# Lattice QCD spectroscopy of heavy mesons

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Marc Wagner

Goethe-Universität Frankfurt am Main, Institut für Theoretische Physik mwagner@th.physik.uni-frankfurt.de

 $http://th.physik.uni-frankfurt.de/{\sim}mwagner/$ 

in collaboration with Abdou Abdel-Rehim, Constantia Alexandrou, Joshua Berlin, Pedro Bicudo, Mattia Dalla Brida, Mario Gravina, Martin Kalinowski, Giannis Koutsou, Annabelle Uenver, Björn Fröhlich Wagenbach, Philipp Wolf

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## Goals, motivation (1)

- Compute the heavy meson spectrum as fully as possible and study the structure of poorly understood candidates using lattice QCD:
  - D mesons (charm-light mesons, D,  $D^*$ ,  $D^{**} = \{D_0^*, D_1, D_2^*\}, ...$ ),
  - $D_s$  mesons (charm-strange mesons,  $D_s$ ,  $D_s^*$ ,  $D_{s0}^*$ ,  $D_{s1}$ ,  $D_{s2}^*$ , ...),
  - charmonium (charm-charm mesons,  $\eta_c$ ,  $J/\psi$ , ...),
  - "strangeonium" (strange-strange mesons,  $a_0(980)$ ,  $f_0(980)$ , ...),
  - static-static-light-light systems (to improve the understanding of possibly existing tetraquarks).
  - Consider parity  $\pm,$  charge conjugation  $\pm,$  radial and orbital excitations.
- Lattice QCD  $\equiv$  from first principles (QCD), (ideally) all systematic errors quantified.

# Goals, motivation (2)

- Why are such lattice investigations important?
  - Some mesons, e.g.  $D_s$ ,  $\eta_c$ ,  $J/\psi$ , have been measured experimentally with high precision and can also be computed on the lattice very accurately  $\rightarrow$  ideal candidates to test QCD by means of lattice QCD.
  - Some mesons are only poorly understood
    - $\rightarrow$  lattice QCD is the perfect tool to clarify the situation:
      - \* Around 20 D,  $D_s$  and charmonium states labeled with "omitted from summary table", i.e. vague experimental signals, experimental contradictions, states not well established, ...
      - \* Example X(3872) ( $\bar{c}c$  state): mass not as expected from quark models; could be a D-D\* molecule, a bound diquark-antidiquark, ...
      - \* Example  $D_{s0}^{*}(2317)$ ,  $D_{s1}(2460)$ : masses significantly lower than expected from quark models, almost equal or even lower than the corresponding D mesons; could be tetraquarks, ...
  - Lattice QCD predictions could give valuable input for future experiments.





Path: \*For Users \*Experiments \*CBM \*Introduction

#### For Users

#### Experiments

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The goal of the research program on nucleus-nucleus collisions at the Facility for Antiproton and Ion Research (FAIR) is the investigation of highly compressed nuclear matter. Matter at very high densities exists in neutron stars and in the core of supernova explosions. In the laboratory, super-dense nuclear matter can be created in the reaction volume of relativistic heavy-ion collisions. The baryon density and the temperature of the fireball reached in such collisions depend on the beam energy. In other words, by varying the beam energy one may, within certain limits, produce different states and phases of strongly interacting matter.

Quark-Gluon Plasma

sQGF

The Compressed Barvonic Matter Experiment



CBM 3D model

In particular, the research program is focused on the investigation of:

- short-lived light vector mesons (e.g. the p-meson) which decay into electron-positron pairs. These penetrating probes carry undistorted information from the dense fireball;
- strange particles, in particular baryons (anti-baryons) containing more than one strange (anti-strange) quark, so called multistrange hyperons (Λ, Ξ, Ω);
- mesons containing charm or anti-charm quarks (D, J/Ψ);
- collective flow of all observed particles. event-by-event fluctuations.

In the CBM experiment, particle multiplicities and phase-space distributions, the collision centrality and the reaction plane will be determined. For example, the study of collective flow of charmonium and multi-strange hyperons will shed light on the production and propagation of these rare probes in dense baryonic matter. The simultaneous measurement of various particles permits the study of cross correlations. This synergy effect opens a new perspective for the experimental investigation of nuclear matter under extreme conditions.

#### Outline

- A brief introduction into lattice QCD hadron spectroscopy.
  - QCD (quantum chromodynamics).
  - Meson spectroscopy.
  - Lattice QCD.
- Some of our ongoing lattice projects:
  - (1) Spectrum of D,  $D_s$ , charmonium.
  - (2) Unstable mesons, tetraquarks, etc.
  - (3) Static-static-light-light tetraquarks.

# **QCD** (quantum chromodynamics)

- Quantum field theory of quarks (six flavors *u*, *d*, *s*, *c*, *t*, *b*, which differ in mass) and gluons.
- Part of the standard model explaining the formation of hadrons (usually mesons =  $q\bar{q}$  and baryons =  $qqq/\bar{q}\bar{q}\bar{q}$ ) and their masses; essential for decays involving hadrons.
- Definition of QCD simple:

$$S = \int d^4x \left( \sum_{f \in \{u,d,s,c,t,b\}} \bar{\psi}^{(f)} \left( \gamma_\mu \left( \partial_\mu - iA_\mu \right) + m^{(f)} \right) \psi^{(f)} + \frac{1}{2g^2} \operatorname{Tr} \left( F_{\mu\nu} F_{\mu\nu} \right) \right)$$
  
$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu - i[A_\mu, A_\nu].$$
 D meson proton

 However, no analytical solutions for low energy QCD observables, e.g. hadron masses, known, because of the absence of any small parameter (i.e. perturbation theory not applicable).



 $\rightarrow$  Solve QCD numerically by means of lattice QCD.

#### Meson spectroscopy

- Proceed as follows:
  - (1) Compute the temporal correlation function C(t) of a mesonic  $q\bar{q}$ operator  $\mathcal{O}$ .
  - (2) Determine the meson mass of interest from the asymptotic exponential decay in time.
- Example: D meson mass  $m_D$  (valence quarks  $\bar{c}$  and u,  $J^P = 0^-$ ),

$$\mathcal{O} \equiv \int d^3 r \, \bar{c}(\mathbf{r}) \gamma_5 u(\mathbf{r})$$
  

$$C(t) \equiv \langle \Omega | \mathcal{O}^{\dagger}(t) \mathcal{O}(0) | \Omega \rangle \overset{t \to \infty}{\propto}$$
  

$$\overset{t \to \infty}{\propto} \exp\left(-m_D t\right).$$





# Lattice QCD (1)

• To compute a temporal correlation function C(t), use the path integral formulation of QCD,

$$C(t) = \langle \Omega | \mathcal{O}^{\dagger}(t) \mathcal{O}(0) | \Omega \rangle = = \frac{1}{Z} \int \left( \prod_{f} D\psi^{(f)} D\bar{\psi}^{(f)} \right) DA_{\mu} \mathcal{O}^{\dagger}(t) \mathcal{O}(0) e^{-S[\psi^{(f)}, \bar{\psi}^{(f)}, A_{\mu}]}$$

- $|\Omega\rangle$ : ground state/vacuum.
- $\mathcal{O}^{\dagger}(t), \mathcal{O}(0)$ : functions of the quark and gluon fields (cf. previous slides).
- $\int (\prod_f D\psi^{(f)} D\overline{\psi}^{(f)}) DA_{\mu}$ : integral over all possible quark and gluon field configurations  $\psi^{(f)}(\mathbf{x}, t)$  and  $A_{\mu}(\mathbf{x}, t)$ .
- $e^{-S[\psi^{(f)}, \bar{\psi}^{(f)}, A_{\mu}]}$ : weight factor containing the QCD action.

# Lattice QCD (2)

- Numerical implementation of the path integral formalism in QCD:
  - Discretize spacetime with sufficiently small lattice spacing  $a\approx 0.05\,{\rm fm}\ldots 0.10\,{\rm fm}$ 
    - $\rightarrow$  "continuum physics".
  - "Make spacetime periodic" with sufficiently large extension  $L \approx 2.0 \text{ fm} \dots 4.0 \text{ fm}$  (4-dimensional torus)

 $\rightarrow\,$  "no finite size effects" .



# Lattice QCD (3)

- Numerical implementation of the path integral formalism in QCD:
  - After discretization the path integral becomes an ordinary multidimensional integral:

$$\int D\psi \, D\bar{\psi} \, DA \, \dots \quad \rightarrow \quad \prod_{x_{\mu}} \left( \int d\psi(x_{\mu}) \, d\bar{\psi}(x_{\mu}) \, dU(x_{\mu}) \right) \, \dots$$

- Typical present-day dimensionality of a discretized QCD path integral: \*  $x_{\mu}$ :  $32^4 \approx 10^6$  lattice sites.
  - \*  $\psi = \psi_A^{a,(f)}$ : 24 quark degrees of freedom for every flavor (×2 particle/antiparticle, ×3 color, ×4 spin), 2 flavors.
  - \*  $U = U^{ab}_{\mu}$ : 32 gluon degrees of freedom (×8 color, ×4 spin).
  - \* In total:  $32^4 \times (2 \times 24 + 32) \approx 83 \times 10^6$  dimensional integral.
  - $\rightarrow$  standard approaches for numerical integration not applicable
  - $\rightarrow$  sophisticated algorithms mandatory (stochastic integration techniques, so-called Monte-Carlo algorithms).

### Spectrum of D, $D_s$ , charmonium (1)

• In the following masses for D mesons,  $D_s$  mesons and charmonium states using quark-antiquark hadron creation operators, e.g. for D

$$\mathcal{O} \equiv \int d^3x \, \bar{c}(\mathbf{x}) \gamma_5 u(\mathbf{x}).$$

- Accurate QCD results only for rather stable mesons, which are predominantly quark-antiquark states.
- Unstable mesons (e.g.  $D_0^*$ ,  $D_1(2430)$ ) or mesons, which might not predominantly be quark-antiquark states (e.g. the tetraquark candidates  $D_{s0}^*$ ,  $D_{s1}$ ), require more sophisticated techniques and computations:
  - \* The correlation functions computed by means of lattice QCD provide the low-lying energy eigenvalues of the QCD Hamiltonian, which correspond to the masses of stable hadronic states (single or multi-particle).
  - \* In lattice QCD the hadron creation operators may not be too different from the state, which is investigated.

#### Spectrum of D, $D_s$ , charmonium (2)

- First preliminary results of a large scale project.
  - [M. Kalinowski and M.W. [ETM Collaboration], PoS Confinement10, 303 (2012) [arXiv:1212.0403]]
  - [M. Kalinowski and M.W. [ETM Collaboration], Acta Phys. Polon. Supp. 6, 991 (2013) [arXiv:1304.7974]]
  - [M. Kalinowski and M.W. [ETM Collaboration], PoS LATTICE2013 [arXiv:1310.5513]]
- D,  $D_s$ , charmonium states computed (in the plots from left to right):

$$-J^{P} = 0^{-}: D, D_{s}, \eta_{c}.$$
  

$$-J^{P} = 0^{+}: D_{0}^{*}, D_{s0}^{*}, \chi_{c0}.$$
  

$$-J^{P} = 1^{-}: D^{*}, D_{s}^{*}, J/\Psi.$$
  

$$-J^{P} = 1^{+}: D_{1}(2430), D_{1}(2420), D_{s1}, D_{s1}, h_{c}, \chi_{c1}$$



### Spectrum of D, $D_s$ , charmonium (3)

- Experimental meson masses: gray points.
- Different lattice discretizations (circles and crosses) indicate that discretization errors are  $\lesssim 2\%$  (will be removed in the near future).
- Different values of the light u/d quark mass (corresponding to  $m_{\pi} = 285 \text{ MeV}, 325 \text{ MeV}, 457 \text{ MeV}$ :
  - Some states are quite stable (solid trustworthy results), ...
  - ... others exhibit a clear dependence on the light quark mass (presumably unstable hadrons, mesonic molecules, tetraquarks containing light quarks; further investigations necessary and ongoing).



#### Unstable mesons, tetraquarks, etc. (1)

- Unstable mesons, e.g. D<sup>\*</sup><sub>0</sub> → D + π, ...
   For a proper treatment of such states, i.e. for a computation of their resonance mass and width using lattice QCD, one has to employ further hadron creation operators of two-meson structure.
- To study  $D_0^*$  ( $J^P = 0^+$ ), in addition to a quark-antiquark operator

$$\mathcal{O}_{D_0^*}^{q\bar{q}} \quad = \quad \int d^3x \, \bar{c}(\mathbf{x}) u(\mathbf{x})$$

also a two-meson operator

$$\mathcal{O}_{D_0^*}^{\mathsf{two-meson}} = \underbrace{\left(\int d^3 x \, \bar{c}(\mathbf{x}) \gamma_5 l(\mathbf{x})\right)}_{\equiv D} \underbrace{\left(\int d^3 y \, \bar{l}(\mathbf{y}) \gamma_5 l(\mathbf{y})\right)}_{\equiv \pi} \overset{\pi}{\longrightarrow} D$$

(l = u, d) is required (both generate the same quantum numbers  $J^P = 0^+$ , when applied to the vacuum).

#### Unstable mesons, tetraquarks, etc. (2)

• E.g.  $D_{s0}^*$  and  $D_{s1}$  do not seem to be ordinary quark-antiquark states ... could be four quark states, for example mesonic molecules (*K*-*D*, ...), diquark-antidiquark states (tetraquarks), ...?

To investigate the structure of such mesons using lattice QCD one has to employ further hadron creation operators of mesonic molecule or of diquark-antidiquark structure.

### Unstable mesons, tetraquarks, etc. (3)

• To study  $D^*_{s0}$   $(J^P=0^+)$ , in addition to the quark-antiquark operator

$$\mathcal{O}_{D_{s0}^*}^{q\bar{q}} = \int d^3x \, \bar{c}(\mathbf{x}) s(\mathbf{x})$$

also four-quark operators

$$\mathcal{O}_{D_{0}^{*}}^{\text{mesonic molecule}} = \int d^{3}x \underbrace{\left(\bar{c}(\mathbf{x})\gamma_{5}\boldsymbol{l}(\mathbf{x})\right)}_{\equiv D} \underbrace{\left(\bar{l}(\mathbf{x})\gamma_{5}s(\mathbf{x})\right)}_{\equiv K}$$
$$\mathcal{O}_{D_{0}^{*}}^{\text{diquark}} = \int d^{3}x \left(\epsilon^{abc}\bar{c}^{b}(\mathbf{x})C\gamma_{5}\bar{\boldsymbol{l}}^{c,T}(\mathbf{x})\right) \left(\epsilon^{ade}\boldsymbol{l}^{d,T}(\mathbf{x})C\gamma_{5}s^{e}(\mathbf{x}$$

diquark

(l = u, d) are required (both generate the same quantum numbers  $J^P = 0^+$ , when applied to the vacuum).

• Further examples of heavy mesons, which are tetraquark candidates: charmonium states X(3872),  $Z(4430)^{\pm}$ ,  $Z(4050)^{\pm}$ ,  $Z(4250)^{\pm}$ , ...

### Unstable mesons, tetraquarks, etc. (4)

- When several operators are used one has to compute correlation matrices, not only a single correlation function.
- Many contributions (similar to Feynman diagrams, but with non-perturbative propagators) need to be computed, which require different techniques.
   → Solid results require years of collaborative work.
- At the moment: preliminary results for  $a_0(980)$  (" $a_0(980)$  is not a rather stable four-quark state.").

 $[\mathsf{C}. \ \mathsf{Alexandrou} \ \mathit{et al.} \ [\mathsf{ETM} \ \mathsf{Collaboration}], \ \mathsf{JHEP} \ \mathbf{1304}, \ \mathbf{137} \ (\mathbf{2013}) \ [\mathsf{arXiv:} \mathbf{1212.1418}]]$ 

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#### Unstable mesons, tetraquarks, etc. (5)



# Static-static-light-light tetraquarks (1)

- Study possibly existing  $QQ\bar{q}\bar{q}$  (heavy-heavy-light-light) tetraquark states:
  - Use the static approximation for the heavy quarks QQ (reduces the necessary computation time significantly).
  - Most appropriate for  $QQ \equiv bb$ .
  - Could also yield information about  $QQ \equiv cc$ .
- Proceed in two steps:
  - (1) Compute the potential of two heavy quarks QQ in the background of two light antiquarks  $\bar{q}\bar{q}$  by means of lattice QCD.

 $\rightarrow$  Many different channels/quantum numbers.

[M.W., PoS LATTICE **2010**, 162 (2010) [arXiv:1008.1538]] [M.W., Acta Phys. Polon. Supp. **4**, 747 (2011) [arXiv:1103.5147]]

(2) Solve the non-relativistic Schrödinger equation for the relative coordinate of the heavy quarks QQ.



#### Static-static-light-light tetraquarks (2)

- Clear indication for a bound state for  $QQ \equiv bb$  in a specific channel:
  - Quantum numbers:  $I(J^P) = 0(0^+), 0(1^+)$  (degeneracy with respect to the heavy quark spin).
  - Binding energy:  $E \approx -50 \,\mathrm{MeV}$ .
  - [P. Bicudo, M.W., Phys. Rev. D 87, 114511 (2013) [arXiv:1209.6274]]
- No four-quark binding in other channels.



# Static-static-light-light tetraquarks (3)

- Ongoing work in the same direction:
  - Extend these investigations to the experimentally more interesting case of  $Q\bar{Q}$  (instead of QQ):
    - \* More difficult than QQ: the light quarks  $q\bar{q}$  can annihilate, one has to distinguish a  $B-\bar{B}/D-\bar{D}$  from a bottonium/charmonium- $\pi$  state, ...
  - Relate the static-static-light-light case to the previously discussed case of four quarks of finite mass:
    - \* First interesting insights: pseudoscalar mesons are not sufficient to generate attractive hadronic forces, a suitable linear combination including also vector mesons is needed.
  - Use a similar approach to study the existence and spectrum of hybrid mesons.

#### Conclusions

- The lattice results for mesons and tetraquark candidates presented are:
  - Preliminary,
    - $\rightarrow$  certain systematic errors need to be studied and quantified, e.g. lattice discretization errors, unphysically heavy u/d quark masses,
    - $\rightarrow$  statistical errors need to be reduced.
  - Promising,
    - $\rightarrow$  contact to experimental results established (e.g. D,  $D_s$ , charmonium spectrum),
    - $\rightarrow$  first statements about states, which are presently not well understood (tetraquark candidates,  $a_0(980)$ ,  $\kappa$ , ...).
- Long-term goal: meson spectroscopy/structure from first principles (QCD), where all sources of systematic errors are investigated and quantified, which is relevant and important in the context of FAIR.