

Parton orbital dynamics from exclusive vector meson production

ExtreMe Matter Institute EMMI

EMMI Rapid Reaction Task Force

Impact of Vector Mesons on the Studies
of the 3D Structure of the Nucleon

June 15-18, 2026
CERN, Switzerland



Shohini Bhattacharya

University of Connecticut

June 17, 2026

Based on: 2601.17506 (26)

In Collaboration with:

David DeAngelo (UConn)

Lei Yang (Shandong U)

Duxin Zheng (Shandong U)

Jian Zhou (Shandong U)



Outline

- Generalized **TMDs** (GTMDs) & connection to spin physics
- Observable(s) for gluon **OAM** & **spin-orbit correlations**
- Opportunities (My wish list)
- Summary



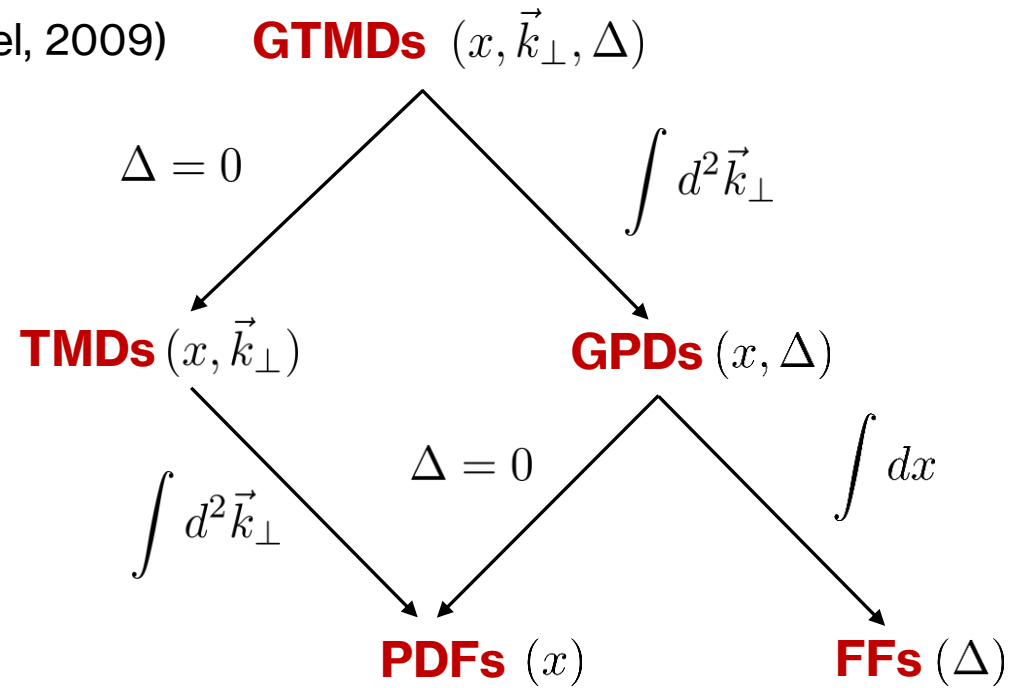
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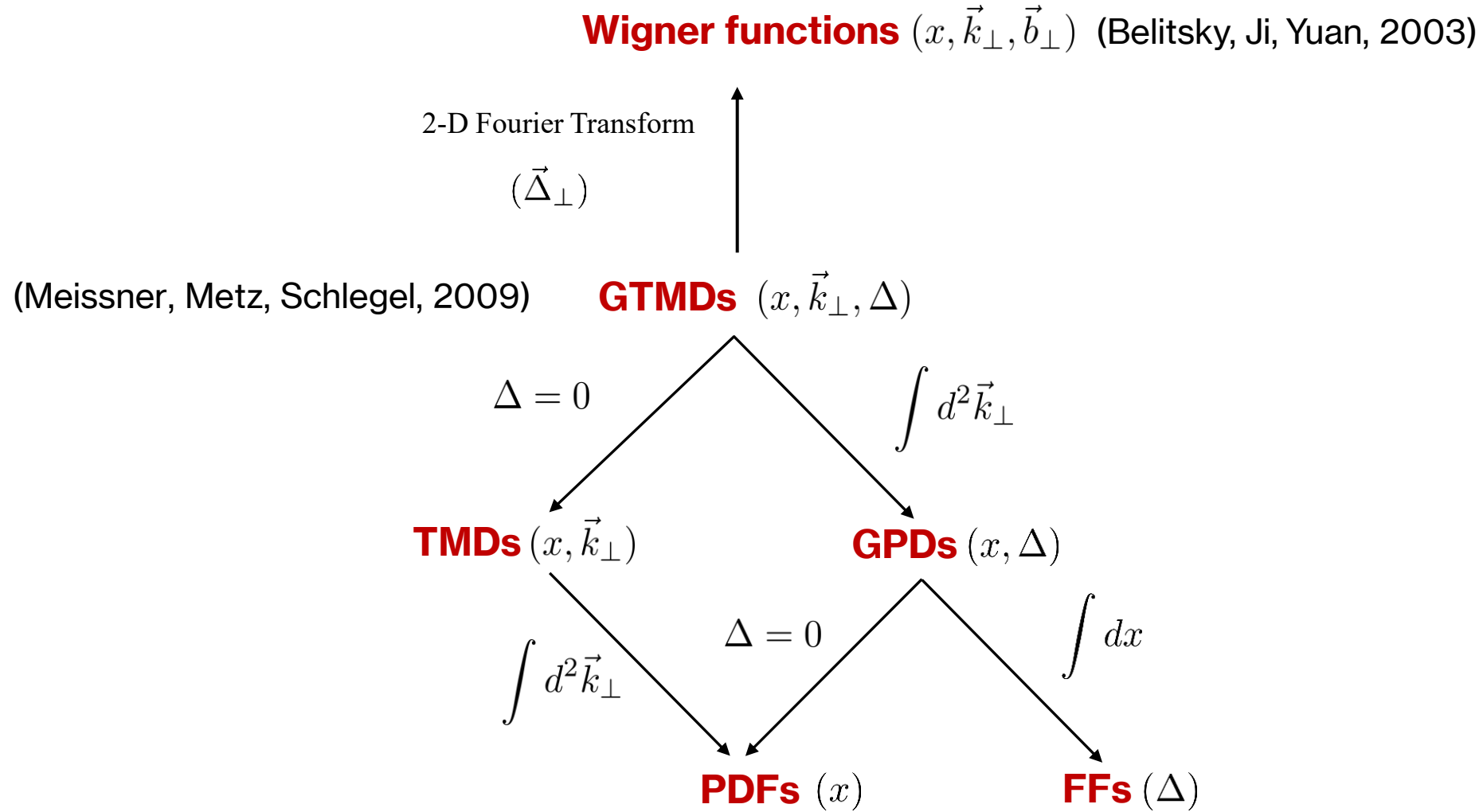
GTMDs - The “mother function”

Generalized **T**ransverse **M**omentum-dependent **D**istributions

(Meissner, Metz, Schlegel, 2009)



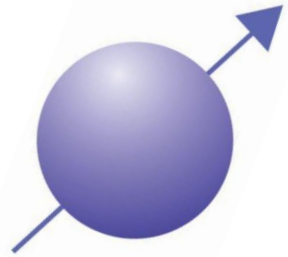
GTMDs/Wigner functions - The “mother function”



Spin of proton

Jaffe-Manohar spin decomposition

An incomplete story:



$$\frac{1}{2} = \frac{1}{2} \Delta \Sigma + \Delta G + L^q + L^g$$

Best known

How well do we know?

???????

Quark helicity $\sim 30\%$

Gluon helicity $\sim 40\%$

OAM of quarks & gluons

So far, no experimental constraints on OAM of quarks & gluons

Wigner functions & Orbital Angular Momentum



Wigner functions in Quantum Mechanics (Wigner, 1932)

- Expectation value of observables:

$$\langle \mathcal{O} \rangle = \int dx \int dk \mathcal{O}(x, k) W(x, k)$$

Wigner functions in parton physics (Belitsky, Ji, Yuan, 2003)

- Calculate from fourier transform of GTMD correlator:

$$W^{[\Gamma]}(x, \vec{k}_\perp, \vec{b}_\perp)$$

- Application: **O**rbital **A**ngular **M**omentum (**OAM**)

$$L_z^{q,g} = \int dx \int d^2 k_\perp d^2 b_\perp (\vec{b}_\perp \times \vec{k}_\perp)_z W^{q,g}(x, \vec{b}_\perp, \vec{k}_\perp)$$

(Lorcé, Pasquini, 2011 / Hatta, 2011)

GTMDs & Orbital Angular Momentum



Wigner functions in Quantum Mechanics

(Wigner, 1932)

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Wigner functions in parton physics

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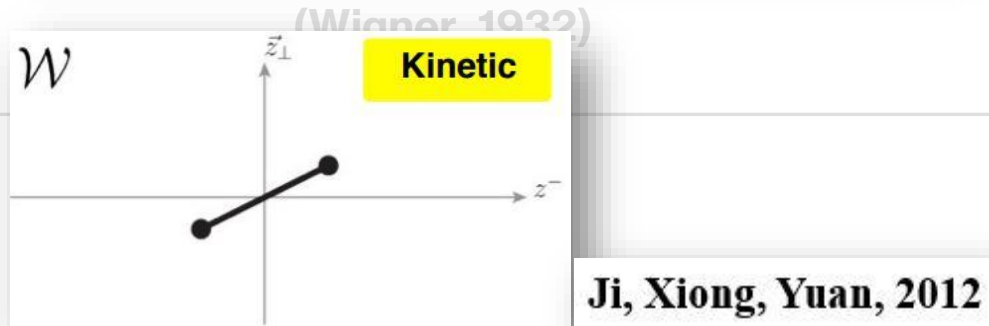
- Application: Relation between GTMD $F_{1,4}^{q,g}$ & OAM

$$L_z^{q,g} = - \int dx \int d^2 \vec{k}_\perp \frac{\vec{k}_\perp^2}{M^2} F_{1,4}^{q,g}(x, \vec{k}_\perp, \xi = 0, \Delta_\perp = 0)$$

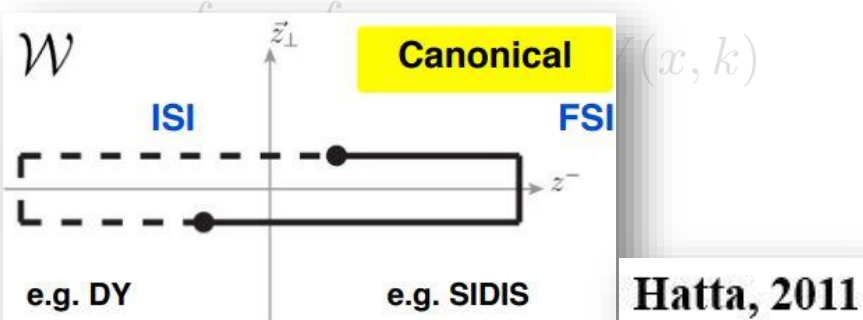
(Lorcé, Pasquini, 2011 / Hatta, 2011)

GTMDs & Orbital Angular Momentum

Gauge-invariant extension



- Expectation value of observables:



Wigner functions in parton physics

(Belitsky, Ji, Yuan, 2003)

- Calculate from fourier transform of GTMD correlator:

$$W^{[\Gamma]}(x, \vec{k}_\perp, \vec{b}_\perp)$$

- Application: Relation between GTMD $F_{1,4}^{q,g}$ & OAM

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(Lorcé, Pasquini, 2011 / Hatta, 2011)

GTMDs & Orbital Angular Momentum

Wigner functions in Quantum Mechanics

(Wigner, 1932)

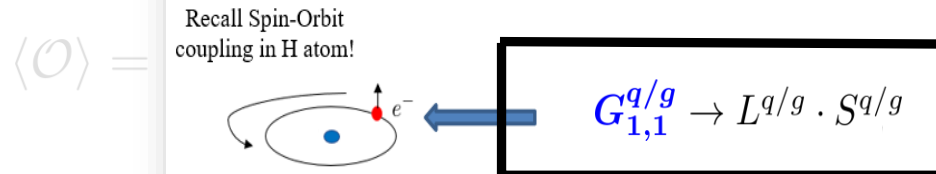
Wigner functions in parton physics

(Belitsky, Ji, Yuan, 2003)

Spin-orbit entanglement in the Color Glass Condensate

Shohini Bhattacharya,^{1,*} Renaud Boussarie,^{2,†} and Yoshitaka Hatta^{3,4,‡}

2404.04208



- Calculate from fourier transform of GTMD correlator:

$$W^{[\Gamma]}(x, \vec{k}_{\perp}, \vec{b}_{\perp})$$

- Application: Relation between GTMD $G_{1,1}^{q,g}$ & spin-orbit correlations

$$C^{q,g} = \int dx \int d^2 \vec{k}_{\perp} \frac{\vec{k}_{\perp}^2}{M^2} G_{1,1}^{q,g}(x, k_{\perp}, \xi = 0, \Delta_{\perp} = 0)$$

(Lorcé, Pasquini, 2011 /
SB, Boussarie, Hatta, 2024)

GTMDs & Orbital Angular Momentum



Wigner functions in Quantum Mechanics

(Wigner, 1932)

Wigner functions in parton physics

(Belitsky, Ji, Yuan, 2003)

Big question:
Experimental observable?

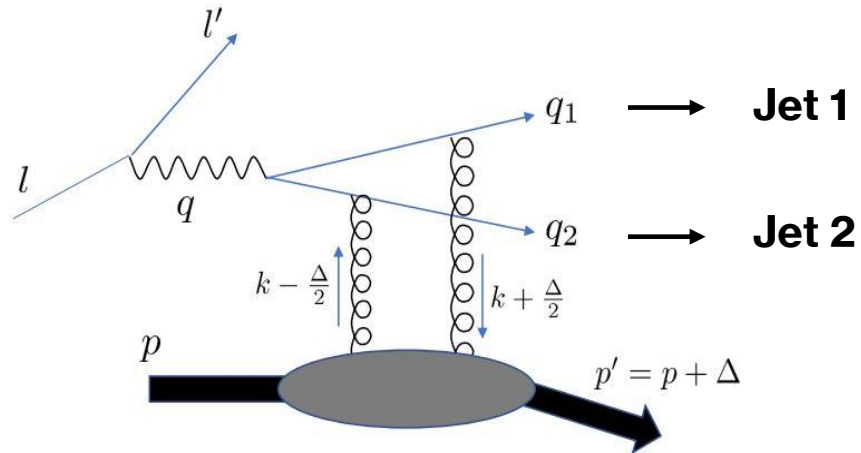
$$C^{q,g} = \int dx \int d^2 \vec{k}_\perp \frac{\vec{k}_\perp^2}{M^2} G_{1,1}^{q,g}(x, k_\perp, \xi = 0, \Delta_\perp = 0)$$

(Lorcé, Pasquini, 2011 /
SB, Boussarie, Hatta, 2024)

arXiv: 1612.02438 (2016)

Hunting the Gluon Orbital Angular Momentum at the Electron-Ion Collider

Xiangdong Ji,^{1,2} Feng Yuan,³ and Yong Zhao^{1,3}



Developments



arXiv: 1612.02438 (2016)

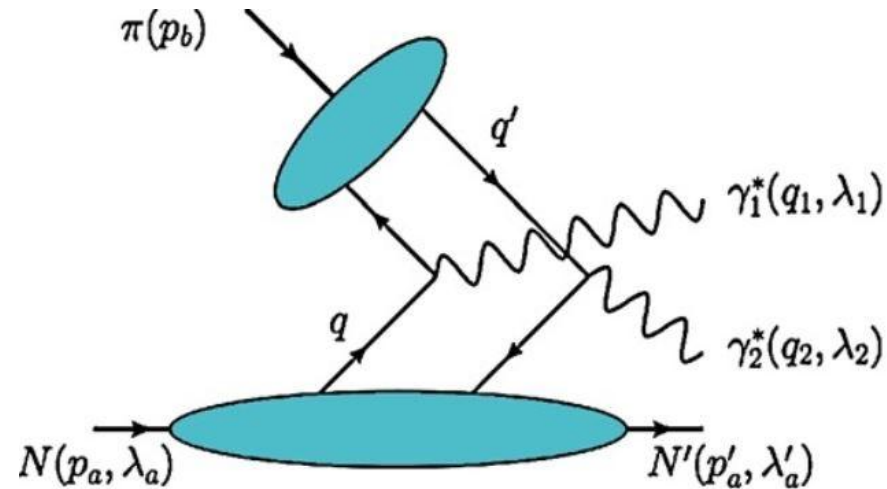
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Generalized TMDs and the exclusive double Drell-Yan process

Shohini Bhattacharya,¹ Andreas Metz,¹ and Jian Zhou²



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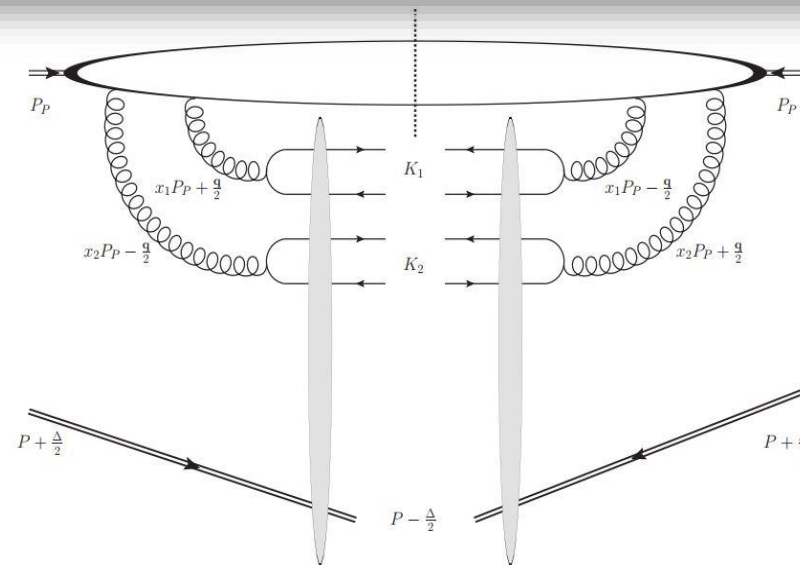
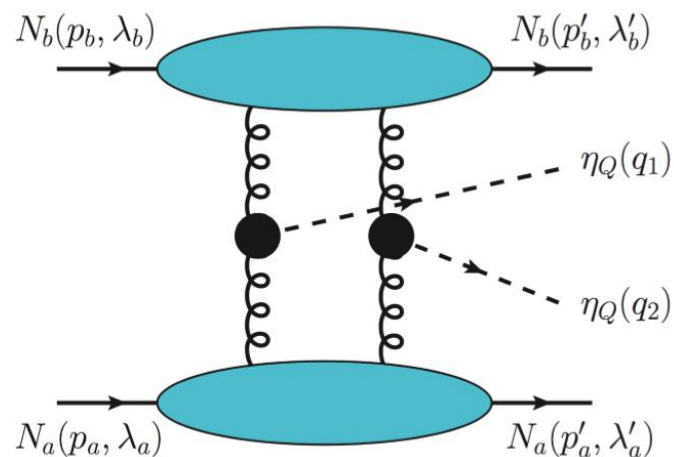
Exclusive double quarkonium production and generalized TM

Shohini Bhattacharya,¹ Andreas Metz,¹ Vikash Kumar Ojha,² Jeng-Yuan Tsai,¹

arXiv: 1807.08697 (2018)

Probing the Weizsäcker-Williams gluon Wigner distribution in pp collisions

Renaud Boussarie,¹ Yoshitaka Hatta,² Bo-Wen Xiao,^{3,4} and Feng Yuan⁵



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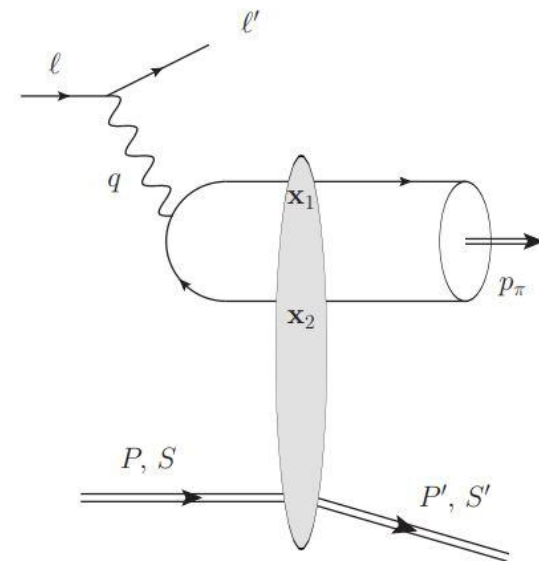
Probing the Weizs

Renaud Bous

arXiv: 1912.08182 (2019)

Probing the gluon Sivers function with an unpolarized target: GTMD distributions and the Odderons

Renaud Boussarie,¹ Yoshitaka Hatta,¹ Lech Szymanowski,² and Samuel Wallon^{3,4}



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FIG. 1. Electroproduction of a π^0 meson at small x . The gray blob represents the interaction with the background field, and the white blob represents the Distribution Amplitude of the meson.

Developments



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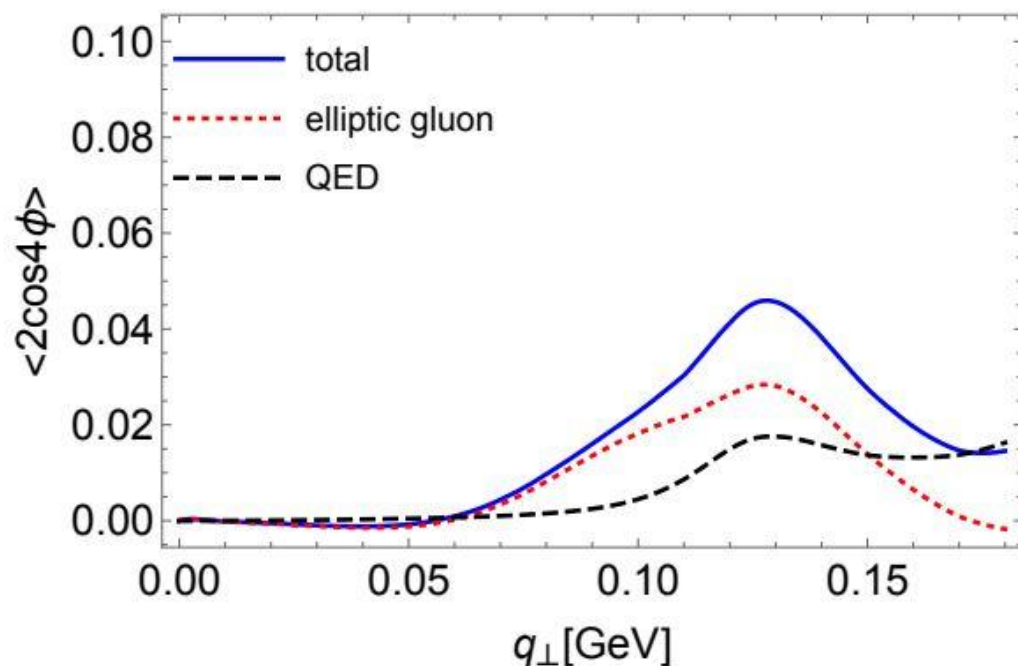
Renaud Boussarie,¹ Yoshitaka Hatta,² Bo-Wen Xiao,^{3,4} and Feng Yuan⁵

arXiv: 2106.13466 (2021)

Probing the gluon tomography in photoproduction of di-pions

Yoshikazu Hagiwara, Cheng Zhang, Jian Zhou, and Ya-jin Zhou

Elliptic Wigner distribution contributes to
 $\cos 4\phi$ asymmetry



Developments



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arXiv: 2201.08709 (2022/2024)

Signature of the gluon orbital angular momentum

Shohini Bhattacharya,^{1,*} Renaud Boussarie,^{2,†} and Yoshitaka Hatta^{1,3,‡}

Developments



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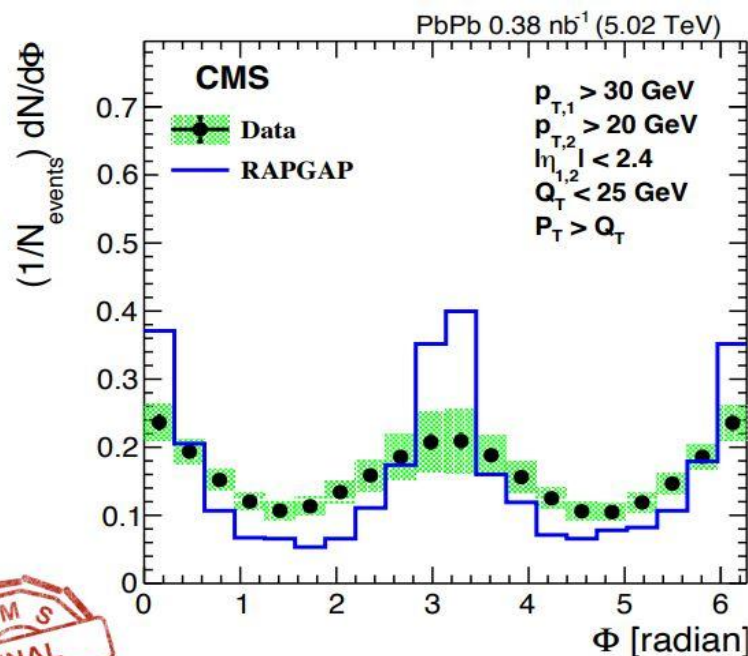
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arXiv: 2106.13466 (2021)

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Yoshikazu Hagiwara, Cheng Zhang, Jian Zhou, and Ya-jin Zhou

arXiv: 2205.00045 (2022)

Angular correlations in exclusive dijet photoproduction in ultra-peripheral PbPb collisions at $\sqrt{s_{NN}} = 5.02$ TeV

arXiv: 1612.02438 (2016)

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Xiangdong Ji,^{1,2} Feng Yuan,³ and Yong Zhao^{1,3}

arXiv: 1702.04387 (2017)

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Shohini Bhattacharya,¹ Andreas Metz,¹ and Jian Zhou²

arXiv: 1802.10550 (2018)

Exclusive double quarkonium production and angular TMDs

Shohini Bhattacharya,¹ Andreas Metz,¹ Vikas Kumar,² Ojha,³ Feng-Yuan Tsai,⁴

arXiv: 1807.08697 (2018)

Generalized TMDs and the exclusive double Drell-Yan process

Renaud Boussarie,¹ Yoshitaka Hatta,² Bo Wen Xiao,^{3,4} and Feng Yuan⁵

Opportunities through exclusive meson production?

arXiv: 1912.08191 (2019)

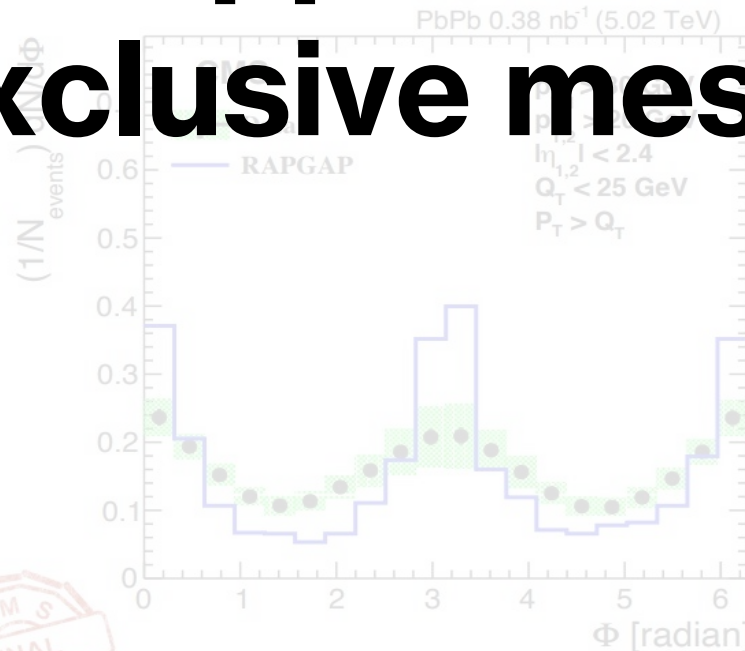
Probing the gluon tomography in photoproduction of di-pions

Renaud Boussarie,¹ Yoshitaka Hatta,² Bo Wen Xiao,^{3,4} and Feng Yuan⁵

arXiv: 2201.08191 (2022)

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arXiv: 2106.13466 (2021)

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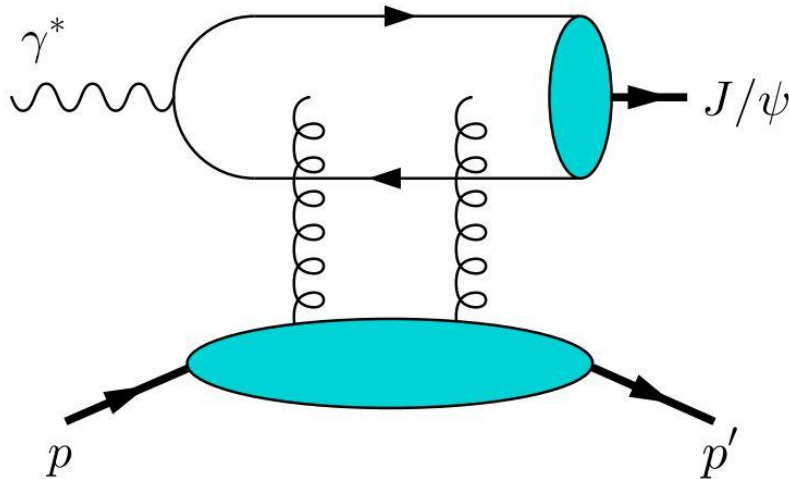
Our work



Gluon Generalized TMD signatures at the EIC from exclusive heavy (axial-)vector meson production

Shohini Bhattacharya,^{1,*} David DeAngelo,^{1,†} Lei Yang,^{2,‡} Duxin Zheng,^{3,§} and Jian Zhou^{2,4,¶}

ArXiv: 2601.17506 (accepted in PRD)



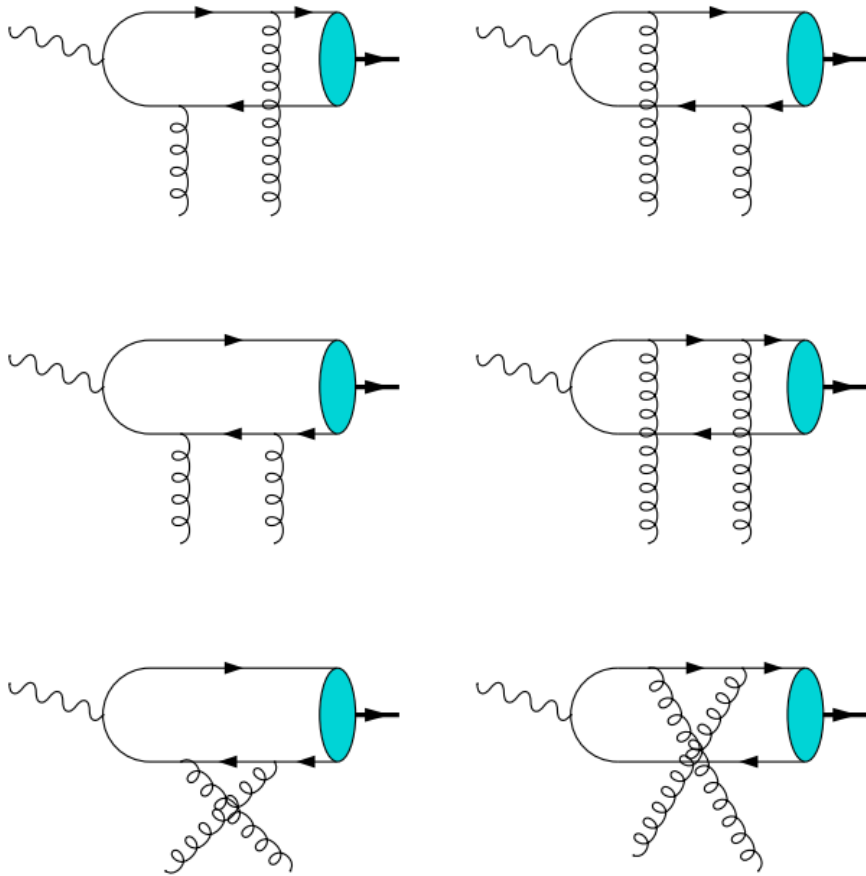
Main Observable:

Interference between different virtual-photon polarizations leads to distinct azimuthal modulations, including $\cos 2\phi$ and $\sin 2\phi$ terms, that are sensitive to gluon OAM and spin-orbit correlations

Probing gluon OAM & spin-orbit correlations via vector-mesons



Scattering amplitude

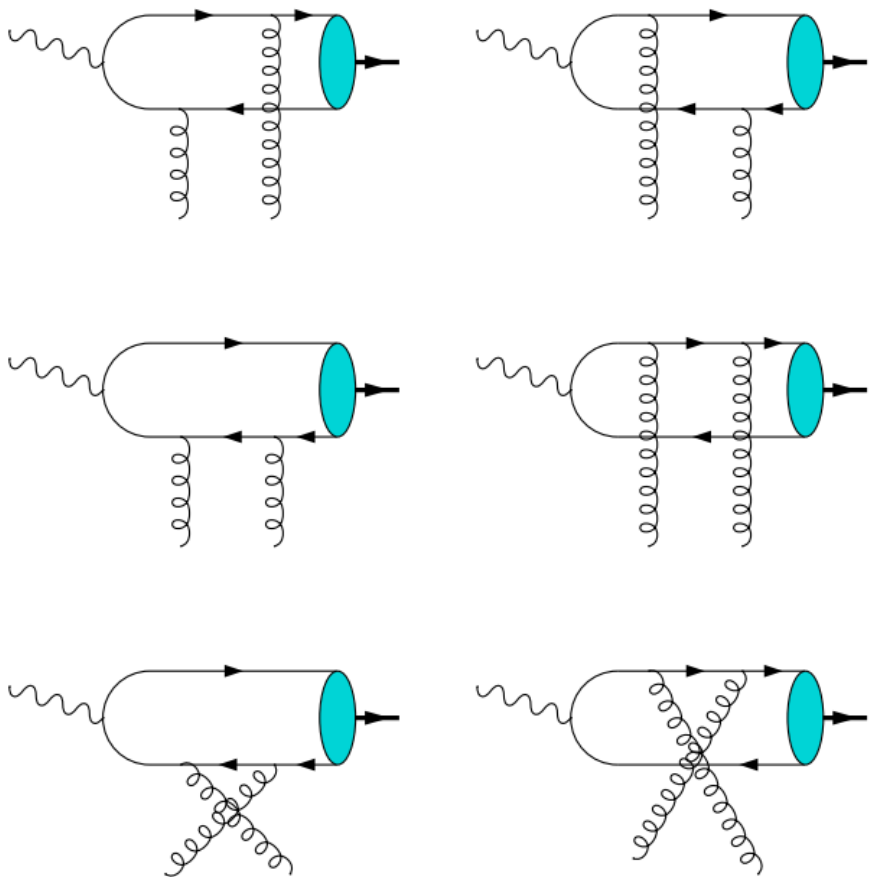


6 leading-order Feynman diagrams

Probing gluon OAM & spin-orbit correlations via vector-mesons



Scattering amplitude



6 leading-order Feynman diagrams

Scattering amplitude:

Hard part

$$\mathcal{M} \propto \int dx \int d^2 k_{\perp} H(x, \xi, z, k_{\perp}, \Delta_{\perp}) f^g(x, \xi, k_{\perp}, \Delta_{\perp}) \int dz \phi_V(z)$$

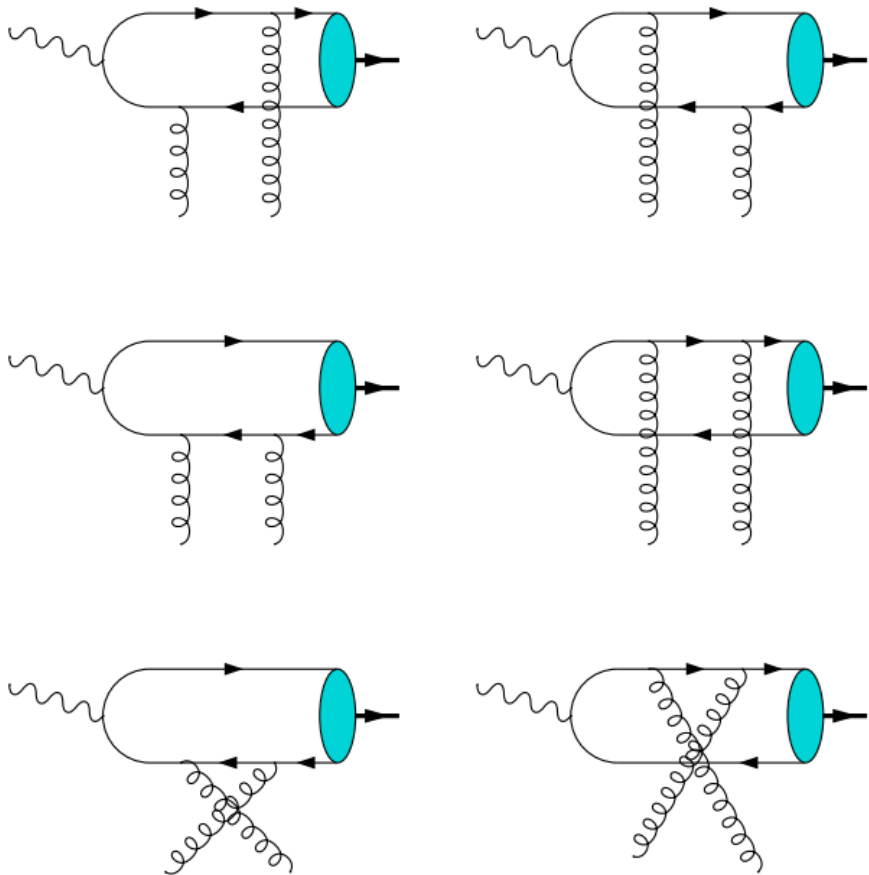
Soft part from
proton

Meson
Distribution Amplitude

Probing gluon OAM & spin-orbit correlations via vector-mesons



Scattering amplitude



6 leading-order Feynman diagrams

Scattering amplitude:

Hard part

$$\mathcal{M} \propto \int dx \int d^2 k_{\perp} H(x, \xi, z, k_{\perp}, \Delta_{\perp}) f^g(x, \xi, k_{\perp}, \Delta_{\perp}) \int dz \phi_V(z)$$

Soft part from
proton

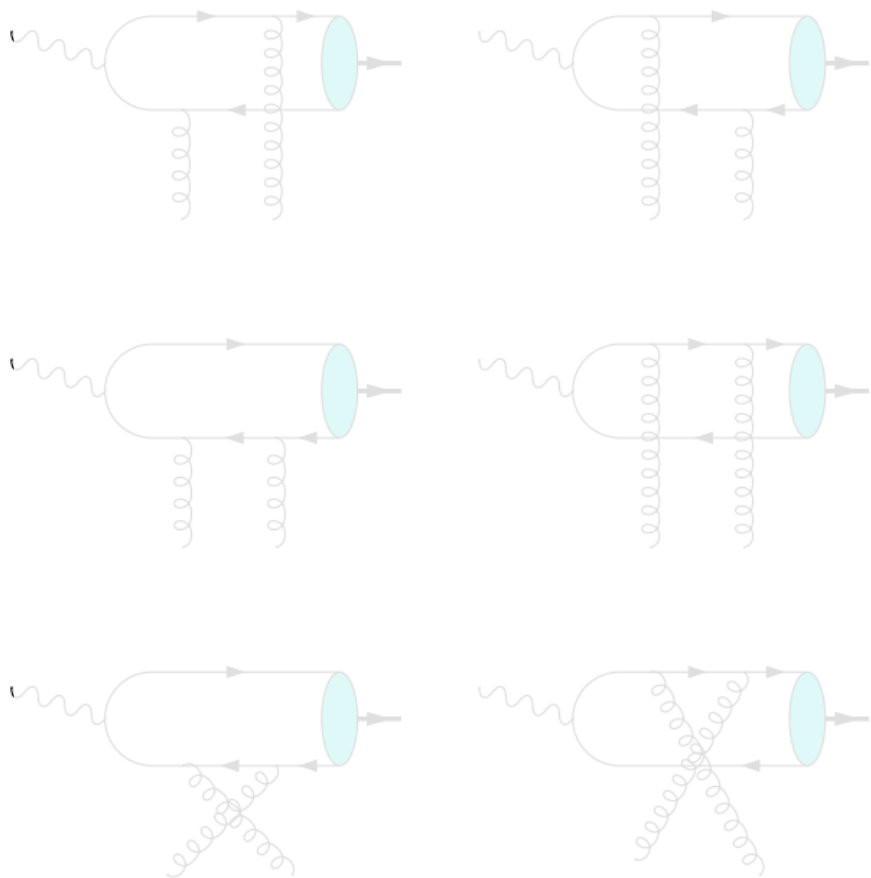
Meson
Distribution Amplitude

(No transverse-momentum
dependence here –
I will comment on this later)

Probing gluon OAM & spin-orbit correlations via vector-mesons



Scattering amplitude



6 leading-order Feynman diagrams

Scattering amplitude:

$$\mathcal{M} \propto \int dx \int d^2 k_{\perp} H(x, \xi, z, k_{\perp}, \Delta_{\perp}) f^g(x, \xi, k_{\perp}, \Delta_{\perp}) \int dz \phi_V(z)$$

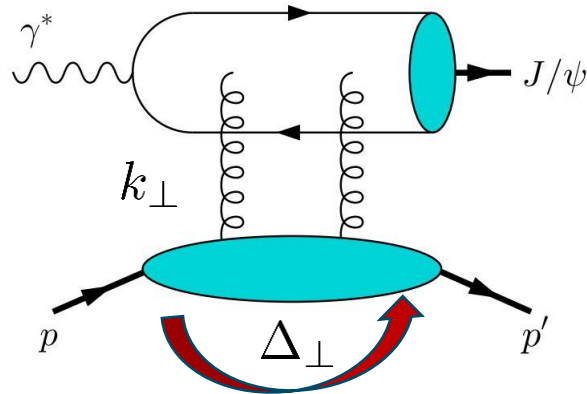
Hard part

Let us now unpack the calculation of the hard scattering kernel step by step.

Probing gluon OAM & spin-orbit correlations via vector-mesons



Sketch of calculation



$$\mathcal{M} \propto \int dx \int d^2 k_{\perp} H(x, \xi, z, k_{\perp}, \Delta_{\perp}) f^g(x, \xi, k_{\perp}, \Delta_{\perp}) \int dz \phi_V(z)$$

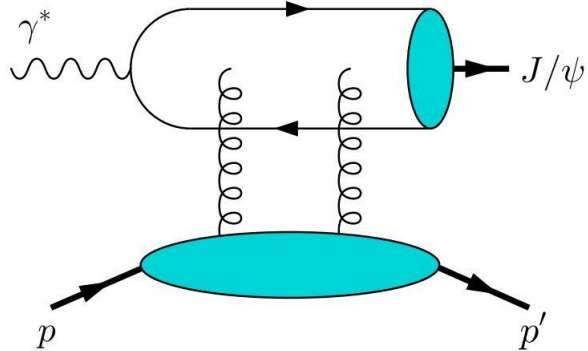
Collinear twist-expansion of hard part:

$$H(k_{\perp}, \Delta_{\perp}) = H(k_{\perp} = 0, \Delta_{\perp} = 0) + \frac{\partial H(k_{\perp}, \Delta_{\perp} = 0)}{\partial k_{\perp}^{\mu}} \Big|_{k_{\perp}=0} k_{\perp}^{\mu} + \frac{\partial H(k_{\perp} = 0, \Delta_{\perp})}{\partial \Delta_{\perp}^{\mu}} \Big|_{\Delta_{\perp}=0} \Delta_{\perp}^{\mu} + \dots$$

Probing gluon OAM & spin-orbit correlations via vector-mesons



Sketch of calculation



$$\mathcal{M} \propto \int dx \int d^2 k_{\perp} H(x, \xi, z, k_{\perp}, \Delta_{\perp}) f^g(x, \xi, k_{\perp}, \Delta_{\perp}) \int dz \phi_V(z)$$

Collinear twist-expansion of hard part:

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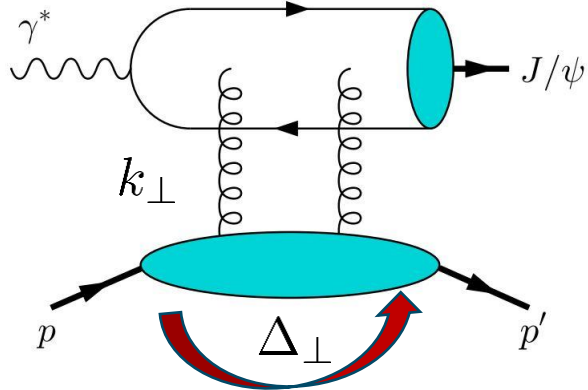


Twist 2 term $\neq 0$

Probing gluon OAM & spin-orbit correlations via vector-mesons



Sketch of calculation



$$\mathcal{M} \propto \int dx \int d^2 k_{\perp} H(x, \xi, z, k_{\perp}, \Delta_{\perp}) f^g(x, \xi, k_{\perp}, \Delta_{\perp}) \int dz \phi_V(z)$$

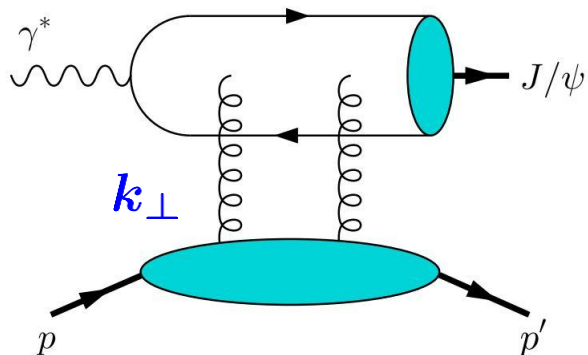
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Probing gluon OAM & spin-orbit correlations via vector-mesons



Sketch of calculation



$$\mathcal{M} \propto \int dx \int d^2 k_{\perp} H(x, \xi, z, k_{\perp}, \Delta_{\perp}) \textcolor{red}{f}^g(x, \xi, k_{\perp}, \Delta_{\perp}) \int dz \phi_V(z)$$

Collinear twist-expansion of hard part:

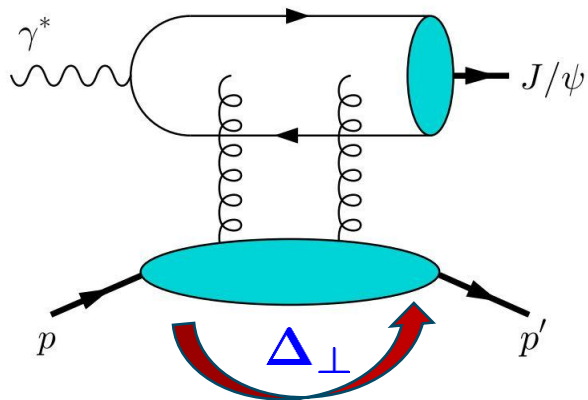
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$$A \propto \int d^2 k_{\perp} k_{\perp}^2 \textcolor{red}{GTMD}$$

Probing gluon OAM & spin-orbit correlations via vector-mesons



Sketch of calculation



$$\mathcal{M} \propto \int dx \int d^2 k_{\perp} H(x, \xi, z, k_{\perp}, \Delta_{\perp}) f^g(x, \xi, k_{\perp}, \Delta_{\perp}) \int dz \phi_V(z)$$

Collinear twist-expansion of hard part:

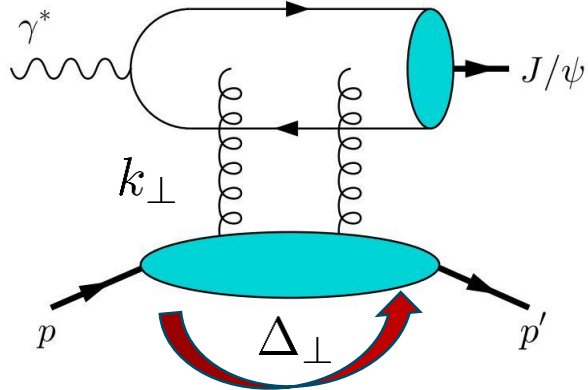
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$$A \propto \text{GPD}$$

Probing gluon OAM & spin-orbit correlations via vector-mesons



Sketch of calculation



$$\mathcal{M} \propto \int dx \int d^2 k_\perp H(x, \xi, z, k_\perp, \Delta_\perp) f^g(x, \xi, k_\perp, \Delta_\perp) \int dz \phi_V(z)$$

Collinear twist-expansion of hard part:

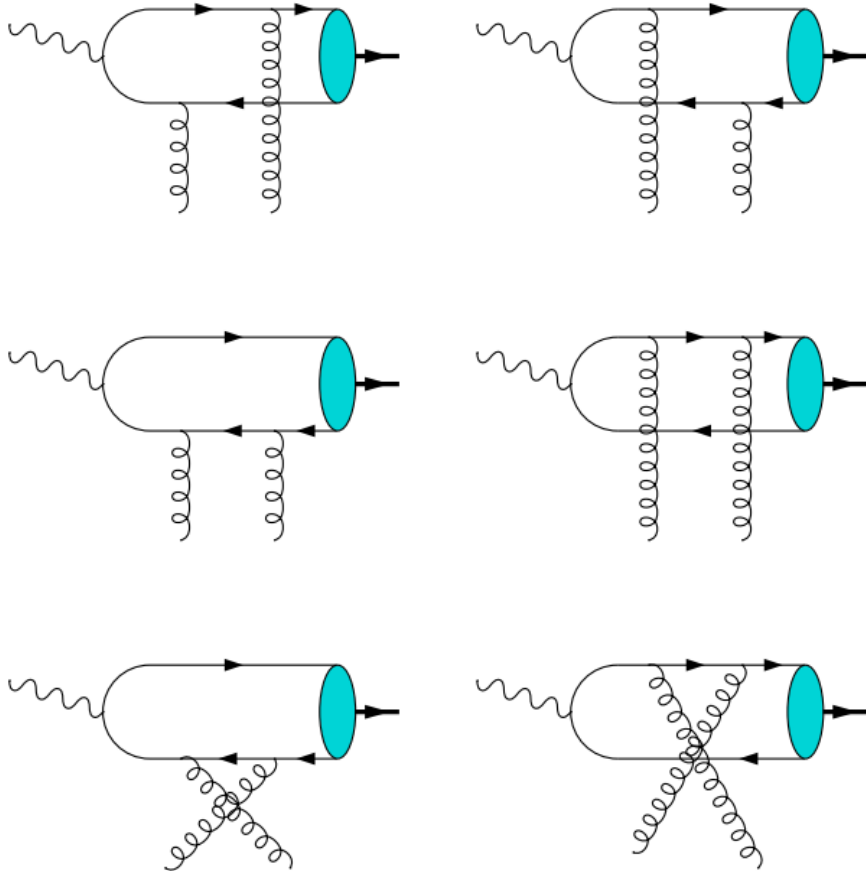
The scattering amplitudes are a linear combination of **moments of GTMDs** and **GPDs**

- This **interplay** is itself physically significant, as it connects two complementary descriptions of hadron structure
- An opportunity to simultaneously constrain both GTMD and GPD dynamics within a unified exclusive framework

Probing gluon OAM & spin-orbit correlations via vector-mesons



Scattering amplitude



6 leading-order Feynman diagrams

Scattering amplitude:

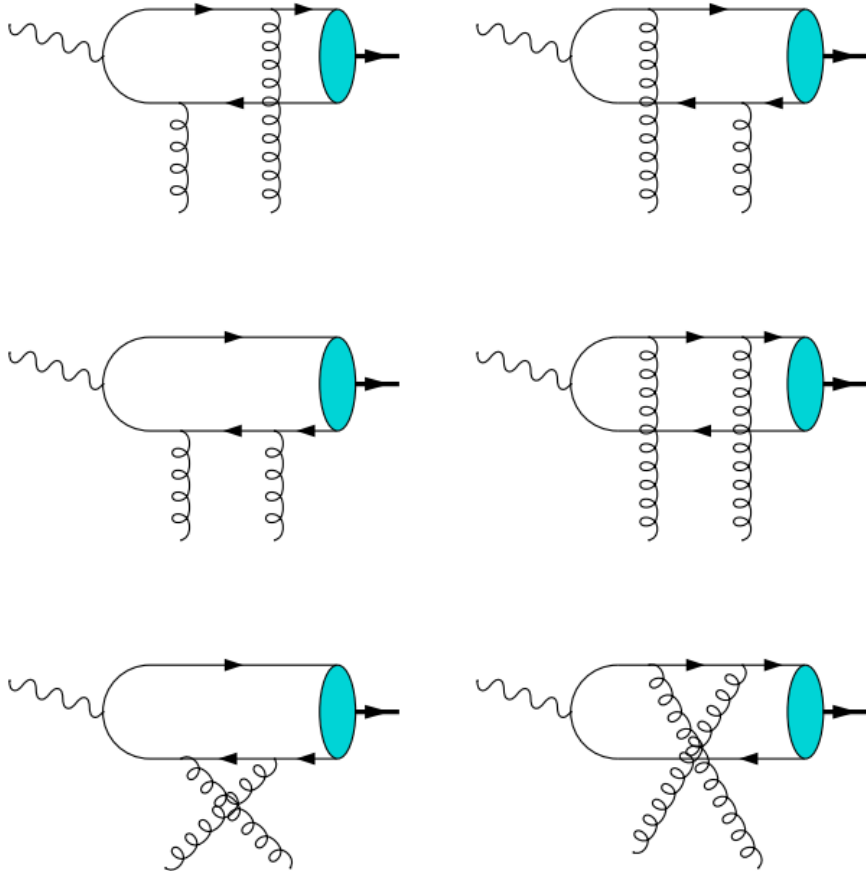
$$\mathcal{M} \propto \int dx \int d^2 k_{\perp} H(x, \xi, z, k_{\perp}, \Delta_{\perp}) f^g(x, \xi, k_{\perp}, \Delta_{\perp}) \int dz \phi_V(z)$$

- Upon performing the collinear twist expansion of the hard scattering kernel, we find contributions from both twist-2 and twist-3 terms.

Probing gluon OAM & spin-orbit correlations via vector-mesons



Scattering amplitude



6 leading-order Feynman diagrams

Scattering amplitude:

$$\mathcal{M} \propto \int dx \int d^2 k_{\perp} H(x, \xi, z, k_{\perp}, \Delta_{\perp}) f^g(x, \xi, k_{\perp}, \Delta_{\perp}) \int dz \phi_V(z)$$

- Upon performing the collinear twist expansion of the hard scattering kernel, we find contributions from both twist-2 and twist-3 terms.
- Calculated scattering amplitudes for specific polarization configurations of the **virtual photon** and the **vector meson**.

Notation: $\mathcal{M}^{\gamma^*}, M$

Probing gluon OAM & spin-orbit correlations via vector-mesons



Scattering amplitude

$$\mathcal{M}^{\gamma^*, M}$$

γ^* = Photon Polarization

M = Meson Polarization

Proton helicity-conserving amplitudes (z=1/2):

$$\mathcal{M}^{LL} = C \delta_{\lambda, \lambda'} \{ \mathcal{H}_{eff}^{LL} \}$$

$$\mathcal{M}^{TT} = C \delta_{\lambda, \lambda'} (\epsilon_{\perp}^{\gamma^*} \cdot \epsilon_{\perp}^V) \{ \mathcal{H}_{eff}^{TT} \}$$

$$\mathcal{M}^{TL} = C \delta_{\lambda, \lambda'} \left\{ (\epsilon_{\perp}^{\gamma^*} \cdot \Delta_{\perp}) (\mathcal{H}_{eff}^{TL} + \mathcal{F}_{1,1}^{TL}) + \lambda (\epsilon_{\perp}^{\gamma^*} \times \Delta_{\perp}) \mathcal{F}_{1,4}^{TL} \right. \\ \left. + (\epsilon_{\perp}^{\gamma^*} \cdot \Delta_{\perp}) \mathcal{G}_{1,1}^{TL} - \lambda (\epsilon_{\perp}^{\gamma^*} \times \Delta_{\perp}) (\mathcal{H}'_{eff}^{TL} + \mathcal{G}_{1,4}^{TL}) \right\}$$

$$\mathcal{M}^{LT} = 0$$

$$C = \frac{16 i \pi e e_q \alpha_s f_V m_V}{N_c (m_V^2 + Q^2)}$$

CFFs:

$$\mathcal{H}_{eff}^{LL} = - \int_{-1}^1 \frac{dx}{(x - \xi + i\varepsilon)(x + \xi - i\varepsilon)} \frac{Q \sqrt{1 - \xi^2}}{m_V} H_{eff}^g$$

$$\mathcal{H}_{eff}^{TT} = \int_{-1}^1 \frac{dx}{(x - \xi + i\varepsilon)(x + \xi - i\varepsilon)} \sqrt{1 - \xi^2} H_{eff}^g$$

$$\mathcal{H}_{eff}^{TL} = \int_{-1}^1 \frac{dx}{(x - \xi + i\varepsilon)^2 (x + \xi - i\varepsilon)^2} \frac{\xi^2 \sqrt{1 - \xi^2}}{m_V} H_{eff}^g$$

$$\mathcal{H}'_{eff}^{TL} = \int_{-1}^1 \frac{dx}{(x - \xi + i\varepsilon)^2 (x + \xi - i\varepsilon)^2} \frac{i x \xi \sqrt{1 - \xi^2}}{m_V} H'_{eff}^g$$

$$\mathcal{F}_{1,1}^{TL} = \int_{-1}^1 \frac{dx}{(x - \xi - i\varepsilon)^2 (x + \xi + i\varepsilon)^2} \frac{2x\xi}{m_V} \frac{1}{\sqrt{1 - \xi^2}} \int d^2 k_{\perp} \left(\frac{k_{\perp} \cdot \Delta_{\perp}}{\Delta_{\perp}^2} \right) F_{1,1}^g$$

$$\mathcal{G}_{1,1}^{TL} = - \frac{1}{\sqrt{1 - \xi^2}} \int_{-1}^1 dx \frac{\xi^2}{m_V (x + \xi - i\varepsilon)^2 (x - \xi + i\varepsilon)^2} \int d^2 k_{\perp} \left(\frac{k_{\perp}^2}{M^2} \right) G_{1,1}^g$$

$$\mathcal{F}_{1,4}^{TL} = \frac{1}{\sqrt{1 - \xi^2}} \int_{-1}^1 dx \frac{i x \xi}{m_V (x + \xi - i\varepsilon)^2 (x - \xi + i\varepsilon)^2} \int d^2 k_{\perp} \left(\frac{k_{\perp}^2}{M^2} \right) F_{1,4}^g$$

$$\mathcal{G}_{1,4}^{TL} = \int \frac{dx}{(x - \xi + i\varepsilon)^2 (x + \xi - i\varepsilon)^2} \frac{2i\xi^2}{m_V} \frac{1}{\sqrt{1 - \xi^2}} \int d^2 k_{\perp} \left(\frac{k_{\perp} \cdot \Delta_{\perp}}{\Delta_{\perp}^2} \right) G_{1,4}^g$$

$$H_{eff}^g = H^g - \frac{\xi^2}{1 - \xi^2} E^g$$

$$H'_{eff}^g = \tilde{H}^g - \frac{\xi^2}{1 - \xi^2} \tilde{E}^g$$

Probing gluon OAM & spin-orbit correlations via vector-mesons



Scattering amplitude

$$\mathcal{M}^{\gamma^*, M}$$

γ^* = Photon Polarization

M = Meson Polarization

Gluon OAM

Proton helicity-conserving amplitudes (z=1/2):

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$$\mathcal{M}^{TT} = C \delta_{\lambda, \lambda'} (\epsilon_{\perp}^{\gamma^*} \cdot \epsilon_{\perp}^V) \{ \mathcal{H}_{eff}^{TT} \}$$

$$\mathcal{M}^{TL} = C \delta_{\lambda, \lambda'} \{ (\epsilon_{\perp}^{\gamma^*} \cdot \Delta_{\perp}) (\mathcal{H}_{eff}^{TL} + \mathcal{F}_{1,1}^{TL}) + \lambda (\epsilon_{\perp}^{\gamma^*} \times \Delta_{\perp}) \mathcal{F}_{1,4}^{TL} + (\epsilon_{\perp}^{\gamma^*} \cdot \Delta_{\perp}) \mathcal{G}_{1,1}^{TL} - \lambda (\epsilon_{\perp}^{\gamma^*} \times \Delta_{\perp}) (\mathcal{H}'_{eff}^{TL} + \mathcal{G}_{1,4}^{TL}) \}$$

$$\mathcal{M}^{LT} = 0$$

$$C = \frac{16 i \pi e e_g \alpha_s f_V m_V}{N_c (m_V^2 + q^2)}$$

Gluon's spin-orbit correlations

CFFs:

$$\mathcal{H}_{eff}^{LL} = - \int_{-1}^1 \frac{dx}{(x - \xi + i\varepsilon)(x + \xi - i\varepsilon)} \frac{Q \sqrt{1 - \xi^2}}{m_V} H_{eff}^g$$

$$\mathcal{H}_{eff}^{TT} = \int_{-1}^1 \frac{dx}{(x - \xi + i\varepsilon)(x + \xi - i\varepsilon)} \sqrt{1 - \xi^2} H_{eff}^g$$

$$\mathcal{H}_{eff}^{TL} = \int_{-1}^1 \frac{dx}{(x - \xi + i\varepsilon)^2 (x + \xi - i\varepsilon)^2} \frac{\xi^2 \sqrt{1 - \xi^2}}{m_V} H_{eff}^g$$

$$\mathcal{H}'_{eff}^{TL} = \int_{-1}^1 \frac{dx}{(x - \xi + i\varepsilon)^2 (x + \xi - i\varepsilon)^2} \frac{i x \xi \sqrt{1 - \xi^2}}{m_V} H_{eff}^g$$

$$\mathcal{F}_{1,1}^{TL} = \int_{-1}^1 \frac{dx}{(x - \xi - i\varepsilon)^2 (x + \xi + i\varepsilon)^2} \frac{2x\xi}{m_V} \frac{1}{\sqrt{1 - \xi^2}} \int d^2 k_{\perp} \left(\frac{k_{\perp} \cdot \Delta_{\perp}}{\Delta_{\perp}^2} \right) F_{1,1}^g$$

$$\mathcal{G}_{1,1}^{TL} = - \frac{1}{\sqrt{1 - \xi^2}} \int_{-1}^1 dx \frac{\xi^2}{m_V (x + \xi - i\varepsilon)^2 (x - \xi + i\varepsilon)^2} \int d^2 k_{\perp} \left(\frac{k_{\perp}^2}{M^2} \right) G_{1,1}^g$$

$$\mathcal{F}_{1,4}^{TL} = \frac{1}{\sqrt{1 - \xi^2}} \int_{-1}^1 dx \frac{i x \xi}{m_V (x + \xi - i\varepsilon)^2 (x - \xi + i\varepsilon)^2} \int d^2 k_{\perp} \left(\frac{k_{\perp}^2}{M^2} \right) F_{1,4}^g$$

$$\mathcal{G}_{1,4}^{TL} = \int \frac{dx}{(x - \xi + i\varepsilon)^2 (x + \xi - i\varepsilon)^2} \frac{2i\xi^2}{m_V} \frac{1}{\sqrt{1 - \xi^2}} \int d^2 k_{\perp} \left(\frac{k_{\perp} \cdot \Delta_{\perp}}{\Delta_{\perp}^2} \right) G_{1,4}^g$$

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$$\mathcal{M}^{LT} = 0 \quad \leftarrow \quad \text{See Bakur's slides from yesterday}$$

$$C = \frac{16 i \pi e e_q \alpha_s f_V m_V}{N_c (m_V^2 + Q^2)}$$

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$$\mathcal{H}_{eff}^{LL} = - \int_{-1}^1 \frac{dx}{(x - \xi + i\varepsilon)(x + \xi - i\varepsilon)} \frac{Q \sqrt{1 - \xi^2}}{m_V} H_{eff}^g$$

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$$\mathcal{H}'_{eff}^{TL} = \int_{-1}^1 \frac{dx}{(x - \xi + i\varepsilon)^2 (x + \xi - i\varepsilon)^2} \frac{i x \xi \sqrt{1 - \xi^2}}{m_V} H'_{eff}^g$$

$$\mathcal{F}_{1,1}^{TL} = \int_{-1}^1 \frac{dx}{(x - \xi - i\varepsilon)^2 (x + \xi + i\varepsilon)^2} \frac{2x\xi}{m_V} \frac{1}{\sqrt{1 - \xi^2}} \int d^2 k_{\perp} \left(\frac{k_{\perp} \cdot \Delta_{\perp}}{\Delta_{\perp}^2} \right) F_{1,1}^g$$

$$\mathcal{G}_{1,1}^{TL} = - \frac{1}{\sqrt{1 - \xi^2}} \int_{-1}^1 dx \frac{\xi^2}{m_V (x + \xi - i\varepsilon)^2 (x - \xi + i\varepsilon)^2} \int d^2 k_{\perp} \left(\frac{k_{\perp}^2}{M^2} \right) G_{1,1}^g$$

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Scattering amplitude

$$\mathcal{M}^{\gamma^*, M}$$

γ^* = Photon Polarization
 M = Meson Polarization

Proton helicity-non conserving amplitudes (z=1/2):

$$\begin{aligned}\widetilde{\mathcal{M}}^{LL} &= C \delta_{\lambda, -\lambda'} \left\{ (\Delta_{\perp} \times S_{\perp}) \widetilde{\mathcal{E}}^{LL} \right\} \\ \widetilde{\mathcal{M}}^{TT} &= C \delta_{\lambda, -\lambda'} (\epsilon_{\perp}^V \cdot \epsilon_{\perp}^{\gamma^*}) \left\{ (\Delta_{\perp} \times S_{\perp}) \widetilde{\mathcal{E}}^{TT} \right\} \\ \widetilde{\mathcal{M}}^{TL} &= C \delta_{\lambda, -\lambda'} \left\{ (\epsilon_{\perp}^{\gamma^*} \times S_{\perp}) \widetilde{\mathcal{F}}_{1,2}^{TL} - (\epsilon_{\perp}^{\gamma^*} \times S_{\perp}) \widetilde{\mathcal{G}}_{1,2}^{TL} \right\} \\ \widetilde{\mathcal{M}}^{LT} &= 0\end{aligned}$$

$$C = \frac{16 i \pi e e_q \alpha_s f_V m_V}{N_c (m_V^2 + Q^2)}$$

CFFs:

$$\widetilde{\mathcal{E}}^{LL} = - \int_{-1}^1 \frac{dx}{(x - \xi + i\varepsilon)(x + \xi - i\varepsilon)} \frac{Q}{m_V} \frac{1}{2M\sqrt{1 - \xi^2}} E^g$$

$$\widetilde{\mathcal{E}}^{TT} = \int_{-1}^1 \frac{dx}{(x - \xi + i\varepsilon)(x + \xi - i\varepsilon)} \frac{1}{2M\sqrt{1 - \xi^2}} E^g$$

$$\widetilde{\mathcal{F}}_{1,2}^{TL} = M\sqrt{1 - \xi^2} \int_{-1}^1 dx \frac{x\xi}{m_V(x + \xi - i\varepsilon)^2(x - \xi + i\varepsilon)^2} \int d^2k_{\perp} \left(\frac{k_{\perp}^2}{M^2} \right) F_{1,2}^g$$

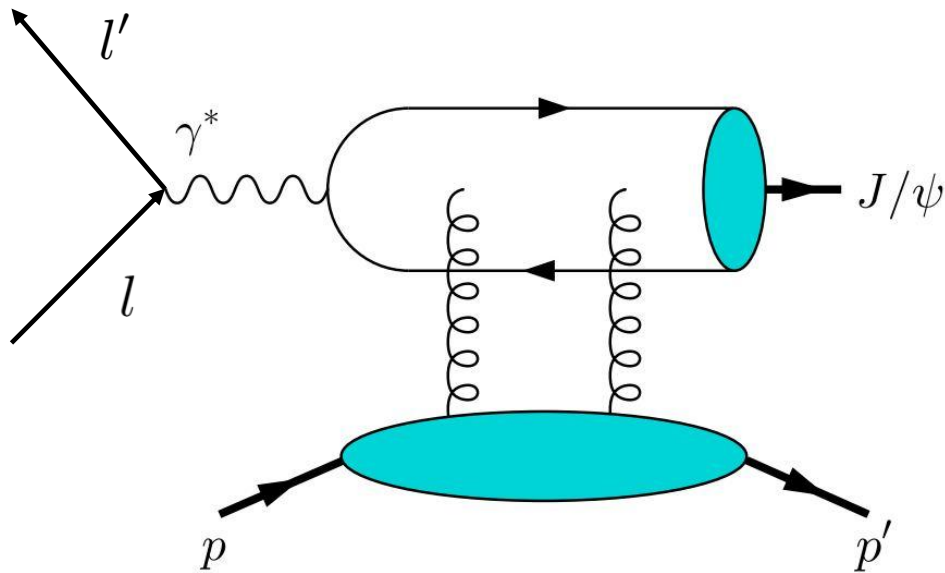
$$\widetilde{\mathcal{G}}_{1,2}^{TL} = -M\sqrt{1 - \xi^2} \int_{-1}^1 dx \frac{\xi^2}{m_V(x + \xi - i\varepsilon)^2(x - \xi + i\varepsilon)^2} \int d^2k_{\perp} \left(\frac{k_{\perp}^2}{M^2} \right) G_{1,2}^g$$

Probing gluon OAM & spin-orbit correlations via vector-mesons



Cross section

$$d\sigma = \frac{\mathcal{C}}{2s} L_{\mu\nu} \mathcal{M}^\mu \mathcal{M}^{*\nu}$$

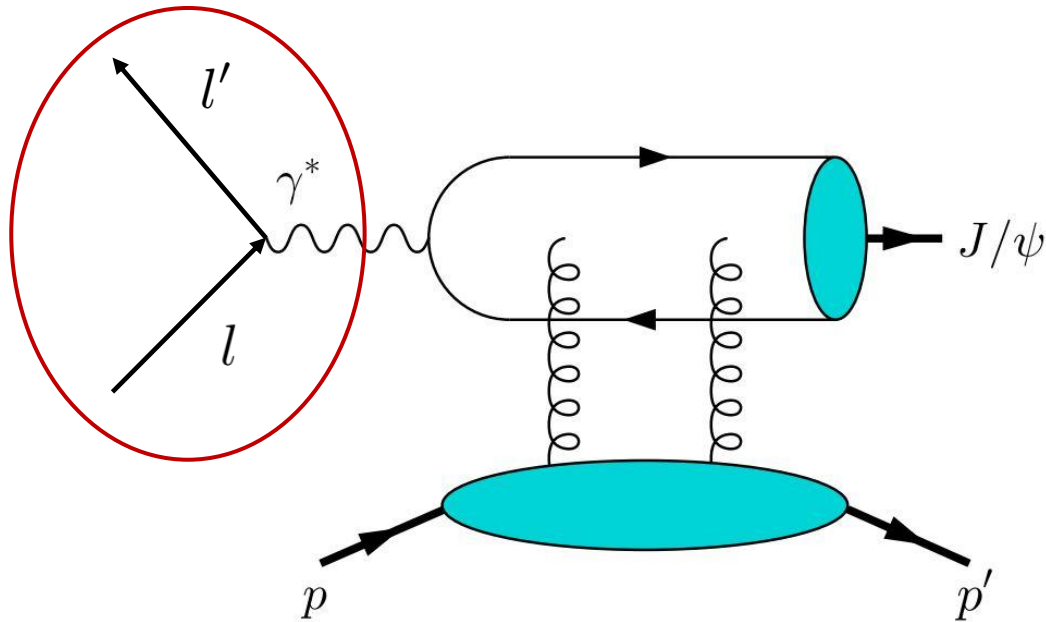


Probing gluon OAM & spin-orbit correlations via vector-mesons



Cross section

$$d\sigma = \frac{C}{2s} L_{\mu\nu} \mathcal{M}^\mu \mathcal{M}^{*\nu}$$



Unpolarized leptonic tensor:

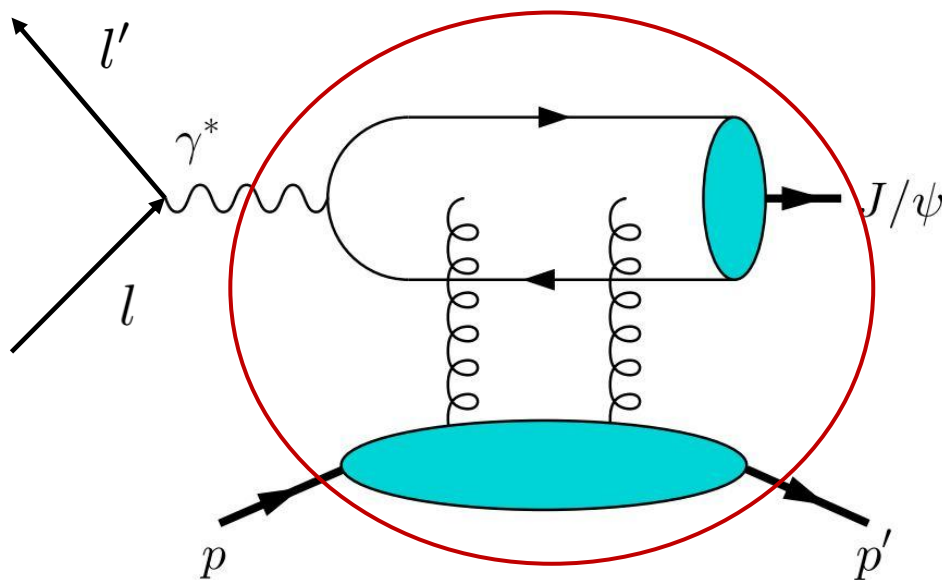
$$L_{\mu\nu} = 2(l_\mu l'_\nu + l_\nu l'_\mu) - g_{\mu\nu} Q^2$$

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Cross section

$$d\sigma = \frac{C}{2s} L_{\mu\nu} \mathcal{M}^\mu \mathcal{M}^{*\nu}$$



Unpolarized leptonic tensor:

$$L_{\mu\nu} = 2(l_\mu l'_\nu + l_\nu l'_\mu) - g_{\mu\nu} Q^2$$

**Hadronic tensor
(photon-helicity amplitudes):**

$$\mathcal{M}^\mu = \sum_{\lambda=0,\pm} \epsilon_\lambda^\mu \mathcal{M}_\lambda^{\gamma^* p}$$

$$\mathcal{M}^{*\nu} = \sum_{\lambda=0,\pm} \epsilon_\lambda^{*\nu} (\mathcal{M}_\lambda^{\gamma^* p})^*$$

Probing gluon OAM & spin-orbit correlations via vector-mesons



Cross section

Next, expand the hadronic tensor into 9 linearly-independent tensor structures (Koike, Nagashima, 2003):

$$\mathcal{M}^\mu \mathcal{M}^{*\nu} = \sum_{k=1}^9 \mathcal{V}_k^{\mu\nu} \left[\tilde{\mathcal{V}}_k^{\rho\sigma} \mathcal{M}_\rho \mathcal{M}_\sigma^* \right].$$

Probing gluon OAM & spin-orbit correlations via vector-mesons



Cross section

Next, expand the hadronic tensor into 9 linearly-independent tensor structures (Koike, Nagashima, 2003):

$$\mathcal{M}^\mu \mathcal{M}^{*\nu} = \sum_{k=1}^9 \mathcal{V}_k^{\mu\nu} [\tilde{\mathcal{V}}_k^{\rho\sigma} \mathcal{M}_\rho \mathcal{M}_\sigma^*].$$

Virtual-photon polarization states:

$$T^\mu = \epsilon_0^\mu, \quad X^\mu = -\frac{1}{\sqrt{2}} (\epsilon_+^\mu - \epsilon_-^\mu), \quad Y^\mu = -\frac{i}{\sqrt{2}} (\epsilon_+^\mu + \epsilon_-^\mu).$$



L polarized



T polarized
(positive/negative helicity)

$$\begin{aligned} \mathcal{V}_1^{\mu\nu} &= X^\mu X^\nu + Y^\mu Y^\nu, \\ \mathcal{V}_2^{\mu\nu} &= T^\mu T^\nu, \\ \mathcal{V}_3^{\mu\nu} &= T^\mu X^\nu + X^\mu T^\nu, \\ \mathcal{V}_4^{\mu\nu} &= X^\mu X^\nu - Y^\mu Y^\nu, \\ \mathcal{V}_5^{\mu\nu} &= i (T^\mu X^\nu - X^\mu T^\nu), \\ \mathcal{V}_6^{\mu\nu} &= i (X^\mu Y^\nu - Y^\mu X^\nu), \\ \mathcal{V}_7^{\mu\nu} &= i (T^\mu Y^\nu - Y^\mu T^\nu), \\ \mathcal{V}_8^{\mu\nu} &= T^\mu Y^\nu + Y^\mu T^\nu, \\ \mathcal{V}_9^{\mu\nu} &= X^\mu Y^\nu + Y^\mu X^\nu. \end{aligned}$$

Linearly
independent
tensor structures

Probing gluon OAM & spin-orbit correlations via vector-mesons



Cross section

Next, expand the hadronic tensor into 9 linearly-independent tensor structures (Koike, Nagashima, 2003):

$$\mathcal{M}^\mu \mathcal{M}^{*\nu} = \sum_{k=1}^9 \mathcal{V}_k^{\mu\nu} \left(\tilde{\mathcal{V}}_k^{\rho\sigma} \mathcal{M}_\rho \mathcal{M}_\sigma^* \right).$$

Virtual-photon polarization states:

$$T^\mu = \epsilon_0^\mu, \quad X^\mu = -\frac{1}{\sqrt{2}} (\epsilon_+^\mu - \epsilon_-^\mu), \quad Y^\mu = -\frac{i}{\sqrt{2}} (\epsilon_+^\mu + \epsilon_-^\mu).$$



L polarized



T polarized
(positive/negative helicity)

$$\begin{aligned} \tilde{\mathcal{V}}_1^{\mu\nu} &= \frac{1}{2} (X^\mu X^\nu + Y^\mu Y^\nu), \\ \tilde{\mathcal{V}}_2^{\mu\nu} &= T^\mu T^\nu, \\ \tilde{\mathcal{V}}_3^{\mu\nu} &= -\frac{1}{2} (T^\mu X^\nu + X^\mu T^\nu), \\ \tilde{\mathcal{V}}_4^{\mu\nu} &= \frac{1}{2} (X^\mu X^\nu - Y^\mu Y^\nu), \\ \tilde{\mathcal{V}}_5^{\mu\nu} &= \frac{i}{2} (T^\mu X^\nu - X^\mu T^\nu), \\ \tilde{\mathcal{V}}_6^{\mu\nu} &= -\frac{i}{2} (X^\mu Y^\nu - Y^\mu X^\nu), \\ \tilde{\mathcal{V}}_7^{\mu\nu} &= \frac{i}{2} (T^\mu Y^\nu - Y^\mu T^\nu), \\ \tilde{\mathcal{V}}_8^{\mu\nu} &= -\frac{1}{2} (T^\mu Y^\nu + Y^\mu T^\nu), \\ \tilde{\mathcal{V}}_9^{\mu\nu} &= \frac{1}{2} (X^\mu Y^\nu + Y^\mu X^\nu). \end{aligned}$$

Linearly
independent
Inverse tensor
structures

Probing gluon OAM & spin-orbit correlations via vector-mesons



Cross section

Using the hadronic decomposition, we can write our cross-section as:

$$d\sigma = \frac{\mathcal{C}}{2s} \sum_{k=1}^9 (L_{\mu\nu} \mathcal{V}_k^{\mu\nu}) \left(\tilde{\mathcal{V}}_k^{\rho\sigma} \mathcal{M}_\rho \mathcal{M}_\sigma^* \right).$$

Probing gluon OAM & spin-orbit correlations via vector-mesons



Cross section

Using the hadronic decomposition, we can write our cross-section as:

$$d\sigma = \frac{\mathcal{C}}{2s} \sum_{k=1}^9 (L_{\mu\nu} \mathcal{V}_k^{\mu\nu}) (\tilde{\mathcal{V}}_k^{\rho\sigma} \mathcal{M}_\rho \mathcal{M}_\sigma^*).$$

$$L_{\mu\nu} \mathcal{V}_1^{\mu\nu} = \frac{4Q^2(1-y+\frac{y^2}{2})}{y^2} * 1$$

$$L_{\mu\nu} \mathcal{V}_2^{\mu\nu} = \frac{4Q^2(1-y+\frac{y^2}{2})}{y^2} * \epsilon$$

$$L_{\mu\nu} \mathcal{V}_3^{\mu\nu} = \frac{4Q^2(1-y+\frac{y^2}{2})}{y^2} * (-1)\sqrt{2\epsilon(1+\epsilon)} * \cos\phi$$

$$L_{\mu\nu} \mathcal{V}_4^{\mu\nu} = \frac{4Q^2(1-y+\frac{y^2}{2})}{y^2} * \epsilon * \cos 2\phi$$

$$L_{\mu\nu} \mathcal{V}_8^{\mu\nu} = \frac{4Q^2(1-y+\frac{y^2}{2})}{y^2} * (-1)\sqrt{2\epsilon(1+\epsilon)} * \sin\phi$$

$$L_{\mu\nu} \mathcal{V}_9^{\mu\nu} = \frac{4Q^2(1-y+\frac{y^2}{2})}{y^2} * \epsilon * \sin 2\phi$$

Contraction of leptonic tensor with linearly-independent tensors yields distinct angular modulations

$$y = \frac{P \cdot q}{P \cdot l}$$

$$\epsilon = \frac{1-y}{1-y+\frac{1}{2}y^2}$$

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Contraction of hadronic tensor
with
linearly-independent
Inverse tensors

$$\tilde{\mathcal{V}}_1^{\mu\nu} \mathcal{M}_\mu \mathcal{M}_\nu^* = \frac{1}{2} (|\mathcal{M}_{Vp', \oplus+}|^2 + |\mathcal{M}_{Vp', \ominus+}|^2)$$

**Transversely-polarized
photon cross-section**

$$\tilde{\mathcal{V}}_2^{\mu\nu} \mathcal{M}_\mu \mathcal{M}_\nu^* = |\mathcal{M}_{Vp', 0+}|^2$$

$$\tilde{\mathcal{V}}_3^{\mu\nu} \mathcal{M}_\mu \mathcal{M}_\nu^* = -\frac{1}{\sqrt{2}} \text{Re}[\mathcal{M}_{Vp', 0+} (\mathcal{M}_{Vp', -+}^* - \mathcal{M}_{Vp', ++}^*)]$$

$$\tilde{\mathcal{V}}_4^{\mu\nu} \mathcal{M}_\mu \mathcal{M}_\nu^* = -\text{Re}[\mathcal{M}_{Vp', ++} \mathcal{M}_{Vp', -+}^*]$$

$$\tilde{\mathcal{V}}_8^{\mu\nu} \mathcal{M}_\mu \mathcal{M}_\nu^* = -\frac{1}{\sqrt{2}} \text{Im}[\mathcal{M}_{Vp', 0+}^* (\mathcal{M}_{Vp', -+} + \mathcal{M}_{Vp', ++})]$$

$$\tilde{\mathcal{V}}_9^{\mu\nu} \mathcal{M}_\mu \mathcal{M}_\nu^* = \text{Im}[\mathcal{M}_{Vp', ++} \mathcal{M}_{Vp', -+}^*]$$

$\mathcal{M}_{Vp', \gamma^* p}$

V = meson polarization

p' = outgoing proton helicity

γ^* = photon polarization

p = incoming proton helicity

Probing gluon OAM & spin-orbit correlations via vector-mesons



Cross section

Using the hadronic decomposition, we can write our cross-section as:

$$d\sigma = \frac{\mathcal{C}}{2s} \sum_{k=1}^9 (L_{\mu\nu} \mathcal{V}_k^{\mu\nu}) \left(\tilde{\mathcal{V}}_k^{\rho\sigma} \mathcal{M}_\rho \mathcal{M}_\sigma^* \right).$$

Contraction of hadronic tensor
with
linearly-independent
Inverse tensors

$$\tilde{\mathcal{V}}_1^{\mu\nu} \mathcal{M}_\mu \mathcal{M}_\nu^* = \frac{1}{2} (|\mathcal{M}_{Vp',++}|^2 + |\mathcal{M}_{Vp',-+}|^2)$$

$$\tilde{\mathcal{V}}_2^{\mu\nu} \mathcal{M}_\mu \mathcal{M}_\nu^* = |\mathcal{M}_{Vp',0+}|^2$$



**Longitudinally-polarized
photon cross-section**

$$\tilde{\mathcal{V}}_3^{\mu\nu} \mathcal{M}_\mu \mathcal{M}_\nu^* = -\frac{1}{\sqrt{2}} \text{Re}[\mathcal{M}_{Vp',0+} (\mathcal{M}_{Vp',-+}^* - \mathcal{M}_{Vp',++}^*)]$$

$$\tilde{\mathcal{V}}_4^{\mu\nu} \mathcal{M}_\mu \mathcal{M}_\nu^* = -\text{Re}[\mathcal{M}_{Vp',++} \mathcal{M}_{Vp',-+}^*]$$

$$\tilde{\mathcal{V}}_8^{\mu\nu} \mathcal{M}_\mu \mathcal{M}_\nu^* = -\frac{1}{\sqrt{2}} \text{Im}[\mathcal{M}_{Vp',0+}^* (\mathcal{M}_{Vp',-+} + \mathcal{M}_{Vp',++})]$$

$$\tilde{\mathcal{V}}_9^{\mu\nu} \mathcal{M}_\mu \mathcal{M}_\nu^* = \text{Im}[\mathcal{M}_{Vp',++} \mathcal{M}_{Vp',-+}^*]$$

$$\mathcal{M}_{Vp',\gamma^*p}$$

V = meson polarization

p' = outgoing proton helicity

γ^* = photon polarization

p = incoming proton helicity

Probing gluon OAM & spin-orbit correlations via vector-mesons



Cross section

Using the hadronic decomposition, we can write our cross-section as:

$$d\sigma = \frac{\mathcal{C}}{2s} \sum_{k=1}^9 (L_{\mu\nu} \mathcal{V}_k^{\mu\nu}) \left(\tilde{\mathcal{V}}_k^{\rho\sigma} \mathcal{M}_\rho \mathcal{M}_\sigma^* \right).$$

Contraction of hadronic tensor
with
linearly-independent
Inverse tensors

$$\tilde{\mathcal{V}}_1^{\mu\nu} \mathcal{M}_\mu \mathcal{M}_\nu^* = \frac{1}{2} (|\mathcal{M}_{Vp',++}|^2 + |\mathcal{M}_{Vp',-+}|^2)$$

$$\tilde{\mathcal{V}}_2^{\mu\nu} \mathcal{M}_\mu \mathcal{M}_\nu^* = |\mathcal{M}_{Vp',0+}|^2$$

$$\tilde{\mathcal{V}}_3^{\mu\nu} \mathcal{M}_\mu \mathcal{M}_\nu^* = -\frac{1}{\sqrt{2}} \text{Re}[\mathcal{M}_{Vp',0+} (\mathcal{M}_{Vp',-+}^* - \mathcal{M}_{Vp',++}^*)]$$

$$\tilde{\mathcal{V}}_4^{\mu\nu} \mathcal{M}_\mu \mathcal{M}_\nu^* = -\text{Re}[\mathcal{M}_{Vp',++} \mathcal{M}_{Vp',-+}^*]$$

$$\tilde{\mathcal{V}}_8^{\mu\nu} \mathcal{M}_\mu \mathcal{M}_\nu^* = -\frac{1}{\sqrt{2}} \text{Im}[\mathcal{M}_{Vp',0+}^* (\mathcal{M}_{Vp',-+} + \mathcal{M}_{Vp',++})]$$

$$\tilde{\mathcal{V}}_9^{\mu\nu} \mathcal{M}_\mu \mathcal{M}_\nu^* = \text{Im}[\mathcal{M}_{Vp',++} \mathcal{M}_{Vp',-+}^*]$$

**Interference terms
between
different photon
polarizations**

$$\mathcal{M}_{Vp',\gamma^*p}$$

V = meson polarization

p' = outgoing proton helicity

γ^* = photon polarization

p = incoming proton helicity

Probing gluon OAM & spin-orbit correlations via vector-mesons



Cross section

$$\frac{d\sigma}{dydQ^2dtd\phi} = \frac{\alpha_{\text{em}}}{2\pi^2} \frac{\left(1 - y + \frac{y^2}{2}\right)}{yQ^2} \left[\frac{d\sigma_T}{dt} + \epsilon \frac{d\sigma_L}{dt} + \sqrt{2\epsilon(1+\epsilon)} \cos \phi \frac{d\sigma_{LT}}{dt} + \epsilon \cos 2\phi \frac{d\sigma_{TT}}{dt} \right. \\ \left. + \sqrt{2\epsilon(1+\epsilon)} \sin \phi \frac{d\sigma_{\sin\phi}}{dt} + \epsilon \sin 2\phi \frac{d\sigma_{\sin 2\phi}}{dt} \right]$$

Probing gluon OAM & spin-orbit correlations via vector-mesons



Cross section

$$\frac{d\sigma}{dydQ^2dtd\phi} = \frac{\alpha_{em}}{2\pi^2} \frac{\left(1 - y + \frac{y^2}{2}\right)}{yQ^2} \left(\left[\frac{d\sigma_T}{dt} + \epsilon \frac{d\sigma_L}{dt} + \sqrt{2\epsilon(1+\epsilon)} \cos \phi \frac{d\sigma_{LT}}{dt} + \epsilon \cos 2\phi \frac{d\sigma_{TT}}{dt} + \sqrt{2\epsilon(1+\epsilon)} \sin \phi \frac{d\sigma_{\sin\phi}}{dt} + \epsilon \sin 2\phi \frac{d\sigma_{\sin 2\phi}}{dt} \right] \right)$$

Remarks:

1) For definite circular polarization of the meson, only one term survives:

$$\frac{d\sigma_T^+}{dt} = \frac{x_B^2}{32\pi Q^4} C^2 \left\{ \delta_{\lambda,\lambda'} |\mathcal{H}_{eff}^{TT}|^2 + \delta_{\lambda,-\lambda'} |\vec{\Delta}_\perp|^2 |\tilde{\mathcal{E}}^{TT}|^2 \right\}$$

$$\frac{d\sigma_T^-}{dt} = \frac{x_B^2}{32\pi Q^4} C^2 \left\{ \delta_{\lambda,\lambda'} |\mathcal{H}_{eff}^{TT}|^2 + \delta_{\lambda,-\lambda'} |\vec{\Delta}_\perp|^2 |\tilde{\mathcal{E}}^{TT}|^2 \right\}$$

**With transversely-polarized mesons,
the cross section is
only sensitive to GPDs**

Probing gluon OAM & spin-orbit correlations via vector-mesons



Cross section

$$\frac{d\sigma}{dydQ^2dtd\phi} = \frac{\alpha_{\text{em}}}{2\pi^2} \frac{\left(1 - y + \frac{y^2}{2}\right)}{yQ^2} \left[\frac{d\sigma_T}{dt} + \epsilon \frac{d\sigma_L}{dt} + \sqrt{2\epsilon(1+\epsilon)} \cos \phi \frac{d\sigma_{LT}}{dt} + \epsilon \cos 2\phi \frac{d\sigma_{TT}}{dt} \right. \\ \left. + \sqrt{2\epsilon(1+\epsilon)} \sin \phi \frac{d\sigma_{\sin\phi}}{dt} + \epsilon \sin 2\phi \frac{d\sigma_{\sin 2\phi}}{dt} \right]$$

Remarks:

- 2) **For the longitudinally polarized vector meson case, the cross section can be expressed in terms of GTMDs.**
In general, all structure functions contribute; here, we present two representative examples.

Probing gluon OAM & spin-orbit correlations via vector-mesons



Cross section

$$\frac{d\sigma}{dydQ^2dtd\phi} = \frac{\alpha_{em}}{2\pi^2} \frac{\left(1 - y + \frac{y^2}{2}\right)}{yQ^2} \left[\frac{d\sigma_T}{dt} + \epsilon \frac{d\sigma_L}{dt} + \sqrt{2\epsilon(1+\epsilon)} \cos \phi \frac{d\sigma_{LT}}{dt} + \epsilon \cos 2\phi \frac{d\sigma_{TT}}{dt} + \sqrt{2\epsilon(1+\epsilon)} \sin \phi \frac{d\sigma_{\sin\phi}}{dt} + \epsilon \sin 2\phi \frac{d\sigma_{\sin 2\phi}}{dt} \right]$$

Remarks:

2) Examples of longitudinally-polarized vector meson cross section:

$$\frac{d\sigma_{TT}^0}{dt} = -\frac{x_B^2}{16\pi Q^4} C^2 \left\{ \delta_{\lambda,\lambda'} \left[-\frac{1}{2} |\vec{\Delta}_\perp|^2 |\mathcal{H}_{eff}^{TL} + \mathcal{F}_{1,1}^{TL} + \boxed{\mathcal{G}_{1,1}^{TL}}|^2 + \frac{1}{2} |\vec{\Delta}_\perp|^2 \left| \boxed{\mathcal{F}_{1,4}^{TL}} - \mathcal{H}'_{eff}^{TL} - \mathcal{G}_{1,4}^{TL} \right|^2 \right] \right\},$$

$$\frac{d\sigma_{\sin 2\phi}^0}{dt} = \frac{x_B^2}{16\pi Q^4} C^2 \left\{ -\delta_{\lambda,\lambda'} \lambda |\vec{\Delta}_\perp|^2 \operatorname{Re} \left[(\mathcal{H}_{eff}^{TL} + \mathcal{F}_{1,1}^{TL} + \boxed{\mathcal{G}_{1,1}^{TL}}) (\boxed{\mathcal{F}_{1,4}^{TL*}} - \mathcal{H}'_{eff}^{TL*} - \mathcal{G}_{1,4}^{TL*}) \right] \right\}.$$

Gluon OAM

Gluon's spin-orbit correlations

Probing gluon OAM & spin-orbit correlations via vector-mesons



Cross section

$$\frac{d\sigma}{dydQ^2dtd\phi} = \frac{\alpha_{\text{em}}}{2\pi^2} \frac{\left(1 - y + \frac{y^2}{2}\right)}{yQ^2} \left[\frac{d\sigma_T}{dt} + \epsilon \frac{d\sigma_L}{dt} + \sqrt{2\epsilon(1+\epsilon)} \cos \phi \frac{d\sigma_{LT}}{dt} + \epsilon \cos 2\phi \frac{d\sigma_{TT}}{dt} + \sqrt{2\epsilon(1+\epsilon)} \sin \phi \frac{d\sigma_{\sin\phi}}{dt} + \epsilon \sin 2\phi \frac{d\sigma_{\sin 2\phi}}{dt} \right]$$

Remarks:

2) Examples of longitudinally-polarized vector meson cross section:

$$\frac{d\sigma_{TT}^0}{dt} = -\frac{x_B^2}{16\pi Q^4} C^2 \left\{ \delta_{\lambda,\lambda'} \left[-\frac{1}{2} |\vec{\Delta}_\perp|^2 |\mathcal{H}_{eff}^{TL} + \mathcal{F}_{1,1}^{TL} + \boxed{\mathcal{G}_{1,1}^{TL}|^2} + \frac{1}{2} |\vec{\Delta}_\perp|^2 \left| \boxed{\mathcal{F}_{1,4}^{TL}} - \mathcal{H}'_{eff}^{TL} - \mathcal{G}_{1,4}^{TL} \right|^2 \right] \right\},$$

The **cos 2φ** modulation stands out as a particularly striking signal: it can be measured with an unpolarized proton. Thus, this signal provides a clean laboratory signal for probing the **gluon OAM** and **gluon spin-orbit correlations**.

Axial-vector production



Brief remarks:

- For the vector meson results, we assumed that the quark anti-quark pair shared an equal amount of the meson's momentum ($z=1/2$).

Axial-vector production



Brief remarks:

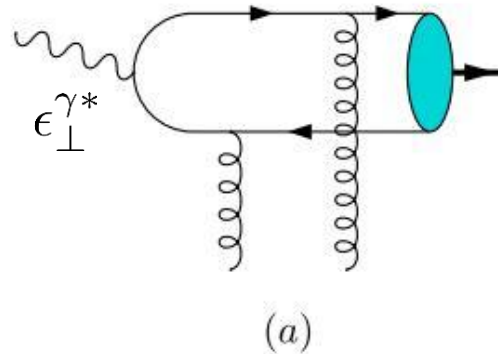
- For the vector meson results, we assumed that the quark anti-quark pair shared an equal amount of the meson's momentum ($z=1/2$). However, for the axial-vector case, both the twist-2 and twist-3 contributions **vanish** at the symmetric point $z = 1/2$. Thus, obtaining a nonvanishing result requires working away from this point (see Appendix of arXiv: 2601.17506).

Axial-vector production

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Example: Diagram “a”



Notation: $\mathcal{M}_{twist}^{\gamma*, M}$

$$\mathcal{H}_{t3, k_{\perp}}^{T, L} = - \frac{8im_{AV} (m_{AV}^2 - Q^2) \xi^2 (1-2z)^2 \epsilon_{\perp}^{ij} (\epsilon_{\perp}^{\gamma* i} k_{\perp}^j)}{(m_{AV}^2 (2x(z-1) + \xi(4z^2 - 6z + 1)) + 2Q^2(z-1)(\xi + x)) (m_{AV}^2 (2xz + \xi(4z^2 - 2z - 1)) + 2Q^2 z(x - \xi))}$$

Axial-vector production



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- For the vector meson results, we assumed that the quark anti-quark pair shared an equal amount of the meson's momentum ($z=1/2$). However, for the axial-vector case, both the twist-2 and twist-3 contributions **vanish** at the symmetric point $z = 1/2$. Thus, obtaining a nonvanishing result requires working away from this point.
- Complementary opportunities in axial-vector meson production



Outline

- Generalized **TMDs** (GTMDs) & connection to spin physics
- Observable(s) for gluon **OAM** & **spin-orbit correlations**
- Opportunities (My wish list)
- Summary

Opportunities (My wish list)

- 1) A fully transverse-momentum-dependent treatment of the meson side would more naturally be formulated in terms of a meson light-front wave function rather than a purely collinear distribution amplitude.

Recall:

Scattering amplitude:

Hard part

$$\mathcal{M} \propto \int dx \int d^2 k_{\perp} H(x, \xi, z, k_{\perp}, \Delta_{\perp}) f^g(x, \xi, k_{\perp}, \Delta_{\perp}) \int dz \phi_V(z)$$

Soft part from
proton

Meson
Distribution Amplitude

(No transverse-momentum
dependence here –
I will comment on this later)

Opportunities (My wish list)



- 1) A fully transverse-momentum-dependent treatment of the meson side would more naturally be formulated in terms of a meson light-front wave function rather than a purely collinear distribution amplitude.

In particular, for light vector meson production, phenomenological approaches often retain the quark transverse momentum in the meson wave function and in the hard subprocess, together with Sudakov suppression (to handle end-point singularities): **Kornelija's talk**

Opportunities (My wish list)



- 1) A fully transverse-momentum-dependent treatment of the meson side would more naturally be formulated in terms of a meson light-front wave function rather than a purely collinear distribution amplitude.

In particular, for light vector meson production, phenomenological approaches often retain the quark transverse momentum in the meson wave function and in the hard subprocess, together with Sudakov suppression:

Vector Meson Electroproduction at small Bjorken- x and Generalized Parton Distributions

S.V. Goloskokov ¹

*Bogoliubov Laboratory of Theoretical Physics, Joint Institute for Nuclear Research,
Dubna 141980, Moscow region, Russia*

P. Kroll ²

Fachbereich Physik, Universität Wuppertal, D-42097 Wuppertal, Germany

The Longitudinal Cross Section of Vector Meson Electroproduction

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Dubna 141980, Moscow region, Russia*

P. Kroll ²

Fachbereich Physik, Universität Wuppertal, D-42097 Wuppertal, Germany

Deeply virtual meson production on the nucleon^{*}

L. Favart¹, M. Guidal^{2,a}, T. Horn^{3,4}, and P. Kroll⁵

Opportunities (My wish list)



- 1) A fully transverse-momentum-dependent treatment of the meson side would more naturally be formulated in terms of a meson light-front wave function rather than a purely collinear distribution amplitude.
- 2) The heavy-quarkonium side is represented at leading order in the velocity expansion by the appropriate NRQCD matrix element (or, equivalently, the wave function at the origin), leaving relativistic corrections for future work.

Opportunities (My wish list)



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Relativistic corrections to exclusive photoproduction of Quarkonia near-threshold

Sarah K. Blask,¹ Sean Fleming,¹ Thomas Mehen,² Jyotirmoy Roy,²
Iain W. Stewart,^{3,4} Fanyi Zhao³

Opportunities (My wish list)



- 1) A fully transverse-momentum-dependent treatment of the meson side would more naturally be formulated in terms of a meson light-front wave function rather than a purely collinear distribution amplitude.
- 2) The heavy-quarkonium side is represented at leading order in the velocity expansion by the appropriate NRQCD matrix element (or, equivalently, the wave function at the origin), leaving relativistic corrections for future work.
- 3) Here we focused on the nucleon-side kinematic twist-3 effects that gave access to gluon GTMD moments, leaving genuine twist-3 effects for future work.

Opportunities (My wish list)



- 1) A fully transverse-momentum-dependent treatment of the meson side would more naturally be formulated in terms of a meson light-front wave function rather than a purely collinear distribution amplitude.
- 2) The heavy-quarkonium side is represented at leading order in the velocity expansion by the appropriate NRQCD matrix element (or, equivalently, the wave function at the origin), leaving relativistic corrections for future work.
- 3) Here we focused on the nucleon-side kinematic twist-3 effects that gave access to gluon GTMD moments, leaving genuine twist-3 effects for future work.

Twist-3 contributions to wide-angle photoproduction of pions

P. Kroll

Fachbereich Physik, Universität Wuppertal, D-42097 Wuppertal, Germany

K. Passek-Kumerički

*Theoretical Physics Division, Rudjer Bošković Institute, HR-10002 Zagreb,
Croatia*

Opportunities (My wish list)



4) Comparison between theory and data

Opportunities (My wish list)



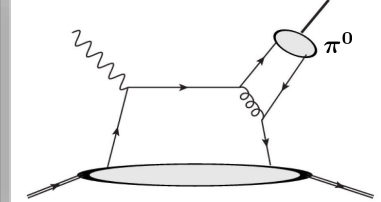
4) Comparison between theory and data

Example:

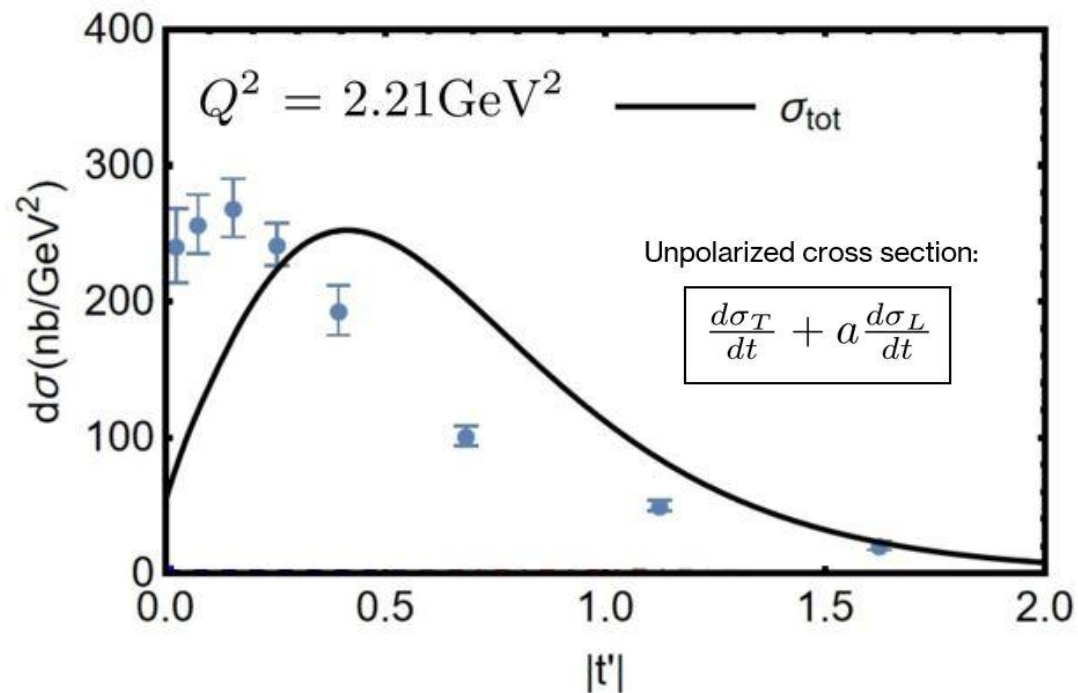
PHYSICAL REVIEW LETTERS **133**, 051901 (2024)

Probing the Quark Orbital Angular Momentum at Electron-Ion Colliders Using Exclusive π^0 Production

Shohini Bhattacharya¹, Duxin Zheng², and Jian Zhou³



Comparison with CLAS data (I. Bedlinskiy et al., Phys. Rev. C 90, 025205 (2014))



Findings:

- Our theoretical model is in reasonable agreement with experimental data

More opportunities with CLAS's recent data on single spin asymmetry

Opportunities (My wish list)

4) Comparison between theory and data

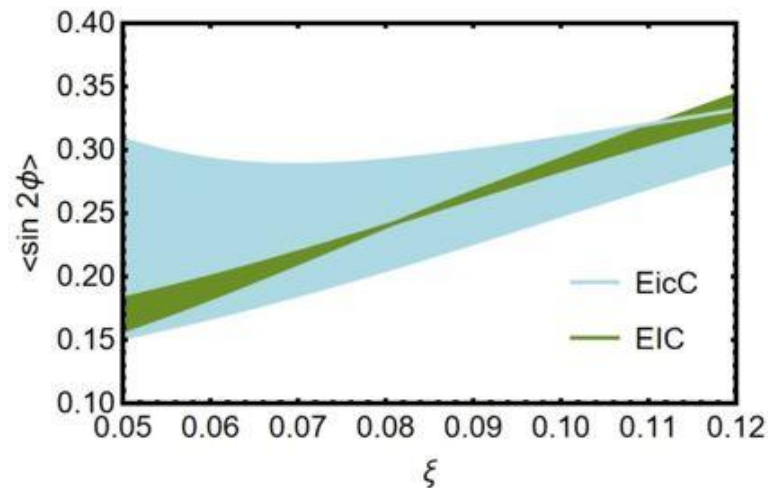
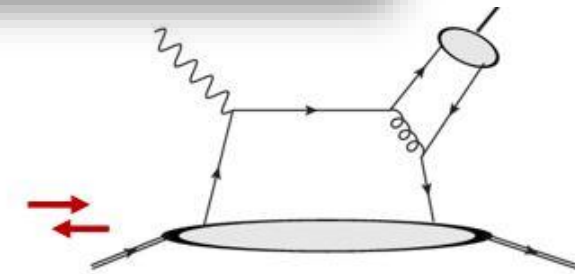
Example of a prediction:

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Probing the Quark Orbital Angular Momentum at Electron-Ion Colliders Using Exclusive π^0 Production

Shohini Bhattacharya¹, Duxin Zheng², and Jian Zhou³

Probe **quark OAM** via exclusive π^0 production in ep collisions



- Asymmetry is substantial & thus exclusive π^0 production in ep collisions maybe a promising route to constrain quark OAM

$$\frac{d\sigma}{dtdQ^2 dx_B d\phi} = \frac{(N_c^2 - 1)^2 \alpha_{em}^2 \alpha_s^2 f_\pi^2 \xi^3 \Delta_\perp^2}{2N_c^4 (1 - \xi^2) Q^{10} (1 + \xi)} [1 + (1 - y)^2] \times \left\{ |\mathcal{F}_{1,1} + \mathcal{G}_{1,1}|^2 + |\mathcal{F}_{1,4} + \mathcal{G}_{1,4}|^2 + 2 \frac{M^2}{\Delta_\perp^2} |\mathcal{F}_{1,2} + \mathcal{G}_{1,2}|^2 \right. \\ \left. + \lambda \sin(2\phi) 2a \operatorname{Re} [(i\mathcal{F}_{1,4} + i\mathcal{G}_{1,4}) (\mathcal{F}_{1,1}^* + \mathcal{G}_{1,1}^*)] \right\}$$

Opportunities (My wish list)



4) Comparison between theory and data:

Data from COMPASS and CLAS are needed to pursue exciting opportunities and test theory across a wide range of vector meson channels.

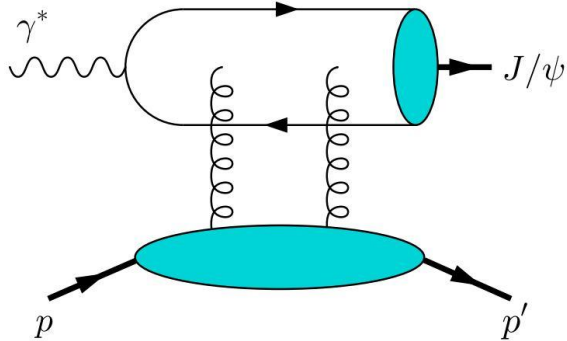


Outline

- Generalized **TMDs** (GTMDs) & connection to spin physics
- Observable(s) for gluon **OAM** & **spin-orbit correlations**
- Opportunities
- **Summary**

Summary

- Probe **gluon OAM and spin-orbit correlations** via exclusive heavy (axial-) vector production



$$\frac{d\sigma}{dydQ^2dtd\phi} = \frac{\alpha_{\text{em}}}{2\pi^2} \frac{\left(1 - y + \frac{y^2}{2}\right)}{yQ^2} \left[\frac{d\sigma_T}{dt} + \epsilon \frac{d\sigma_L}{dt} + \sqrt{2\epsilon(1+\epsilon)} \cos \phi \frac{d\sigma_{LT}}{dt} + \epsilon \cos 2\phi \frac{d\sigma_{TT}}{dt} + \sqrt{2\epsilon(1+\epsilon)} \sin \phi \frac{d\sigma_{\sin \phi}}{dt} + \epsilon \sin 2\phi \frac{d\sigma_{\sin 2\phi}}{dt} \right]$$

$$\frac{d\sigma_{TT}^0}{dt} = -\frac{x_B^2}{16\pi Q^4} C^2 \left\{ \delta_{\lambda,\lambda'} \left[-\frac{1}{2} |\vec{\Delta}_\perp|^2 |\mathcal{H}_{eff}^{TL} + \mathcal{F}_{1,1}^{TL} + \mathcal{G}_{1,1}^{TL}|^2 + \frac{1}{2} |\vec{\Delta}_\perp|^2 \left| \mathcal{F}_{1,4}^{TL} - \mathcal{H}_{eff}'^{TL} - \mathcal{G}_{1,4}^{TL} \right|^2 \right] \right\}$$

The $\cos 2\phi$ modulation stands out as a striking signal: it can be measured with an unpolarized proton

- Rich prospects with COMPASS and CLAS data

Backup slides

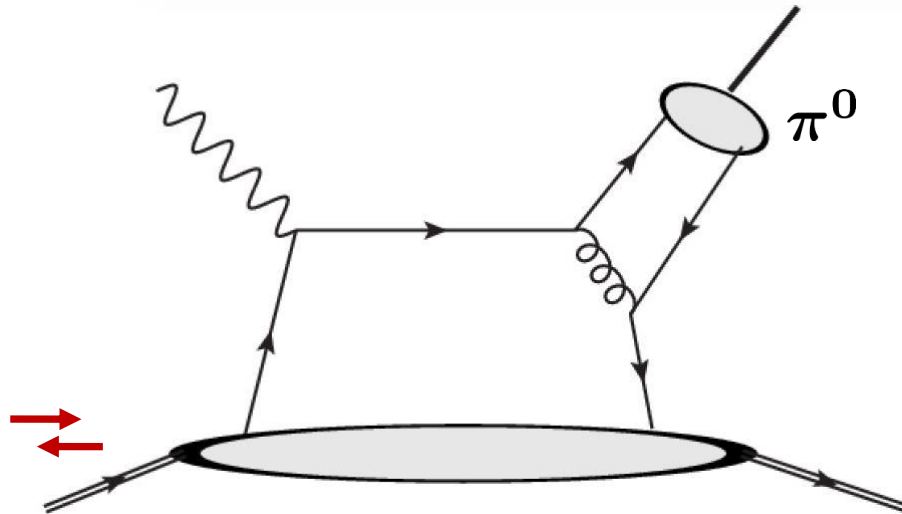
Our work



PHYSICAL REVIEW LETTERS **133**, 051901 (2024)

Probing the Quark Orbital Angular Momentum at Electron-Ion Colliders Using Exclusive π^0 Production

Shohini Bhattacharya¹, Duxin Zheng², and Jian Zhou³



Main Observable:

Longitudinal single-target spin asymmetry

Probing quark OAM through π^0 production in ep collisions



Cross section

$$\begin{aligned} \frac{d\sigma}{dt dQ^2 dx_B d\phi} &= \frac{(N_c^2 - 1)^2 \alpha_{em}^2 \alpha_s^2 f_\pi^2 \xi^3 \Delta_\perp^2}{2N_c^4 (1 - \xi^2) Q^{10} (1 + \xi)} [1 + (1 - y)^2] \\ &\times \left\{ \left[|\mathcal{F}_{1,1} + \mathcal{G}_{1,1}|^2 + |\mathcal{F}_{1,4} + \mathcal{G}_{1,4}|^2 + 2 \frac{M^2}{\Delta_\perp^2} |\mathcal{F}_{1,2} + \mathcal{G}_{1,2}|^2 \right] + \cos(2\phi) a \left[-|\mathcal{F}_{1,1} + \mathcal{G}_{1,1}|^2 + |\mathcal{F}_{1,4} + \mathcal{G}_{1,4}|^2 \right] \right. \\ &\quad \left. + \lambda \sin(2\phi) 2a \operatorname{Re} \left[(i\mathcal{F}_{1,4} + i\mathcal{G}_{1,4}) (\mathcal{F}_{1,1}^* + \mathcal{G}_{1,1}^*) \right] \right\} \end{aligned}$$

Unpolarized

Single spin asymmetry

$a = \frac{2(1-y)}{1+(1-y)^2}$

Probing quark OAM through π^0 production in ep collisions



Cross section

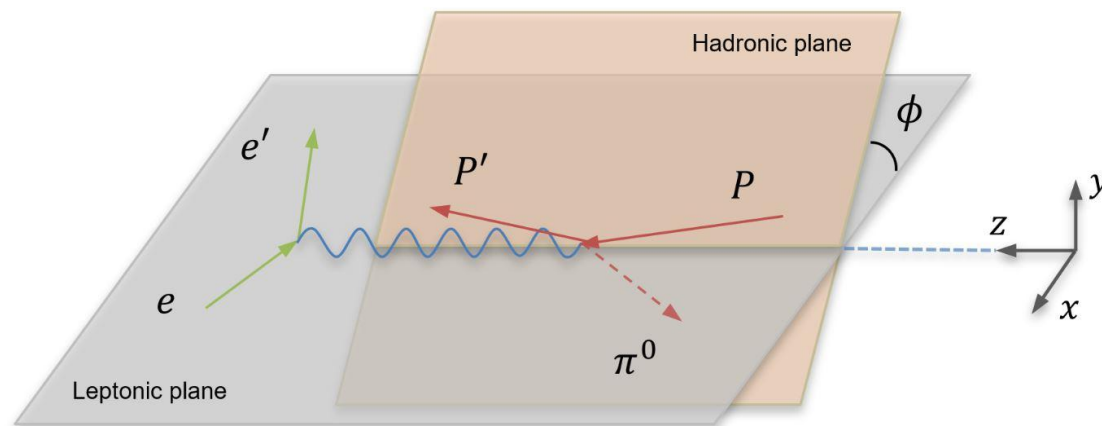
$$\frac{d\sigma}{dtdQ^2dx_Bd\phi} = \frac{(N_c^2 - 1)^2 \alpha_{em}^2 \alpha_s^2 f_\pi^2 \xi^3 \Delta_\perp^2}{2N_c^4 (1 - \xi^2) Q^{10} (1 + \xi)} [1 + (1 - y)^2]$$

$$\times \left\{ \left[|\mathcal{F}_{1,1} + \mathcal{G}_{1,1}|^2 + |\mathcal{F}_{1,4} + \mathcal{G}_{1,4}|^2 + 2 \frac{M^2}{\Delta_\perp^2} |\mathcal{F}_{1,2} + \mathcal{G}_{1,2}|^2 \right] + \cos(2\phi) a \left[-|\mathcal{F}_{1,1} + \mathcal{G}_{1,1}|^2 + |\mathcal{F}_{1,4} + \mathcal{G}_{1,4}|^2 \right] \right.$$

$$\left. + \lambda \sin(2\phi) 2a \operatorname{Re} \left[(i\mathcal{F}_{1,4} + i\mathcal{G}_{1,4}) (\mathcal{F}_{1,1}^* + \mathcal{G}_{1,1}^*) \right] \right\}$$

Distinguished experimental signature of quark OAM

$$\phi = \phi_{l_\perp} - \phi_{\Delta_\perp}$$



Probing quark OAM through π^0 production in ep collisions



Cross section

$$\frac{d\sigma}{dtdQ^2dx_Bd\phi} = \frac{(N_c^2 - 1)^2 \alpha_{em}^2 \alpha_s^2 f_\pi^2 \xi^3 \Delta_\perp^2}{2N_c^4 (1 - \xi^2) Q^{10} (1 + \xi)} [1 + (1 - y)^2]$$

$$\times \left\{ \left[|\mathcal{F}_{1,1} + \mathcal{G}_{1,1}|^2 + |\mathcal{F}_{1,4} + \mathcal{G}_{1,4}|^2 + 2 \frac{M^2}{\Delta_\perp^2} |\mathcal{F}_{1,2} + \mathcal{G}_{1,2}|^2 \right] + \cos(2\phi) a \left[-|\mathcal{F}_{1,1} + \mathcal{G}_{1,1}|^2 + |\mathcal{F}_{1,4} + \mathcal{G}_{1,4}|^2 \right] \right.$$

$$\left. + \lambda \sin(2\phi) 2a \operatorname{Re} \left[(i\mathcal{F}_{1,4} + i\mathcal{G}_{1,4}) (\mathcal{F}_{1,1}^* + \mathcal{G}_{1,1}^*) \right] \right\}$$

Surprise!

- Probe quark Sivers function through an unpolarized target

$$\operatorname{Im} [\mathcal{F}_{1,2}]|_{\Delta=0} = -f_{1T}^\perp$$

(Similar to the gluon GTMD $F_{1,2}$, as discussed in Boussarie, Hatta, Szymanowski, Wallon, 2019)

Probing quark OAM through π^0 production in ep collisions



Cross section

$$\frac{d\sigma}{dtdQ^2dx_Bd\phi} = \frac{(N_c^2 - 1)^2 \alpha_{em}^2 \alpha_s^2 f_\pi^2 \xi^3 \Delta_\perp^2}{2N_c^4 (1 - \xi^2) Q^{10} (1 + \xi)} [1 + (1 - y)^2]$$

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$$\left. + \lambda \sin(2\phi) 2a \operatorname{Re} [(i\mathcal{F}_{1,4} + i\mathcal{G}_{1,4}) (\mathcal{F}_{1,1}^* + \mathcal{G}_{1,1}^*)] \right\} \quad \uparrow \quad \text{Surprise!}$$

- Probe quark Sivers function through an unpolarized target

$$\operatorname{Im} [\mathcal{F}_{1,2}]|_{\Delta=0} = -f_{1T}^\perp$$

- Probe quark worm gear function through an unpolarized target

$$\operatorname{Re} [\mathcal{G}_{1,2}]|_{\Delta=0} = g_{1T}$$

Probing quark OAM through π^0 production in ep collisions



Cross section

$$\frac{d\sigma}{dtdQ^2dx_Bd\phi} = \frac{(N_c^2 - 1)^2 \alpha_{em}^2 \alpha_s^2 f_\pi^2 \xi^3 \Delta_\perp^2}{2N_c^4 (1 - \xi^2) Q^{10} (1 + \xi)} [1 + (1 - y)^2]$$

$$\times \left\{ \left[|\mathcal{F}_{1,1} + \mathcal{G}_{1,1}|^2 + |\mathcal{F}_{1,4} + \mathcal{G}_{1,4}|^2 + 2 \frac{M^2}{\Delta_\perp^2} |\mathcal{F}_{1,2} + \mathcal{G}_{1,2}|^2 \right] + \cos(2\phi) a \left[-|\mathcal{F}_{1,1} + \mathcal{G}_{1,1}|^2 + |\mathcal{F}_{1,4} + \mathcal{G}_{1,4}|^2 \right] \right.$$

$$\left. + \lambda \sin(2\phi) 2a \operatorname{Re} \left[(i\mathcal{F}_{1,4} + i\mathcal{G}_{1,4}) (\mathcal{F}_{1,1}^* + \mathcal{G}_{1,1}^*) \right] \right\}$$



Helicity flip terms persist even when $\Delta_\perp \rightarrow 0$

Probing quark OAM through π^0 production in ep collisions



Cross section

$$\begin{aligned} \frac{d\sigma}{dtdQ^2dx_Bd\phi} = & \frac{(N_c^2 - 1)^2 \alpha_{em}^2 \alpha_s^2 f_\pi^2 \xi^3 \Delta_\perp^2}{2N_c^4 (1 - \xi^2) Q^{10} (1 + \xi)} [1 + (1 - y)^2] \\ & \times \left\{ \left[|\mathcal{F}_{1,1} + \mathcal{G}_{1,1}|^2 + |\mathcal{F}_{1,4} + \mathcal{G}_{1,4}|^2 + 2 \frac{M^2}{\Delta_\perp^2} |\mathcal{F}_{1,2} + \mathcal{G}_{1,2}|^2 \right] + \cos(2\phi) a \left[-|\mathcal{F}_{1,1} + \mathcal{G}_{1,1}|^2 + |\mathcal{F}_{1,4} + \mathcal{G}_{1,4}|^2 \right] \right. \\ & \left. + \lambda \sin(2\phi) 2a \operatorname{Re} \left[(i\mathcal{F}_{1,4} + i\mathcal{G}_{1,4}) (\mathcal{F}_{1,1}^* + \mathcal{G}_{1,1}^*) \right] \right\} \end{aligned}$$

Since both unpolarized and polarized cross sections contribute at twist-3, the magnitudes of the asymmetries are not power-suppressed

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Theoretical complications

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Theoretical complications

End-point singularity

$$\mathcal{F}_{1,4} = \int_{-1}^1 dx \frac{x\xi \int d^2 k_{\perp} k_{\perp}^2 F_{1,4}^{u+d}(x, \xi, \Delta_{\perp}, k_{\perp})}{M^2(x + \xi - i\epsilon)^2(x - \xi + i\epsilon)^2} \times \int_0^1 dz \frac{\phi_{\pi}(z)(1 + z^2 - z)}{z^2(1 - z)^2}$$

Model-dependent method:

$$\int_{\langle p_{\perp}^2 \rangle / Q^2}^{1 - \langle p_{\perp}^2 \rangle / Q^2} dz$$

$\langle p_{\perp}^2 \rangle = 0.04 \text{ GeV}^2$ determined based on a fit to CLAS data

S. V. Goloskokov and P. Kroll, 2005

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Theoretical complications

End-point singularity & discontinuity:

$$\mathcal{F}_{1,4} = \int_{-1}^1 dx \frac{x\xi \int d^2 k_{\perp} k_{\perp}^2 F_{1,4}^{u+d}(x, \xi, \Delta_{\perp}, k_{\perp})}{M^2 (\mathbf{x} + \boldsymbol{\xi} - i\epsilon)^2 (\mathbf{x} - \boldsymbol{\xi} + i\epsilon)^2} \times \int_0^1 dz \frac{\phi_{\pi}(z)(1+z^2-z)}{z^2(1-z)^2}$$

Model-dependent method:

$$\int_{\langle p_{\perp}^2 \rangle / Q^2}^{1 - \langle p_{\perp}^2 \rangle / Q^2} dz$$

S. V. Goloskokov and P. Kroll, 2005

$$\frac{1}{(x - \xi + i\epsilon)^2} \rightarrow \frac{1}{(x - \xi - \langle p_{\perp}^2 \rangle / Q^2 + i\epsilon)^2}$$

I. V. Anikin, O. V. Teryaev, 2003

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Numerical results

Kinematics:

	$Q^2(\text{GeV}^2)$	$\sqrt{s}_{ep}(\text{GeV})$
EIC	10	100
EicC	3	16

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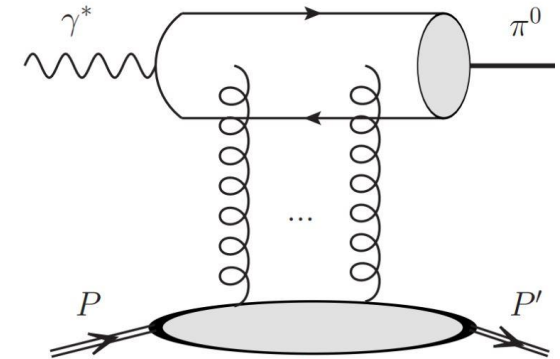


Numerical results

Kinematics:

	$Q^2(\text{GeV}^2)$	$\sqrt{s_{ep}}(\text{GeV})$
EIC	10	100
EicC	3	16

- We focus on large skewness (ξ) region to suppress gluon contribution



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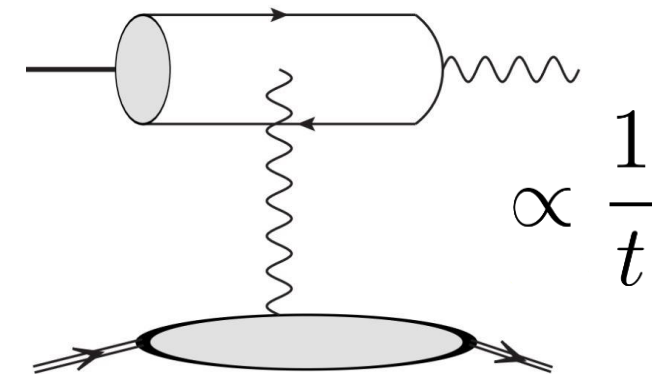


Numerical results

Kinematics:

	$Q^2(\text{GeV}^2)$	$\sqrt{s_{ep}}(\text{GeV})$
EIC	10	100
EicC	3	16

- We focus on large skewness (ξ) region to suppress gluon contribution
- We focus on large momentum transfer (t) region to suppress contribution from Primakoff process



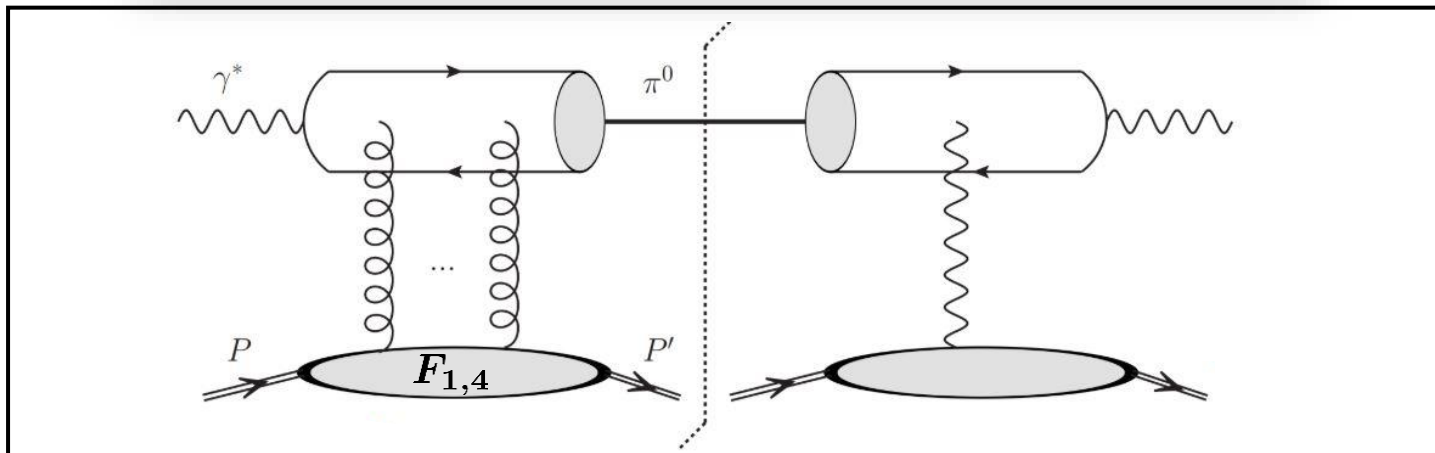
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Remark:

Accessing the gluon GTMD $F_{1,4}$ in exclusive π^0 production in ep collisions

Shohini Bhattacharya,¹ Duxin Zheng,² and Jian Zhou³



$$\frac{d\Delta\sigma}{dtdQ^2dx_Bd\phi} = -\sin(2\phi) \frac{\alpha_{em}^3 \alpha_s f_\pi^2 (1-y) \xi x_B \mathcal{F}(t)}{3Q^8 N_c} \left[\int_0^1 dz \frac{\phi_\pi(z)}{z(1-z)} \right]^2 \text{Im} \left[\int_{-1}^1 dx \frac{F_{1,4}^{(1)}(x, \xi, \Delta_\perp)/M^2}{(x + \xi - i\epsilon)^2 (x - \xi + i\epsilon)^2} \right]$$

The same azimuthal asymmetry, precisely mirroring what we observe in this study, also emerges from the interference between the Primakoff process and the contribution from the gluon GTMD

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Model input for numerical estimations

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Model input for numerical estimations

Ingredients for non-perturbative functions:

- Model $(H^q, \tilde{H}^q)$ according to the Double distribution approach (see Radyushkin, 9805342)

Example:

$$H^q(x, \xi, t) = \int_{-1}^1 d\beta \int_{-1+|\beta|}^{1-|\beta|} d\alpha \delta(\beta + \xi\alpha - x) \times \frac{3}{4} |\beta|^{-1.3t} \frac{[(1 - |\beta|)^2 - \alpha^2]}{(1 - |\beta|)^3} q(|\beta|)$$



2022 PDFs

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Model input for numerical estimations

Ingredients for non-perturbative functions:

- Model $(H^q, \tilde{H}^q)$ according to the Double distribution approach (see Radyushkin, 9805342)

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The t-dependence is determined based on a fit to CLAS data

Probing quark OAM through π^0 production in ep collisions



Model input for numerical estimations

Ingredients for non-perturbative functions:

- Model $(H^q, \tilde{H}^q)$ according to the Double distribution approach (see Radyushkin, 9805342)
- Model for OAM:
 1. “OAM density”: (Hatta, Yoshida, 1207.5332)

$$L_{can}^q(\textcolor{blue}{x}) = x \int_x^1 \frac{dx'}{x'} q(x') - x \int_x^1 \frac{dx'}{x'^2} \Delta q(x') + \text{genuine twist-three}$$

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Model input for numerical estimations

Ingredients for non-perturbative functions:

- Model $(H^q, \tilde{H}^q)$ according to the Double distribution approach (see Radyushkin, 9805342)
- Model for OAM:
 1. “OAM density”: (Hatta, Yoshida, 1207.5332)

$$L_{can}^q(\textcolor{blue}{x}) \stackrel{\text{WW approx}}{=} x \int_x^1 \frac{dx'}{x'} q(x') - x \int_x^1 \frac{dx'}{x'^2} \Delta q(x') + \text{genuine twist-three}$$

Probing quark OAM through π^0 production in ep collisions



Model input for numerical estimations

Ingredients for non-perturbative functions:

- Model $(H^q, \tilde{H}^q)$ according to the Double distribution approach (see Radyushkin, 9805342)
- Model for OAM:
 1. “OAM density”: (Hatta, Yoshida, 1207.5332)

$$L_{can}^q(\mathbf{x}) \stackrel{\text{WW approx}}{=} x \int_x^1 \frac{dx'}{x'} q(x') - x \int_x^1 \frac{dx'}{x'^2} \Delta q(x') + \text{genuine twist-three}$$

2. Use the Double distribution approach to construct $xL^q(x, \xi)e^{t/\Lambda}$ from $xL^q(x)$

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Model input for numerical estimations

Ingredients for non-perturbative functions:

- Pion distribution amplitude:

Asymptotic form

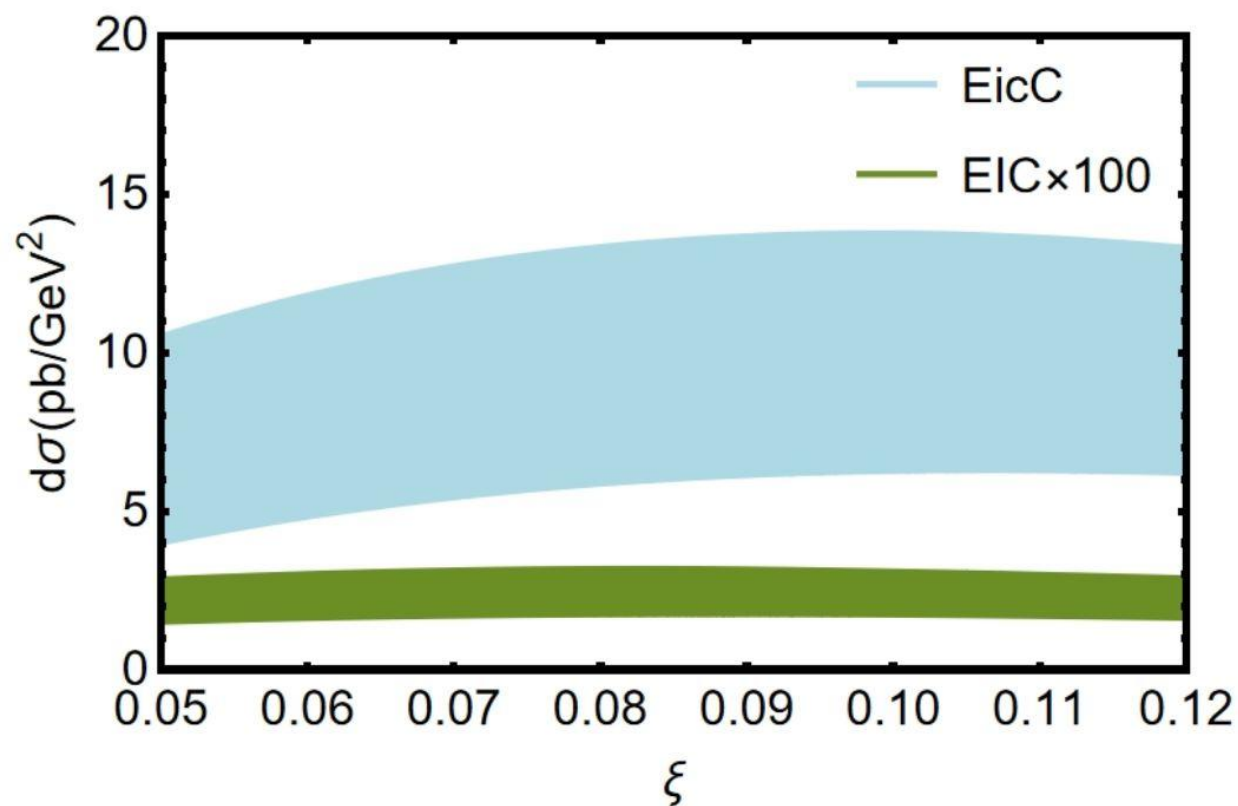
$$\phi_{\pi}(z) = 6z(1 - z)$$

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Numerical results

Unpolarized cross section



Findings:

- The unpolarized cross section exhibits a notable magnitude at EicC energy
- Relatively small at EIC energy

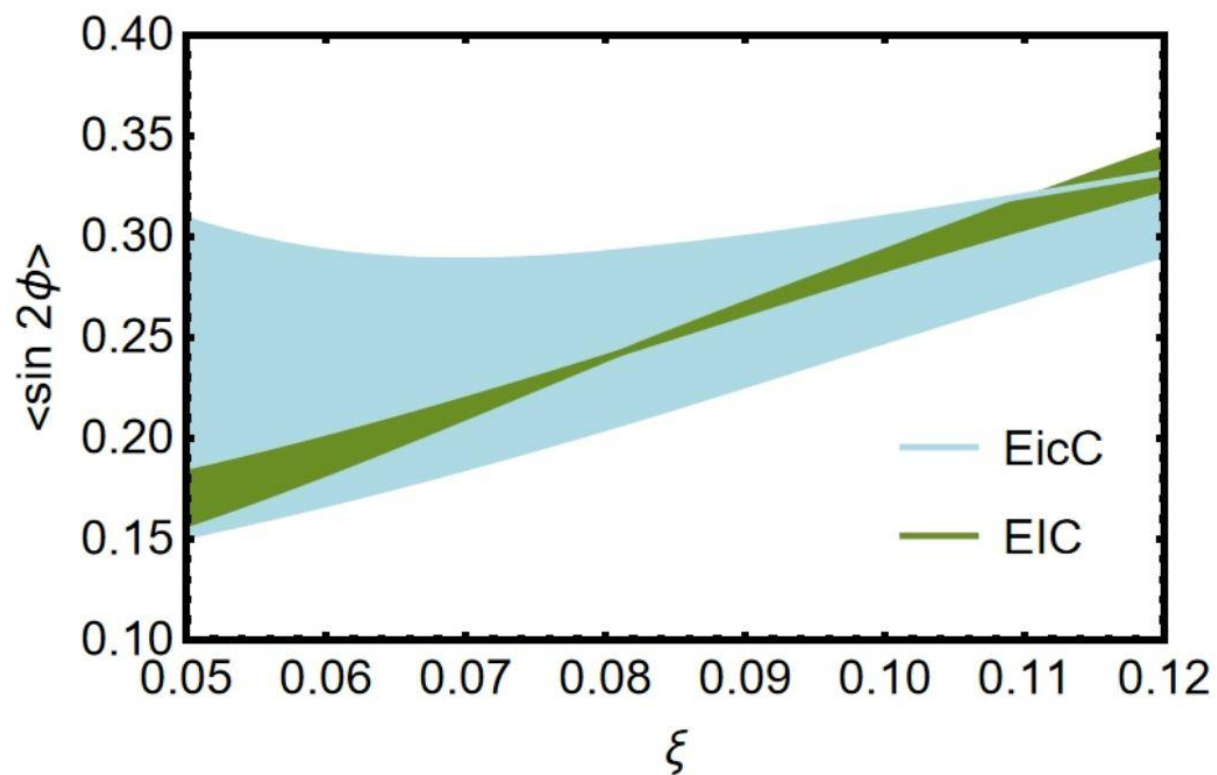
This strong suppression with increasing Q^2 suggests that it may be particularly interesting to study this reaction in the kinematic range accessible at JLab.

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Numerical results

Asymmetry



$$\langle \sin(2\phi) \rangle = \frac{\int \frac{d\Delta\sigma}{d\mathcal{P}.S.} \sin(2\phi) d\mathcal{P}.S.}{\int \frac{d\sigma}{d\mathcal{P}.S.} d\mathcal{P}.S.}$$

Findings:

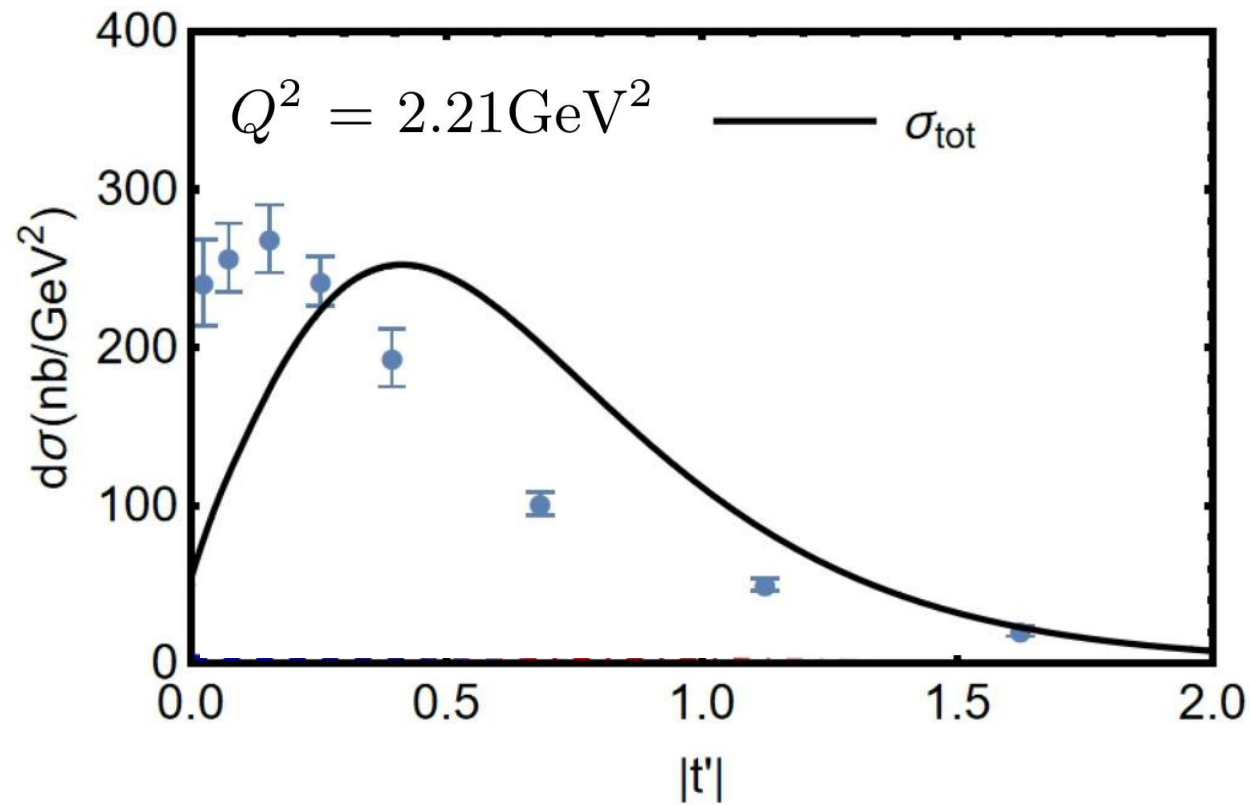
The asymmetries are substantial for both EIC & EicC kinematics

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Numerical results

Comparison with CLAS data (I. Bedlinskiy et al., Phys. Rev. C 90, 025205 (2014))



Unpolarized cross section:

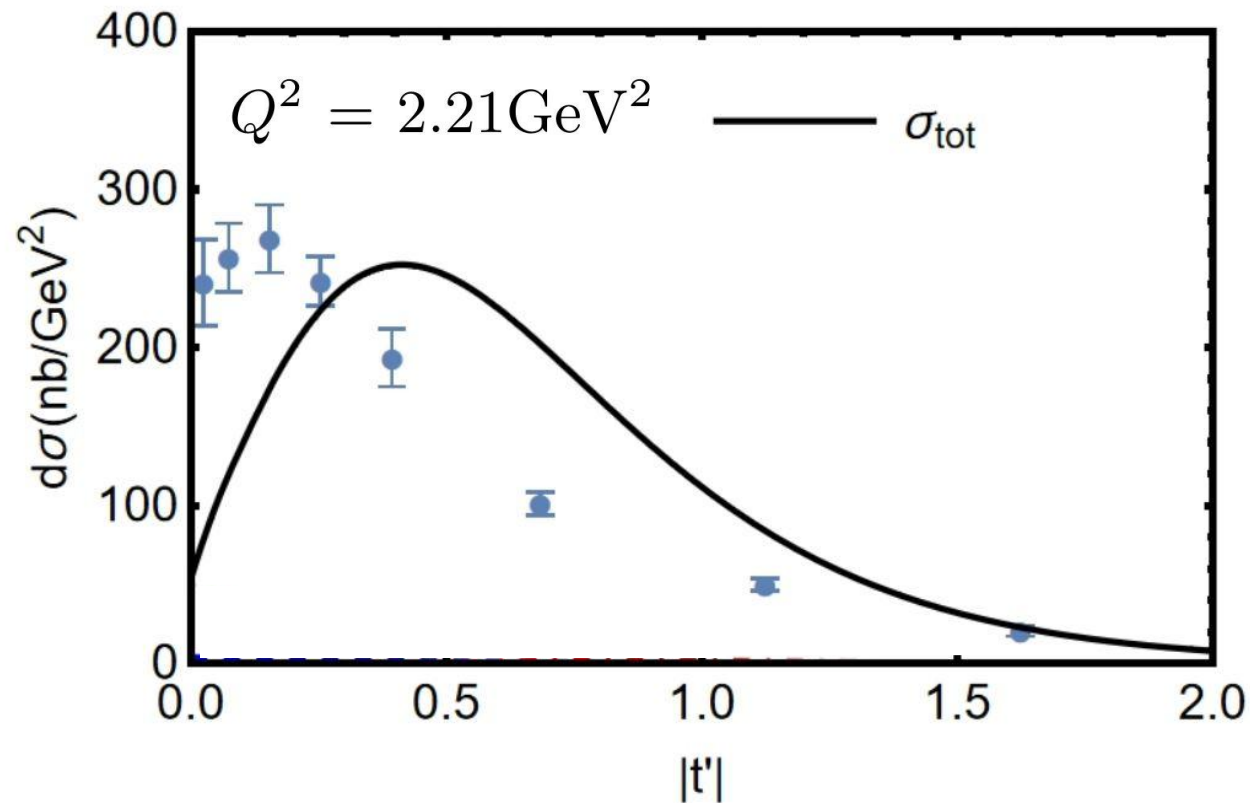
$$\frac{d\sigma_T}{dt} + a \frac{d\sigma_L}{dt}$$

Probing quark OAM through π^0 production in ep collisions



Numerical results

Comparison with CLAS data (I. Bedlinskiy et al., Phys. Rev. C 90, 025205 (2014))



Findings:

- Our theoretical model is in reasonable agreement with experimental data

More opportunities with CLAS's recent data on single spin asymmetry

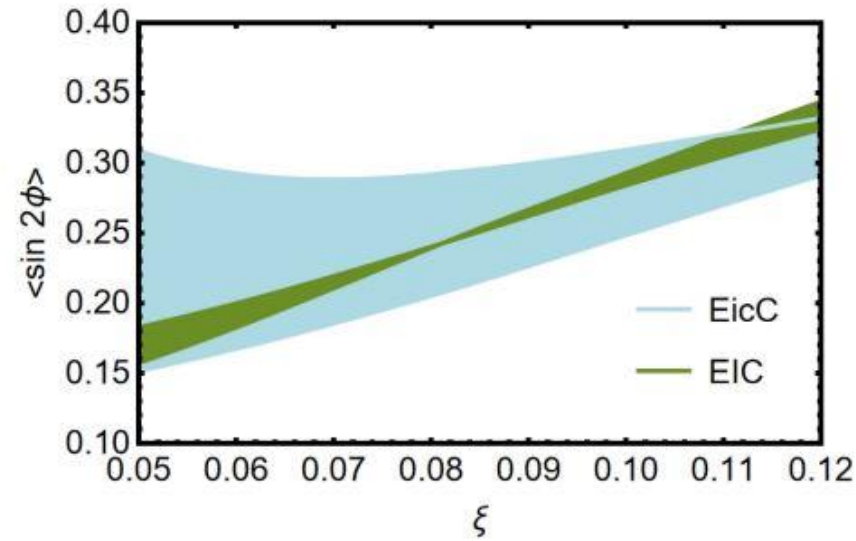


FIG. 3: The unpolarized cross section, as given by Eq. (15), is displayed in the top plot for EIC kinematics with $Q^2 = 10 \text{ GeV}^2$ and $\sqrt{s_{ep}} = 100 \text{ GeV}$, as well as for EicC kinematics with $Q^2 = 3 \text{ GeV}^2$ and $\sqrt{s_{ep}} = 16 \text{ GeV}$. The unpolarized cross section for the EIC case is re-scaled by a factor of 100. The bottom plot shows the average value of $\langle \sin(2\phi) \rangle$ given by Eq. (16). The variable t is integrated over the range $[-0.5 \text{ GeV}^2, -\frac{4\xi^2 M^2}{1-\xi^2}]$. The error bands are obtained by varying the value of $\sqrt{\langle p_{\perp}^2 \rangle}$ from 150 MeV to 250 MeV and the value of α' , which determines the t -dependence of the various distributions in the double distribution approach (see supplementary material), from 1.2 to 1.4.

We notice that other fitting for $g_{1T}^{(1)}(x)$ exist too [82]. Meanwhile, the $k_{\perp}$ moment of $F_{1,2}$ can be related to the Qiu-Sterman function,

$$\begin{aligned} & \int d^2 k_{\perp} \frac{k_{\perp}^2}{M} \text{Im}[F_{1,2}(x, \xi = 0, \Delta_{\perp} = 0, k_{\perp})] \\ &= - \int d^2 k_{\perp} \frac{k_{\perp}^2}{M} f_{1T}^{\perp}(x, k_{\perp}) = T_F(x, x) \end{aligned} \quad (6)$$

where the Qiu-Sterman function is parametrized as [83],

$$T_F(x, x) = N_q \frac{(\alpha_q + \beta_q)^{(\alpha_q + \beta_q)}}{\alpha_q^{\alpha_q} \beta_q^{\beta_q}} x^{\alpha_q} (1 - x)^{\beta_q} q(x) \quad (7)$$

with $\alpha_u = 1.051, \alpha_d = 1.552, \beta_u = \beta_d = 4.857$, and $N_u = 1.06, N_d = -0.163$. See also Refs. [84], [85], and [86] for the state-of-the-art extractions of the Sivers functions. Once the x -dependence of the $k_{\perp}$ moments of $F_{1,2}$ and $G_{1,2}$ is reconstructed as explained above, we reconstruct their (ξ, t) -dependence in accordance with the double distribution

$$\mathcal{H}_{eff}^{LL} = - \int_{-1}^1 \frac{dx}{(x - \xi + i\varepsilon)(x + \xi - i\varepsilon)} \frac{Q\sqrt{1 - \xi^2}}{m_V} H_{eff}^g, \quad (25)$$

$$\tilde{\mathcal{E}}^{LL} = - \int_{-1}^1 \frac{dx}{(x - \xi + i\varepsilon)(x + \xi - i\varepsilon)} \frac{Q}{m_V} \frac{1}{2M\sqrt{1 - \xi^2}} E^g, \quad (26)$$

$$\mathcal{H}_{eff}^{TT} = \int_{-1}^1 \frac{dx}{(x - \xi + i\varepsilon)(x + \xi - i\varepsilon)} \sqrt{1 - \xi^2} H_{eff}^g, \quad (27)$$

$$\tilde{\mathcal{E}}^{TT} = \int_{-1}^1 \frac{dx}{(x - \xi + i\varepsilon)(x + \xi - i\varepsilon)} \frac{1}{2M\sqrt{1 - \xi^2}} E^g, \quad (28)$$

CFFs:

$$\mathcal{H}_{eff}^{TL} = \int_{-1}^1 \frac{dx}{(x - \xi + i\varepsilon)^2(x + \xi - i\varepsilon)^2} \frac{\xi^2\sqrt{1 - \xi^2}}{m_V} H_{eff}^g, \quad (29)$$

$$\mathcal{H}'_{eff}{}^{TL} = \int_{-1}^1 \frac{dx}{(x - \xi + i\varepsilon)^2(x + \xi - i\varepsilon)^2} \frac{ix\xi\sqrt{1 - \xi^2}}{m_V} H'_{eff}{}^g, \quad (30)$$

$$\mathcal{F}_{1,1}^{TL} = \int_{-1}^1 \frac{dx}{(x - \xi - i\varepsilon)^2(x + \xi + i\varepsilon)^2} \frac{2x\xi}{m_V} \frac{1}{\sqrt{1 - \xi^2}} \int d^2k_{\perp} \left(\frac{k_{\perp} \cdot \Delta_{\perp}}{\Delta_{\perp}^2} \right) F_{1,1}^g, \quad (31)$$

$$\mathcal{G}_{1,1}^{TL} = - \frac{1}{\sqrt{1 - \xi^2}} \int_{-1}^1 dx \frac{\xi^2}{m_V(x + \xi - i\varepsilon)^2(x - \xi + i\varepsilon)^2} \int d^2k_{\perp} \left(\frac{k_{\perp}^2}{M^2} \right) G_{1,1}^g, \quad (32)$$

$$\tilde{\mathcal{F}}_{1,2}^{TL} = M\sqrt{1 - \xi^2} \int_{-1}^1 dx \frac{x\xi}{m_V(x + \xi - i\varepsilon)^2(x - \xi + i\varepsilon)^2} \int d^2k_{\perp} \left(\frac{k_{\perp}^2}{M^2} \right) F_{1,2}^g, \quad (33)$$

$$\tilde{\mathcal{G}}_{1,2}^{TL} = -M\sqrt{1 - \xi^2} \int_{-1}^1 dx \frac{\xi^2}{m_V(x + \xi - i\varepsilon)^2(x - \xi + i\varepsilon)^2} \int d^2k_{\perp} \left(\frac{k_{\perp}^2}{M^2} \right) G_{1,2}^g, \quad (34)$$

$$\mathcal{F}_{1,4}^{TL} = \frac{1}{\sqrt{1 - \xi^2}} \int_{-1}^1 dx \frac{ix\xi}{m_V(x + \xi - i\varepsilon)^2(x - \xi + i\varepsilon)^2} \int d^2k_{\perp} \left(\frac{k_{\perp}^2}{M^2} \right) F_{1,4}^g, \quad (35)$$

$$\mathcal{G}_{1,4}^{TL} = \int \frac{dx}{(x - \xi + i\varepsilon)^2(x + \xi - i\varepsilon)^2} \frac{2i\xi^2}{m_V} \frac{1}{\sqrt{1 - \xi^2}} \int d^2k_{\perp} \left(\frac{k_{\perp} \cdot \Delta_{\perp}}{\Delta_{\perp}^2} \right) G_{1,4}^g. \quad (36)$$

in the lepton plane. The outgoing lepton momentum follows from momentum conservation. Upon contracting the resulting leptonic tensor with the tensor structures of the basis constructed from the photon polarization vectors, as described above, we obtain the following distinct angular modulations:

$$\begin{aligned} L_{\mu\nu} \mathcal{V}_3^{\mu\nu} &\propto \cos \phi, & L_{\mu\nu} \mathcal{V}_4^{\mu\nu} &\propto \cos 2\phi, \\ L_{\mu\nu} \mathcal{V}_8^{\mu\nu} &\propto \sin \phi, & L_{\mu\nu} \mathcal{V}_9^{\mu\nu} &\propto \sin 2\phi. \end{aligned} \quad (66)$$

Here, ϕ may equivalently be identified as the angle between the lepton scattering plane and the hadron production plane provided we choose $\Delta_\perp = (|\vec{\Delta}_\perp|, 0)$, which we adopt to keep the expressions simple (and for this reason we henceforth drop the subscript $l_\perp$ from the angle). These angular modulations are well known in the literature [45–47] and originate from specific interference patterns: the $\cos \phi$ and $\sin \phi$ terms arise from the interference between twist-2 and twist-3 amplitudes, while the $\cos 2\phi$ and $\sin 2\phi$ terms arise from the interference between twist-3 amplitudes.

Next, we work, without loss of generality, with an initial proton of positive helicity and proceed to calculate the second term in parentheses of Eq. (61). Below, the photon-helicity amplitudes will be denoted by $\mathcal{M}_{Vp',\gamma^*p}$. It then follows straightforwardly that the differential cross section for exclusive vector-meson electroproduction can be written as

$$\begin{aligned} \frac{d\sigma}{dydQ^2dt d\phi} &= \frac{\alpha_{\text{em}}}{2\pi^2} \frac{\left(1-y+\frac{y^2}{2}\right)}{yQ^2} \left[\frac{d\sigma_T}{dt} + \epsilon \frac{d\sigma_L}{dt} + \sqrt{2\epsilon(1+\epsilon)} \cos \phi \frac{d\sigma_{LT}}{dt} + \epsilon \cos 2\phi \frac{d\sigma_{TT}}{dt} \right. \\ &\quad \left. + \sqrt{2\epsilon(1+\epsilon)} \sin \phi \frac{d\sigma_{\sin\phi}}{dt} + \epsilon \sin 2\phi \frac{d\sigma_{\sin 2\phi}}{dt} \right] \end{aligned} \quad (67)$$

where the individual structure functions are expressed as

$$\frac{d\sigma_T}{dt} = \frac{x_B^2}{32\pi Q^4} (|\mathcal{M}_{Vp',++}|^2 + |\mathcal{M}_{Vp',-+}|^2), \quad (68)$$

$$\frac{d\sigma_L}{dt} = \frac{x_B^2}{16\pi Q^4} |\mathcal{M}_{Vp',0+}|^2, \quad (69)$$

$$\frac{d\sigma_{LT}}{dt} = \frac{\sqrt{2}x_B^2}{32\pi Q^4} \text{Re}[\mathcal{M}_{Vp',0+} (\mathcal{M}_{Vp',-+}^* - \mathcal{M}_{Vp',++}^*)], \quad (70)$$

$$\frac{d\sigma_{TT}}{dt} = -\frac{x_B^2}{16\pi Q^4} \text{Re}[\mathcal{M}_{Vp',++} \mathcal{M}_{Vp',-+}^*], \quad (71)$$

$$\frac{d\sigma_{\sin\phi}}{dt} = \frac{\sqrt{2}x_B^2}{32\pi Q^4} \text{Im}[\mathcal{M}_{Vp',0+}^* (\mathcal{M}_{Vp',-+} + \mathcal{M}_{Vp',++})], \quad (72)$$

$$\frac{d\sigma_{\sin 2\phi}}{dt} = \frac{x_B^2}{16\pi Q^4} \text{Im}[\mathcal{M}_{Vp',++} \mathcal{M}_{Vp',-+}^*]. \quad (73)$$

Here we have expressed our results in terms of the virtual-photon flux parameter ϵ , defined as, $\epsilon = \frac{1-y}{1-y+\frac{1}{2}y^2}$. The

meson with positive circular polarization ($V = +$), the results read:

$$\frac{d\sigma_T^+}{dt} = \frac{x_B^2}{32\pi Q^4} C^2 \left\{ \delta_{\lambda,\lambda'} |\mathcal{H}_{eff}^{TT}|^2 + \delta_{\lambda,-\lambda'} |\vec{\Delta}_\perp|^2 |\tilde{\mathcal{E}}^{TT}|^2 \right\}, \quad (74)$$

and vanishing in all other cases. Similarly, in the case of a vector meson with negative circular polarization ($V = -$), the results read:

$$\frac{d\sigma_T^-}{dt} = \frac{x_B^2}{32\pi Q^4} C^2 \left\{ \delta_{\lambda,\lambda'} |\mathcal{H}_{eff}^{TT}|^2 + \delta_{\lambda,-\lambda'} |\vec{\Delta}_\perp|^2 |\tilde{\mathcal{E}}^{TT}|^2 \right\}, \quad (75)$$

and vanishing in all other cases. Both of the above results arise from the interference between twist-2 contributions.

Finally, for longitudinally polarized vector meson ($V = 0$), the results read:

$$\begin{aligned} \frac{d\sigma_T^0}{dt} = \frac{x_B^2}{32\pi Q^4} C^2 \left\{ \delta_{\lambda,\lambda'} \left[|\vec{\Delta}_\perp|^2 |\mathcal{H}_{eff}^{TL} + \mathcal{F}_{1,1}^{TL} + \mathcal{G}_{1,1}^{TL}|^2 + |\vec{\Delta}_\perp|^2 |\mathcal{F}_{1,4}^{TL} - \mathcal{H}'_{eff}^{TL} - \mathcal{G}_{1,4}^{TL}|^2 \right] \right. \\ \left. + 2 \delta_{\lambda,-\lambda'} |\tilde{\mathcal{F}}_{1,2}^{TL} - \tilde{\mathcal{G}}_{1,2}^{TL}|^2 \right\}, \end{aligned} \quad (76)$$

$$\frac{d\sigma_L^0}{dt} = \frac{x_B^2}{16\pi Q^4} C^2 \left\{ \delta_{\lambda,\lambda'} |\mathcal{H}_{eff}^{LL}|^2 + \delta_{\lambda,-\lambda'} |\vec{\Delta}_\perp|^2 |\tilde{\mathcal{E}}^{LL}|^2 \right\}, \quad (77)$$

$$\begin{aligned} \frac{d\sigma_{LT}^0}{dt} = \frac{\sqrt{2} x_B^2}{32\pi Q^4} C^2 \left\{ \delta_{\lambda,\lambda'} \sqrt{2} |\vec{\Delta}_\perp| \operatorname{Re} \left[\mathcal{H}_{eff}^{LL} (\mathcal{H}_{eff}^{TL*} + \mathcal{F}_{1,1}^{TL*} + \mathcal{G}_{1,1}^{TL*}) \right] \right. \\ \left. + \delta_{\lambda,-\lambda'} \sqrt{2} |\vec{\Delta}_\perp| \operatorname{Re} \left[\tilde{\mathcal{E}}^{LL} (\tilde{\mathcal{F}}_{1,2}^{TL*} - \tilde{\mathcal{G}}_{1,2}^{TL*}) \right] \right\}, \end{aligned} \quad (78)$$

$$\frac{d\sigma_{TT}^0}{dt} = -\frac{x_B^2}{16\pi Q^4} C^2 \left\{ \delta_{\lambda,\lambda'} \left[-\frac{1}{2} |\vec{\Delta}_\perp|^2 |\mathcal{H}_{eff}^{TL} + \mathcal{F}_{1,1}^{TL} + \mathcal{G}_{1,1}^{TL}|^2 + \frac{1}{2} |\vec{\Delta}_\perp|^2 |\mathcal{F}_{1,4}^{TL} - \mathcal{H}'_{eff}^{TL} - \mathcal{G}_{1,4}^{TL}|^2 \right] \right\}, \quad (79)$$

$$\begin{aligned} \frac{d\sigma_{\sin\phi}^0}{dt} = \frac{\sqrt{2} x_B^2}{32\pi Q^4} C^2 \left\{ -\delta_{\lambda,\lambda'} \lambda |\vec{\Delta}_\perp| \operatorname{Re} \left[\sqrt{2} \mathcal{H}_{eff}^{LL*} (\mathcal{F}_{1,4}^{TL} - \mathcal{H}'_{eff}^{TL} - \mathcal{G}_{1,4}^{TL}) \right] \right. \\ \left. - \lambda \delta_{\lambda,-\lambda'} |\vec{\Delta}_\perp| \operatorname{Im} \left[\sqrt{2} \tilde{\mathcal{E}}^{LL*} (\tilde{\mathcal{F}}_{1,2}^{TL} - \tilde{\mathcal{G}}_{1,2}^{TL}) \right] \right\}, \end{aligned} \quad (80)$$

$$\frac{d\sigma_{\sin 2\phi}^0}{dt} = \frac{x_B^2}{16\pi Q^4} C^2 \left\{ -\delta_{\lambda,\lambda'} \lambda |\vec{\Delta}_\perp|^2 \operatorname{Re} \left[(\mathcal{H}_{eff}^{TL} + \mathcal{F}_{1,1}^{TL} + \mathcal{G}_{1,1}^{TL}) (\mathcal{F}_{1,4}^{TL*} - \mathcal{H}'_{eff}^{TL*} - \mathcal{G}_{1,4}^{TL*}) \right] \right\}. \quad (81)$$

Twist nature:

Here, $d\sigma_L^0/dt$ arises from twist-2 $\times$ twist-2 contributions, while the $d\sigma_{LT}^0/dt$ and $d\sigma_{\sin\phi}^0/dt$ terms result from the interference between twist-2 and twist-3 contributions. All remaining terms involve purely twist-3 $\times$ twist-3 contributions. From these expressions, we conclude that the polarization-independent $\cos 2\phi$ and polarization-dependent $\sin 2\phi$ terms provide exceptionally clean signatures of the gluon GTMDs $F_{1,4}$ and $G_{1,1}$. While other observables, such as $d\sigma_T/dt$

Final cross section: longitudinal meson



$$\frac{d\sigma_T^0}{dt} = \frac{x_B^2}{32\pi Q^4} C^2 \left\{ \delta_{\lambda,\lambda'} \left[|\vec{\Delta}_\perp|^2 |\mathcal{H}_{eff}^{TL} + \mathcal{F}_{1,1}^{TL} + \mathcal{G}_{1,1}^{TL}|^2 + |\vec{\Delta}_\perp|^2 |\mathcal{F}_{1,4}^{TL} - \mathcal{H}'_{eff}^{TL} - \mathcal{G}_{1,4}^{TL}|^2 \right] + 2 \delta_{\lambda,-\lambda'} |\tilde{\mathcal{F}}_{1,2}^{TL} - \tilde{\mathcal{G}}_{1,2}^{TL}|^2 \right\}$$

$$\frac{d\sigma_L^0}{dt} = \frac{x_B^2}{16\pi Q^4} C^2 \left\{ \delta_{\lambda,\lambda'} |\mathcal{H}_{eff}^{LL}|^2 + \delta_{\lambda,-\lambda'} |\vec{\Delta}_\perp|^2 |\tilde{\mathcal{E}}^{LL}|^2 \right\}$$

$$\frac{d\sigma_{LT}^0}{dt} = \frac{\sqrt{2} x_B^2}{32\pi Q^4} C^2 \left\{ \delta_{\lambda,\lambda'} \sqrt{2} |\vec{\Delta}_\perp| \operatorname{Re} \left[\mathcal{H}_{eff}^{LL} (\mathcal{H}_{eff}^{TL*} + \mathcal{F}_{1,1}^{TL*} + \mathcal{G}_{1,1}^{TL*}) \right] \delta_{\lambda,-\lambda'} \sqrt{2} |\vec{\Delta}_\perp| \operatorname{Re} \left[\tilde{\mathcal{E}}^{LL} (\tilde{\mathcal{F}}_{1,2}^{TL*} - \tilde{\mathcal{G}}_{1,2}^{TL*}) \right] \right\}$$

**First ever results for
heavy vector DVMP
cross section in terms of
GTMDs**

$$\frac{d\sigma_{TT}^0}{dt} = -\frac{x_B^2}{16\pi Q^4} C^2 \left\{ \delta_{\lambda,\lambda'} \left[-\frac{1}{2} |\vec{\Delta}_\perp|^2 |\mathcal{H}_{eff}^{TL} + \mathcal{F}_{1,1}^{TL} + \mathcal{G}_{1,1}^{TL}|^2 + \frac{1}{2} |\vec{\Delta}_\perp|^2 |\mathcal{F}_{1,4}^{TL} - \mathcal{H}'_{eff}^{TL} - \mathcal{G}_{1,4}^{TL}|^2 \right] \right\}$$

$$\frac{d\sigma_{\sin\phi}^0}{dt} = \frac{\sqrt{2} x_B^2}{32\pi Q^4} C^2 \left\{ -\delta_{\lambda,\lambda'} \lambda |\vec{\Delta}_\perp| \operatorname{Re} \left[\sqrt{2} \mathcal{H}_{eff}^{LL*} (\mathcal{F}_{1,4}^{TL} - \mathcal{H}'_{eff}^{TL} - \mathcal{G}_{1,4}^{TL}) \right] - \lambda \delta_{\lambda,-\lambda'} |\vec{\Delta}_\perp| \operatorname{Im} \left[\sqrt{2} \tilde{\mathcal{E}}^{LL*} (\tilde{\mathcal{F}}_{1,2}^{TL} - \tilde{\mathcal{G}}_{1,2}^{TL}) \right] \right\}$$

$$\frac{d\sigma_{\sin 2\phi}^0}{dt} = \frac{x_B^2}{16\pi Q^4} C^2 \left\{ -\delta_{\lambda,\lambda'} \lambda |\vec{\Delta}_\perp|^2 \operatorname{Re} \left[(\mathcal{H}_{eff}^{TL} + \mathcal{F}_{1,1}^{TL} + \mathcal{G}_{1,1}^{TL}) (\mathcal{F}_{1,4}^{TL*} - \mathcal{H}'_{eff}^{TL*} - \mathcal{G}_{1,4}^{TL*}) \right] \right\}$$