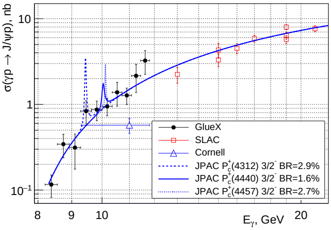
Helicity Amplitude Formalism for Vector Meson Leptonic Decays in Electroproduction

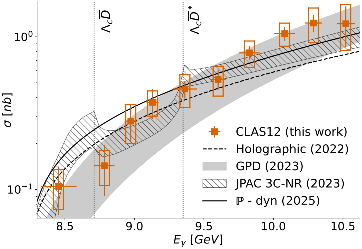
Alessandro Pilloni

June 15, 2026

Quarkonium photo- and electroproduction



GlueX 1905.10811

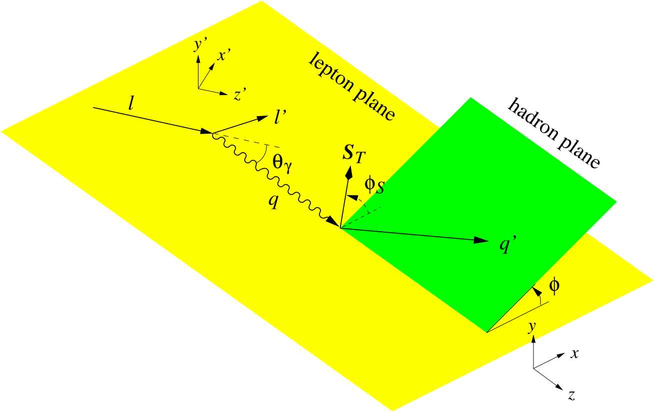


CLAS 2602.22128

Quarkonium photo- and electroproduction has received lots of attention in recent years, since it was proposed as a way to access gluonic proton matrix elements.

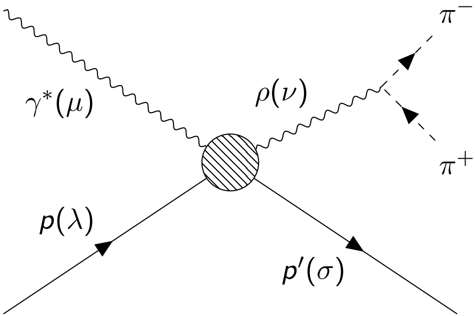
While the attention is mostly focused on differential and total cross sections, little is known about the SDMEs, which could allow to validate the model assumed for matrix element extractions.

Rho vs. ψ electroproduction



Diehl 0704.1565

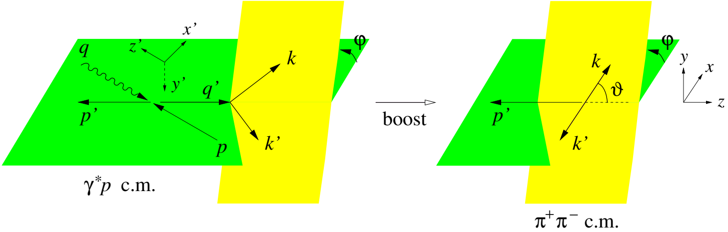
Rho vs. ψ electroproduction



The full information on the $\gamma^* p \to \rho p$ process is contained in the helicity amplitude $T^{\nu\sigma}_{\mu\lambda}(x_B, t; Q^2)$. Including the decay $\rho \to \pi^+\pi^-$, one gets

$$\sum_\nu T^{\nu\sigma}_{\mu\lambda}(x_B, t; Q^2)\, Y^1_\nu(\varphi, \vartheta)$$

Rho vs. ψ electroproduction

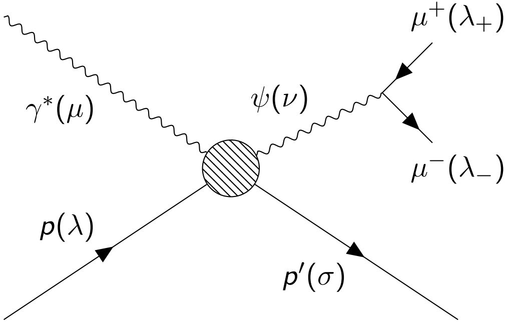


The full information on the $\gamma^* p \to \rho p$ process is contained in the helicity amplitude $T^{\nu\sigma}_{\mu\lambda}(x_B, t; Q^2)$. Including the decay $\rho \to \pi^+\pi^-$, one gets

$$\sum_\nu T^{\nu\sigma}_{\mu\lambda}(x_B, t; Q^2)\, Y^1_\nu(\varphi, \vartheta)$$

We consider here the decay angle defined in the helicity frame, suitable for, say, extract the baryon parent of the ρ. Other definitions (e.g. GJ) are possible.

Rho vs. ψ electroproduction



Quarkonium cleanest decay channel is into lepton pair. The amplitude is described by

$$\sum_{\nu} T^{\nu\sigma}_{\mu\lambda}(x_B, t; Q^2) \sqrt{\frac{3}{4\pi}} D^{1*}_{\nu,\lambda_+-\lambda_-}(\varphi, \vartheta, 0) H_{\lambda_+,\lambda_-}$$

Rho vs. ψ electroproduction

Quarkonium cleanest decay channel is into lepton pair. The amplitude is described by

$$\sum_{\nu} T^{\nu\sigma}_{\mu\lambda}(x_B, t; Q^2)\sqrt{\frac{3}{4\pi}} D^{1*}_{\nu,\lambda_+-\lambda_-}(\varphi,\vartheta,0) H_{\lambda_+,\lambda_-}$$

However, parity and the fact that the leptonic decay is driven by the known QED vertex, one gets

$$H_{\frac{1}{2}-\frac{1}{2}} = H_{-\frac{1}{2}\ \frac{1}{2}} = H \qquad H_{\frac{1}{2}\ \frac{1}{2}} = H_{-\frac{1}{2}-\frac{1}{2}} \simeq 0$$

Therefore the amplitude reduces to

$$\propto \sum_{\nu} T^{\nu\sigma}_{\mu\lambda}(x_B, t; Q^2)\sqrt{\frac{3}{4\pi}} D^{1*}_{\nu\Delta}(\varphi,\vartheta,0)$$

with $\Delta = \pm 1$.

The ψ SDMEs

Following Diehl, we define the SDMEs as

$$\rho^{\nu\nu'}_{\mu\mu',\lambda\lambda'} = (N_T + \varepsilon N_L)^{-1} \sum_\sigma T^{\nu\sigma}_{\mu\lambda}\, T^{*\nu'\sigma}_{\mu'\lambda'}$$

with $N_T = \sum_{\lambda\nu\sigma} \left| T^{\nu\sigma}_{+,\lambda} \right|^2$, $N_L = \sum_{\lambda\nu\sigma} \left| T^{\nu\sigma}_{0,\lambda} \right|^2$ proportional to the transverse and longitudinal cross sections. The decay can thus be included as $\rho_{\mu\mu',\lambda\lambda'} = \sum_{\nu\nu'} \rho^{\nu\nu'}_{\mu\mu',\lambda\lambda'} Y_{\nu\nu'}$, with

$$Y_{\nu\nu'}(\varphi,\vartheta) = \begin{cases} Y^1_\nu\, Y^{1*}_{\nu'} & \text{for } \rho \\ \sum_\Delta D^{1*}_{\nu\Delta}\, D^1_{\nu'\Delta} & \text{for } \psi \end{cases}$$

The ψ SDMEs

$$\rho: \quad \begin{pmatrix} \frac{\sin^2\vartheta}{2} & -\frac{e^{i\varphi}\cos\vartheta\,\sin\vartheta}{\sqrt{2}} & -\frac{1}{2}e^{2i\varphi}\sin^2\vartheta \\ -\frac{e^{-i\varphi}\cos\vartheta\,\sin\vartheta}{\sqrt{2}} & \cos^2\vartheta & \frac{e^{i\varphi}\cos\vartheta\,\sin\vartheta}{\sqrt{2}} \\ -\frac{1}{2}e^{-2i\varphi}\sin^2\vartheta & \frac{e^{-i\varphi}\cos\vartheta\,\sin\vartheta}{\sqrt{2}} & \frac{\sin^2\vartheta}{2} \end{pmatrix}$$

$$\psi: \quad \begin{pmatrix} 1-\frac{\sin^2\vartheta}{2} & \frac{e^{i\varphi}\cos\vartheta\,\sin\vartheta}{\sqrt{2}} & \frac{1}{2}e^{2i\varphi}\sin^2\vartheta \\ \frac{e^{-i\varphi}\cos\vartheta\,\sin\vartheta}{\sqrt{2}} & \sin^2\vartheta & -\frac{e^{i\varphi}\cos\vartheta\,\sin\vartheta}{\sqrt{2}} \\ \frac{1}{2}e^{-2i\varphi}\sin^2\vartheta & -\frac{e^{-i\varphi}\cos\vartheta\,\sin\vartheta}{\sqrt{2}} & 1-\frac{\sin^2\vartheta}{2} \end{pmatrix}$$

The ψ SDMEs

$$\rho:\quad \begin{pmatrix} \dfrac{\sin^2\vartheta}{2} & -\dfrac{e^{i\varphi}\cos\vartheta\,\sin\vartheta}{\sqrt{2}} & -\dfrac{1}{2}e^{2i\varphi}\sin^2\vartheta \\ -\dfrac{e^{-i\varphi}\cos\vartheta\,\sin\vartheta}{\sqrt{2}} & \cos^2\vartheta & \dfrac{e^{i\varphi}\cos\vartheta\,\sin\vartheta}{\sqrt{2}} \\ -\dfrac{1}{2}e^{-2i\varphi}\sin^2\vartheta & \dfrac{e^{-i\varphi}\cos\vartheta\,\sin\vartheta}{\sqrt{2}} & \dfrac{\sin^2\vartheta}{2} \end{pmatrix}$$

$$\psi:\quad \begin{pmatrix} 1-\dfrac{\sin^2\vartheta}{2} & \dfrac{e^{i\varphi}\cos\vartheta\,\sin\vartheta}{\sqrt{2}} & \dfrac{1}{2}e^{2i\varphi}\sin^2\vartheta \\ \dfrac{e^{-i\varphi}\cos\vartheta\,\sin\vartheta}{\sqrt{2}} & \sin^2\vartheta & -\dfrac{e^{i\varphi}\cos\vartheta\,\sin\vartheta}{\sqrt{2}} \\ \dfrac{1}{2}e^{-2i\varphi}\sin^2\vartheta & -\dfrac{e^{-i\varphi}\cos\vartheta\,\sin\vartheta}{\sqrt{2}} & 1-\dfrac{\sin^2\vartheta}{2} \end{pmatrix}$$

Cross section decomposition

The cross section can be written as

$$\frac{d\sigma}{d\psi\, d\phi\, d\varphi\, d(\cos\vartheta)\, dx_B\, dQ^2\, dt} = \frac{1}{(2\pi)^2}\, \frac{d\sigma}{dx_B\, dQ^2\, dt}$$
$$\times \Big(W_{UU} + P_\ell W_{LU} + S_L W_{UL} + P_\ell S_L W_{LL} + S_T W_{UT} + P_\ell S_T W_{LT} \Big)$$

with

$$\frac{d\sigma}{dx_B\, dQ^2\, dt} = \frac{\alpha_{\rm em}}{2\pi}\, \frac{y^2}{1-\varepsilon}\, \frac{1-x_B}{x_B}\, \frac{1}{Q^2} \left(\frac{d\sigma_T}{dt} + \varepsilon\, \frac{d\sigma_L}{dt} \right),$$

where $d\sigma_T/dt$ and $d\sigma_L/dt$ are the usual $\gamma^* p$ cross sections for a transverse and longitudinal photon and an unpolarized proton, with Hand's convention for virtual photon flux.

Example for unpolarized beam and target: ρ

For unpolarized target and beam we have

$$W_{UU}(\phi,\varphi,\vartheta) =$$
$$\frac{3}{4\pi}\left[\cos^2\vartheta\; W_{UU}^{LL}(\phi) + \sqrt{2}\cos\vartheta\,\sin\vartheta\; W_{UU}^{LT}(\phi,\varphi) + \sin^2\vartheta\; W_{UU}^{TT}(\phi,\varphi)\right]$$

in terms of unpolarized SDMEs, $u_{\mu\mu'}^{\nu\nu'} = \frac{1}{2}\left(\rho_{\mu\mu',++}^{\nu\nu'} + \rho_{\mu\mu',--}^{\nu\nu'}\right)$

Example for unpolarized beam and target: ρ

For unpolarized target and beam we have

$$W_{UU}^{LL}(\phi) = \left(u_{++}^{0\,0} + \varepsilon u_{0\,0}^{0\,0}\right) - 2\cos\phi\,\sqrt{\varepsilon(1+\varepsilon)}\,\mathrm{Re}\,u_{0+}^{0\,0} - \cos(2\phi)\,\varepsilon u_{-+}^{0\,0}\,,$$

$$W_{UU}^{LT}(\phi,\varphi) = \cos(\phi+\varphi)\,\sqrt{\varepsilon(1+\varepsilon)}\,\mathrm{Re}\left(u_{0+}^{0+} - u_{0+}^{-0}\right)$$

$$- \cos\varphi\;\mathrm{Re}\left(u_{++}^{0+} - u_{++}^{-0} + 2\varepsilon u_{0\,0}^{0+}\right) + \cos(2\phi+\varphi)\,\varepsilon\,\mathrm{Re}\,u_{-+}^{0+}$$

$$- \cos(\phi-\varphi)\,\sqrt{\varepsilon(1+\varepsilon)}\,\mathrm{Re}\left(u_{0+}^{0-} - u_{0+}^{+0}\right) + \cos(2\phi-\varphi)\,\varepsilon\,\mathrm{Re}\,u_{-+}^{+0}\,,$$

$$W_{UU}^{TT}(\phi,\varphi) = \tfrac{1}{2}\left(u_{++}^{++} + u_{++}^{--} + 2\varepsilon u_{0\,0}^{++}\right) + \tfrac{1}{2}\cos(2\phi+2\varphi)\,\varepsilon u_{-+}^{-+}$$

$$- \cos\phi\,\sqrt{\varepsilon(1+\varepsilon)}\,\mathrm{Re}\left(u_{0+}^{++} + u_{0+}^{--}\right) + \cos(\phi+2\varphi)\,\sqrt{\varepsilon(1+\varepsilon)}\,\mathrm{Re}\,u_{0+}^{-+}$$

$$- \cos(2\varphi)\;\mathrm{Re}\left(u_{++}^{-+} + \varepsilon u_{0\,0}^{-+}\right) - \cos(2\phi)\,\varepsilon\,\mathrm{Re}\,u_{-+}^{++}$$

Example for unpolarized beam and target: ψ

For unpolarized target and beam we have

$$W_{UU}(\phi,\varphi,\vartheta) =$$
$$\frac{3}{4\pi}\Big[\sin^2\vartheta\; W^{LL}_{UU}(\phi) - \sqrt{2}\cos\vartheta\,\sin\vartheta\; W^{LT}_{UU}(\phi,\varphi)$$
$$- \sin^2\vartheta\; W^{TT}_{UU}(\phi,\varphi) + W'^{TT}_{UU}(\phi)\Big]$$

in terms of unpolarized SDMEs, $u^{\nu\nu'}_{\mu\mu'} = \frac{1}{2}\left(\rho^{\nu\nu'}_{\mu\mu',++} + \rho^{\nu\nu'}_{\mu\mu',--}\right)$

Example for unpolarized beam and target: ψ

For unpolarized target and beam we have

$$
\begin{aligned}
W'^{TT}_{UU}(\phi) = &\left(u^{++}_{++} + u^{--}_{++} + 2\,\mathrm{Re}\,u^{-+}_{++} + 2\varepsilon u^{++}_{0\,0} + 2\varepsilon\,\mathrm{Re}\,u^{-+}_{0\,0}\right) \\
&+ \cos(2\phi)\,\varepsilon\left(u^{-+}_{-+} + u^{+-}_{-+} + 2\,\mathrm{Re}\,u^{++}_{-+}\right) \\
&- 2\cos\phi\,\sqrt{\varepsilon(1+\varepsilon)}\,\mathrm{Re}\left(u^{++}_{0+} + u^{--}_{0+} + u^{-+}_{0+} + u^{+-}_{0+}\right)
\end{aligned}
$$

Conclusions

- Lepton spin reshuffles the terms in the SDMEs
- In the unpolarized case, some terms have similar ϑ dependence, might be hard to disentangle
- Calculation of polarized terms come straightforwardly from the same formalism

Thank you!