markov chains:

$$A(i,j) = L(i)L(j)$$



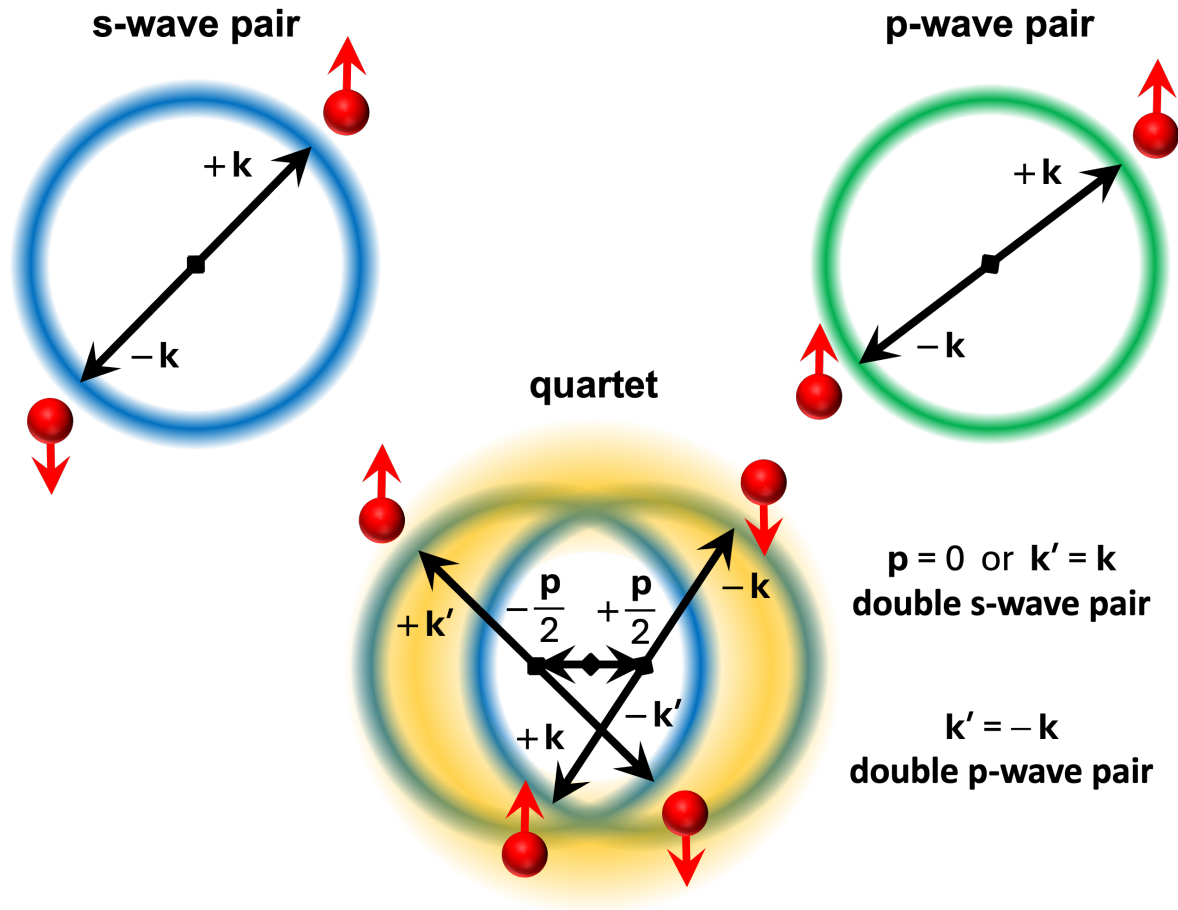
Mirror approximation:



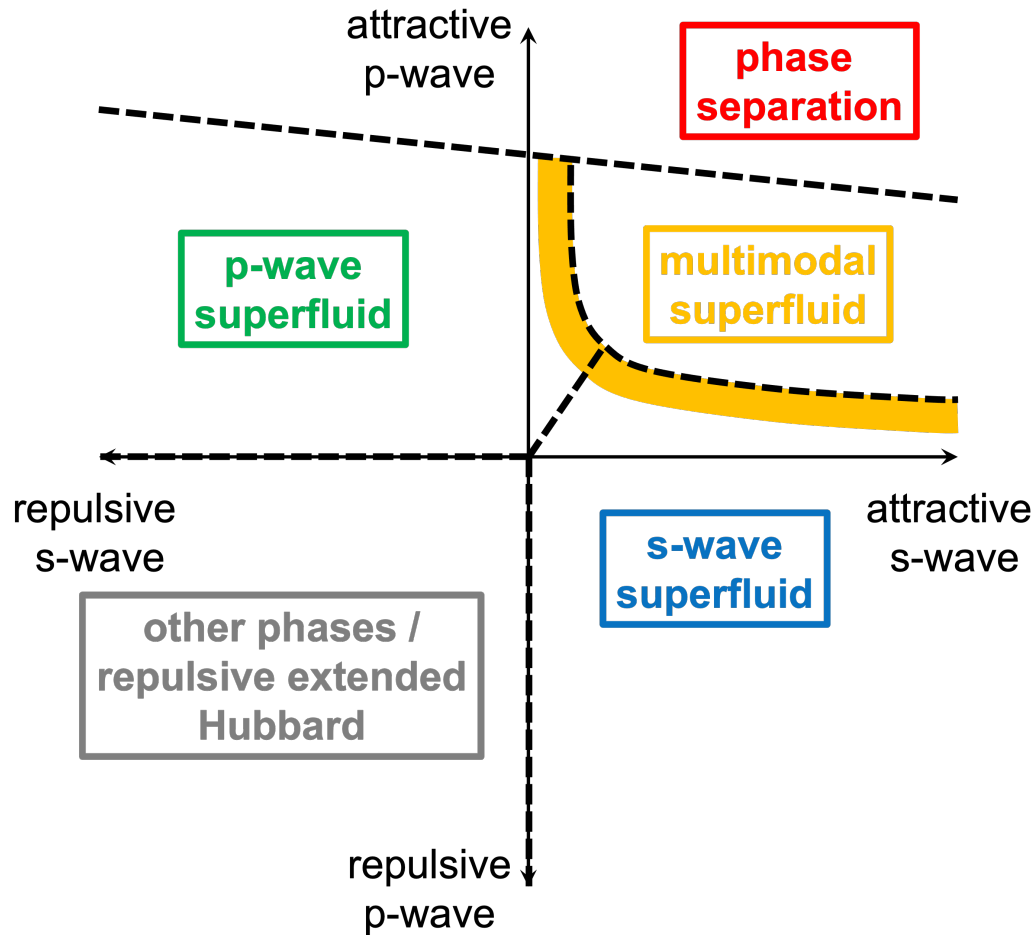


Shen, Ma, Elhatisari, et al., work in progress

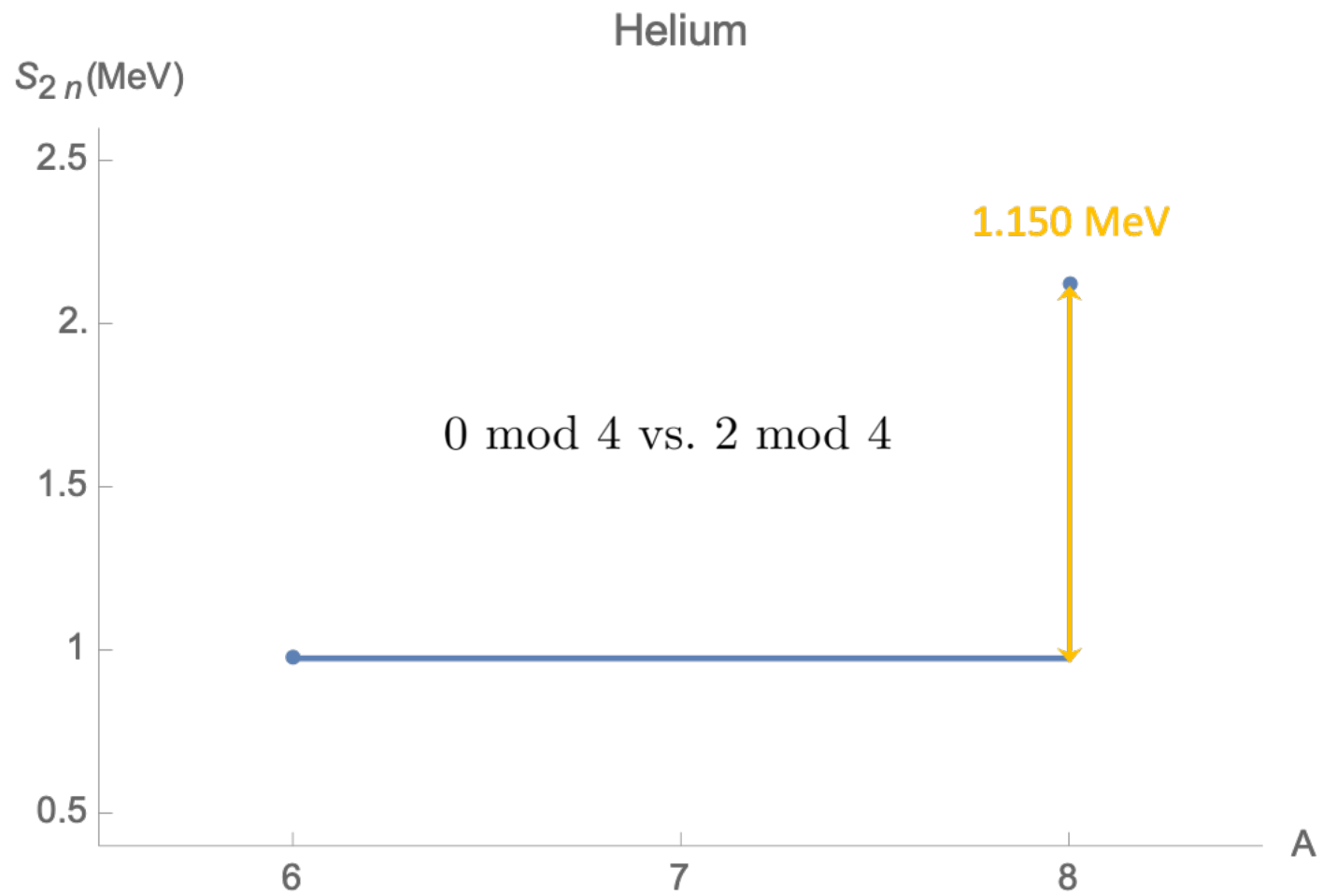
Multimodal superfluidity

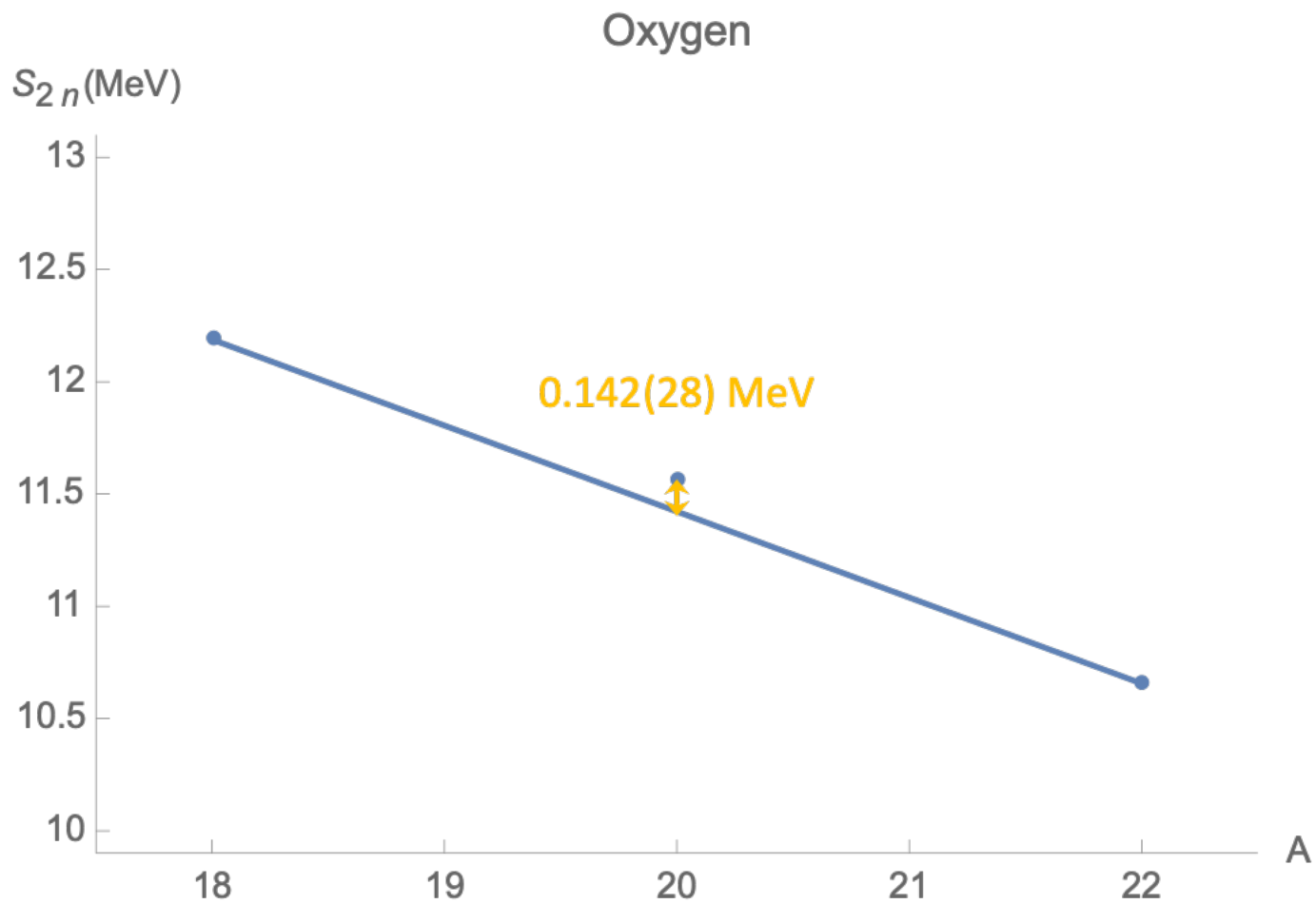


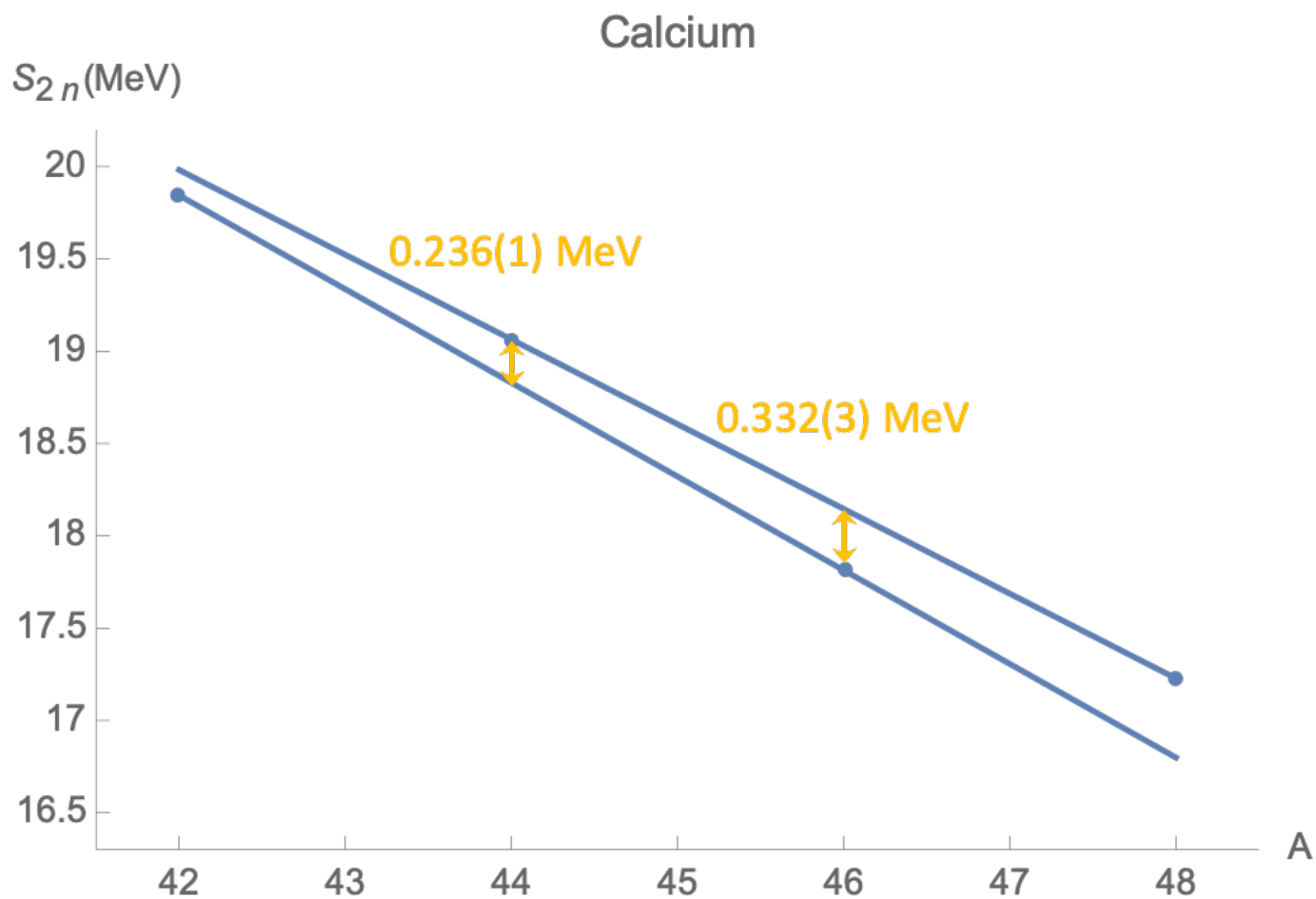
Quantum phase diagram



a. Neutron matter (chiral N³LO)		
Density (fm ⁻³)		2Δ _S (MeV)
0.033		5.58(92)
0.085		2.61(76)
Density (fm ⁻³)	channel	2Δ _P (MeV)
0.033	³ P ₀	1.25(86)
0.033	³ P ₁	0.59(113)
0.033	³ P ₂ [T ₂ ⁻]	1.36(86)
0.033	³ P ₂ [E ⁻]	2.16(90)
0.085	³ P ₀	1.95(95)
0.085	³ P ₁	-0.24(81)
0.085	³ P ₂ [T ₂ ⁻]	2.10(80)
0.085	³ P ₂ [E ⁻]	2.03(87)
Density (fm ⁻³)		4Δ _Q (MeV)
0.033		3.90(167)
0.085		2.01(86)







Experimental evidence

Quartet binding in MeV

Nuclei	Predominant Orbitals	$4\Delta_Q$
$S_{2n}\{^6\text{He}_4, ^8\text{He}_4\}+$	$(1p_{3/2})^4$	1.150
$\frac{1}{2}S_{2n}\{^{18}_8\text{O}_{10}, ^{20}_8\text{O}_{12}, ^{22}_8\text{O}_{14}\}-$	$(1d_{5/2})^4$	0.142(28)
$\frac{1}{2}S_{2n}\{^{42}_{20}\text{Ca}_{22}, ^{44}_{20}\text{Ca}_{24}, ^{46}_{20}\text{Ca}_{26}\}-$	$(1f_{7/2})^4$	0.236(1)
$\frac{1}{2}S_{2n}\{^{44}_{20}\text{Ca}_{24}, ^{46}_{20}\text{Ca}_{26}, ^{48}_{20}\text{Ca}_{28}\}+$	$(1f_{7/2})^4$	0.332(3)
$\frac{1}{2}S_{2n}\{^{106}_{50}\text{Sn}_{56}, ^{108}_{50}\text{Sn}_{58}, ^{110}_{50}\text{Sn}_{60}\}+$	combination	0.033(11)
$\frac{1}{2}S_{2n}\{^{108}_{50}\text{Sn}_{58}, ^{110}_{50}\text{Sn}_{60}, ^{112}_{50}\text{Sn}_{62}\}-$	combination	0.007(17)
$\frac{1}{2}S_{2n}\{^{110}_{50}\text{Sn}_{60}, ^{112}_{50}\text{Sn}_{62}, ^{114}_{50}\text{Sn}_{64}\}+$	combination	0.025(16)
$\frac{1}{2}S_{2n}\{^{112}_{50}\text{Sn}_{62}, ^{114}_{50}\text{Sn}_{64}, ^{116}_{50}\text{Sn}_{66}\}-$	combination	0.015(7)
$\frac{1}{2}S_{2n}\{^{114}_{50}\text{Sn}_{64}, ^{116}_{50}\text{Sn}_{66}, ^{118}_{50}\text{Sn}_{68}\}+$	combination	0.050(30)
$\frac{1}{2}S_{2n}\{^{194}_{82}\text{Pb}_{112}, ^{196}_{82}\text{Pb}_{114}, ^{198}_{82}\text{Pb}_{116}\}+$	combination	0.047(22)
$\frac{1}{2}S_{2n}\{^{196}_{82}\text{Pb}_{114}, ^{198}_{82}\text{Pb}_{116}, ^{200}_{82}\text{Pb}_{118}\}-$	combination	0.057(16)
$\frac{1}{2}S_{2n}\{^{198}_{82}\text{Pb}_{116}, ^{200}_{82}\text{Pb}_{118}, ^{202}_{82}\text{Pb}_{120}\}+$	combination	0.021(15)
$\frac{1}{2}S_{2n}\{^{200}_{82}\text{Pb}_{118}, ^{202}_{82}\text{Pb}_{120}, ^{204}_{82}\text{Pb}_{122}\}-$	combination	0.013(13)
$\frac{1}{2}S_{2n}\{^{202}_{82}\text{Pb}_{120}, ^{204}_{82}\text{Pb}_{122}, ^{206}_{82}\text{Pb}_{124}\}+$	combination	0.014(7)
$\frac{1}{2}S_{2n}\{^{204}_{82}\text{Pb}_{122}, ^{206}_{82}\text{Pb}_{124}, ^{208}_{82}\text{Pb}_{126}\}-$	combination	0.110(2)
$\frac{1}{2}S_{2n}\{^{210}_{82}\text{Pb}_{128}, ^{212}_{82}\text{Pb}_{130}, ^{214}_{82}\text{Pb}_{132}\}-$	$(2g_{9/2})^4$	0.014(2)
$\frac{1}{2}S_{2n}\{^{212}_{84}\text{Po}_{128}, ^{214}_{84}\text{Po}_{130}, ^{216}_{84}\text{Po}_{132}\}-$	$(2g_{9/2})^4$	0.019(1)
$\frac{1}{2}S_{2n}\{^{214}_{84}\text{Po}_{130}, ^{216}_{84}\text{Po}_{132}, ^{218}_{84}\text{Po}_{134}\}+$	$(2g_{9/2})^4$	0.015(2)
$\frac{1}{2}S_{2n}\{^{214}_{86}\text{Rn}_{128}, ^{216}_{86}\text{Rn}_{130}, ^{218}_{86}\text{Rn}_{132}\}-$	$(2g_{9/2})^4$	0.079(12)

Summary and outlook

We reviewed the framework of nuclear lattice effective field theory, detailing methods such as auxiliary-field Monte Carlo, wavefunction matching, the pinhole algorithm, and the mirror approximation. We have also presented evidence for multimodal superfluidity in neutron matter together with experimental evidence in neutron-rich nuclei. We will be working to produce improved pinhole configurations for several nuclei of interest for relativistic ion collisions.