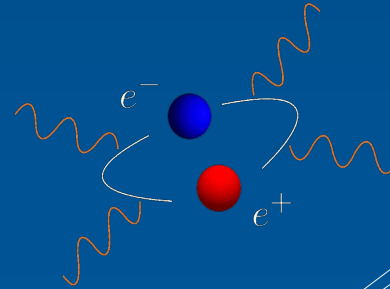


Quantitative Theoretical Predictions of the Signals in Quantum Vacuum Experiments

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- 1 Introduction
- 2 Photonic Quantum Vacuum Signals
- 3 High-Intensity **Pump** + X-ray **Probe** (→ “Vacuum Birefringence”)
- 4 High-Intensity **Pump** + High-Intensity **Probe**
- 5 Conclusions

1 Introduction

Classical electrodynamics (ED): $[\bar{F}^{\mu\nu} = \partial^\mu \bar{A}^\nu - \partial^\nu \bar{A}^\mu; \bar{F}^{0i} = E_i, \bar{F}^{ij} = \epsilon_{ijk} B_k]$

→ accurate description of **macroscopic electromagnetic fields** $\bar{F}^{\mu\nu}$ in laboratory

$$\mathcal{L}_{\text{ED}} = -\frac{1}{4} \bar{F}_{\mu\nu} \bar{F}^{\mu\nu}$$

[Maxwell: Philos. Trans. R. Soc. **155** (1865)]



$$\mathcal{L} = \mathcal{L}(\bar{F}^{\mu\nu}) \xrightarrow{\text{EoM}} \partial_\mu \frac{\partial \mathcal{L}}{\partial \bar{F}_{\mu\nu}} = 0$$

$$\partial_\mu \bar{F}^{\mu\nu} = 0 \simeq \text{Maxwell equations}$$

→ linear equations of motion (EoM) → superposition principle holds

↔ our world is quantum → there should be corrections (depending on $\hbar$)

1 Introduction

Quantum electrodynamics (QED):

$[\hbar = c = 1; \text{quantum fields } A^\mu, \bar{\psi}, \psi]$

$\hat{=}$ quantum theory of light and matter

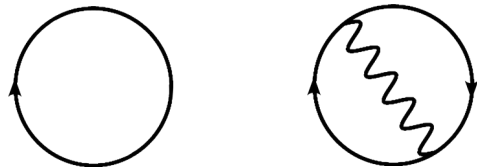
$$\mathcal{L}_{\text{QED}} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \bar{\psi}(i\gamma^\mu\partial_\mu - m)\psi + \bar{\psi}\gamma_\mu\psi eA^\mu$$

apply **electromagnetic field**

$$+ \bar{\psi}\gamma_\mu\psi e\bar{A}^\mu$$



$\rightarrow$ notion of the “quantum vacuum” $\hat{=}$ diagrams without external lines



$\rightarrow$ “know” of m and e

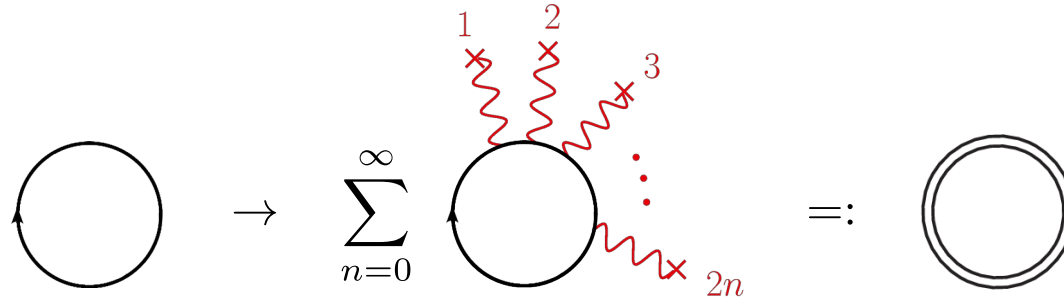
$$\alpha = \frac{e^2}{4\pi} \simeq \frac{1}{137}$$

1 Introduction

Heisenberg-Euler (HE) theory:

[Heisenberg, Euler: Z. Phys. **98** (1936)]

→ vacuum diagrams are “dressed” by $e\bar{F}^{\mu\nu}$



$$\rightarrow \mathcal{L}_{\text{HE}} = -\frac{1}{4} \bar{F}_{\mu\nu} \bar{F}^{\mu\nu} + \text{[double circle diagram]} + \mathcal{O}(\alpha)$$

[Heisenberg, Euler: Z. Phys. **98** (1936)],
 [Weisskopf: Mat.-fys. Medd. **14** (1936)],
 [Schwinger: Phys. Rev. **82** (1951)]

1 Introduction

Weak-field expansion:

[Heisenberg, Euler: Z. Phys. **98** (1936)]

$$\mathcal{L}_{\text{HE}} = -\frac{1}{4} \bar{F}_{\mu\nu} \bar{F}^{\mu\nu} + \frac{m^4}{5760\pi^2} \left(\frac{e}{m^2}\right)^4 \left[\mathbf{a} (\bar{F}_{\mu\nu} \bar{F}^{\mu\nu})^2 + \mathbf{b} (\bar{F}_{\mu\nu} {}^* \bar{F}^{\mu\nu})^2 \right] + \dots$$

→ applicable for $\{E/E_{\text{cr}}, B/B_{\text{cr}}\} \ll 1$ with $E_{\text{cr}} \simeq 1.3 \times 10^{18} \text{ V/m}$, $B_{\text{cr}} \simeq 4 \times 10^9 \text{ T}$

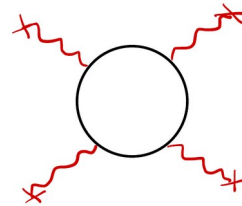
→ QED up to two loops yields

$$\mathbf{a} = 4 \left(1 + \frac{40}{9} \frac{\alpha}{\pi} + \dots \right)$$

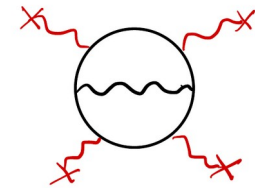
$$\mathbf{b} = 7 \left(1 + \frac{1315}{252} \frac{\alpha}{\pi} + \dots \right)$$



corrections on 1% level



[Euler, Kockel: Naturwiss. **23** (1935)],
[Euler: Ann. Phys. **418** (1936)]



[Ritus: JETP **42** (1975)]

2 Photonic Quantum Vacuum Signals

Weak-field expansion:

[Heisenberg, Euler: Z. Phys. **98** (1936)]

$$\mathcal{L}_{\text{HE}} = -\frac{1}{4}\bar{F}_{\mu\nu}\bar{F}^{\mu\nu} + \frac{m^4}{5760\pi^2}\left(\frac{e}{m^2}\right)^4 \left[\mathbf{a} (\bar{F}_{\mu\nu}\bar{F}^{\mu\nu})^2 + \mathbf{b} (\bar{F}_{\mu\nu}\star\bar{F}^{\mu\nu})^2 \right] + \dots$$

$$\xrightarrow{\text{EoM}} \quad \partial_\mu \bar{F}^{\mu\nu} = \underbrace{\frac{e^4}{720\pi^2 m^4} \partial_\mu \left[\mathbf{a} (F_{\alpha\beta}\bar{F}^{\alpha\beta}) \bar{F}^{\mu\nu} + \mathbf{b} (\bar{F}_{\alpha\beta}\star\bar{F}^{\alpha\beta}) \star\bar{F}^{\mu\nu} \right]}_{:= j^\nu(x|\bar{F}) = \mathcal{O}(\hbar)} + \dots$$

$$\downarrow \quad \bar{F}^{\mu\nu} \rightarrow \bar{F}^{\mu\nu} + f_{\text{signal}}^{\mu\nu} \quad \text{with} \quad \|f_{\text{signal}}^{\mu\nu}\| \ll \|\bar{F}^{\mu\nu}\|$$

→ quantum vacuum signals $\partial_\mu f_{\text{signal}}^{\mu\nu} \simeq j^\nu(x|\bar{F})$ [Mckenna, Platzman: Phys. Rev. D **129** (1963)], ...

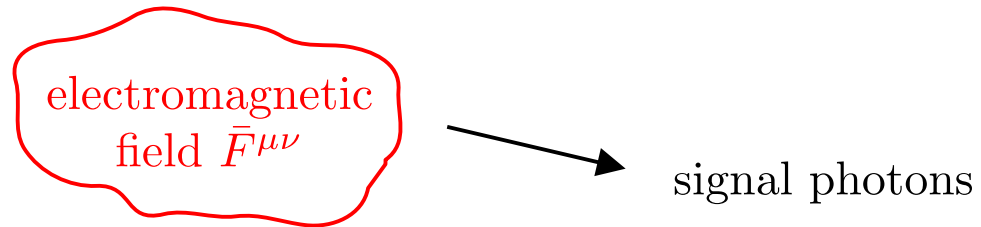
2 Photonic Quantum Vacuum Signals

Look for signal photons:

[FK, Shaisultanov: Phys. Rev. D **91** (2015)]

$$\partial_\mu f_{\text{signal}}^{\mu\nu} \simeq j^\nu(x|\bar{\mathbf{F}})$$

- energy $|\vec{k}|$
- emission direction $\vec{k}/|\vec{k}|$
- polarization p (vector $\epsilon_{(p)}^\mu(\vec{k})$)



$$\text{amplitude } \mathcal{S}_p(\vec{k}) \sim j_\mu(\vec{k}|\bar{\mathbf{F}}) \epsilon_{(p)}^\mu(\vec{k})$$

$$\rightarrow \text{differential signal photon number } d^3N_p = \frac{d^3k}{(2\pi)^3} |\mathcal{S}_p(\vec{k})|^2$$

2 Photonic Quantum Vacuum Signals

Laser beam collisions:

→ consider 2 beam scenario electromagnetic field → 1 pump + 1 probe field

→ quasi-elastic scattering signal

$$N_{\parallel,\perp} \sim c_{\parallel,\perp} (1 - \cos \vartheta_{\text{coll}})^4 \left(\frac{I_{0,\text{pump}}}{I_S} \frac{\omega_{\text{probe}}}{mc^2} \right)^2 N_{\text{probe}} \quad \text{with} \quad I_S = \left(\frac{m^2}{e} \right)^2 \simeq 10^{29} \frac{\text{W}}{\text{cm}^2}$$

$$c_{\parallel} = (\mathbf{a} + \mathbf{b})^2, \quad c_{\perp} = (\mathbf{a} - \mathbf{b})^2$$

Signal-to-background separation?

→ measure signal scattered in $\perp$ -polarized mode!

[Aleksandrov, Anselm, Moskalev: JETP **62** (1985)],
[Heinzl, *et al.*: Opt. Commun. **267** (2006)],
[Di Piazza, *et al.*: Phys. Rev. Lett. **97** (2006)], ...

3 High-Intensity Pump + X-ray Probe

Very hard to detect $\perp$ -signal in XFEL + high-intensity laser collisions

$\leftrightarrow$ polarization purity $\mathcal{P} = \mathcal{O}(10^{-11})$ @ $\omega_{\text{probe}} = \mathcal{O}(10)$ keV achieved experimentally

$$\rightarrow N_{\text{bgr},\perp} = \mathcal{P} N_{\text{probe}}$$

[Schulze, *et al.*: Phys. Rev. Res. 4 (2022)]

Various reasons (including):

- only lasers providing $\lesssim 0.5$ petawatt installed at XFELs
- standard Be lenses deteriorate polarization purity
- losses in optimal elements, bandwidth mismatch XFEL $\leftrightarrow$ polarizers, etc.

$\rightarrow$ signals are strongly background dominated

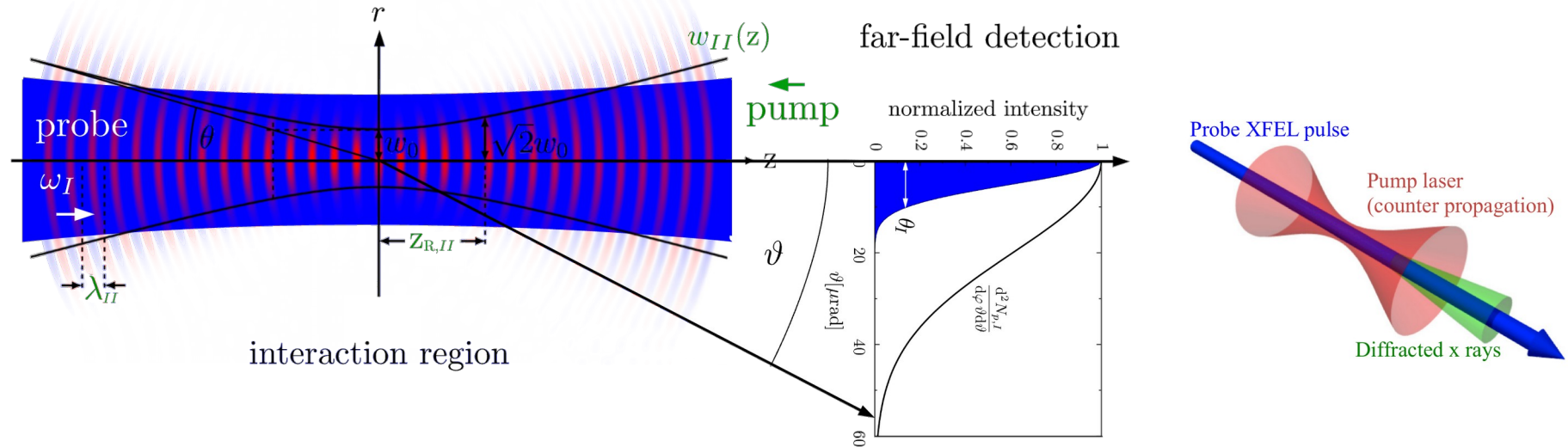
[Mosman, FK: Phys. Rev. D 104 (2021)]

3 High-Intensity Pump + X-ray Probe

Signal enhancement strategies in single **high-intensity laser** + **XFEL** collisions?

→ diffraction assisted measurement:

[FK, Gies, Reuter, Zepf: Phys. Rev. D **92** (2015)],
[FK, Sundqvist: Phys. Rev. D **94** (2016)],
[FK: Phys. Rev. D **98** (2018)]



→ attempts to measure $N_{||}$ with this schema at SACLA, Japan

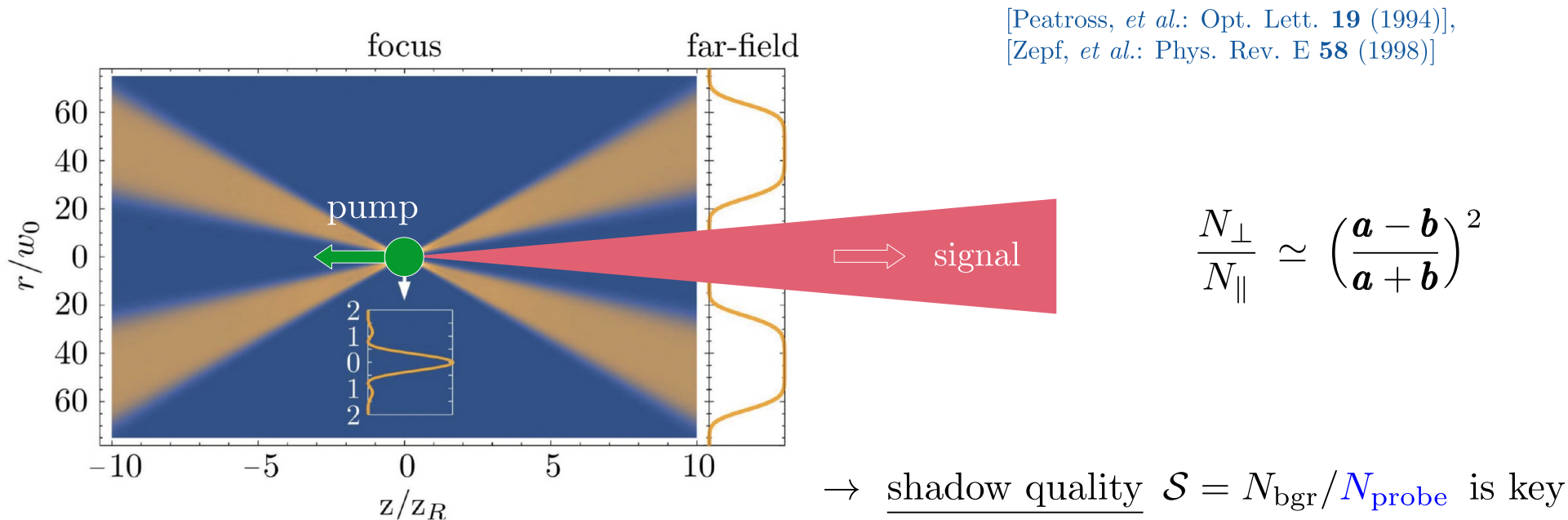
[Inada, *et al.*: Appl. Sci. **7** (2017)],
[Seino, *et al.*: PTEP **7** (2020)]

3 High-Intensity Pump + X-ray Probe

Dark-field concept:

[FK, Mosman: Phys. Rev. D **101** (2020)],
[FK, Ullmann, Mosman, Zepf: Phys. Rev. Lett. **129** (2022)]

→ modify the **probe field** to make the forward signal experimentally accessible:



3 High-Intensity Pump + X-ray Probe

ReLaX

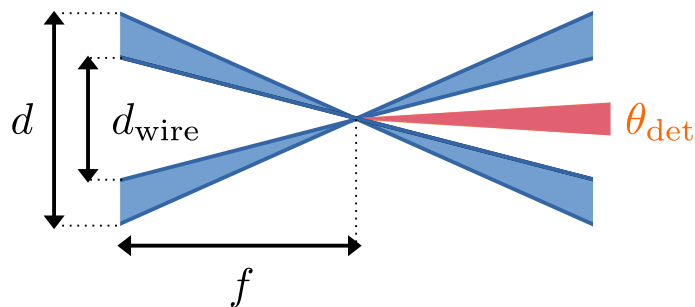


- $W = 4.8 \text{ J}$
- $\tau_{\text{FWHM}} = 30 \text{ fs}$
- $w_{\text{FWHM}} = 1.3 \text{ }\mu\text{m}$

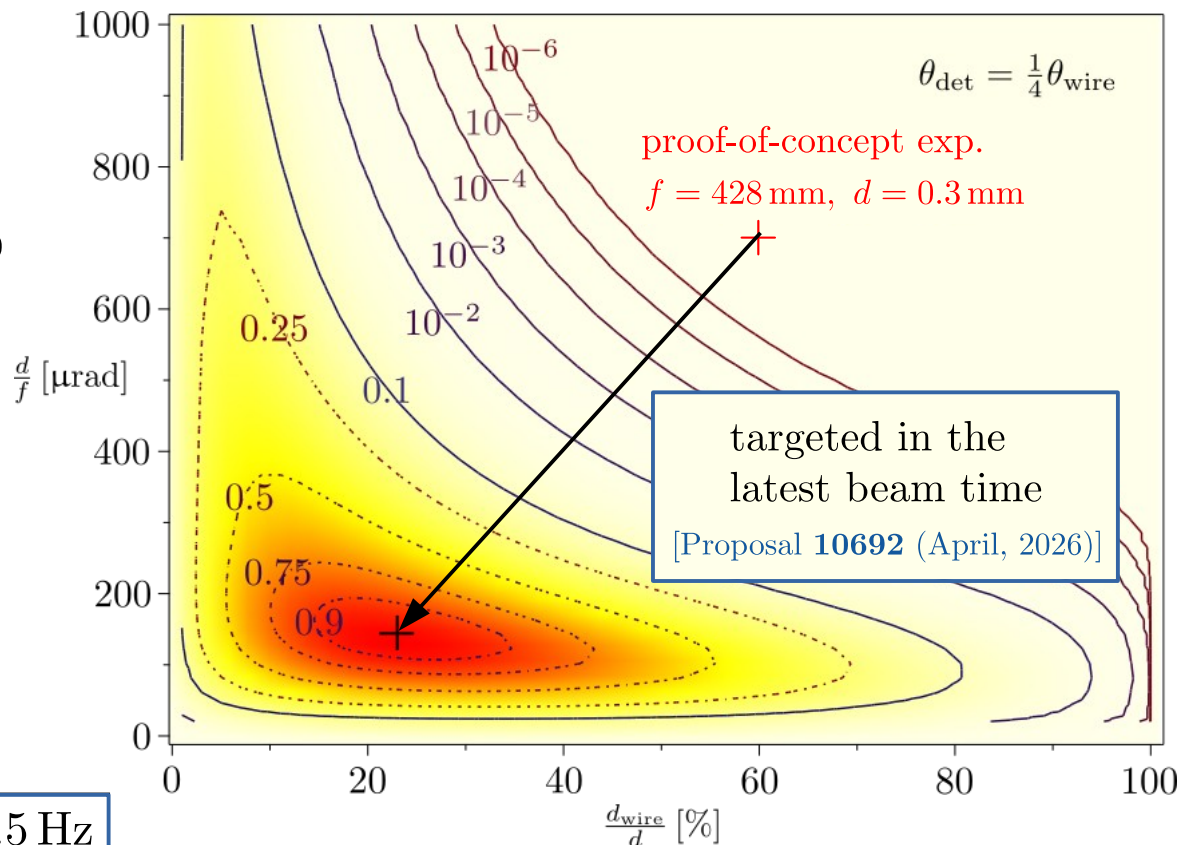
EuXFEL



- $N_{\text{probe}} = 7.57 \times 10^{10}$
- $\omega = 8766 \text{ eV}$
- $\tau_{\text{FWHM}} = 25 \text{ fs}$



$$N_{\text{sig},\perp}^{\text{max}} = 1/h, \quad N_{\text{sig},\parallel}^{\text{max}} = 10/h \quad @ \text{ 0.5 Hz}$$



3 High-Intensity Pump + X-ray Probe

Number of successful shots needed to achieve a significance of $\#\sigma$:

$$n_{\text{shot}} N_{\text{sig}} > \#^2 \frac{1}{2} \frac{1}{(1 + 1/\mathcal{R}) \ln(1 + \mathcal{R}) - 1} = \#^2 \frac{N_{\text{bgr}}}{N_{\text{sig}}} \left(1 + \mathcal{O}(\mathcal{R})\right)$$

[Cowan, Cranmer, Gross, Vitells: Eur. Phys. J. C **71** (2011)]

with signal-to-background ratio $\mathcal{R} = N_{\text{sig}}/N_{\text{bgr}}$

→ require this criterion to be fulfilled for the $\perp$ -signal component

→ for $N_{\text{sig},\parallel} \ll N_{\text{bgr},\parallel}$ the uncertainty in $\mathbf{a}$ and $\mathbf{b}$ is dominated by $N_{\text{bgr},\parallel}$

$$\frac{\Delta \mathbf{a}}{\mathbf{a}} \sim \frac{\Delta \mathbf{b}}{\mathbf{b}} \sim \frac{N_{\text{sig},\parallel}}{\sqrt{N_{\text{bgr},\parallel}}}$$

[Šmíd, *et al.*: Phys. Rev A **112** (2025)]

3 High-Intensity Pump + X-ray Probe

ReLaX



- $W = 4.8 \text{ J}$
- $\tau_{\text{FWHM}} = 30 \text{ fs}$
- $w_{\text{FWHM}} = 1.3 \mu\text{m}$

$$\mathcal{S}_{\parallel} = \mathcal{S} \simeq 6.8 \times 10^{-12}$$

$$\mathcal{S}_{\perp} = \mathcal{SP} \simeq 3.9 \times 10^{-16}$$

[Šmíd, *et al.*: Phys. Rev A **112** (2025)]

EuXFEL



- $N_{\text{probe}} = 7.57 \times 10^{10}$
- $\omega = 8766 \text{ eV}$
- $\tau_{\text{FWHM}} = 25 \text{ fs}$

$$\rightarrow \parallel \text{-mode} \quad N_{\text{sig},\parallel} \simeq 5.7 \times 10^{-3} \quad \text{and} \quad N_{\text{bgr},\parallel} \simeq 0.64$$

$$\rightarrow \perp \text{-mode} \quad N_{\text{sig},\perp} \simeq 4.2 \times 10^{-4} \quad \text{and} \quad N_{\text{bgr},\perp} \simeq 3.5 \times 10^{-5}$$

$$\rightarrow n_{\text{shot}} > 1.7 \times 10^4 \xrightarrow{\text{@ } 0.5 \text{ Hz}} 10 \text{ hours needed for } 5\sigma \text{ confirmation}$$

3 High-Intensity Pump + X-ray Probe

ReLaX



- $W = 4.0 \text{ J}$
- $\tau_{\text{FWHM}} = 30 \text{ fs}$
- $w_{\text{FWHM}} = 1.5 \times 1.3 \mu\text{m}$

$$\mathcal{S}_{\parallel} = \mathcal{S} \simeq 3 \times 10^{-8}$$

$$\mathcal{S}_{\perp} = \mathcal{SP} \simeq 3 \times 10^{-12}$$

[FK, Ullmann, Mosman, Zepf: Phys. Rev. Lett. **129** (2022)]

EuXFEL



- $N_{\text{probe}} = 1 \times 10^{10}$
- $\omega = 8766 \text{ eV}$
- $\tau_{\text{FWHM}} = 25 \text{ fs}$

$\leftrightarrow$ assume **worse actual performance**
and $n_{\text{shot}} = 100$ successful shots

$$\rightarrow \parallel\text{-mode} \quad N_{\text{sig},\parallel} \simeq 1.2 \times 10^{-4} \quad \text{and} \quad N_{\text{bgr},\parallel} \simeq 380$$

$$\rightarrow \perp\text{-mode} \quad N_{\text{sig},\perp} \simeq 9.1 \times 10^{-6} \quad \text{and} \quad N_{\text{bgr},\perp} \simeq 3.8 \times 10^{-2}$$

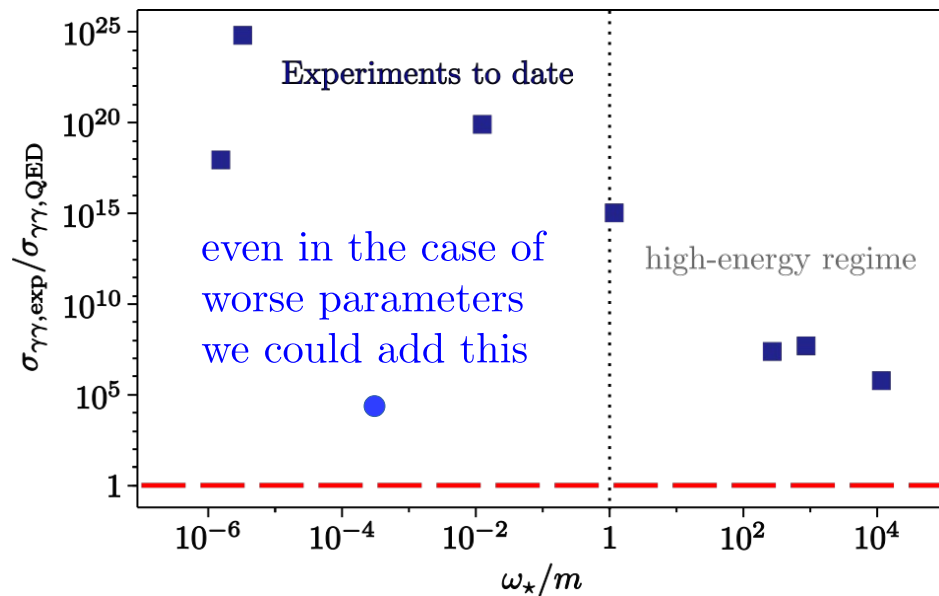
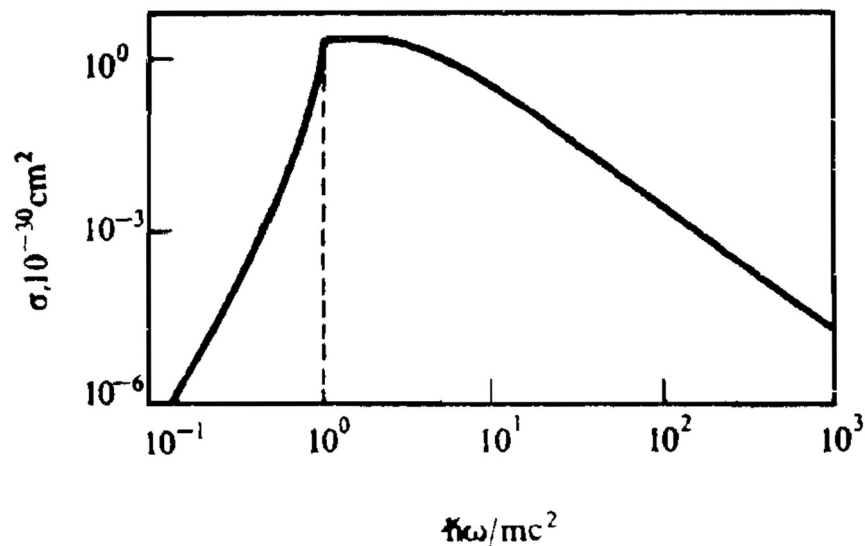
$$\rightarrow 5\sigma\text{-bounds} \quad N_{\text{sig},\parallel}^{\text{excl}}/N_{\text{sig},\parallel} > 8.2 \times 10^4, \quad N_{\text{sig},\perp}^{\text{excl}}/N_{\text{sig},\perp} > 1.5 \times 10^4$$

3 High-Intensity Pump + X-ray Probe

Bounds on elastic photon-photon scattering:

$$\leftrightarrow N_{\text{sig}} \sim \sigma_{\gamma\gamma}$$

[Berestetskii, Lifshitz, Pitaevskii: QED (1982)]

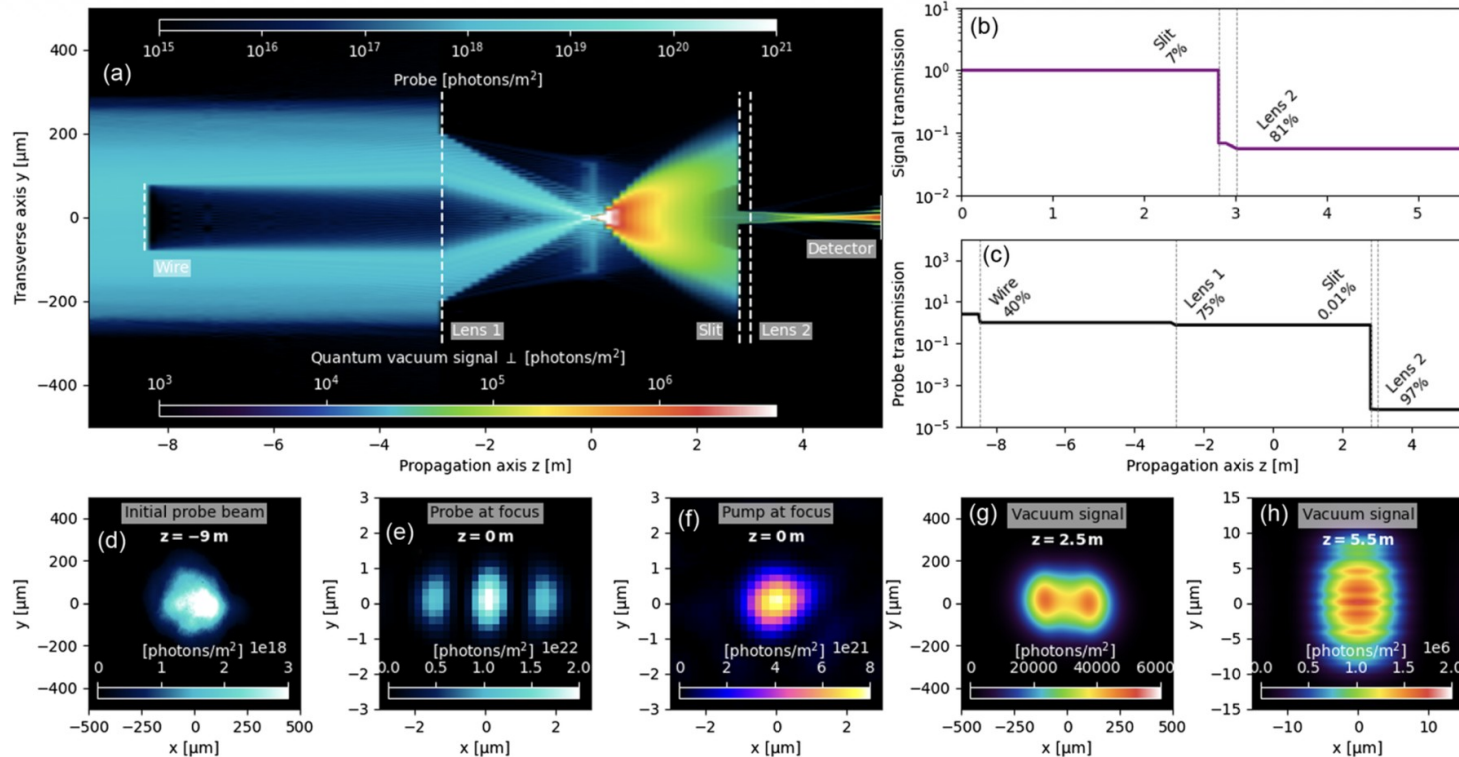


→ we are at a center-of-mass photon energy of $\omega_* = 116 \text{ eV}$

3 High-Intensity Pump + X-ray Probe

Towards consistent numerical simulations:

[Matheron, Šmíd, Zepf, FK: Phys. Rev. D **113** (2026)]



4 High-Intensity Pump + High-Intensity Probe

Towards an all-optical implementation of the dark-field approach

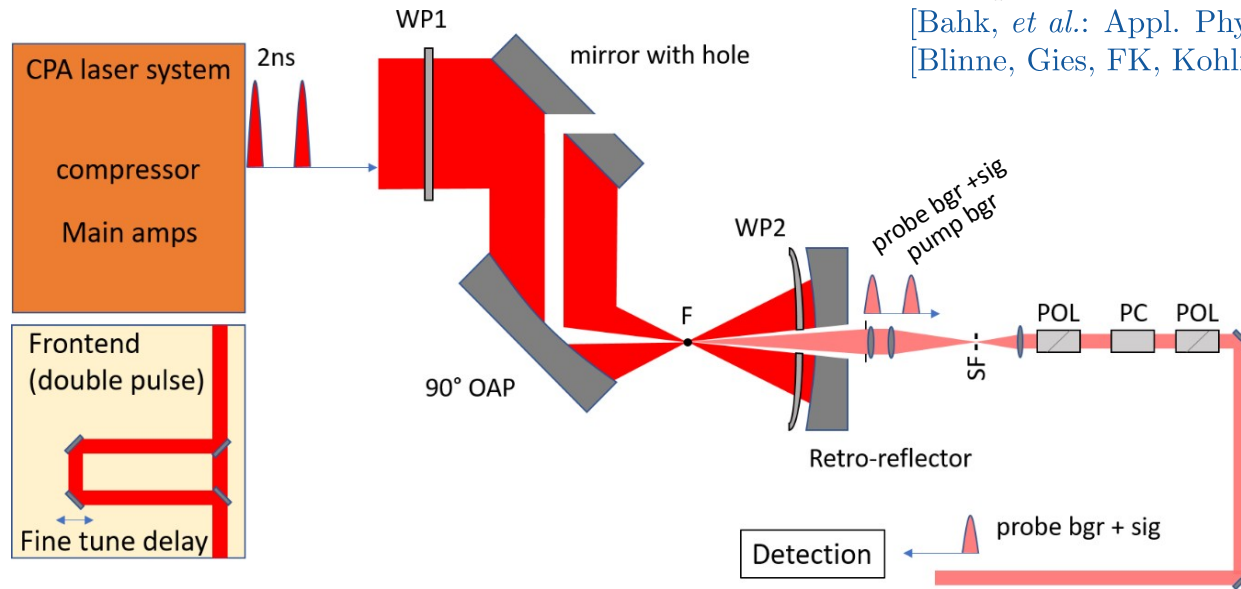
→ schematic layout of envisioned setup:

[Schütze, Doyle, Schreiber, Zepf, FK: Phys. Rev. D **109** (2024)]

Laser pulse model & field evolution:

[Bahk, *et al.*: Appl. Phys. B **80** (2005)],

[Blinne, Gies, FK, Kohlfürst, Zepf: Phys. Rev. D **99** (2019)]



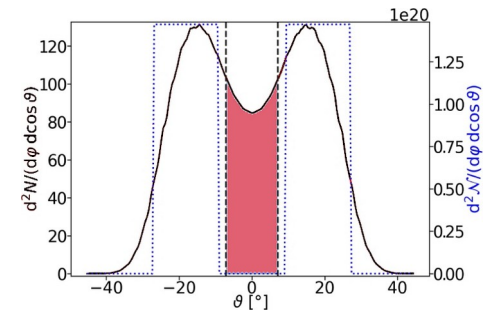
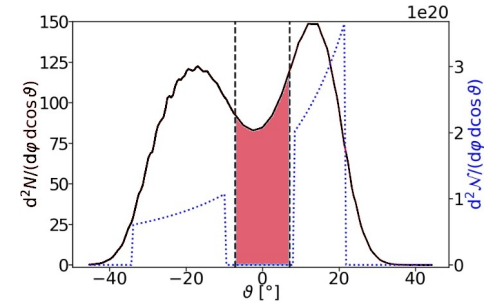
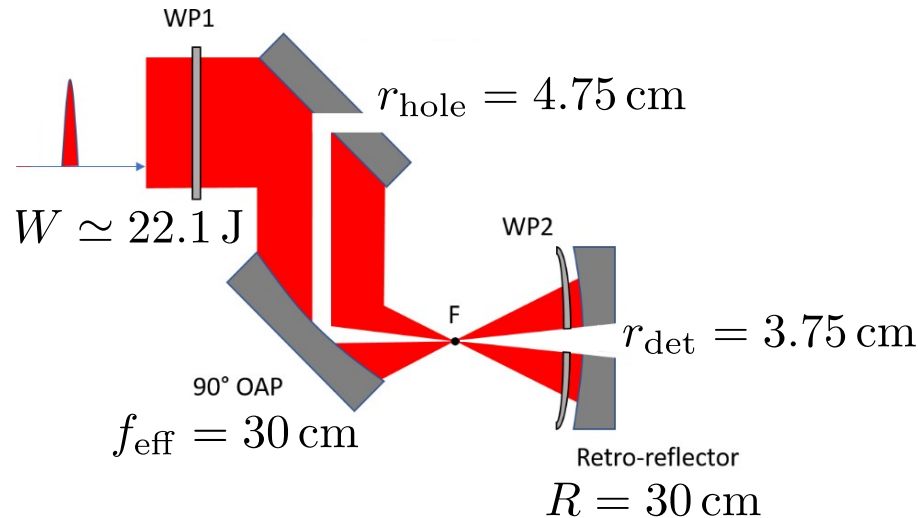
4 High-Intensity Pump + High-Intensity Probe

Predictions for *quantum vacuum experiment* (ideal shots):

→ laser pulse parameters (ATLAS 3000 @ CALA):



- $W_0 = 25 \text{ J}$
- $\lambda = 800 \text{ nm}$
- $\tau_{\text{FWHM}} = 30 \text{ fs}$
- $r_{\text{beam}} = 14 \text{ cm}$



→ $N_{\text{sig, total}}^{\text{max}} \simeq 4.3/\text{shot}$, $N_{\text{sig, } \perp}^{\text{max}} \simeq 0.2/\text{shot}$

5 Conclusions

In this talk, I have

- reminded you of nonlinear quantum vacuum corrections to classical Maxwell theory
- pointed out that measuring quantum vacuum signals is hard and needs new ideas
- emphasized that quantitative predictions require a detailed modeling of the experiment
- shown that progress is only possible in close collaboration of theory and experiment

Thank you very much for your attention!