Partial Wave Analysis at HADES

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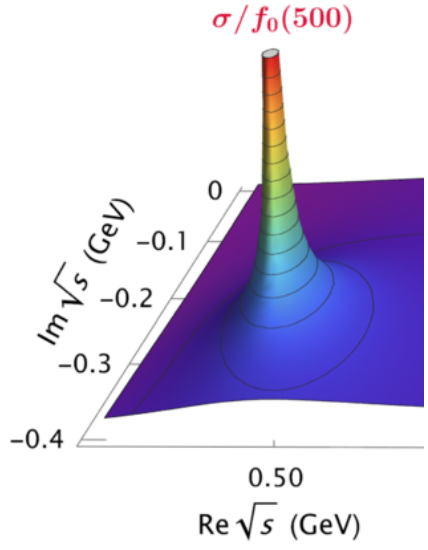
Concepts of Partial Wave Analysis

- **The Basic Idea of PWA:** $\sigma = \sum$ states.
- **Pole:** A singularity at $\sqrt{s} = M_R - i\frac{\Gamma_R}{2}$.
- **Pole Residue:** couplings and channel decay rates.
- **Right-Hand Cut (RHC):** open thresholds, unitarity, self-energy loops.
- **Left-Hand Cut (LHC):** t & u channels.



(https://doi.org/10.1103/fdv7-k23j)

Baryon Spectroscopy: Strategy

Goal: resonance dynamics in the dense nuclear medium.

How: Modular, open-source PWA framework (`AmpTools` [1]) unitary + analytic.

$\pi^- p$: K/D-Matrix Formalism

- Overlapping resonances (2nd/3rd region).
- Chew-Mandelstam unitarization + conformal Left-Hand Cuts (LHC).

pp: JüBo DCC Model

- Production couplings in baryon-meson channels.
- Dynamic Coupled-Channel; exact 2-body unitarity, $L \leq 5$, M=[1,2.5] GeV.

Framework available on GSI GitLab
Users can add their own reactions/amplitudes.

[1] M. Shepherd *et al*, https://doi.org/10.5281/zenodo.7336113

Upcoming Pion Beam
($\sqrt{s} \sim 1.7\ GeV$)

Baryon-Meson couplings
($\pi^- p \rightarrow (\pi\pi/\omega/\eta)\ N$ / $K^0\Lambda$ / $K\Sigma$)

- Sparse π-beam database (PWA)
- Baryon Structure: Cascade decays, ηn couplings

E.M. Baryon Transitions
($\pi^-\ p \rightarrow n\ e^+e^-$)

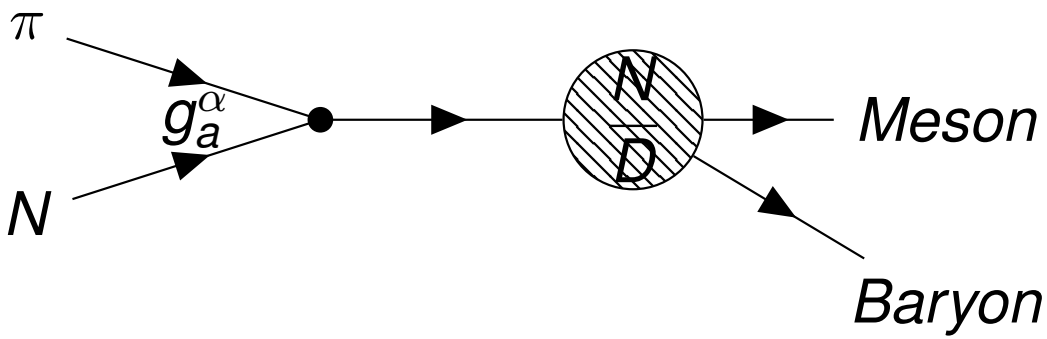
- Broad range of $q^2 = (M_{ee})^2$ $\rightarrow$ Time-Like
- Confirmation for VMD (ρ, ω)
- Extract SDMEs
- Limited data available

Cold Matter Studies (C, Ag Targets)

- ω absorption, ρ spectral function
- Strangeness production
- Limited data available

N/D-Matrix [a]

[a] PhysRevD.84.076001



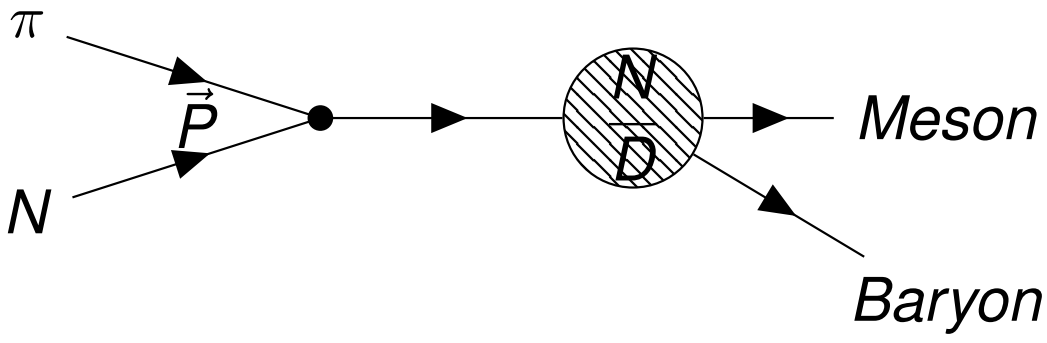
$$A_{ab} = \sum_{\alpha,\beta} g_a^\alpha d_{\alpha\alpha} D_{\alpha\beta} g_b^\beta$$

$$\hat{D} = \hat{d}(\hat{I} - \hat{B}\hat{d})^{-1}, \qquad B(s) = B_{\mathrm{RHC}}(s) + B_{\mathrm{LHC}}(s)$$

$d_{\alpha\alpha}$ propagators (resonant & non-resonant)
g_a^α coupling of channel a to state α a, b, j: channels, α, β: bare states

Production Vector[a]

[a]PhysRevD.84.076001



$$A_{ab} = \sum_{\alpha,\beta} P_a^{(\alpha)} d_{\alpha\alpha} D_{\alpha\beta} g_b^{L(\beta)}$$

$$\hat{D} = \hat{d}(\hat{I} - \hat{B}\hat{d})^{-1}, \qquad B(s) = B_{\mathrm{RHC}}(s) + B_{\mathrm{LHC}}(s)$$

$d_{\alpha\alpha}$ propagators (resonant & non-resonant)
g_a^{α} coupling of channel a to state α a, b, j: channels, α, β: bare states

AmpTools Fit

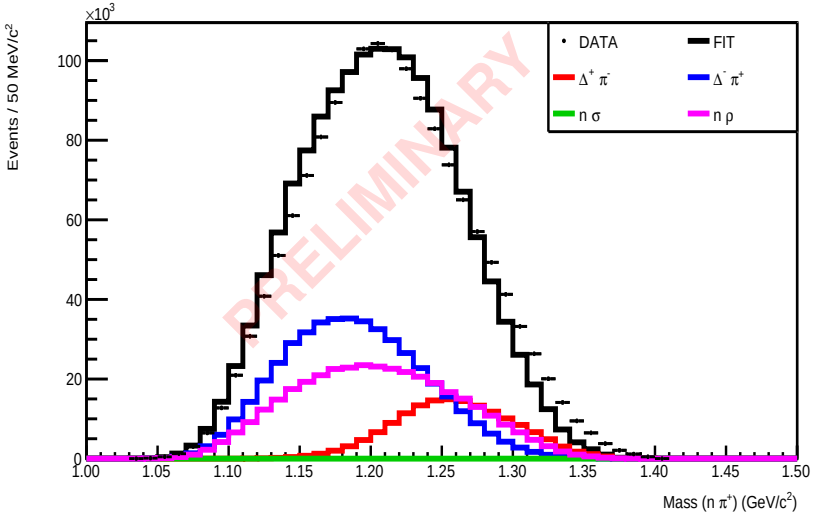
$$\pi^- p \to n\pi^+\pi^-$$

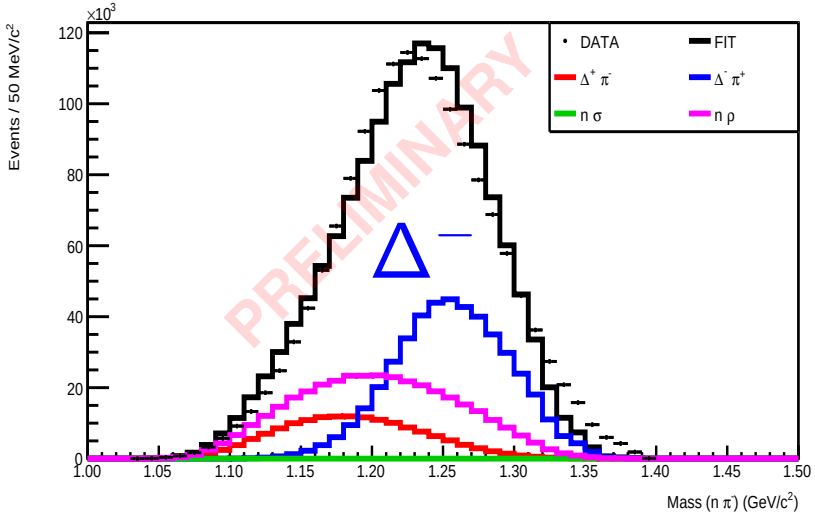
Channels: $\Delta^+\pi^-$, $\Delta^-\pi^+$, $n\sigma$, $n\rho$

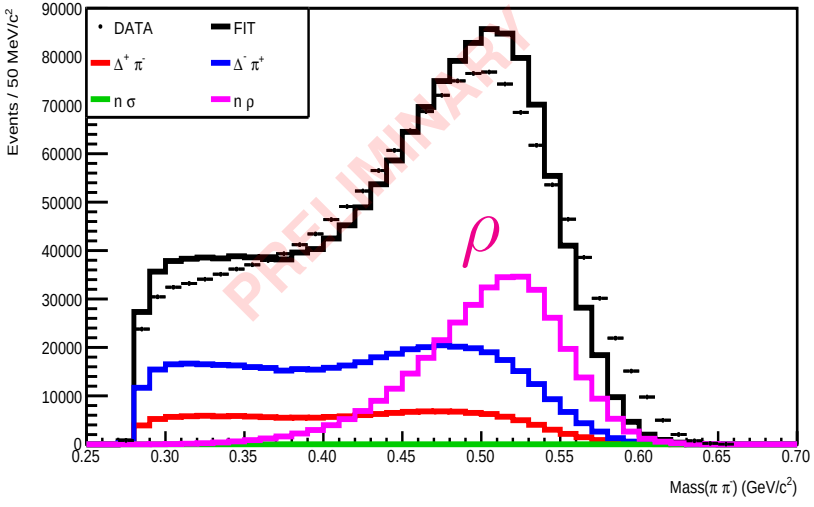
Resonances (Poles): N(1440), N(1520) & N(1535)

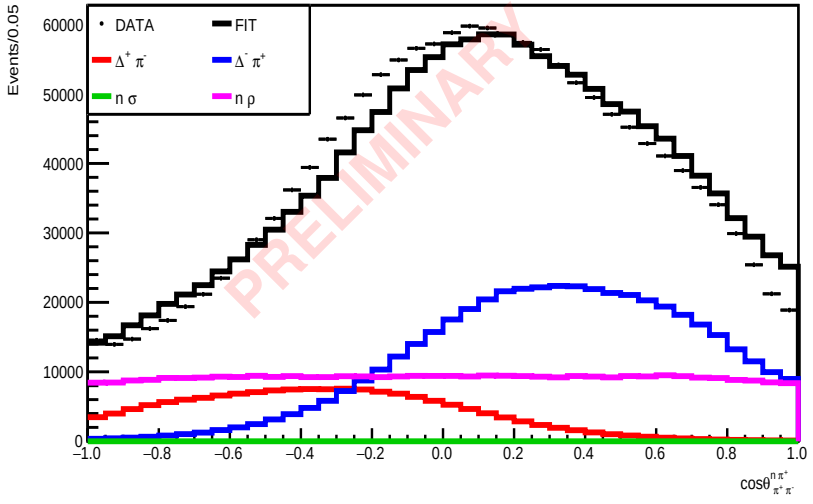
- 4 energy bins: $\sqrt{s} = 1.47 - 1.55$ GeV
- Spectral function description of Δ, σ & ρ.
- Dispersion Matrix: - bare resonance mass
  - Total width = Σ partial widths.
- Fit Parameters: production & decay couplings (energy independent).
- Left-hand cuts disabled.

Example Fit


DATA
FIT
Δ⁺ π⁻
Δ⁻ π⁺
n σ
n ρ
Events / 50 MeV/c²
Mass (n π⁺) (GeV/c²)
PRELIMINARY


DATA
FIT
Δ⁺ π⁻
Δ⁻ π⁺
n σ
n ρ
Events / 50 MeV/c²
Mass (n π⁻) (GeV/c²)
Δ⁻
PRELIMINARY


DATA
FIT
Δ⁺ π⁻
Δ⁻ π⁺
n σ
n ρ
Events / 50 MeV/c²
Mass(π π⁻) (GeV/c²)
ρ
PRELIMINARY


DATA
FIT
Δ⁺ π⁻
Δ⁻ π⁺
n σ
n ρ
Events/0.05
PRELIMINARY

Baseline Fit Results (Zero-Background)

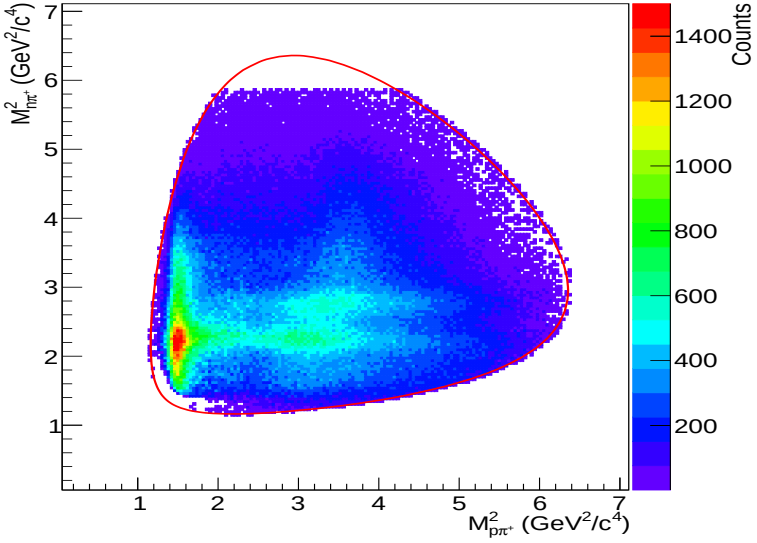
- ~~non-resonant terms~~ & ~~LHC~~ (set to zero).

Resonance	Pole Mass (MeV)	Pole Width (MeV)
$N(1440)$	1387	67
$N(1520)$	1513	2.5
$N(1535)$	1532	1.4

Proton-Proton Reactions

- **Exclusive production** N^* Δ**:** Differential cross-section & coupling strengths.
- **Rescattering effects:** Resolve overlapping contributions.

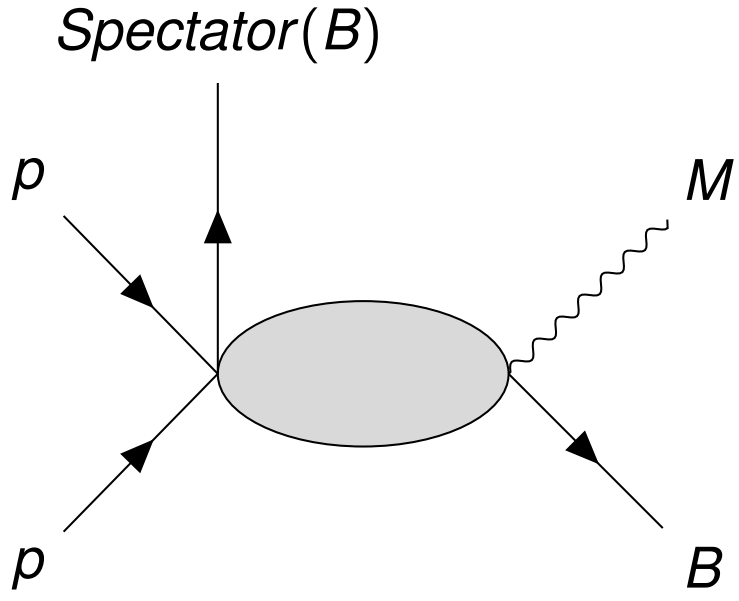
Adapted from S. K. Sahu



p p $\rightarrow p\pi^+(n)$

Julich-Bonn Model

- JuBo: Dynamical coupled-channel (DCC) model for meson-baryon reactions.
- πN, ηN, $K\Lambda$, $K\Sigma$, $\pi\Delta$, ρN, σN [2].
- Partial Waves: Up to L=5, Resonances: 1.07-2.5 GeV
- Exact 2 body unitarity, 3-body unitarity through $\sigma N, \pi\Delta, \rho N$.
- Left-hand cuts: *t*- and *u*-channel exch. (ρ, σ, K^*, N, Λ, Σ).



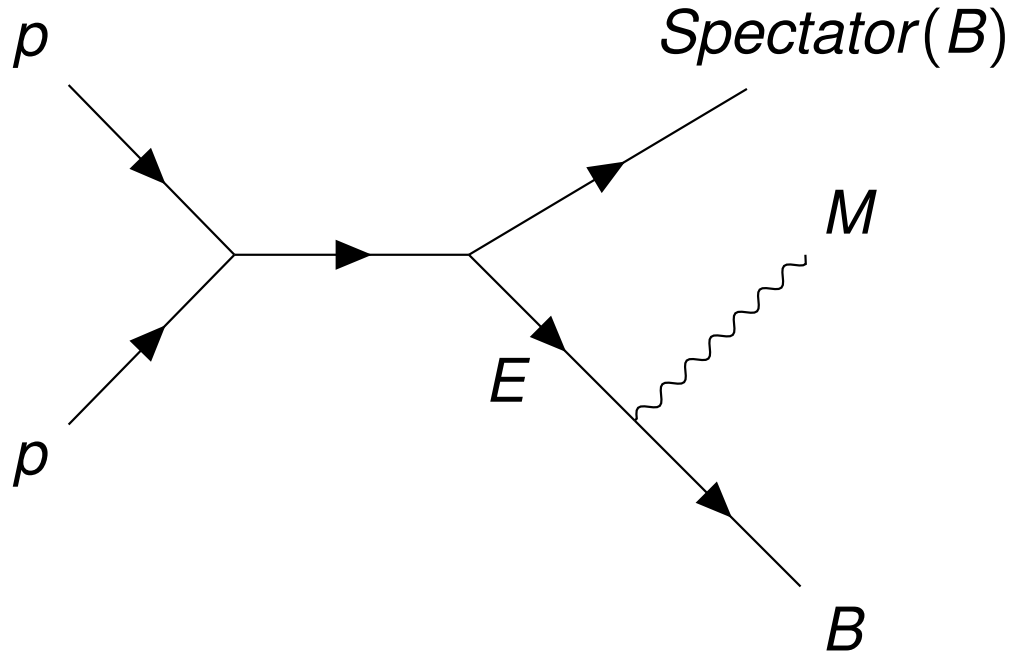
[2] doi:10.1140/epja/s10050-022-00852-1

Amplitude Structure

- The total amplitude is constructed as:

$$\mathcal{A}(L, I, J, s, E) = \mathcal{A}_{\text{bgd}}(s, E) + \mathcal{A}_{\text{res}}(s, E)$$

- **Background** and **resonance** amplitudes are pre-calculated based on JuBo fit.
- Each amplitude is a complex number.

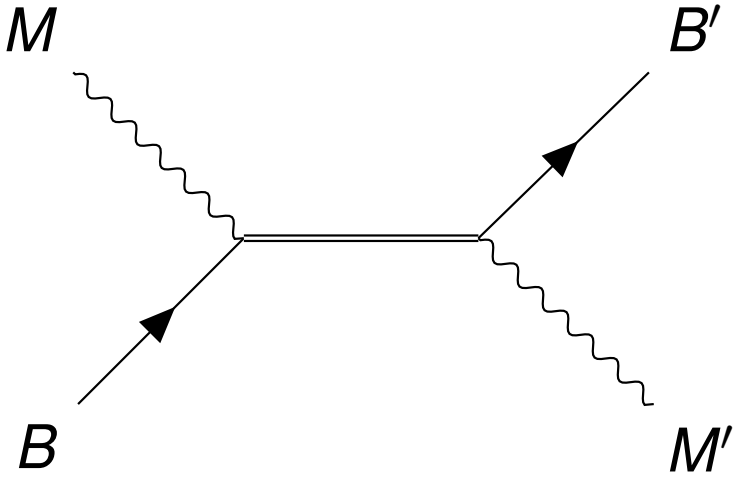


Background Contribution

- The background contribution is a calculated as:

$$\mathcal{A}_{\text{bgd}} = \alpha_0 + \alpha_1 (E - E_{\text{thr}}) + \sum_{k}^{\textit{Channels}} \alpha_k \, \Gamma_k(E)$$

- $\Gamma_k(E)$: complex amplitudes.
- α_k: decay channel fit parameters.
- α_k: same for all analyses.

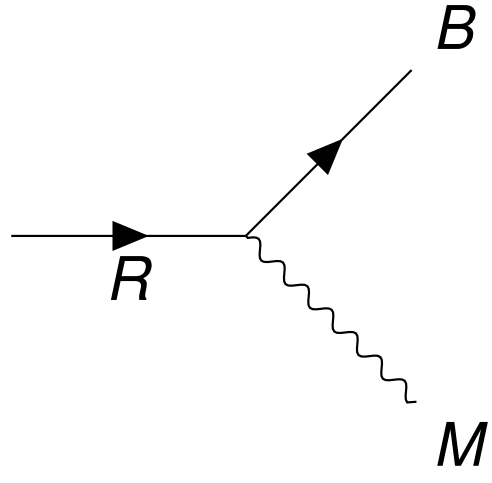


Resonance Contribution

- The resonance contribution to the amplitude is constructed as:

$$\mathcal{A}_{\text{res}} = \sum_{i=1}^{\text{Resonances}} \gamma_i \, \Gamma_i^{(R)}(E)$$

- $\Gamma_i^{(R)}(E)$: complex resonance amplitudes.
- γ_i: resonance coupling.
- User can choose which resonances to include.



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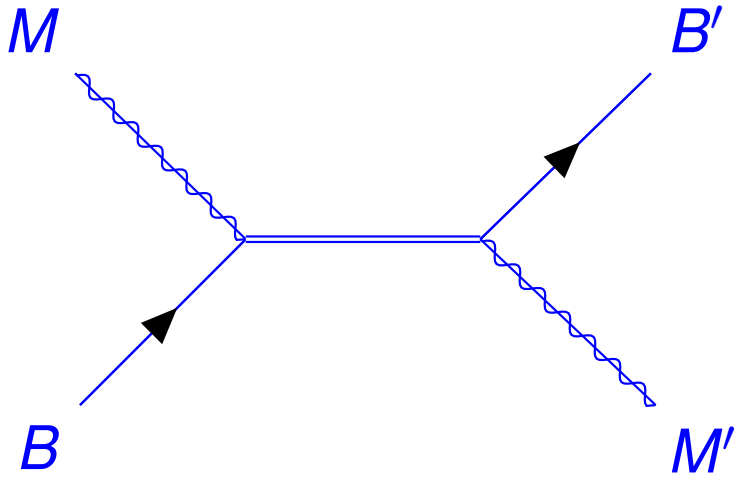
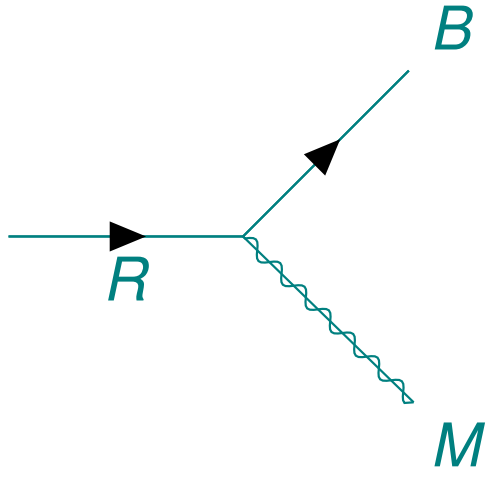
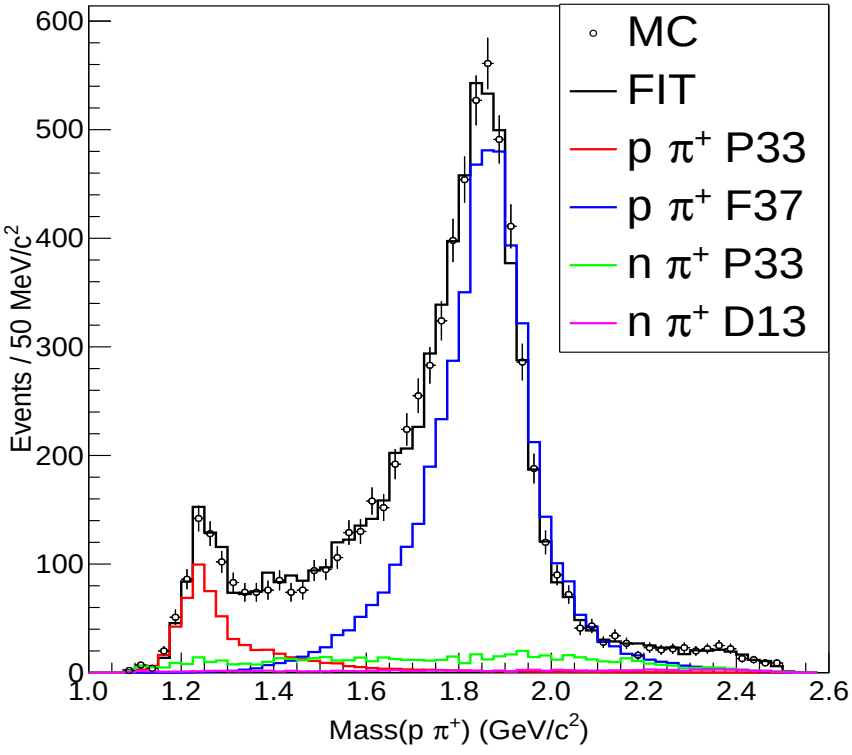
$$\mathcal{A}(L, I, J, s, E) = \alpha_0 + \alpha_1 (E - E_{\text{thr}}) + \sum_{k}^{\text{Channels}} \alpha_k \Gamma_k(E) + \sum_{i=1}^{\text{Resonances}} \gamma_i \Gamma_i^{(R)}(E)$$

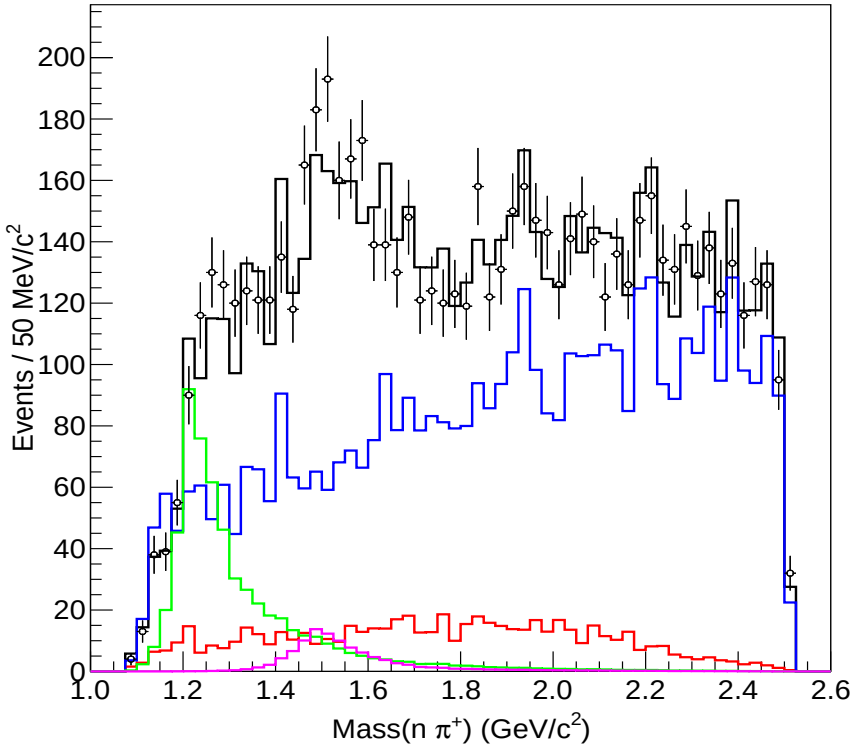




JuBo Model - Proof of Concept

- MC simulation for pp $\rightarrow$ p π^+ (n) ($\sqrt{s} = 3.47$ GeV).
- pπ^+: $\Delta^{++}(1232)$ (P_{33}), $\Delta(1950)$ (F_{37})
  $n\pi^+$: $\Delta^{+}(1232)$ (P_{33}), N(1520) (D_{13})
- Perfect acceptance.





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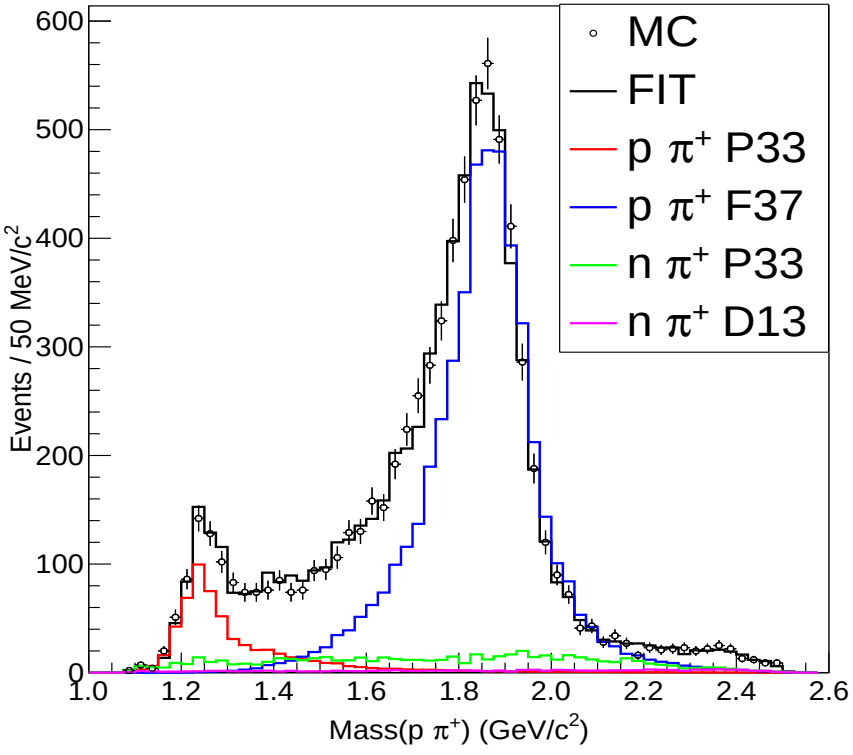
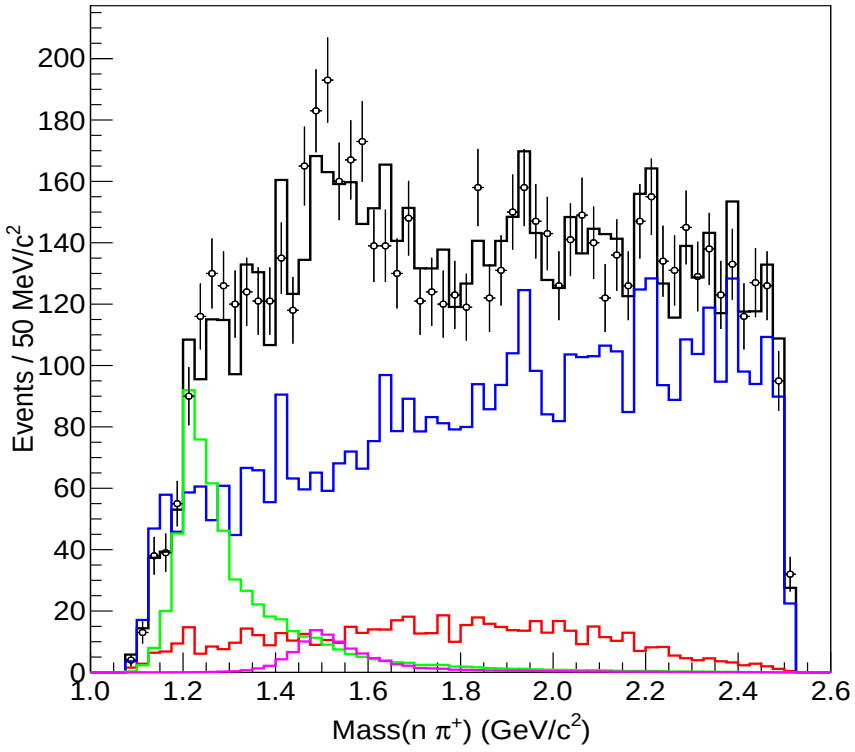
$p\pi^+ n \rightarrow$ S. K. Sahu
$\Lambda k^0_S\, p\pi^+ \rightarrow$ S. Pattnaik





Summary

PWA @ HADES: Modular partial wave analysis

- K-/Dispersion-matrix amplitudes, Chew-Mandelstam self-energy, Left-Hand Cuts, Production Vector.
- **Channels:** $\pi p \to \{\pi N, \pi\Delta, \rho N, \sigma N\}$
- **In progress:** HADES 2014 data

JuBoAmp: Event-by-event unitary analytic low comp cost coupled-channel analysis.

- **Channels:** $pp \to \{\pi N, \eta N, K\Lambda, K\Sigma, \pi\Delta, \rho N, \sigma N\}$
- **Resonance Range:** 1.07–2.5 GeV; Partial waves $L = 5$
- **In progress:** pp(@4.5 GeV)$\to$: $\{p\pi^+n, p\pi^0p, \Delta K\Lambda\}$

Outlook

PWA @ HADES: Modular partial wave analysis

- Fine Tune 2014 Fit: Left-hand cuts
- Covariant Tensor Formalism
- $p\pi^-\pi^0$ data, More channels ($\pi^0\Delta^0, \eta N$)

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- $\pi^0 p$, $K^+\Lambda$

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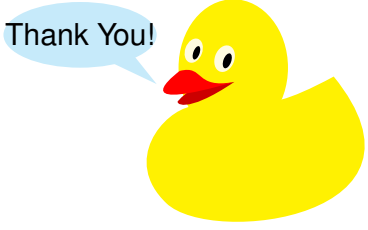

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