Impact of Vector Mesons on the Studies of the 3D Structure of the Nucleon

The Role of Longitudinal Photons in the Factorization of Hard Scattering Processes

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Preamble Motivation

Owing to the kinematic reach & dynamical mechanisms underlying TMD observables in SIDIS, the extraction of the independent structure functions from measured cross sections, and their quantitative mapping onto the nucleon's nonperturbative partonic structure, continue to present significant phenomenological challenges (talks of H. Avakian, A. Bacchetta, V. Benesova, . . .)

A quantitatively reliable description requires among other things

- **consistent control of power corrections**
- QED radiative effects, Vector meson background
- ***Matching of small & large transverse momentum regimes within QCD Factorization***
- *Consider here role* of longitudinal structure function on extraction of TMDs in SIDIS
  - $\longrightarrow$ entails subleading power TMD factorization
  - $\longrightarrow$ entails matching small & large-$P_{h\perp}$ regimes within QCD factorization

SIDIS basics

$$\ell\, P \longrightarrow \gamma^*_{T/L} \longrightarrow \ell'\, h\, X$$

$$\frac{d\sigma}{dx\,dy\,dz\,dq_T^2\,d\varphi} = \frac{\alpha^2 y}{Q^2(1-\varepsilon)}\left[F_{UU,T} + \varepsilon F_{UU,L} - \sqrt{2\varepsilon(1+\varepsilon)}F_{UU}^{\cos\varphi}\cos\varphi + \varepsilon F_{UU}^{\cos 2\varphi}\cos 2\varphi\right]$$

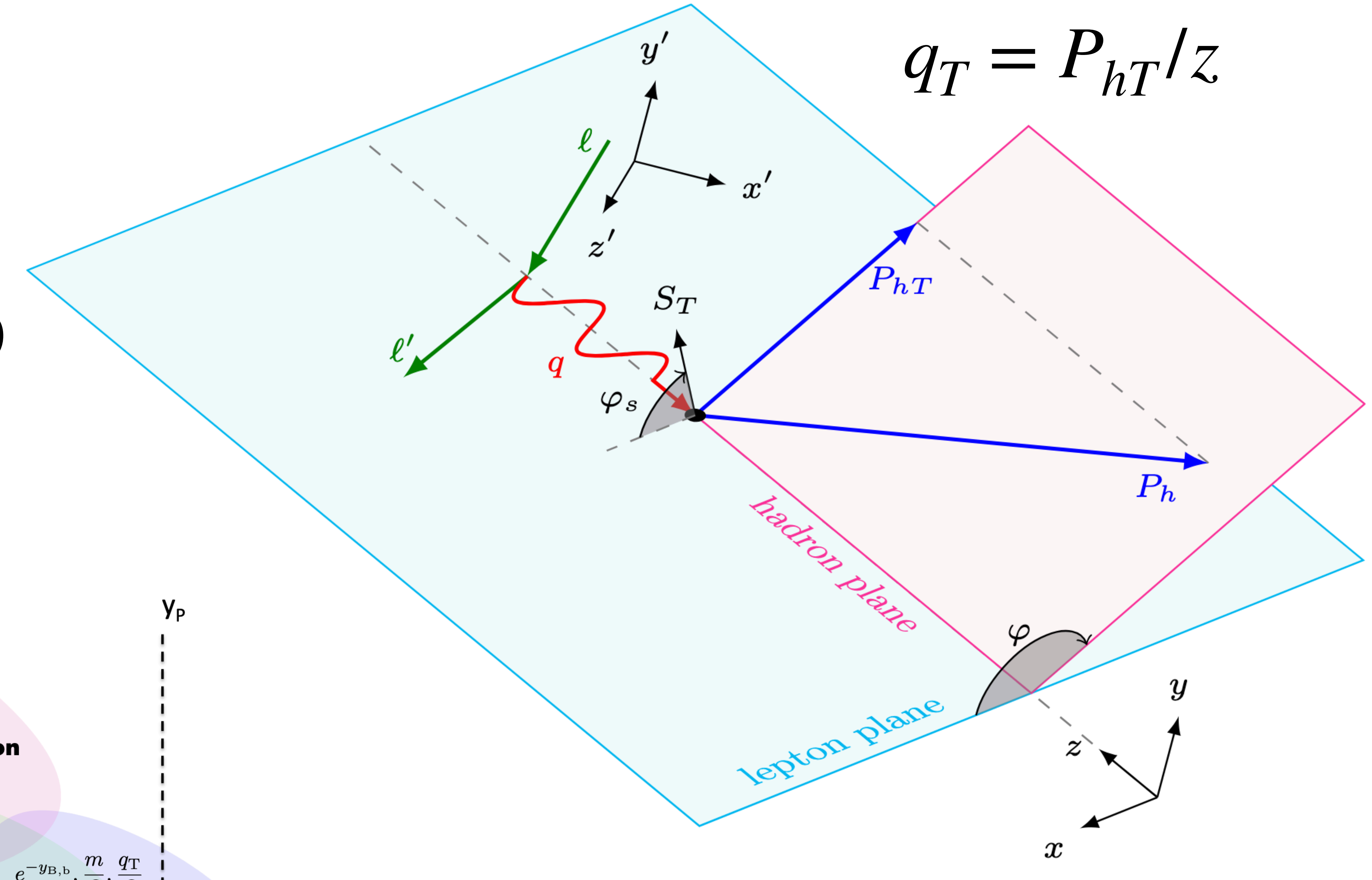
Observation of final-state hadron*

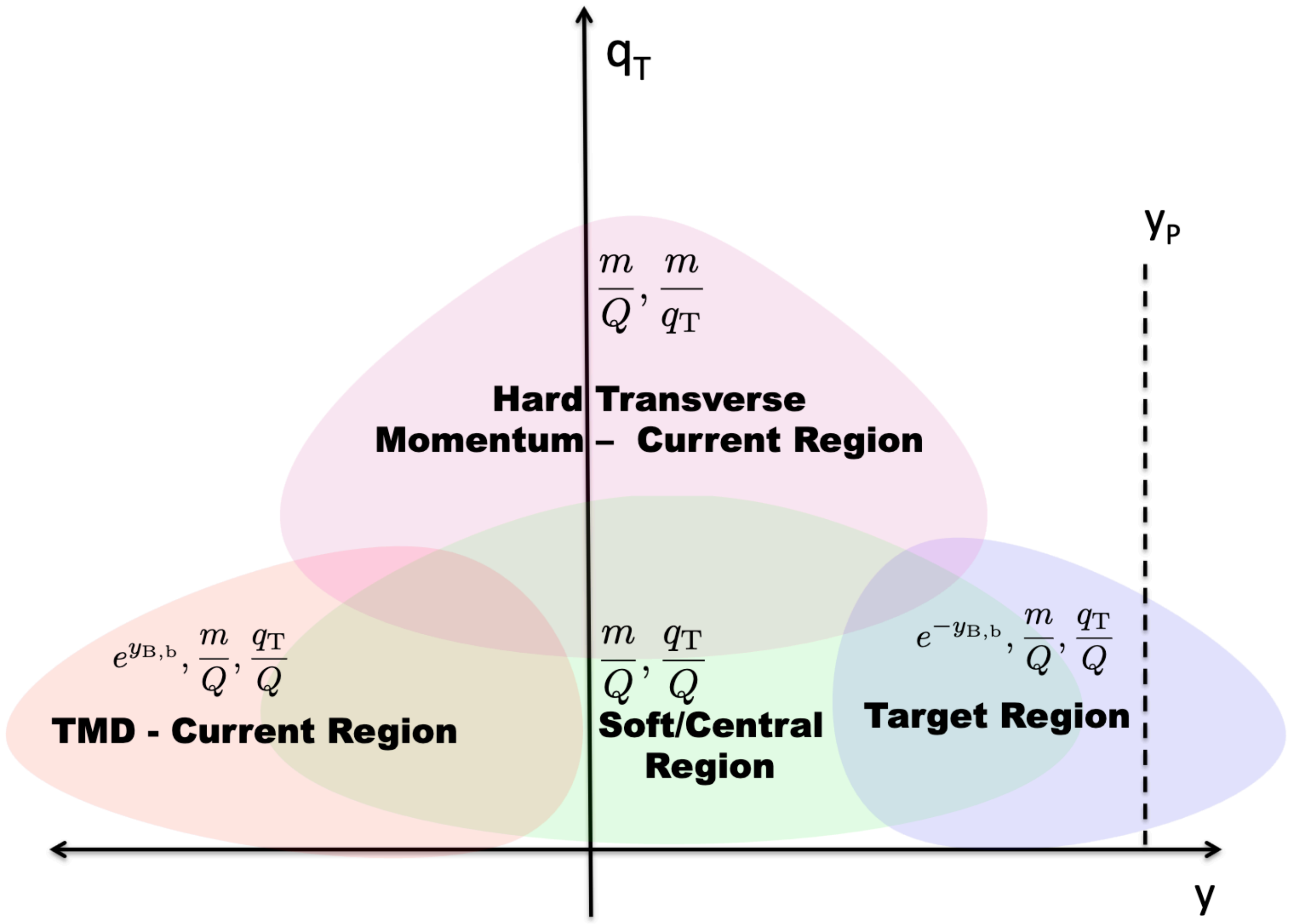
→ Unique correlations encoded in azimuthal modulations

→ Bridge between hadron structure, hadronization, pQCD

*produced via current fragmentation

$$q_T = P_{hT}/z$$





Boglione, Collins, LG, J.-O. Hernandez Rogers, Sato, Phys.Lett.B 766 (2017)

Preamble Motivation

- In particular underexplored ingredient is contribution/role of longitudinally polarized virtual photons on unpolarized SIDIS cross section $F_{UU,L}$ impact on multiplicities

$$M^h(Q^2, x, z, P_{h\perp}) = \frac{F_{UU,T}}{F_T} \longrightarrow \frac{F_{UU,T} + \epsilon\, F_{UU,L}}{F_T + \epsilon\, F_L}$$

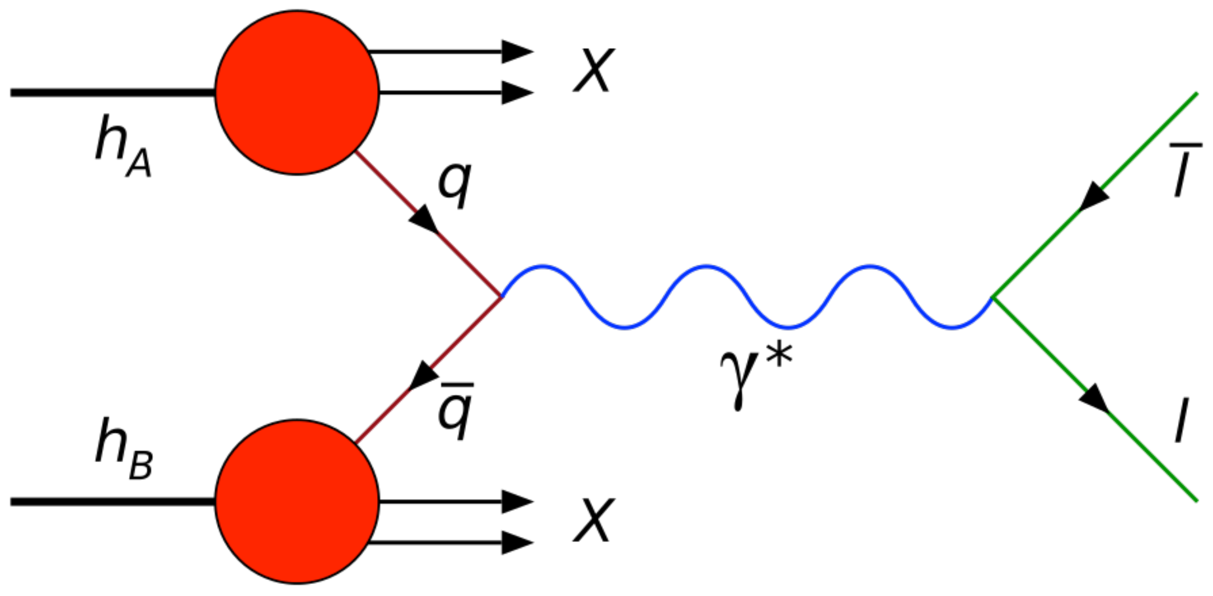
given challenge to describe unpolarized SIDIS cross section in TMD region $q_T \ll Q$

- Could directly impact phenomenological extractions of unpolarized TMDs

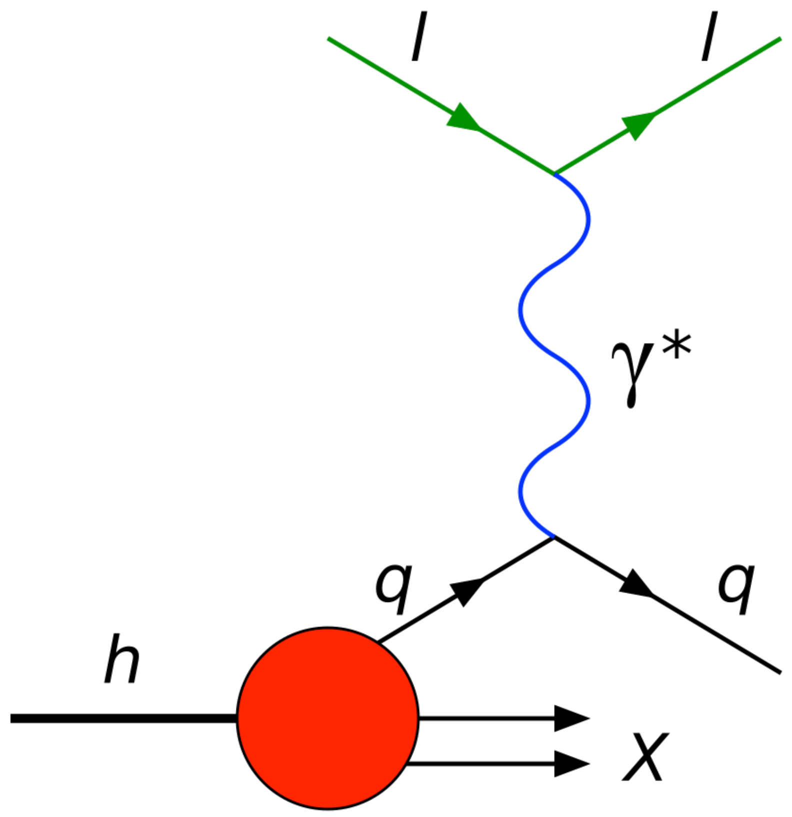
Impact on TMD extractions ?

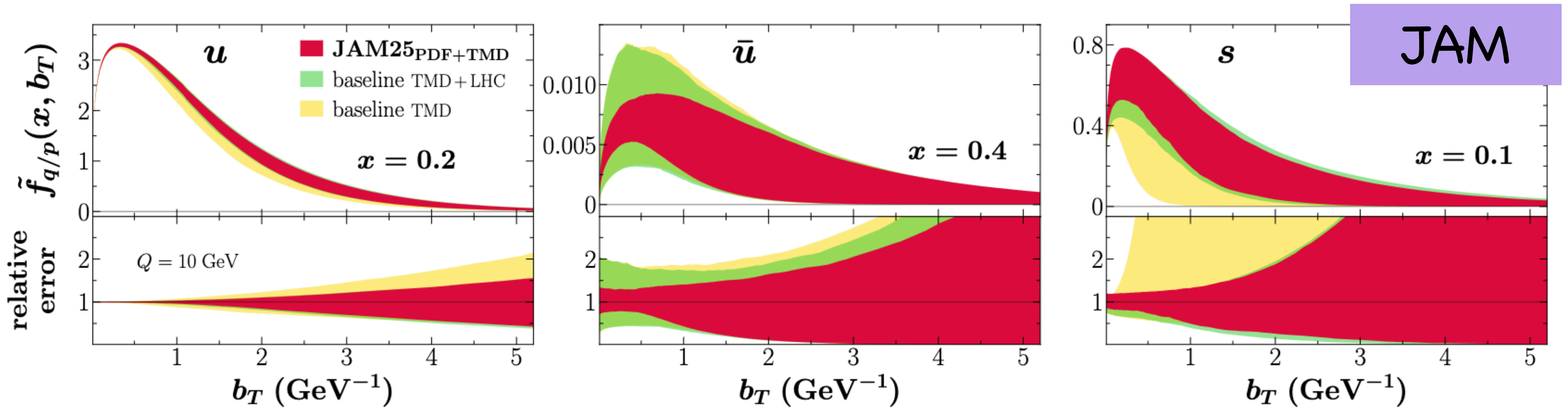
1) Universality* and factorization
2) Global Analysis

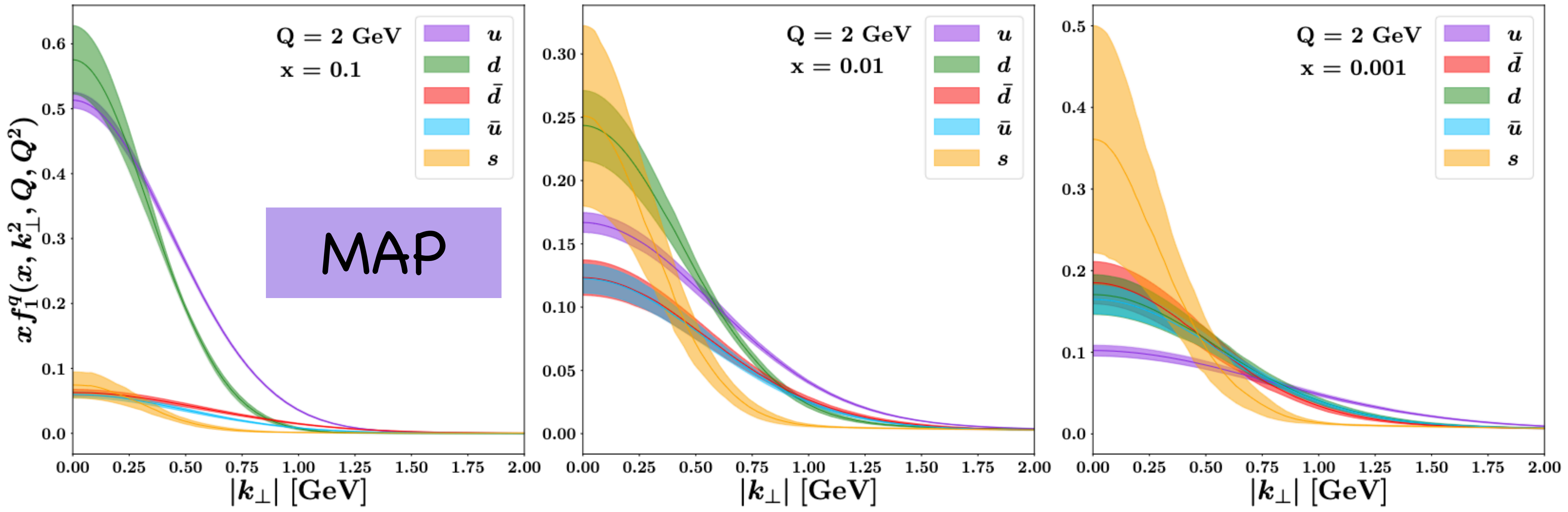
Drell-Yan

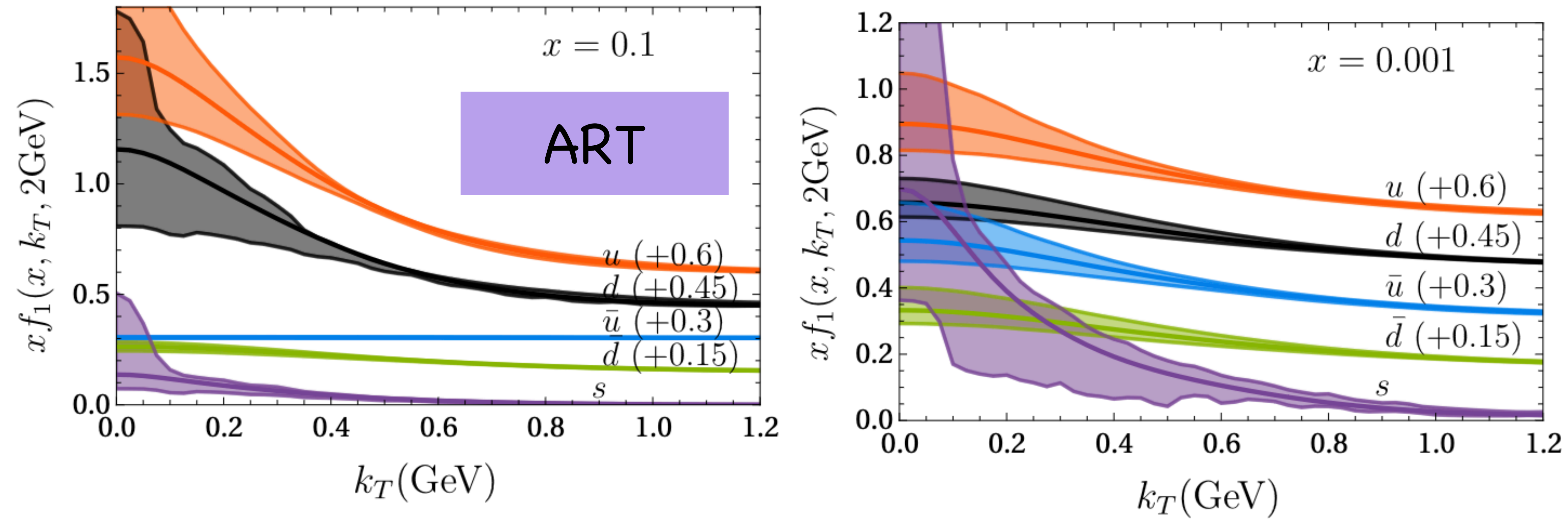


SIDIS









Preamble Motivation

- Also related & underexplored contribution $F_{UU,L}$ @ $q_T \gg Q$

$$M^h(Q^2, x, z, P_{h\perp}) = \frac{F_{UU,T}}{F_T} \longrightarrow \frac{F_{UU,T} + \epsilon\, F_{UU,L}}{F_T + \epsilon\, F_L}$$

- extend global SIDIS analyses to incorporate longitudinal photon effects in a consistent factorization framework *spanning available q_T range*

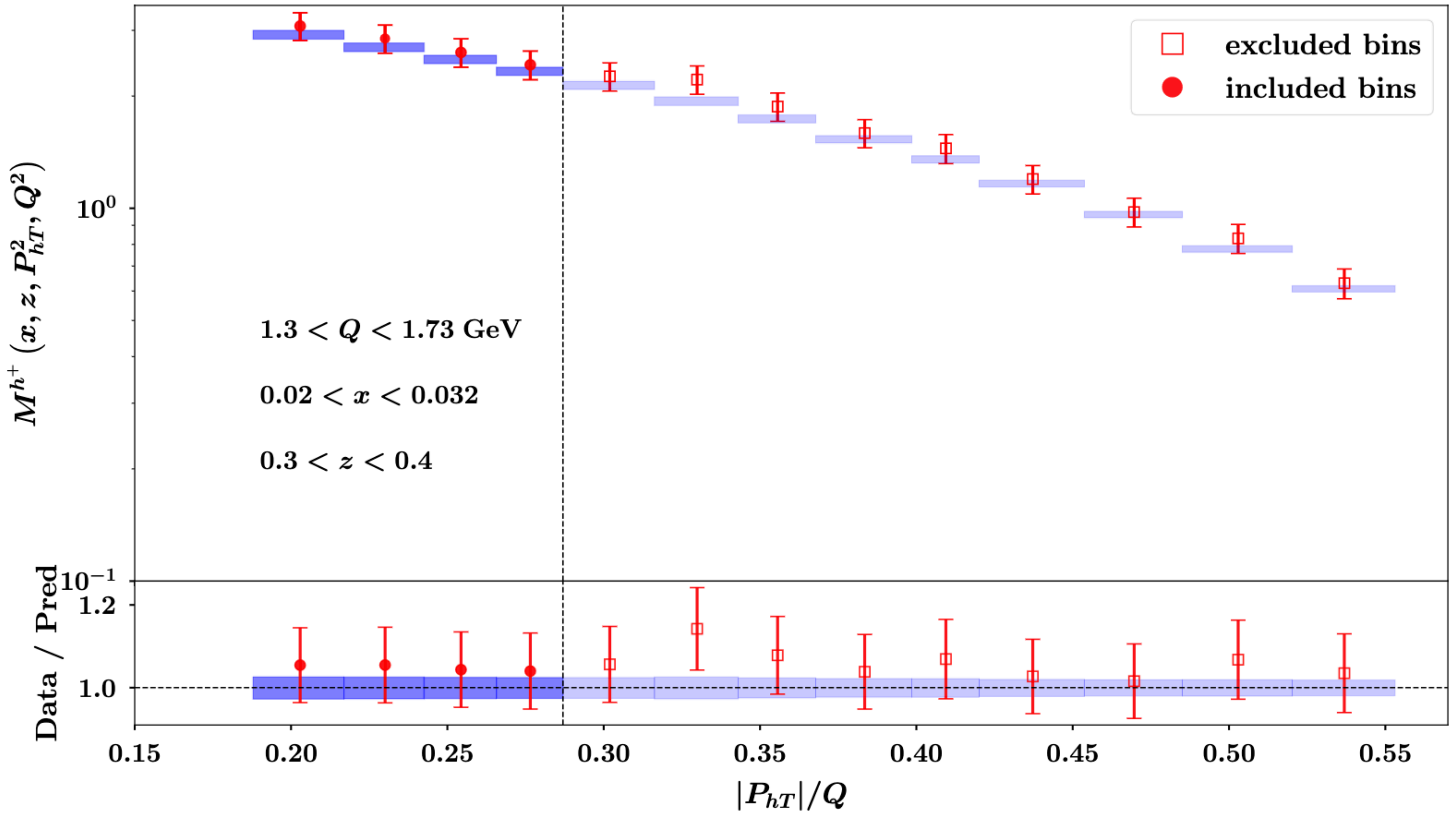
- In this study consider the $F_{UU,L}$ in a consistent factorization framework spanning the entire transverse-momentum range
- Lever arm to probe "TMD contribution" of γ_L^* contributions

Preamble Motivation

Multiplicity: SIDIS cross section normalized to DIS

$$M^h(Q^2, x, z, P_{h\perp}) \equiv \frac{dxdQ^2}{d^2\sigma^{\text{DIS}}(Q^2, x)} \frac{d^4\sigma^h(Q^2, x, z, P_{h\perp})}{dxdQ^2dzdP_{h\perp}}$$

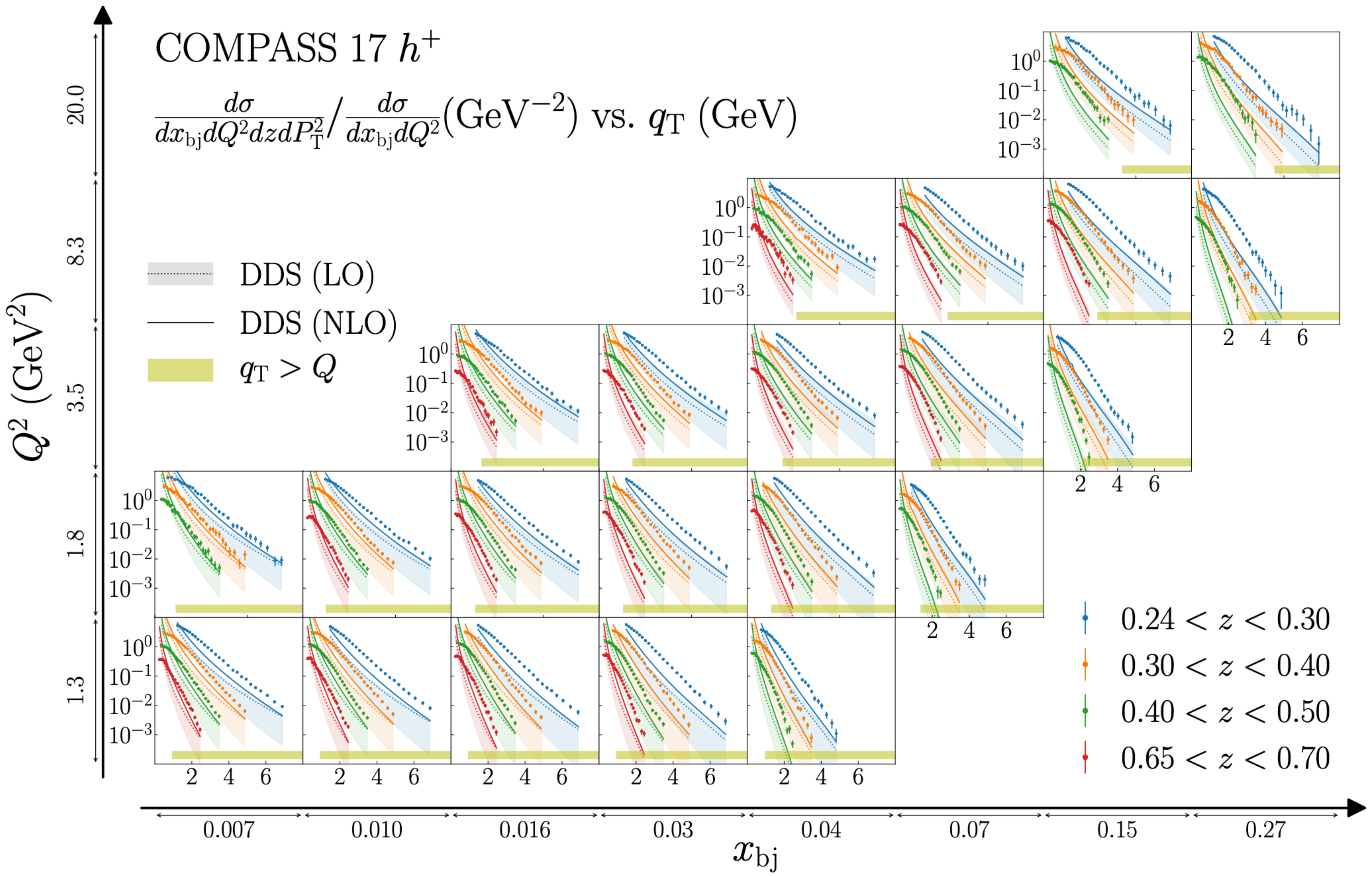
MAP Bacchetta et. al., arXiv:2206.07598v2 [hep-ph] (2022)



W term describes "large" P_{hT} data surprisingly well well beyond "TMD region"? MAPTMD22

Accuracy	H and C	K and γ_F	γ_K	PDF and α_s evolution	FF evolution
NLL	0	1	2	LO	LO
N^2LL	1	2	3	NLO	NLO
N^3LL^-	2	3	4	NNLO	NLO
N^3LL	2	3	4	NNLO	NNLO

Gonzalez-Hernandez et. al., arXiv:1808.04396v2 [hep-ph] (2018)



FO theory does not describe "large" P_{hT} data so well

Take away: Interplay of large & small P_{hT}

Preamble Motivation

Necessitates a unified treatment of

- low-P_T region governed by TMD factorization
- high-P_T region governed by collinear factorization necessitates a controlled implementation of small & large--transverse-momentum matching

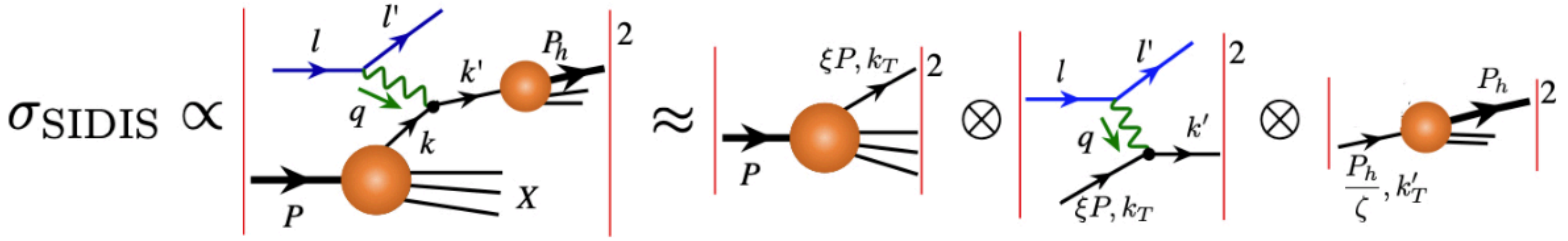
- " $W + Y$ " in the language of CSS (nb so far only at leading power, possibly at NLP e.g. Cahn etc ...)

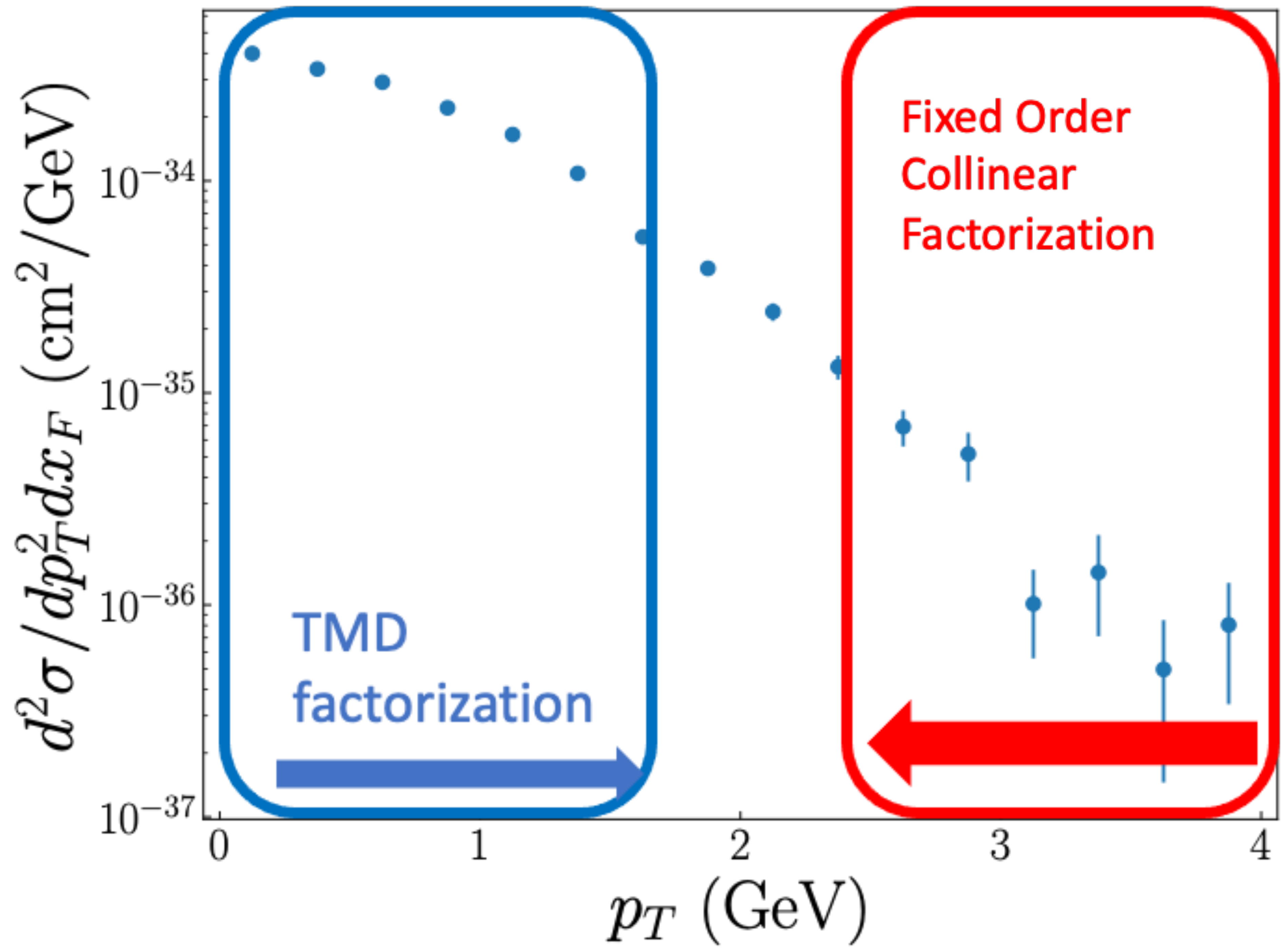
Preamble

Entails discussion factorization: TMDs & collinear factorization

- Collinear $M \ll q_T \sim Q$
- TMD $M \sim q_T \ll Q$

$$E'E_h \frac{d\sigma_{ep\to e'hX}}{d^3l' d^3P_h} \approx \hat{\sigma}_{eq\to e'q'} \otimes f_1 \,\widetilde{\otimes}\, D_{h/q'}.$$



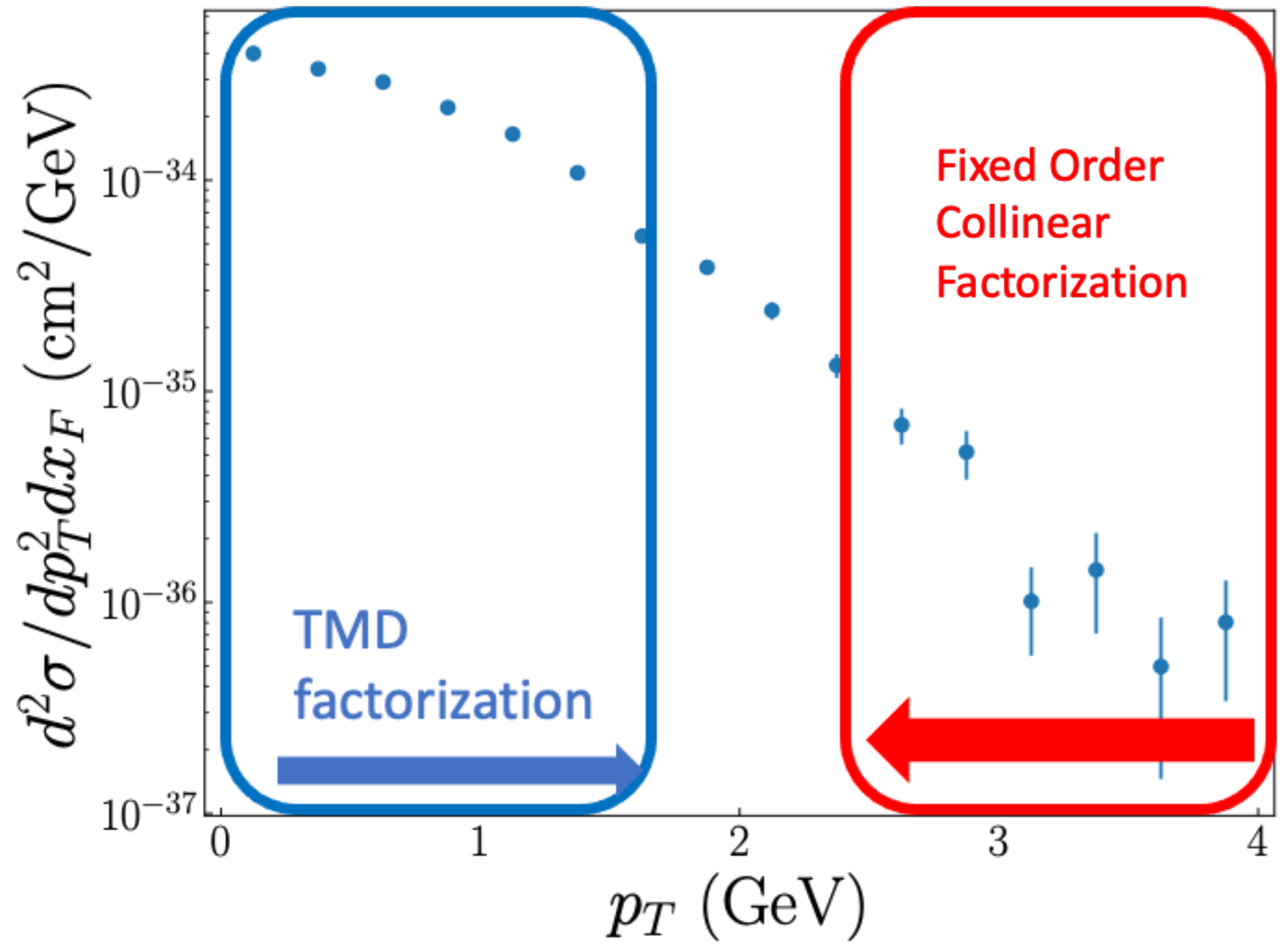


1. $\Longrightarrow$Matching "low" (TMD) to "high" (collinear) $P_T\,(\text{or}\; q_T)$ spectrum
2. Discussion: power counting & $\exists$ a resummation formalism generaized CSS?
3. Challenge of Factorization at LP NLP NNLP in the hard scale Q

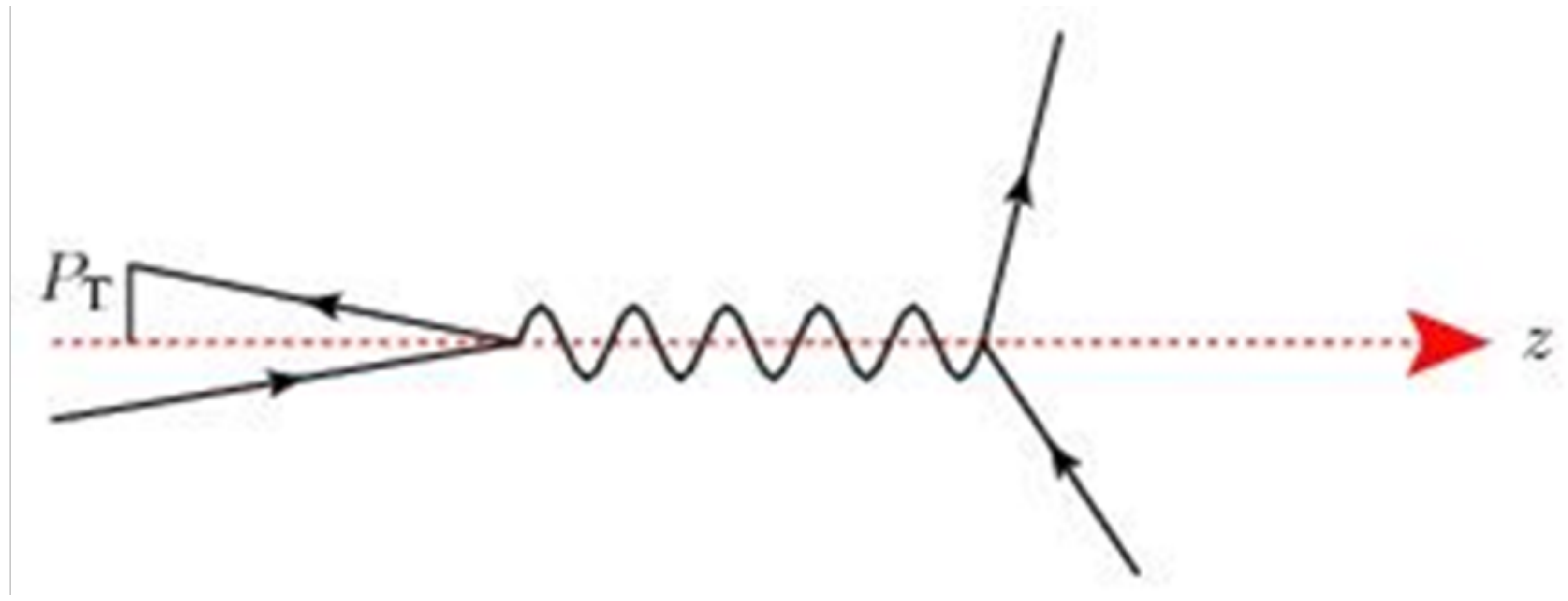
Break into steps

Framework Collins Soper Sterman (CSS) , Catani et. al match in “AY” region

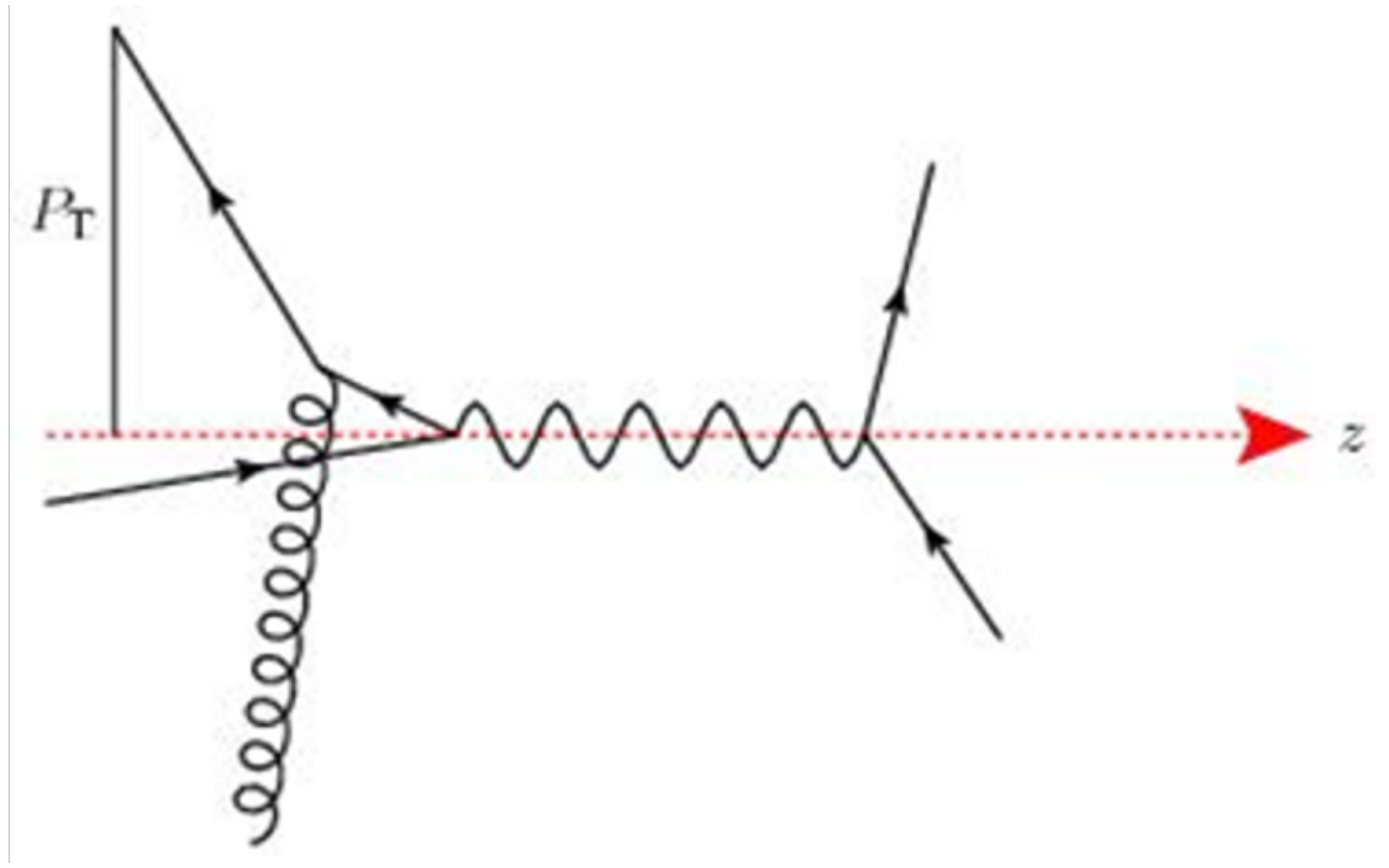
Scale separation: $F = W + \mathrm{FO} - \mathrm{ASY}$



W term probes *intrinsic* transverse motion



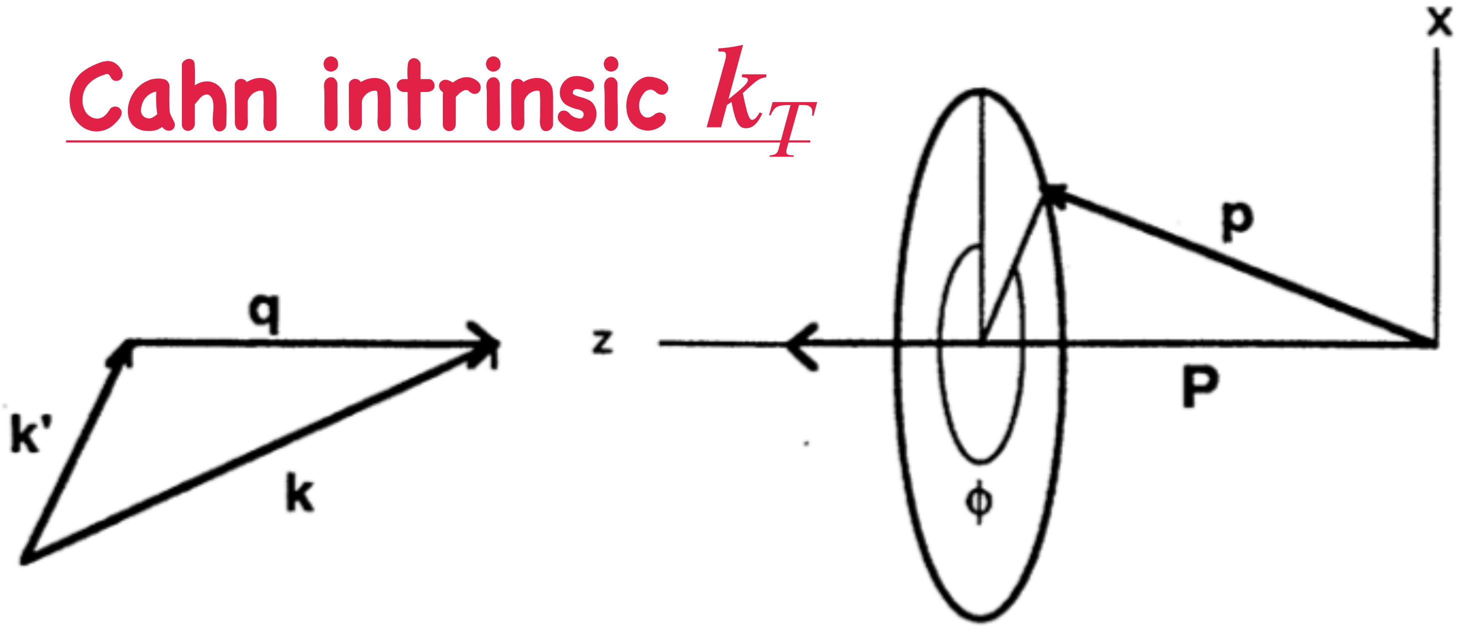
FO captures *hard* radiation



Two mechanisms ? *Matching* ...

Factorization & Matching collinear to TMD unpolarized/angle independent Collins Soper Sterman NPB 1985

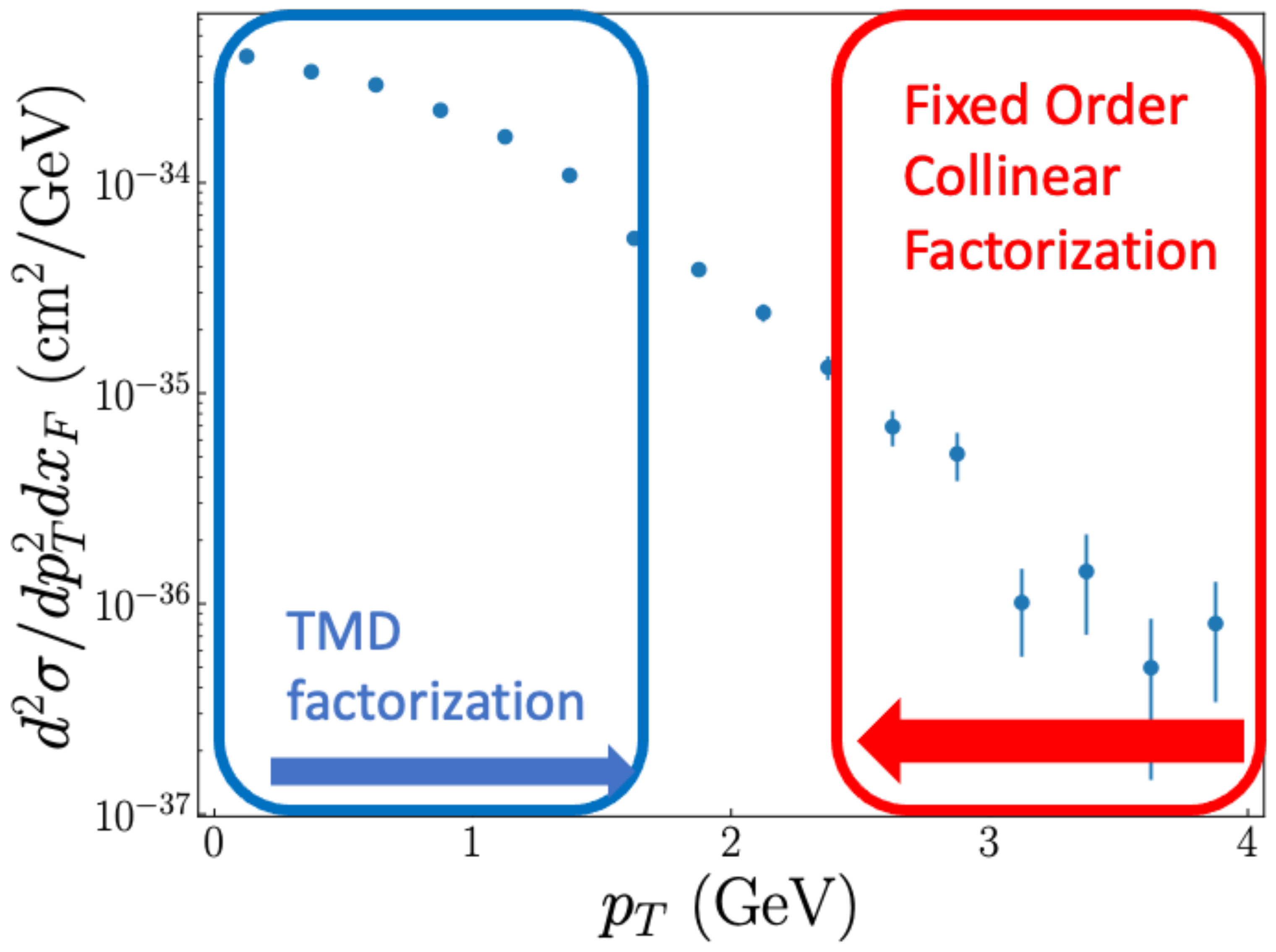
Cahn intrinsic k_T

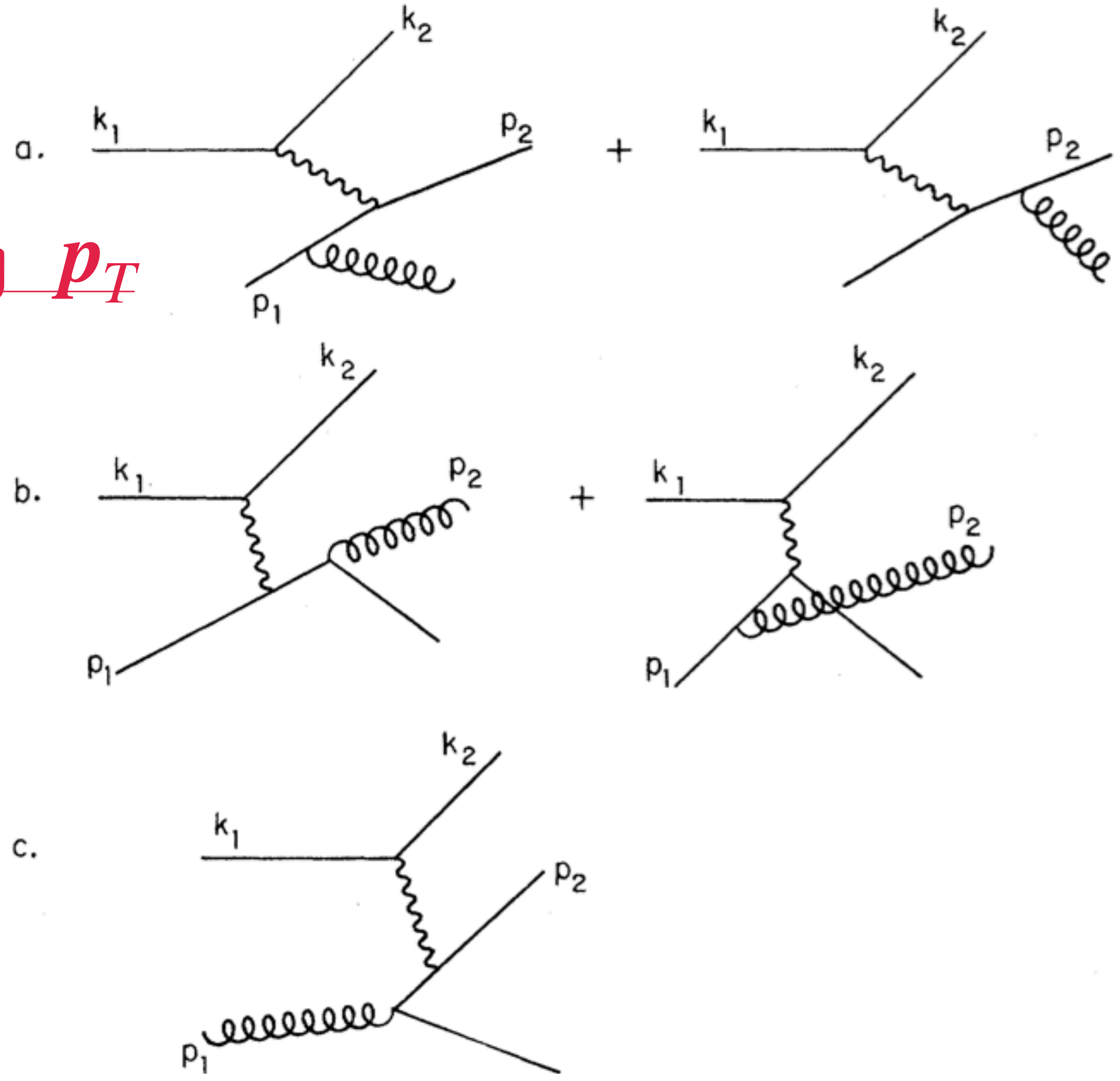


- **"TMD" region**

$(p_T \sim k_T) \sim q_T \ll Q$

Georgi & Politzer
hard gluon bremsstrahlung p_T





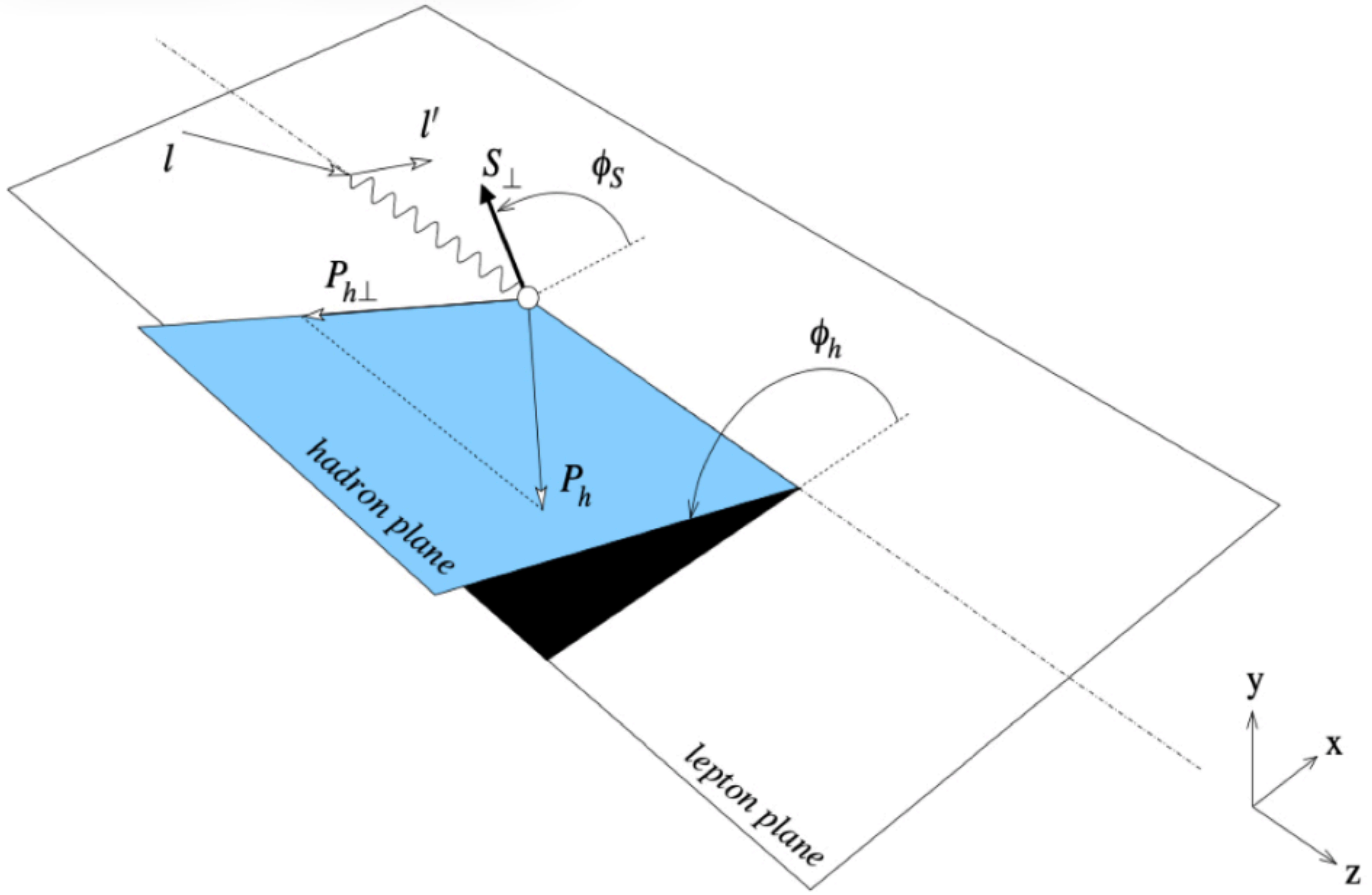
- **"Collinear " region**

$\Lambda_{qcd} \ll q_T \sim Q$

Comprehensive study of matching the hi & low Q_T in the AY (overlap) region in SIDIS Bacchetta, Boer, Diehl, Mulders JHEP (2008) attention given to azimuthal and polarization dependence

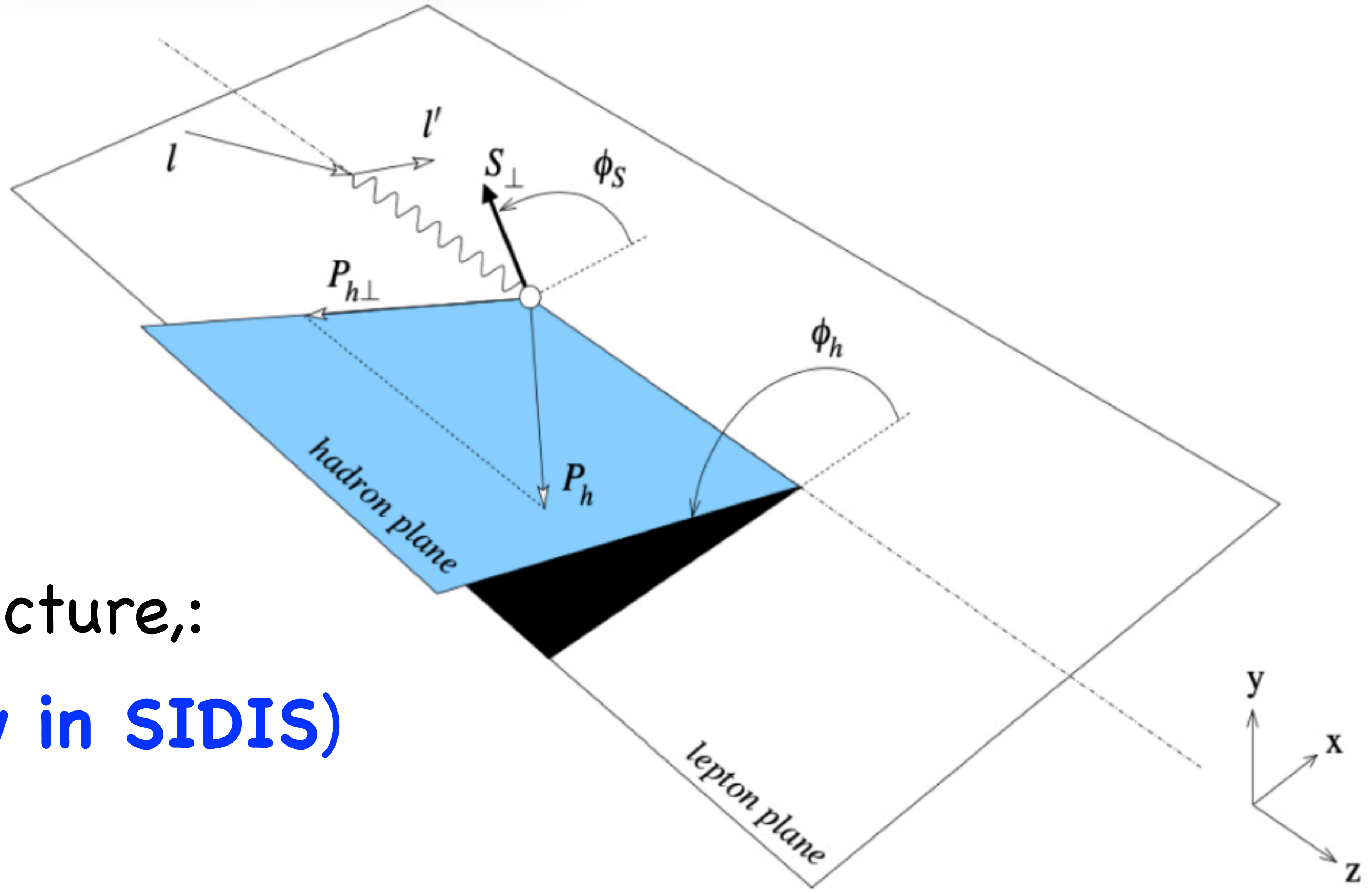
Focus on two related $F_{UU,L}$ NNLP observables

- $F_{UU,L}$ & $R_{SIDIS} = \frac{\sigma_L}{\sigma_T} \sim \frac{F_{UU,L}}{F_{UU,T}}$



Focus on two related $F_{UU,L}$ NNLP observables

- $F_{UU,L}$ & $R_{SIDIS} \stackrel{?}{=} \dfrac{\sigma_L}{\sigma_T} \sim \dfrac{F_{UU,L}}{F_{UU,T}}$



Feynman "Photon-Hadron Phys." 1972, Ravndal, PLB 1973

- Age old topic: interesting opportunity to study hadron structure,:

Ratio of longitudinal and transverse $\gamma^*_{T/L}$ cross section **(now in SIDIS)**

classic power supressed $\sim (1/Q)^2$ in DIS ...

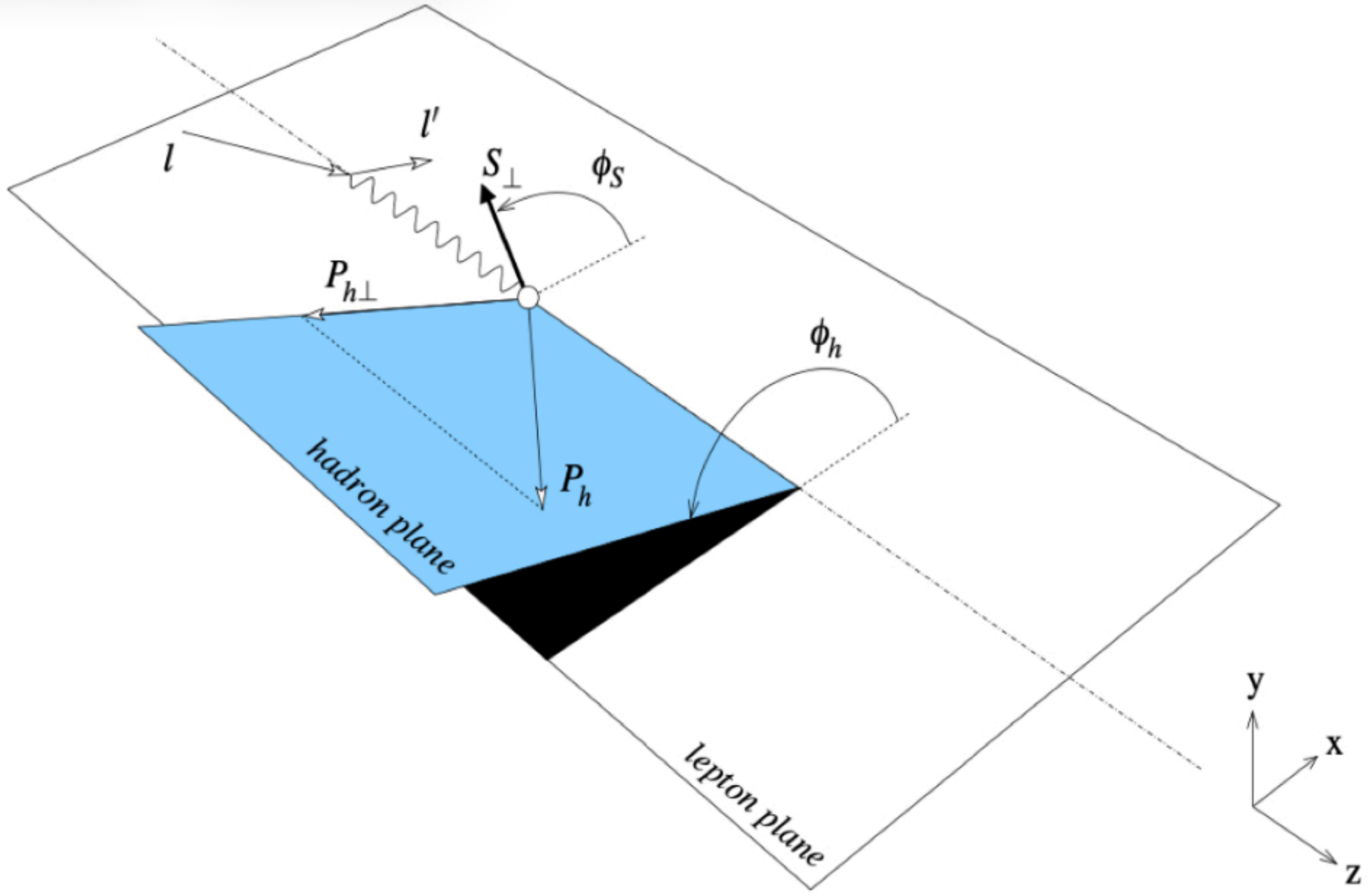
- Suggested extension beyond DIS: Feynman 1972 "Photon Hadron Interactions" & Ravndal PLB 1973, Cahn 1989 PRD 1989:

$$R = \frac{\sigma_L}{\sigma_T} = \frac{4\left(m^2 + \langle p_\perp^2 \rangle\right)}{Q^2} \quad \stackrel{?}{\Longrightarrow} \quad \frac{F_L}{F_T} \stackrel{?}{\sim} \frac{F_{UU,L}}{F_{UU,T}}$$

- $\langle p_\perp^2 \rangle$ intrinsic parton transverse momentum

Another related topic NLP unpol. observables

Another talk ...



- $F_{UU}^{\cos\phi_h} \to \langle \cos\phi_h \rangle$ n.b. $\gamma^*_{T/L}$ L/T inteference

Georgi & Cahn, PRL 1978, PLB 1978

- Critique of perturbative QCD calculation of azimuthal dependence in SIDIS
- Emphasize importance intrinsic k_T the early days/birth of "TMD physics"

R_{SIDIS} What do we know

$$F_{UU,L} \quad \& \quad R_{SIDIS} \equiv \frac{F_{UU,L}}{F_{UU,T}} \overset{?}{\sim} \frac{\sigma_L}{\sigma_T} = R_{DIS}$$

- Often assumed $R_{SIDIS} \approx R_{DIS}$ however the latter independent of z , P_T, φ
- Few measurements of $F_{UU,L}(x, z, q_T)$ – JLab Hall C in process
- In TMD pheno @ low transverse momentum often assumed negligible

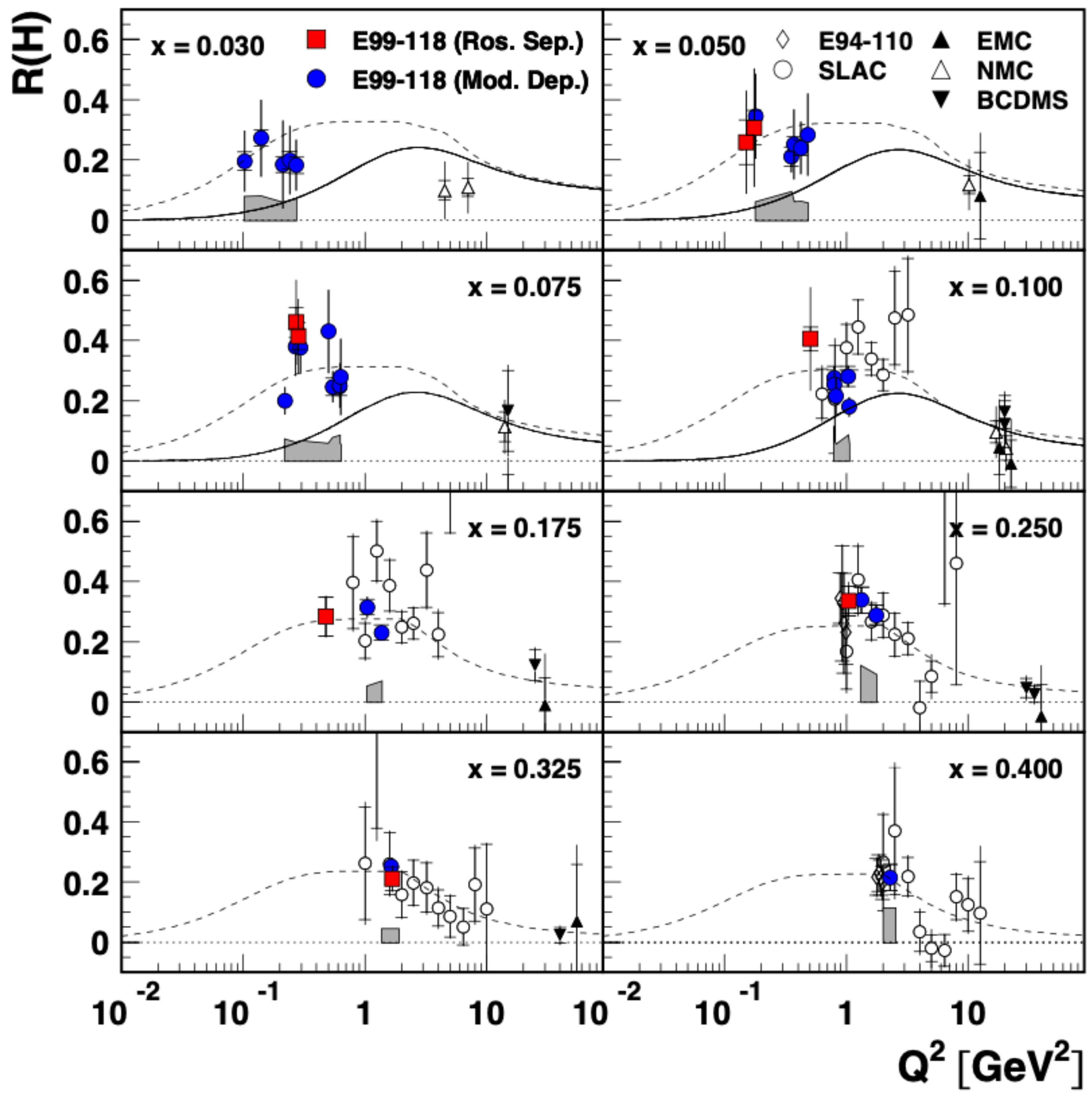
20^{th} century interpretation collinear DIS physics

Recall inclusive cross section: σ_T and σ_L
or structure functions $F_L\,(F_2\,\&\,F_1)\;\&\;F_T\,(F_1)$,
via absorption of longitudinal vs. transverse photons

$$R = \frac{\sigma_L}{\sigma_T} = \frac{F_L}{F_T} = \frac{1}{2xF_1}\left\{F_2\left(1+\frac{4x^2M^2}{Q^2}\right) - 2xF_1\right\}$$

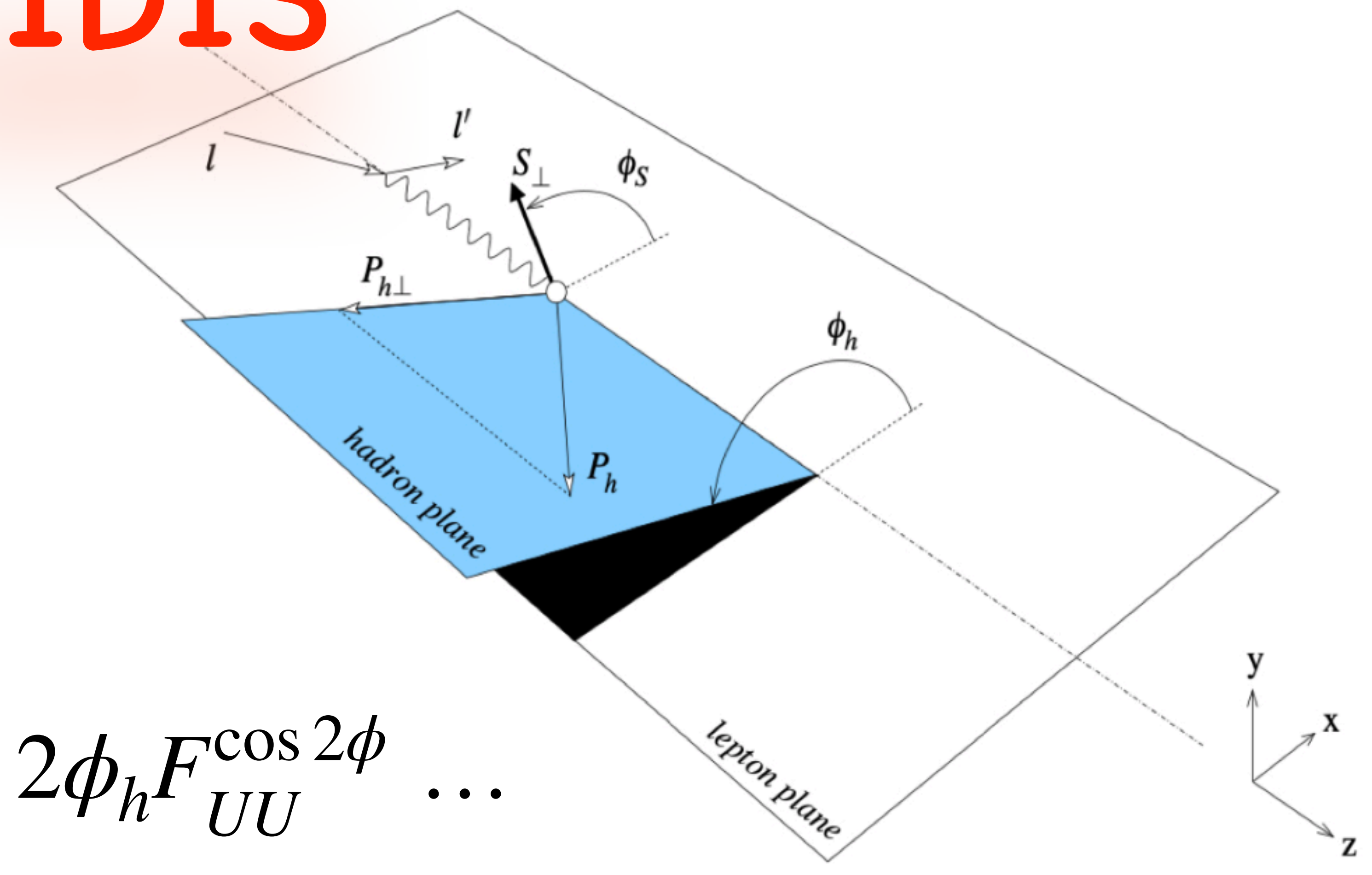
- Zero in parton model scaling limit $\lim_{Q^2\to\infty} R \approx \frac{4x^2M^2}{Q^2} \to 0$,
- non-trivial pQCD process $\{F_2 - 2xF_1\} \sim \frac{\alpha_s(Q)}{2\pi} C_2(F)\,x$

JLab exp. (E99-118) Prl 2007



Comparison of values of $R(x,Q^2)$ for hydrogen from JLab exp. (E99-118) to results of other exps.

21st century interpretation: from DIS to SIDIS



$$R = \frac{\sigma_L}{\sigma_T} \longrightarrow \frac{F_{UU,L}}{F_{UU,T}}$$

$$\frac{d\sigma}{dxdydzd\phi_h dP_{h\perp}} = \frac{\alpha^2}{xQ^2}\frac{y}{2(1-\varepsilon)}\left\{F_{UU,T} + \varepsilon F_{UU,L} + \sqrt{2\varepsilon(1+\varepsilon)}\cos\phi_h F_{UU}^{\cos\phi} + \varepsilon\cos 2\phi_h F_{UU}^{\cos 2\phi} \ \ldots\right.$$

- Does $\exists$ TMD observable for $F_{UU,L}$?
- Tree level TMD factorization Mulders et al. NPB 1996 Bacchetta et al. JHEP 2007

$$F_{UU,T} = C\,[f_1\, D_1]\ , \qquad F_{UU,L} = C\,[\,.\,?\,.\,]$$

$$\mathcal{C}\big[wfD\big] = \sum_a x\, e_a^2 \int d^2\boldsymbol{p}_T\, d^2\boldsymbol{k}_T\, \delta^{(2)}\big(\boldsymbol{p}_T - \boldsymbol{k}_T + \boldsymbol{q}_T\big)\, w(\boldsymbol{p}_T, \boldsymbol{k}_T)\, f^a(x, p_T^2)\, D^a(z, k_T^2)$$

$\Updownarrow$ decomposition

$$W^{\mu\nu} \propto \sum_a e_z^2 \int d^2p_T\, d^2k_T \delta^2\left(\mathbf{p}_T - \mathbf{k}_T + \mathbf{q}_T\right) \mathrm{Tr}\left\{\Phi^a(x, \mathbf{p}_T)\gamma^\mu \Delta^a(z, \mathbf{k}_T)\gamma^\nu\right\}$$

Reminder: recall Quark correlator LP, NLP

n.b. "tree level" Mulders Tangerman 1996 Bacchetta 2007

$$\Phi(x, p_T) = \frac{1}{2}\left\{ f_1 \not{n} + f_{1T}^{\perp} \frac{S_{Ti}\epsilon^{ij}p_{Tj}}{M} \not{n} + \dots \right\} + \frac{M}{2P^+}\left\{ e + \dots + f^{\perp}\frac{\not{p}_T}{M} \dots \right\}$$

Leading Quark TMDPDFs

Nucleon Spin — Quark Spin

Nucleon Polarization	Quark Polarization: Un-Polarized (U)	Quark Polarization: Longitudinally Polarized (L)	Quark Polarization: Transversely Polarized (T)
U	f_1 = Unpolarized		$h_1^{\perp}$ = − Boer-Mulders
L		g_1 = − Helicity	$h_{1L}^{\perp}$ = − Worm-gear
T	$f_{1T}^{\perp}$ = − Sivers	$g_{1T}^{\perp}$ = − Worm-gear	h_1 = − Transversity $h_{1T}^{\perp}$ = − Pretzelosity

Subleading Quark TMDPDFs

Nucleon Polarization	Quark Chirality: Chiral Even	Quark Chirality: Chiral Odd
U	$f^{\perp}, g^{\perp}$	$e\,,\ h$
L	$f_L^{\perp}, g_L^{\perp}$	e_L, h_L
T	$f_T, f_T^{\perp}, g_T, g_T^{\perp}$	$e_T, e_T^{\perp}, h_T, h_T^{\perp}$

Reminder: recall Quark correlator LP, NLP, NNLP

n.b. “tree level” Mulders Tangerman 1996 Bacchetta 2007

$$\Phi(x,p_T) = \frac{1}{2}\left\{ f_1 \not{n} + f_{1T}^{\perp}\frac{S_{Ti}\varepsilon^{ij}p_{Tj}}{M}\not{n} + \ldots \right\} + \frac{M}{2P^+}\left\{ e + \ldots \quad + f^{\perp}\frac{\not{p}_T}{M}\ldots \right\} + \frac{M^2}{(2P^+)^2}\left\{ \ldots \quad ?? \quad \ldots \right\}$$

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???

Context TMD Correlator at tree level NNLP "twist 4"

Goeke, Metz, Schlegel PLB 2005

NNLP:

$$\Phi^{[\gamma^-]} = \frac{M^2}{(P^+)^2}\left[f_3(x,\vec{k}_T^2) - \frac{\varepsilon_T^{ij} k_{Ti} S_{Tj}}{M} f_{3T}^{\perp}(x,\vec{k}_T^2)\right]$$

- Correlator at tree level @ "twist" 4
- previously of academic interest
- factorization relatively unexplored

Recall *LP* & *NLP* :

$$\Phi^{[\gamma^+]} = f_1(x,\vec{k}_T^2) - \frac{\varepsilon_T^{ij} k_{Ti} S_{Tj}}{M} f_{1T}^{\perp}(x,\vec{k}_T^2),$$

$$\Phi^{[\gamma^i]} = \frac{M}{P^+}\left[\frac{k_T^i}{M}\left(f^{\perp}(x,\vec{k}_T^2) - \frac{\varepsilon_T^{jk} k_{Tj} S_{Tk}}{M} f_T^{\perp\prime}(x,\vec{k}_T^2)\right) + \quad \dots\right.$$

Leading Quark TMDPDFs — Nucleon Spin — Quark Spin

Nucleon Polarization	Quark Polarization: Un-Polarized (U)	Longitudinally Polarized (L)	Transversely Polarized (T)
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Context TMD Correlator at tree level NNLP "twist 4"

NNLP:

NNLP: some discussion in Bacchetta et al.
Matches and mismatches JHEP 2008

$$F_{UU,L} = \frac{4M^2}{Q^2} C \left[\frac{p_T^2}{M^2} f_1 D_1 \right]$$

- Correlator at tree level @ "twist" 4
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R_{SIDIS} NLP TMDs ?

R_{sidis} esitmated MAP coll. Sizable up to 20%

$$\frac{d\sigma}{dx\, dy\, dz\, d\phi_S\, d\phi_h\, dP_{h\perp}} = \frac{\alpha^2}{xQ^2}\frac{y}{2(1-\varepsilon)}\left\{F_{UU,T} + \varepsilon F_{UU,L}\right.$$

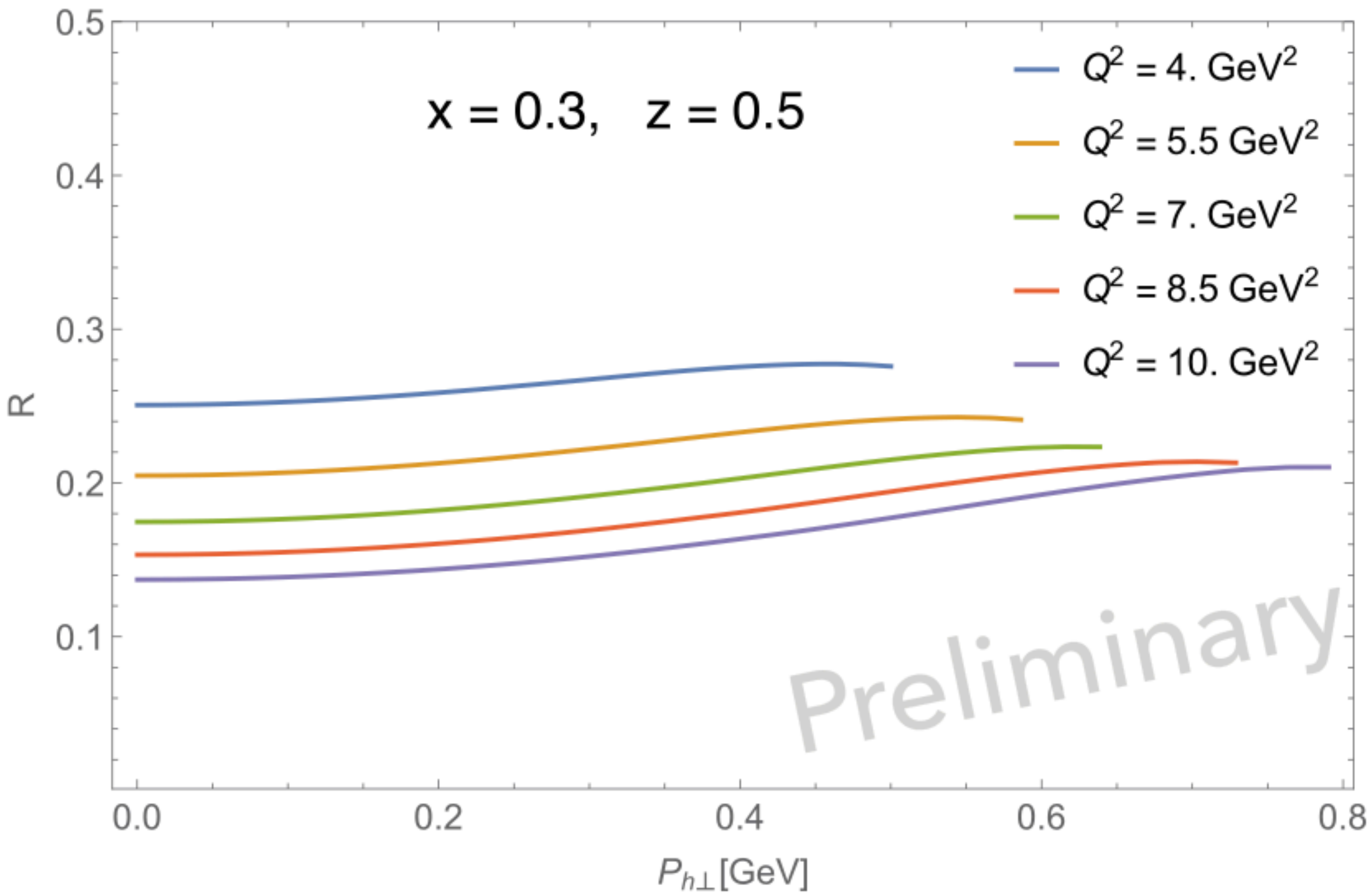
$$F_{UU,L} = \frac{4M^2}{Q^2} C\left[\frac{p_T^2}{M^2} f_1 D_1\right]$$

ε ratio of longitudinal and transverse photon flux

$$\varepsilon = \frac{1 - y - \frac{1}{4}\gamma^2 y^2}{1 - y + \frac{1}{2}y^2 + \frac{1}{4}\gamma^2 y^2}, \quad \gamma^2 \equiv \frac{4M^2x^2}{Q^2}$$

- **Findings demonstrate $F_{UU,L}$ can't be ignored**
- substantial & essential for an accurate interpretation of $F_{UU,T}$
- which is associated with LP TMDs
- SIDIS normalization in TMD factorization ?

Bacchetta & Cerruti MAP Eur. Phys. J. A (2024) 60:173



Estimate of $R_{SIDIS} = F_{UU,L}/F_{UU,T}$ versus hadron transverse momentum P_{hT} @ fixed x and z and for values Q2, compatible with JLab22 kinematics, using MAP22 TMD analysis

To learn more we consider also lg q_T $F_{UU,L}(x, z, q_T, Q)$

Take "granular" look: consider both @ small & large $q_T \approx \frac{P_{hT}}{z}$ structure functions

- $F_{UU,T}(x, z, q_T, Q)$ &
- $F_{UU,L}(x, z, q_T, Q)$

Key point for SIDIS & global analysis (GA) and whether the low & high q_T physics necessary for GA ?

- **Explore thru factorization low & high q_T "matching"**

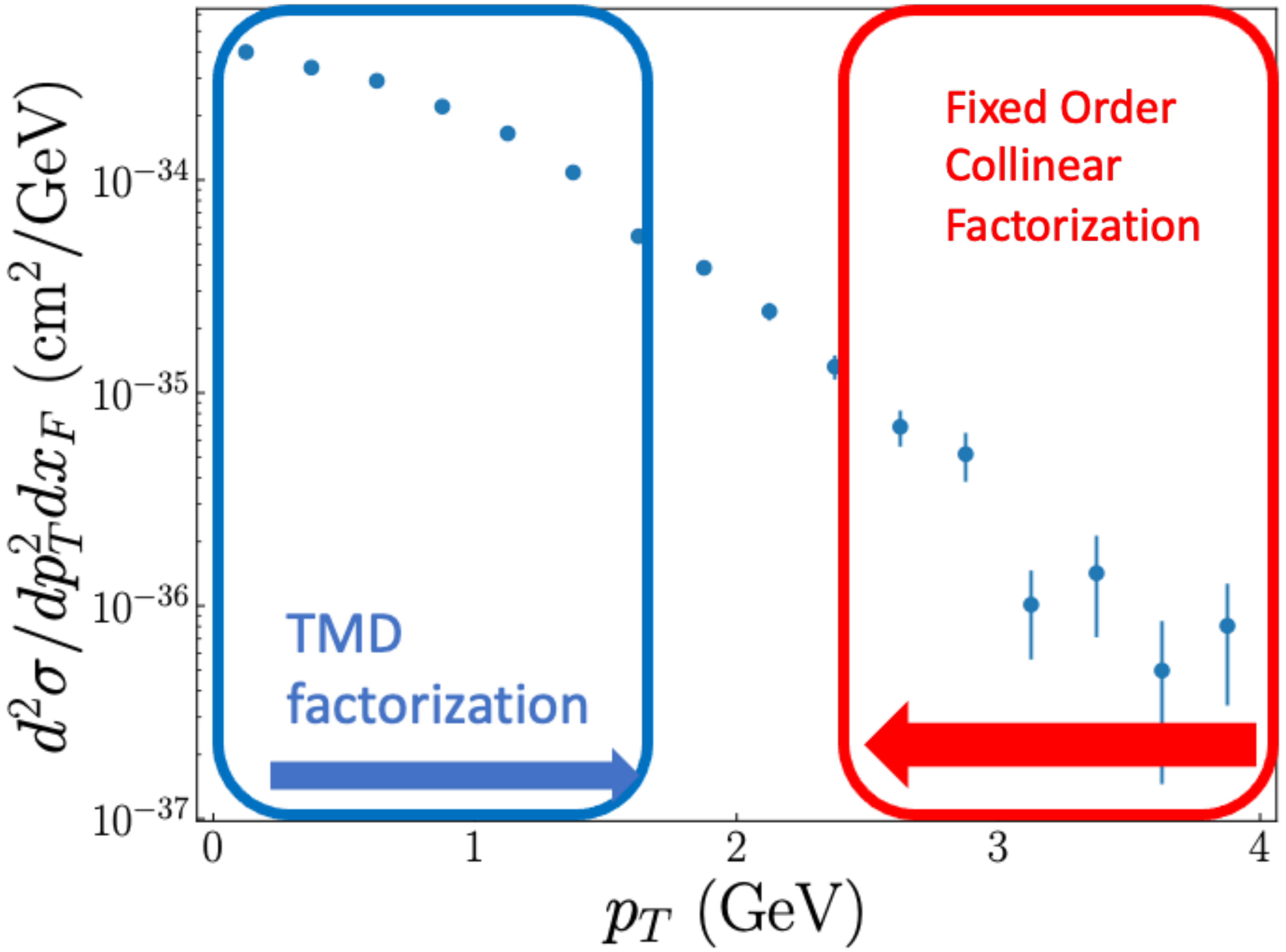
i) consider power counting: <u>necessary conditions for match</u>
<u>ii</u>) direct calculation: <u>sufficient conditions for match</u>

e.g. Allows extension of simulataneous fits of colliner pdfs & TMDs JAM <u>arXiv:2510.13771</u>
Barry, Prokudin, Anderson, Cocuzza, LG, Melnitchouk, Moffat, Pitonyak, Qiu, Sato, Vladimirov, Whitehill

$$\tilde{f}(x, b_T; \mu, \zeta) = \left[C \otimes f\right](x, b_T; \mu_0, \zeta_0) \times e^{S(b_T;\mu,\mu_0,\zeta,\zeta_0)} f_{NP}(x, b_T)$$

Matching of TMD & FO-large $q_T \approx P_{hT}/z$ Rigorous proof @ Leading power

- Factorization & Matching unpolarized Collins Soper Sterman NPB (1985), Bozzi Catani Floraian Grazzini NPB (2006), Bacchetta, Boer, Diehl, Mulders JHEP (2008) Collins, Gamberg, Prokudin, Rogers, Sato, Wang PRD (2016) ....
- N.B. Transverse polarization Ji, Qiu, Vogelsang Yuan PRL (2006); PRD (2006)



- **Cross section in terms of different "regions"**
- W valid for $q_T \sim k_T \ll Q$ TMD factorization
- FO valid for $k_T \ll p_T \sim Q$ Collinear factorization
- AY subtracts d.c. & in principle,

  $AY \to W,\ p_T \to \infty$ and $AY \to FO, p_T \to 0$
- $Y \equiv \to FO - AY$

$$\frac{d\sigma(m \lesssim p_T \lesssim Q, Q)}{dy\, dq^2 d^2 p_T} = W(p_T, Q) - AY(p_T, Q) + FO(p_T, Q) + \mathcal{O}\left(\frac{M}{Q}\right)^c$$

Explore thru factorization low & high q_T "matching"

i) consider power counting: necessary conditions for match

ii) direct calculation: sufficient conditions for match

Power counting in regions

Bacchetta et al. JHEP 2008

Power counting $F_{XY,Z}$ $\longrightarrow$ $F_{UU,L}$ & $F_{UU,T}$ $\dots$

- Low TMD factorization

$M \sim q_T \ll Q$

$$M^2 F_{UU,T} \sim \mathscr{F}\left[f_1 D_1\right]$$ Leading power

$$M^2 F_{UU,L} \sim \frac{M^2}{Q^2}\mathscr{F}\left[\,..?..\,\right]$$ Sub sub leading

- High collinear factorization

$M \ll q_T \sim Q$

$$Q^2 F_{UU,T} = \alpha_s \mathscr{F}\left[f_1 D_1\right]$$ Leading power

$$Q^2 F_{UU,L} = \alpha_s \mathscr{F}\left[f_1 D_1\right]$$ Leading power

Power counting in "intermediate region

Power counting $F_{UU,L}$

- Low TMD factorization $\longrightarrow M \ll q_T \ll Q \longleftarrow$ - High collinear factorization

$$M^2 F_{UU,L}^{M \ll q_T \ll Q} \sim \alpha_s \frac{q_T^2}{Q^2} \frac{M^2}{q_T^2} \mathscr{F}[f_1 D_1]$$

$$M^2 F_{UU,L}^{M \ll q_T \ll Q} \sim \alpha_s \frac{M^2}{q_T^2} \frac{q_T^2}{Q^2} \mathscr{F}[f_1 D_1]$$

There could be a Match

$$F_{UU,L} \sim \frac{1}{Q^2} \alpha_s \mathcal{F}[f_1 D_1]$$

Explore thru factorization low & high q_T "matching"

i) consider power counting: necessary conditions for match

ii) direct calculation: sufficient conditions for match

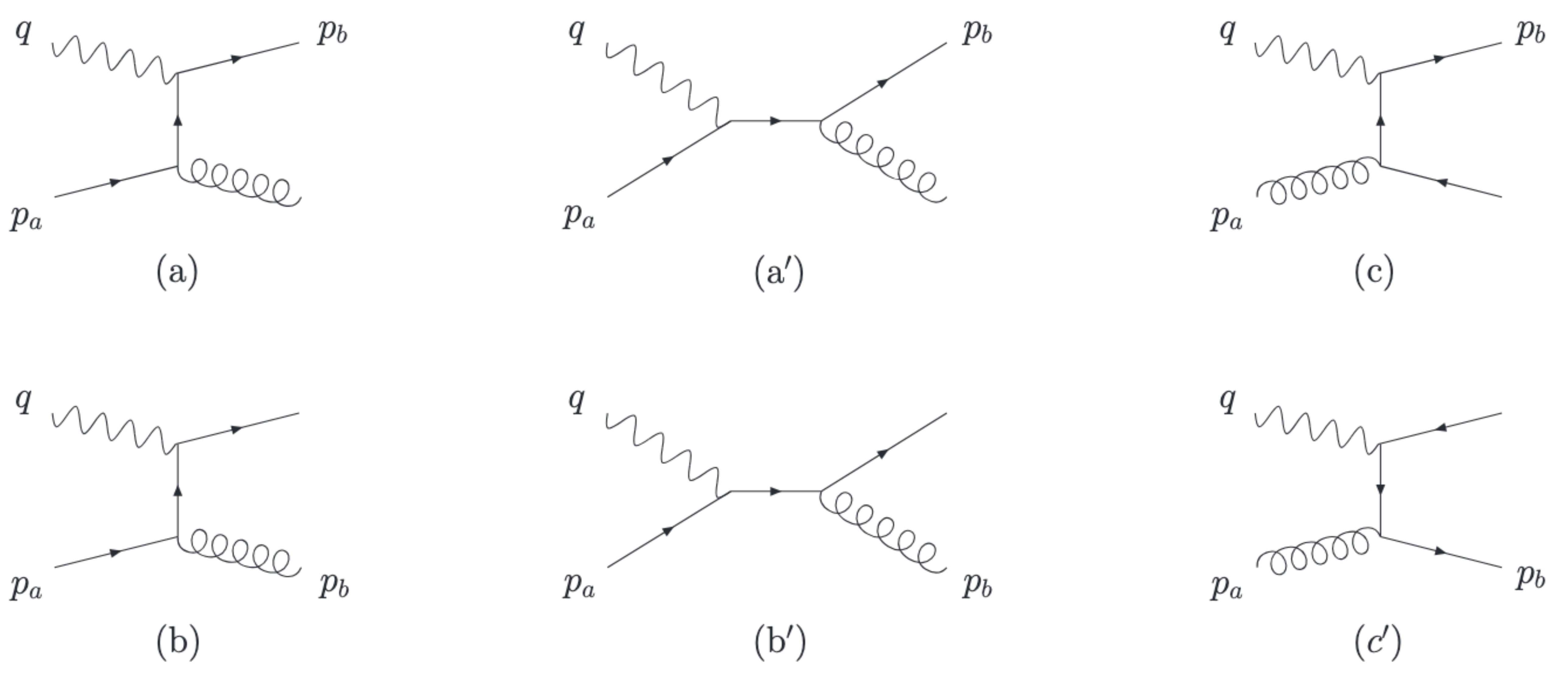
Consider $F_{UU,L}$ & R_{SIDIS} @ large P_T

- @ large P_T , $F_{UU,L} \sim F_{UU}^{\cos 2\phi_h}$ — see Bacchetta et al. JHEP 2008 "Matches & Mis-matches":

so in principle hard gluon raditation — "collinear P_T factorization applies CSS 1985

Catani et al. 1997-2015, Nadolsky, Vogelsang Koike NPB 2005 … many others

$$\frac{d\sigma}{dxdydzd\phi_h dP_{h\perp}} = \frac{\alpha^2}{xQ^2}\frac{y}{2(1-\varepsilon)}\left\{F_{UU,T} + \varepsilon F_{UU,L} + \dots\right.$$



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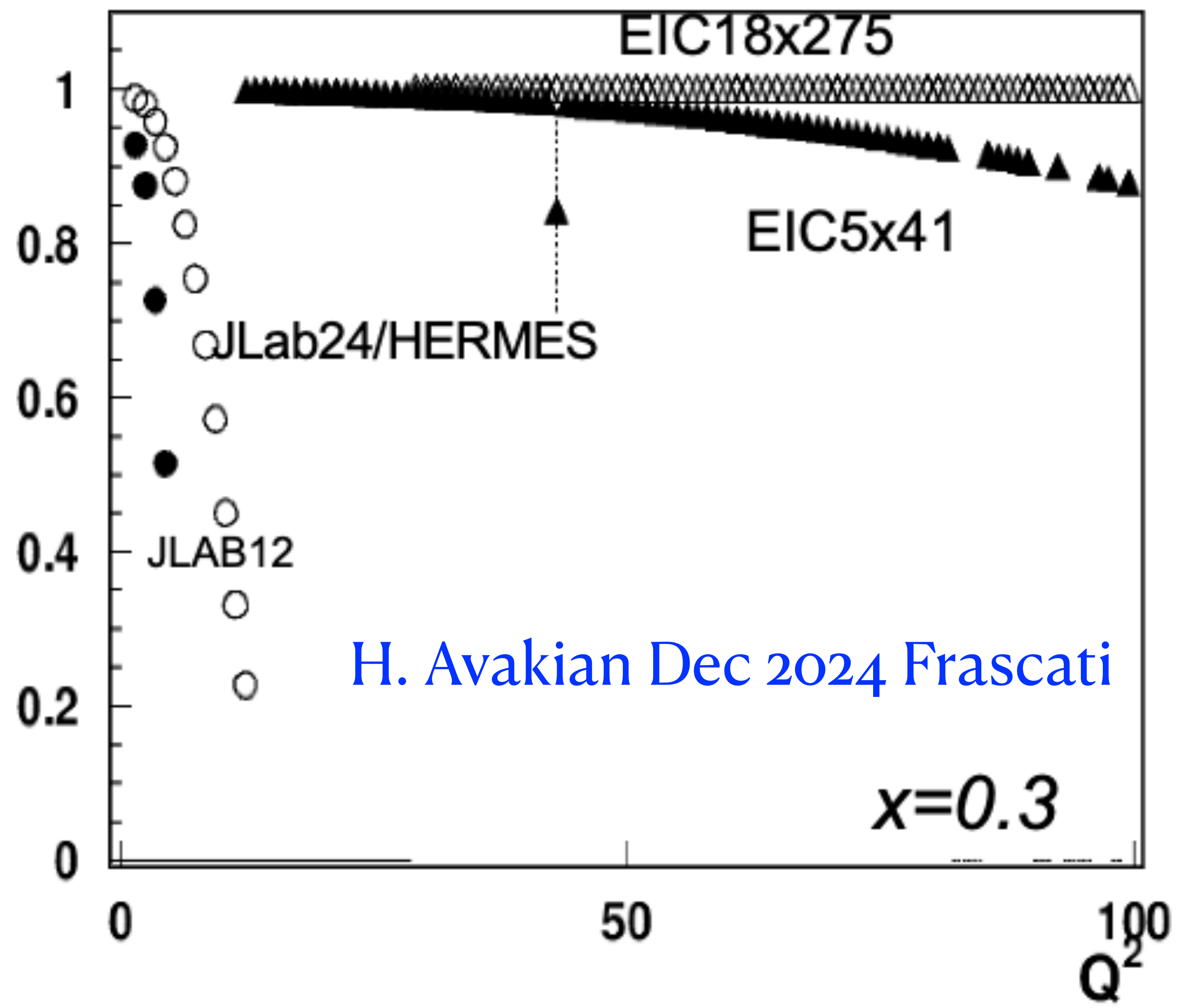
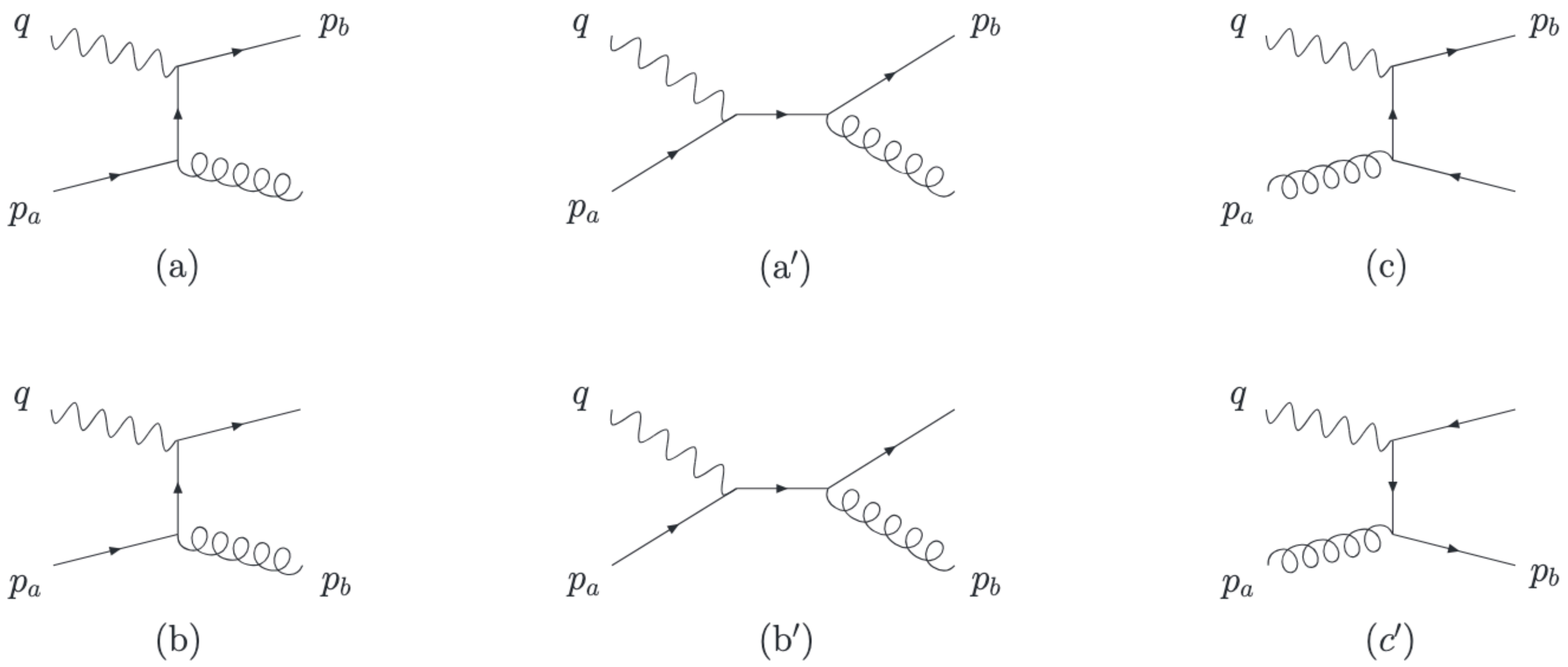


Large $q_T \sim P_{hT}/z$ well established factorization Leading POWER $F_{UU,T}$

$M \ll q_T \sim Q$

i.e.

$$F_{UU,T/L} = \frac{1}{Q^2}\frac{\alpha_s}{2\pi^2 z}\int_{\xi_{\min}}^{1}\frac{\mathrm{d}\xi}{\xi}\int_{\zeta_{\min}}^{1}\frac{\mathrm{d}\zeta}{\zeta^2}\,\delta\!\left(\frac{q_T^2}{Q^2} - \frac{(1-\hat{x})(1-\hat{z})}{\hat{x}\hat{z}}\right) \times \sum_{ij} C_{UU,T/L}^{(ij)}(\hat{x}, Q^2, \hat{z}, q_T) f_{i/N}(\xi, Q^2) D_{h/j}(\zeta, Q^2),$$



- $\gamma^* q \to qg$

$$C_{UU,T}^{(qq)} = 2C_F e_q^2\left\{2\hat{x}\hat{z} + \left[2 + \frac{(1-\hat{x})^2 + (1-\hat{z})^2}{\hat{x}\hat{z}}\right]\frac{Q^2}{q_T^2}\right\}$$

- $\gamma^* q \to gq$

$$C_{UU,T}^{(qg)} = 2C_F e_q^2 (1-\hat{z})\left\{2\hat{x} + \frac{1 + (\hat{x}-\hat{z})^2}{\hat{x}\hat{z}^2}\frac{Q^2}{q_T^2}\right\}$$

- $\gamma^* g \to q\bar{q}$

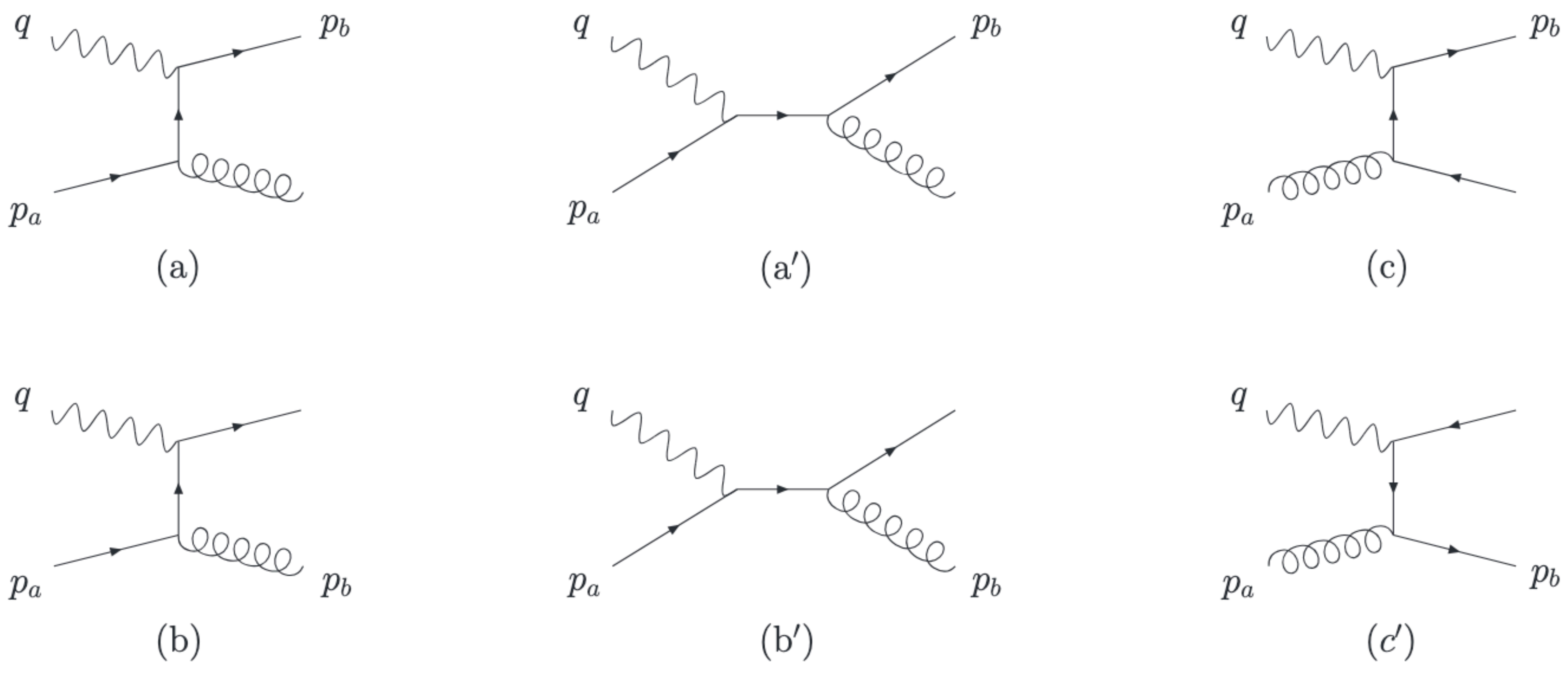
$$C_{UU,T}^{(gq)} = 2T_R e_q^2 (1-\hat{x})\left\{4\hat{x} + \frac{1 - 2\hat{x}(1-\hat{x}) - 2\hat{z}(1-\hat{z})}{\hat{x}\hat{z}^2}\frac{Q^2}{q_T^2}\right\}$$

Large $q_T \sim P_{hT}/z$ well established factorization Leading POWER $F_{UU,L}$

$M \ll q_T \sim Q$

i.e.

$$F_{UU,T/L} = \frac{1}{Q^2}\frac{\alpha_s}{2\pi^2 z}\int_{\xi_{\min}}^{1}\frac{\mathrm{d}\xi}{\xi}\int_{\zeta_{\min}}^{1}\frac{\mathrm{d}\zeta}{\zeta^2}\,\delta\Big(\frac{q_T^2}{Q^2}-\frac{(1-\hat{x})(1-\hat{z})}{\hat{x}\hat{z}}\Big) \times \sum_{ij} C_{UU,T/L}^{(ij)}(\hat{x},Q^2,\hat{z},q_T) f_{i/N}(\xi,Q^2) D_{h/j}(\zeta,Q^2),$$



- $\gamma^* q \to qg$ $\quad C_{UU,L}^{(qq)} = 8 C_F e_q^2 \hat{x}\hat{z}$
- $\gamma^* q \to gq$ $\quad C_{UU,L}^{(qg)} = 8 C_F e_q^2 \hat{x}(1-\hat{z})$
- $\gamma^* g \to q\bar{q}$ $\quad C_{UU,L}^{(gq)} = 16 T_R e_q^2 \hat{x}(1-\hat{x})$

Power behavior $F_{UU,L}$ & $F_{UU,T}$ in AY region, $M \ll q_T \ll Q$

$M \ll q_T \sim Q$

$$F_{UU,T/L} = \frac{1}{Q^2}\frac{\alpha_s}{2\pi^2 z}\int_{\xi_{\min}}^{1}\frac{d\xi}{\xi}\int_{\zeta_{\min}}^{1}\frac{d\zeta}{\zeta^2}\,\delta\!\left(\frac{q_T^2}{Q^2} - \frac{(1-\hat{x})(1-\hat{z})}{\hat{x}\hat{z}}\right) \times \sum_{ij} C_{UU,T/L}^{(ij)}(\hat{x}, Q^2, \hat{z}, q_T)\, f_{i/N}(\xi, Q^2)\, D_{h/j}(\zeta, Q^2),$$

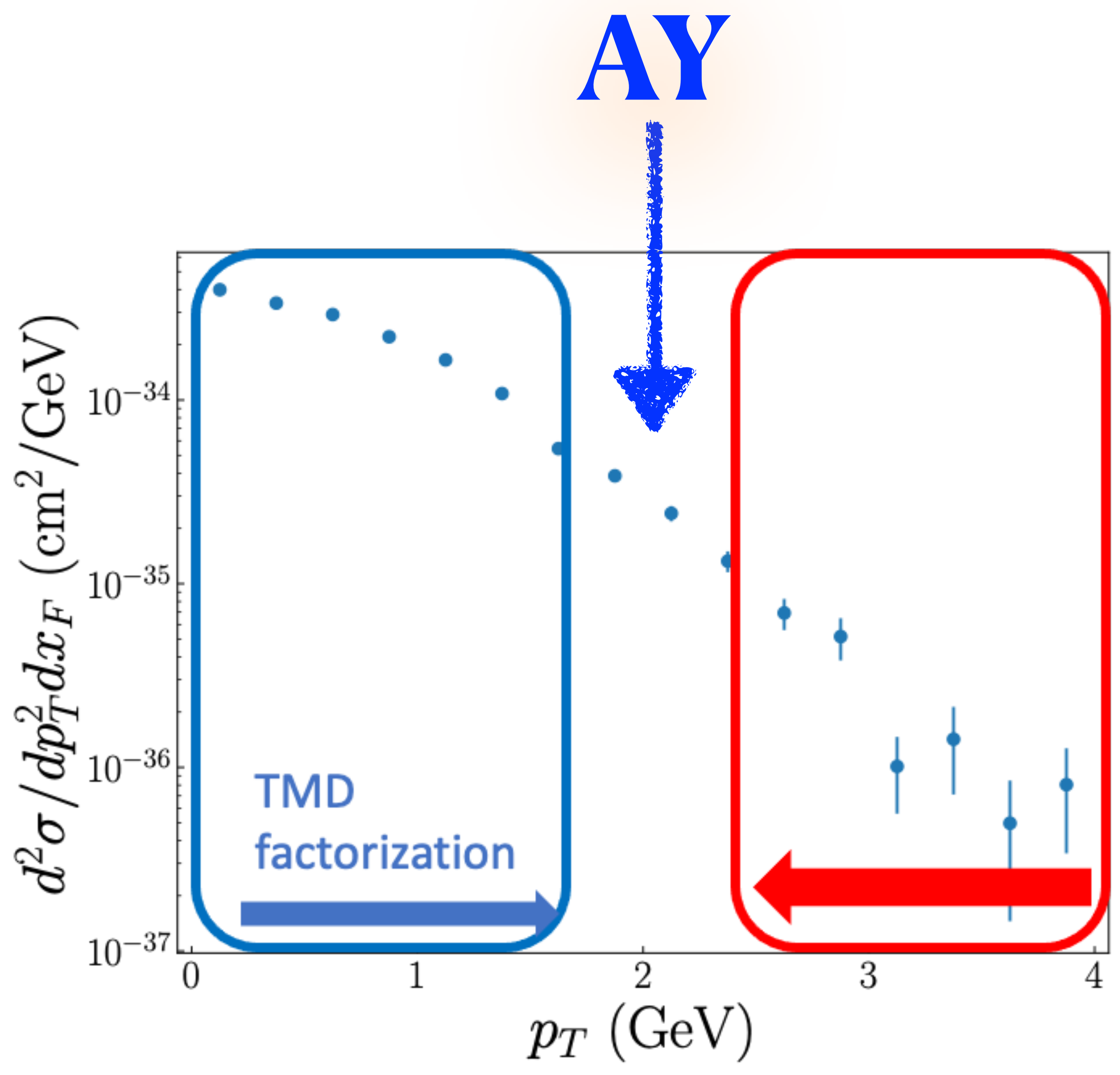
$M \ll q_T \ll Q$

Asymptotic region

$$F_{UU,T} = \frac{1}{q_T^2}\frac{\alpha_s}{2\pi^2 z^2}\sum_a x\, e_a^2\Big[f_1^a(x)\, D_1^a(z)\, L\!\left(\frac{Q^2}{q_T^2}\right) + f_1^a(x)\,(D_1^a \otimes P_{qq} + D_1^g \otimes P_{gq})(z) + (P_{qq}\otimes f_1^a + P_{qg}\otimes f_1^g)(x)\, D_1^a(z)\Big]$$

$$F_{UU,L} = \frac{1}{Q^2}\frac{\alpha_s}{2\pi^2 z^2}\sum_a x\, e_a^2\Big[f_1^a(x)\, D_1^a(z)\, L\!\left(\frac{Q^2}{q_T^2}\right) + f_1^a(x)\,(D_1^a \otimes P''_{qq} + D_1^g \otimes P''_{gq})(z) + (P''_{qq}\otimes f_1^a + P''_{qg}\otimes f_1^g)(x)\, D_1^a(z)\Big]$$

$$L\!\left(\frac{Q^2}{q_T^2}\right) \equiv C_F\left[2\ln\!\left(\frac{Q^2}{q_T^2}\right) - 3\right]$$



Can $F_{UU,L}$ be resummed ?

See e.g. Berger Qiu Zhang, PRD D65 (2002)

Probe of sub-leading physics

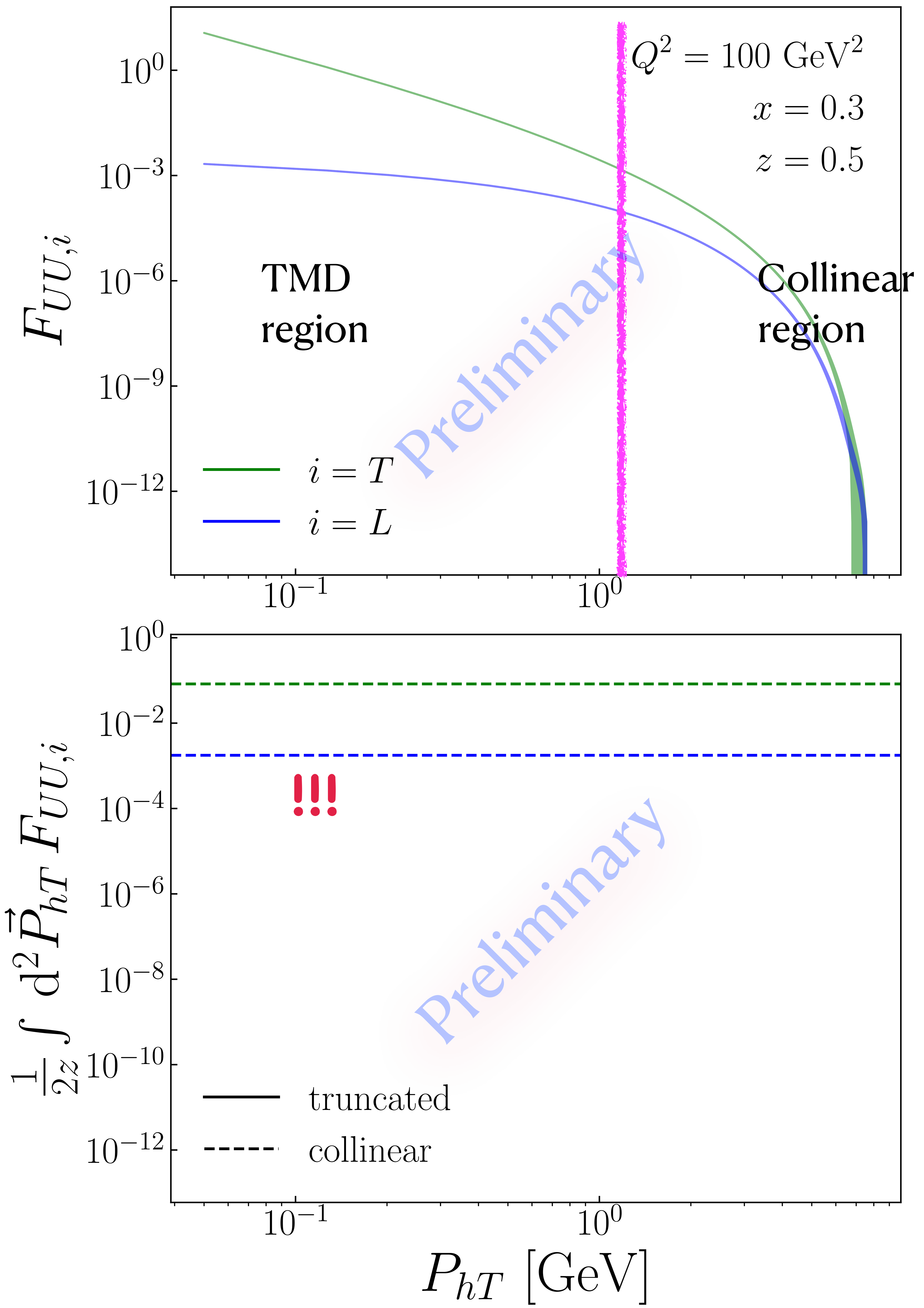
Large → intermediate → low q_T regions

- $F_{UU,T}(x, z, q_T, Q)$ & $F_{UU,L}(x, z, q_T, Q)$

- **Collinear SIDIS vs. truncated moments the dashed line for $F_{UU,L}$ is a leading power statement**

$$\int_{P^2_{hT\,\mathrm{min}}/z^2}^{q^2_{T\,\mathrm{max}}} F(x, z, Q^2, P_{hT})$$

- $F_{UU,L}$ truncated moment converges "P_T **integrable**"
- $F_{UU,T}$ truncated moment as expected **diverges**
- Small TMD contribution ? Small power corrections ?



Probe for sub-leading physics

Truncated moments of R_{SIDIS} from large p_T

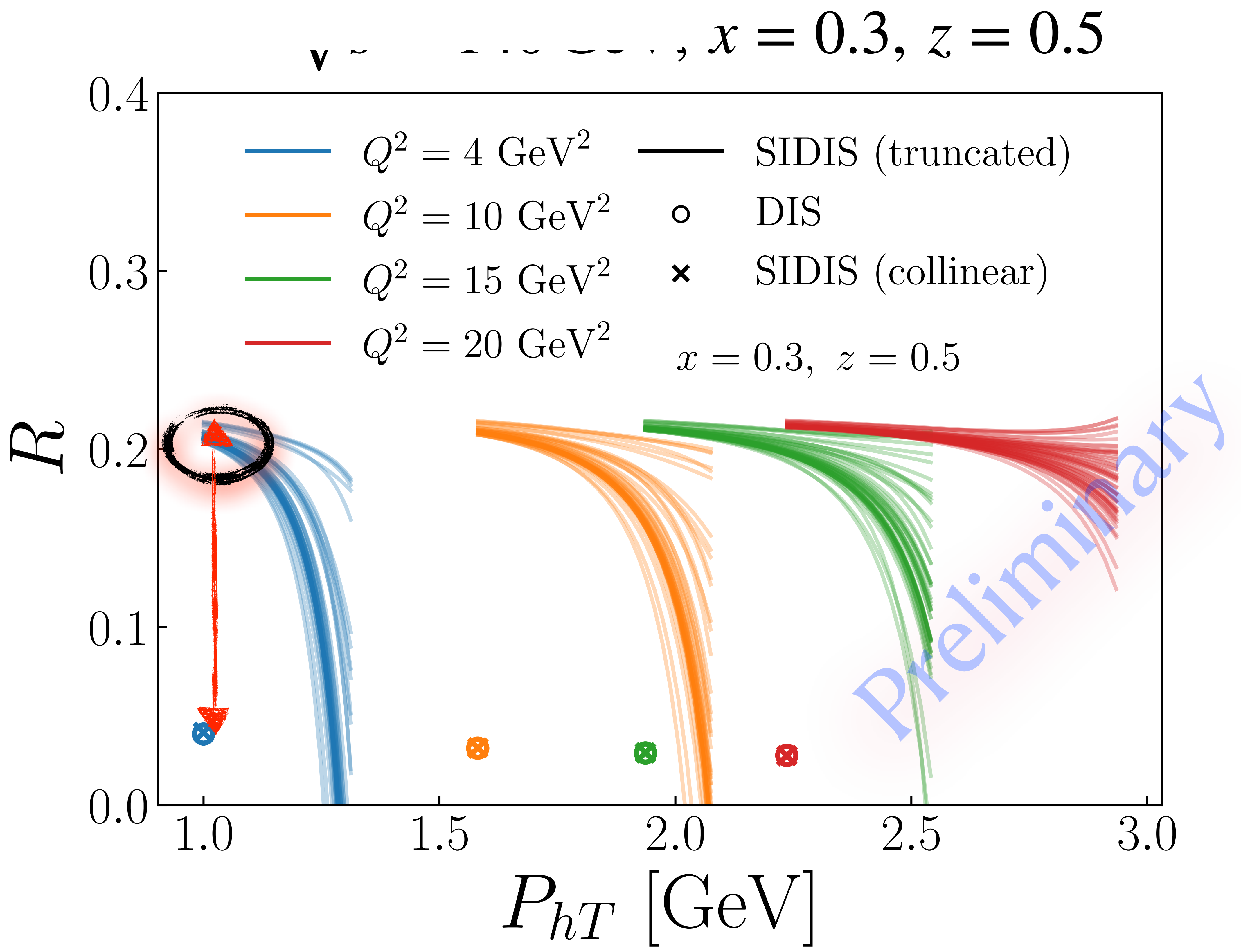
i.e.

$$F_{UU,T/L} = \frac{1}{Q^2}\frac{\alpha_s}{2\pi^2 z}\int_{\xi_{\min}}^{1}\frac{d\xi}{\xi}\int_{\zeta_{\min}}^{1}\frac{d\zeta}{\zeta^2}\,\delta\!\left(\frac{q_T^2}{Q^2}-\frac{(1-\hat{x})(1-\hat{z})}{\hat{x}\hat{z}}\right) \times \sum_{ij} C^{(ij)}_{UU,T/L}(\hat{x},Q^2,\hat{z},q_T)\,f_{i/N}(\xi,Q^2)\,D_{h/j}(\zeta,Q^2),$$

$$R_{SIDIS} = \int_{P_{hT}} d^2\vec{P}'_{hT}\,F^{(FO)}_{UU,L} \Big/ \int_{P_{hT}} d^2\vec{P}'_{hT}\,F^{(FO)}_{UU,T}$$

SIDIS truncated moments

$$\int_{P^2_{hT\,\min}/z^2}^{q^2_{T\,\max}} F(x,z,Q^2,P_{hT})$$



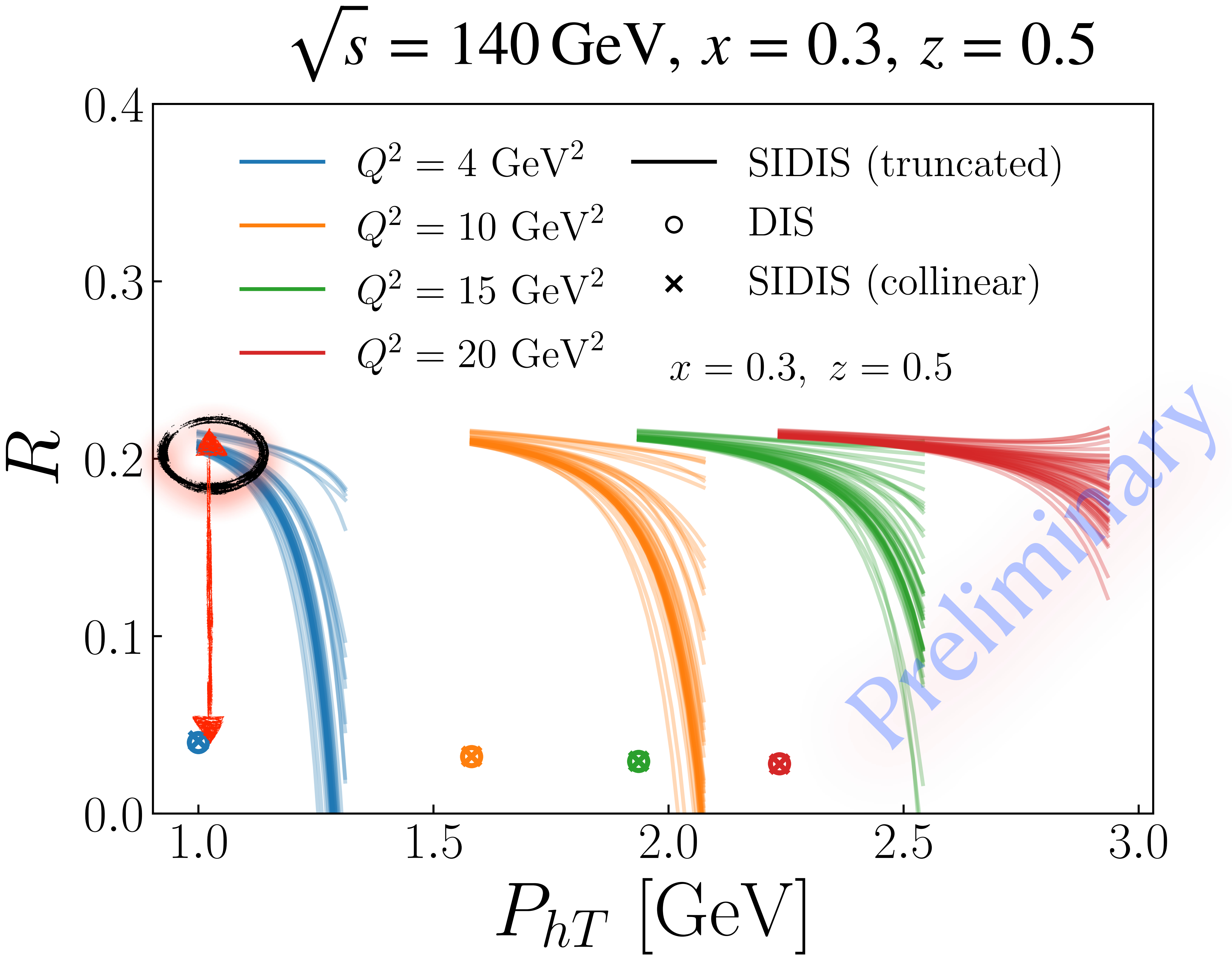
Nb: Bands are generated by computing the observable on subset of JAM replicas (from recent W+charm analysis) & taking the mean ± standard deviation

Probe for sub-leading physics

Truncated moments of R_{SIDIS} from large p_T

$$F_{UU,T} = \frac{1}{Q^2}\frac{\alpha_s}{(2\pi z)^2}\sum_a x\,e_a^2 \int_x^1 \frac{d\hat{x}}{\hat{x}}\int_z^1 \frac{d\hat{z}}{\hat{z}}\,\delta\left(\frac{q_T^2}{Q^2} - \frac{(1-\hat{x})(1-\hat{z})}{\hat{x}\,\hat{z}}\right)$$
$$\times\left[f_1^a\left(\frac{x}{\hat{x}}\right)D_1^a\left(\frac{z}{\hat{z}}\right)C_{UU,T}^{(\gamma^* q\to qg)} + f_1^a\left(\frac{x}{\hat{x}}\right)D_1^g\left(\frac{z}{\hat{z}}\right)C_{UU,T}^{(\gamma^* q\to gq)} + f_1^g\left(\frac{x}{\hat{x}}\right)D_1^a\left(\frac{z}{\hat{z}}\right)C_{UU,T}^{(\gamma^* g\to q\bar{q})}\right]$$

$$R_{SIDIS} = \frac{F_{UU,L}^{FO}}{F_{UU,T}^{FO}}$$



Comments:

- Truncated moment is sig. larger than P_T integrated SIDIS—indication of
- power corrections ?
- TMD contribution ?

W-term is important for R_{SIDIS}!

What role does sub-leading $F_{UU,L}^{(W)}$ play?

Stitching together TMD + FO with goal of estimating subleading power "resummed W" term

Since $\mathrm{W} + \mathrm{Y}$ not feasible phenomenologically yet …

→ approximate ASY by interpolating W & FO

→ use intersection points between W & FO to separate scales

→ ensure analyticity point-by-point

$$F_{UU,L} = \left(1 - w_L\right) F_{UU,L}^{[M\sim q_T \ll Q]} + w_L\, F_{UU,L}^{[M \ll q_T \sim Q]} \overset{?}{\Longrightarrow} W + Y + \mathcal{O}\left(\frac{M}{Q}\right)^c$$

Where w_L is-transition function

$$F_{UU,L}^{[M\sim q_T \ll Q]} = \frac{M^2}{Q^2} C\left[\frac{p_T^2}{M^2} f_1 D_1\right] \qquad F_{UU,L}^{[M \ll q_T \sim Q]} = Fixed\,order$$

Building our model

Since $W + Y$ not feasible phenomenologically yet …

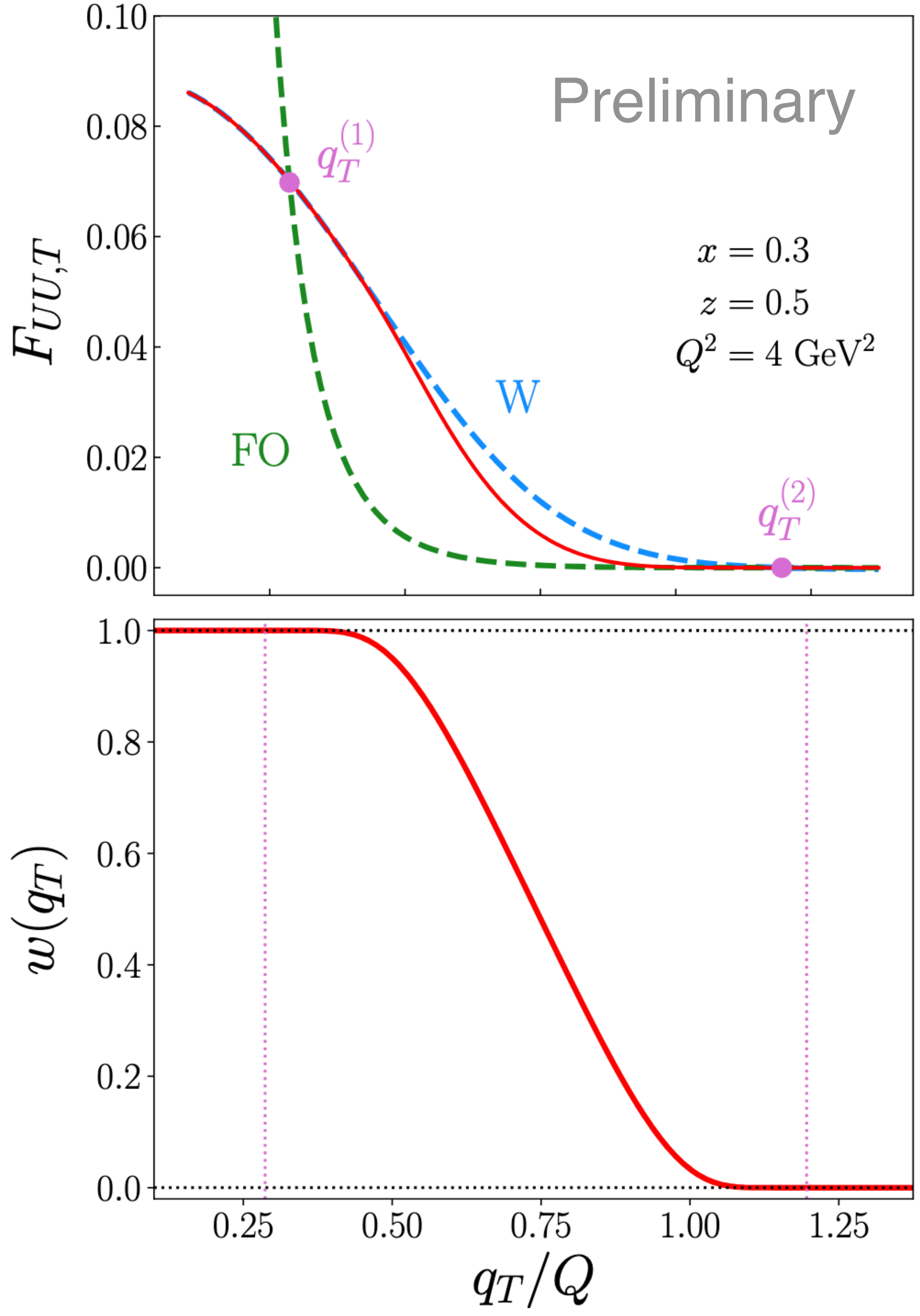
→ approximate ASY by interpolating W & FO

→ use intersection points between W & FO to separate scales

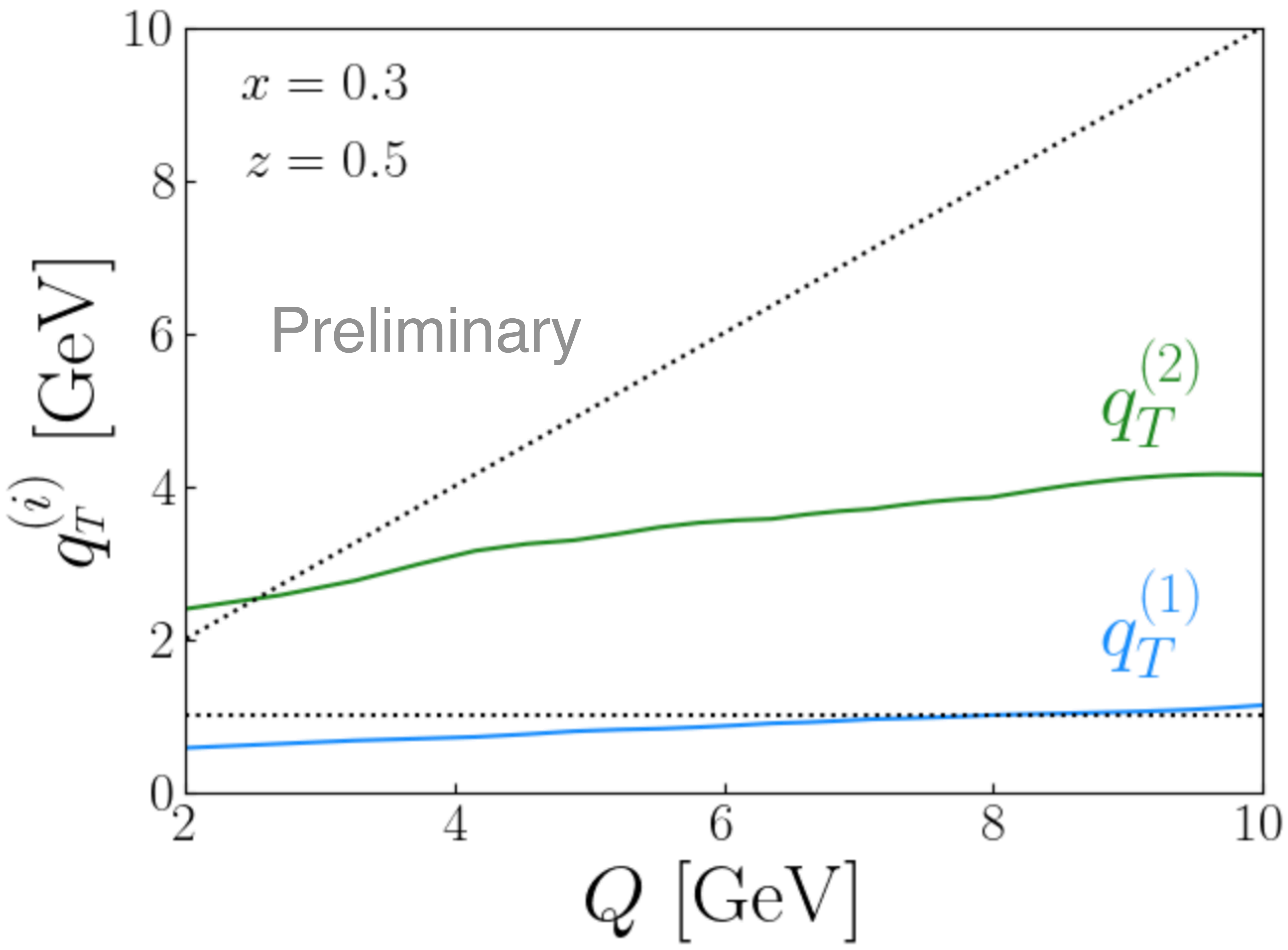
→ ensure analyticity point-by-point

$$F = w(q_T)\,\mathrm{W} + [1 - w(q_T)]\,\mathrm{FO}$$

$$w(q_T) = \begin{cases} 1 & q_T < q_T^{(1)} \\ f_{\mathrm{interp}}(q_T^{(c)}, \sigma) & q_T^{(1)} < q_T < q_T^{(2)} \\ 0 & q_T > q_T^{(1)} \end{cases}$$



Behavior of the intersections



Regions of validity

→ W: $M \sim q_T \ll Q$

→ FO: $M \ll q_T \sim Q$

Intersections roughly follow this trend!

→ weak slope for W term near M

→ stronger slope for FO but < 1

→ intriguing linear behavior …

Caution: region separation difficult when $M \sim Q$

SETUP

Our determination of R_{SIDIS} based on use TMD PDF and TMD FF grids from the MAP22 extraction Bacchetta:2022awv, which are used to compute the W-term contribution to both $F_{UU,T}$ and $F_{UU,L}$

Use the fixed-order calculation at $\mathcal{O}(\alpha_S)$ for both structure functions, and we use the MMHT2014 and DSS14 sets at NLO for PDFs and FFs, respectively.

To achieve a consistent matching between the W and FO terms we use TMD grids at NNLL accuracy.

Focus here on the differential multiplicities in transverse momentum, and study the size of R_{SIDIS} in the kinematic region covered by COMPASS data.

Uncertainty quantification

Use same interpolation for $F_{UU,T/L}$

$\rightarrow$

$$W_L = \frac{k_T^2}{k_T^2 + Q^2/4} \rightarrow \frac{k_T^2}{4Q^2} \text{ when } k_T \sim 0$$

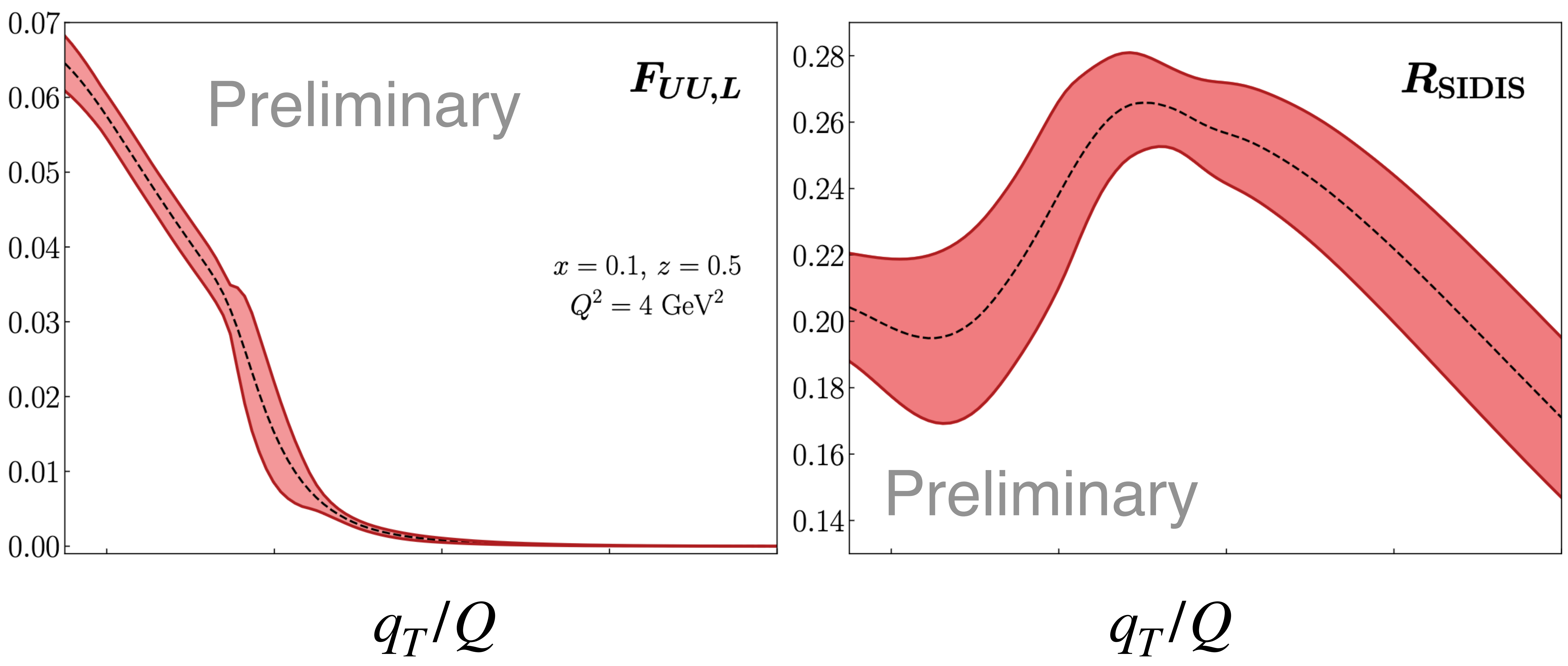
$\rightarrow$ 4 types of uncertainty:

(1) interpolation between W & FO

(2) Scale variation

(3) TMD replicas

(4) Collinear hessian sets



Pheno impacts/opportunities

R_{SIDIS} predictions at Compass kinematics

→ full error budget

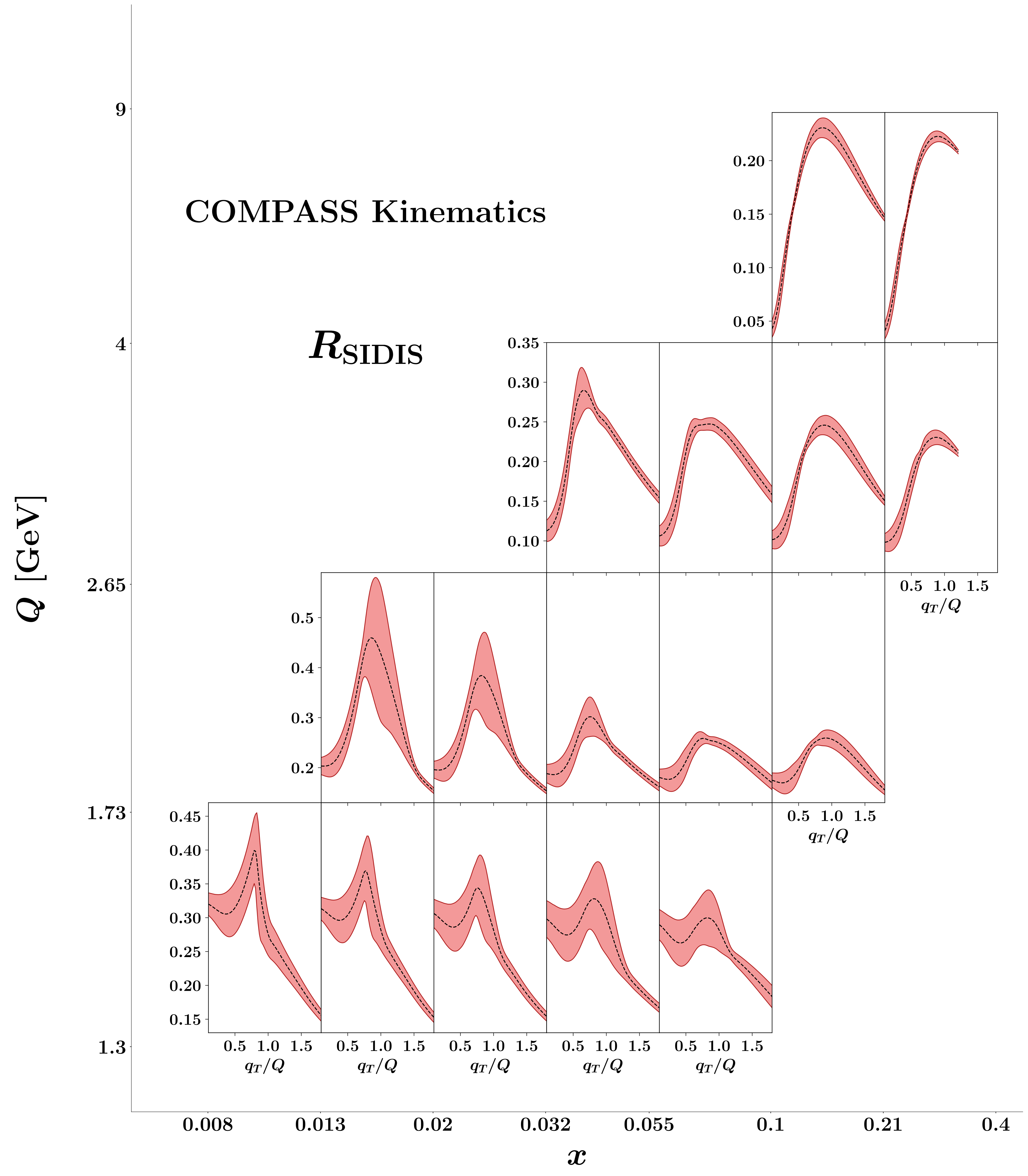
→ trends with increasing Q

(1) Magnitude of $R_{\text{SIDIS}}(q_T \to 0)$ decreases

(2) $R_{\text{SIDIS}}(q_T \to 0) \sim (q_T/Q)^2$

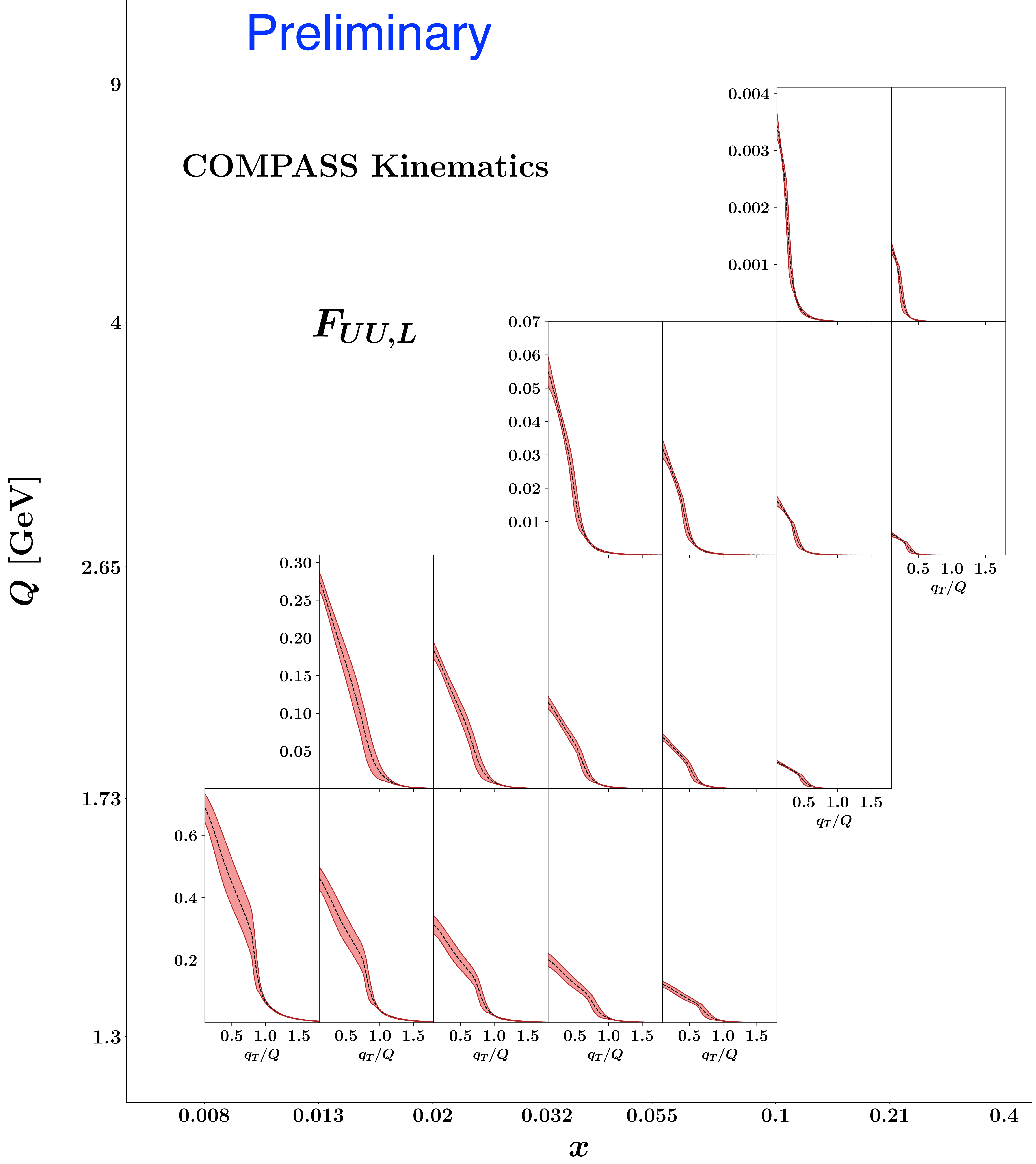
(3) Uncertainty decreases

How do theory & experimental uncertainties compare?



Pheno cont.

$F_{UU,L}$ predictions @ Compass kinematics



Summary

Longitudinal Structure function $F_{UU,L}$

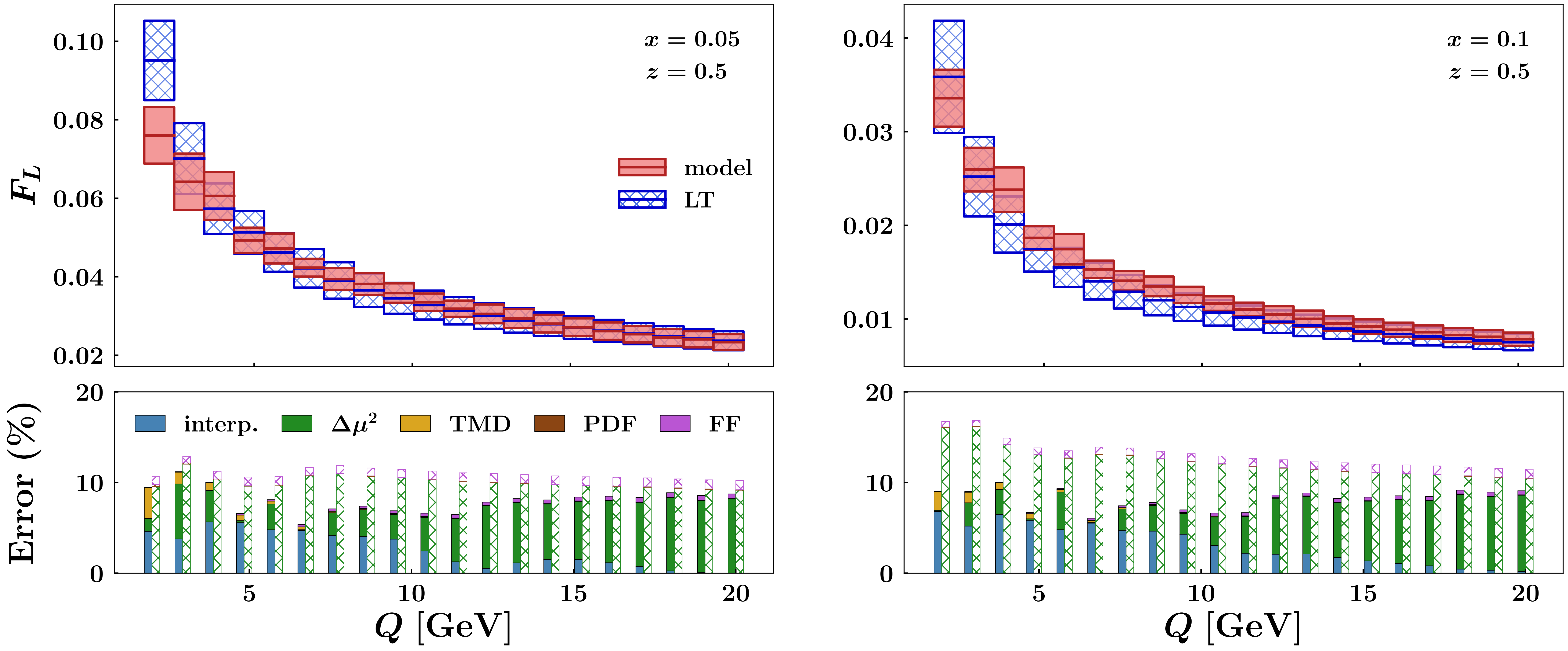
→ indispensable to probe transverse proton structure SIDIS multiplicites ?

→ lots of challenges in formal/phenomenological description of q_T spectrum

→Progress in sub-leading TMD physics

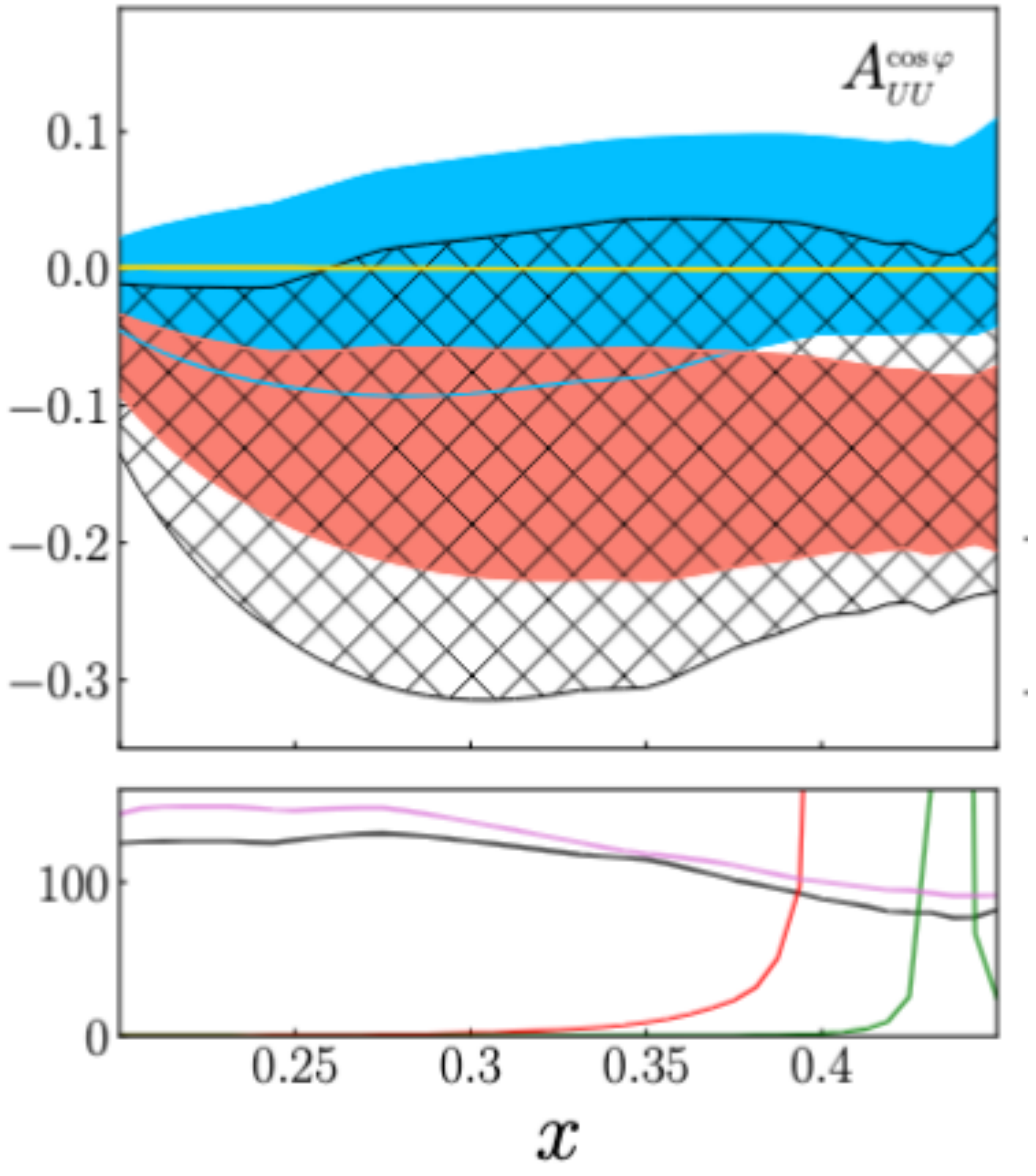
→ pheno estimates with uncertainties of $F_{UU,L}$ and R_{SIDIS} considering interplay of *W* & FO

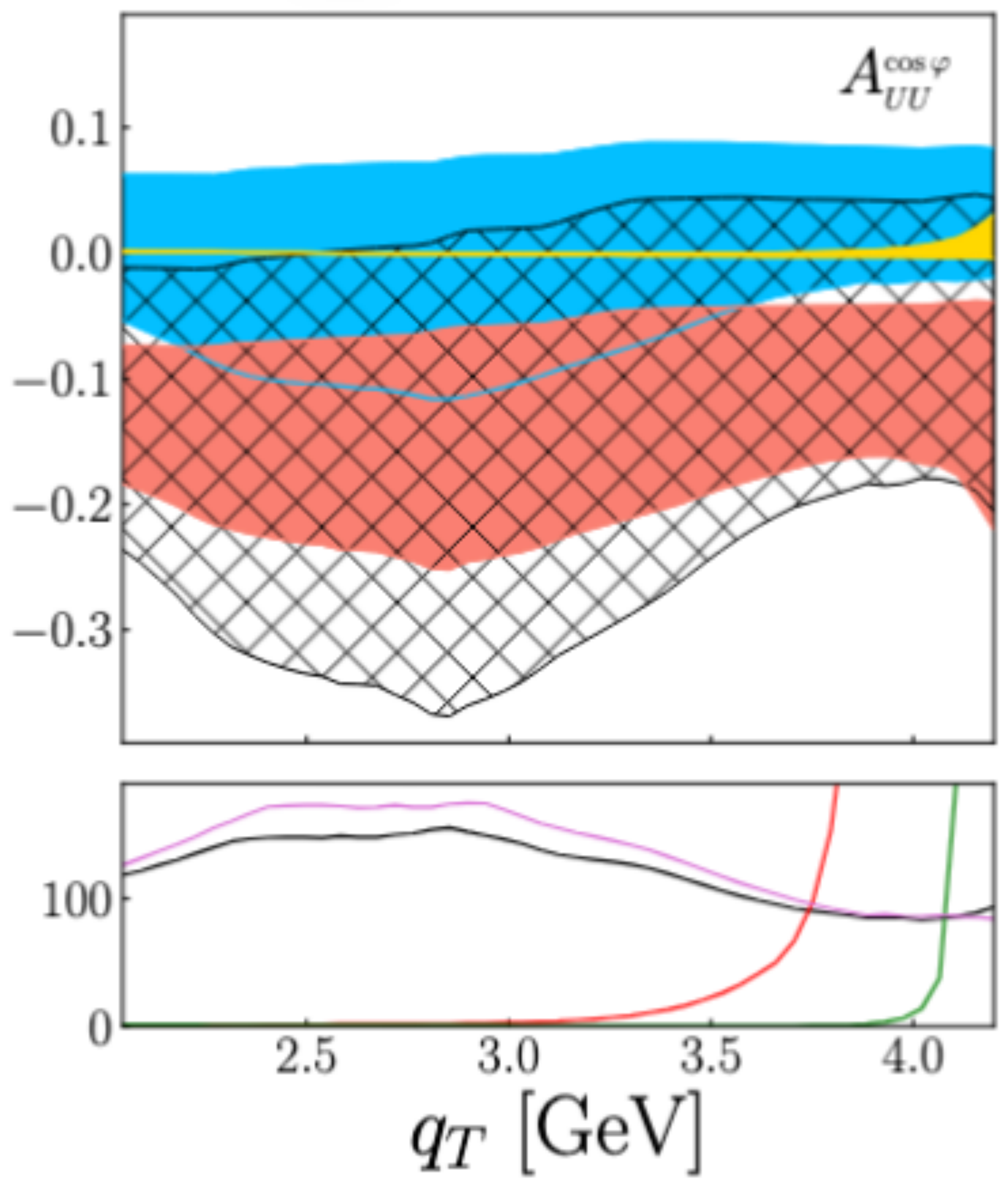
→ Future — impacts on multiplicities/collinear SIDIS

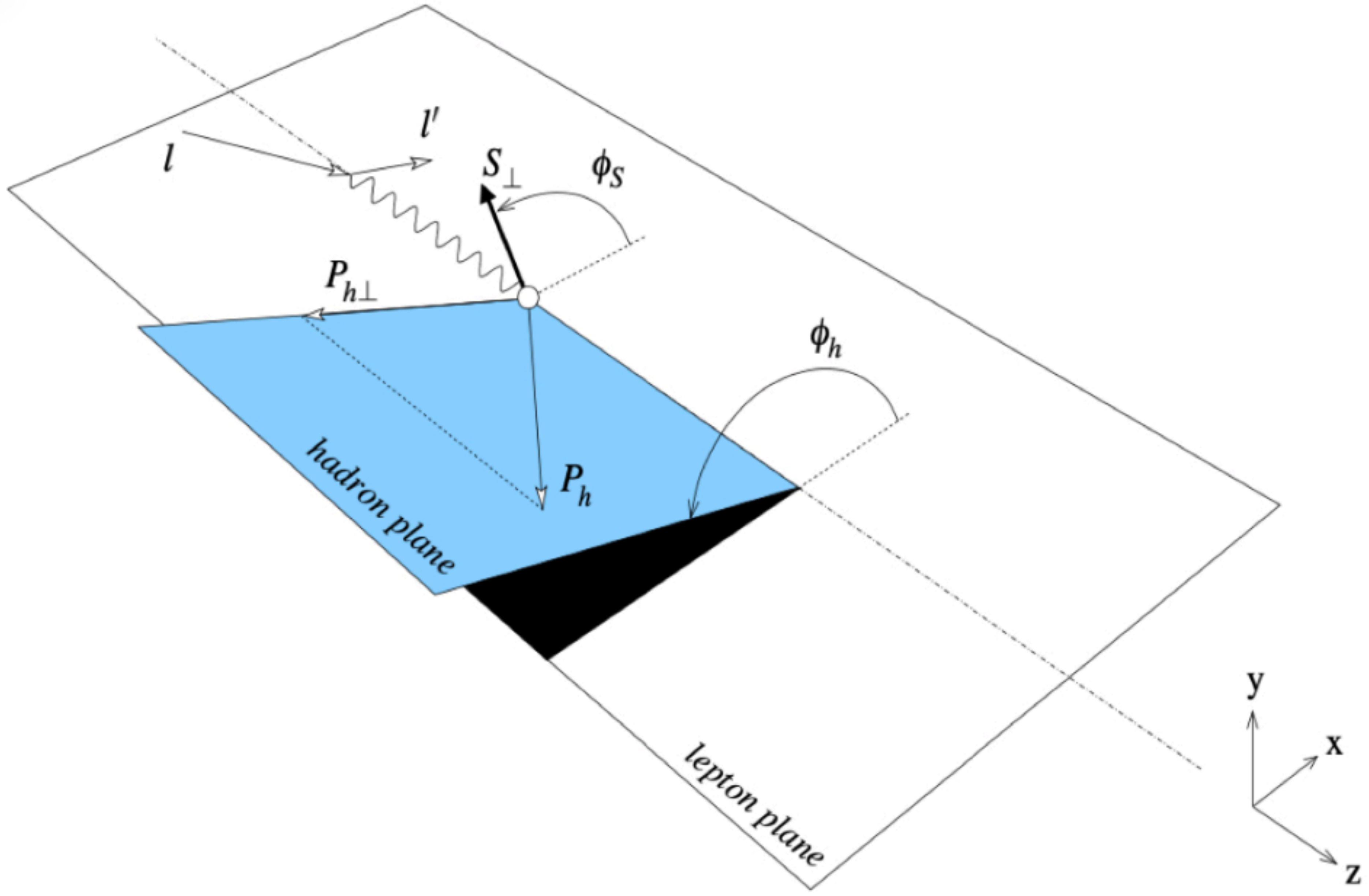


The integrated longitudinal structure function over transverse momentum as a function of $ Q at x=0.1 and z=0.5 (left panel) and at x=0.01 and z=0.5 (right panel). The red band corresponds to the integrated structure function obtained from our model, while the blue band corresponds to the integral of only the fixed-order calculation. In the upper plot we show the integrated structure function, while in the lower plot we show the breakdown and the size of the different sources of uncertainties.

A related NLP unpol. observables







- $F_{UU}^{\cos\phi_h} \to \langle \cos\phi_h \rangle$ n.b. L/T inteference

Georgi & Cahn, PRL 1978, PLB 1978

- Critique of perturbative QCD calculation of azimuthal dependence in SIDIS
- Emphasize importance intrinsic k_T the early days/birth of "TMD physics"

Literature on NLP

*First prelim SIDIS studies beyond tree level:
"Matches & Mis-matches" Bacchetta et al. JHEP 2008, Chen & Ma PLB 2017

Literature
Bacchetta et al. PLB 2019
MIT group, Gao, Ebert, Stewart JHEP 2022
Gamberg, Kang, Shao, Terry, Zhao arXiv:221.13209 … & …Page Gamberg Kang Stasto 2026…
Madrid group Vladimirov, Rodini, Scimemi, Piloñeta et al JHEP 2021, 2022, 2023, 2024 2025
Balitsky 2023 rapidity TMD evolution Balitsky & Prokudin 2026

TMD Collaboration

Preprints: JLAB-THY-23-3780, LA-UR-21-20798, MIT-CTP/5386

TMD Handbook

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