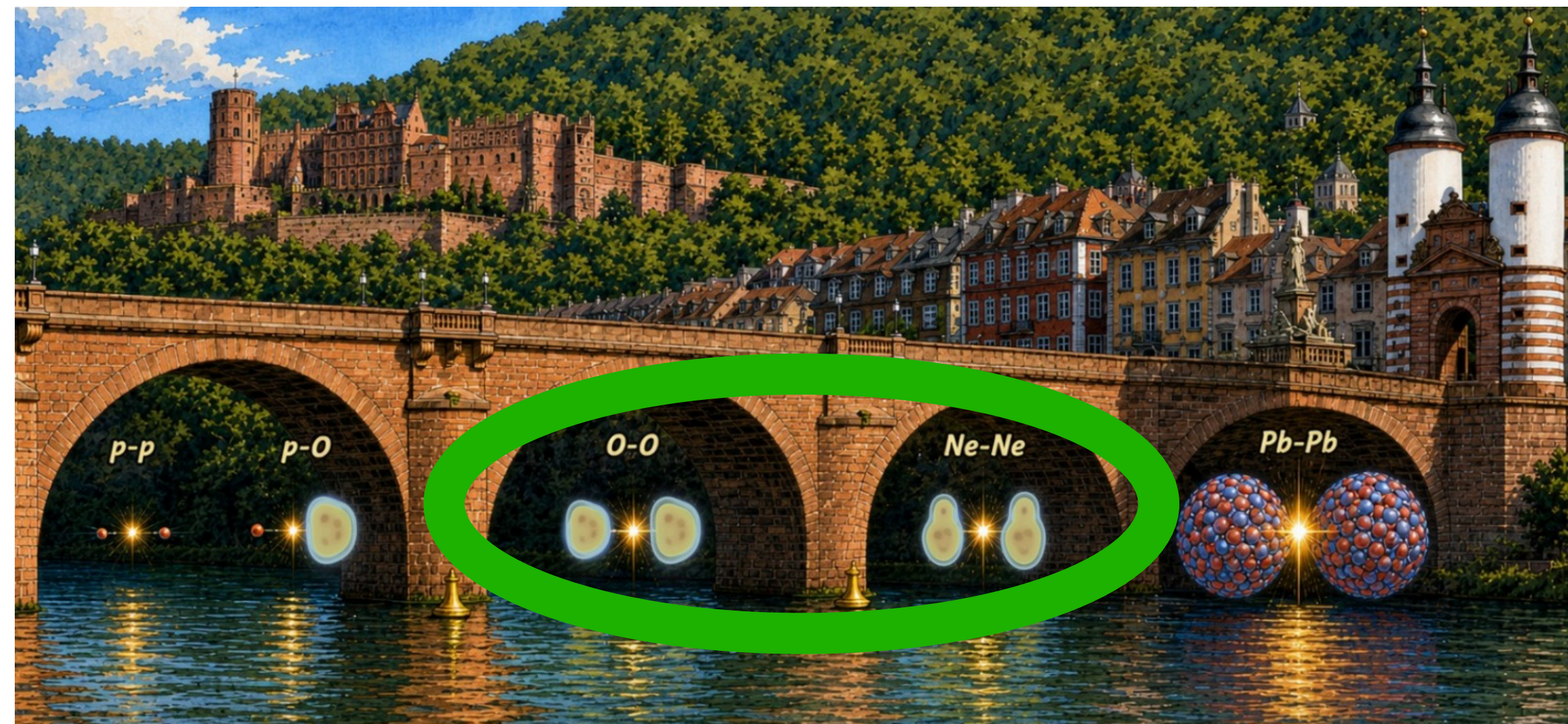


Towards calibrating chiral interactions across energy scales

Andreas Ekström



Overview

- Chiral Hamiltonians and *ab initio* nuclear physics.
- Global sensitivity analysis and NeNe / OO collisions.
- Summary.

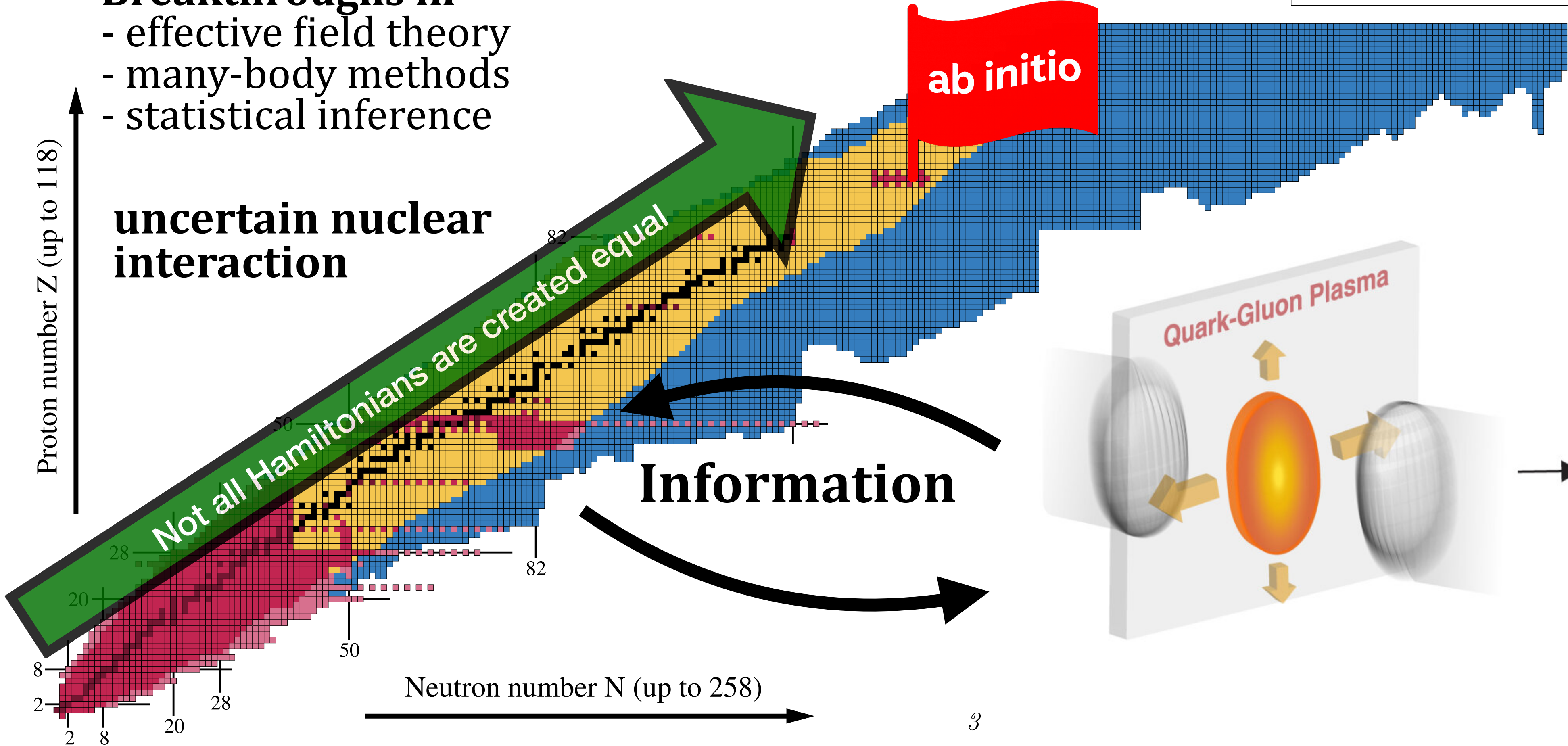
Thanks to my collaborators: Thomas Duguet, Giuliano Giacalone, Gaute Hagen, Takayuki Miyagi, Alexander Tichai.

Ab initio nuclear theory

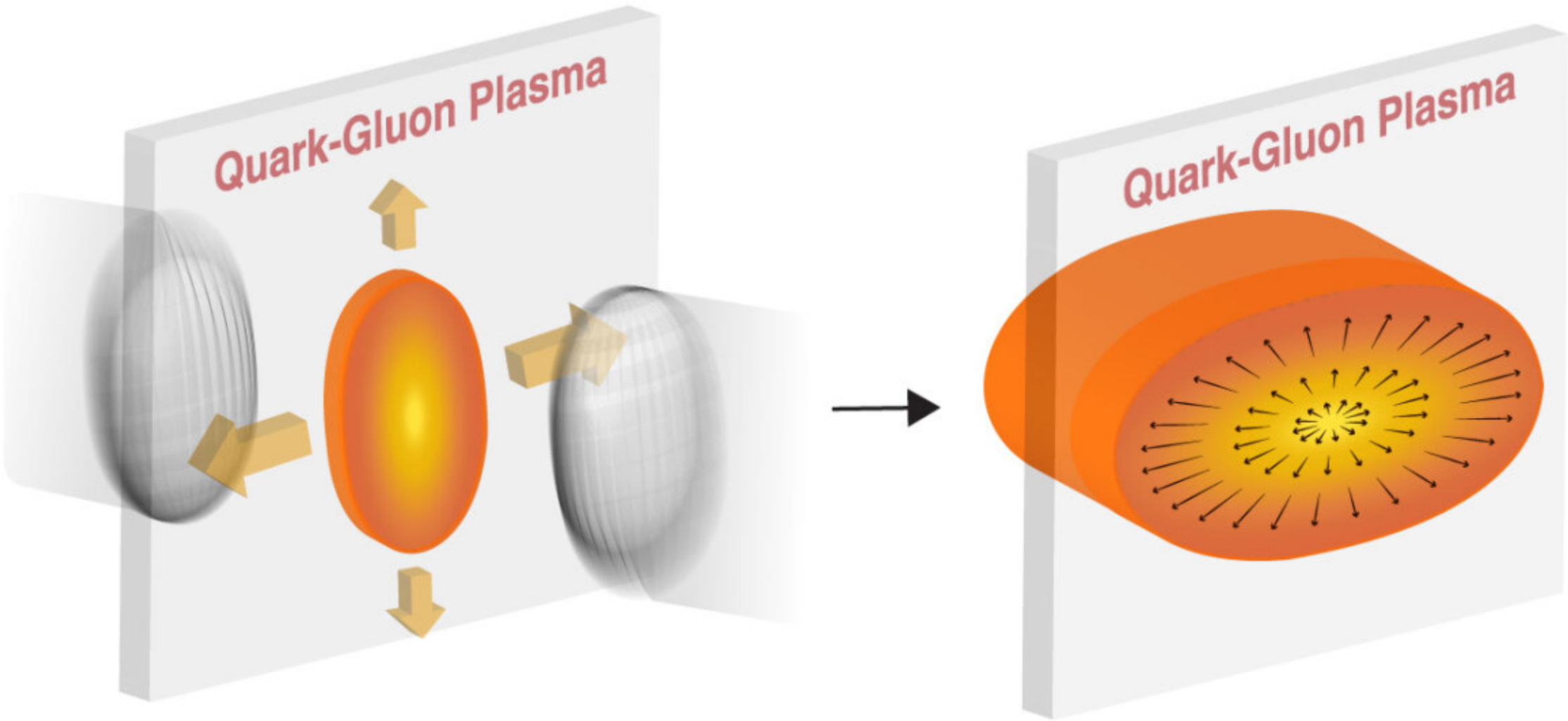
$$H(\vec{\alpha})|\Psi\rangle = E|\Psi\rangle$$

Breakthroughs in
- effective field theory
- many-body methods
- statistical inference

**uncertain nuclear
interaction**

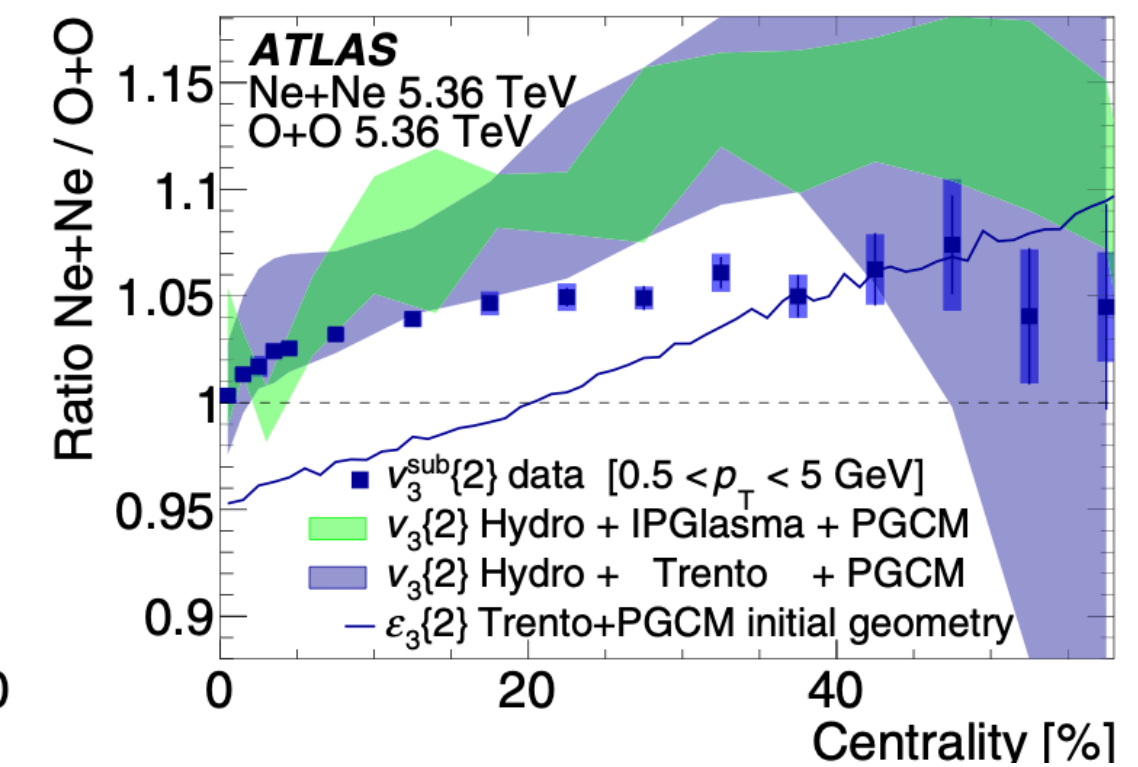
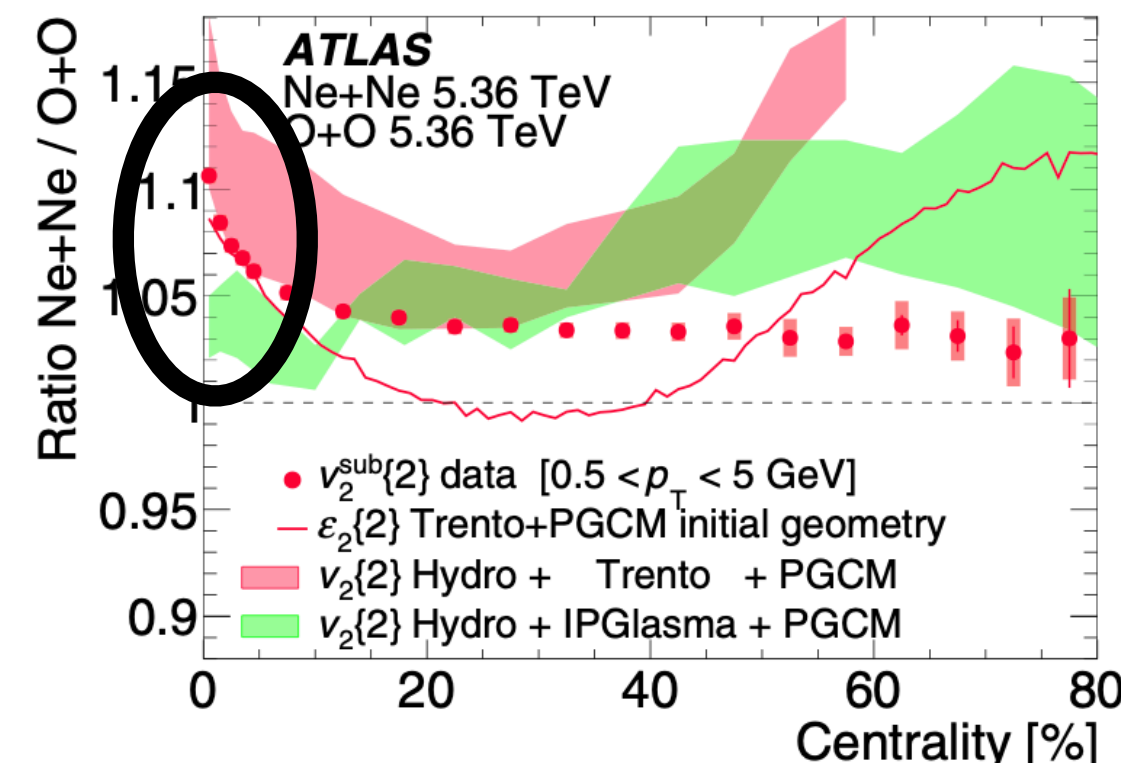
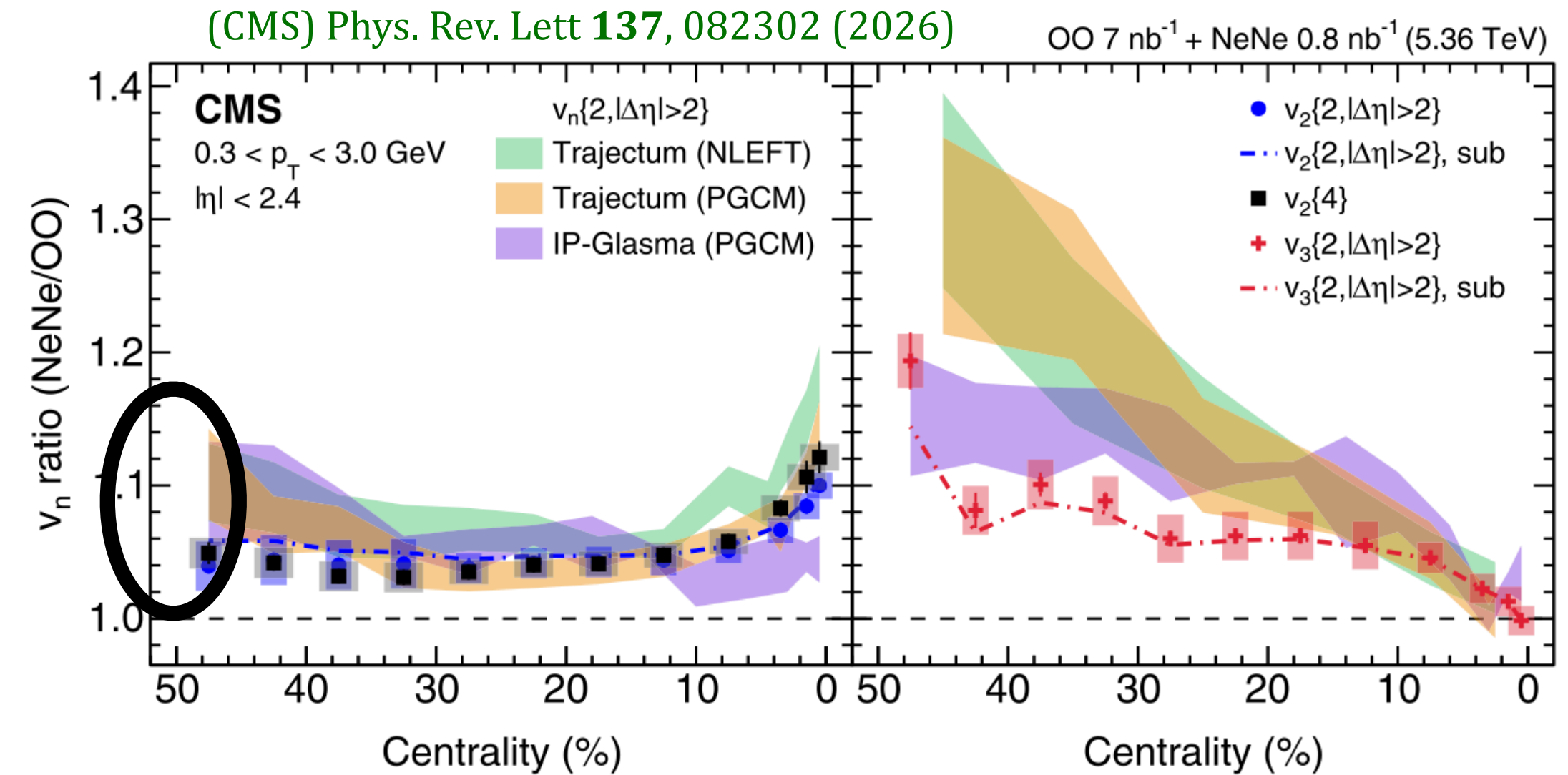
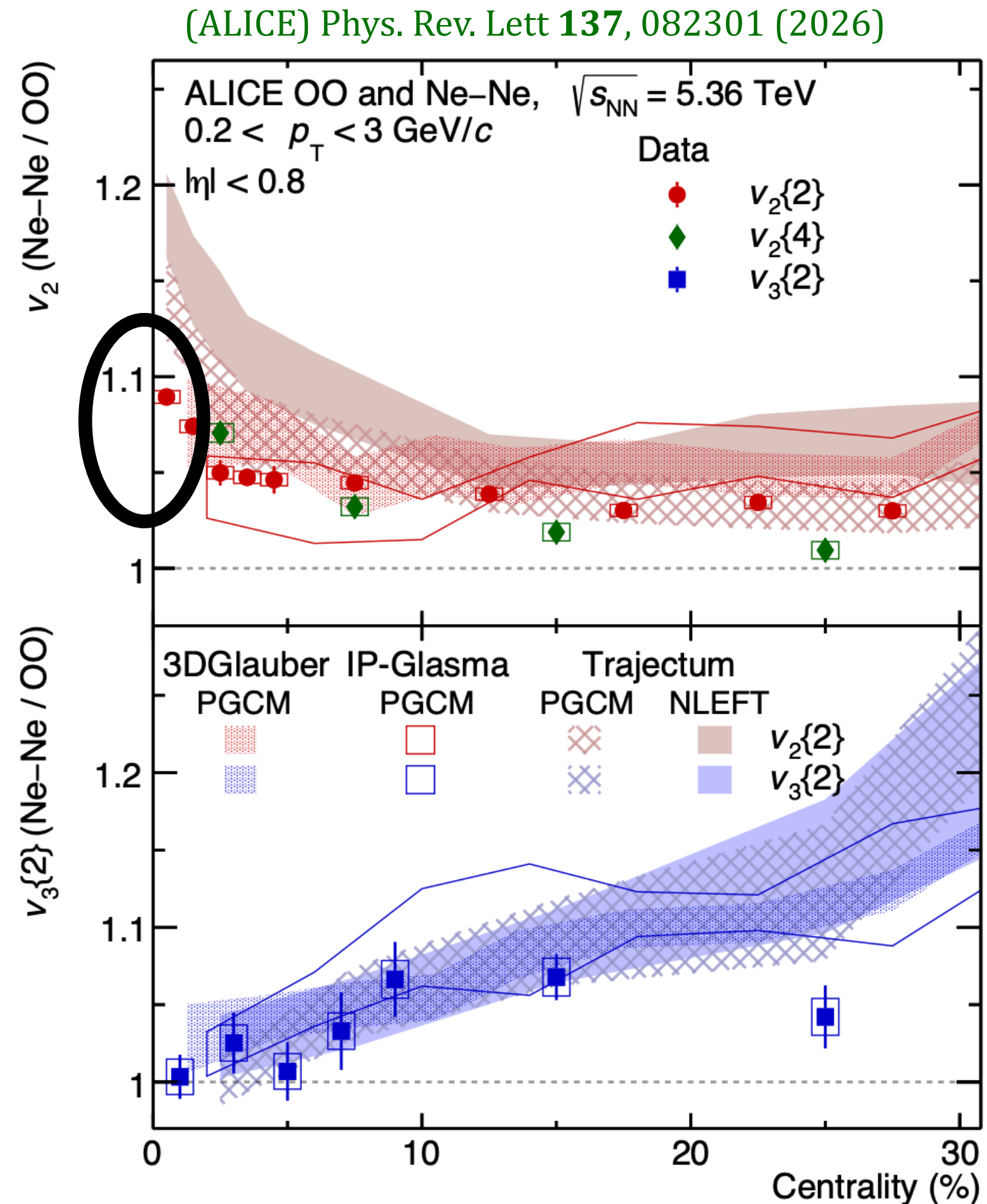


- Stable
- Atomic mass evaluation 2020
- Energy density functional (Gogny D1M)
- *Ab initio* 2026



New data from CERN: $^{20}\text{Ne}+^{20}\text{Ne}$ and $^{16}\text{O}+^{16}\text{O}$

How sensitive are NeNe/OO ratios to nuclear Hamiltonian parameters $\vec{\alpha}$?



0-1% central $v_2(\text{NeNe/OO}) \sim 1.09-1.11$

HIC data, *ab initio* many-body, and chiral Hamiltonians

See Duguet (monday), Hagen (today)

- Momentum harmonics $v_2, v_3, \dots$ extracted from HIC data. Linear hydrodynamic response $v_\ell = \kappa_\ell \epsilon_\ell$ yields quadrupolar (elliptic) ϵ_2 , octupolar (triangular) $\epsilon_3, \dots$ anisotropies.
- On the *ab initio* side, the eccentricity operator $\mathcal{E}_\ell^{(2)} = \mathcal{E}_\ell^{(1)} \mathcal{E}_{-\ell}^{(1)}$ scaled by the nuclear transverse (multipole) radius $\langle R_\ell \rangle^2$ yield a re-scaled anisotropy $\langle \delta \epsilon_\ell^2 \rangle$. It splits into a 1-body part, about $1/A$, and a 2-body part proportional to $B_\ell^2(HE)$.
- We compute $B_2^2(HE)$ and $B_3^2(HE)$ for ^{16}O and ^{20}Ne using coupled cluster. Fast and accurate emulation using sub-space projected coupled cluster. ^{20}Ne is angular-momentum projected. Parity is not broken. In this talk, I focus on $B_2^2(HE)$.

G. Giacalone Phys. Rev. C **99**, 024910 (2019)
G. Giacalone et al Phys. Rev. Lett. **135**, 012302 (2025)
T. Duguet et al Phys. Rev. Lett **135**, 182301 (2025)
J.-P. Blaizot et al, arXiv 2512.18926 (2025)
S. Bofos et al, Phys. Lett. B **879**, 140649 (2026)
C. Ding et al, arXiv 2608.22699 (2026)

AE and G. Hagen Phys. Rev. Lett. **123**, 252501 (2019)

D. Frame, et al. Phys. Rev. Lett. **121**, 032501 (2018)

T. Duguet, AE, R. J. Furnstahl, S. König, D. Lee Rev. Mod. Phys **96**, 031002 (2024)

Chiral EFT at NNLO: sixteen low-energy constants (LECs)

The parameters this talk is about

	2N Force	3N Force	4N Force
LO $(Q/\Lambda_\chi)^0$		$\tilde{C}_{1S0}^{np}, \tilde{C}_{1S0}^{pp}, \tilde{C}_{1S0}^{nn}, \tilde{C}_{3S1}$	
NLO $(Q/\Lambda_\chi)^2$		$C_{1S0}, C_{3S1}, C_{E1}, C_{1P1}$ $C_{3P0}, C_{3P1}, C_{3P2}$	
NNLO $(Q/\Lambda_\chi)^3$			C_1, C_3, C_4 C_D, C_E

- EFTs promise a very useful systematicity

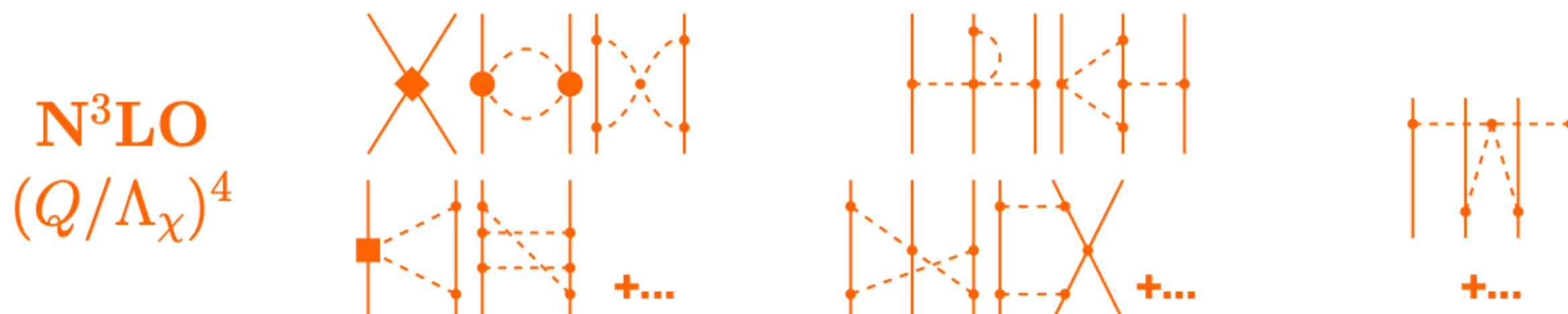
$$H(\vec{\alpha}) = T + V^{(0)}(\vec{\alpha}_{(0)}) + V^{(1)}(\vec{\alpha}_{(1)}) + V^{(2)}(\vec{\alpha}_{(2)}) + \dots$$

- Order-by-order expansion** in a dimensionless ratio (Q/Λ_χ) constructed from the separation of scales. A soft scale ($Q \sim m_\pi$) and a hard scale ($\Lambda_\chi \sim m_N$), separating the resolved and unresolved dynamics of the nuclear interaction.
- The 16 LECs up to NNLO are not fixed by the theory, and they must be calibrated to data.**

$$y_{\text{th}}^{(k)} = y_{\text{ref}} \sum_{\nu=0}^k c_\nu \left(\frac{Q}{\Lambda_\chi} \right)^\nu \quad \text{and} \quad \delta y_{\text{th}}^{(k)} = y_{\text{ref}} \sum_{\nu=k+1}^{\infty} c_\nu \left(\frac{Q}{\Lambda_\chi} \right)^\nu$$

**EFT predictions
order-by-order**

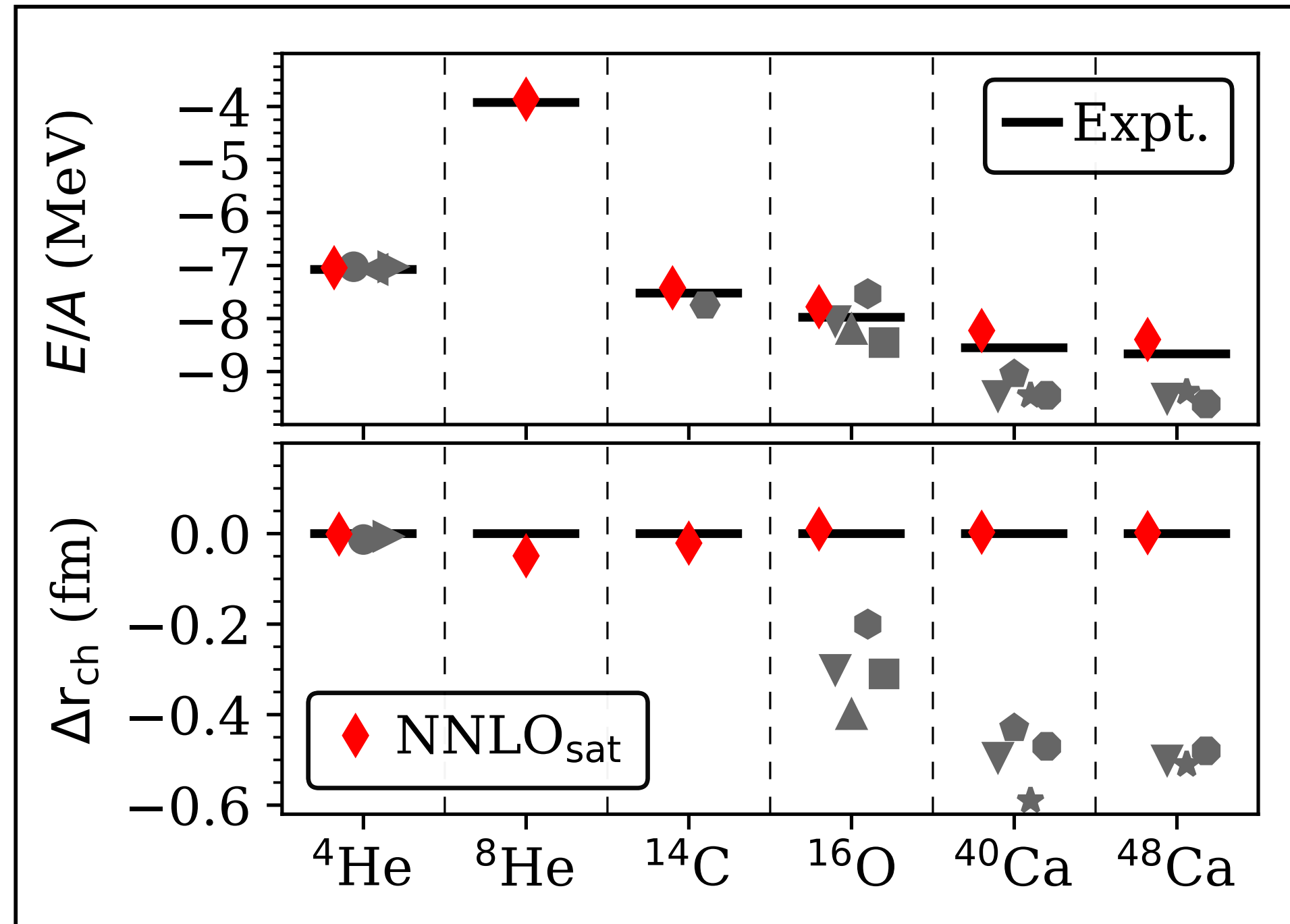
**EFT truncation
error**



N³LO
 $(Q/\Lambda_\chi)^4$

NNLO_{sat} interaction as a starting point

Hamiltonian parameter calibration is an unsettled business



AE et al. Phys. Rev. C **91**, 051301(R) (2015)

Low-energy NN scattering + binding energies and radii of ${}^2\text{H}$ - ${}^4\text{He}$, ${}^{14}\text{C}$, ${}^{16}\text{O}$ included in fit.

For this work we use the NNLO_{sat} parametrization as a starting point for analyzing the sensitivity of HIC data to low-energy constants in the chiral Hamiltonian.

We vary the LEC values in the ranges below. A conservative box with the shape and roughly the scale of published posteriors and non-implausible volumes.

Uncorrelated LEC ranges, centered on NNLO_{sat}

LO contact LECs $\widetilde{C} \pm 0.005 \times 10^4 \text{GeV}^{-2}$

NLO contact LECs $C \pm 0.05 \times 10^4 \text{GeV}^{-4}$

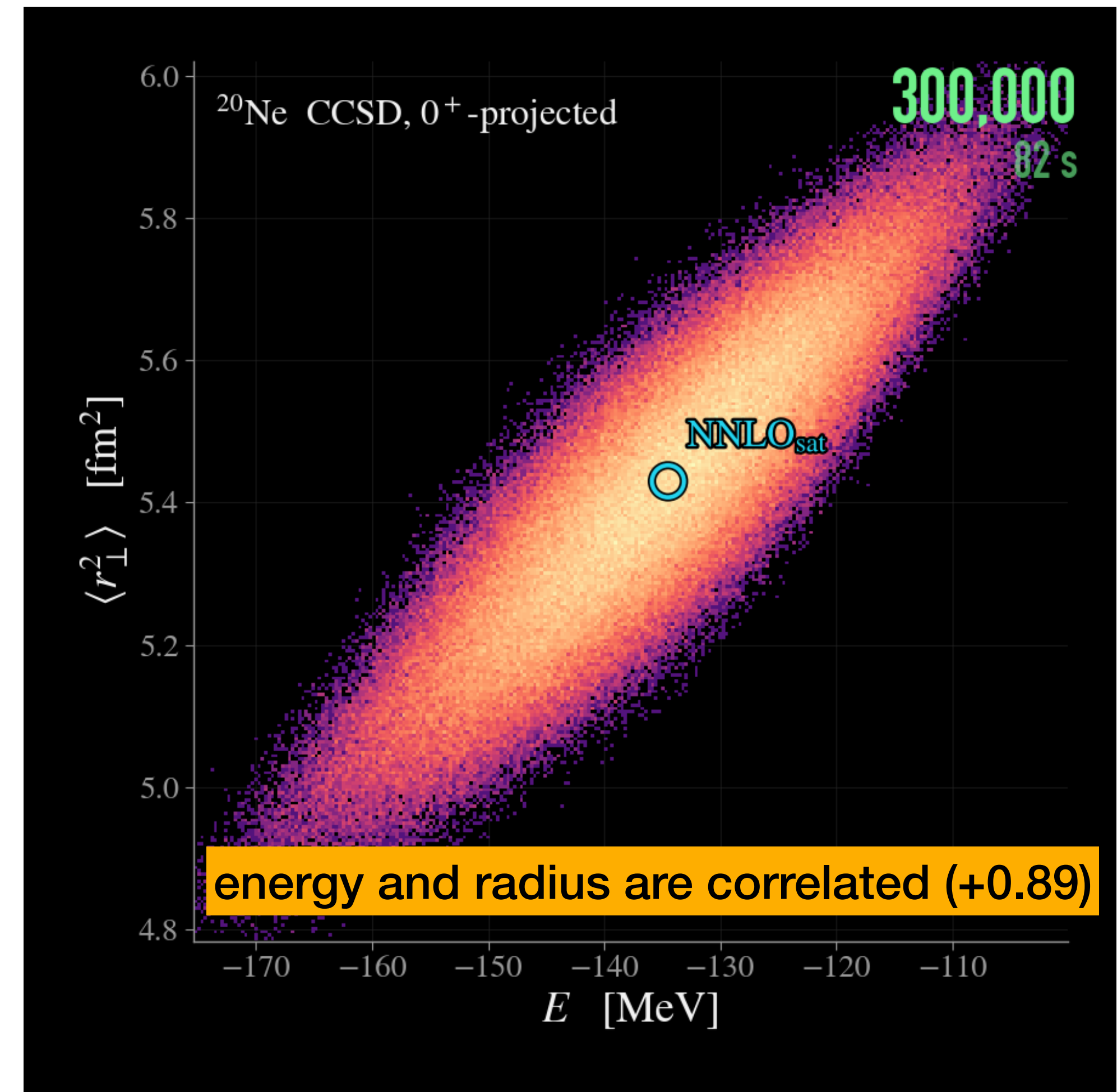
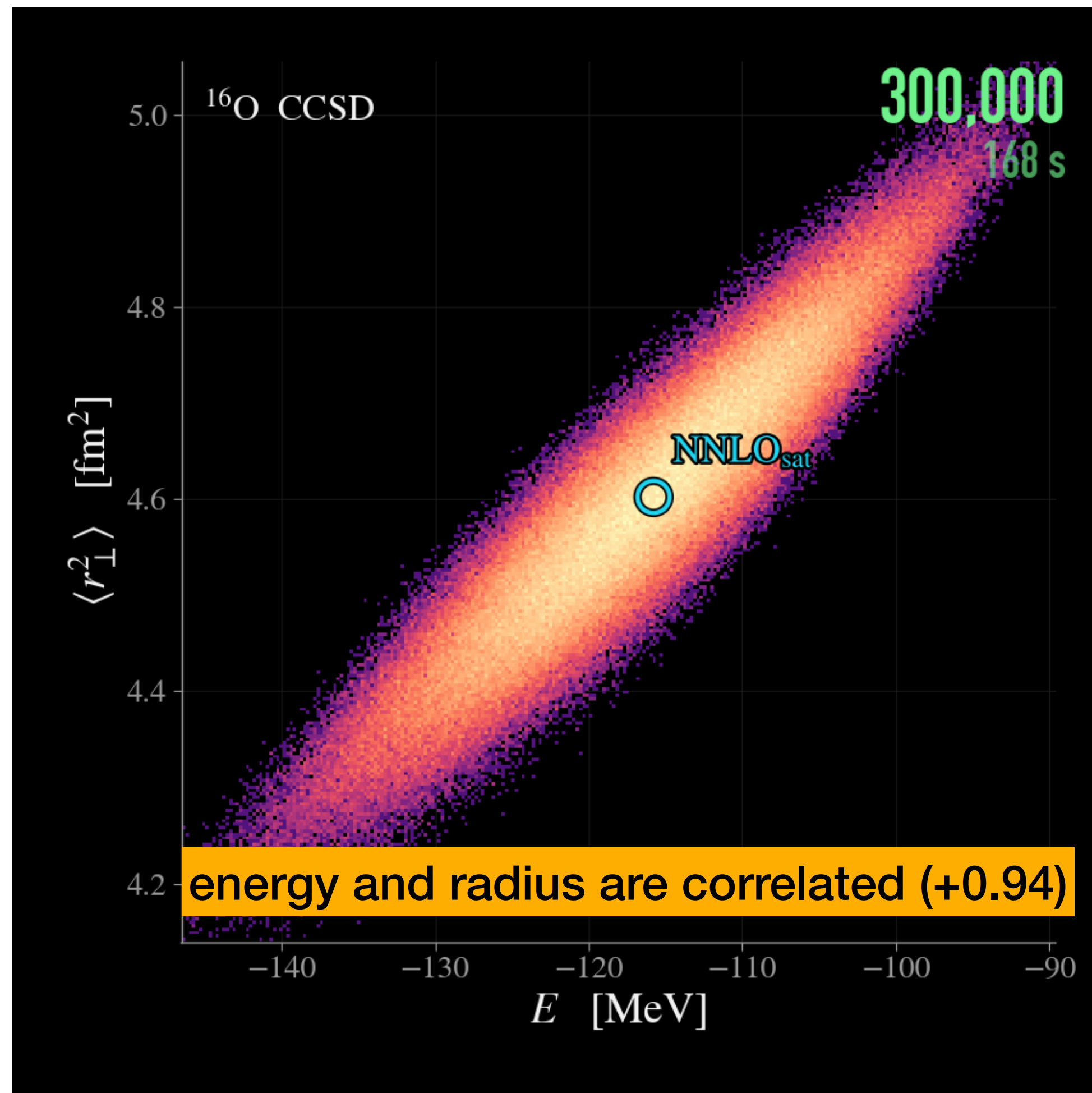
NNLO πN LECs $c \pm 0.05 \text{GeV}^{-1}$

NNLO 3N LECs $cD, cE \pm 0.05$

All results conditional on this choice of domain

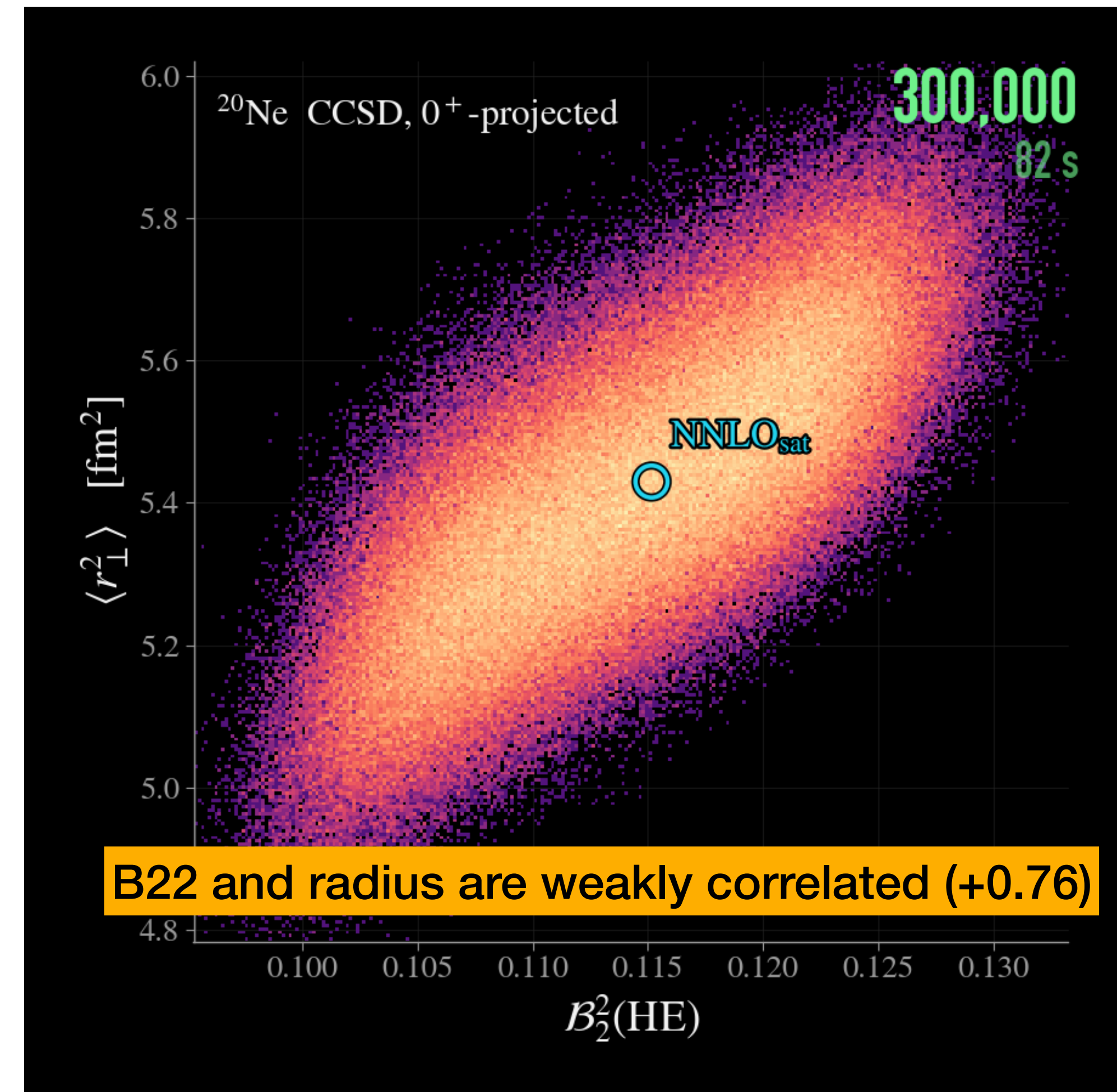
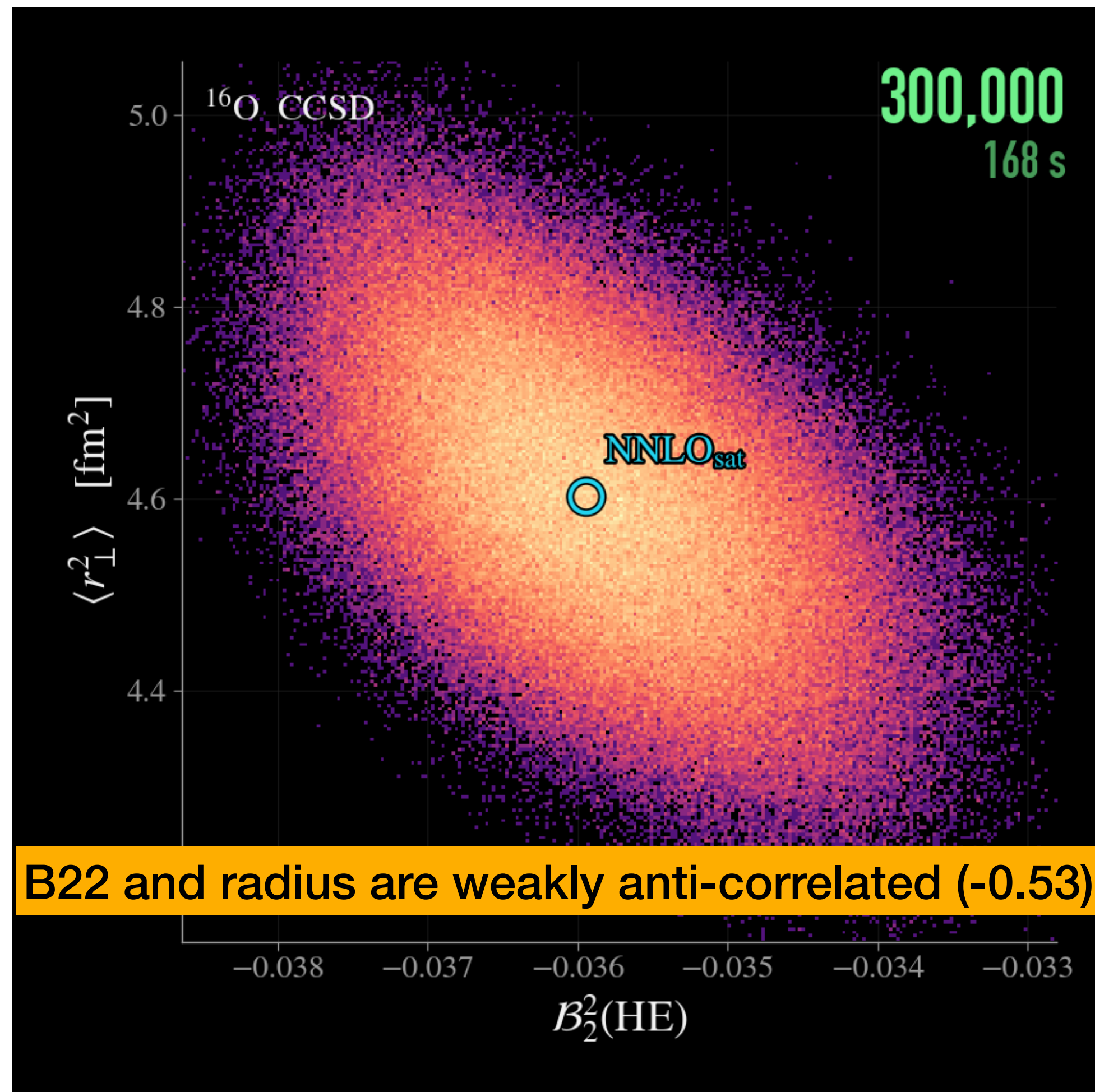
Energy and radius similarly correlated in ^{16}O and ^{20}Ne

Emulators make the 16-dimensional problem tractable (G. Hagen later today)



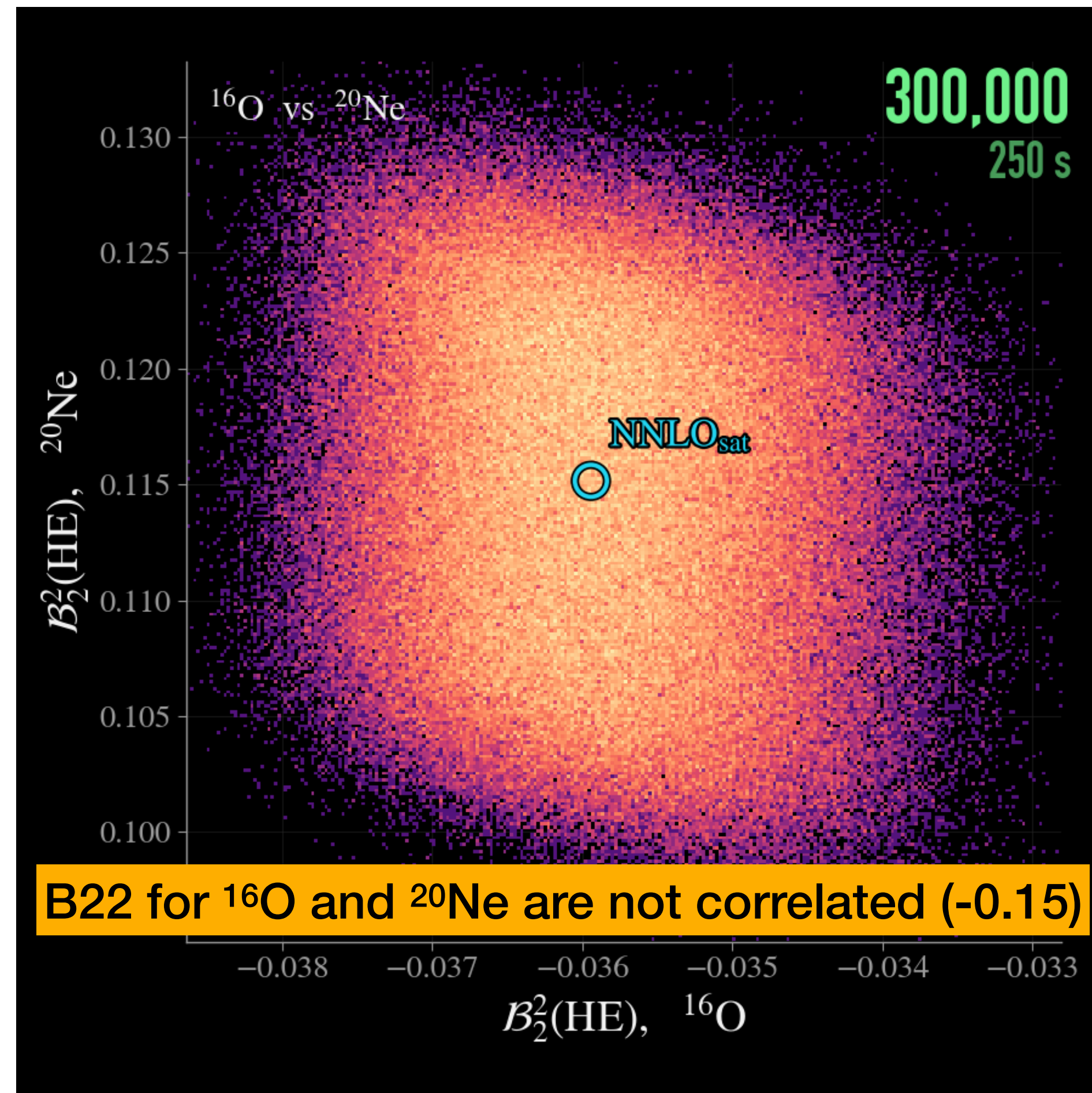
$B_2^2(HE)$ and radii correlate differently in ^{16}O and ^{20}Ne

Emulators make the 16-dimensional problem tractable (G. Hagen later today)



$B_2^2(HE)$ in ^{16}O and ^{20}Ne are not correlated at all

Emulators make the 16-dimensional problem tractable (G. Hagen later today)



Which LECs do the HIC observables see?

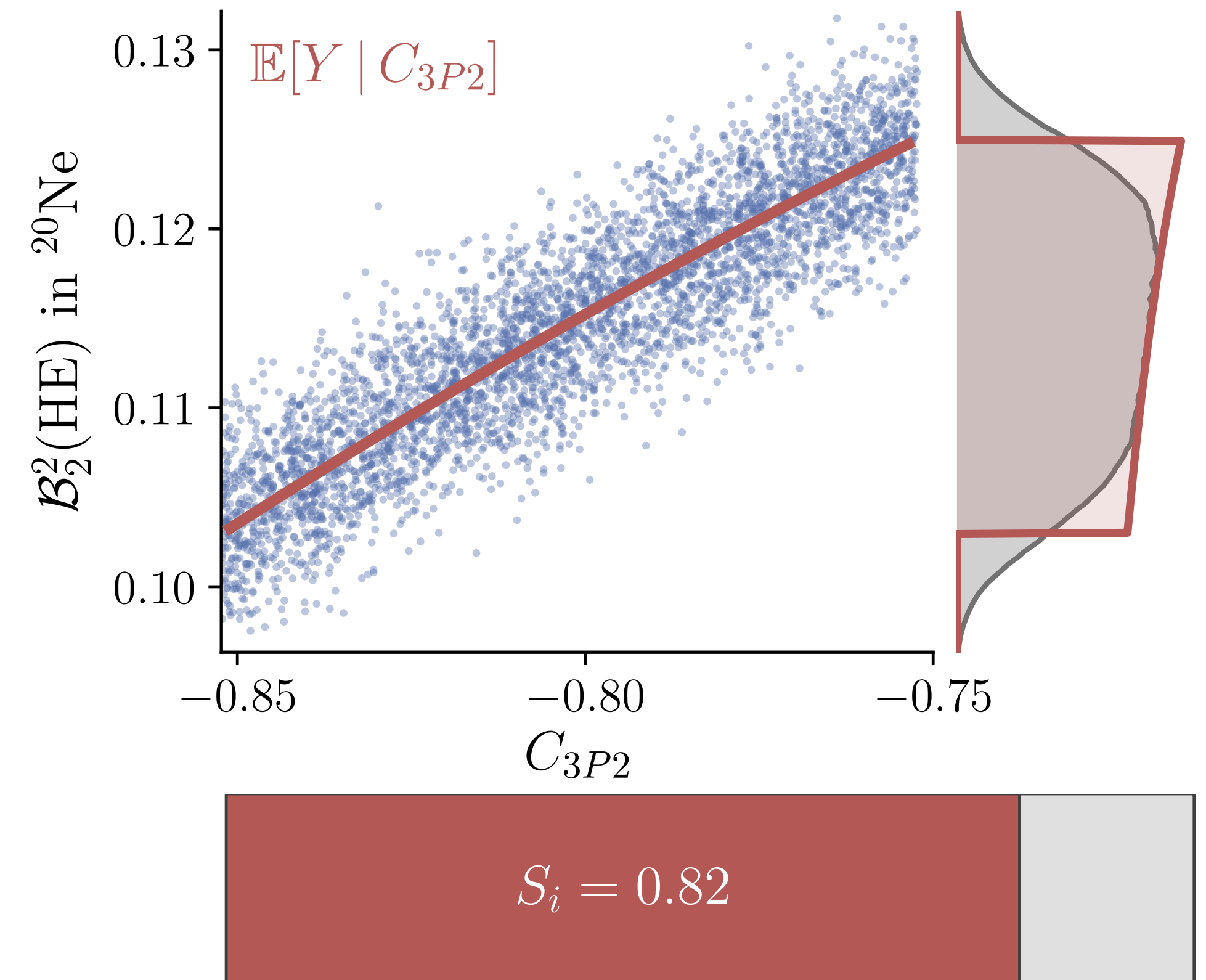
Global sensitivity analysis

A sensitivity analysis addresses the question ‘*How much does each model parameter contribute to the variance in the prediction?*’

GSA decompose the variance of a certain model output in terms of each input and their combinations.

Global = over the whole domain.

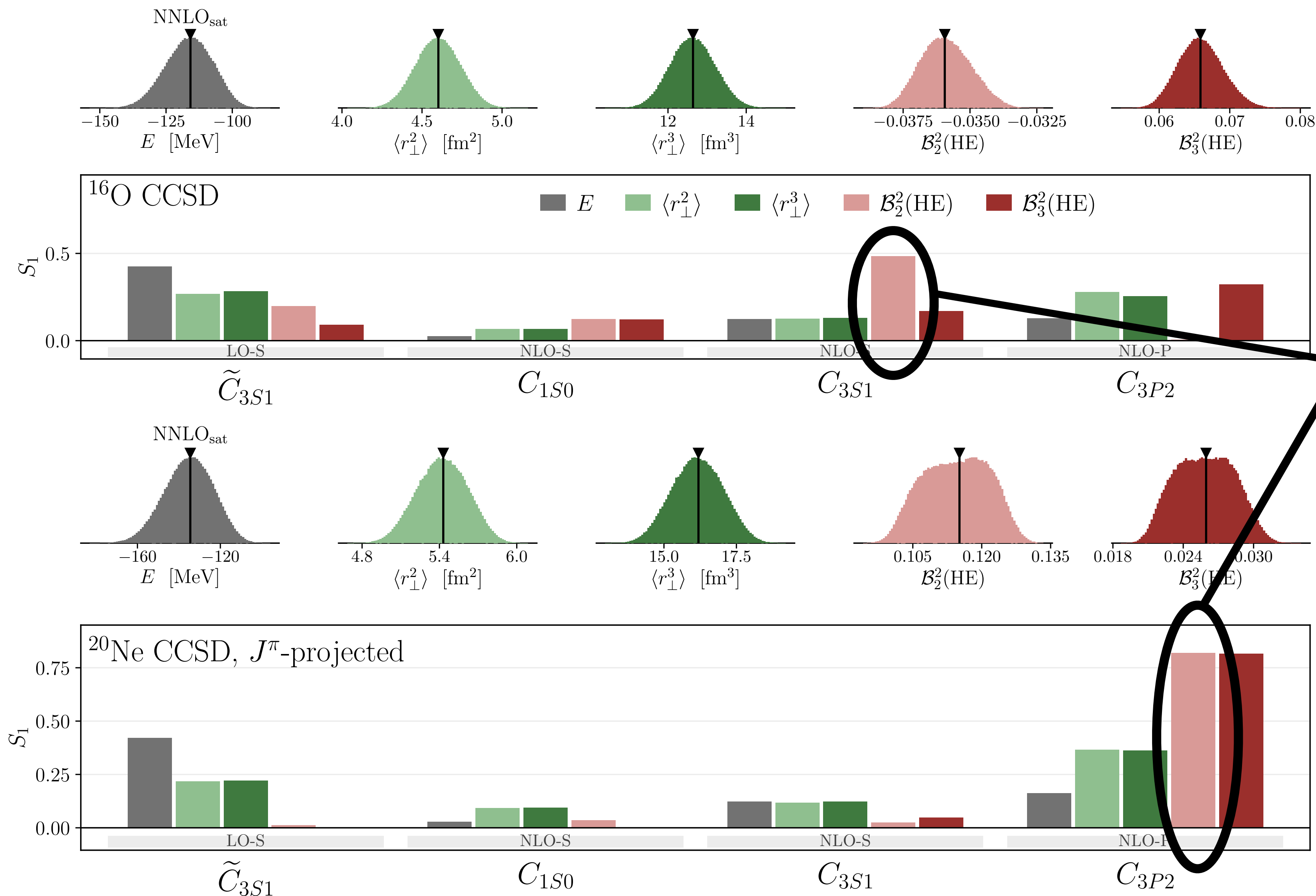
Cost: more than 100k samples. Only possible with emulators



$$S_i = \frac{\text{Var } \mathbb{E}[Y | \alpha_i]}{\text{Var}(Y)}$$

$$\sum_i S_i = 0.997 \Rightarrow \text{additive}$$

Which LECs are the HIC observables sensitive to?



- **Energy:** LO $^3\text{S}_1$ sets the scale of binding in both nuclei.
- **Radius:** Similar LO/NLO-S and NLO-P wave sensitivities in both nuclei.

Quadrupolar (B_2^2) shape:

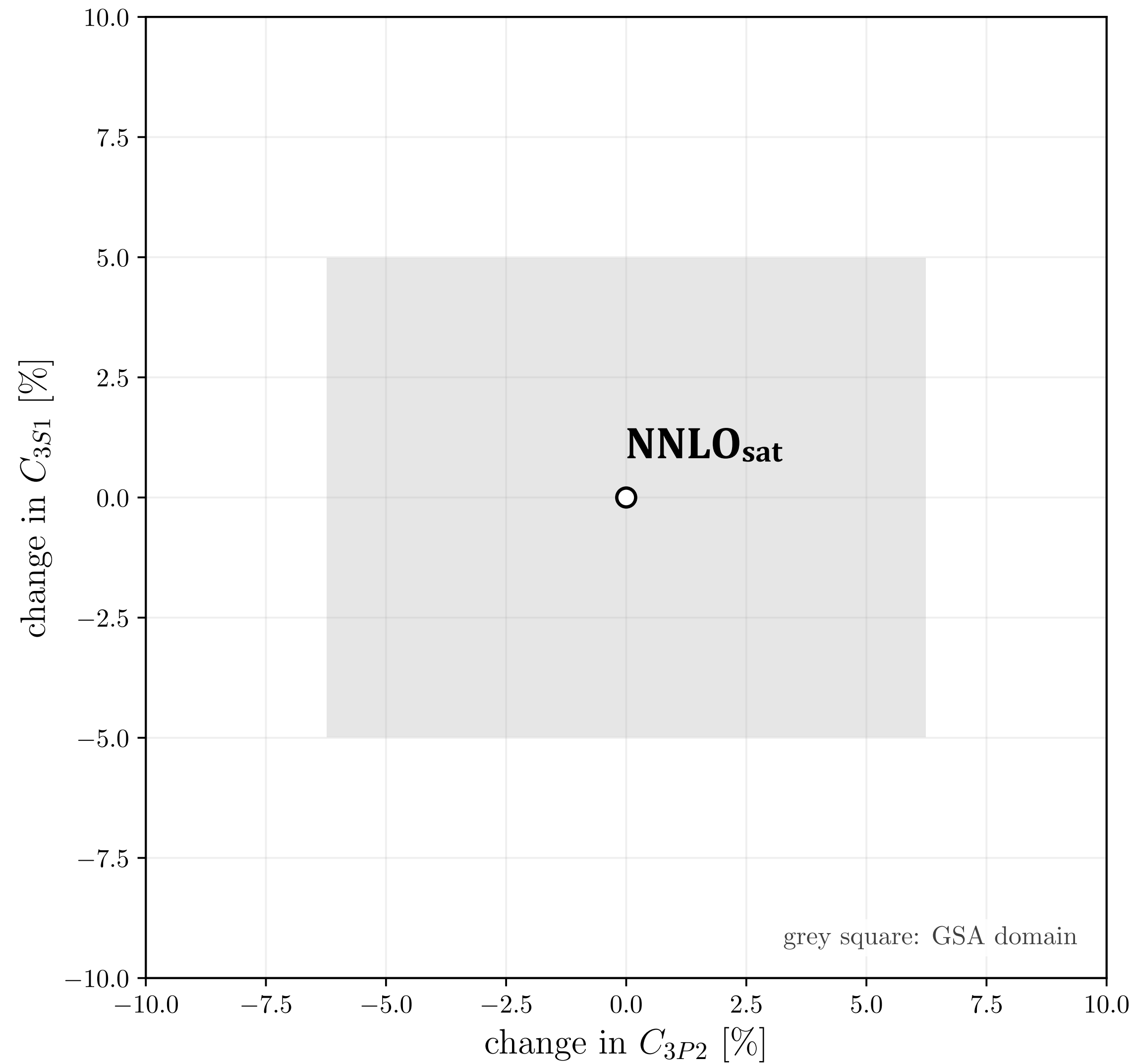
- ^{16}O NLO $^3\text{S}_1$ 49% + LO $^3\text{S}_1$ 20%
- ^{20}Ne NLO $^3\text{P}_2$ 82%

Octupolar (B_3^2) shape:

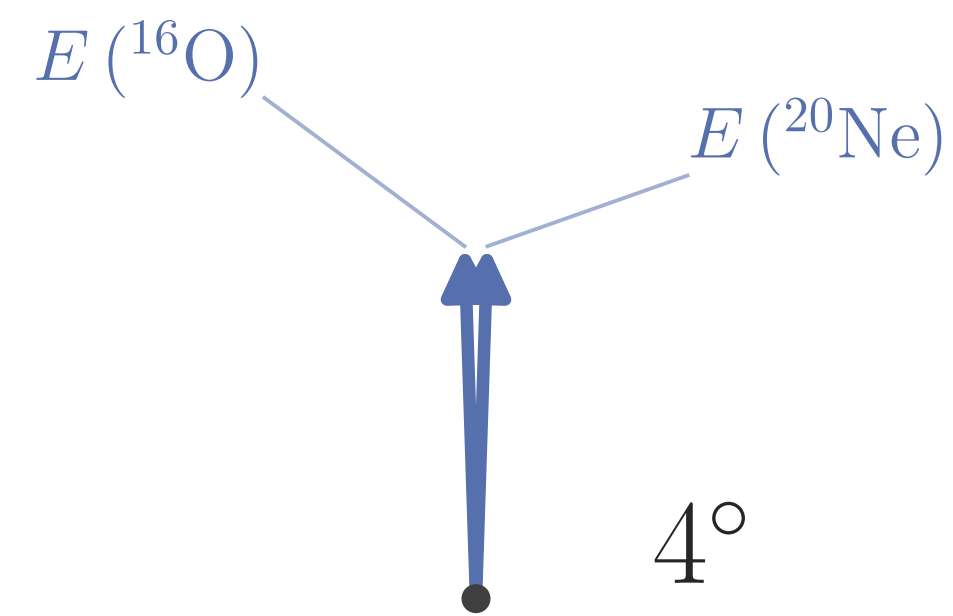
- ^{16}O NLO $^3\text{P}_2$ 32% $C_{3S1}/^1\text{S}_0$ 29%
- ^{20}Ne NLO $^3\text{P}_2$ 82%

- No LEC combinations needed to explain sensitivities (additive model).

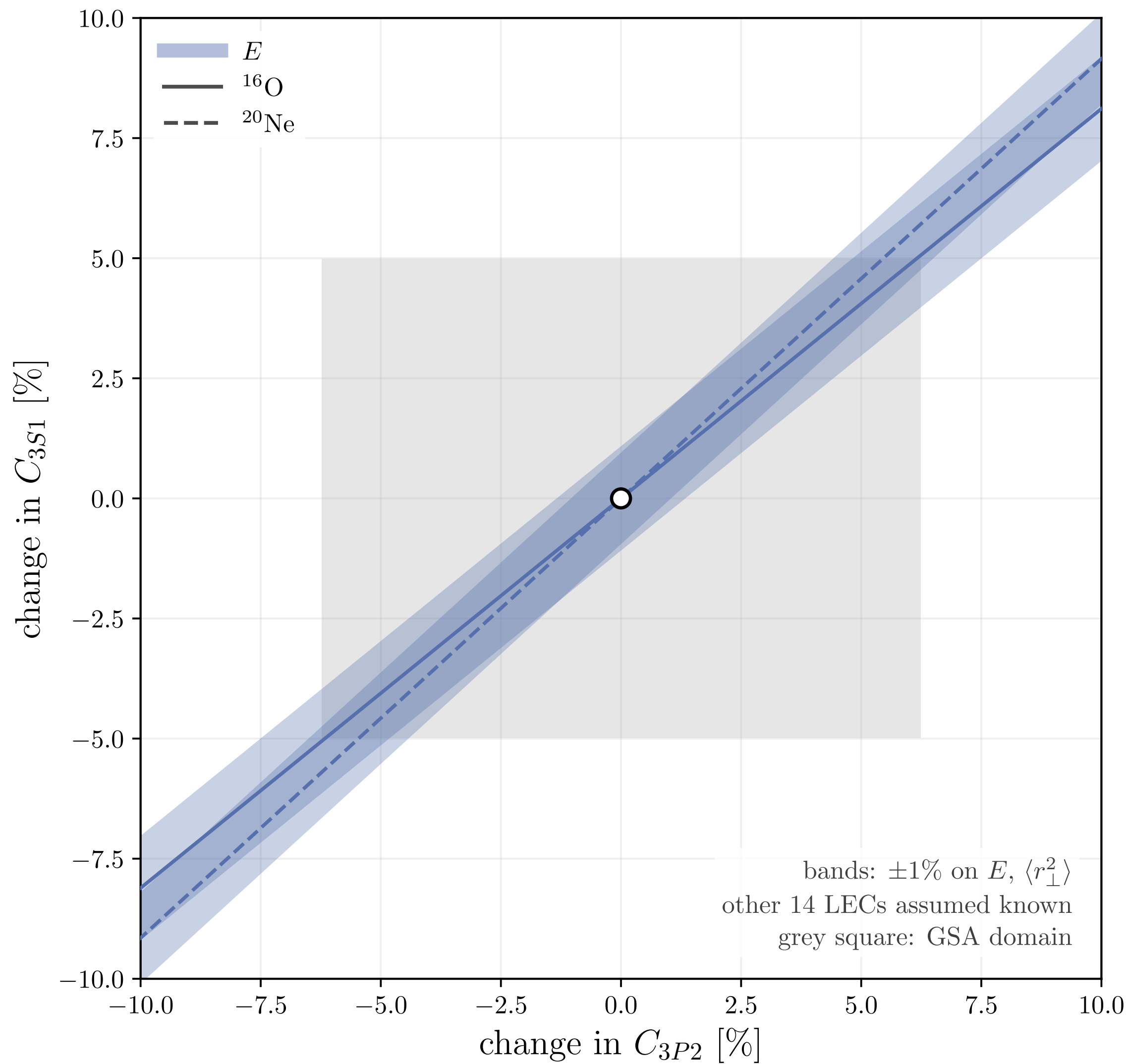
How different is B_2^2 relative to energy and radius?



How different is B_2^2 relative to energy and radius?



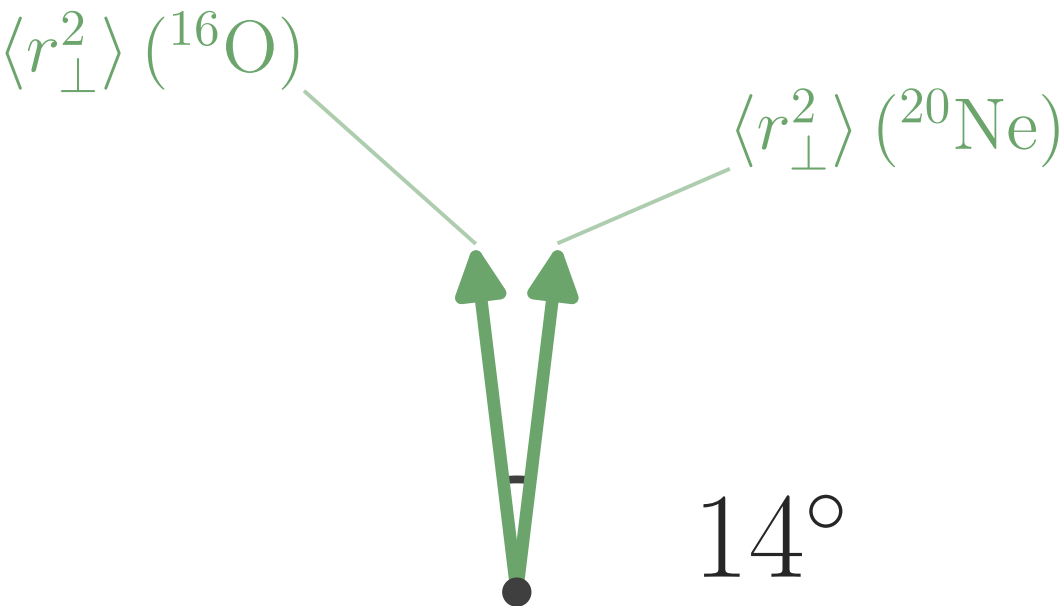
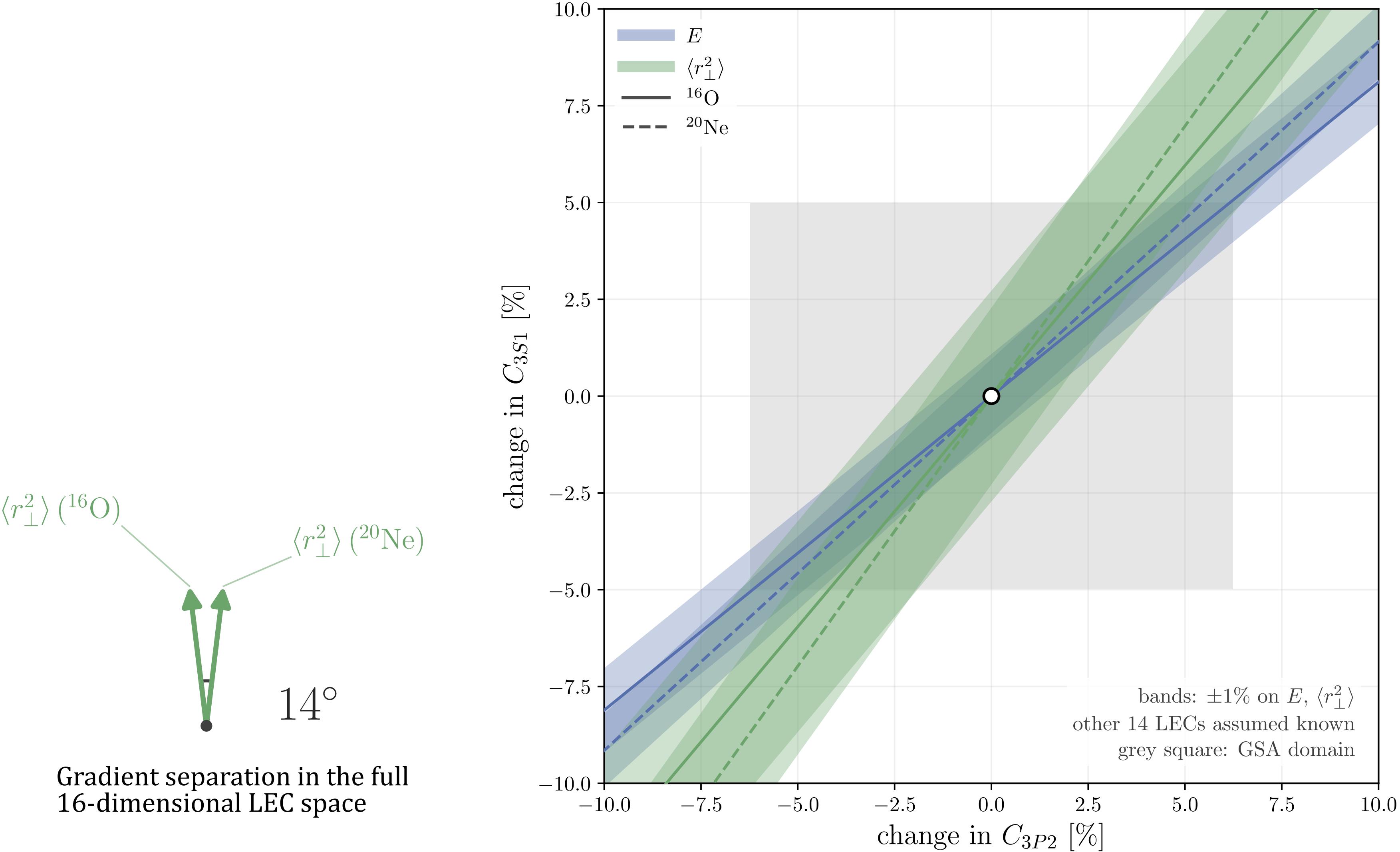
Gradient separation in the full 16-dimensional LEC space



Lines trace out the values of C_{3P2} and C_{3S1} that keep the value of the observable to within 1% of the NNLOsat value

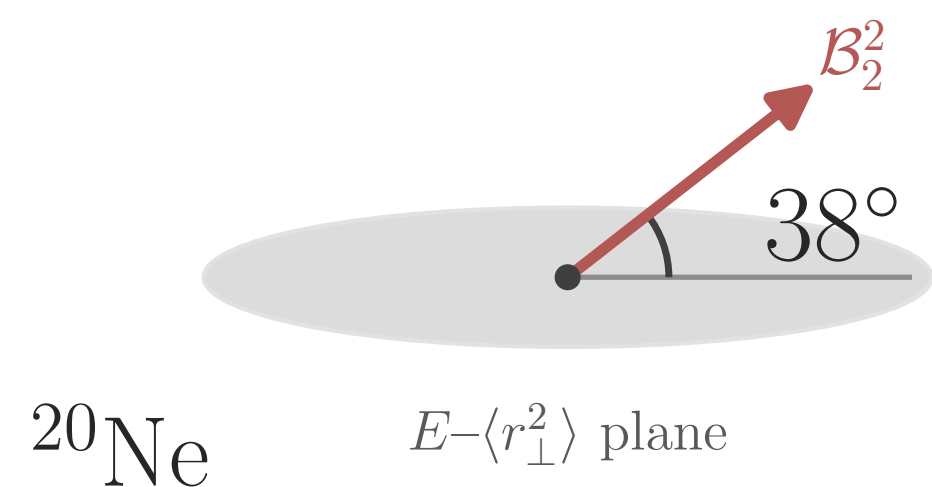
All other LECs fixed.

How different is B_2^2 relative to energy and radius?

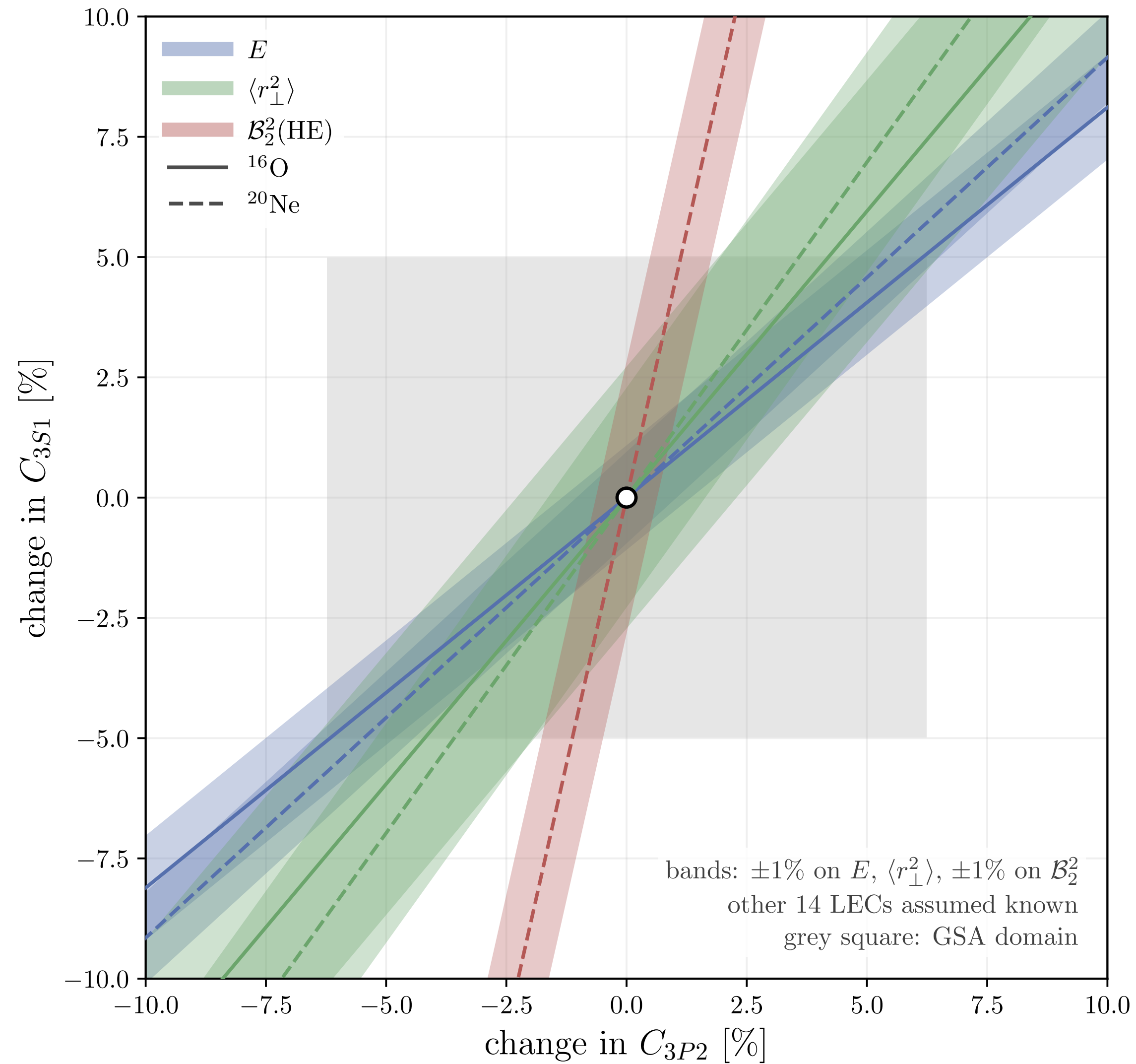


Gradient separation in the full 16-dimensional LEC space

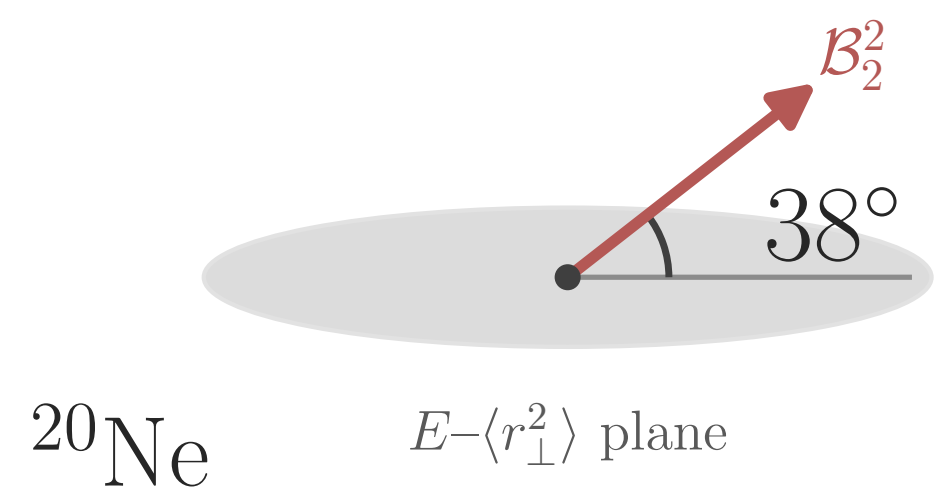
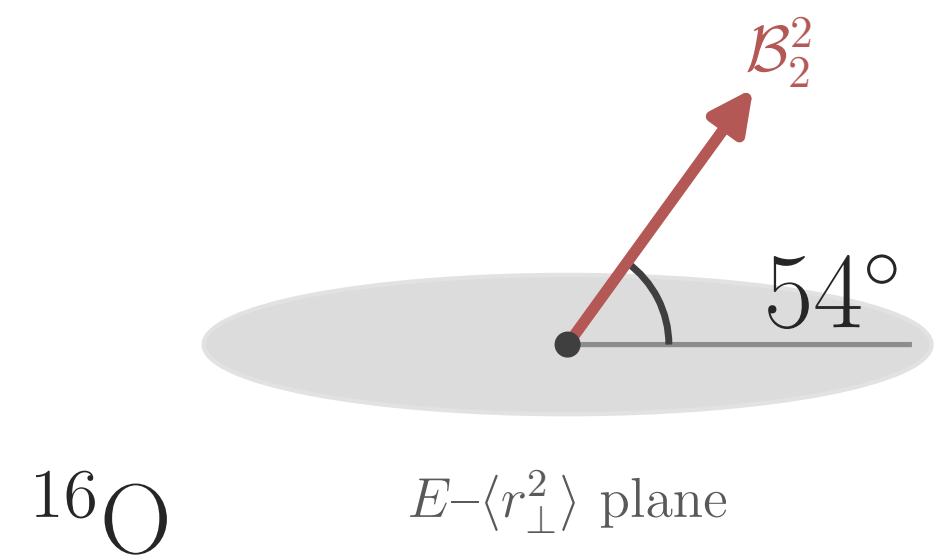
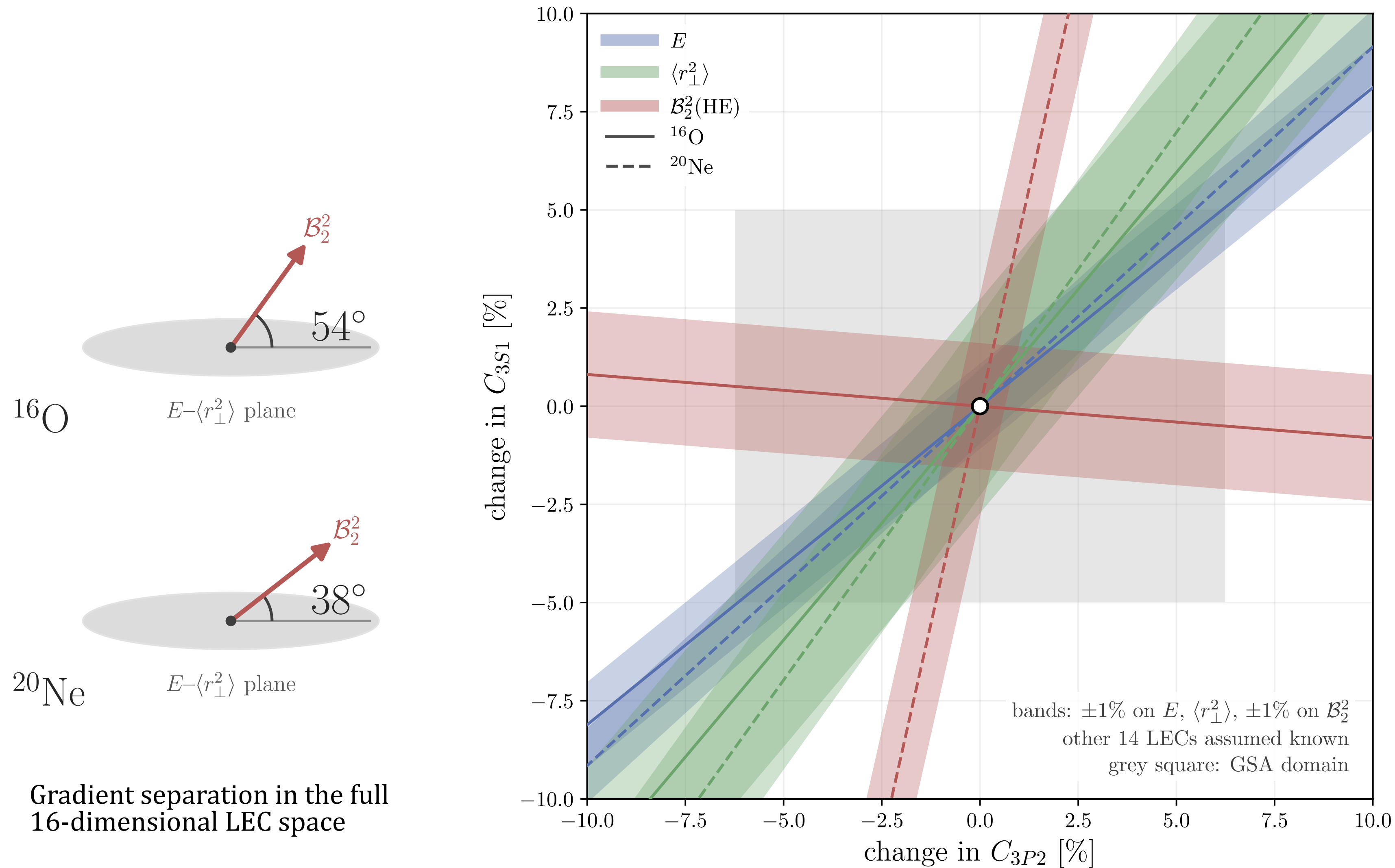
How different is B_2^2 relative to energy and radius?



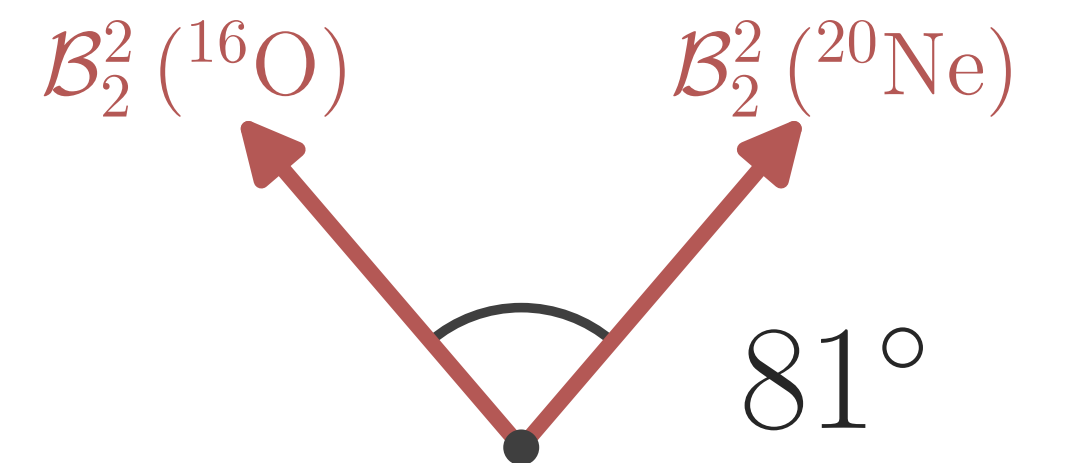
Gradient separation in the full
16-dimensional LEC space



How different is B_2^2 relative to energy and radius?

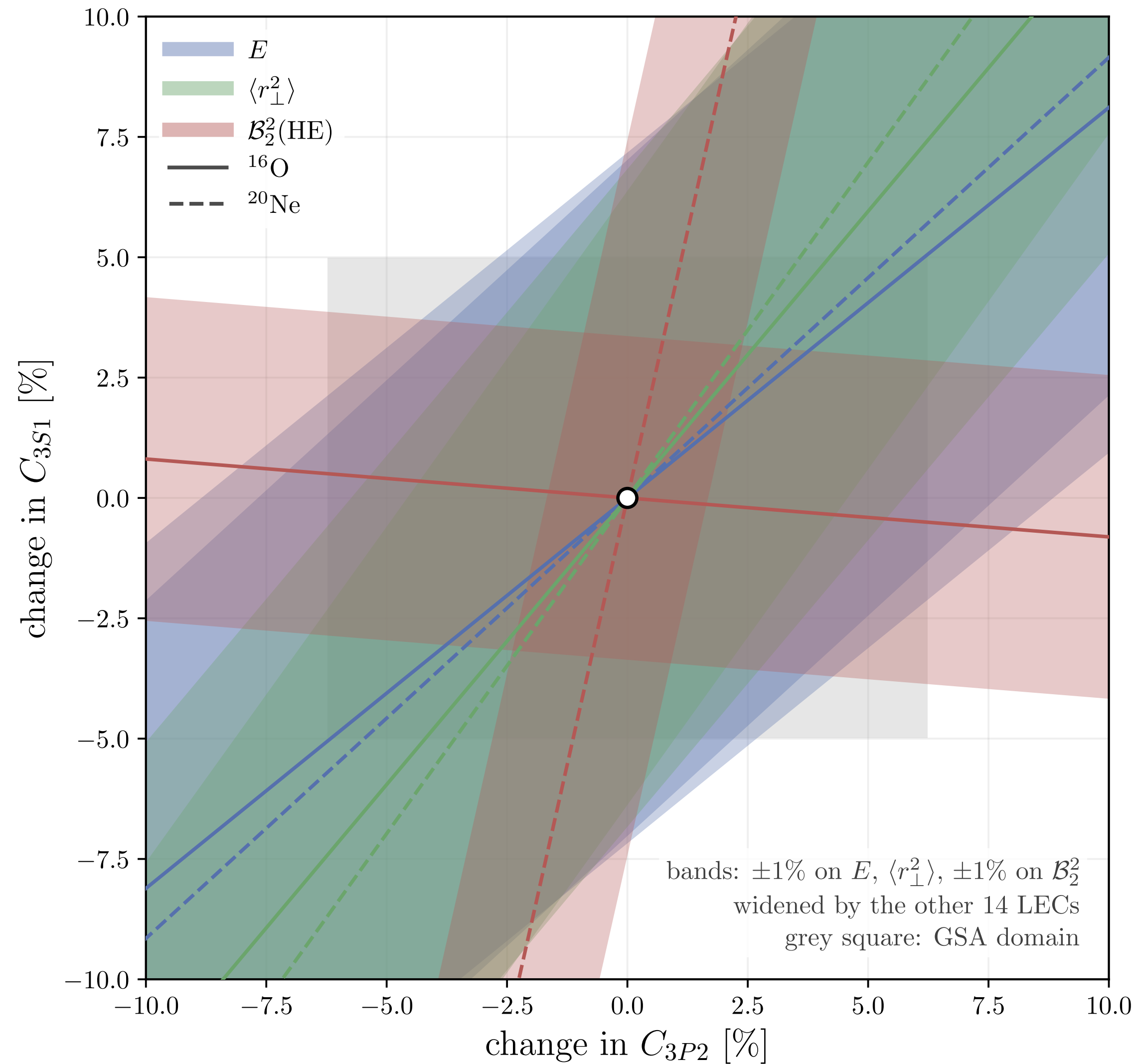


Gradient separation in the full 16-dimensional LEC space



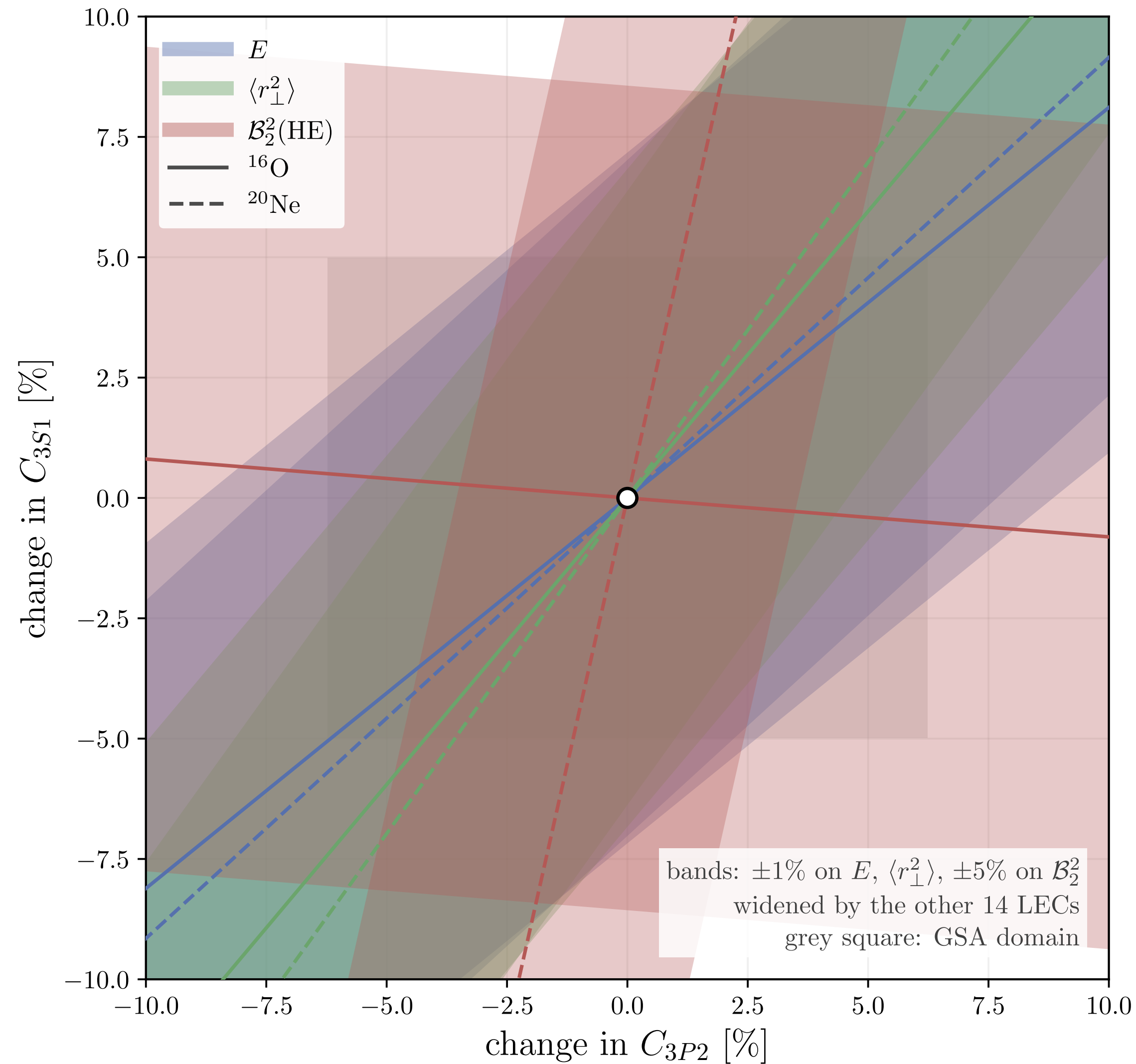
$B_2^2(HE)$ in Ne and O are nearly orthogonal.
 Different information!

How different is B_2^2 relative to energy and radius?



All other LECs can vary (1σ) within the GSA domain.

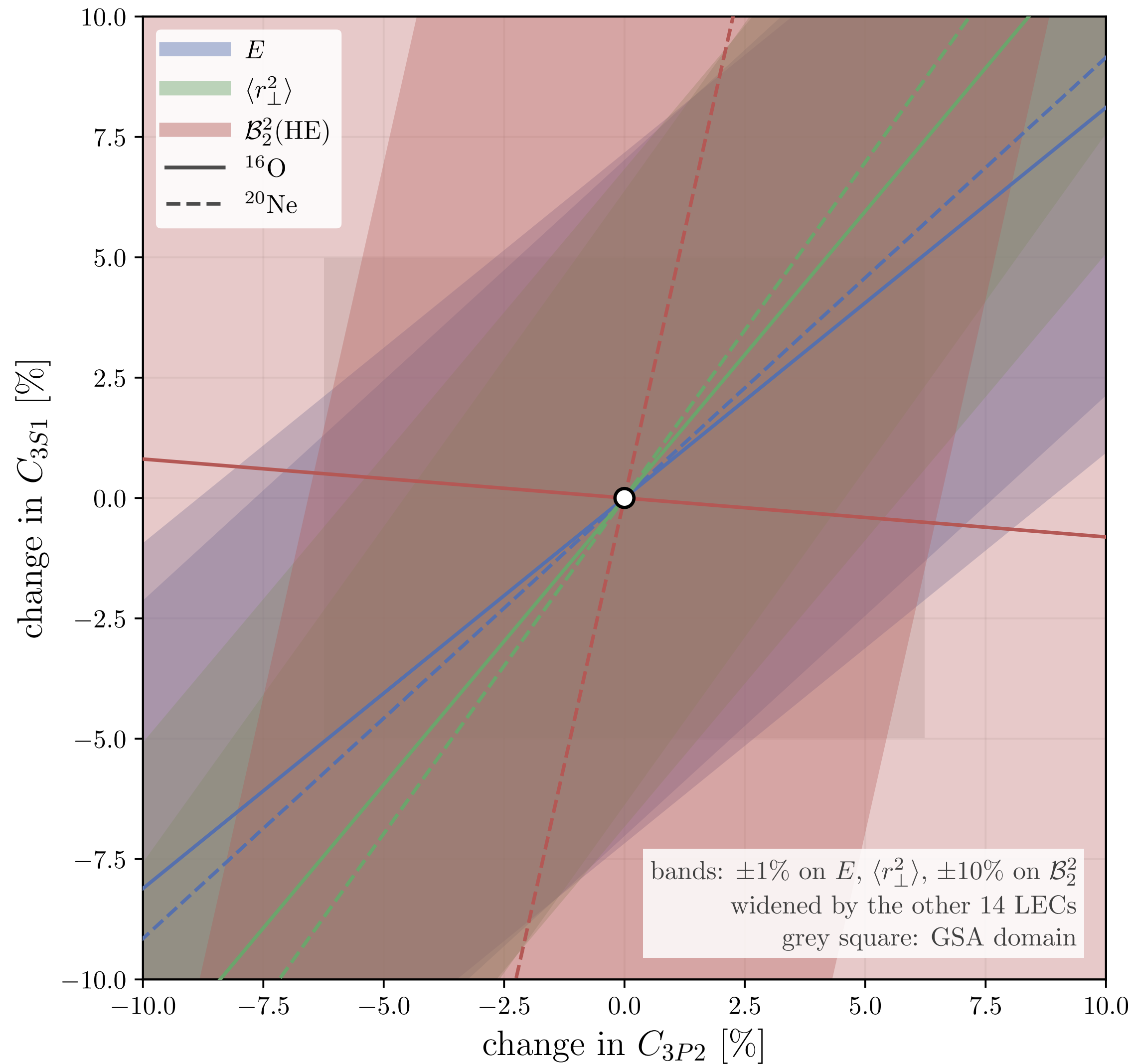
How different is B_2^2 relative to energy and radius?



Now assuming B_2^2 with a 5% total uncertainty.

All other LECs can vary (1σ) within the GSA domain.

How different is B_2^2 relative to energy and radius?



Now assuming B_2^2 with a 10% total uncertainty.

All other LECs can vary (1σ) within the GSA domain.

guesstimate: need to know $B_2^2(HE)$ to $\lesssim 5 - 10\%$ total uncertainty for it to inform chiral Hamiltonians

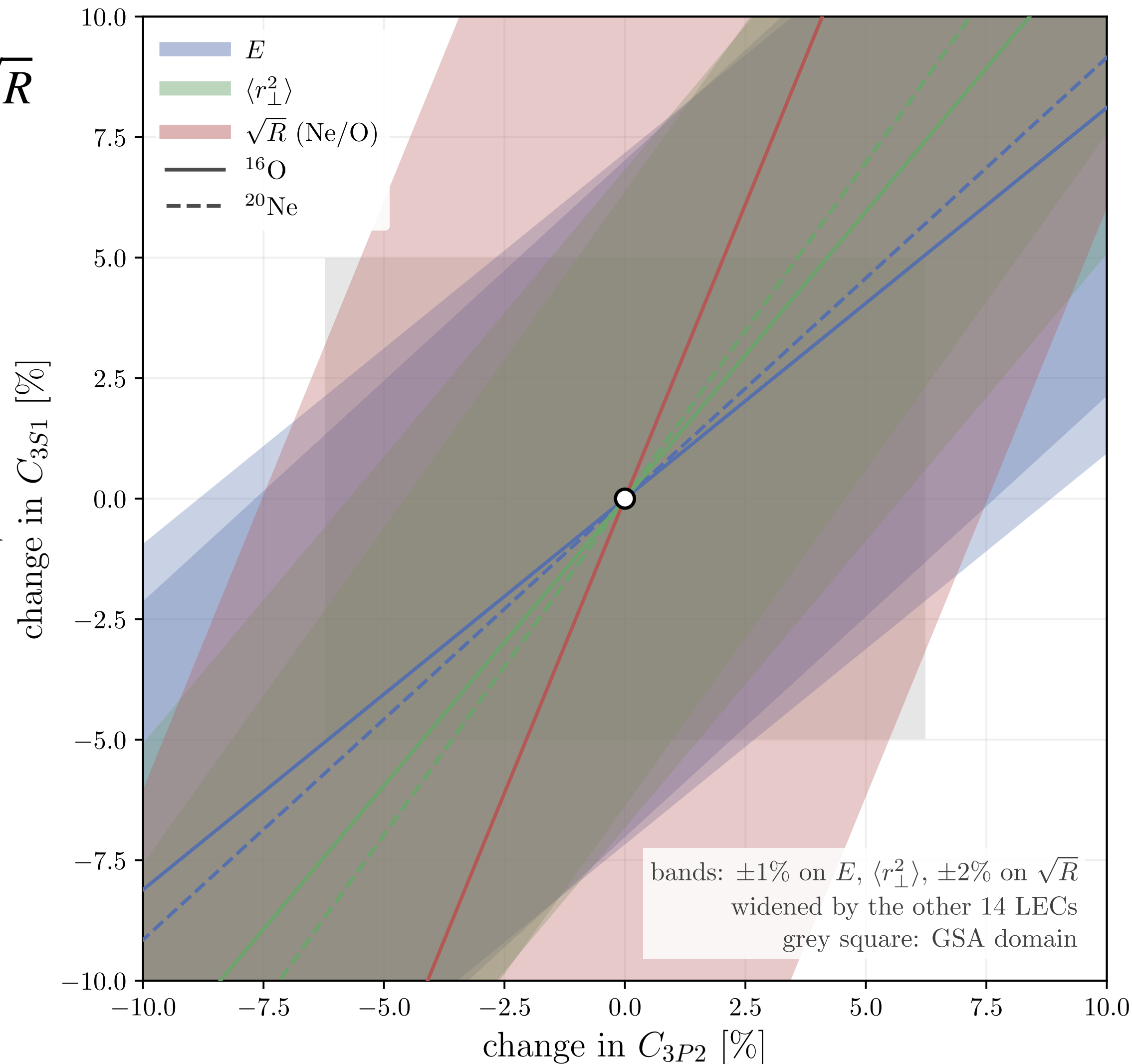
A ratio of HIC measurements

$$\frac{\kappa_2 \epsilon_2 \{2\}({}^{20}\text{Ne})}{\kappa_2 \epsilon_2 \{2\}({}^{16}\text{O})} = \sqrt{\frac{x_{20\text{Ne}}}{x_{16\text{O}}}} \equiv \sqrt{R}$$

😊 *The NeNe/OO ratio reduces the impact of hydrodynamic modeling.*

$$x_A \simeq \frac{1}{A} + \frac{3}{4\pi} \mathcal{B}_2^2(\text{HE})_A$$

😞 *Challenge: identify the 2b-part, where the LEC signal resides, on top of the 1b-part*



Now assuming a ratio $v_2(\text{NeNe}/\text{OO})$ with a 2% total uncertainty.

All other LECs can vary (1σ) within the GSA domain.

guesstimate: need to know the ratio $v_2(\text{NeNe}/\text{OO})$ to $\lesssim 1 - 2\%$ total uncertainty for it to inform chiral Hamiltonians

Summary and outlook

- **^{20}Ne & ^{16}O HIC data have very different LEC sensitivities to $B_2^2(HE)$.**
Great to have measured both of these light ions!
 - ^{20}Ne is $\sim 10\%$ S-waves and $\sim 80\%$ P-waves.
 - ^{16}O is $\sim 90\%$ S-waves and $\sim 1\%$ P-waves.
 - **First estimate:** need to compute $B_2^2(HE)$ with total uncertainty $\lesssim 5 - 10\%$ and $v_2(NeNe/OO)$ with total uncertainty $\lesssim 1 - 2\%$ to inform chiral Hamiltonians
- **Outlook I** $B_2^2(HE)$ relatively small compared to the 1b-part in light-mass systems. Stronger signal for heavier mass?
- **Outlook II** Go beyond GSA. The present domain is an estimated box, not a posterior, it ignores LEC correlations. EFT truncation and many-body errors are not accounted for in the analysis.
- **Outlook III** Compare the GSA results for B_2^2 with other signatures of nuclear deformation, e.g., the $4+/2+$ energy ratio (See G. Hagen talk)

Thank you for your attention!