Heavy-meson scattering off hadrons in hot and dense matter: benefits from unitarity, chiral and heavy-quark symmetries

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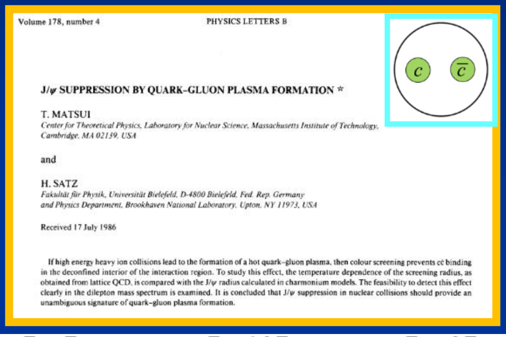
Introduction and motivation

Why **charm** in dense matter


Volume 178, number 4 PHYSICS LETTERS B

J/ψ SUPPRESSION BY QUARK–GLUON PLASMA FORMATION ☆

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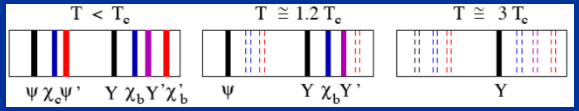
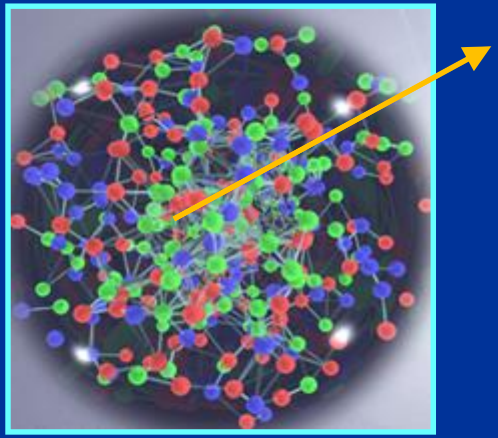
If high energy heavy ion collisions lead to the formation of a hot quark–gluon plasma, then colour screening prevents $c\bar{c}$ binding in the deconfined interior of the interaction region. To study this effect, the temperature dependence of the screening radius, as obtained from lattice QCD, is compared with the J/ψ radius calculated in charmonium models. The feasibility to detect this effect clearly in the dilepton mass spectrum is examined. It is concluded that J/ψ suppression in nuclear collisions should provide an unambiguous signature of quark–gluon plasma formation.

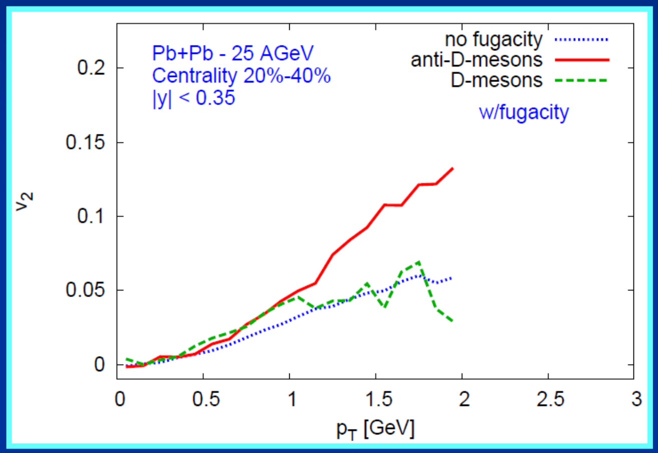


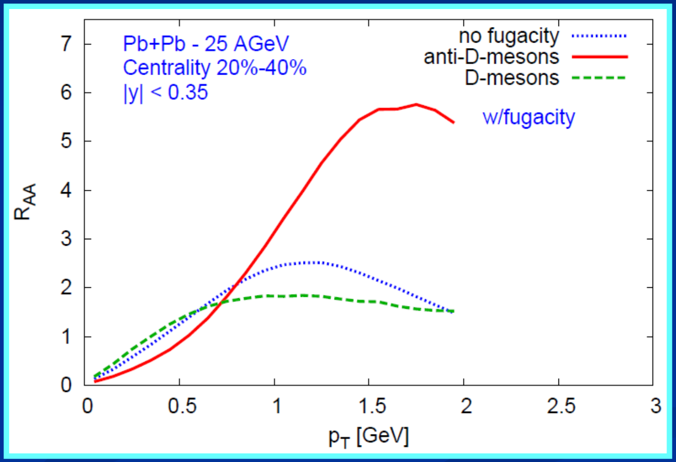


- Initial motivation: J/ψ suppression in HICs
- Dynamics in hadronic phase relevant!
- Sector not constrained by Ch.Symmetry
- Heavy flavor relaxation in hadronic phase

Introduction and motivation

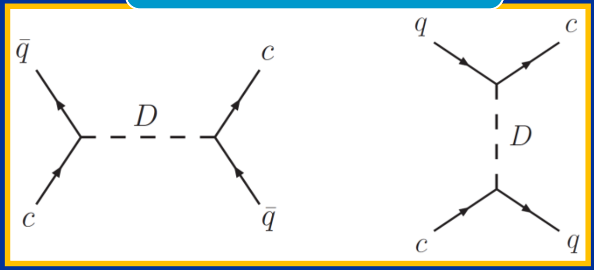
- **Recent results from transport theory at FAIR:** (UrQMD Hybrid) *Lang, van Hees, Steinheimer, Bleicher, arXiv:1305.1797*



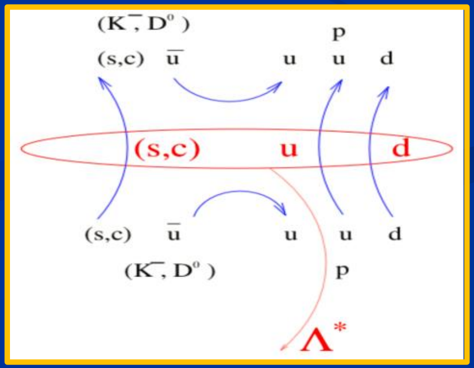


- Interaction: *Heavy-light **resonant** interaction + "quark-hadron" duality*

partonic world



hadronic world



Introduction and motivation

Heavy mesons through hadronic matter

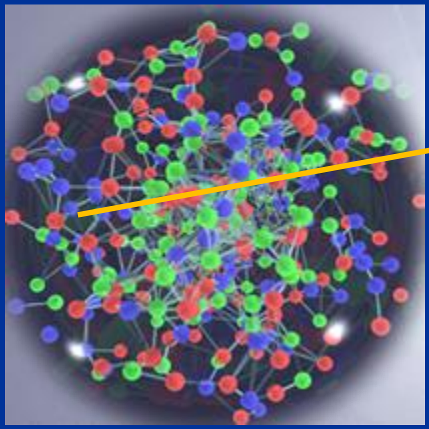
- Produced in *early stages*: witness full evolution

- In the hadronic phase: cannot be thermally excited

- Interactions with medium can distort their properties

D **=** $(c\ \bar{q})$

B **=** $(q\ \bar{b})$



D, B

HEAVY MESON transport phenomena (diffusion)

Introduction and motivation

QUESTION: *how much "memory of the initial state" do heavy quarks loose in their way through the hadronic phase of a heavy-ion collision?*

- **Or, in other words:**
  - How much do they "relax", do they become "thermal"?
  - What is their relaxation length / time?
  - How quickly does their momentum distribution evolves towards that of a gas in thermal equilibrium?

Introduction and motivation

Two key points:

1. *Realistic model for heavy-quark interactions with the hadronic gas constituents*
2. *Suitable transport theory approach for heavy-quark diffusion*

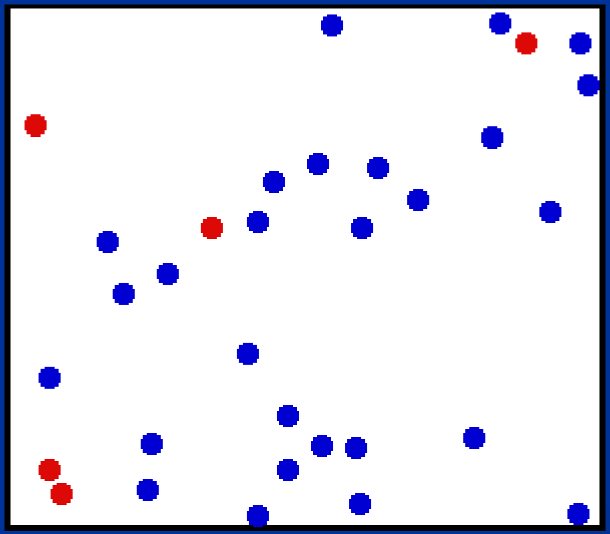
Outline

- *D and B meson interactions in matter*
- *D and B meson relaxation in hadronic gas*
- *Summary*

Modelling heavy-meson interactions in the hadron gas

Set-up of the approach

- Hadronic phase of HIC ≈ *light hadron gas* [mostly π's, also K's , η's]
- Effective d.o.f.: ***HEAVY-FLAVORED MESONS → D, B***
- Heavy – light meson interactions → *eff. theories of QCD, models*
- Propagate off-equilibrium → Transport approach



Legend:

- light mesons (pions)
- Heavy mesons (D, B)

$D = (c\, \bar{q})$

$M_c = 1.27$ GeV ↔ $M_D = 1.87$ GeV

$B = (q\, \bar{b})$

$M_b = 4.67$ GeV ↔ $M_B = 5.28$ GeV

Modelling heavy-meson interactions in the hadron gas

Two distinct scenarios…

- $\mu_B \simeq 0$ *RHIC, LHC conditions*

  Hadronic phase of HIC ≈ *light meson gas* [mostly π's, also *K*'s , η's]

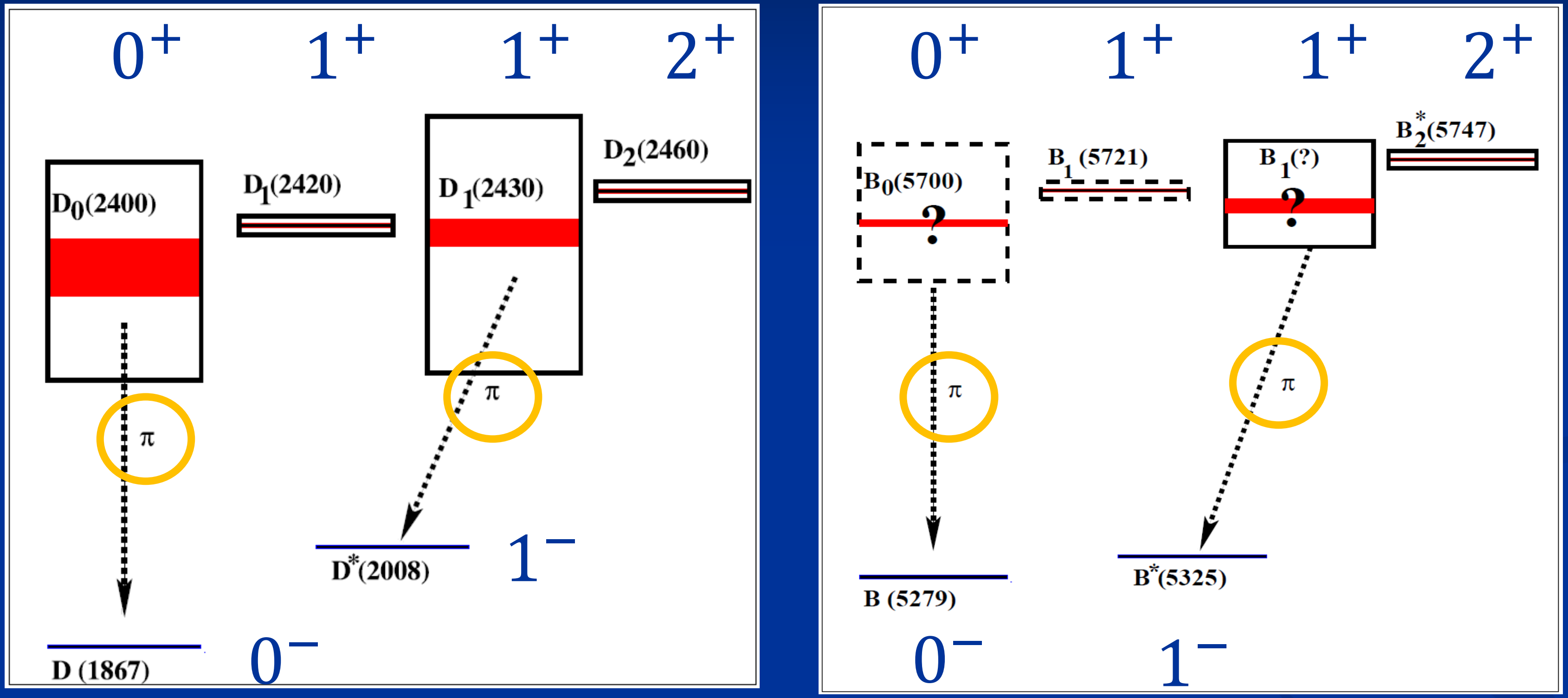
- $\mu_B \neq 0$ *FAIR conditions, production reactions in nuclei*

  Hadronic phase of HIC ≈ *hot nuclear matter* [π's… **AND** *N*, Δ, *N**…]

Effect of baryons: See Laura Tolos' talk

Modelling heavy-meson interactions in a hadron gas

D- and *B*-meson spectra



- *D*, *D** and *B*, *B** mesons as **fundamental** (***stable***) degrees of freedom
- HQ limit: 4 degenerate heavy-quark modes
- *S*-wave pion decays ⟹ πD and πB interaction ***is RESONANT!!***

Modelling heavy-meson interactions in a hadron gas

Effective Lagrangian for $D(B)\pi$ and $D^*(B^*)\pi$ interactions

Lutz, Soyeur, NPA813 (2008) 14; Guo, Hanhart, Meissner, EPJA 40 (2009) 171; Geng, Kaiser, Martin-Camalich, Weise, PRD82 (2010) 054022

Chiral symmetry (NLO) + Heavy-quark symmetry (LO)

LO

$$\mathcal{L}^{(1)} = \underbrace{Tr[\nabla^\mu D \nabla_\mu D^\dagger] - M_D^2 Tr[DD^\dagger]}\, - \underbrace{Tr[\nabla^\mu D^{*\nu} \nabla_\mu D_\nu^{*\dagger}] + M_{D^*}^2 Tr[D^{*\mu} D_\mu^{*\dagger}]}$$
$$+ igTr\left[\left(D^{*\mu} u_\mu D^\dagger - D u^\mu D_\mu^{*\dagger}\right)\right] + \frac{g}{2M_D} Tr\left[\left(D_\mu^* u_\alpha \nabla_\beta D_\nu^{*\dagger} - \nabla_\beta D_\mu^* u_\alpha D_\nu^{*\dagger}\right)\epsilon^{\mu\nu\alpha\beta}\right]$$

NLO

$$\mathcal{L}^{(2)} = -h_0 Tr[DD^\dagger]Tr[\chi_+] + h_1 Tr[D\chi_+ D^\dagger] + h_2 Tr[DD^\dagger]Tr[u^\mu u_\mu] + h_3 Tr[D u^\mu u_\mu D^\dagger]$$
$$+ h_4 Tr[\nabla_\mu D \nabla_\nu D^\dagger]Tr[u^\mu u^\nu] + h_5 Tr[\nabla_\mu D\{u^\mu, u^\nu\}\nabla_\nu D^\dagger] + \{D \to D^\mu\}$$

$$D^{(*)} = (D^{(*)0}, D^{(*)+}, D_s^{(*)+})$$
$$u_\mu = i(u^\dagger \partial_\mu u - u \partial_\mu u^\dagger)$$
$$\nabla_\mu = \partial_\mu - \frac{1}{2}(u^\dagger \partial_\mu u - u \partial_\mu u^\dagger)$$

$$U = exp\left(\frac{\sqrt{2} i \Phi}{F}\right)$$

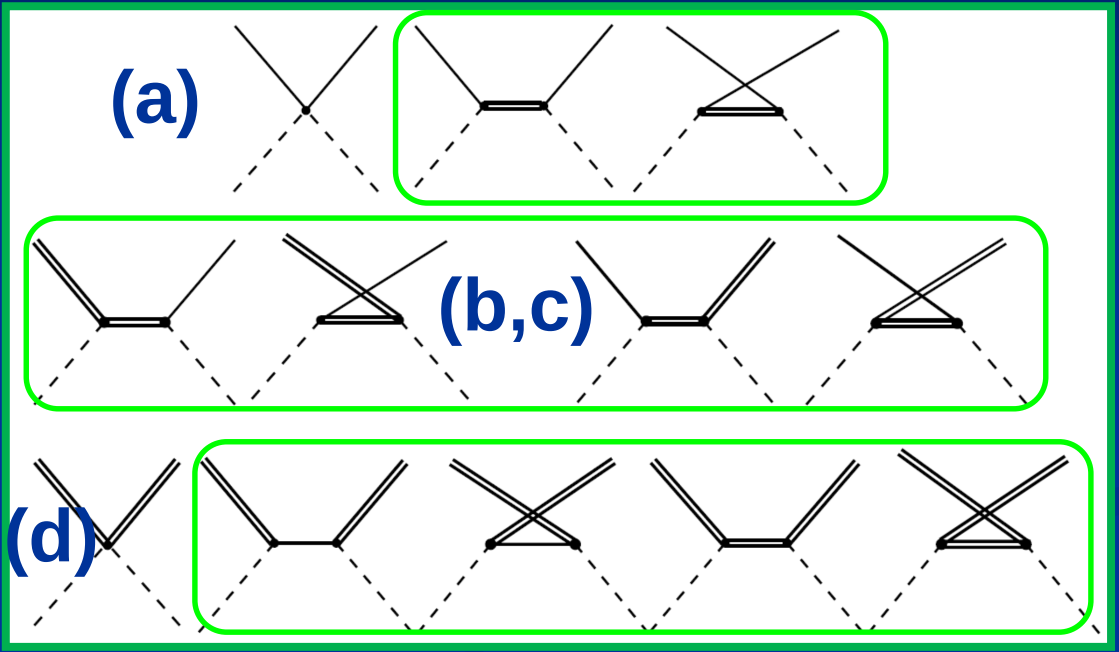
$$\Phi = \begin{pmatrix} \frac{1}{\sqrt{2}}\pi^0 + \frac{1}{\sqrt{6}}\eta & \pi^+ & K^+ \\ \pi^- & -\frac{1}{\sqrt{2}}\pi^0 + \frac{1}{\sqrt{6}}\eta & K^0 \\ K^- & \bar{K}^0 & -\frac{2}{\sqrt{6}}\eta \end{pmatrix}$$

Modelling heavy-meson interactions in a hadron gas

Effective Lagrangian for $D(B)\pi$ and $D^*(B^*)\pi$ interactions

(a) : $D\phi \to D\phi$
(b) : $D^*\phi \to D\phi$
(c) : $D\phi \to D^*\phi$
(d) : $D^*\phi \to D^*\phi$

$(\phi = \pi, K, \eta, \ldots)$



- ***Constraints from HQ symmetry:***
  - *Spin-flip and Born exchange terms subleading in HQ*
  - *LECs: $h_i = \widetilde{h}_i$, i = 0,…,5*
  - *Large N_c: only h_1, h_3, h_5*

Guo, Hanhart, Meissner, EPJA 40 (2009) 171

Modelling heavy-meson interactions in a hadron gas

Effective Lagrangian for $D(B)\pi$ and $D^*(B^*)\pi$ interactions

- ***Amplitude for scattering off a heavy quark in a light meson gas***

  *(**NLO in chiral expansion** and **LO in HQ expansion**)*

$$V \simeq \frac{C_0}{2F^2}(s-u) + \frac{2C_1}{F^2}h_1 + \frac{2C_2}{F^2}h_3(p_2 \cdot p_4) + \frac{2C_3}{F^2}h_5[(p_1 \cdot p_2)(p_3 \cdot p_4) + (p_1 \cdot p_4)(p_2 \cdot p_3)]$$

- C_i: ***isospin-channel coefficients***
- h_j: ***low-energy constants***

Coupled-channel dynamics

C_i	$D\pi(\frac{1}{2})$	$D\pi(\frac{3}{2})$	$D\bar{K}(0)$	$D\bar{K}(1)$	$DK(0)$	$DK(1)$	$D\eta(\frac{1}{2})$	$D\pi \leftrightarrow D\eta(\frac{1}{2})$
C_0	-2	1	-1	1	-2	0	0	0
C_1	$-m_\pi^2$	$-m_\pi^2$	m_K^2	$-m_K^2$	$-2m_K^2$	0	$-m_\pi^2/3$	$-m_\pi^2$
C_2	1	1	-1	1	2	0	$1/3$	1
C_3	1	1	-1	1	2	0	$1/3$	1

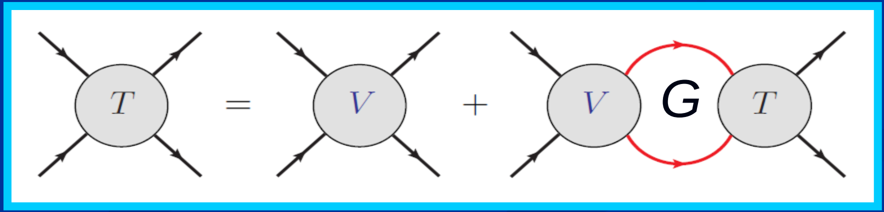
Modelling heavy-meson interactions in a hadron gas

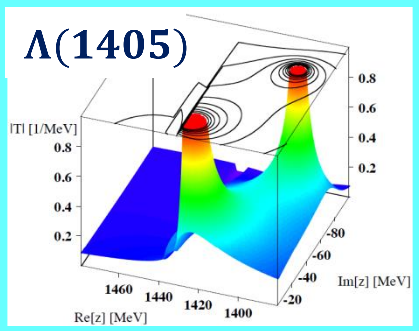
Role of unitarization of NLO scattering amplitudes

- *ChPT amplitudes valid at low energies*
- ***Resonances:* more efficient diffusion in hot meson gas**
- Reach high temperatures (up to ≈150 MeV)
- **Unitarization is required !**

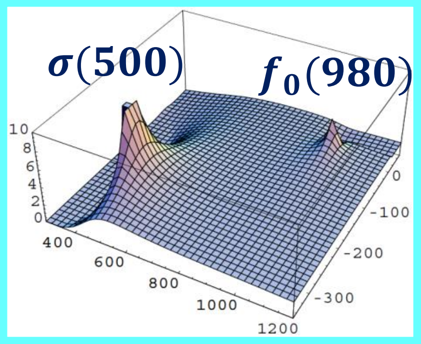
Relativistic Bethe-Salpeter equation in partial waves (S-wave)

$$T = V + \overline{VGT}$$





Hyodo, Jido, Prog. Part. Nucl. Phys 67 (2012) 55

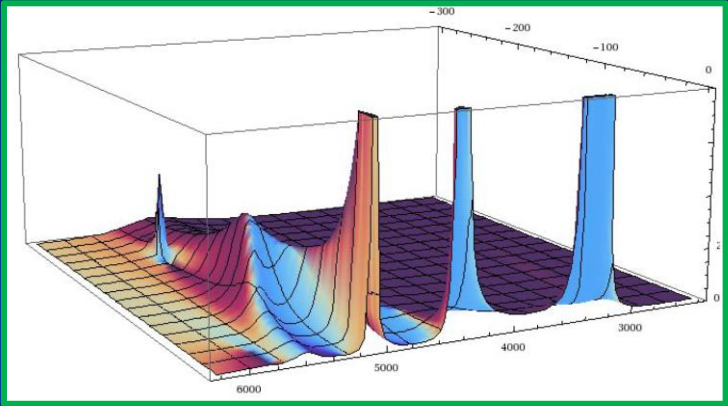


Ollet, Oset, Pelaez, Phys.Rev. D59 (1999) 074001; Phys. Rev. Lett. 80 (1998) 3452

Modelling heavy-meson interactions in a hadron gas

Pole analysis in unphysical sheets

$$T_{ij} \approx \frac{g_i g_j}{z - z_R} \qquad z_R = M_R - i\,\Gamma_R/2$$



- *Resonance parameters:*

$(S, I) = (0, 1/2)$

State	M_0 (MeV)	Γ_0 (MeV)	M_1 (MeV)	Γ_1 (MeV)
	$J = 0$		$J = 1$	
$D_{0,1}$	2300	250	2400	300-350
$B_{0,1}$	5540	210	5590	245

Kolomeitsev, Lutz, PLB 582, 39 (2004)
Guo et al, PLB 641, 27 (2006)
Flynn and Nieves, PRD 75 (2007) 074024
Altenbuchinger et al, PRD 89 (2014) 014026
Tolos et al, PRD 89 (2014) 074042

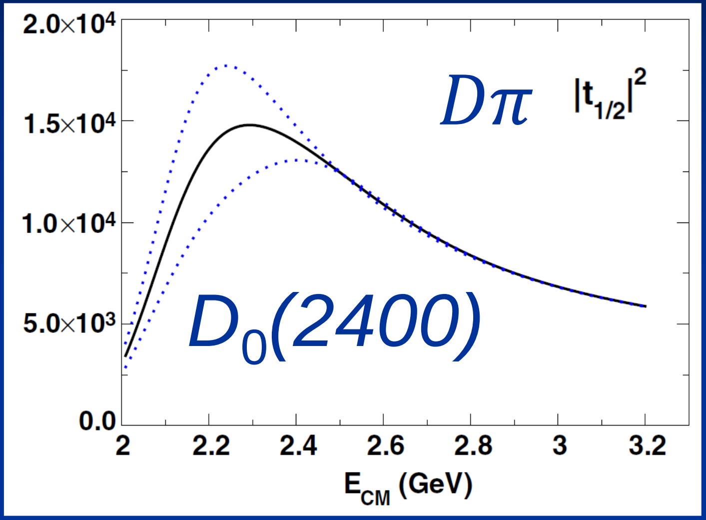
$(S, I) = (-1, 0)$

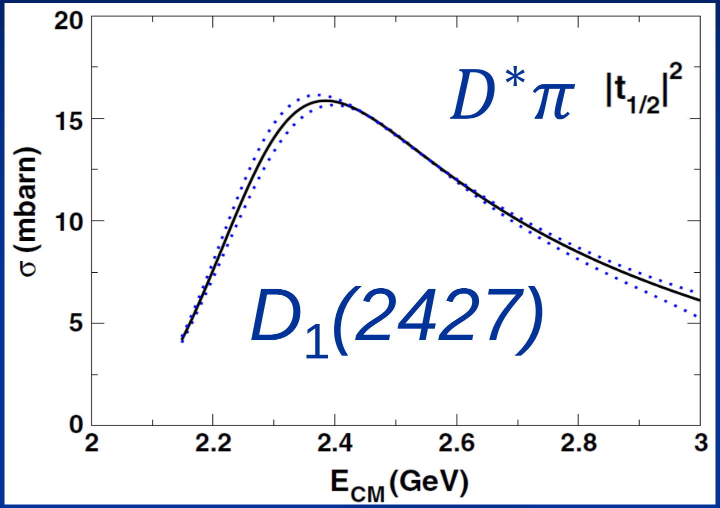
State	M_0 (MeV)	Γ_0 (MeV)	M_1 (MeV)	Γ_1 (MeV)
$D^*_{s0,1}$	2317	≈ 0	2457	≈ 0
$B^*_{s0,1}$	5748	≈ 0	5799	≈ 0

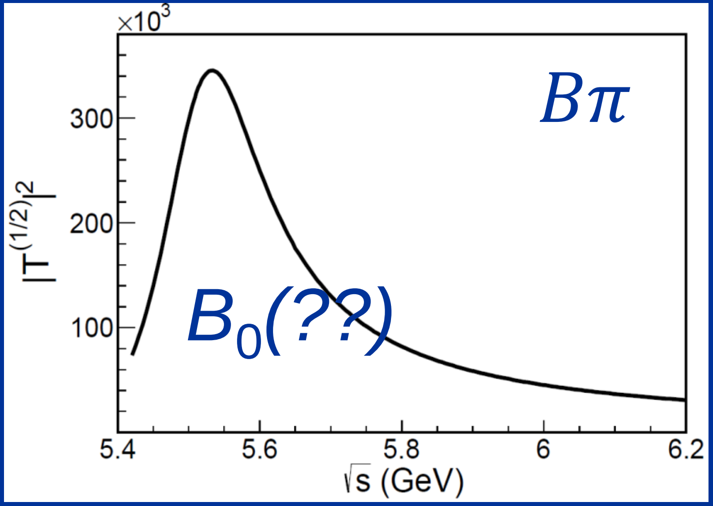
⋮

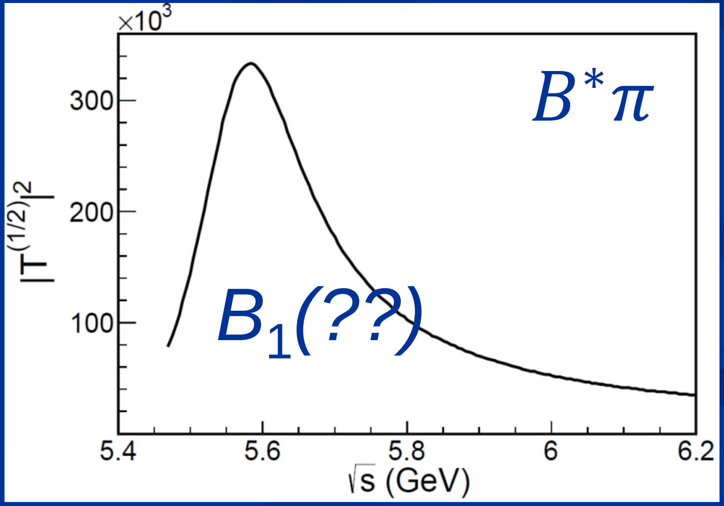
Modelling heavy-meson interactions in a hadron gas

Amplitudes and Cross sections









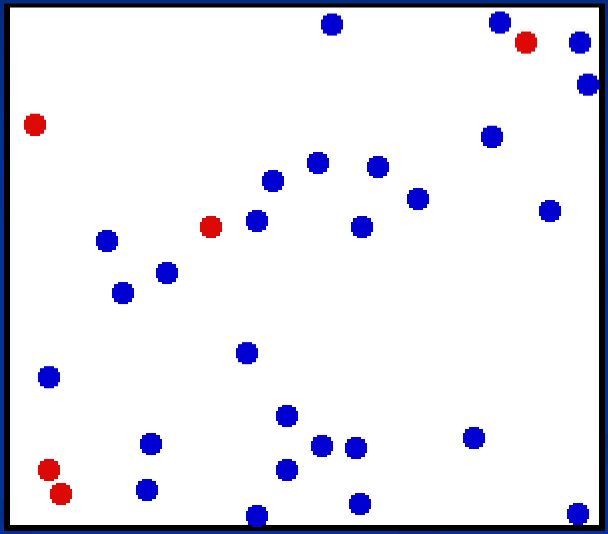
Kolomeitsev, Lutz, PLB 582, 39 (2004); Guo et al, PLB 641, 27 (2006); Flynn and Nieves, PRD 75 (2007) 074024

Heavy-meson transport coefficients in a hadron gas

Transport approach for heavy mesons:

- D/B distribution not in equilibrium with the hot hadron gas
- Relax via Boltzmann-Uehling-Uhlenbeck equation

$$\frac{df_c(\mathbf{p})}{dt} = C[f_c(\mathbf{p})]$$



- Small number of heavy mesons
- $M_{D/B} \gg m_{\pi \dots}$

Fokker-Planck equation

Heavy-meson transport coefficients in a hadron gas

Fokker-Planck equation:

$$D(\mathbf{p}) + \pi(\mathbf{q}) \to D(\mathbf{p}-\mathbf{k}) + \pi(\mathbf{q}+\mathbf{k})$$

Momentum distribution function

$$\frac{\partial f_c(t,\mathbf{p})}{\partial t} = \frac{\partial}{\partial p_i}\left\{F_i(\mathbf{p}) f_c(t,\mathbf{p}) + \frac{\partial}{\partial p_j}\left[\Gamma_{ij}(\mathbf{p}) f_c(t,\mathbf{p})\right]\right\}$$

Friction term (“drag force”)

Diffusion term

Collision rate:

$$w(\mathbf{p},\mathbf{k}) = g_\pi \int \frac{d\mathbf{q}}{(2\pi)^9} f_\pi(\mathbf{q})[1 + f_\pi(\mathbf{q}+\mathbf{k})] \frac{1}{2E_q^\pi} \frac{1}{2E_p^c} \frac{1}{2E_{q+k}^\pi} \frac{1}{2E_{p-k}^c} \times (2\pi)^4 \delta(E_p^c + E_q^\pi - E_{p-k}^c - E_{q+k}^\pi) \sum |\mathcal{M}_{\pi c}(s,t,\chi)|^2$$

(and similarly for bottomed mesons and other species in the gas)

Isospin-averaged D/D π amplitude*

Heavy-meson transport coefficients in a hadron gas

Models for D (and B) interaction with hadronic medium

- *Phenomenological approaches*

  He, Fries, Rapp, Phys. Lett. B 701, 445 (2001)

  "empirical" scattering amplitudes of D/D off $\pi, K, \eta, \rho, \omega, K^*, N, \Delta$*

- *Effective field theory: Heavy-meson ChPT* *Lutz, Soyeur, NPA813(2008)14; Guo, Hanhart, Meissner, EPJA 40 (2009) 171; Geng, Kaiser, Martin-Camalich, Weise, PRD82 (2010) 054022*

  D/D off (π, K, η) SU(3) octet + constraints from **HQ symmetry***

  - *Perturbative expansion of amplitudes (NLO)*

    Ghosh, Das, Sarkar, Alam, Phys. Rev D 84, 011503 (2011)

    Das, Ghosh, Sarkar, Alam, Phys. Rev D 85, 074017 (2012)

  - *Coupled-channel unitarized approach*

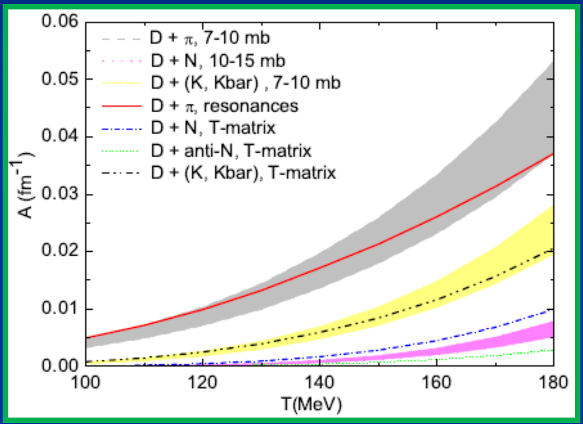
    Abreu, DC, Llanes-Estrada, Torres-Rincon, Ann. Phys. 326, 2737 (2011)

    Abreu, DC, Torres-Rincon, Phys.Rev. D87 (2013) 3, 034019

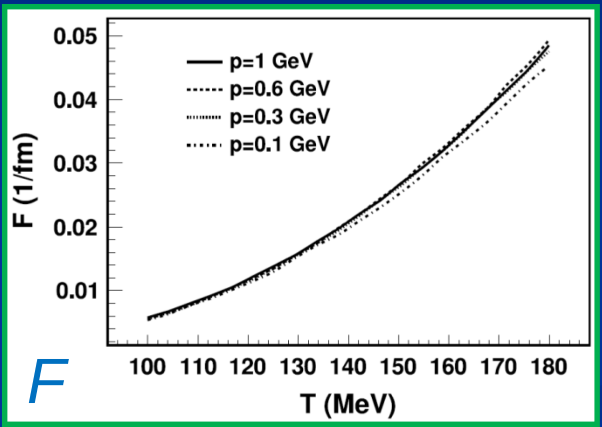
See also: Fuchs, Martemyanov, Faessler, Krivoruchenko, PRC73, 035204 (2006)

Heavy-meson transport coefficients in a hadron gas

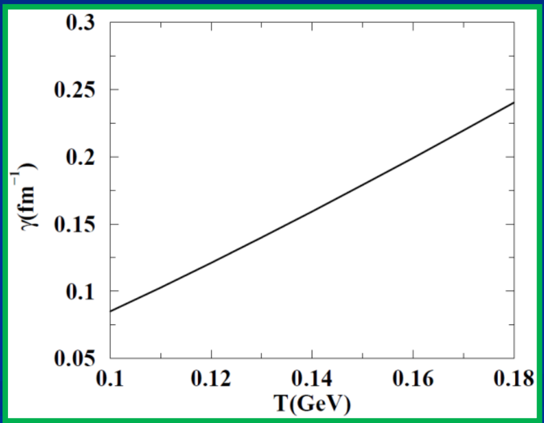
Drag and Diffusion coefficients at $\mu_B \simeq 0$: charmed mesons



Phenomenological
(He, Fries, Rapp)

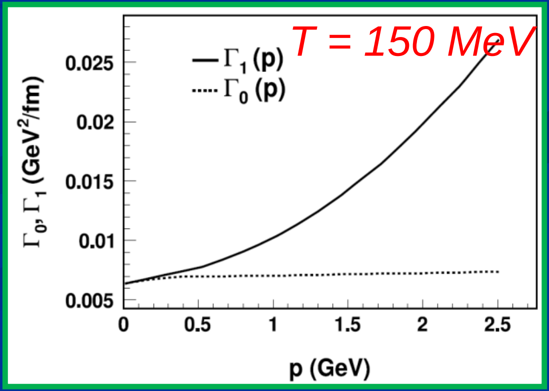


Unitarized ChPT
(Abreu, DC, Llanes, Torres)



Perturbative ChPT
(Ghosh, Das, Sarkar, Alam)

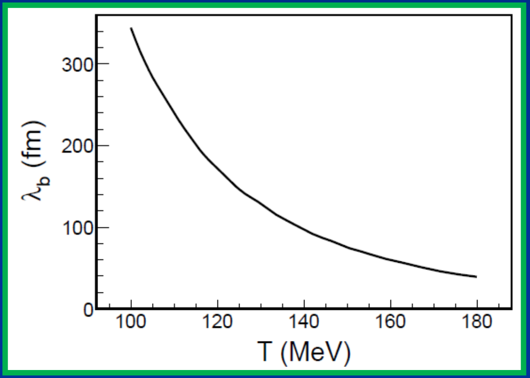
- *Drag and diffusion strengthened in* ***hotter stages***
- ***Resonant interaction!***
- Sizable momentum dependence of Γ_1 vs Γ_0
- **Sensitivity of anisotropic observables (elliptic flow)**



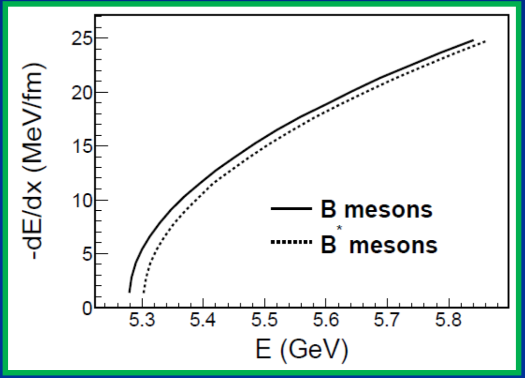
Heavy-meson transport coefficients in a hadron gas

Heavy-quark ***relaxation time*** *and* ***energy loss*** *in π gas*

$\lambda = 1/F$



$-dE/dx = F * p$



At p = 1 GeV/c , T = 150 MeV

State	λ(fm)	- dp/dx (MeV/fm)
D	40	≈ 50
B	100	≈ 70

"It is reasonable to expect that the lifetime of the pion gas is smaller than the relaxation time of *D* and *B* mesons, meaning that heavy quarks DO NOT completely relax and will carry information from QGP"

"Thus, if the pion gas is in existence for, say, 4 fm, a *D* meson measured in the final state with a momentum of 800 MeV/c will have been emitted from the quark–gluon plasma phase with 1 GeV/c"

Heavy-meson transport coefficients in a hadron gas

Comparison to other approaches

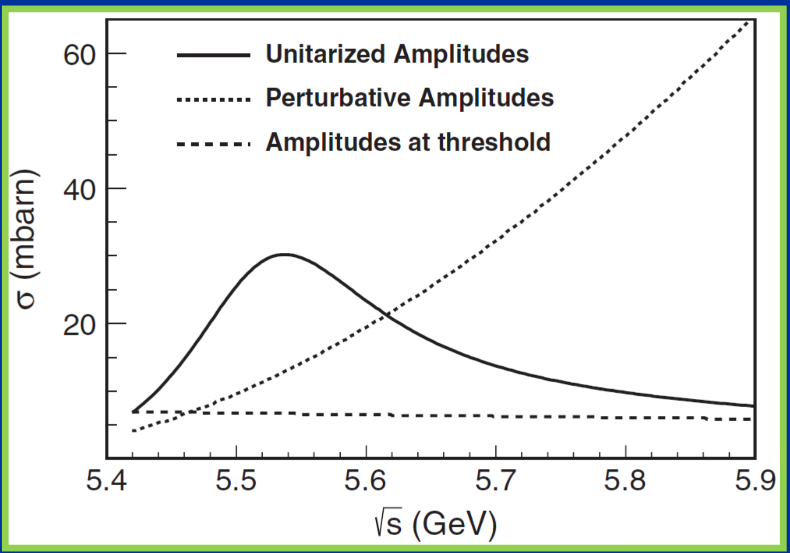
Effect of baryons: See Laura Tolos' talk

Value of the drag coefficient at $p \to 0$ and $T = 100$ MeV.

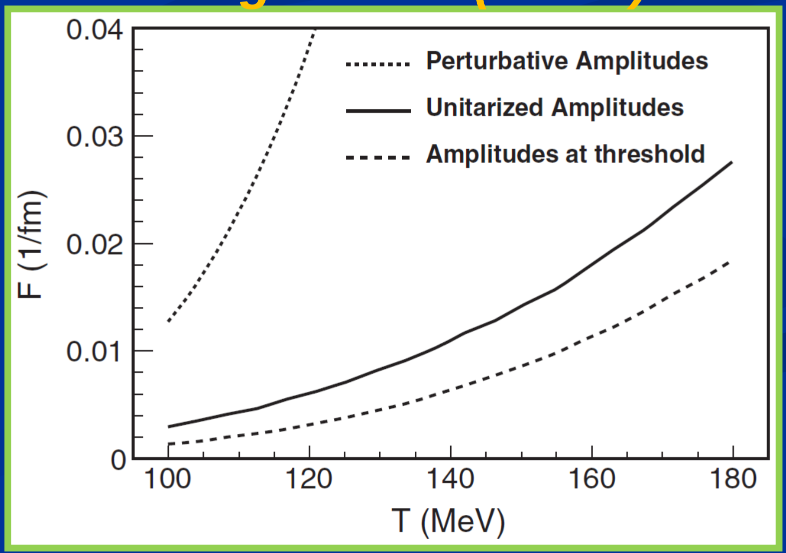
Authors	F (fm^{-1})
Laine	0.05
He, Fries, Rapp	5×10^{-3}
Ghosh et al.	0.11
This work	$\mathbf{3.5 \times 10^{-3}}$

Resonant interactions!

$B\pi$ total cross section



B drag coef. F (static)



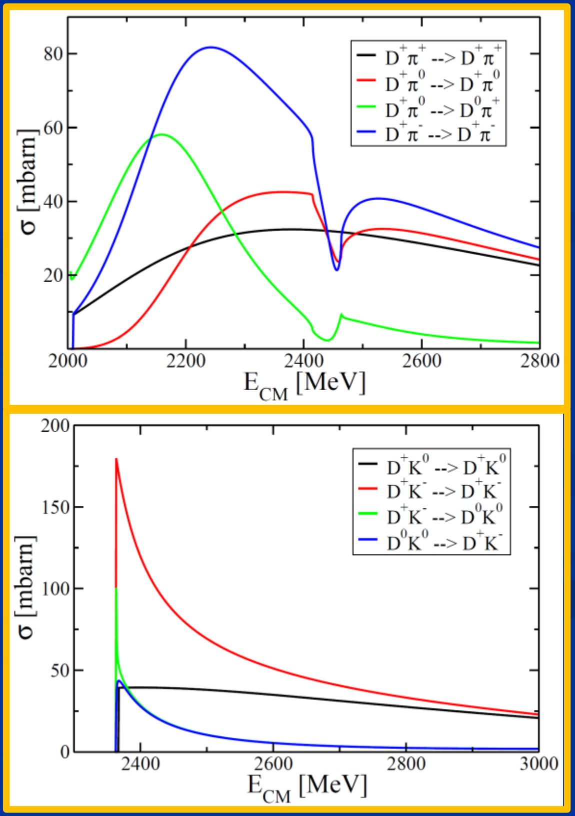
On-going and future work…

- *Towards implementation in "fully-fledged" microscopic transport simulations: e.g.* ***PHSD, IQMD***

Taesoo Song **Pol Gossiaux**

- *Issues to be addressed:*

  - ***p-waves*** ⇨ *anisotropic cross sections*
  - ***high-energy of $\sigma(s)$: Regge?***
  - $D^{(*)} - \rho, \quad D^{(*)} - K^{*}$
    Molina et al., PRD 80 (2009) 014025
  - **Medium effects on cross sections**



Summary

- Heavy mesons stand as unique probes for testing strong interactions at FAIR conditions, as long as we understand their dynamics in vacuum and in the *hot and dense nuclear medium*
- A lot of theoretical work is required, within *realistic approaches* to interactions in the hadronic gas, in connection with experimental information
- Transport simulations with the input of realistic interactions: understand the production and propagation of charm in HICs
- **It is a very exciting moment!**

Backup slides

Modelling heavy-meson interactions in a hadron gas

$$V_a = \frac{C_0}{4F^2}(s-u) + \frac{2C_1\, m_\pi^2}{F^2} h_1 + \frac{2C_2}{F^2} h_3 (p_2 \cdot p_4) + \frac{2C_3}{F^2} h_5 \left[(p_1 \cdot p_2)(p_3 \cdot p_4) + (p_1 \cdot p_4)(p_2 \cdot p_3)\right]$$
$$+ \frac{2i\, g^2}{F^2} p_2^\mu \left[C_4 D_{\mu\nu}(p_1+p_2) + C_5 D_{\mu\nu}(p_2 - p_3)\right] p_4^\nu \ ,$$

$$V_b = \frac{i\, g^2}{m_D F^2} \left[C_4\, p_2^\alpha \left(2p_1^\beta + p_2^\beta\right) p_{4\rho} D^{\nu\rho}(p_1+p_2) + C_5\, p_4^\alpha \left(p_2^\beta - p_3^\beta - p_1^\beta\right) p_{2\rho} D^{\nu\rho}(p_2 - p_3)\right] \varepsilon_{\alpha\beta\mu\nu} \epsilon^\mu(p_1) \ ,$$

$$V_c = \frac{i\, g^2}{m_D F^2} \left[C_4\, p_4^\alpha \left(p_1^\beta + p_2^\beta + p_3^\beta\right) p_{2\rho} D^{\rho\nu}(p_1+p_2) + C_5\, p_2^\alpha \left(p_2^\beta - 2p_3^\beta\right) p_{4\rho} D^{\nu\rho}(p_2 - p_3)\right] \varepsilon_{\alpha\beta\mu\nu} \epsilon^{*\mu}(p_3) \ ,$$

$$V_d = -\left\{\frac{C_0}{4F^2}(s-u) + \frac{2C_1\, m_\pi^2}{F^2} \tilde{h}_1 + \frac{2C_2}{F^2} \tilde{h}_3 (p_2 \cdot p_4) + \frac{2C_3}{F^2} \tilde{h}_5 \left[(p_1 \cdot p_2)(p_3 \cdot p_4) + (p_1 \cdot p_4)(p_2 \cdot p_3)\right]\right\} \epsilon^\mu(p_1) \epsilon^*_\mu(p_3)$$
$$+ \frac{2i\, g^2}{F^2} \left[C_4 D(p_1+p_2) + C_5 D(p_2 - p_3)\right] p_2^\mu \epsilon_\mu(p_1) p_4^\nu \epsilon^*_\nu(p_3)$$
$$+ \frac{i g^2}{3 m_D^2 F^2} \left[C_6\, p_2^\alpha \left(2p_1^\beta + p_2^\beta\right) p_4^\rho \left(p_1^\sigma + p_2^\sigma + p_3^\sigma\right) D^{\nu\gamma}(p_1+p_2) + C_7\, p_2^\alpha \left(p_2^\beta - 2p_3^\beta\right) p_4^\rho \left(p_2^\sigma - p_3^\sigma - p_1^\sigma\right) D^{\nu\gamma}(p_2 - p_3)\right]$$
$$\times \varepsilon_{\alpha\beta\mu\nu} \varepsilon_{\rho\sigma\gamma\delta} \epsilon^\mu(p_1) \epsilon^{*\delta}(p_3) \ ,$$

Modelling heavy-meson interactions in a hadron gas

A word on determination of free parameters (LECs)

- *LO: fixed by Chiral Symmetry breaking*

$$V \simeq \frac{C_0}{2F^2}(s-u) + \frac{2C_1}{F^2} h_1 + \frac{2C_2}{F^2} h_3 (p_2 \cdot p_4) + \frac{2C_3}{F^2} h_5 [(p_1 \cdot p_2)(p_3 \cdot p_4) + (p_1 \cdot p_4)(p_2 \cdot p_3)]$$

C_i	$B\pi(\frac{1}{2})$	$B\pi(\frac{3}{2})$	$BK(0)$	$BK(1)$	$B\bar{K}(0)$	$B\bar{K}(1)$	$B\eta(\frac{1}{2})$
C_0	-2	1	-1	1	-2	0	0
C_1	$-m_\pi^2$	$-m_\pi^2$	m_K^2	$-m_K^2$	$-2m_K^2$	0	$-\frac{m_\pi^2}{3}$
C_2	1	1	-1	1	2	0	$\frac{1}{3}$
C_3	1	1	-1	1	2	0	$\frac{1}{3}$

- *NLO: h_1, h_3, h_5 → fit to experimental data on D (B) resonances*

Heavy Spin, J^π	D (quark model)	D (experimental)	(M,Γ) MeV	B (quark model)	B (experimental)	(M,Γ) MeV
1/2, 0^+	D_0	$D_0^*(2400)$	2318,267	$\mathbf{B_0}$	?	?
1/2, 1^+	D_1	$D_1(2430)$	2427,384	$\mathbf{B_1}$	?	?
3/2, 1^+	D_1	$D_1(2420)^0$	2421, 27	B_1	$B_1(5721)$	5723,?
3/2, 2^+	D_2	$D_2^*(2460)$	2466,49	B_2	B_2^* (5747)	5743,23

- *Resonance parameters*

State	M_0 (MeV)	Γ_0 (MeV)	M_1 (MeV)	Γ_1 (MeV)
D	2300	250	2400	300-350
B	5540	210	5590	245

Kolomeitsev, Lutz, PLB 582, 39 (2004)
Guo et al, PLB 641, 27 (2006)
Flynn and Nieves, PRD 75 (2007) 074024

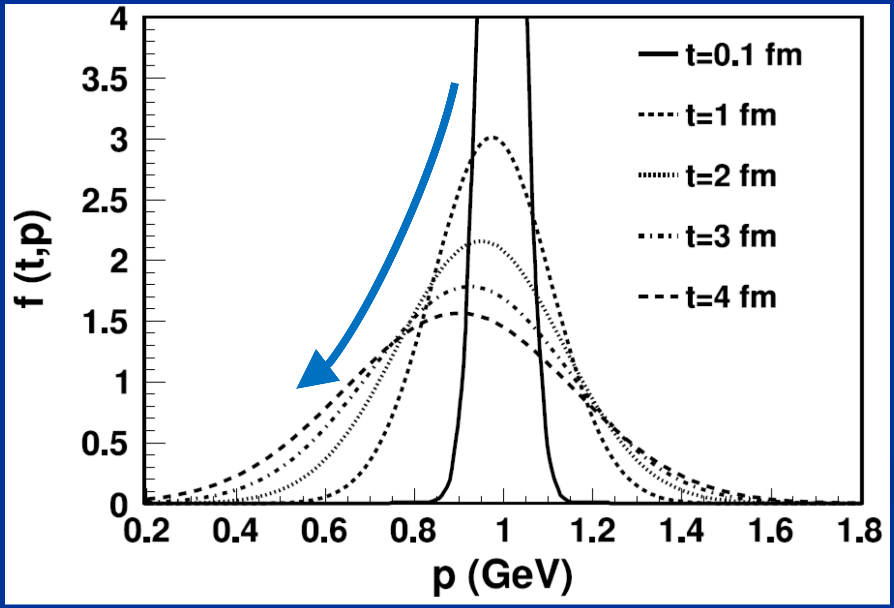
Heavy-meson transport coefficients in a hadron gas

Fokker-Plank equation approach:

$$\frac{\partial f_c(t,\mathbf{p})}{\partial t} = \frac{\partial}{\partial p_i}\left\{F_i(\mathbf{p}) f_c(t,\mathbf{p}) + \frac{\partial}{\partial p_j}\left[\Gamma_{ij}(\mathbf{p}) f_c(t,\mathbf{p})\right]\right\}$$

$$\begin{aligned} F(p^2) &= \int d\mathbf{k}\; w(\mathbf{p},\mathbf{k})\, \frac{k_i p^i}{p^2}\,, \\ \Gamma_0(p^2) &= \frac{1}{4}\int d\mathbf{k}\; w(\mathbf{p},\mathbf{k}) \left[\mathbf{k}^2 - \frac{(k_i p^i)^2}{p^2}\right] \\ \Gamma_1(p^2) &= \frac{1}{2}\int d\mathbf{k}\; w(\mathbf{p},\mathbf{k})\, \frac{(k_i p^i)^2}{p^2}\,. \end{aligned}$$

- F is the "drag force"
- Γ_0, Γ_1 are the diffusion coefficients



With time evolution…

- *Initial momentum is "dragged" to zero*
- *Shape broadens towards Boltzmann distribution*

Can one extend the range of applicability to higher energies?

The problem stems from the lack of unitarity of a perturbative expansion

*Non-perturbative methods implementing **unitarity** are needed:*

*A very important and successful effort: Chiral unitary approach (or Unitarized Chiral Perturbation Theory), **UChPT***

Oller, Oset, Dobado, Pelaez, Meissner, Kaiser, Weise, Ramos, Vicente-Vacas, Nieves, Ruiz-Arriola, Lutz,...

***UChPT**:*

With the only input of the lowest order ChPT Lagrangians, the exploitation of the analytical properties of the scattering amplitudes and the implementation of unitarity in coupled channels obtains a good reproduction of many MM and MB interactions

→ *many resonances appear naturally!*

Three different ways to implement the idea:

- *N/D method*
- *IAM (Inverse Amplitude Method)*
- *Bethe-Salpeter equation*

} *Give equivalent results*

Basic idea of UChPT:

Unitarity *of the S-matrix implies:*

$$Im T_{i,j} = T_{i,l} \rho_l T^*_{l,j}$$

$$(S = 1 - iT)$$

Different channels for a given L, I

(for instance: for L=0, I=0 they are $\pi\pi$, $K\overline{K}$, $\eta\eta$)

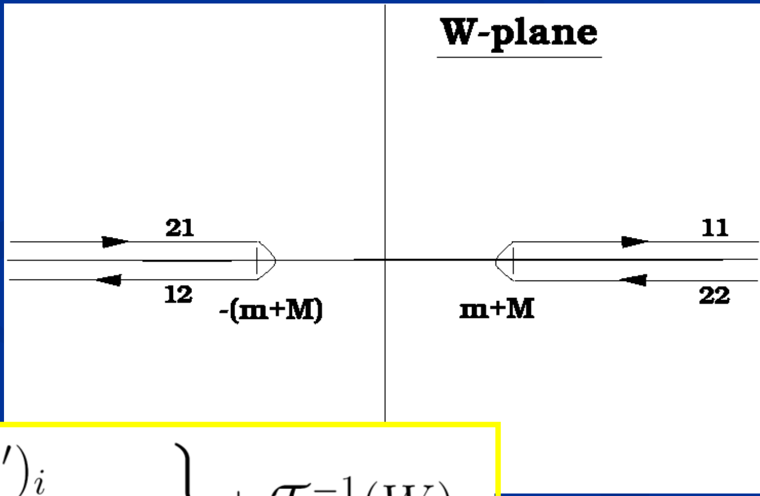
$$\rho_i \equiv q_i/(8\pi W)$$

CM momentum *CM energy*

More convenient in terms of the inverse of T:

$$\mathrm{Im}\ T^{-1}(W)_{ij} = -\rho(W)_i \delta_{ij}$$

T *is real below the lowest threshold and complex above*



One can write a dispersion relation for T^{-1}:

$$T^{-1}(W)_{ij} = -\delta_{ij} \left\{ \widetilde{a}_i(s_0) + \frac{s - s_0}{\pi} \int_{s_i}^{\infty} ds' \frac{\rho(s')_i}{(s' - s)(s' - s_0)} \right\} + \mathcal{T}^{-1}(W)_{ij}$$

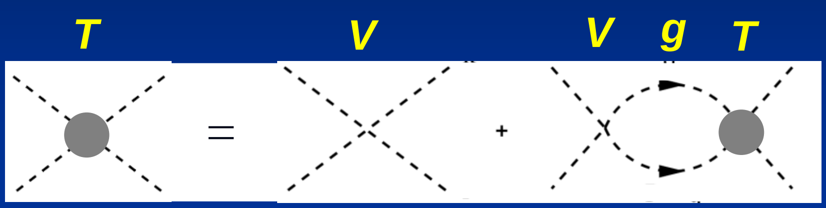
$$g(s)_i = \int \frac{d^4q}{(2\pi)^4} \frac{1}{(q^2 - M_i^2 + i\epsilon)((P-q)^2 - m_i^2 + i\epsilon)}$$

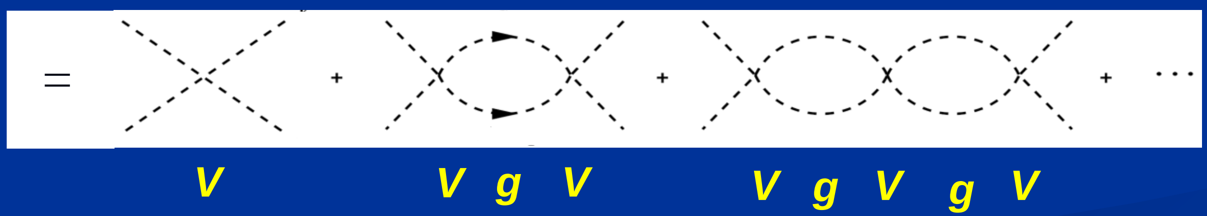
(Note that this is the familiar two meson loop function)

Hence, T can be written as

$$T(W) = [I - \mathcal{T}(W) \cdot g(s)]^{-1} \cdot \mathcal{T}(W) \quad \Longrightarrow \quad T(W) = \mathcal{T}(W) + \mathcal{T}(W) g(s) T(W)$$

Bethe-Salpeter (BS) equation





*The kernel of the BS equation, **V**, is the lowest order ChPT Lagrangian*

Effectively, one is summing this infinite series of diagrams

In summary:

With the only input of the lowest order ChPT Lagrangian + implemenatation of unitarity in coupled channels one gets the full scattering amplitude