# Theoretical Uncertainties and Systematic Effects in Exclusive Vector Meson Production

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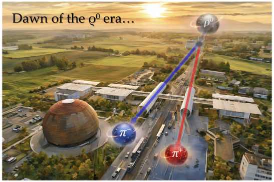
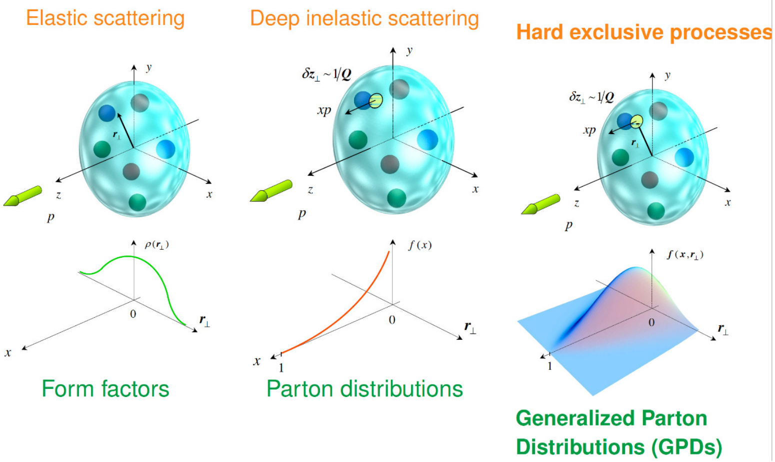


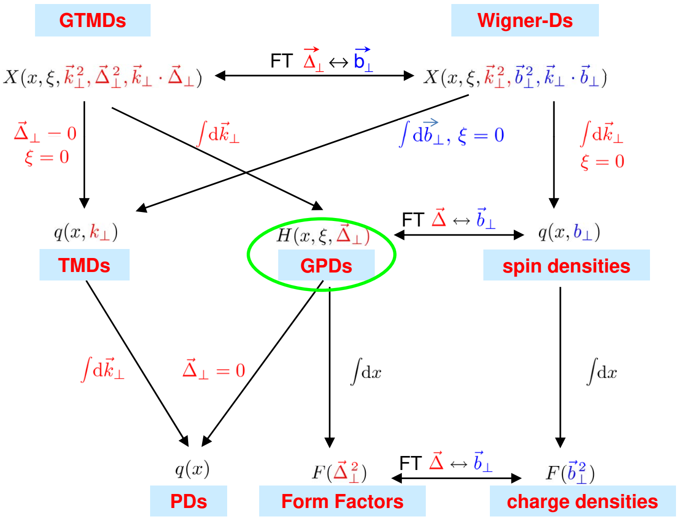

*Impact of Vector Mesons . . .*
*CERN, June 15th 2026*

# Hard exclusive processes and GPDs



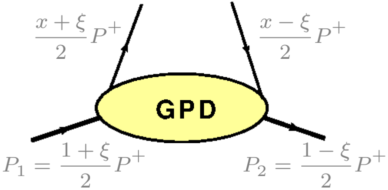

▷ GPDs combine momentum and spatial information

# Nucleon distributions




*[B. Pasquini, 2010]*

# Generalized Parton Distributions




$P = P_1 + P_2$

$\Delta = P_2 - P_1$

GPDs: $\boxed{H^a(x, \xi, t; \mu) \qquad a \in \{q, g\}}$

| | | |
|---|---|---|
| $x$ | | average longitudinal momentum fraction |
| $\xi$ | $= -\frac{\Delta^+}{P^+}$ | longitudinal momentum transfer (skewness) |
| $t$ | $= \Delta^2$ | momentum transfer |

[Müller et al. '94, Radyushkin, Ji '96]

# Generalized parton distributions: properties

GPDs: $\boxed{F^a(x,\xi,t), \qquad a \in \{q,g\}}$

| (twist-2) GPDs | | Partons | | |
|---|---|---|---|---|
| | | helicity conserving (vector) | helicity conserving (axial) | helicity flip (transversity) |
| Nucleon | non-flip | $H$ | $\widetilde{H}$ | $\bar{E}_T$, $\widetilde{E}_T$ |
| | spin flip | $E$ | $\tilde{E}$ | $H_T$, $\widetilde{H}_T$ |

- ▷ $H, \widetilde{H}, H_T(x,0,0) \longrightarrow$ PDFs(x)
- ▷ $\int_{-1}^{1} dx \;\; F(x,\xi,t) \rightarrow$ form factors(t)

▷ higher moments $\rightarrow$ angular momentum and EMT

# How do we unravell GPDs?

- theory input:
  - factorization
  - perturbative calculation
  - GPD models
  - meson distribution amplitudes (DAs)

- deconvolution problem (GPDs are not measured directly)
  - amplitudes = convolutions of GPDs with hard part:

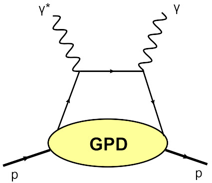
$$\mathcal{A} \sim \int dx\, C(x, \xi, Q^2)\, H(x, \xi, t)$$

  $\rightarrow$ observables $\sim |\mathcal{A}|^2$

- GPDs are universal (same GPDs $\rightarrow$ different processes )

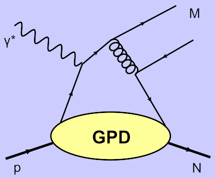
# Hard exclusive processes: focus on meson production

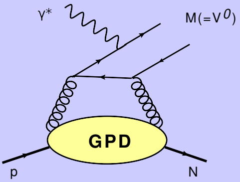
Compton scattering
$\rightarrow$ deeply virtual (DVCS)




$$\boxed{\gamma^* p \rightarrow \gamma p}$$

meson production
$\rightarrow$ deeply virtual (DVMP)




$$\boxed{\gamma^* p \rightarrow M N}$$

$\rightarrow$ meson distribution amplitude (DA): $\phi_{\mathrm{M}}(z)$

$z$ . . . longitudinal momentum fraction

# Factorization for hard exclusive processes

| | | | | |
|---|---|---|---|---|
| $\mathcal{A}$mplitude | = | Hard-scattering amplitude (parton interactions) | $\otimes$ | Hadron wave functions (GPD, DA) |

$\Rightarrow$ observables

- hard scale (e.g. $Q^2$, $s$, $M^2_{J/\Psi}$, $\ldots$) $\rightarrow$ perturbative QCD
  - expansion in $\alpha_s(Q)$
  - power corrections $\sim 1/Q$ (higher twist)

# Generic questions

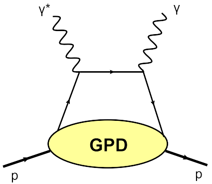
- factorization proven/assumed? broken?

- pQCD calculation:
  - accessible energies high enough ?
  - finite order predictions $\Rightarrow$ theoretical uncertainties ?, dependence on factorization and renormalization scales ?
  - higher-orders in $\alpha_S$ ?
  - higher-twist effects ?

- nonperturbative input:
  - form/modeling of GPDs and DAs

# Hard exclusive processes: focus on meson production

Compton scattering
$\to$ deeply virtual (DVCS)




$$\boxed{\gamma_T^* N \to \gamma N}$$

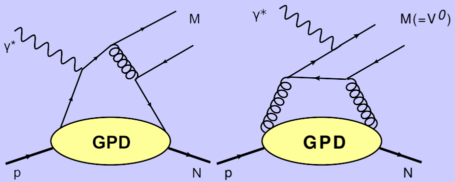
[Collins, Freund '99]

$H^q$, $E^q$, $\widetilde{H}^q$, $\widetilde{E}^q$

NLO:

$H^G$, $E^G$, $\widetilde{H}^G$, $\widetilde{E}^G$, $F_T^G$

meson production
$\to$ deeply virtual (DVMP)




$$\boxed{\gamma_L^* N \to M N'}$$ twist-2

[Collins, Frankfurt, Strikman '97]

$M = V_L$: $H^{q_i}$, $E^{q_i}$; $H^G$, $E^G$

$M = PS$: $\widetilde{H}^{q_i}$, $\widetilde{E}^{q_i}$

$$\boxed{\gamma_T^* N \to M N'}$$ twist-3

$M_{\text{twist-3}} \Rightarrow F_T^q$

# Meson production status

- experimental data from HERA, COMPASS and JLab, expected from EIC ...
- DV$\rho^0$P, DV$\phi$P, DV$\omega$P:
  - data show importance of $\gamma_L^*$ contributions $(Q^2 < 100$ GeV$^2)$

    $\rightarrow$ twist-2 predictions to describe $\sigma_L$
- in contrast to DV$\pi$P:
  - data show suppression of $\gamma_L^*$ contributions $(Q^2 < 10$ GeV$^2)$

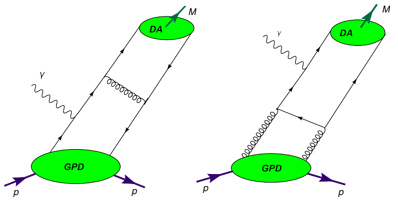
    $\rightarrow$ twist-3 predictions describing $\gamma_T^*$ contributions needed

# DVMP $\boxed{\gamma_L^* \, p \to M N}$ to NLO

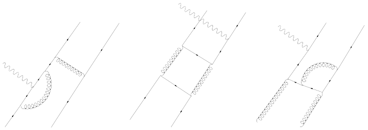
$$\mathcal{A} \sim \text{hard amplitude}(\mu_R, \mu_F) \otimes GPD(\mu_F) \otimes DA(\mu_F)$$

$\mu_R, \mu_F \to f(Q)$

$\boxed{\text{LO} \sim \alpha_S(\mu_R)}$




$\boxed{\text{NLO} \sim \alpha_S^2(\mu_R)}$



[Belitsky, Müller '01, Ivanov et al '04, Duplančić, Müller, KPK '17]

- large NLO corrections [Diehl, Kugler '07, Müller, Lautenschläger, KPK, Schäfer '14] , (and large $\ln(1/\xi)$ terms for $\xi \ll$)
- strong renormalization scale dependence at LO but improved stability at NLO (due to $\alpha_S^2(\mu_R) \log \mu_R$ terms)
- GPD, DA (NLO) evolution to factorization scale $\mu_F$
- few DVMP analysis to NLO [Lautenschläger, Müller, KPK, Schäfer '14 (unpublished), Čuić, Duplančić, Kumerički, KPK '23, Guo et al. '24 '25]

# From x-space to j-space: CPaW formalism

$$\mathcal{A} \sim \text{hard amplitude} \otimes GPD \otimes DA$$

$\overset{x}{\otimes} \equiv \int_{-1}^{1} \frac{dx}{2\xi}$ x-space (momentum fraction)

$\to 2\sum_{j=0}^{\infty} \xi^{-j-1}$ j-space (conformal moments)

(as Mellin moments in DIS $x^n \to C_n^{3/2}(x), C_n^{5/2}(x)$)

$\to \overset{j}{\otimes} \equiv \frac{1}{2i}\int_{c-i\infty}^{c+i\infty} dj\ \xi^{-j-1}\left[i \pm \left\{ \begin{matrix}\tan\\ \cot\end{matrix} \right\} \left(\frac{\pi j}{2}\right)\right]$

(Mellin-Barnes integral)

CPaW(conformal partial wave): [Müller '06, Müller, Schäfer '06], [Kumerički, Müller, KPK, Schäfer '07, Müller, Lautenschläger, KPK, Schäfer '14]

- widely used, intuitive
  $\to$ see P. Sznajder talk
  (PARTONS software)
- simpler evolution
- efficient phenomenology
  (Gepard software) [Kumerički '22]

$\to$ existing NLO DVMP phenomenology: conformal representation

# NLO analysis: global NLO fits DIS, DVCS, DV$\rho^0_L$P

[Čuić, Duplančić, Kumerički, KPK '23]

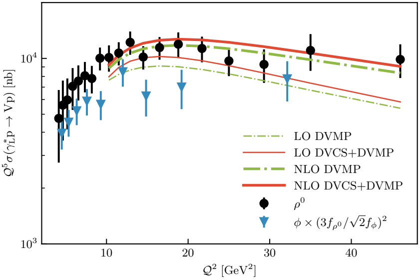
- fits to small-$x$ HERA collider data
- CPaW formalism
- small-$x$ kinematics $\Rightarrow$ only dominant GPD $H^a$, $a \in \{\mathsf{sea}, \mathsf{G}\}$,

  updated [Kumerički, Müller '10] model, 11 parameters
- LT separation: $R \equiv \dfrac{\sigma_{\mathsf{L}}^{\rho^0}}{\sigma_{\mathsf{T}}^{\rho^0}} \to R(W, \mathcal{Q}^2)$ fit

# NLO analysis: global NLO fits DIS, DVCS, DV$\rho_L^0$P

[Čuić, Duplančić, Kumerički, KPK '23]

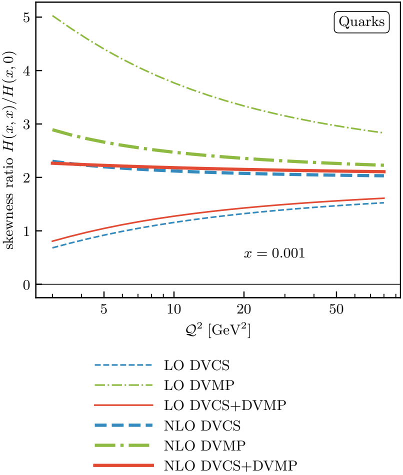
- successful description of $Q^2$ dependence at NLO




- $\sigma_L$ asymptotically $\sim 1/Q^6$
- exp: $\sim 1/Q^4$ for fixed $x_B$,
  $\sim 1/Q^5$ for fixed $W$

- **!** only NLO fit successful

goodness of GPD extraction:



# NLO analysis: global NLO fits DIS, DVCS, DV$\rho_L^0$P

- global DIS+DVCS+DVMP fits show importance of NLO

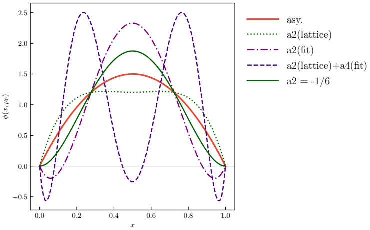
  $\Rightarrow$ universal GPDs

- DVV$_L$P can only be described at NLO

- ! meson DA additional nontrivial nonperturbative input

  $\Rightarrow$ end-point supressed $\rho_L^0$ DA favoured?

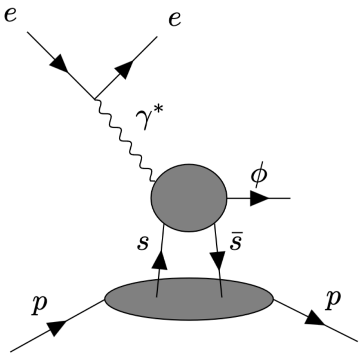
$\rightarrow$ work in progress (Kishore, Kumerički, KPK)



# Near threshold kinematics: DV $\phi$ production proposed

$$\boxed{\gamma^* p \to \phi\, p}$$

- near threshold $\Rightarrow$ large $\xi$ ($\xi \to 1$) $\Rightarrow$ dominance of first conformal moment $\to \sim D(t)$ $\to$ presure in $p$
- $\phi \simeq s\bar{s}$ $\Rightarrow$ sensitivity to $D_s(t)$
- complementary to DVCS $(u, d)$ and $J/\psi$ production (gluons) [Guo, Ji, Yuan '24]
- proposed measurement at JLab




[Hatta, Klest, KPK, Schoenleber '25]

$$D(0) = \underbrace{D_g(0)}_{J/\psi} + \underbrace{D_u(0) + D_d(0)}_{\text{DVCS}} + \underbrace{D_s(0)}_{\phi\ \text{DVMP}} + \ldots .$$

# Beyond leading-twist collinear DVMP

$\rightarrow$ x-space representation

- **Modified perturbative approach (MPA)** [Goloskokov, Kroll '05]
  - meson-side $k_\perp$ + Sudakov suppression; nucleon side unchanged
  - realistic $Q^2$ dependence already at LO
  - broad phenomenology and GPD-model development
  - difficult to extend consistently to NLO

- **Higher twist /transverse photons**
  - [Anikin, Teryaev '02] $\gamma_T^* N \rightarrow V_T N'$: endpoint singularities; collinear factorization breaks; no pheno
  - [Goloskokov, Kroll '13] $\gamma_T^* N \rightarrow V_L N'$: partial twist-3 + MPA; phenomenology for $\rho$, $\omega$ and polarization observables

# Concluding remarks

## Theoretical uncertainties

- Factorization: controlled channel $\gamma_L^* N \to V_L N'$ (twist-2)
- Perturbative accuracy: NLO and GPD/DA evolution needed $\to$ scale dependence, possibly resummation needed
- Higher-twist: transverse photons $\gamma_T^* N \to V N'$ $\to$ endpoint singularities, factorization breaking
- Nonperturbative input: GPD ansatz, vector-meson DA

## Systematics effects

- Measured vs. controlled object: experiments measure $\sigma_T + \epsilon\sigma_L$; theory controls $\sigma_L$
- Separation assumptions: $R = \sigma_L/\sigma_T$, SDMEs, SCHC
- Input to GPD fits: the extracted $\sigma_L$ carries the assumptions used in the separation

# Concluding remarks

## Theoretical uncertainties

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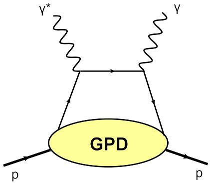
Thank you.

# Appendix

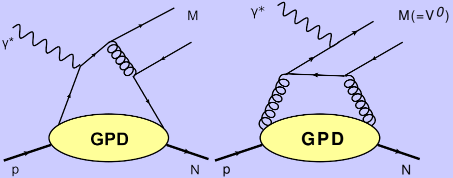
# Hard exclusive processes: focus on meson production

Compton scattering
$\rightarrow$ deeply virtual (DVCS)



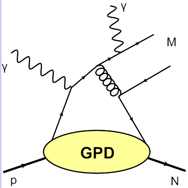

$\boxed{\gamma^* p \rightarrow \gamma p}$

meson production
$\rightarrow$ deeply virtual (DVMP)




$\boxed{\gamma^* p \rightarrow MN}$

$(\gamma M)$ photoproduction



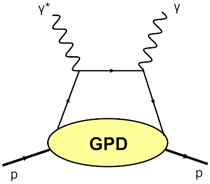

$\boxed{\gamma p \rightarrow (\gamma M) N}$

$\rightarrow$ meson distribution amplitude (DA): $\phi_{\mathrm{M}}(z)$

$z$ . . . longitudinal momentum fraction

# Hard exclusive processes: focus on meson production

Compton scattering
$\to$ deeply virtual
(DVCS)




$\boxed{\gamma_T^* N \to \gamma N}$

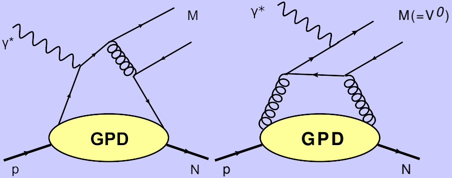
[Collins, Freund '99]

$H^q$, $E^q$, $\widetilde{H}^q$, $\widetilde{E}^q$

NLO:

$H^G$, $E^G$, $\widetilde{H}^G$, $\widetilde{E}^G$, $F_T^G$

meson production
$\to$ deeply virtual (DVMP)




$\boxed{\gamma_L^* N \to M N'}$ twist-2

[Collins, Frankfurt, Strikman '97]

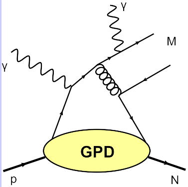
$M = V_L$: $H^{q_i}$, $E^{q_i}$; $H^G$, $E^G$

$M = PS$: $\widetilde{H}^{q_i}$, $\widetilde{E}^{q_i}$

$\boxed{\gamma_T^* N \to M N'}$ twist-3

$M_{\text{twist-3}} \Rightarrow F_T^q$

$(\gamma M)$
photopro-
duction




$\boxed{\gamma N \to (\gamma M) N'}$

[Qiu, Yu '22]
(quarks)

$F^a$, $F_T^a$

# Meson production status

- experimental data from HERA, JLab and COMPASS (CERN), expected from EIC ...
- DV$\rho^0$P:
  - data show importance of $\gamma_L^*$ contributions $(Q^2 < 100$ GeV$^2)$
    $\rightarrow$ twist-2 predictions describe $\sigma_L$
  - global DIS+DVCS+DV$V_L$P fits at NLO [Čuić, Duplančić, Kumerički, P-K. '23]
- DV$\pi$P:
  - data show suppression of $\gamma_L^*$ contributions $(Q^2 < 10$ GeV$^2)$
    $\rightarrow$ twist-3 predictions describing $\gamma_T^*$ contributions needed
  - twist-3 contributions analyzed [Duplančić, Kroll, P-K., Szymanowski '24]

# Global NLO fits (DIS+DVCS+DV$V_L$P)

small-x global fits to HERA collider data ($\rho_0$)

- only NLO predecessor: [Lautenschlager, Müller, Schäfer '13 unpublished]
- hard scattering amplitude corrected [Duplančić, Müller, P-K. '17]
- new NLO fit [Čuić, Duplančić, Kumerički, P-K. '23]: improved treatment of experimental data

GPD model: [Kumerički, Müller, P-K., Schäfer '07, Kumerički, Müller '10]

- $H_j^a(\xi,t) = q_j^a \dfrac{1+j-\alpha_0^a}{1+j-\alpha_0^a-\alpha'^a t}\left(1-\dfrac{t}{m_a^2}\right)^{-2}\left(1+s_2^a\,\xi^2+s_4^a\,\xi^4\right)$

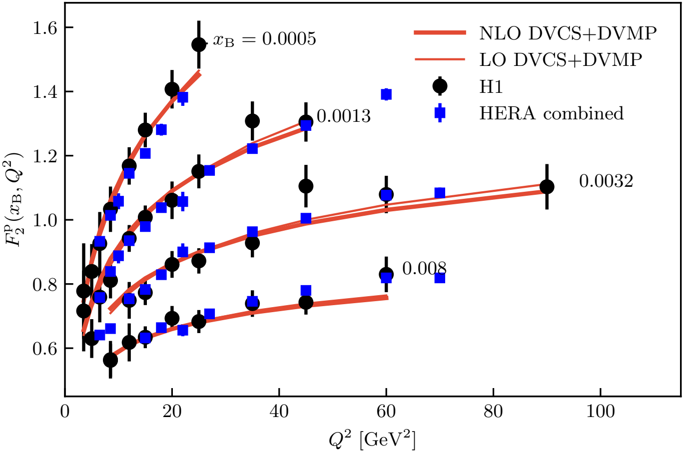
$$q_j^a = N_a \frac{B(1-\alpha_0^a+j,\beta^a+1)}{B(2-\alpha_0^a,\beta^a+1)}$$

- small-$x$ kinematics $\Rightarrow a \in \{\mathsf{sea},\mathsf{G}\}$, only dominant $H$ GPD

Fit parameters:

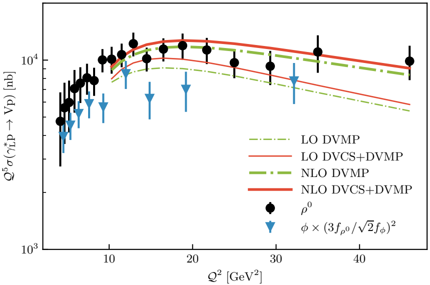
- DIS: $\{N_{\mathsf{sea}},\, \alpha_0^{\mathsf{sea}},\, \alpha_0^{\mathsf{G}}\}$
- DVCS+DVMP: $\{\alpha'^{\mathsf{sea}},\, \alpha'^{\mathsf{G}},\, m_{\mathsf{sea}}^2,\, m_{\mathsf{G}}^2,\, s_2^{\mathsf{sea}},\, s_2^{\mathsf{G}},\, s_4^{\mathsf{sea}},\, s_4^{\mathsf{G}}\}$

# Global NLO fits (DIS+DVCS+DV$V_L$P)




- may seem trivial, but not all popular models describe DIS

# Global NLO fits (DIS+DVCS+DVV$_L$P)



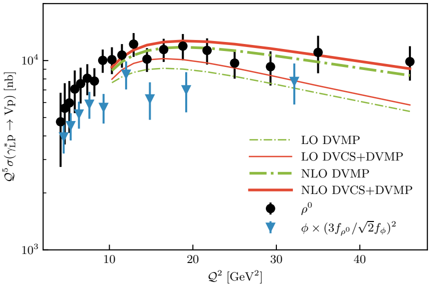

$\sigma_L$ asymptoticaly: $\dfrac{1}{Q^6}$

experimental data for fixed $x_B$: $\approx \dfrac{1}{Q^4}$, for fixed $W$: $\approx \dfrac{1}{Q^5}$

- successful description of $Q^2$ dependence

# NLO analysis: global NLO fits DIS, DVCS, DV$\rho^0_L$P

[Čuić, Duplančić, Kumerički, KPK ’23]




$\sigma_L$ asymptoticaly $\dfrac{1}{Q^6}$ while experimental data $\approx \dfrac{1}{Q^5}$

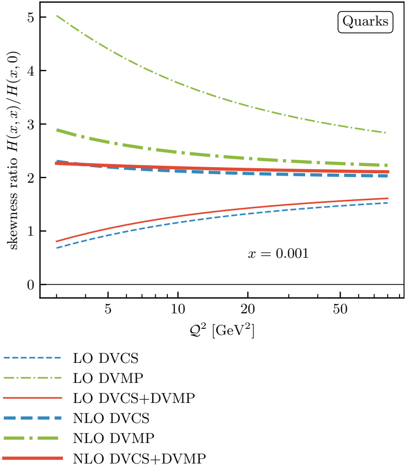
$\to$ **!** successful description of $Q^2$ dependence at NLO

# NLO analysis: global NLO fits DIS, DVCS, DV$\rho_L^0$P

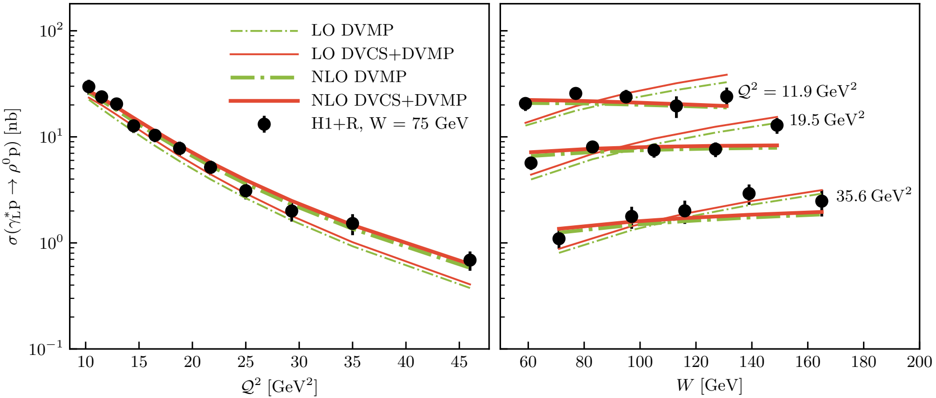
[Čuić, Duplančić, Kumerički, KPK '23]

- strong constraints on GPDs
- **!** only NLO fit successful

measures goodness of GPD extraction:



# Global NLO fits (DIS+DVCS+DV$V_L$P)




$$\boxed{R \equiv \frac{\sigma_\mathsf{L}^{\rho^0}}{\sigma_\mathsf{T}^{\rho^0}}} \to R(W, \mathcal{Q}^2) = \frac{\mathcal{Q}^2}{m_{\rho^0}^2}\left(1 + a\frac{\mathcal{Q}^2}{m_{\rho^0}^2}\right)^{-p}\left(1 + b\frac{\mathcal{Q}^2}{W}\right) \text{ fit}$$

# Global NLO fits (DIS+DVCS+DVV$_L$P)

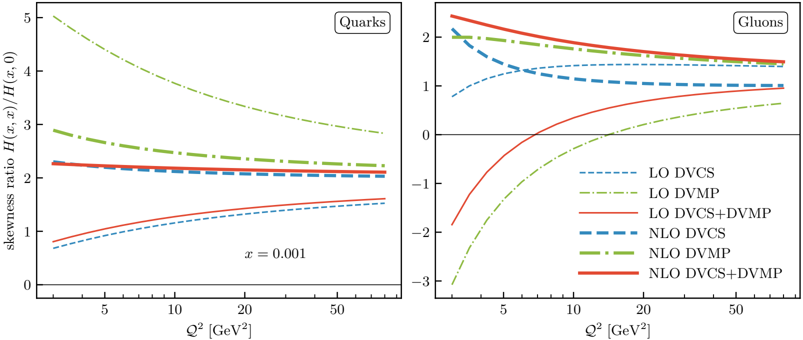
| Dataset | Refs. | $n_{\text{pts}}$ | LO- DVCS | LO- DVMP | LO- DVCS-DVMP | NLO- DVCS | NLO- DVMP | NLO- DVCS-DVMP |
|---|---|---|---|---|---|---|---|---|
| DIS | [90] | 85 | 0.6 | 0.6 | 0.6 | 0.8 | 0.8 | 0.8 |
| DVCS | [92–95] | 27 | 0.4 | $\gg 1$ | 0.6 | 0.6 | $\gg 1$ | 0.8 |
| DVMP | [88, 89] | 45 | $\gg 1$ | 3.1 | 3.3 | $\gg 1$ | 1.5 | 1.8 |
| Total | | 157 | $\gg 1$ | $\gg 1$ | 1.4 | 3.7 | $\gg 1$ | 1.1 |

**Table 3**. Values of $\chi^2/n_{\text{pts}}$ for each LO or NLO model (columns) for the total DIS + DVCS + DVMP dataset and for subsets corresponding to different processes (rows). (The values denoted by $\gg 1$ are greater than 10.).

- NLO DVCS-DVMP fit describes the data well

# Global NLO fits (DIS+DVCS+DVV$_L$P)

Skewness ratio $r = \dfrac{H(x,x)}{H(x,0)}$




- $r$ measures goodness of GPD extraction $\Rightarrow$ NLO fit successful

# NLO for DV $V_L$ production

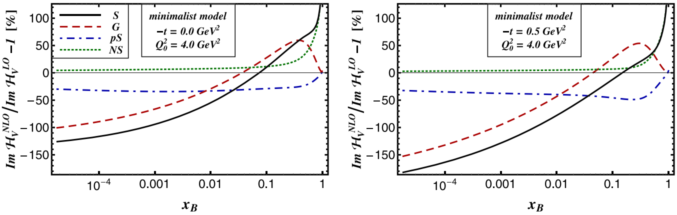



Fig. 6. Relative NLO corrections to the imaginary part of the flavor singlet TFF $\mathcal{F}_V^S$ (solid) broken down to the gluon (dashed), pure singlet quark (dash-dotted) and 'non-singlet' quark (dotted) at $t = 0$ GeV$^2$ (left panel) and $t = -0.5$ GeV$^2$ (right panel) at the initial scale $\mathcal{Q}_0^2 = 4$ GeV$^2$.

[Müller, Lautenschlager, P-K., Schäfer '14]

- large NLO corrections for small $x_B$, i.e., $\xi$

$$\Uparrow$$

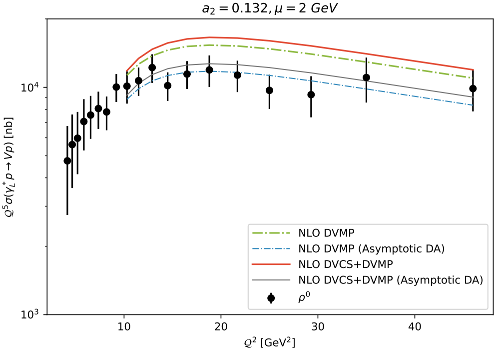
big $\ln(1/\xi)$ terms for $\xi \ll$, i.e, $j = 0$ pole, in gluon evolution and gluon coefficient function

# Improving DA description

→ work in progress with Raj Kishore, K. Kumerički

$$\phi^\rho(u,\mu_F) = 6u(1-u)\left[1 + a_2(\mu_F)\, C_2^{3/2}(2u-1) + \dots\right]$$

$a_2(\mu_0) = 0.132,\ \mu_0 = 2$ GeV [Braun et al. '16]




⇒ significant impact of the DA form

# Improving DA description

→ work in progress with Raj Kishore, K. Kumerički

- new GPD fit:

| Dataset | Refs. | $n_{\text{pts}}$ | LO | NLO | | | |
|---|---|---|---|---|---|---|---|
| | | | asy. | asy. | $a_2$ (lat.) | $a_2$ (fit) | $a_2$ (lat.) $+a_4$ (fit) |
| DIS | [5] | 85 | 0.6 | 0.8 | 0.8 | 0.8 | 0.8 |
| DVCS | [6, 7, 8, 9] | 27 | 0.6 | 0.8 | 0.9 | 0.9 | 0.9 |
| DVMP | [1, 10] | 45 | 3.3 | 1.8 | 2.1 | 0.9 | 0.7 |
| Total | | 157 | 1.4 | 1.1 | 1.2 | 0.9 | 0.8 |

- $a_2$(fit)$= -0.369$
- $a_2$[lattice: Braun et al. '16]$= 0.132$, $a_4$(fit)$= -0.519$

⇒ dominant influence of LO $\sim \sum a_n$

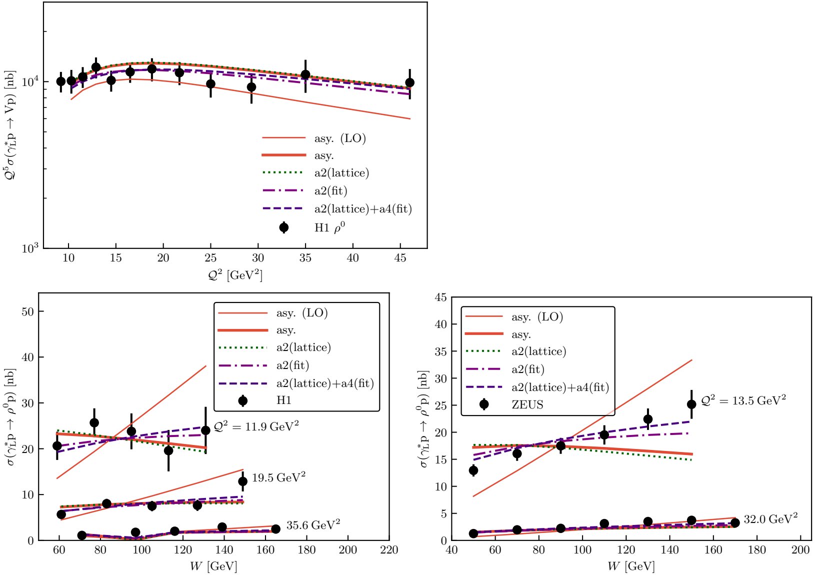
⇒ indication of end-point supressed $\rho_L$ DA [Liu, Shuryak, Zahed '24]

⇒ GPDs not changed much

(similar skewness ratio, $W$ dependence improved)

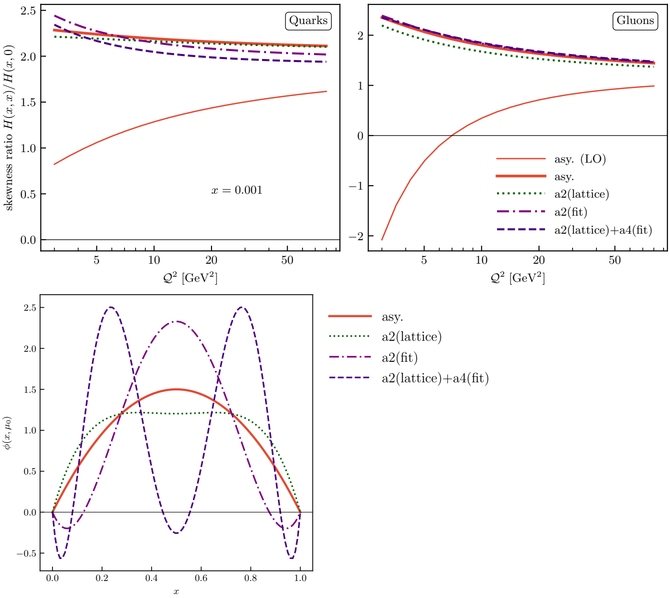
# Improving DA description

→ work in progress with Raj Kishore, K. Kumerički



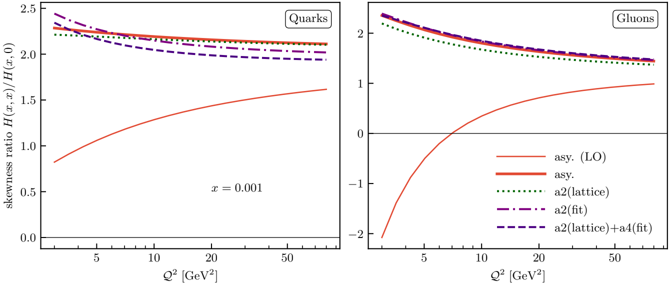
# Improving DA description

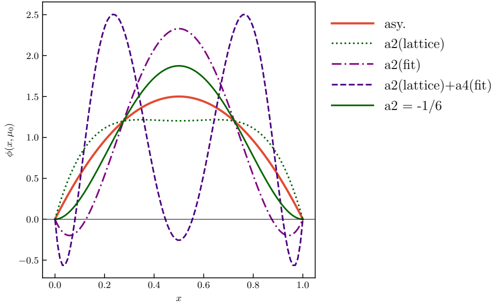
→ work in progress with Raj Kishore, K. Kumerički



# Improving DA description

→ work in progress with Raj Kishore, K. Kumerički

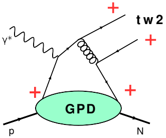


# When the simple picture fails: twist-3 effects in DVMP

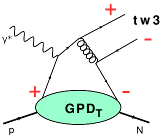
$$\boxed{\gamma^* \, p \to \pi p}$$

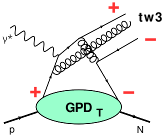
twist-2 . . . non-flip subprocess amplitudes




$\widetilde{H}$, $\widetilde{E}$

twist-3 . . . flip subprocess amplitudes




$H_T$, $\bar{E}_T = 2\widetilde{H}_T + E_T$, . . .

- twist-2 contribution insufficient
- twist-3 contributions $\sim \mu_\pi / Q$ ($\mu_\pi = 2$ GeV) important
  - factorization challenging

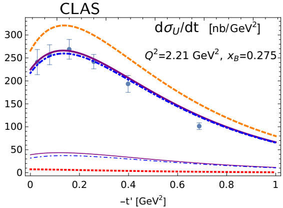
[Kroll, KPK '21]

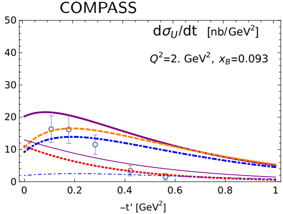
# DV$\pi$P at twist-3

! $\sigma_T$ (twist-3) dominant

! $\sigma_L$ (twist-2) negligible

(except for COMPASS kinematics at $x_B < 0.1$)




red curves: tw2

blue curves: tw3

exp data:

full circles [CLAS '14]

open circles [COMPASS '19]

[Duplančić, Kroll, KPK, Szymanowski '24]

# Treatment of end-point singularities: MPA

$\Rightarrow$ Modified perturbative approach (MPA) [Goloskov, Kroll, '10]

- $k_\perp$ quark transverse momenta in pion

$$\frac{1}{((x+\xi)\tau - k_T^2/Q^2(2\xi) + i\epsilon)} \frac{1}{(x+\xi+i\epsilon)}$$

- $\phi_\pi \to$ light-cone wave function $\Psi_\pi \sim \phi_\pi \exp\left[-a_\pi^2 k_\perp^2\right]$
- $\int_0^1 d\tau \quad \to \int d^2\mathbf{k}_T \int_0^1 d\tau \quad \overset{\text{FT}}{\to} \int d^2\mathbf{b} \int_0^1 d\tau$
- Sudakov form factor $\exp\left[-S(\tau, \mathbf{b}, Q^2)\right]$

- consistently treated 2- and 3-body tw3 contributions, as well as tw2
- involved multidimensional integrations
- calculation of NLO corrections would be complicated

# Appendix: EMT matrix element

$$
\begin{aligned}
\langle p', s' | T_a^{\mu\nu} | p, s \rangle \quad = \quad & \bar{u}(p', s') \Bigg[ A_a(t)\, \gamma^{(\mu} P^{\nu)} + B_a(t)\, \frac{P^{(\mu} i\sigma^{\nu)\rho} \Delta_\rho}{2M} \\
& + D_a(t)\, \frac{\Delta^\mu \Delta^\nu - g^{\mu\nu} \Delta^2}{4M} + \bar{C}_a(t)\, M\, g^{\mu\nu} \Bigg] u(p, s)
\end{aligned}
$$

$$
P = \frac{p + p'}{2}, \qquad \Delta = p' - p, \qquad t = \Delta^2
$$

- $A_a \to$ momentum / energy
- $A_a + B_a \to$ angular momentum
- $D_a \to$ mechanical properties

# Appendix: GPD moments and GFFs

$$\int_{-1}^{1} \mathrm{d}x\, x\, H^a(x,\xi,t) = A_a(t) + \xi^2 D_a(t)$$

$$\int_{-1}^{1} \mathrm{d}x\, x\, E^a(x,\xi,t) = B_a(t) - \xi^2 D_a(t)$$

$$\int_{-1}^{1} \mathrm{d}x\, x \left[H^a + E^a\right] = A_a(t) + B_a(t)$$

$$J_a(t) = \frac{1}{2}\left[A_a(t) + B_a(t)\right]$$

$$D(t) = \sum_a D_a(t)$$

# DVMP differential cross-sections

$$\boxed{\begin{aligned}\frac{d^4\sigma}{dW^2dQ^2dtd\varphi} \quad &= \quad \frac{\alpha_{em}(W^2-m_N^2)}{16\pi^2E_L^2m_N^2Q^2(1-\varepsilon)}\left(\frac{d\sigma_T}{dt}+\varepsilon\frac{d\sigma_L}{dt}\right.\\ &\left. + \varepsilon\cos(2\varphi)\frac{d\sigma_{TT}}{dt}+\sqrt{2\varepsilon(1+\varepsilon)}\cos\varphi\frac{d\sigma_{LT}}{dt}\right)\end{aligned}}$$

$$\frac{d\sigma_U}{dt} = \frac{d\sigma_T}{dt} + \epsilon\frac{d\sigma_L}{dt}$$

$$\frac{d\sigma_L}{dt} : \widetilde{H}, \widetilde{E} \qquad \frac{d\sigma_T}{dt} : H_T, \bar{E}_T \qquad \frac{d\sigma_{TT}}{dt} : \bar{E}_T \qquad \frac{d\sigma_{LT}}{dt} : \widetilde{E}, H_T$$

$$\left|\frac{d\sigma_{TT}}{dt}\right| \le \frac{d\sigma_T}{dt}$$