FIAS Frankfurt Institute for Advanced Studies

Charmonium production in antiproton-nucleus reactions close to threshold

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Outline

- Motivation
- Glauber model (probabilistic)
- Results for J/Ψ and Ψ´ production
- Generalized eikonal approximation (quantum)
- Results for polarized χ_{c2} production
- X(3872) production
- Summary and outlook

A.L., M. Bleicher, A. Gillitzer, M. Strikman, PRC 87, 054608 (2013);
A.L., M. Strikman, M. Bleicher, PRC 89, 014621 (2014);
and work in progress

Why to study charmonium-nucleon interactions ?

- Important for the interpretation of J/Ψ suppression in relativistic heavy-ion collisions and separation of the quark-gluon plasma signals from the cold nuclear matter effects.

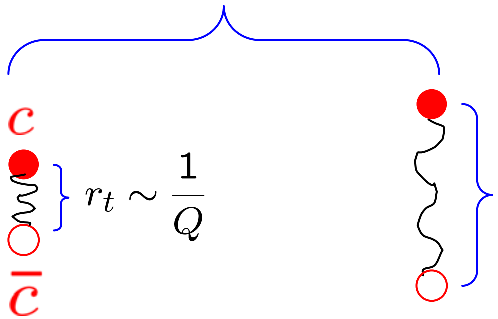
- May constrain the QCD-inspired models of charmonia and of the charmonium-like XYZ mesons.

- Deepens our understanding of the nonperturbative vs perturbative QCD: factorization theorem, color dipole cross section, color transparency …

Color transparency

At high momentum transfer Q the small-size quark-antiquark configuration is created which expands to the normal meson size:

l_f **- formation length**



$r_t \sim r_R$

$R = J/\Psi,\ \Psi',\ \chi_c, \ldots$

z **Beam direction**

E.g., for J/Ψ:

$$l_f \simeq \frac{1}{E_{\Psi'} - E_{J/\Psi}} \simeq \frac{2p_{J/\Psi}}{m_{\Psi'}^2 - m_{J/\Psi}^2} \simeq 0.1 \text{ fm } p_{J/\Psi} \text{ (GeV)}$$

G.R. Farrar, H. Liu, L.L. Frankfurt, M.I. Strikman, PRL 61, 686 (1988)

Color dipole – proton cross section (in the pQCD limit $r_t \to 0$) :

$$\sigma_{q\bar{q}} \propto r_t^2 \propto -Q^{-2} \sim m_R^{-2}$$

Within formation length charmonium-nucleon cross section is small.

At p_{lab} > ~20 GeV the formation length is large: $l_f \sim R_{nucleus}$.

No direct information on J/Ψ N cross section can be extracted from hadron- and photon-induced reactions on nuclei at high energies.

Antiproton-nucleus reactions can be used to determine $\sigma_{J/\Psi N}$:

Formation reaction: $\bar{p}p \to J/\Psi\ ,\ p_{J/\Psi} \simeq p_{lab} \simeq 4\ \mathrm{GeV/c}\ ,\ l_f \simeq 0.4\ \mathrm{fm}$

***J/Ψ is formed inside the nucleus*:**

- **Possible to study the *genuine* J/Ψ N interactions**
- **Difficulty - due to Fermi motion the J/Ψ production cross section on a nucleus is reduced:**

$$\frac{\sigma_{\bar{p}A\to J/\Psi(A-1)^*}}{Z\sigma_{\bar{p}p\to J/\Psi}} \sim 10^{-4}$$

G.R. Farrar et al., NPB 345, 125 (1990)

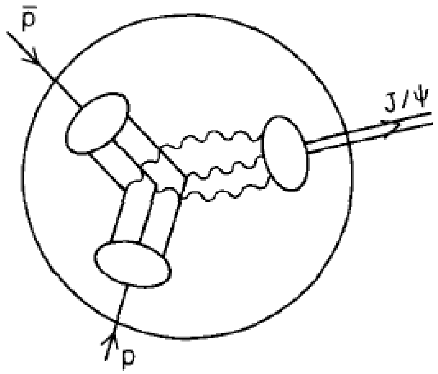


Fig. 2. The dominant mechanism for pp̄ exclusive annihilation into J/ψ.

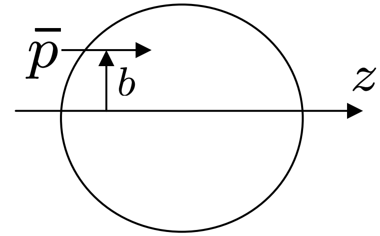
Figure from S.J. Brodsky and A.H. Mueller, PLB 206, 685 (1988)

Other charmonia $(\Psi'(2S), \chi_c(1P), \ldots)$ can be also produced in $\bar{p}A$ reactions at threshold (p_{lab}=4-6 GeV/c). Their internal structure can be tested by interactions with target nucleons.

Possible at PANDA@FAIR: antiproton beam at p_{lab}~1.5-15 GeV/c, luminocity L~ $2 \cdot 10^{32}$ cm^{-2} s^{-1}, proton and nuclear targets.

How good are the antiproton-nucleus reactions to probe the charmonium-nucleon interactions ?

Charmonium $R = J/\Psi,\ \Psi',\ \chi_c, \dots$ production cross section in the Glauber model:



$$\sigma_{\bar{p}A\to R(A-1)^*} = 2\pi \int\limits_0^\infty db\, b \int\limits_{-\infty}^{\infty} \frac{dz}{v_{\bar{p}}} \mathcal{P}_{\bar{p},\mathrm{surv}}(z,b)\Gamma_{\bar{p}\to R}(z,b)\mathcal{P}_{R,\mathrm{surv}}(z,b) \ ,$$

$$\Gamma_{\bar{p}\to R}(z,b) = \int \frac{2d^3p}{(2\pi)^3} v_{\bar{p}p}\sigma_{\bar{p}p\to R}(p,p_{\bar{p}}) f_p(z,b,\mathbf{p}) \ , \quad f_p(z,b,\mathbf{p}) = \Theta(p_{F,p} - |\mathbf{p}|) \ ,$$

$$\mathcal{P}_{\bar{p},\mathrm{surv}}(z,b) = \mathrm{e}^{-\int\limits_{-\infty}^{z} dz' \rho(z',b)\sigma^{\mathrm{inel}}_{\bar{p}N}(p_{\mathrm{lab}})} \ , \quad \mathcal{P}_{R,\mathrm{surv}}(z,b) = \mathrm{e}^{-\int\limits_{z}^{\infty} dz' \rho(z',b)\sigma^{\mathrm{eff}}_{RN}(p_R,z'-z)} \ .$$

$$\sigma^{\mathrm{eff}}_{RN}(p_R,z) = \sigma_{RN}(p_R)\left(\left[\frac{z}{l_f} + \frac{< n^2 k_t^2 >}{m_R^2}\left(1 - \frac{z}{l_f}\right)\right]\Theta(l_f - z) + \Theta(z - l_f)\right)$$

- charmonium-nucleon effective interaction cross section in the color diffusion model G.R. Farrar, L.L. Frankfurt, M.I. Strikman, and H. Liu, NPB 345, 125 (1990)

$\sigma_{RN}(p_R)$ **– total fully-formed-charmonium-nucleon cross section**

$< k_t^2 >^{1/2} \simeq 0.35$ GeV/c **- r.m.s. transverse momentum of a quark in a hadron**

$n = 3$ **- number of intermediate gluons**

Density profiles

For light nuclei (A ≤ 20) — harmonic oscillator model:

$$\rho_q(r) = \rho_q^0 \left[1 + a_q \left(\frac{r}{R_q}\right)^2\right] \exp\{-(r/R_q)^2\} , \quad q = p, n .$$

For heavy nuclei (A > 20) — two-parameter Fermi distribution:

$$\rho_q(r) = \frac{\rho_q^0}{\exp\left(\frac{r-R_q}{a_q}\right) + 1} , \quad q = p, n$$

Charge density parameters: **C. De Jager et al., Atom. Data Nucl. Data Tabl. 14, 479 (1974).**

Neutron density parameters: **J. Nieves et al., NPA 554, 509 (1993);**
V. Koptev et al., Yad. Fiz. 31, 1501 (1980);
R. Schmidt et al., PRC 67, 044308 (2003).

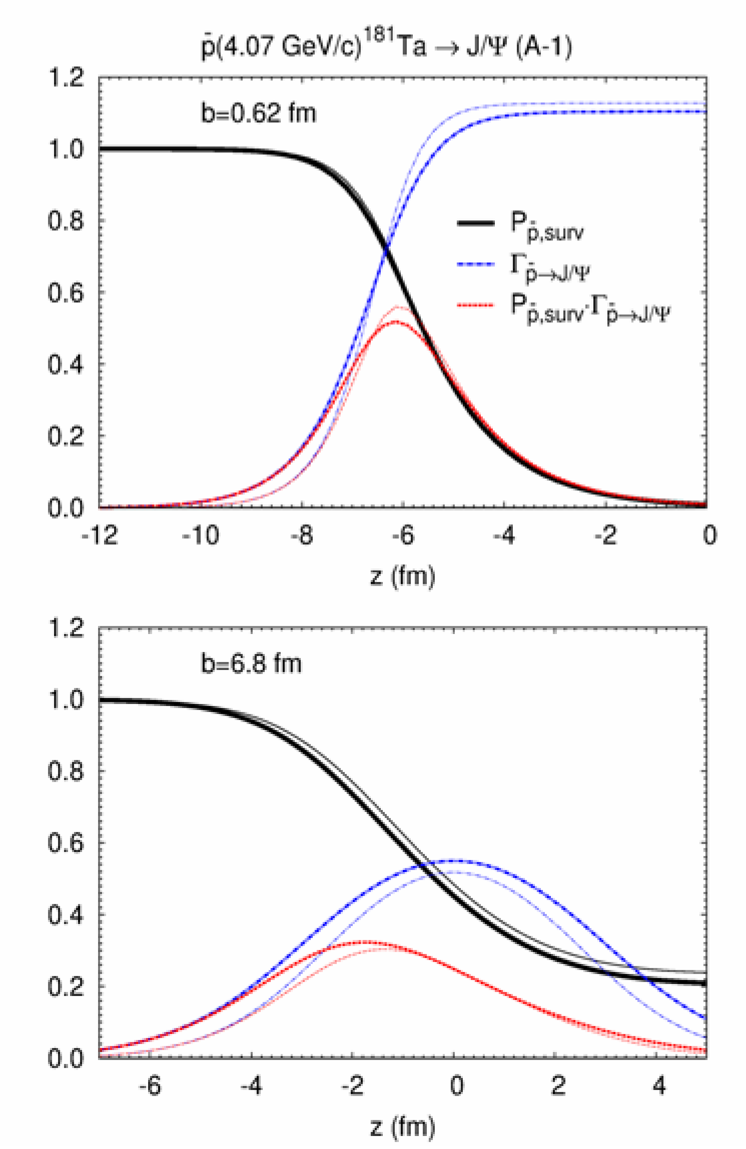
Space distribution of J/Ψ production $\propto \mathcal{P}_{\bar{p},\mathrm{surv}} \Gamma_{\bar{p}\to R}$

Central collision:

Antiproton does not penetrate deeply to the nuclear interior.

Peripheral collision:

Antiproton may pass through and leave the nucleus.



$\Gamma_{\bar{p}\to J/\Psi} \propto \rho_p^{2/3}$ **(in 10^{-8} c/fm)**

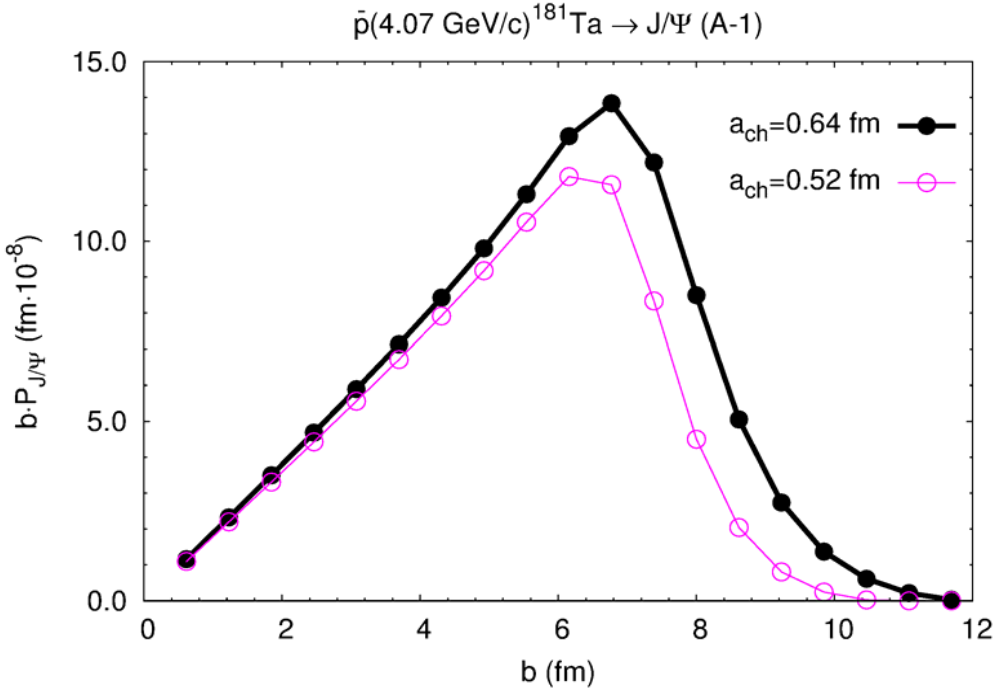
Thick (thin) lines: a_{ch}=0.64 (0.52) fm

The centre of the nucleus at b=0, z=0.

J/Ψ production is always localized in the diffuse surface zone.

Probability of J/Ψ production

$$P_{\mathrm{J/\Psi}}(b) = v_{\bar{p}}^{-1} \int\limits_{-\infty}^{\infty} dz \mathcal{P}_{\bar{p},\mathrm{surv}}(z,b) \Gamma_{\bar{p}\to \mathrm{J/\Psi}}(z,b)$$



- Peripheral collisions contribute mostly (geometrical factor).

- Diffuseness parameter of the charge distribution influences sensitively.

Charmonium dissociation cross sections (expectations):

$$RN \to \Lambda_c \bar{D}(+\text{pions})\ , \qquad R = J/\Psi, \chi_c, \Psi'$$

$\sigma_{J/\Psi N} = 6-7$ mb

- from J/Ψ transparency ratios for AA at $\sqrt{s} = 20$ GeV (except PbPb), $pA, \gamma A, \pi A$ and $\bar{p}A$ reactions on nuclei *without* including sidefeeding effects from χc and Ψ' decays

C. Gerschel and J. Hüfner, Z. Phys. C 56, 171 (1992);
D. Kharzeev et al., Z. Phys. C 74, 307 (1997)

$\sigma_{J/\Psi N} = 3.62$ mb,
$\sigma_{\Psi' N} = 20.0$ mb,
$\sigma_{\chi_{c1}}(L_z = 0) = 6.82$ mb,
$\sigma_{\chi_{c1}}(L_z = \pm 1) = 15.9$ mb

- from QCD factorization theorem and nonrelativistic quarkonium model. Consistent with $\Psi'/J/\Psi$ ratio in pA collisions *with* sidefeeding effects from χc and Ψ' decays

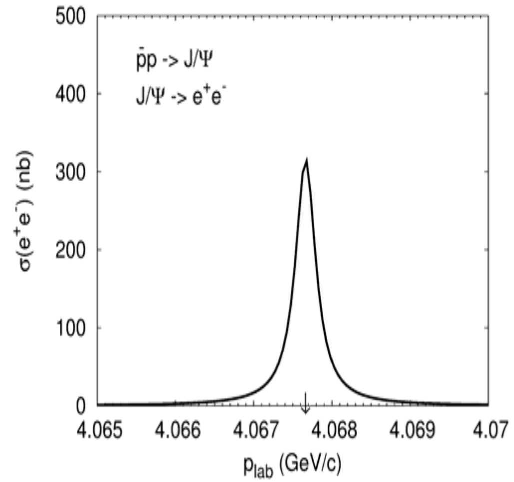
L. Gerland et al., PRL 81, 762 (1998)

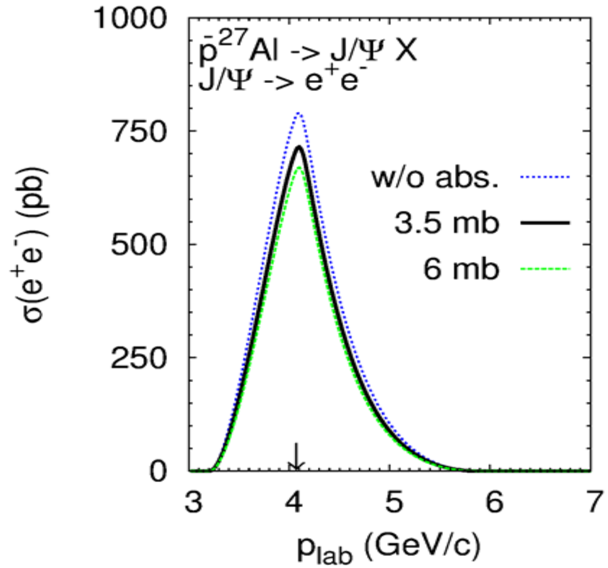
$\sigma_{J/\Psi N} \simeq 5$ mb

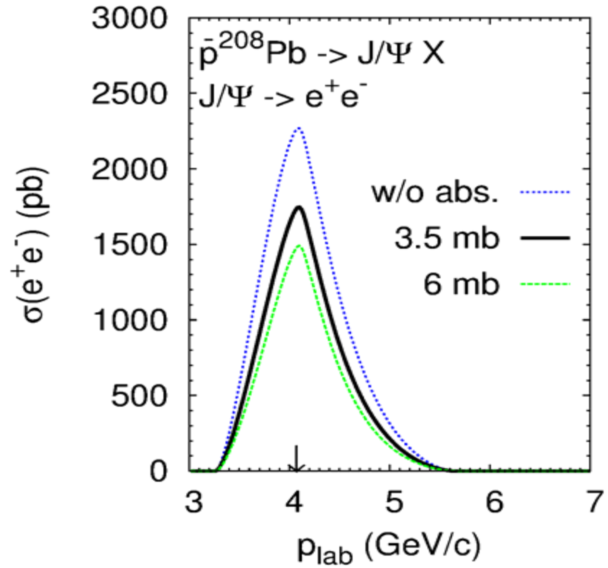
- hadronic model

R. Molina et al., PRC 86, 014604 (2012)

J/Ψ(1S) production cross section on a proton and on nuclei for the different values of dissociation cross section $\sigma_{J/\Psi\ N}$





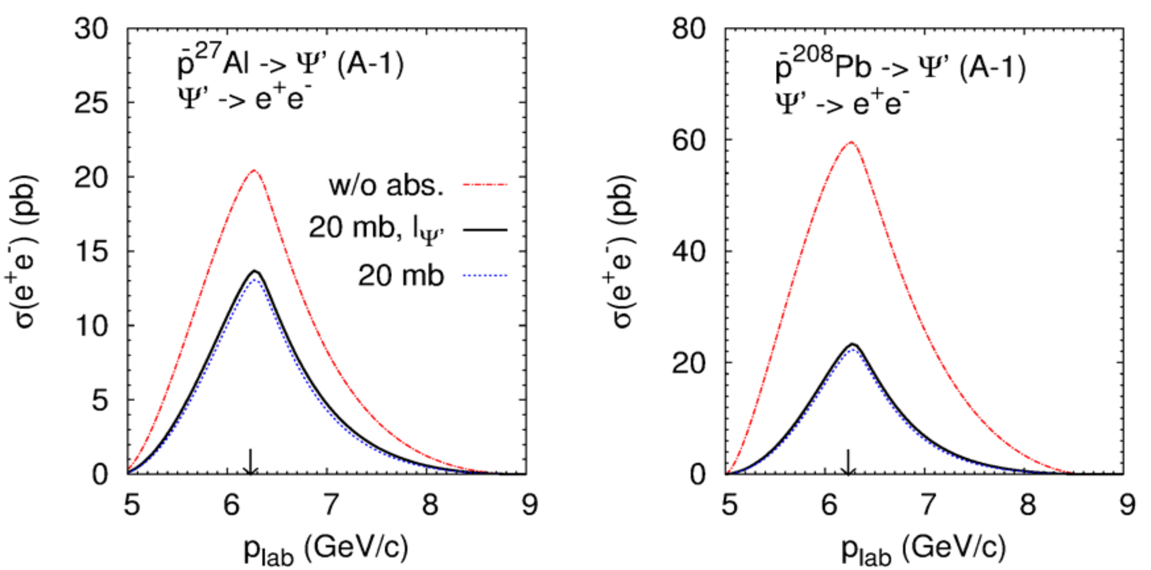


- Broadening and reduction of the on-shell peak due to Fermi motion.

- For heavy nuclei - strong sensitivity to $\sigma_{J/\Psi\ N}$.

Ψ'(2S) production cross section on nuclei for the different values of dissociation cross section $\sigma_{\Psi' N}$ with and w/o formation length effect $l_{\Psi'} \simeq 2 l_{J/\Psi} \simeq 1 \text{ fm}$

QCD factorization theorem with nonrelativistic quark model of charmonium L. Gerland et al., PRL 81, 762 (1998): $\sigma_{\Psi' N} \simeq 20 \text{ mb}$

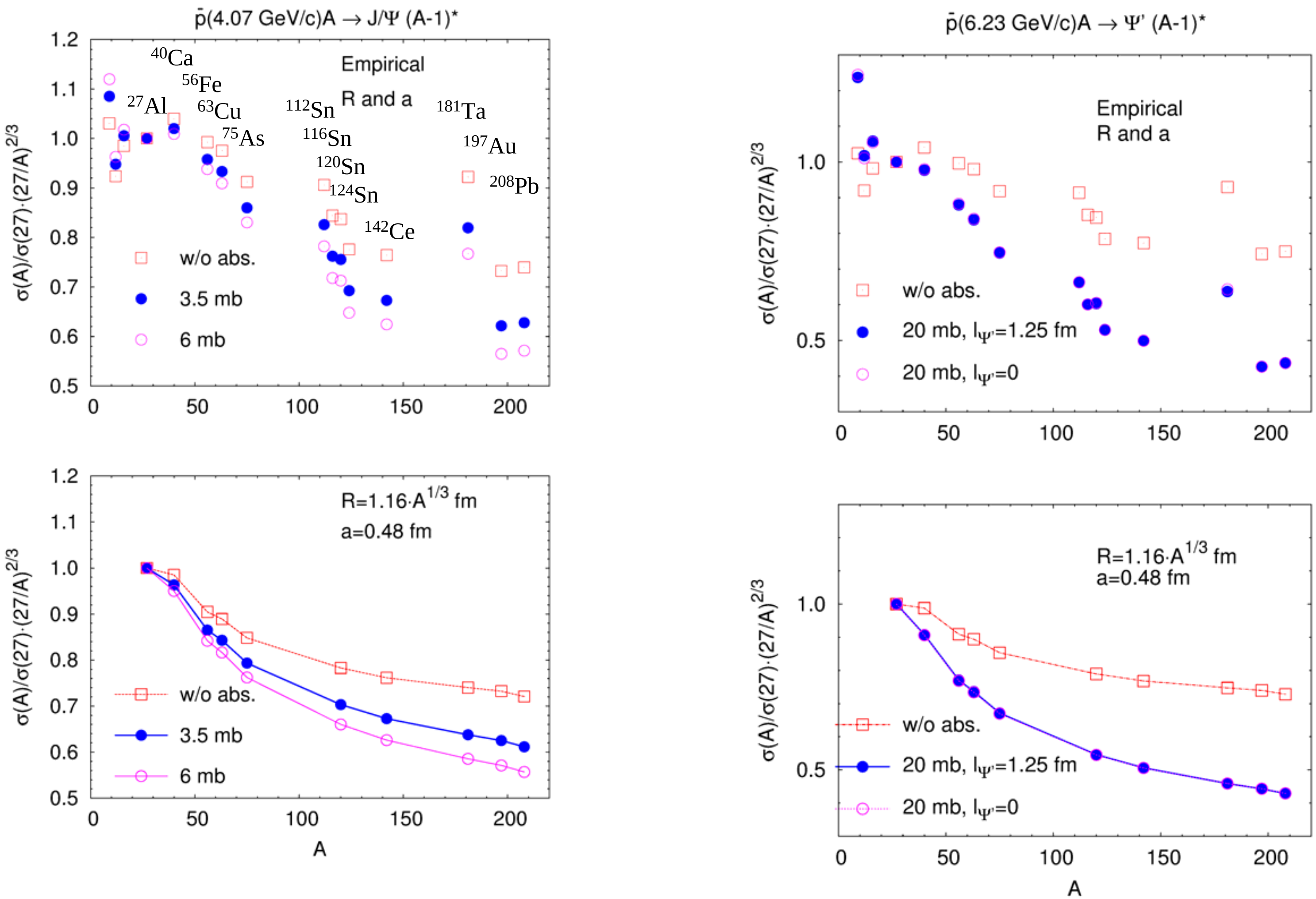


- Strong sensitivity to $\sigma_{\Psi' N}$.

- Almost no sensitivity to formation length.

Transparency ratio $\frac{\sigma_{\bar{p}A\to R(A-1)^*}}{\sigma_{\bar{p}\,^{27}\mathrm{Al}\to R\,^{26}\mathrm{Mg}^*}}\left(\frac{27}{A}\right)^{2/3}$ $(R = J/\Psi, \Psi')$ **at the on-shell peak**

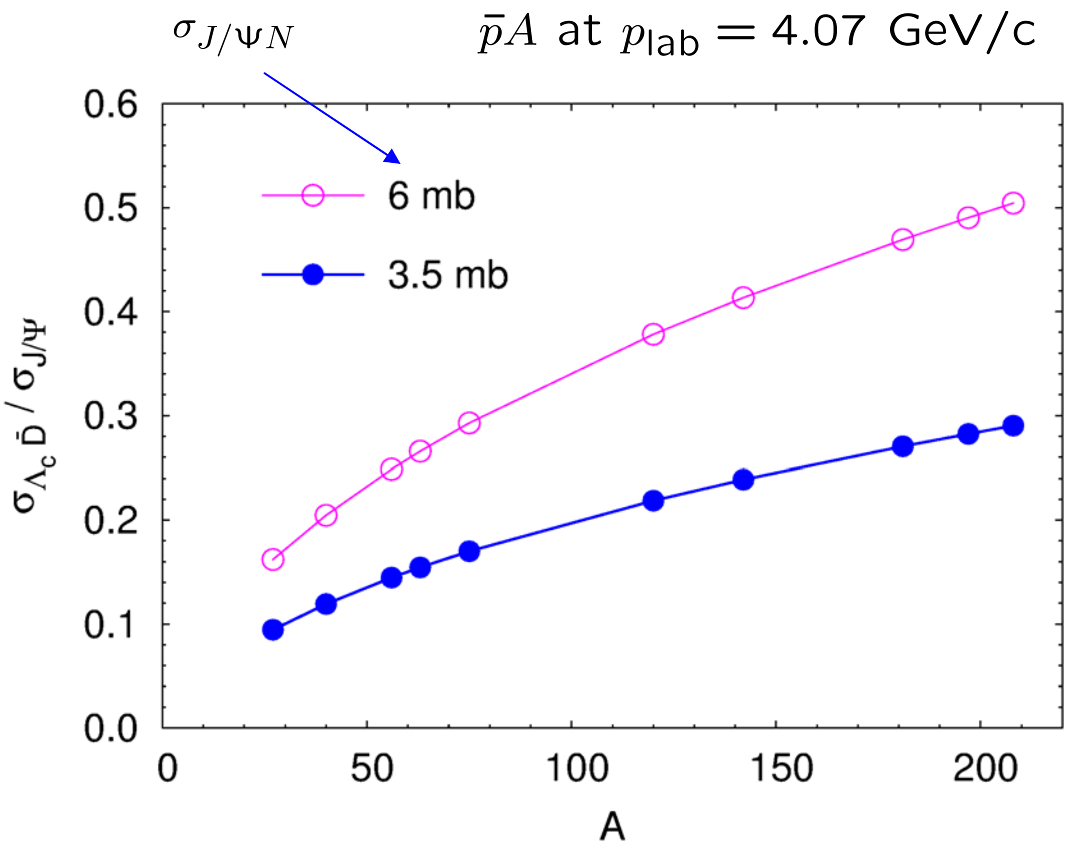
Possible uncertainties in the in-medium production width $\Gamma_{\bar{p}\to R}$ cancel-out



- Local variations of A-dependence due to details of nuclear density profiles.
- Careful selection of the target nuclei needed (e.g. noble gases).

The only channel of $\Lambda_c\bar{D}$-pair production at $p_{lab} < 5.194$ GeV/c (χ_{c0} production threshold in $\bar{p}p$ collisions) is $J/\Psi N \to \Lambda_c\bar{D}$

$$\sigma_{\Lambda_c\bar{D}} = \sigma^{w/o\, J/\Psi abs.}_{\bar{p}A\to J/\Psi(A-1)^*} - \sigma_{\bar{p}A\to J/\Psi(A-1)^*}$$



- Strong sensitivity to the assumed value of $\sigma_{J/\Psi N}$.

$\chi_{cJ}\ (J=0,1,2)$ ***production:***

- Mass splitting between different χ_{cJ} states is small ~ 140 MeV

- Nondiagonal transitions $\chi_{cJ_1}N \to \chi_{cJ}N$ are easily possible

-In the simplest quark model with central (e.g Cornell) potential *the physical* χ_{cJ} state with helicity ν can be decomposed in the basis of $c\bar{c}$ states with fixed orbital and spin angular momentum projections on the charmonium momentum axis:

$$|J\nu\rangle = \sum_{L_z,S_z} |1L_z;1S_z\rangle\langle 1L_z;1S_z|J\nu\rangle \ .$$

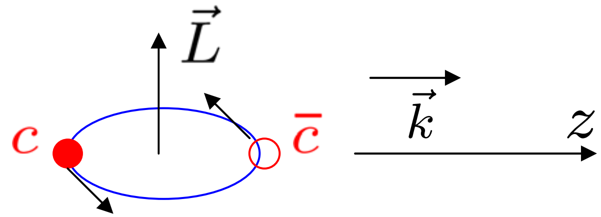
- **For the basis states $|1L_z;1S_z\rangle$ the interaction cross section with a nucleon depends on L_z (QCD factorization theorem and nonrelativistic quarkonium model, L. Gerland et al, PRL 81, 762 (1998)):**

$$\sigma_{L_z} = \int \sigma_{q\bar{q}}(r_t) R^2(r) |Y_{1L_z}(\hat{\vec{r}})|^2 d^3r$$

color dipole cross section (nonperturbative evaluation)

$$\boxed{\sigma_{L_z=0} = 6.8 \text{ mb}, \ \sigma_{L_z=\pm 1} = 15.9 \text{ mb}}$$

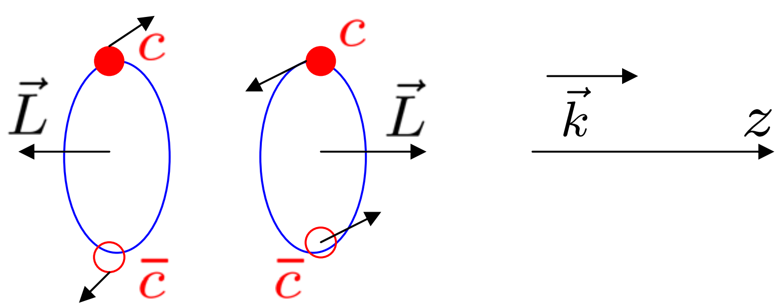
$L_z = 0$



pQCD estimate:

$$\sigma_{\bar{q}q}(r_t \to 0) \propto r_t^2$$

$L_z = \pm 1$



$$\frac{\sigma_1}{\sigma_0} \simeq \frac{< r_t^2 >_{L_z=\pm 1}}{< r_t^2 >_{L_z=0}} = 2$$

Longitudinally polarized $c\bar{c}$ pair has a larger transverse size and, hence, a larger interaction cross section with a nucleon.

Diagonal (elastic) or nondiagonal $\chi_c N$ scattering: $\chi_{J_1\nu} N \to \chi_{J\nu} N$

Assume that the interaction with a nucleon does not change the spin and internal angular momentum of $c\bar{c}$ pair:

→

Invariant matrix element: $M_{J\nu;J_1\nu}(\mathrm{q}_t) = \mathrm{e}^{-B_{\chi N}\mathrm{q}_t^2/2} \sum_{L_z,S_z} \langle J\nu|1L_z;1S_z\rangle M_{L_z}(0)\langle 1L_z;1S_z|J_1\nu\rangle$

$B_{\chi N} \simeq 3\ \mathrm{GeV}^{-2}$ **- two-gluon exchange (L. Gerland et al, PLB 619, 95 (2005))**

Optical theorem: $M_{L_z}(0) = 2ip_{\mathrm{lab}} m_N \sigma_{L_z}(1 - i\rho_{\chi N})$, $\rho_{\chi N} = \dfrac{\mathrm{Re}M_{L_z}(0)}{\mathrm{Im}M_{L_z}(0)} \simeq 0.15 - 0.30$

(soft Pomeron exchange – pQCD limits)

The amplitudes of nondiagnal transitions are proportional to σ_1-σ_0 :

$$
\begin{aligned}
M_{20;00}(0) &= 2ip_{\mathrm{lab}} m_N \frac{\sqrt{2}}{3}(\sigma_1 - \sigma_0)(1 - i\rho_{\chi N}) \ , \\
M_{2,\pm1;1,\pm1}(0) &= \pm 2ip_{\mathrm{lab}} m_N \frac{1}{2}(\sigma_1 - \sigma_0)(1 - i\rho_{\chi N}) \ .
\end{aligned}
$$

Multiple scattering diagrams

— keep only diagrams with elastic rescattering: inelastic diffractive cross sections are small (e.g. $\sigma(\bar{p}p \to \bar{N}^* p + c.c.) \simeq 0.1$ mb at $p_{\mathrm{lab}} \simeq 6$ GeV/c)

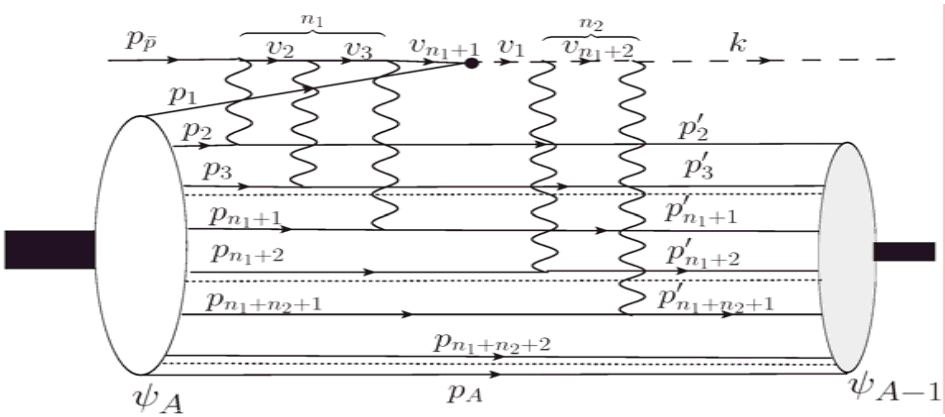
Nondiagonal, i.e. with transition

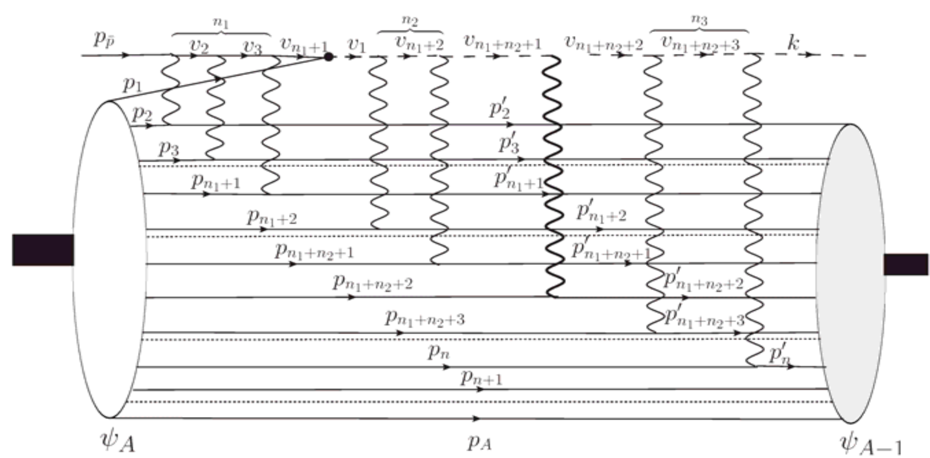
$\chi_{J_1} N_{n_1+n_2+2} \to \chi_J N'_{n_1+n_2+2}$:

Diagonal: **number of involved nucleons**

$M^J(1,2,\ldots,n)$, $n = n_1 + n_2 + 1$

$M^{J_1 J}(1,2,\ldots,n)$, $n = n_1 + n_2 + n_3 + 2$





Generalized eikonal approximation (GEA): **L. Frankfurt, M. Sargsian, M. Strikman, PRC 56, 1124 (1997); M. Sargsian, Int. J. Mod. Phys. E 10, 405 (2001).**

— neglect energy transfer in rescatterings (soft rescatterings on nonrelativistic nucleons);

— eikonal form of propagators (nonrelativistic initial and final nucleons);

— keep only transverse momentum transfer dependence in elementary amplitudes (soft scatterings at high energies);

— quasifree kinematics of the produced charmonium: $|p_{\mathrm{lab}} - k^z| \ll p_{\mathrm{lab}}$ **;**

— systematic expansion of $|M|^2$ in the number of rescatterings.

Additive terms contributing to $|M|^2$:

Optical theorem:

$$\mathrm{Im} M_{JN}(0) = 2 p_{\mathrm{lab}} m \sigma_{JN}^{\mathrm{tot}}$$

Direct term („simple“ Glauber model):

$$\sum_{\mathrm{set1}\neq\mathrm{set2}} \sum_{\psi_{A-1}} M^J(1,\mathrm{set1}) M^{J*}(1,\mathrm{set2})$$

$$= \frac{|M_{J;\bar{p}p}(\mathbf{k}_t)|^2}{2E_1} \int d^3X f_1(\mathbf{X},\mathbf{k}_t,\Delta_J^0) \underbrace{\prod_{i=2}^{A} \left(1 - \sigma_{\bar{p}N}^{\mathrm{tot}} \int\limits_{-\infty}^{Z} dz_i |\phi_i(\mathbf{B},z_i)|^2 - \sigma_{JN}^{\mathrm{tot}} \int\limits_{Z}^{+\infty} dz_i |\phi_i(\mathbf{B},z_i)|^2 \right)}_{\simeq \exp\left(-\sigma_{\bar{p}N}^{\mathrm{tot}} \int\limits_{-\infty}^{Z} dz_2 \rho(\mathbf{B},z_2) - \sigma_{JN}^{\mathrm{tot}} \int\limits_{Z}^{+\infty} dz_2 \rho(\mathbf{B},z_2) \right)}$$

Interference term:

$$\sum_{\mathrm{set1}\neq\mathrm{set2}} \sum_{\psi_{A-1}} M^{J_1 J}(1,2,\mathrm{set1}) M^{J*}(1,\mathrm{set2}) + \mathrm{c.c.}$$

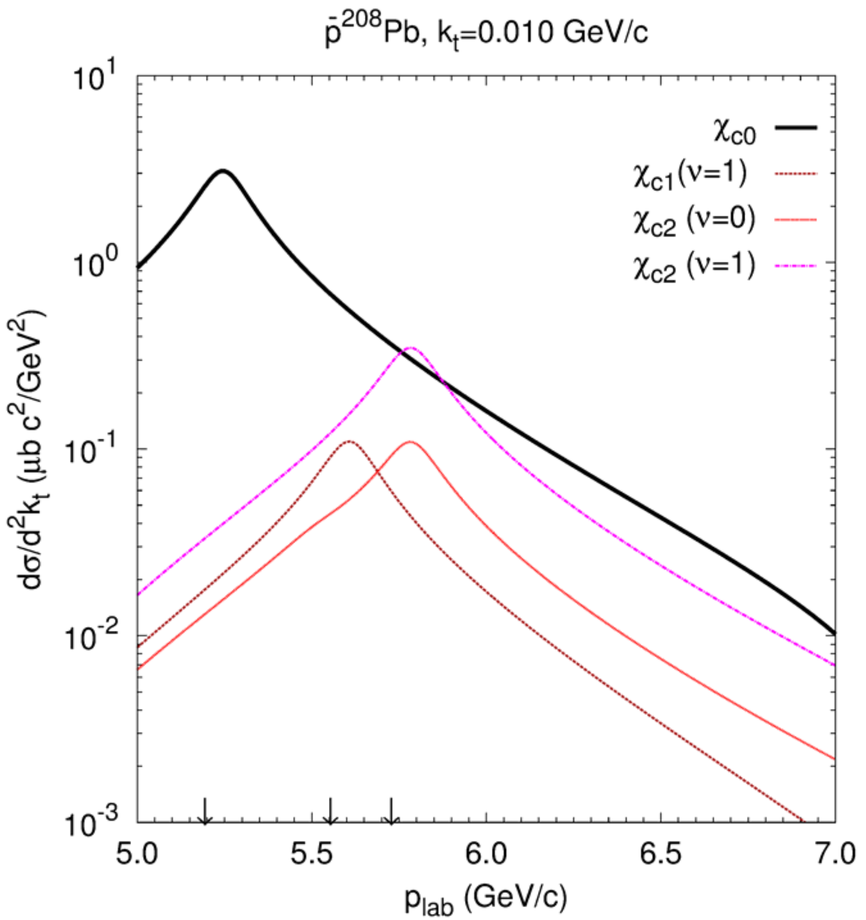
$$= \frac{i M_{J;\bar{p}p}^*(\mathbf{k}_t)}{2E_1 4 m p_{\mathrm{lab}}} M_{JN';J_1N}(0) M_{J_1;\bar{p}p}(\mathbf{k}_t) \int d^3X f_1\left(\mathbf{X},\mathbf{k}_t,\frac{\Delta_J^0+\Delta_{J_1}^0}{2}\right) \int\limits_{Z}^{+\infty} dz_2\, |\phi_2(\mathbf{B},z_2)|^2 \mathrm{e}^{i(\Delta_{J_1}^0-\Delta_J^0)(z_2-Z)}$$

$$\times \prod_{i=3}^{A} \left(1 - \sigma_{\bar{p}N}^{\mathrm{tot}} \int\limits_{-\infty}^{Z} dz_i |\phi_i(\mathbf{B},z_i)|^2 + \frac{i[M_{J_1N}(0) - M_{JN}^*(0)]}{4 m p_{\mathrm{lab}}} \int\limits_{Z}^{z_2} dz_i |\phi_i(\mathbf{B},z_i)|^2 - \sigma_{JN}^{\mathrm{tot}} \int\limits_{z_2}^{+\infty} dz_i |\phi_i(\mathbf{B},z_i)|^2 \right) + \mathrm{c.c.}$$

$$\Delta_J^0 = \frac{m^2 + E_1^2 + 2E_{\bar{p}}E_1 - m_J^2}{2p_{\mathrm{lab}}}$$ **— longitudinal momentum transfer needed for on-shell χ_J**

Differential cross sections:

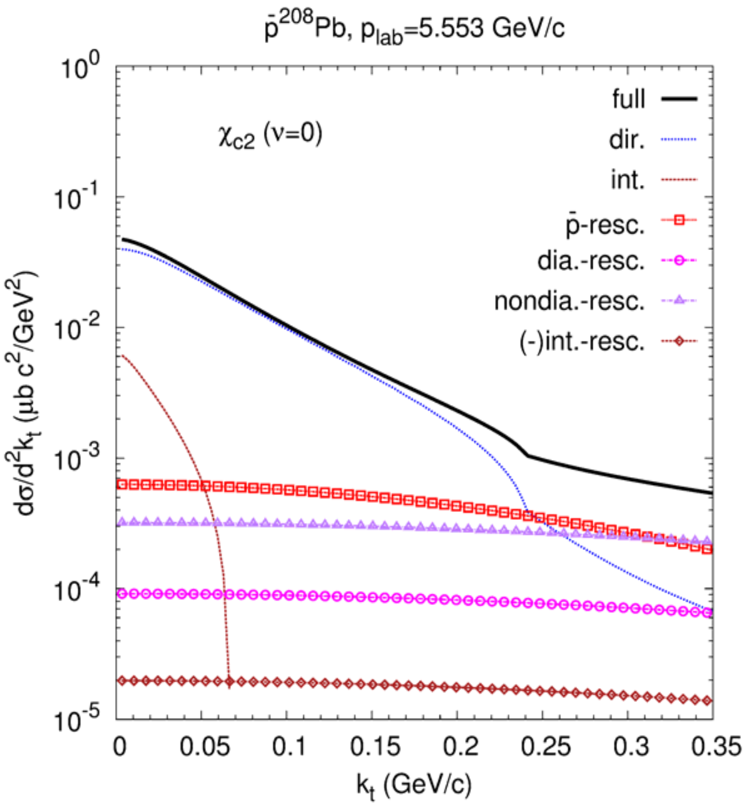
$$\frac{d\sigma_{\bar{p}A\to\chi_{J\nu}(A-1)^*}}{d^2k_t} = \frac{|M|^2}{16\pi^2 p_{\rm lab}^2}$$

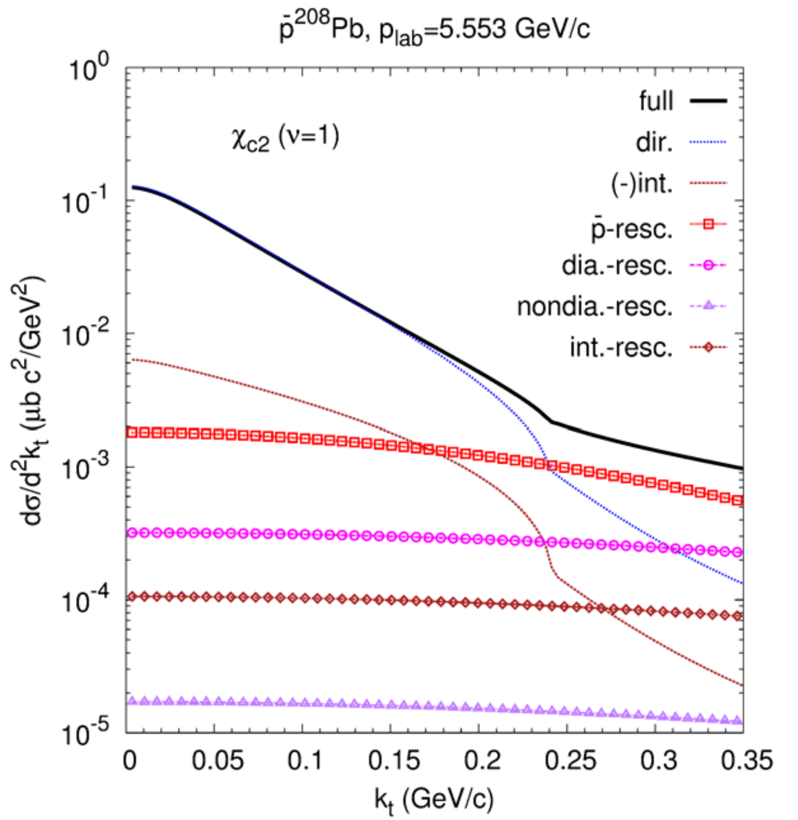


Arrows – on-shell production in $\bar{p}p \to \chi_c$ **:**
$p_{\rm lab} = 5.194,\ 5.553$ and 5.727 GeV/c
for χ_{c0}, χ_{c1} and χ_{c2}, resp.

- Strong overlap in p_{lab} for the different χ_c flavors. Interference is possible.

$\bar{p}p \to \chi_{c1}$ **on-shell**





Significant (~15%) contribution of the interference term of the direct $\bar{p}p \to \chi_{20}$ and two-step $\bar{p}p \to \chi_{00},\ \chi_{00}N \to \chi_{20}N$ amplitudes to the χ_{20} production at $k_t = 0$.

Helicity ratio

$$\mathcal{R} = \frac{\chi_{20}}{(\chi_{20} + 2\chi_{21})|B_0|^2}$$

$\mathcal{R} = 1$ for $\bar{p}p$

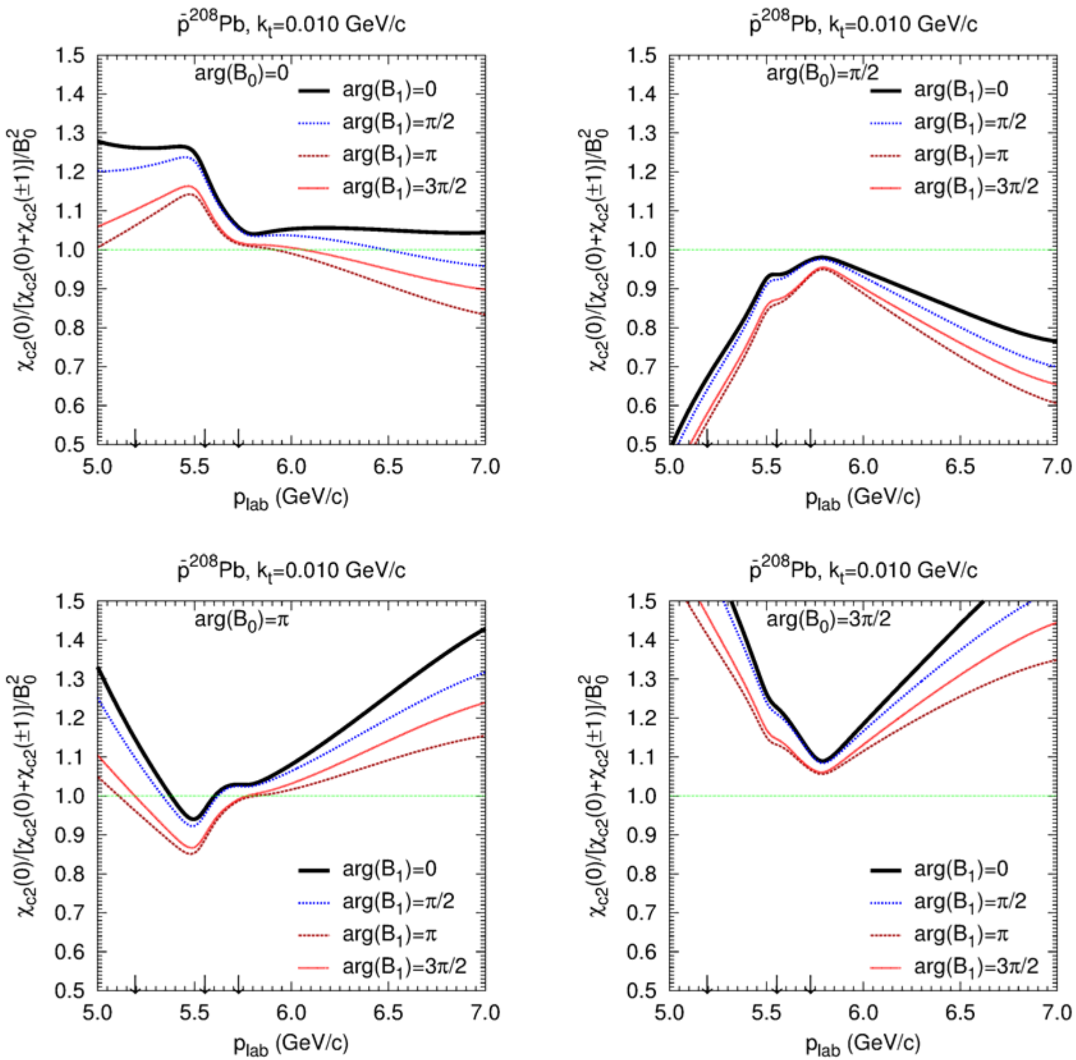
$B_0, \; B_1$ **- helicity amplitudes**

$|B_0|^2 + 2|B_1|^2 = 1$

$|B_0|^2 = 0.13 \pm 0.08$

- from angular distributions for $\bar{p}p \to \chi_{c2} \to J/\Psi\gamma \to e^+e^-\gamma$

M. Ambrogiani et al. (E835), PRD 65, 052002 (2002)



The deviation of $\mathcal{R}$ ***from 1 is due to the interference of the direct*** $\bar{p}p \to \chi_{20}$ ***and the two-step*** $\bar{p}p \to \chi_{00}, \;\chi_{00}N \to \chi_{20}N$ ***amplitudes*** **and proportional to** $\sigma_1 - \sigma_0$ **.**

XYZ production

Noncharmonium mesons containing a $c\bar{c}$ pair:

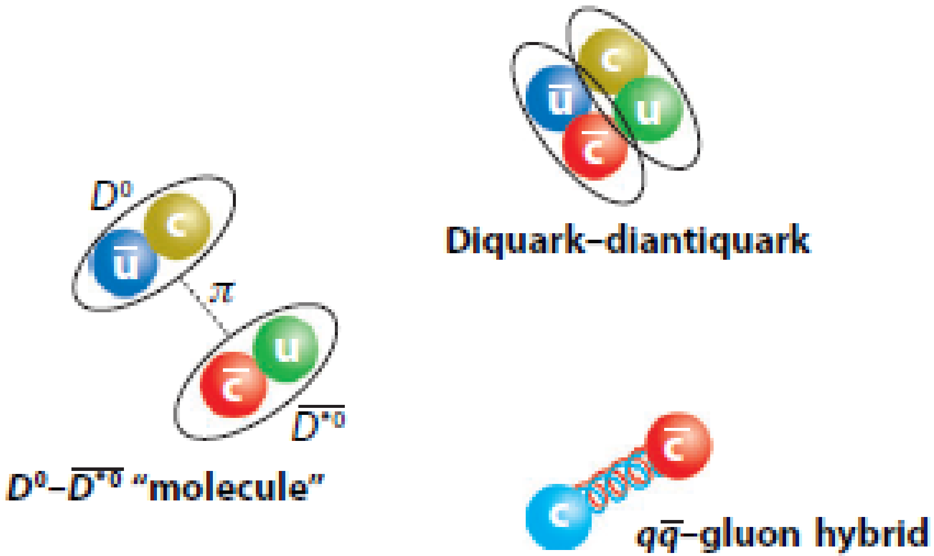


Figure from S. Godfrey and S.L. Olsen, Annu. Rev. Nucl. Part. Sci. 58, 51 (2008)

$D\bar{D}^*$ and $D^*\bar{D}^*$ candidates found at B-factories:

State	M (MeV)	Γ (MeV)	J^{PC}
$X(3872)$	3871.52 ± 0.2	1.3 ± 0.6	$1^{++}/2^{-+}$
$X(3915)$	3915.6 ± 3.1	28 ± 10	$0/2^{?+}$
$X(3940)$	3942^{+9}_{-8}	37^{+27}_{-17}	$?^{?+}$
$G(3900)$	3943 ± 21	52 ± 11	1^{--}
$Y(4008)$	4008^{+121}_{-49}	226 ± 97	1^{--}

Parameters from N. Brambilla et al., EPJ C 71, 1534 (2011)

$$m_{D^0} + m_{D^{*0}} = 3871.85 \pm 0.28 \text{ MeV}$$

$$m_{D^\pm} + m_{D^{*\pm}} = 3879.91 \pm 0.28 \text{ MeV}$$

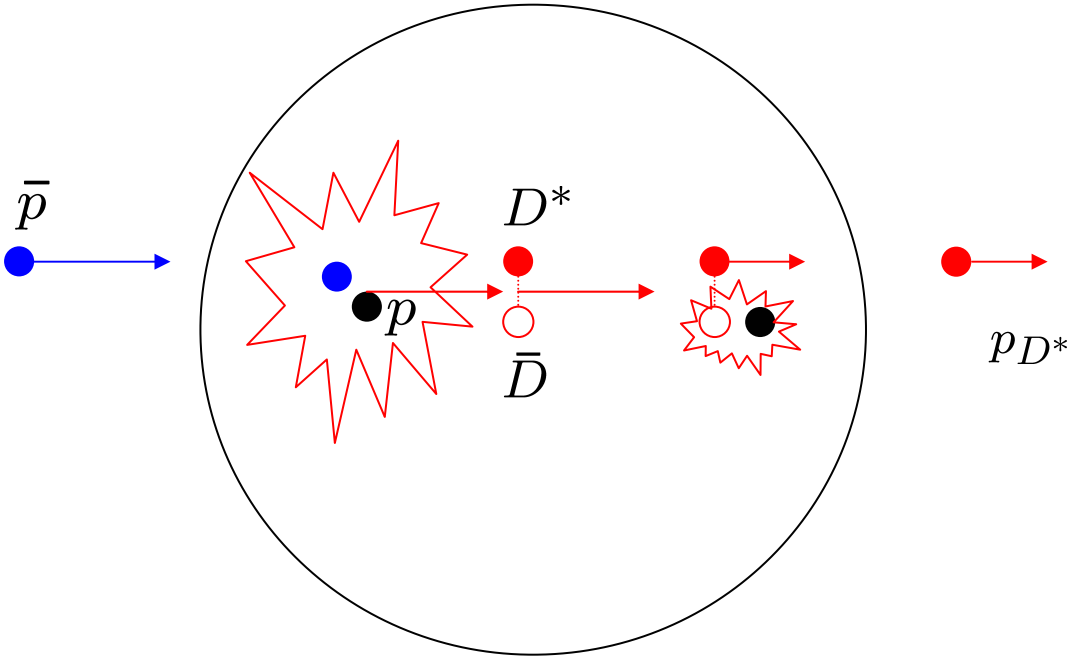
$$2m_{D^{*0}} = 4013.98 \pm 0.30 \text{ MeV}$$

$$2m_{D^{*\pm}} = 4020.58 \pm 0.26 \text{ MeV}$$

Parameters from PDG

- X(3872) is more likely to be a strongly bound $D^+D^{*-} + c.c.$
then weakly bound $D^0\bar{D}^{*0} + c.c.$

Use nucleus to test the possible molecular structure:



$$p_{D^*} \simeq p_{lab}/2$$

$$r_{r.m.s.}(\bar{D}D^*) \sim \frac{\sqrt{2\mu\epsilon_B}}{\hbar} \sim 1 - 10 \text{ fm}$$

$D^- D^{*+}$ $\bar{D}^0 D^{*0}$

$$\mu = \frac{m_{\bar{D}} m_{D^*}}{m_{\bar{D}} + m_{D^*}}$$

Expected elementary cross sections:

$$\frac{\sigma^{\text{tot}}_{Dp}(p_{\text{lab}}/2)}{\sigma^{\text{tot}}_{\pi^+p}(p_{\text{lab}}/2)} \sim \left(\frac{r_{Bohr}(D)}{r_{Bohr}(\pi)}\right)^2 \sim \frac{1}{2}$$

$$\bar{p}p \longrightarrow X(3872), \qquad p_{\text{lab}} \simeq 7 \text{ GeV/c}$$

$$\sigma^{\text{tot}}_{\pi^+p} \simeq 29 \text{ mb}$$

$$\sigma^{\text{tot}}_{Dp} \simeq \sigma^{\text{tot}}_{D^*p} \simeq 14.5 \text{ mb}, \quad \sigma^{\text{tot}}_{Xp} \simeq \sigma^{\text{tot}}_{Dp} + \sigma^{\text{tot}}_{D^*p} \simeq 29 \text{ mb}$$

Inclusive cross section on nucleus:

$$\sigma_{\bar{p}A\to D^*(A-1)^*} = 2\pi \int\limits_0^\infty db\, b\, v_{\bar{p}}^{-1} \int\limits_{-\infty}^{\infty} dz \mathcal{P}_{\bar{p},\mathrm{surv}}(z,b)\Gamma_{\bar{p}\to X}(z,b)$$
$$\times \mathcal{P}_{D^*,\mathrm{surv}}(z,b)\underbrace{(1-\mathcal{P}_{\bar{D},\mathrm{surv}}(z,b))}_{\bar{D}\text{ - stripping probability}}$$

$$\mathcal{P}_{D^*(\bar{D}),\mathrm{surv}}(z,b) = \exp\left\{-\sigma^{\mathrm{tot}}_{D^*(\bar{D})p}(p_{\mathrm{lab}}/2)\int\limits_z^\infty dz'\rho(z',b)\right\}$$

$D^*(\bar{D})$ **survival probability**

X(3872) and D (D*) production cross sections on nuclei

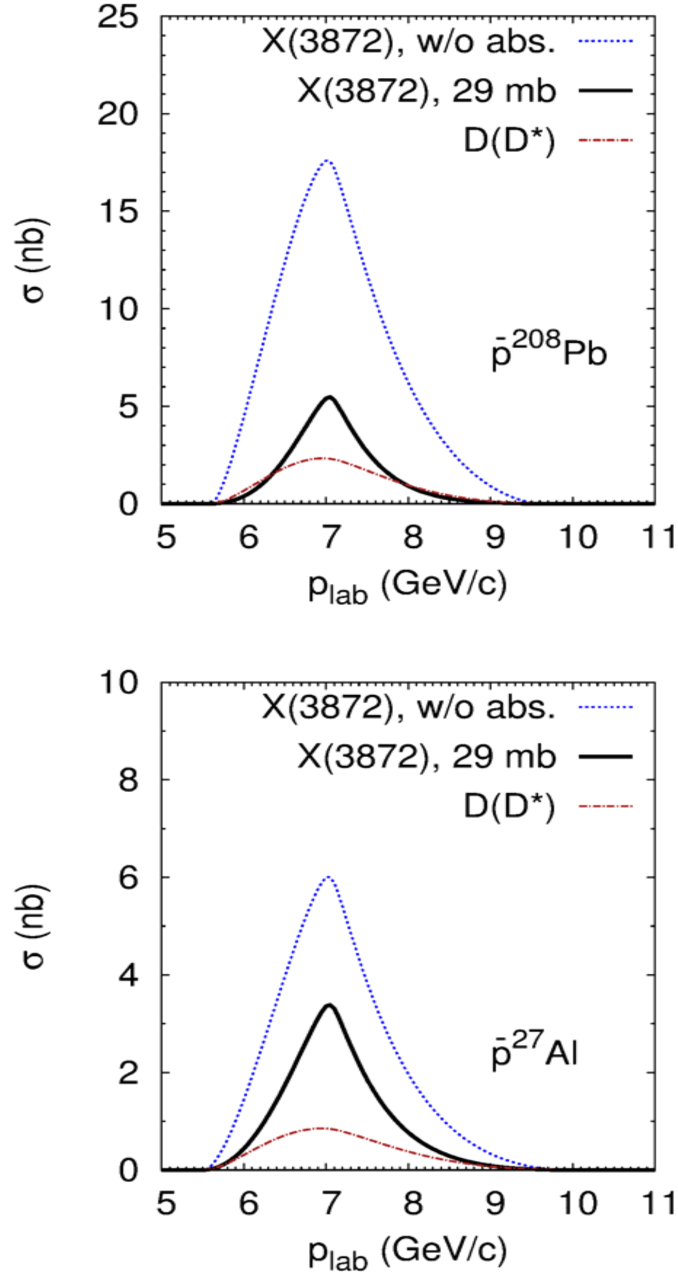
Input:

$$\Gamma_{X(3872)} = 2.3 \text{ MeV}$$

$$\frac{\Gamma_{X(3872)\to\bar{p}p}}{\Gamma_{X(3872)}} = 1 \times 10^{-4}$$

- Strong absorption of X(3872)

- Molecular structure of X(3872) enhances D (D*) production



Summary

— Sensitivity of J/Ψ(Ψ´) production in antiproton-induced reactions to the genuine J/ΨN (Ψ´N) dissociation cross section

— For the quantitative determination of J/ΨN (Ψ´N) cross sections the density profiles are important

— Polarization effects in χ_{c2} production on nuclei due to σ_{L_z}

— Possible molecular structure of X(3872) should result in its strong absorption and enhanced production of D(D*) at forward angles and momentum ≈p_{lab}/2 in lab. frame

Further steps

— X(3872) and D(D*) differential production cross section, shadowing effects

— Deuteron target

Thank you for your attention !

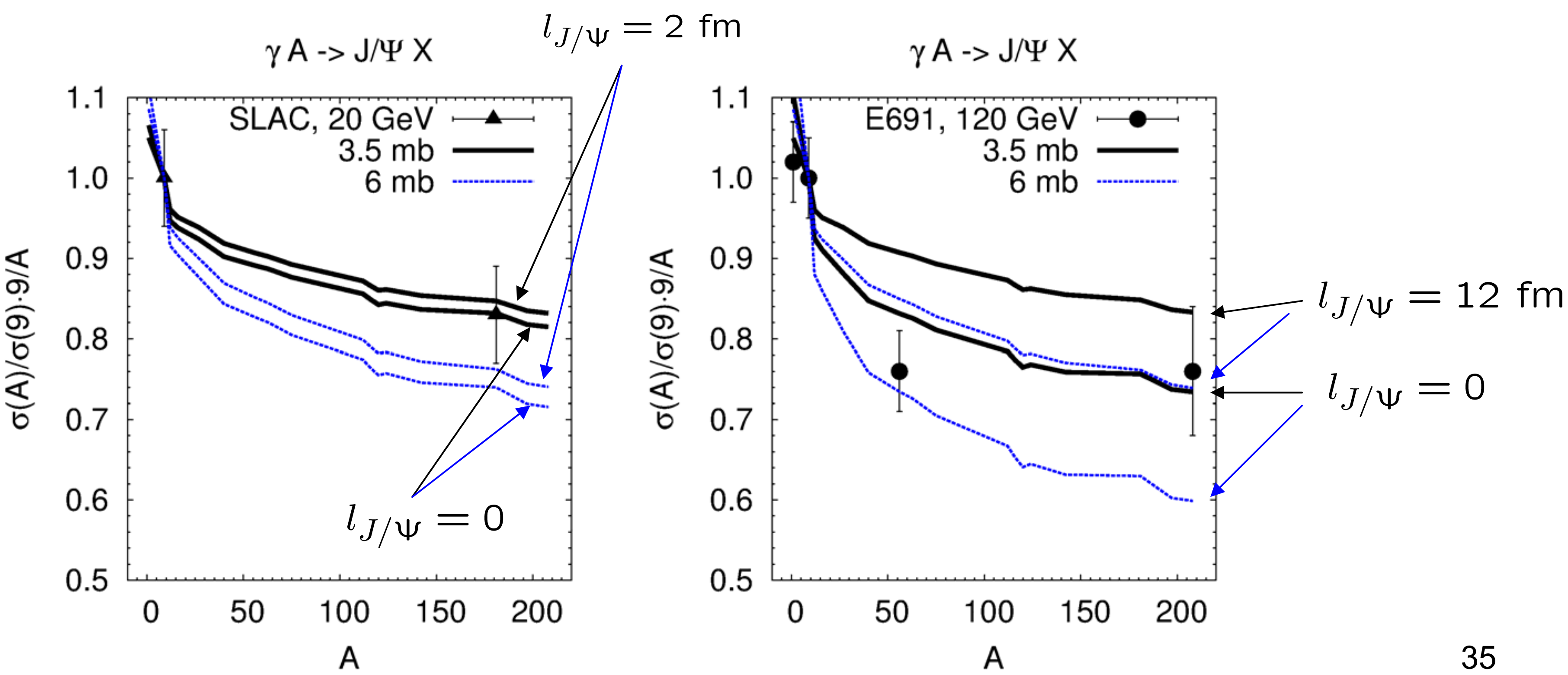
Backup

Influence of J/Ψ formation length on transparency ratio in γ-induced reactions:

$$S_A = \frac{\sigma_{\gamma A \to J/\Psi X}}{A\sigma_{\gamma p \to J/\Psi X}} = \frac{2\pi}{A}\int_0^\infty db\, b \int_{-\infty}^{\infty} dz \rho(z,b) P_{J/\Psi,\mathrm{surv}}(z - l_c, b)$$

$l_c = 2E_\gamma / m^2_{J/\Psi}$ **- coherence length**

$$\mathcal{P}_{J/\Psi,\mathrm{surv}}(z,b) = \exp\left\{ -\int_z^\infty dz' \rho(z',b) \sigma^{\mathrm{eff}}_{J/\Psi N}(p_{J/\Psi}, z'-z) \right\}$$ **- J/Ψ survival probability**



Effective charmonium-nucleon cross section:

$$\sigma_{RN}^{\text{eff}}(p_R, z) = \sigma_{RN}(p_R) \left(\left[\left(\frac{z}{l_R} \right)^{\tau} + \frac{< n^2 k_t^2 >}{m_R^2} \left(1 - \left(\frac{z}{l_R} \right)^{\tau} \right) \right] \Theta(l_R - z) + \Theta(z - l_R) \right) \, ,$$

$$\tau = 1 \, .$$

G.R. Farrar, L.L. Frankfurt, M.I. Strikman, and H. Liu, NPB 345, 125 (1990)

$< k_t^2 >^{1/2} \simeq 0.35 \ \text{GeV/c}$ **— average quark transverse momentum in a hadron**

$n = 3$ **— number of intermediate gluons**

$$l_{J/\Psi} \simeq \frac{2p_{J/\Psi}}{m_{\Psi'}^2 - m_{J/\Psi}^2} \simeq 3\text{fm} \frac{p_{J/\Psi}}{30\text{GeV}} \, ,$$

$$l_{\Psi'} \simeq 6\text{fm} \frac{p_{\Psi'}}{30\text{GeV}} \, ,$$

$$l_{\chi_c} \simeq 3\text{fm} \frac{p_{\chi_c}}{30\text{GeV}} \, .$$

— formation lengths

L. Gerland et al, PRL 81, 762 (1998)

Partial width:

$$p_{\text{lab}} = m_R\sqrt{m_R^2/4m_N^2 - 1} \quad (\text{for } \bar{p}p \to R_{\text{on-shell}}):$$

$$\Gamma_{\bar{p}\to R} = \int \frac{2d^3p}{(2\pi)^3} v_{\bar{p}p}\sigma_{\bar{p}p\to R}(\sqrt{s}) f_p(\mathbf{p}) \simeq \frac{3m_R^2\Gamma_{R\to\bar{p}p}p_{F,p}^2}{8p_{\text{lab}}E_{\bar{p}}E_p q_R} \propto \rho_p^{2/3}$$

$$E_{\bar{p}} = \sqrt{p_{\text{lab}}^2 + m_N^2},\ q_R = \sqrt{m_R^2/4 - m_N^2},\ f_p(\mathbf{p}) = \Theta(p_{F,p} - |\mathbf{p}|).$$

$$\frac{\sigma_{\bar{p}A\to R(A-1)*}}{Z\sigma_{\bar{p}p\to R}} \sim \frac{\Gamma_{\bar{p}\to R}}{v_{\bar{p}}\sigma_{\bar{p}p\to R}(m_R)\rho_p} = \frac{3\pi m_R m_N \Gamma_R}{4(m_R^2 - 2m_N^2)v_{\bar{p}}p_{F,p}} \sim 10^{-4}$$

(for $m_{J/\Psi} = 3.097$ GeV, $\Gamma_{J/\Psi} = 93$ keV,
$p_{F,p} \simeq 0.3$ GeV/c, $p_{\text{lab}} = 4.07$ GeV/c)

Strong reduction of charmonium production due to Fermi motion

Density profiles:

For light nuclei (A ≤ 20) — harmonic oscillator model:

$$\rho_q(r) = \rho_q^0 \left[1 + a_q \left(\frac{r}{R_q}\right)^2\right] \exp\{-(r/R_q)^2\} , \quad q = p, n .$$

For heavy nuclei (A > 20) — two-parameter Fermi distribution:

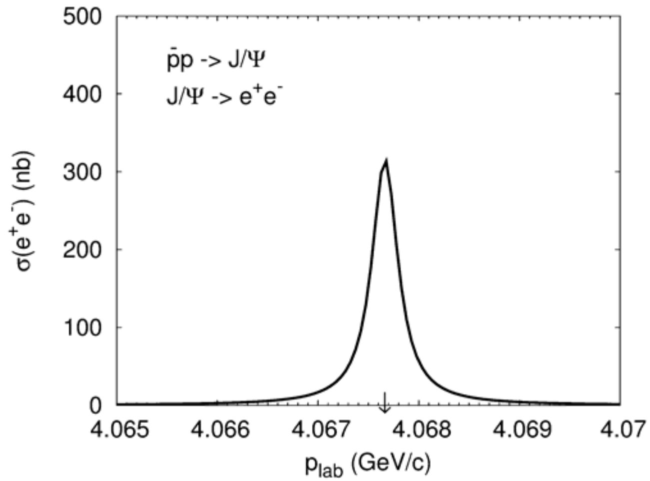
$$\rho_q(r) = \frac{\rho_q^0}{\exp\left(\frac{r-R_q}{a_q}\right) + 1} , \; q = p, n$$

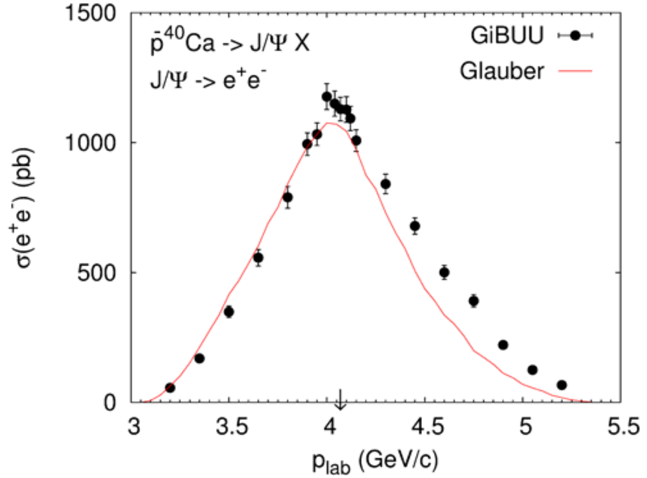
Charge density parameters: **C. De Jager et al., Atom. Data Nucl. Data Tabl. 14, 479 (1974).**

Neutron density parameters: **J. Nieves et al., NPA 554, 509 (1993); V. Koptev et al., Yad. Fiz. 31, 1501 (1980); R. Schmidt et al., PRC 67, 044308 (2003).**

Fermi motion by Monte-Carlo:

Due to Fermi motion
cross section drops
by a factor of ~10^{-3} at the peak





Good agreement between GiBUU and Glauber calculations

GiBUU model review: **O. Buss et al., Phys. Rep. 512, 1 (2012)**

Symmetries of helicity amplitudes **F.L. Ridener et al., PRD 45, 3173 (1992):**

Charge conjugation invariance: $B^J_{\lambda_{\bar{p}}\lambda_1} = \eta_c(-1)^J B^J_{\lambda_1\lambda_{\bar{p}}}\ , \quad \eta_c = (-1)^{L+S}$

Parity invariance: $B^J_{\lambda_{\bar{p}}\lambda_1} = \eta_p(-1)^J B^J_{-\lambda_{\bar{p}},-\lambda_1}\ , \quad \eta_p = (-1)^{L+1}$

For χ-states $L = S = 1$, $\eta_c = \eta_p = 1$.

$$|B^J_{-+}|^2 = |B^J_{+-}|^2 = |B_1|^2\ , \quad B_1 \equiv B^J_{+-}$$

$$|B^J_{++}|^2 = |B^J_{--}|^2 = |B_0|^2/2\ , \quad B_0/\sqrt{2} \equiv B^J_{++}$$

Norma: $2|B_1|^2 + |B_0|^2 = 1$

$B_1 = 0$ for $J = 0$

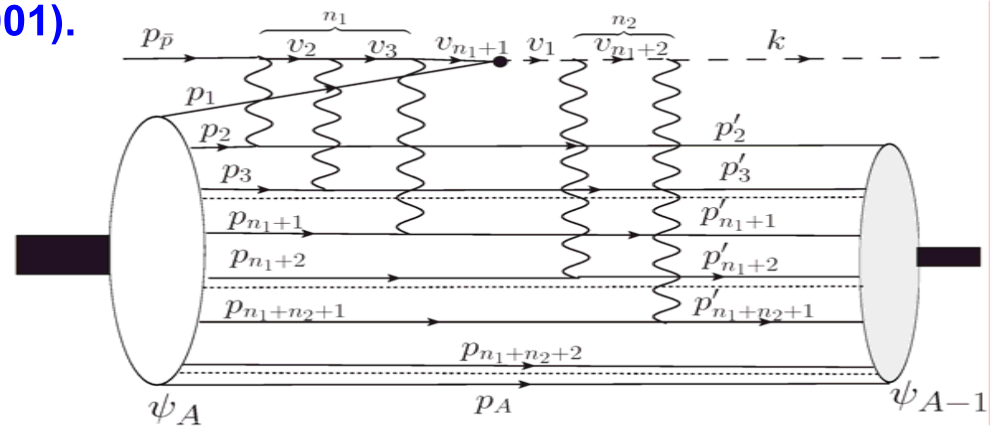
$B_0 = 0$ for $J = 1$ (from C-parity)

$|B_0|^2 = 0.13 \pm 0.08$ for $J = 2$ from angular distributions for $\bar{p}p \to \chi_{c2} \to J/\psi\gamma \to e^+e^-\gamma$

M. Ambrogiani et al., PRD 65, 052002 (2002)

Generalized eikonal approximation (GEA)

L. Frankfurt, M. Sargsian, M. Strikman, PRC 56, 1124 (1997);
M. Sargsian, Int. J. Mod. Phys. E 10, 405 (2001).



Multiple scattering diagram:

$$n = n_1 + n_2 + 1$$

- number of involved nucleons

— neglect energy transfer in rescatterings (soft rescatterings on nonrelativistic nucleons):

$$S^J(1,2,\ldots,n) = \frac{i(2\pi)\delta(E_{\bar p} + E_1 - \omega)}{(2E_{\bar p}V2\omega V)^{1/2}} M^J(1,2,\ldots,n) \ ,$$

$$\begin{aligned} M^J(1,2,\ldots,n) \ = \ & \frac{1}{\sqrt{2E_1}(2m)^{n-1}(2\pi)^{6(n-1)}} \int d^3x'_2 \cdots d^3x'_n \int d^3x_1 \cdots d^3x_A \\ & \times \psi^*_{A-1}(\mathbf{x}'_2,\ldots,\mathbf{x}'_n,\mathbf{x}_{n+1},\ldots,\mathbf{x}_A)\psi_A(\mathbf{x}_1,\ldots,\mathbf{x}_A) \\ & \times \int d^3p'_2 \cdots d^3p'_n \int d^3p_1 \cdots d^3p_n \delta^{(3)}(\mathbf{p}_{\bar p} + \mathbf{p}_1 + \mathbf{q}_2 + \cdots + \mathbf{q}_n - \mathbf{k}) \\ & \times \mathrm{e}^{i\mathbf{p}'_2\mathbf{x}'_2 + \cdots + i\mathbf{p}'_n\mathbf{x}'_n} \frac{M_{JN}(q_n)M_{JN}(q_{n-1})\cdots M_{JN}(q_{n_1+2})M_{J;\bar p p}(p_1)}{D_J(v_{n-1})\cdots D_J(v_{n_1+2})D_J(v_1)} \\ & \times \frac{M_{\bar p N}(q_{n_1+1})\cdots M_{\bar p N}(q_2)}{D_{\bar p}(v_{n_1+1})\cdots D_{\bar p}(v_2)} \mathrm{e}^{-i\mathbf{p}_1\mathbf{x}_1 - \cdots - i\mathbf{p}_n\mathbf{x}_n} \ , \end{aligned}$$

$$q_i = p_i - p'_i \ , \qquad i = 2,\ldots,n \ .$$

Inverse propagators of $\bar{p}$ and χ_J
(for nonrelativistic initial and final nucleons):

$$\Delta_J^0 = \frac{m^2 + E_1^2 + 2E_{\bar{p}}E_1 - m_J^2}{2p_{\mathrm{lab}}}$$

Longitudinal momentum transfer in case of on-shell χ_J production

— **keep only transverse momentum transfer dependence in elementary amplitudes (soft scatterings at high energies), i.e.** $M_{\bar{p}N}(q_i) \to M_{\bar{p}N}(\mathbf{t}_i), \quad \mathbf{t}_i = \mathbf{q}_{it}$ **etc.;**

— **coordinate representation of propagators:** $\dfrac{1}{\Delta_J^0 - p_1^z + i\varepsilon} = -i \int dz^0 \Theta(z^0) \mathrm{e}^{i(\Delta_J^0 - p_1^z) z^0}$

Gribov-Glauber-type expression:

$$
\begin{aligned}
M^J(1,2,\ldots,n) \;=\; & \frac{i^{n-1}}{(2E_1)^{1/2}(2\pi)^{2(n-1)}(4mp_{\mathrm{lab}})^{n-1}} \\
& \times \int d^3x_1 \cdots d^3x_A \psi^*_{A-1}(\mathbf{x}_2,\ldots,\mathbf{x}_A)\psi_A(\mathbf{x}_1,\ldots,\mathbf{x}_A) \\
& \times \Theta(z_3 - z_2)\cdots\Theta(z_{n_1+1} - z_{n_1})\Theta(z_1 - z_{n_1+1})\Theta(z_{n_1+2} - z_1) \\
& \times \Theta(z_{n_1+3} - z_{n_1+2})\cdots\Theta(z_n - z_{n-1}) \\
& \times \exp\{i(p_{\mathrm{lab}} - k^z + \Delta_J^0)z_n - i\Delta_J^0 z_1 - i\mathbf{k}_t\mathbf{b}_1\} \int d^2t_2 \cdots d^2t_n \\
& \times \exp\{-i\mathbf{t}_2(\mathbf{b}_2 - \mathbf{b}_1) - \cdots - i\mathbf{t}_n(\mathbf{b}_n - \mathbf{b}_1)\} M_{\bar{p}N}(\mathbf{t}_2)\cdots M_{\bar{p}N}(\mathbf{t}_{n_1+1}) \\
& \times M_{J;\bar{p}N}(\mathbf{k}_t - \mathbf{t}_2 - \cdots - \mathbf{t}_n) M_{JN}(\mathbf{t}_{n_1+2})\cdots M_{JN}(\mathbf{t}_n) \;.
\end{aligned}
$$

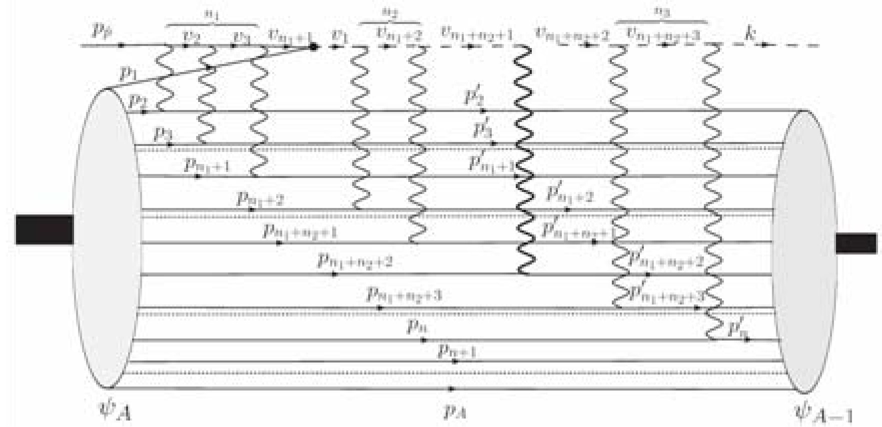
Quasifree production: $|p_{\mathrm{lab}} - k^z| \ll p_{\mathrm{lab}} \;, \quad k^z - p_{\mathrm{lab}} \simeq \Delta_J^0 - \cancel{(\Delta_J^0)^2/2p_{\mathrm{lab}}}$

Sum over different orders of scatterings:

$$\Theta(z_3 - z_2)\cdots\Theta(z_{n_1+1} - z_{n_1})\Theta(z_1 - z_{n_1+1})\Theta(z_{n_1+2} - z_1)\Theta(z_{n_1+3} - z_{n_1+2})\cdots\Theta(z_n - z_{n-1})$$

$$\Theta(z_1 - z_2)\cdots\Theta(z_1 - z_{n_1+1})\Theta(z_{n_1+2} - z_1)\cdots\Theta(z_n - z_1)$$

Diagram with one nondiagonal transition $\chi_{J_1} N_{n_1+n_2+2} \to \chi_J N'_{n_1+n_2+2}$**:**



$$n = n_1 + n_2 + n_3 + 2$$

number of involved nucleons

$\bar{p}$ rescattering term:

$$\sum_{\text{set1}\neq\text{set2}}\sum_{\psi_{A-1}} M^J(2,1,\text{set1})M^{J*}(2,1,\text{set2})$$
$$= \frac{1}{(2\pi)^2 2E_1(4mp_{\text{lab}})^2}\int d^2t_2 |M_{J;\bar{p}p}(\mathbf{k}_t-\mathbf{t}_2)|^2\,|M_{\bar{p}N}(\mathbf{t}_2)|^2\int d^3X f_1(\mathbf{X},\mathbf{k}_t-\mathbf{t}_2,\Delta_J^0)$$
$$\times\int_{-\infty}^{Z} dz_2\,|\phi_2(\mathbf{B},z_2)|^2\prod_{i=3}^{A}\left(1-\sigma_{\bar{p}N}^{\text{tot}}\int_{-\infty}^{Z} dz_i|\phi_i(\mathbf{B},z_i)|^2-\sigma_{JN}^{\text{tot}}\int_{Z}^{+\infty} dz_i|\phi_i(\mathbf{B},z_i)|^2\right)\ .$$

χ_J diagonal rescattering term:

$$\sum_{\text{set1}\neq\text{set2}}\sum_{\psi_{A-1}} M^J(1,2,\text{set1})M^{J*}(1,2,\text{set2})$$
$$= \frac{1}{(2\pi)^2 2E_1(4mp_{\text{lab}})^2}\int d^2t_2 |M_{JN}(\mathbf{t}_2)|^2\,|M_{J;\bar{p}p}(\mathbf{k}_t-\mathbf{t}_2)|^2\int d^3X f_1(\mathbf{X},\mathbf{k}_t-\mathbf{t}_2,\Delta_J^0)$$
$$\times\int_{Z}^{+\infty} dz_2\,|\phi_2(\mathbf{B},z_2)|^2\prod_{i=3}^{A}\left(1-\sigma_{\bar{p}N}^{\text{tot}}\int_{-\infty}^{Z} dz_i|\phi_i(\mathbf{B},z_i)|^2-\sigma_{JN}^{\text{tot}}\int_{Z}^{+\infty} dz_i|\phi_i(\mathbf{B},z_i)|^2\right)\ .$$

$\chi_{J_1} N_2 \to \chi_J N_2'$ **nondiagonal rescattering term:**

$$\sum_{\mathsf{set1} \neq \mathsf{set2}} \sum_{\psi_{A-1}} M^{J_1 J}(1,2,\mathsf{set1}) M^{J_1 J *}(1,2,\mathsf{set2})$$

$$= \frac{1}{(2\pi)^2 2E_1 (4mp_{\mathsf{lab}})^2} \int d^2 t_2 |M_{JN';J_1N}(\mathbf{t}_2)|^2 \, |M_{J_1;\bar{p}p}(\mathbf{k}_t - \mathbf{t}_2)|^2$$

$$\times \int d^3 X f_1(\mathbf{X}, \mathbf{k}_t - \mathbf{t}_2, \Delta^0_{J_1}) \int\limits_{Z}^{+\infty} dz_2 \, |\phi_2(\mathbf{B}, z_2)|^2$$

$$\times \prod_{i=3}^{A} \left(1 - \sigma^{\mathsf{tot}}_{\bar{p}N} \int\limits_{-\infty}^{Z} dz_i |\phi_i(\mathbf{B}, z_i)|^2 - \sigma^{\mathsf{tot}}_{J_1N} \int\limits_{Z}^{z_2} dz_i |\phi_i(\mathbf{B}, z_i)|^2 - \sigma^{\mathsf{tot}}_{JN} \int\limits_{z_2}^{+\infty} dz_i |\phi_i(\mathbf{B}, z_i)|^2 \right) \; .$$

Diagonal-nondiagonal rescattering interference term:

Elastic $\bar{p}N$ scattering amplitude:

$$M_{\bar{p}N}(\mathbf{q}_t) = 2ip_{\text{lab}} m \sigma^{\text{tot}}_{\bar{p}p}(1 - i\rho_{\bar{p}p})\mathrm{e}^{-B_{\bar{p}p}\mathbf{q}_t^2/2}$$

$$B_{\bar{p}p} = 12.5 \pm 1 \ \text{GeV}^{-2} \ \text{at} \ \sqrt{s} \simeq 3.4 - 7.0 \ \text{GeV}$$

exp. data: Yu.M. Antipov et al., NPB 57, 333 (1973)

$$\rho_{\bar{p}p} \simeq -0.05 \ \text{at} \ \sqrt{s} \simeq 3 - 5 \ \text{GeV}$$

Reggeized Pomeron exchange model:
R. Fiore et al., PRD 81, 056001 (2010)

$$\sigma^{\text{tot}}_{\bar{p}p}(p_{\text{lab}}) = 38.4 + 77.6 p_{\text{lab}}^{-0.64} + 0.26 \ln^2(p_{\text{lab}}) - 1.2 \ln(p_{\text{lab}})$$

in mb

in GeV/c

PDG: L. Montanet et al., PRD 50, 1173 (1994)

Proton occupation numbers:

$$2n(\mathbf{X},\mathbf{p}) = \sum_{N_1} f_1(\mathbf{X},\mathbf{p})$$

spin factor

sum over all occupied proton states

$$n(\mathbf{X},\mathbf{p}) = (1-P_2)\Theta(p_F-p)+\boxed{\frac{(2\pi)^3}{2}\rho_p a_2|\psi_D(p)|^2\Theta(p-p_F)}$$

high momentum tail due to short range NN correlations (SRC)

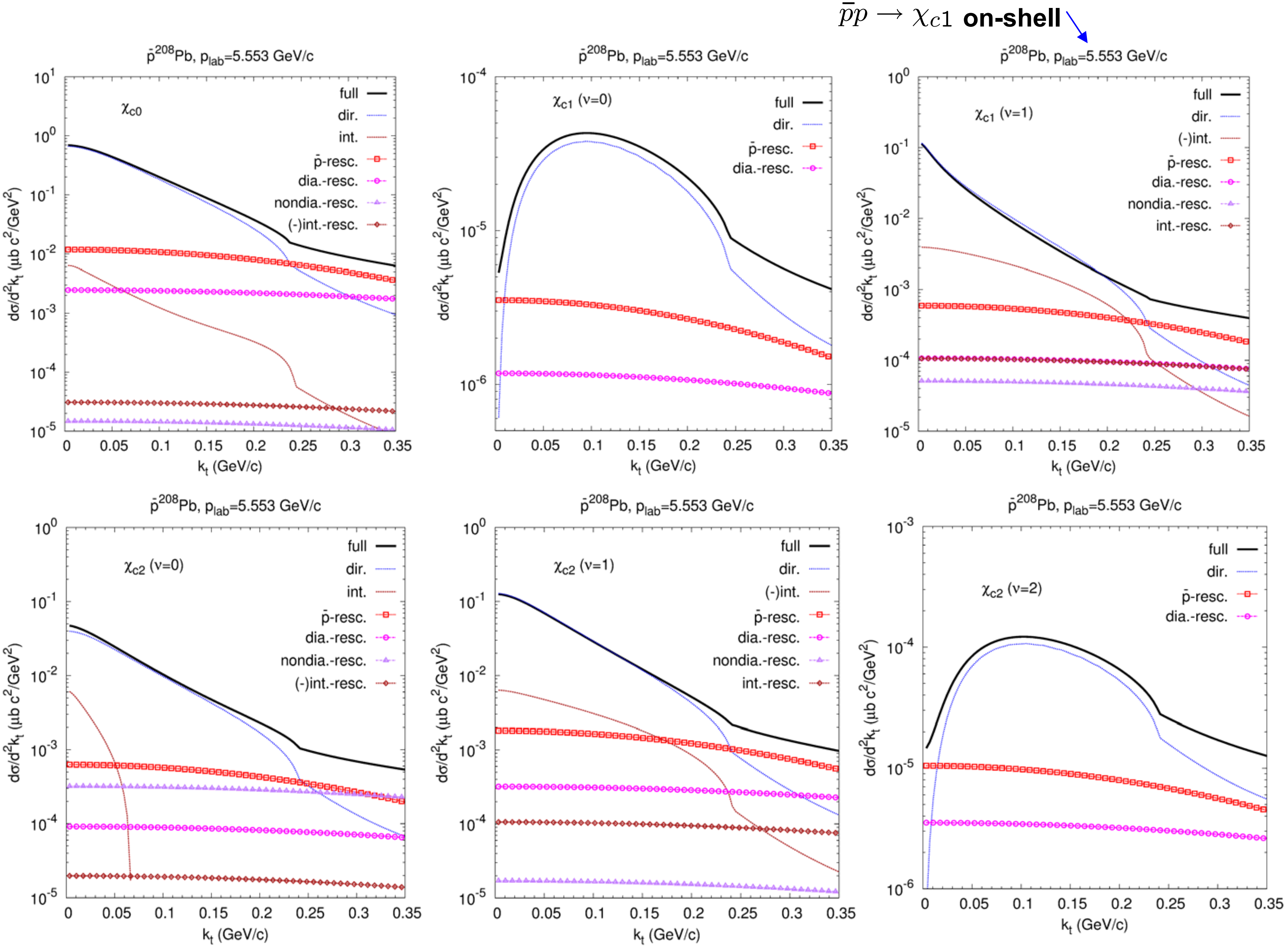
L. Frankfurt, M. Strikman, Phys. Rep. 76, 215 (1981);
L. Frankfurt, M. Sargsian, M. Strikman,
Int. J. Mod. Phys. A23, 2991 (2008)

$p_F(\mathbf{X}) = (3\pi^2\rho_p)^{1/3}$ **— proton Fermi momenum**

$P_2 \simeq 0.25$ **— proton fraction above Fermi surface,** $P_2 = 4\pi a_2 \int\limits_{p_F}^{+\infty} dp p^2 |\psi_D(p)|^2$

$\psi_D(p)$ **— deuteron wave function,** $4\pi \int\limits_0^{+\infty} dp p^2 |\psi_D(p)|^2 = 1$

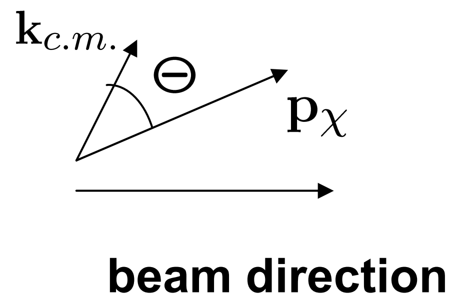
Paris potential: **M. Lacombe et al., PRC 21, 861 (1980)**



$\bar{p}p \to \chi_{c1}$ on-shell
$\bar{p}^{208}$Pb, p_{lab}=5.553 GeV/c
χ_{c0}
χ_{c1} (ν=0)
χ_{c1} (ν=1)
χ_{c2} (ν=0)
χ_{c2} (ν=1)
χ_{c2} (ν=2)
full
dir.
int.
(-)int.
$\bar{p}$-resc.
dia.-resc.
nondia.-resc.
(-)int.-resc.
int.-resc.
$d\sigma/d^2k_t$ (μb c^2/GeV^2)
k_t (GeV/c)

How to measure $\mathcal{R}$ at PANDA ?

- **Use decay** $\chi_c \to J/\Psi\gamma \to e^+e^-\gamma$
- **Double trigger on the photon energy in** $e^+e^-\gamma$ **c.m. frame (E$_\gamma$=303, 389 and 430 MeV for** χ_{c0} **,** χ_{c1} **and** χ_{c2} **, respectively) and on** $M_{\rm inv}(e^+e^-\gamma)$
- **Determine the angle** Θ **between photon momentum in** χ_c **c.m. frame and the** χ_c **momentum in lab. frame**



Distribution in Θ **:**

$$W_J(\Theta) = \sum_{\nu=\pm1,0} P_{J\nu} W_{J\nu}(\Theta) \ ,$$

$$P_{J\nu} = \frac{\chi_{J\nu}}{\chi_{J0} + 2\chi_{J1}} \ ,$$

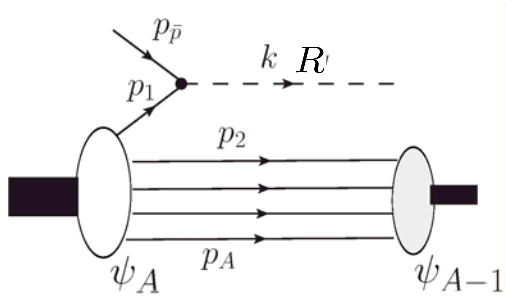
$$P_{20} = \mathcal{R}|B_0|^2, \ P_{2,\pm1} = (1 - \mathcal{R}|B_0|^2)/2$$

$$W_{J\nu}(\Theta) \propto \sum_{\nu'=0}^{J} |A^J_{\nu'}|^2([d^J_{\nu\nu'}(\Theta)]^2 + [d^J_{\nu,-\nu'}(\Theta)]^2) \ .$$

F.L. Ridener et al., PRD 45, 3173 (1992)

$a_1, a_2, \ldots, a_{J+1}$ **- multipole amplitudes of E1,M2,… transitions**

Impulse approximation



$$k = (\omega, \mathbf{k})$$
$$p_{\bar p} = (E_{\bar p}, \mathbf{p}_{\bar p})$$
$$\mathbf{p}_{\bar p} = (0, 0, p_{\mathrm{lab}})$$
$$p_1 = (E_1, \mathbf{p}_1)$$

fixed

$$S_R(1) = \frac{i(2\pi)\delta(E_{\bar p} + E_1 - \omega)}{(2E_{\bar p} V 2\omega V)^{1/2}} M_R(1)$$

$$M_R(1) = \frac{M_{R;\bar p p}(k - p_{\bar p})}{\sqrt{2E_1}} \int d^3x_1 ... d^3x_A \psi^*_{A-1}(\mathbf{x}_2, ..., \mathbf{x}_A) \mathrm{e}^{-i(\mathbf{k}-\mathbf{p}_{\bar p})\mathbf{x}_1} \psi_A(\mathbf{x}_1, \mathbf{x}_2, ..., \mathbf{x}_A)$$

$$\int d^3x_1 ... d^3x_A |\psi_A(\mathbf{x}_1, \mathbf{x}_2, ..., \mathbf{x}_A)|^2 = 1$$

$$d\sigma_{\bar p A \to R(A-1)^*} = \sum_{N_1} \sum_{\psi_{A-1}} \frac{2\pi\delta(E_{\bar p} + E_1 - \omega)}{2p_{\mathrm{lab}}} |M_R(1)|^2 \frac{d^3k}{(2\pi)^3 2\omega} \,,$$

Completeness relation: $$\sum_{\psi_{A-1}} \psi_{A-1}(\tilde{\mathbf{x}}_2, \tilde{\mathbf{x}}_3, ..., \tilde{\mathbf{x}}_A) \psi^*_{A-1}(\mathbf{x}_2, \mathbf{x}_3, ..., \mathbf{x}_A) = \delta^{(3)}(\tilde{\mathbf{x}}_2 - \mathbf{x}_2) \delta^{(3)}(\tilde{\mathbf{x}}_3 - \mathbf{x}_3) \ldots \delta^{(3)}(\tilde{\mathbf{x}}_A - \mathbf{x}_A)$$

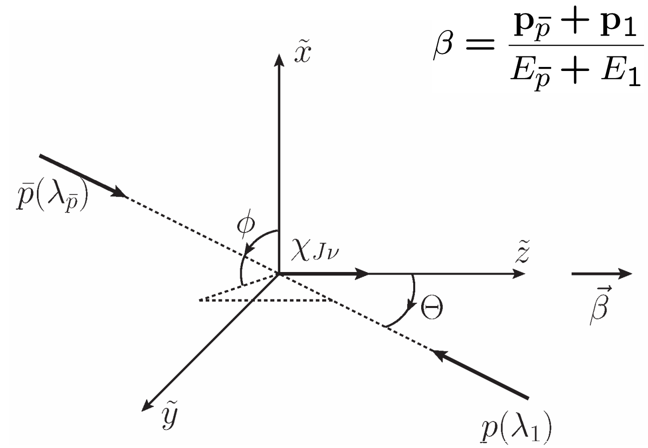
Independent particle model: $$\psi_A(\mathbf{x}_1, \mathbf{x}_2, ..., \mathbf{x}_A) = \prod_{i=1}^{A} \phi_i(\mathbf{x}_i) \,, \qquad \int d^3x_i |\phi_i(\mathbf{x}_i)|^2 = 1$$

$$\sum_{\psi_{A-1}} |M_R(1)|^2 = \frac{|M_{R;\bar p p}(k - p_{\bar p})|^2}{2E_1} \int d^3X f_1(\mathbf{X}, \mathbf{k} - \mathbf{p}_{\bar p})$$

$$f_1(\mathbf{X}, \mathbf{p}) = \int d^3x \phi_1^* \left(\mathbf{X} + \frac{\mathbf{x}}{2}\right) \phi_1 \left(\mathbf{X} - \frac{\mathbf{x}}{2}\right) \mathrm{e}^{i\mathbf{p}\mathbf{x}}$$

- Wigner function (phase space occupation number) of the struck proton

Formation amplitude $\bar{p}(\lambda_{\bar{p}})p(\lambda_1) \to \chi_J(\nu)$ **:**



$$\langle J\nu|B|\Theta\phi, \lambda_{\bar{p}}\lambda_1\rangle = \left(\frac{2J+1}{4\pi}\right)^{1/2} B^J_{\lambda_{\bar{p}}\lambda_1} D^J_{\nu\lambda}(\phi, \Theta, -\phi) \ ,$$

$$\lambda = \lambda_{\bar{p}} - \lambda_1 \ , \qquad \sum_{\lambda_{\bar{p}}\lambda_1} |B^J_{\lambda_{\bar{p}}\lambda_1}|^2 = 1$$

M. Jacob and G.C. Wick, Ann. Phys. 7, 404 (1959);
A.D. Martin et al., PLB 147, 203 (1984);
F.L. Ridener et al., PRD 45, 3173 (1992)

Invariant amplitude: $M_{J\nu;\lambda_{\bar{p}}\lambda_1} = \kappa_J \langle J\nu|B|\Theta\phi, \lambda_{\bar{p}}\lambda_1\rangle \ , \quad \kappa_J = \left(\dfrac{64\pi^2 m_J^2 \Gamma_{\chi_J \to \bar{p}p}}{\sqrt{m_J^2 - 4m^2}}\right)^{1/2} .$

Notations: $\forall J\colon\ B_0/\sqrt{2} \equiv B^J_{++}, \quad B_1 \equiv B^J_{+-}, \quad 2|B_1|^2 + |B_0|^2 = 1.$

$J = 0: \quad |B_0|^2 = 1$

$J = 1: \quad B_0 = 0$ **(charge conjugation invariance)**

$J = 2: \quad |B_0|^2 = 0.13 \pm 0.08$ **- from angular distributions for**

$\bar{p}p \to \chi_{c2} \to J/\Psi\gamma \to e^+e^-\gamma$

M. Ambrogiani et al. (E835), PRD 65, 052002 (2002)

Somewhat different results are obtained in lattice QCD:

$$V(r_{med}) = F_1(r \to \infty, T)$$

$q\bar{q}$ **potential at T=0**

$q\bar{q}$ **color singlet free energy**

Ψ' and χ_c melt at T_c, but J/Ψ may survive above T_c

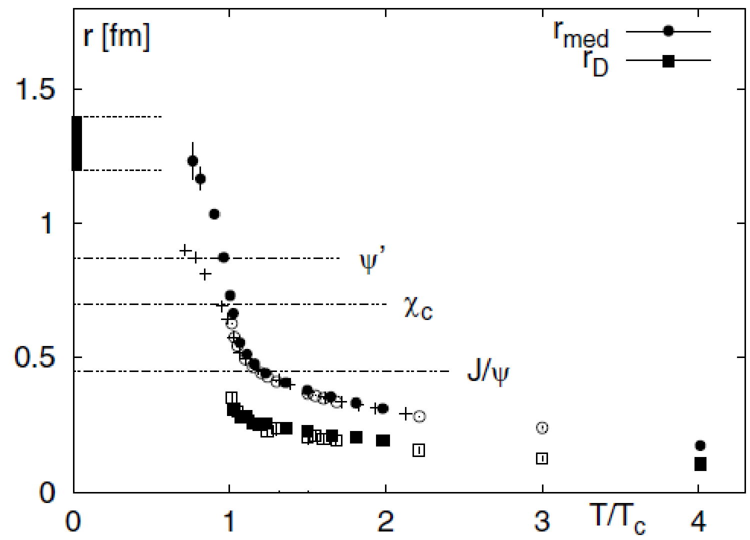


FIG. 11. The screening radius estimated from the inverse Debye mass, $r_D \equiv 1/m_D$ ($N_f = 0$: open squares; $N_f = 2$ filled squares), and the scale r_{med} ($N_f = 0$: open circles; $N_f = 2$: filled circles; $N_f = 3$: crosses) defined in (22) as a function of T/T_c. The dotted line indicates the smallest separation available on lattices with temporal extent $N_\tau = 4$. The horizontal lines give the mean squared charge radii of some charmonium states, J/ψ, χ_c, and $\psi\prime$ (see also [13]) and the band at the left frame shows the distance at which string breaking is expected in 2-flavor QCD at $T = 0$ and quark mass $m_\pi/m_\rho \simeq 0.7$ [37].

Figure from O. Kaczmarek, F. Zantow, PRD 71, 114510 (2005)

Although static potentials at T > 0 are not unambiguosly defined:
O. Philipsen, NPA 820, 33c (2009);
P.W.M. Evans, C.R. Allton, J.-I. Skullerud, PRD 89, 071502 (2014)