Radiative corrections in experiments on vector meson electroproduction

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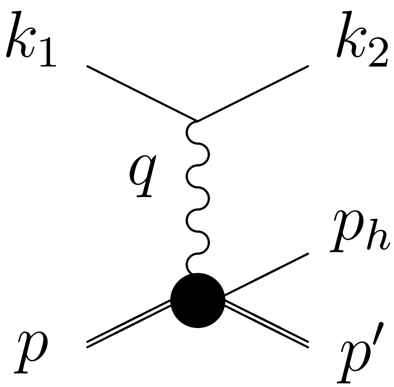
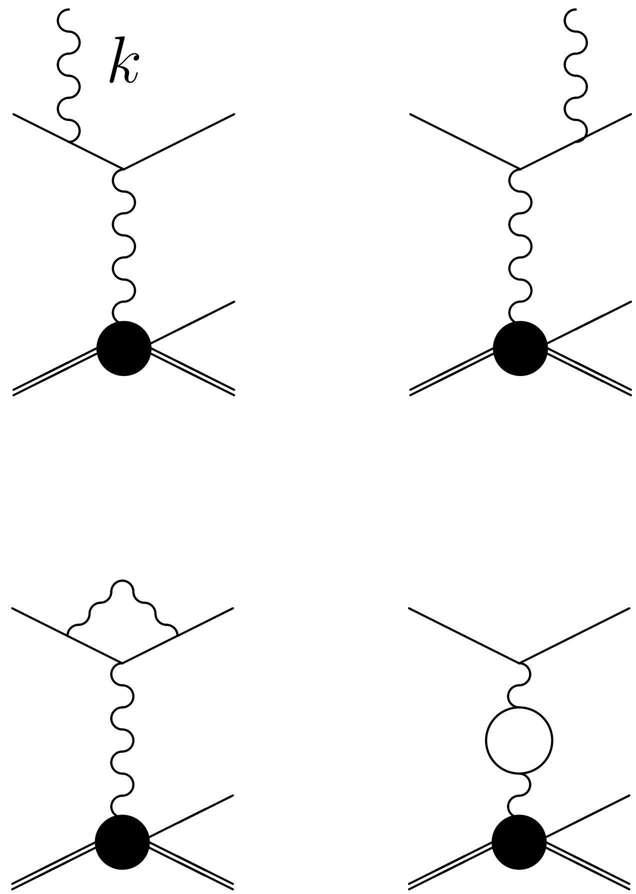
Introduction

- Measurements in experiments on vector meson production provide a uniquely clean and broadly applicable probe of QCD structure and dynamics across multiple regimes.
- A high level of control over systematic uncertainties is essential for precision studies, with radiative corrections constituting one of the most critical sources of these uncertainties.
- In this presentation, we discuss and compare approaches to the calculation of radiative corrections in four related kinematic and theoretical settings:
  - Radiative corrections to the cross section of exclusive vector meson electroproduction;
  - Radiative corrections to spin-density matrix elements in exclusive vector meson production;
  - Radiative corrections to the decay width of vector mesons;
  - Radiative corrections to inclusive vector meson electroproduction.
- We review the methodologies used to compute radiative corrections in these four settings and discuss possible directions for improving these and other available calculations.

The Born and lowest order RC: Feynman graphs

The Feynman diagrams necessary to calculate the lowest order RC





The kinematics and cross section

- Seven kinematical variables are necessary to describe the radiative process of the exclusive vector meson production
- Four of them are the same as for non-radiated case:
  - usual scaling variables x and y, or $Q^2 = -(k_1 - k_2)^2$ and $W^2 = (p + q)^2$,
  - the negative square of momentum transfered from the virtual photon to the proton $t = (q - p_h)^2$, and
  - the angle ϕ_h between the scattering ($\vec{k_1}, \vec{k_2}$) and production ($\vec{q}, \vec{p_h}$) planes in the laboratory frame.
- The kinematics of a real photon is described by three additional variables:
  - the inelasticity $v = \Lambda^2 - M^2$ (M is the proton mass and $\Lambda^2 = (p' + k)^2$ is the squared invariant mass of the system of unobserved particles),
  - $\tau = kq/kp$ and
  - angle ϕ_k between planes ($\vec{k_1}, \vec{k_2}$) and ($\vec{q}, \vec{k}$).

We consider RC to three and four dimensional cross sections $\sigma = d\sigma/dxdydtd\phi_h$ and $\bar{\sigma} = d\sigma/dxdydt$. They are related as

$$\bar{\sigma} = \int_0^{2\pi} d\phi_h \, \sigma \, .$$

RC calculation

The differential cross section of the radiative process has the following form:

$$\sigma_R \sim \frac{|M_i + M_f|^2}{4\ k_1 p} dv \frac{d^3k}{2k_0} \delta((\Lambda - k)^2 - M^2),$$

where $\Lambda = p + q - p_h$, $M_{i,f}$ are matrix elements of the initial and final state radiation. Traditionally, we extract the infrared divergence using the identity

$$\sigma_R = \sigma_R - \sigma_{IR} + \sigma_{IR} = \sigma_F + \sigma_{IR},$$

where σ_F is finite for $k \to 0$ and σ_{IR} is the infrared divergent part

$$\sigma_{IR} = \frac{\alpha}{\pi} \delta_R^{IR} \sigma_0 = \frac{\alpha}{\pi} (\delta_S + \delta_H) \sigma_0.$$

The quantities δ_S and δ_H appear after splitting the integration region over v by the infinitesimal parameter $\bar{v}$:

$$\begin{aligned} \delta_S &= 2\left(P_{IR} + \log\frac{\bar{v}}{\mu M}\right)(l_m - 1) + \log\frac{S'X'}{m^2M^2} + S_\phi, \\ \delta_H &= 2(l_m - 1)\log\frac{v_m}{\bar{v}}, \end{aligned}$$

where $l_m = \log(Q^2/m^2)$, $S' = 2\Lambda k_1 = S - Q^2 - V_1$ and $X' = 2\Lambda k_2 = X + Q^2 - V_2$.

RC calculation

The infrared terms P_{IR}, parameters μ and $\bar{v}$ as well as the squared logarithms containing the mass singularity l_m^2 are completely canceled in the sum of δ_R^{IR} with δ_V coming from a contribution of the vertex function:

$$\delta_V = -2(P_{IR} + \log\frac{m}{\mu})(l_m - 1) - \frac{1}{2}l_m^2 + \frac{3}{2}l_m - 2 + \frac{\pi^2}{6}.$$

For this sum we have

$$\delta_S + \delta_H + \delta_V = \delta_{inf} + \delta_{VR},$$

where

$$\begin{aligned}
\delta_{VR} &= \frac{\alpha}{\pi}\biggl(\frac{3}{2}l_m - 2 - \frac{1}{2}\log^2\frac{X'}{S'} + \mathrm{Li}_2\biggl[1 - \frac{Q^2M^2}{S'X'}\biggr] - \frac{\pi^2}{6}\biggr), \\
\delta_{inf} &= \frac{\alpha}{\pi}(l_m - 1)\log\frac{v_m^2}{S'X'}.
\end{aligned}$$

For the observed cross section of the vector meson electroproduction we obtain

$$\sigma_{obs} = \sigma_0 e^{\delta_{inf}}(1 + \delta_{VR} + \delta_{vac}) + \sigma_F.$$

RC calculation

The contribution of the infrared finite part is:

$$\sigma_F \quad = \quad -\frac{\alpha^2 y}{16\pi^3}\int\limits_0^{2\pi} d\phi_k \int\limits_{\tau_{min}}^{\tau_{max}} d\tau \sum_{i=1}^{2}\sum_{j=1}^{3}\theta_{ij}\int\limits_0^{v_m}\frac{dv}{f}R^{j-2}\left[\frac{\mathcal{F}_i}{\tilde{Q}^4}-\delta_j\frac{\mathcal{F}_i^0}{Q^4}\right],$$

where $R = v/f$, $f = 1+\tau-\mu$ and $2M\tau_{max,min} = S_x \pm \sqrt{\lambda_q}$; $\delta_j = 1$ for $j = 1$ and $\delta_j = 0$ otherwise.

The dependence on the photoproduction cross sections is included in $\mathcal{F}_i$:

$$\mathcal{F}_1 = (S_x - R)\sigma_T^R, \qquad \mathcal{F}_2 = \frac{2\tilde{Q}^2}{S_x - R}(\sigma_T^R + \sigma_L^R),$$

$$\mathcal{F}_1^0 = S_x\sigma_T, \qquad \mathcal{F}_2^0 = 2x(\sigma_T + \sigma_L).$$

The quantities $\sigma_{T,L}$ have to be calculated for Born kinematics, but $\sigma_{T,L}^R$ is calculated in terms of so called true kinematics. It means that they have to be calculated for the tilde variables

$$\tilde{Q}^2 = Q^2 + R\tau, \quad \tilde{W}^2 = W^2 - R(1+\tau), \tilde{t} = t + R(\tau - \mu)$$

instead of usual Q^2, W^2 and t.

Inelasticity

The important point is the dependence of the results on the maximal inelasticity v_m. The inelasticity is calculated in terms of the measured momenta, so it is possible to make a cut on the maximal value of this quantity. If this cut is not applied the maximal inelasticity is defined by kinematics only. Below we give the formulae for v_m in terms of the kinematical invariants

$$4Q^2 v_m = \left(\sqrt{\lambda_q} - \sqrt{t_q^2 + 4m_v^2 Q^2}\right)^2 - (S_x - 2Q^2 + t_q)^2 - 4M^2 Q^2$$

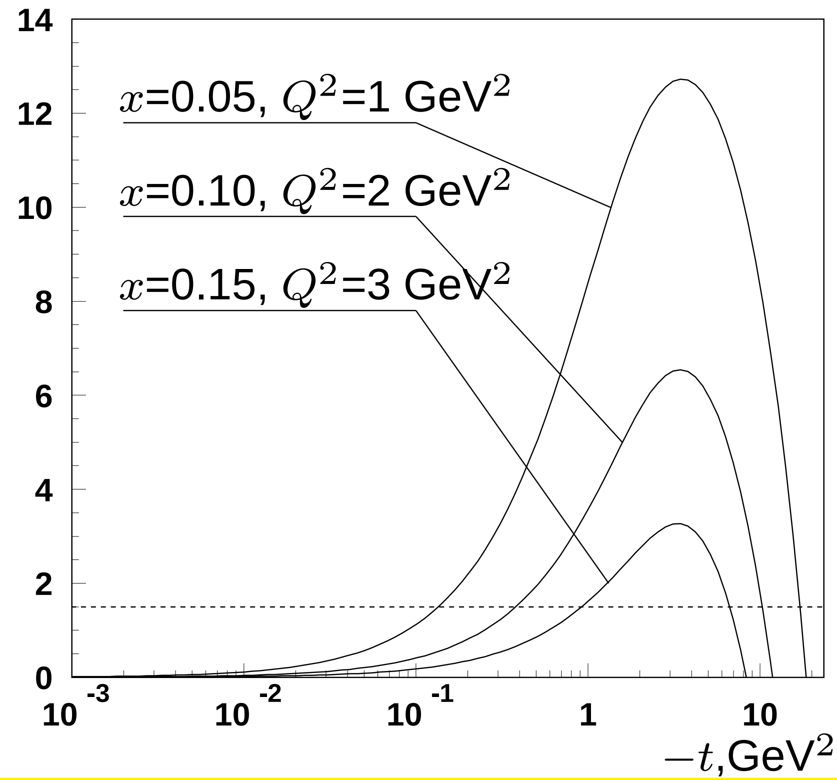
and in terms of kinematical limits on t:

$$v_m = \frac{1}{C}(t_{max} - t)(t - t_{min}),$$

where C behaves for small t as

$$C = \frac{Q^2 + m_v^2}{2W^2}\left(S_x + \sqrt{\lambda_q}\right) + O(t).$$

The maximal inelasticity given by kinematics are (the dashed line gives a possible cut).



The approximation

Assumptions for constructing the approximation:

- The uper limit of inelasticity, v_m, is relatively small, or kinematical cut on v is used.
- The squared momentum t transfered to the hadronic system is small, $M^2 \ll S;\ t \ll Q^2$.

For small v_m, the leading term in the integrand of σ_F is for $j=1$

$$\frac{F^0_{IR}}{v}\left[\frac{\mathcal{F}_i}{\tilde{Q}^4}-\frac{\mathcal{F}^0_i}{Q^4}\right]$$

and it is possible to integrate over ϕ_h, ϕ_k and τ explicitly:

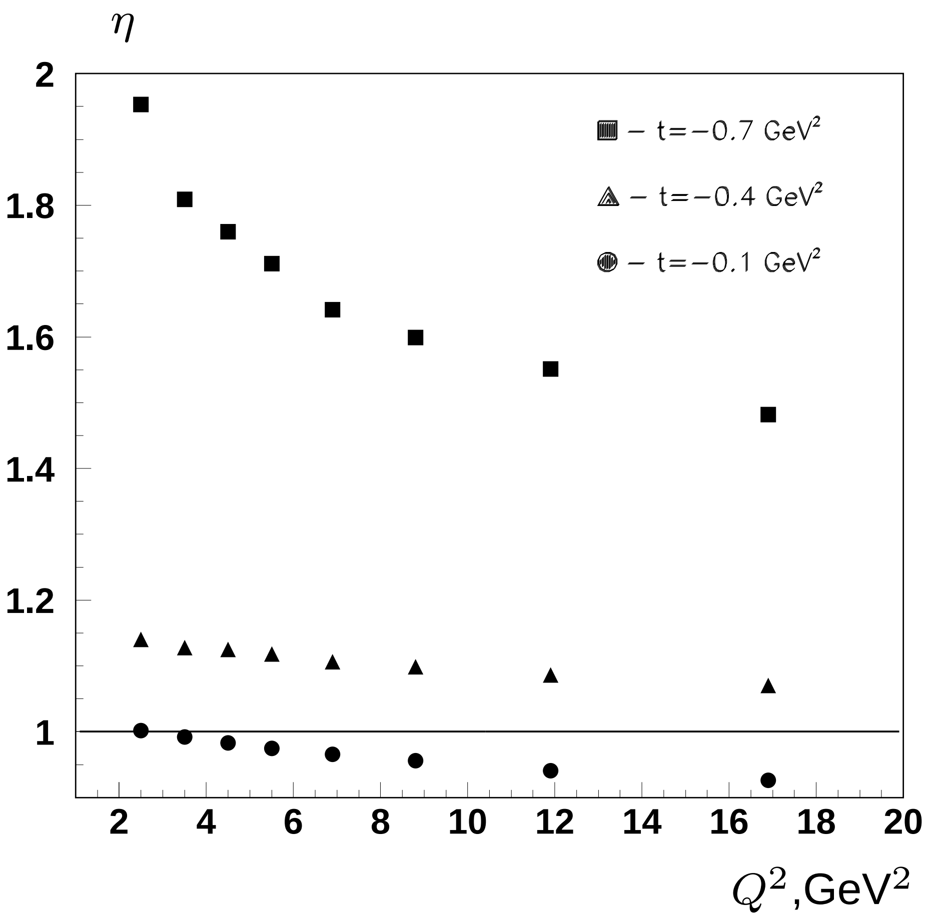
$$\frac{1}{2\pi}\int\limits_0^{2\pi} d\phi_h \frac{1}{v}\left[\frac{Q^4}{\tilde{Q}^4}\frac{\mathcal{F}_i}{\mathcal{F}^0_i}-1\right] \approx b_v \frac{Q^2+m_v^2}{S_x-Q^2-m_v^2},$$

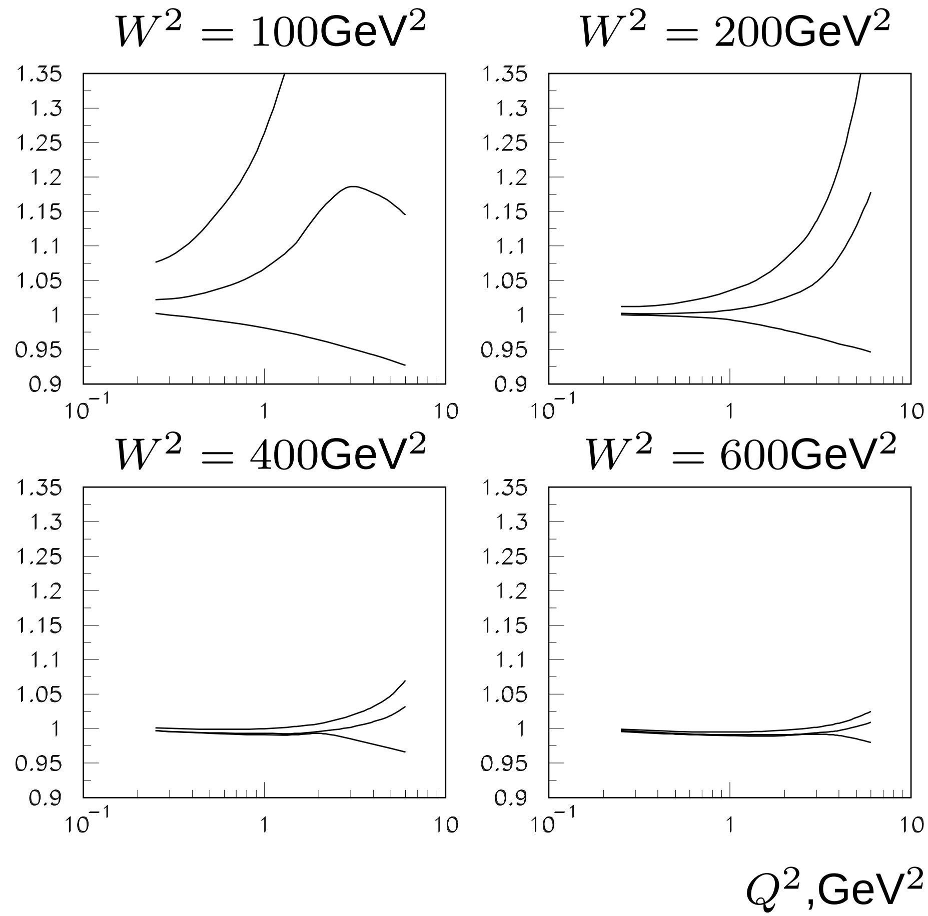
$$\int\limits_0^{2\pi} d\phi_k \int\limits_{\tau_{min}}^{\tau_{max}} d\tau F^0_{IR} = -2\bigl(l_m-1\bigr).$$

Final result for $\bar{\sigma}_F$ is simple

$$\bar{\sigma}_F = \frac{2\alpha b_v v_m}{\pi}(l_m-1)\frac{Q^2+m_v^2}{S_x-Q^2-m_v^2}\bar{\sigma}_0.$$

RC factor in muonoproduction of vector mesons

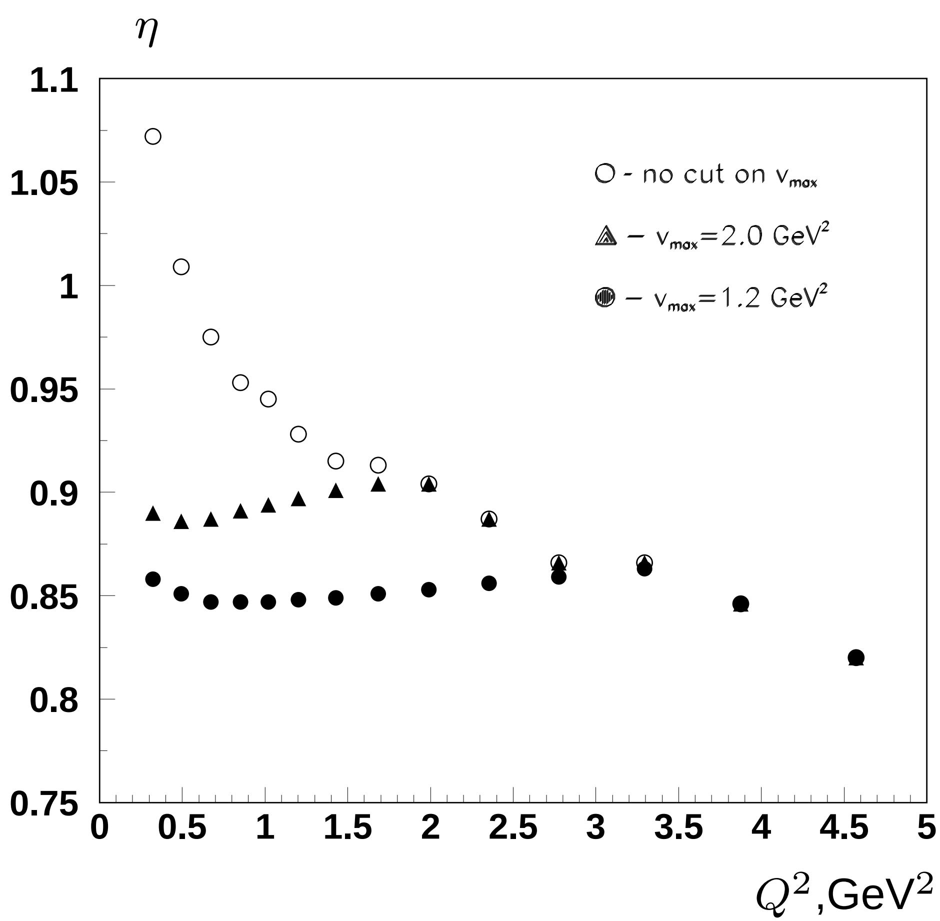




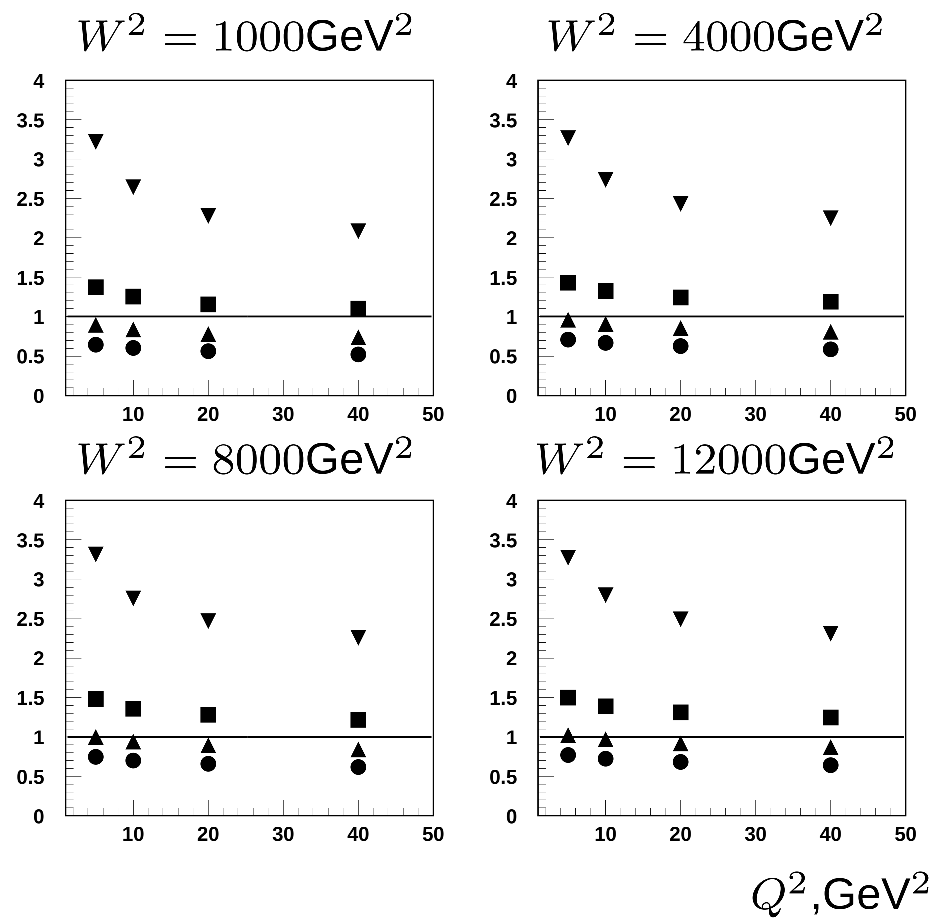
RC factor in kinematical conditions of EMC/NMC ($\mu p \to \mu p \rho$).

RC factor in kinematical conditions of E665 ($\mu p \to \mu p \rho$). Curves from top to bottom correspond to $t =$-0.9, -0.5, -0.1 GeV^2.

RC factor in electroproduction of vector mesons



RC factor in kinematical conditions of HERMES ($ep \to ep\rho$).



RC factor in kinematical conditions of collider experiments at HERA ($ep \to ep\rho$). Symbols from top to bottom correspond to $t =$-0.7, -0.5, -0.3, -0.1 GeV^2

RC to spin-density matrix elements

The angular distribution of vector meson decay is parameterized by fifteen matrix elements $r_{ij}^{\alpha}, r_{ij}^{\alpha\beta}$, including helicity-preserving, single helicity-flip, and double helicity-flip spin-density matrix elements (SDME).

SDME are reconstructed through vector meson decay angular distribution $W\left(\cos\theta, \phi, \Phi\right)$ and the weight coefficients $F_{ij}\left(\cos\theta, \phi, \Phi\right)$:

$$r_{ij}^{obs} = \frac{\int\limits_{0}^{2\pi} \frac{d\Phi}{2\pi} \int\limits_{-1}^{+1} d\cos\theta \int\limits_{0}^{2\pi} d\phi \;\; W\left(\cos\theta, \phi, \Phi\right) F_{ij}\left(\cos\theta, \phi, \Phi\right)(1+\delta)}{\int\limits_{0}^{2\pi} \frac{d\Phi}{2\pi} \int\limits_{-1}^{+1} d\cos\theta \int\limits_{0}^{2\pi} d\phi \;\; W\left(\cos\theta, \phi, \Phi\right)(1+\delta)}.$$

where:

- Φ is the angle between the lepton scattering plane and $\rho-$production plane,
- ϕ is the angle between $\rho-$decay and production plane,
- θ is the polar angle of the direction of flight of the positive decay pion.
- δ is RC factor discussed above.

Explicit results for RC to SDME

The QED corrections ($\Delta r = r_{obs} - r_{Born}$) to SDME are:

$$\Delta \mathrm{r}^{04}_{00} = -\epsilon \mathsf{I}_2 \mathrm{r}^1_{00} + a\mathsf{I}_1 r^5_{00} \qquad \Delta \mathrm{Re}\ \mathrm{r}^{04}_{10} = -\epsilon \mathsf{I}_2 \mathrm{Re}\ \mathrm{r}^1_{10} + a\mathsf{I}_1 \mathrm{Re}\ \mathrm{r}^5_{10} \qquad \Delta \mathrm{r}^{04}_{1-1} = -\epsilon \mathsf{I}_2 \mathrm{r}^1_{1-1} + a\mathsf{I}_1 \mathrm{r}^5_{1-1}$$

$$\Delta \mathrm{r}^1_{00} = \frac{1}{\epsilon}\left[-2\mathsf{I}_2 \mathrm{r}^{04}_{00} + \epsilon \mathsf{I}_4 \mathrm{r}^1_{00} - a(\mathsf{I}_1 + \mathsf{I}_3)\mathrm{r}^5_{00}\right] \qquad \Delta \mathrm{r}^5_{00} = \frac{1}{a}\left[2\mathsf{I}_1 \mathrm{r}^{04}_{00} + a\mathsf{I}_2 \mathrm{r}^5_{00} - \epsilon(\mathsf{I}_1 + \mathsf{I}_3)\mathrm{r}^1_{00}\right]$$

$$\Delta \mathrm{r}^1_{11} = \frac{1}{\epsilon}\left[\mathsf{I}_2(\mathrm{r}^{04}_{00} - 1) + \epsilon \mathsf{I}_4 \mathrm{r}^1_{11} - a(\mathsf{I}_1 + \mathsf{I}_3)\mathrm{r}^5_{11}\right] \qquad \Delta \mathrm{r}^5_{11} = \frac{1}{a}\left[\mathsf{I}_1(1 - \mathrm{r}^{04}_{00}) + a\mathsf{I}_2 \mathrm{r}^5_{11} - \epsilon(\mathsf{I}_1 + \mathsf{I}_3)\mathrm{r}^1_{11}\right]$$

$$\Delta \mathrm{Re}\ \mathrm{r}^1_{10} = \frac{1}{\epsilon}\left[-2\mathsf{I}_2 \mathrm{Re}\ \mathrm{r}^{04}_{10} + \epsilon \mathsf{I}_4 \mathrm{Re}\ \mathrm{r}^1_{10} - a(\mathsf{I}_1 + \mathsf{I}_3)\mathrm{Re}\ \mathrm{r}^5_{10}\right] \qquad \Delta \mathrm{Re}\ \mathrm{r}^5_{10} = \frac{1}{a}\left[\mathsf{I}_1 \mathrm{Re}\ \mathrm{r}^{04}_{10} + a\mathsf{I}_2 \mathrm{Re}\ \mathrm{r}^5_{10} - \epsilon(\mathsf{I}_1 + \mathsf{I}_3)\mathrm{Re}\ \mathrm{r}^1_{10}\right]$$

$$\Delta \mathrm{r}^1_{1-1} = \frac{1}{\epsilon}\left[-2\mathsf{I}_2 \mathrm{r}^{04}_{1-1} + \epsilon \mathsf{I}_4 \mathrm{r}^1_{1-1} - a(\mathsf{I}_1 + \mathsf{I}_3)\mathrm{r}^5_{1-1}\right] \qquad \Delta \mathrm{r}^5_{1-1} = \frac{1}{a}\left[\mathsf{I}_1 \mathrm{r}^{04}_{1-1} + a\mathsf{I}_2 \mathrm{r}^5_{1-1} - \epsilon(\mathsf{I}_1 + \mathsf{I}_3)\mathrm{r}^1_{1-1}\right]$$

$$\Delta \mathrm{Im}\ \mathrm{r}^2_{10} = -\mathsf{I}_4 \mathrm{Im}\ \mathrm{r}^2_{10} + \frac{a}{\epsilon}(\mathsf{I}_1 + \mathsf{I}_3)\mathrm{Im}\ \mathrm{r}^6_{10} \qquad \Delta \mathrm{Im}\ \mathrm{r}^6_{10} = -\mathsf{I}_2 \mathrm{Im}\ \mathrm{r}^6_{10} + \frac{\epsilon}{a}(\mathsf{I}_1 + \mathsf{I}_3)\mathrm{Im}\ \mathrm{r}^2_{10}$$

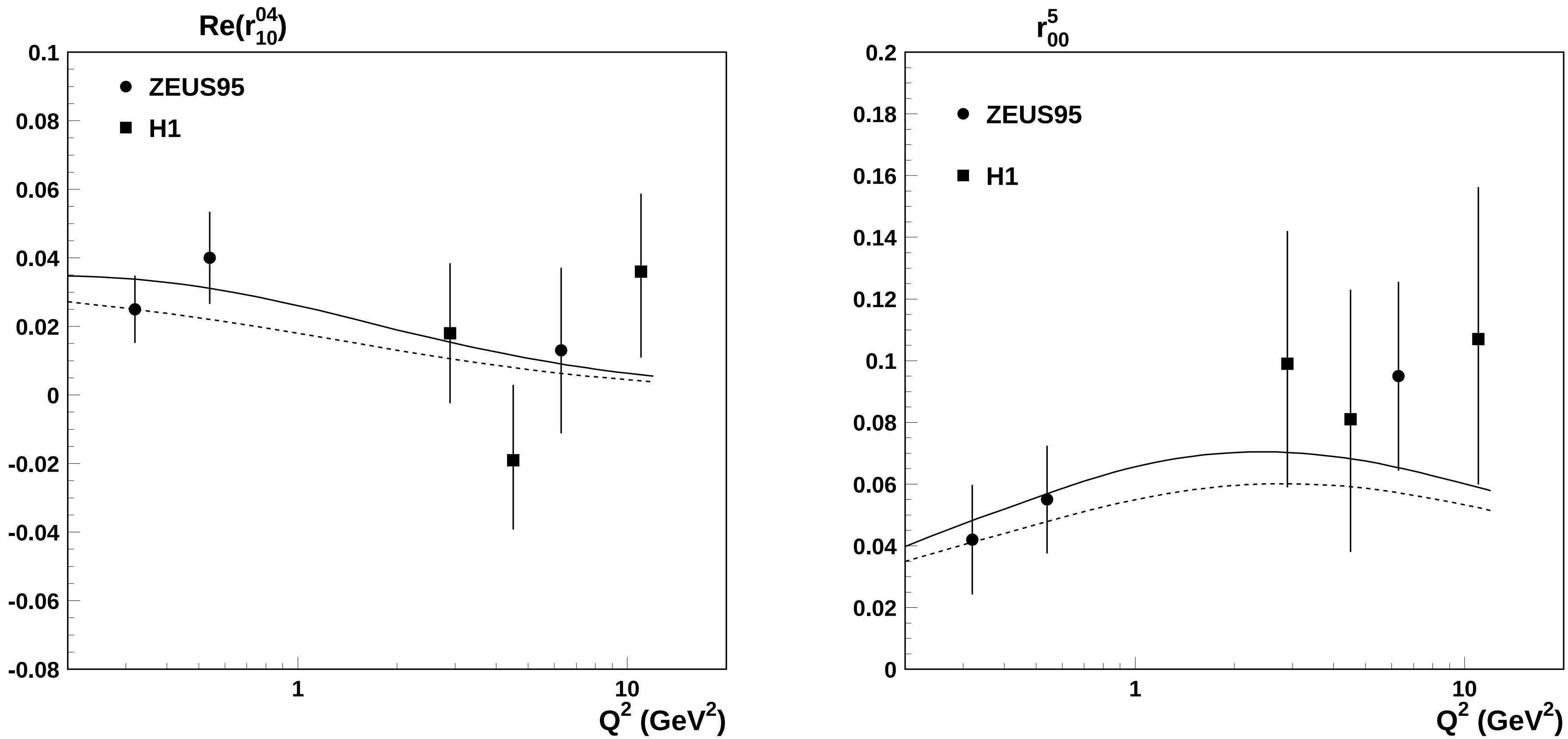
$$\Delta \mathrm{Im}\ \mathrm{r}^2_{1-1} = -\mathsf{I}_4 \mathrm{Im}\ \mathrm{r}^2_{1-1} + \frac{a}{\epsilon}(\mathsf{I}_1 + \mathsf{I}_3)\mathrm{Im}\ \mathrm{r}^6_{1-1} \qquad \Delta \mathrm{Im}\ \mathrm{r}^6_{1-1} = -\mathsf{I}_2 \mathrm{Im}\ \mathrm{r}^6_{1-1} + \frac{\epsilon}{a}(\mathsf{I}_1 + \mathsf{I}_3)\mathrm{Im}\ \mathrm{r}^2_{1-1}.$$

The polarization parameter $\epsilon = \frac{1-y}{1-y-y^2/2}$, $a = \sqrt{2\epsilon(1+\epsilon)}$, and

$$\mathsf{I}_n = \frac{\int_0^{2\pi} \frac{d\Phi}{2\pi}\ \cos n\Phi\ \delta(\Phi)}{\int_0^{2\pi} \frac{d\Phi}{2\pi}\ (1 + \delta(\Phi))}, \quad n = 0, ..., 4.$$

Thus, RC to SDME is linear in the lowest order of $r^{\alpha}_{ij}, r^{\alpha\beta}_{ij}$, and the dependence on $\delta = \delta(\Phi)$ is included in the coefficients I_n.

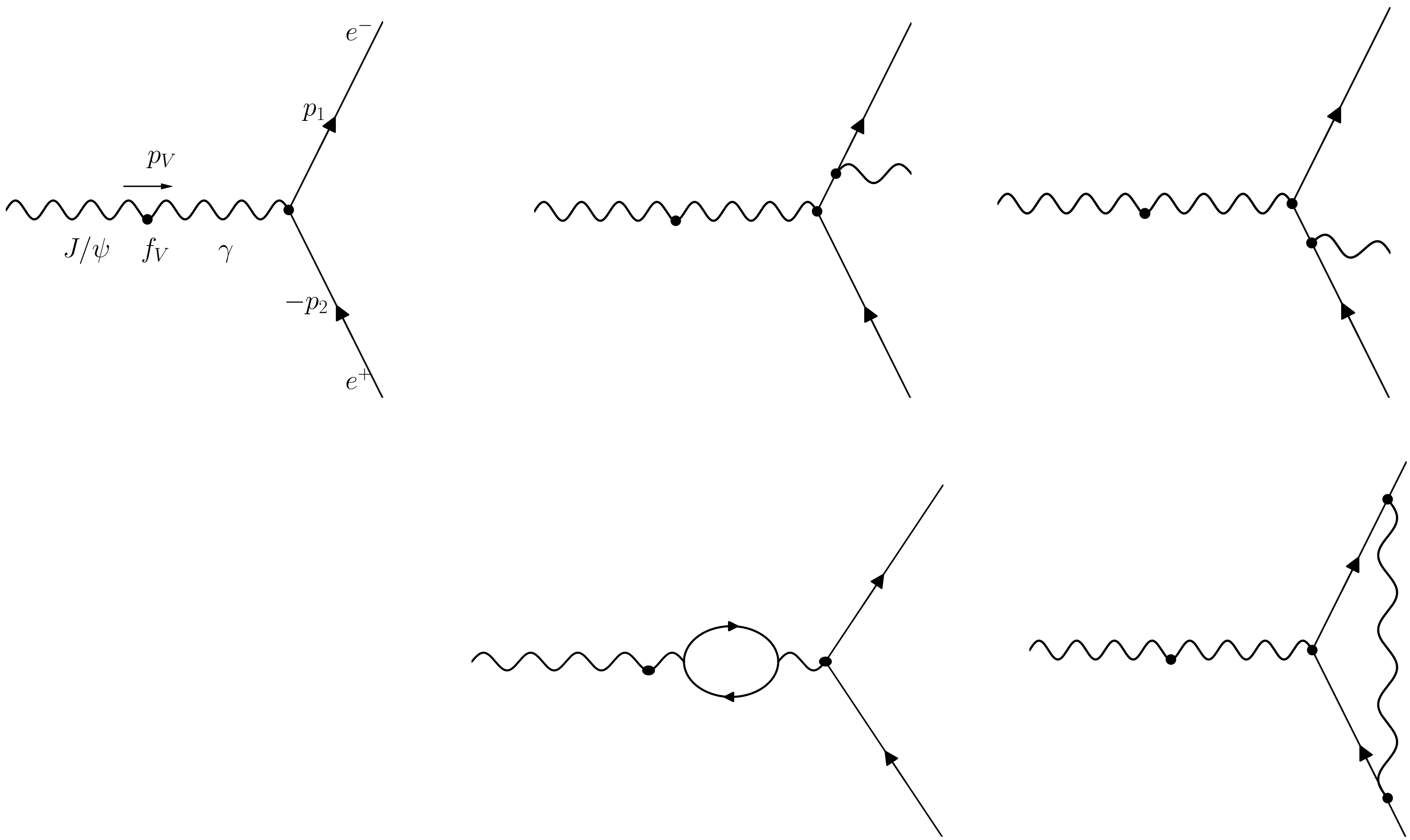
Born abd RC corrected spin-density matrix elements



The dependence of Born (dashed line) and radiative corrected (solid line) spin-density matrix elements on Q^2 under the kinematical conditions of H1/ZEUS experiments.

RC to the width of the vector meson decay

The Feynman diagrams necessary to calculate the lowest order RC to the width of the vector meson decay



RC to the width of the vector meson decay

At the Born level, the vector meson decay width is derived from the following differential formula:

$$d\Gamma_0 = \frac{1}{2E_V}|\mathcal{M}_B|^2 d\Phi_2, \qquad \Gamma_0 = \frac{4\pi\alpha^2}{3f_V^2 M^2}\sqrt{M^2-4m^2}(2m^2+M^2).$$

The total RC defined as $\Gamma = \Gamma_0(1+\delta)$ is: $\delta = \delta_{vac} + \delta_{vert} + \delta_{soft} + \delta_{hard}$ with $L = 2\log\frac{M}{m}$

$$\delta_{vert} = \frac{\alpha}{2\pi}\left(-4\log\frac{M}{\lambda}(L-1) + L + L^2 + \frac{4}{3}\pi^2 - 4\right)$$

$$\delta_{soft} = \frac{\alpha}{\pi}\left(2\log\frac{2\omega}{\lambda}(L-1) + L - \frac{1}{2}L^2 - \frac{\pi^2}{3}\right)$$

$$\delta_{hard} = -\frac{\alpha}{\pi}\left(2\log\frac{2\omega}{M}(L-1) + \frac{3}{2}L + \frac{\pi^2}{3} - \frac{11}{4}\right)$$

resulting in:

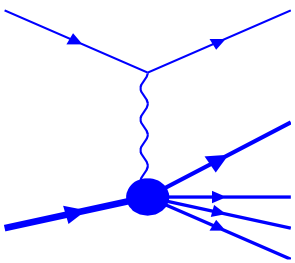
$$\delta_{vert} + \delta_{soft} + \delta_{hard} = \frac{3\alpha}{4\pi}, \qquad \delta_{vac} = \frac{2\alpha}{\pi}\left(\frac{1}{3}L - \frac{5}{9}\right).$$

If the photon energy distribution is required then ($x = 2ME_\gamma$)

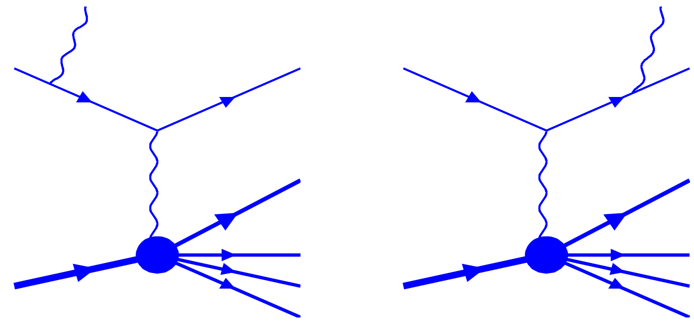
$$\frac{d\delta_{hard}}{dx} = \frac{\alpha}{\pi}\frac{1}{x}\left(\log\frac{M^2-x}{m^2} - 1\right)\left(1 + \left(1 - \frac{x}{M^2}\right)^2\right)$$

RC to inclusive vector meson electroproduction

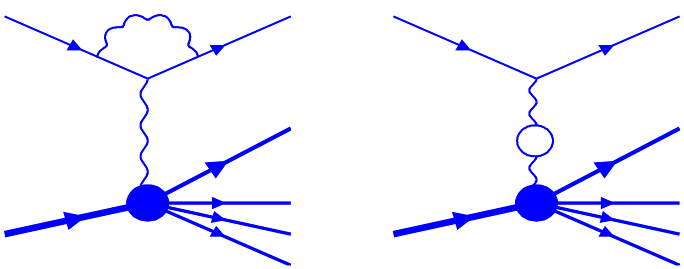
The Born cross section



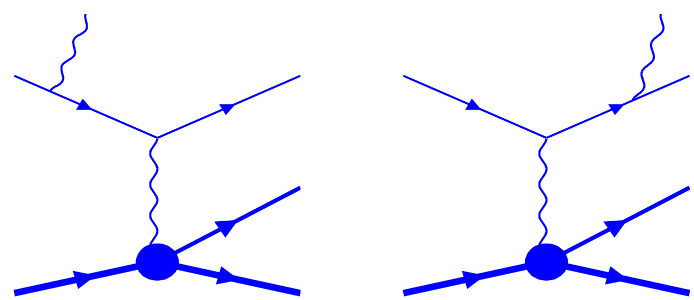
Emission of a radiated photon (inclusive processes)



Loop diagrams



Emission of a radiated photon (exclusive processes)



RC to inclusive vector meson electroproduction

- The cut on the inelasticity v is not applied, therefore no approximations like the soft photon approximation or the approximation we discussed for exclusive vector meson leptoproduction is not possible.
- All theoretical components of RC for semi-inclusive processes and specific calculations of RC including the spin-dependent components and the ultrarelatisistic and leadin-log approaximations, developed by our group are applicable for the case of inclusive vector meson leptoproduction.
- The refs for semi-inclusive RC are:
  - I. Akushevich and A. Ilyichev, Phys. Rev. D 100, 033005 (2019).
  - I. Akushevich, H. Avakian, A. Ilyichev, and S. Srednyak, Eur. Phys. J. A 59, 246 (2023).
  - I. Akushevich, S. Srednyak, and A. Ilyichev, Phys. Rev. D 109, 076028 (2024).
- The main difference is in the hadronic tensor and 18 structure functions. What we need to do is to find models for required SFs. A simple model for unpolirized SFs is implemented in the code DIFFRAD.
- A specific attention has to be paid to the radiative tail from exclusive peak. Different kinematical regions are required for this calculations comparing to the case of RC to exclusive leptoproduction.

Conclusion

- The newly achieved precision in modern experiments on vector-meson leptoproduction necessitates renewed attention to radiative correction (RC) calculations and their implementation in data analysis software.
- We have discussed four experimental designs in the context of RC calculations and their implementation. The main conclusion is that RC calculations for all four designs are well under control.
- For exclusive processes: i) the RC is negative due to the application of cuts on inelasticity, which suppress the phase space available for hard photon radiation with a positive contribution to the cross section; ii) the developed approximation can be useful, particularly for small inelasticity cuts; iii) in this case, the RC effectively reduces to a multiplicative factor applied to the Born contribution.
- For spin-density matrix elements, RCs can be significant; for example, the corrections to $\mathrm{Re}\, r^{04}_{10}$ and r^{5}_{00} can reach the level of about 20%.
- For the width of vector-meson decay, we confirm the classic result for the fully integrated width and provide results for the photon-energy differential distribution.
- For inclusive production of vector mesons, the RC calculation is similar to the well-established case of semi-inclusive processes; particular attention has to be paid to the modeling of hadronic structure functions.