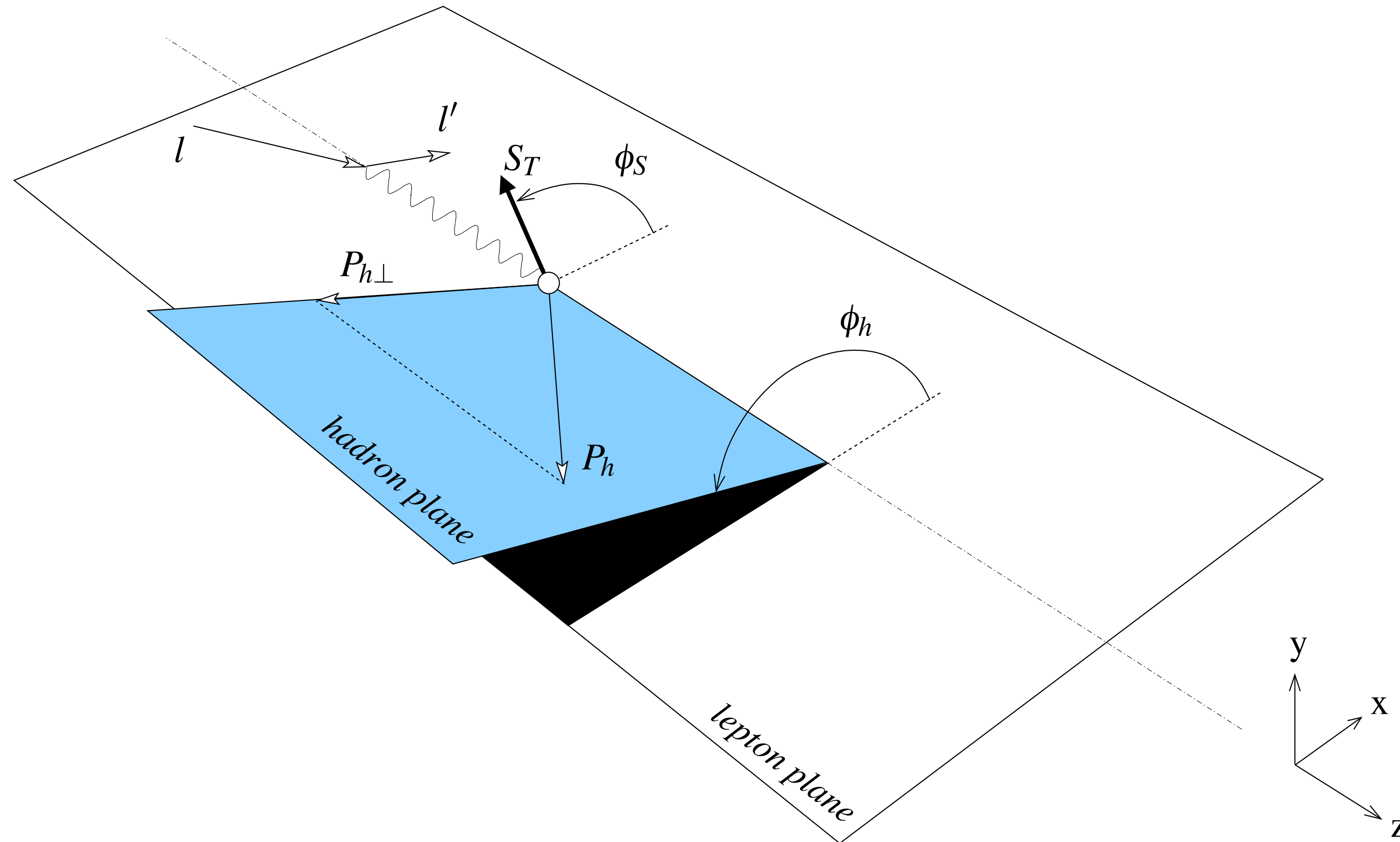


ALESSANDRO BACCHETTA, PAVIA U. AND INFN

SIDIS STRUCTURE FUNCTIONS AND EXCLUSIVE CONTRIBUTIONS

SIDIS KINEMATICS

Bacchetta, Diehl, Goeke, Metz, Mulders, Schlegel, [hep-ph/0611265](https://arxiv.org/abs/hep-ph/0611265)



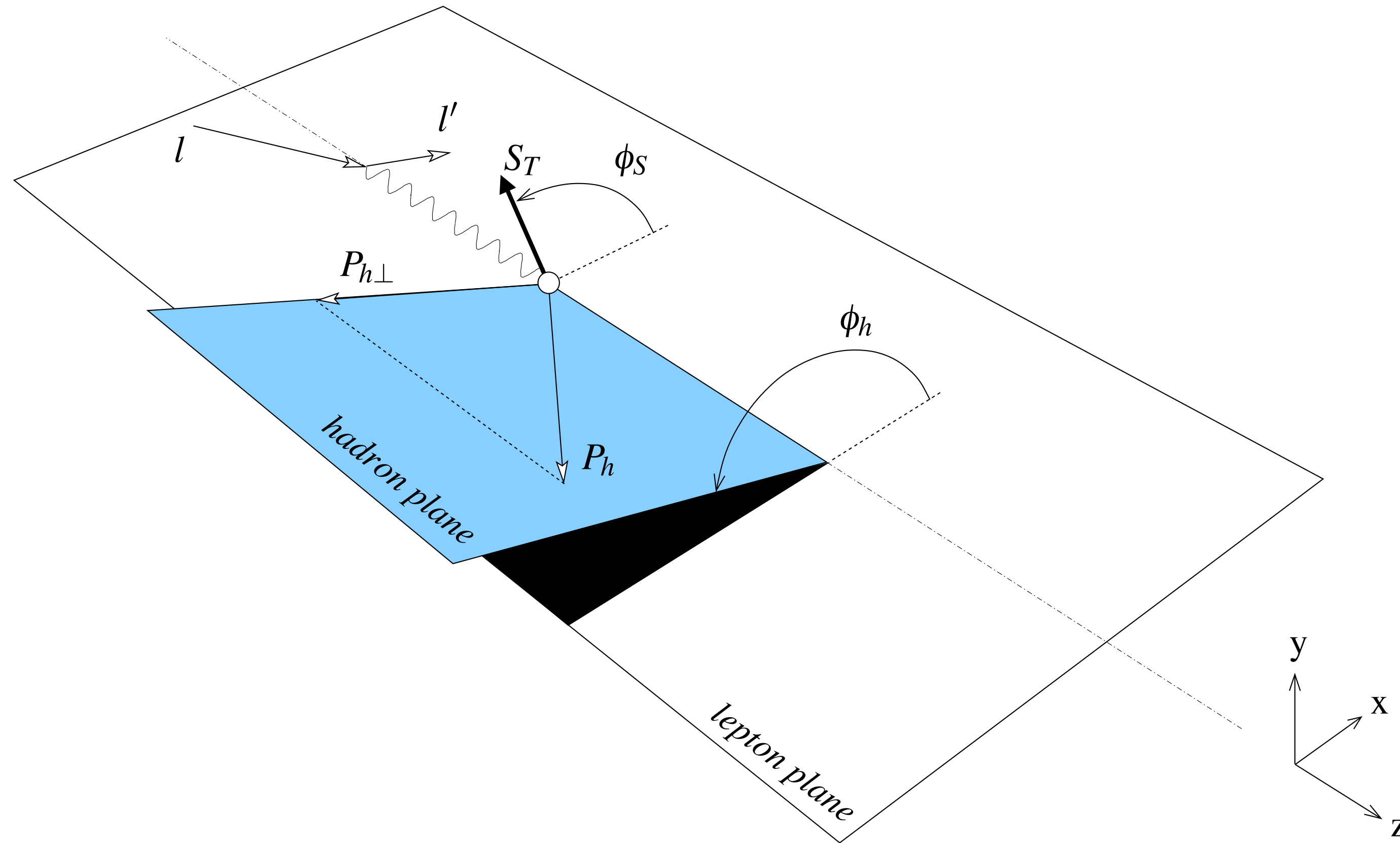
Q = photon virtuality

M = hadron mass

$P_{h\perp}$ = hadron transverse momentum = P_T

SIDIS KINEMATICS

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Q = photon virtuality

M = hadron mass

$P_{h\perp}$ = hadron transverse momentum = P_T $q_T^2 \approx P_{h\perp}^2/z^2$

SIDIS STRUCTURE FUNCTIONS

Bacchetta, Diehl, Goeke, Metz, Mulders, Schlegel, [hep-ph/0611265](https://arxiv.org/abs/hep-ph/0611265)

$$\begin{aligned}
 & \frac{d\sigma}{dx dy d\phi_S dz d\phi_h dP_{h\perp}^2} \\
 &= \frac{\alpha^2}{x y Q^2} \frac{y^2}{2(1-\varepsilon)} \left\{ F_{UU,T} + \varepsilon F_{UU,L} + \sqrt{2\varepsilon(1+\varepsilon)} \cos\phi_h F_{UU}^{\cos\phi_h} + \varepsilon \cos(2\phi_h) F_{UU}^{\cos 2\phi_h} \right. \\
 &+ \lambda_e \sqrt{2\varepsilon(1-\varepsilon)} \sin\phi_h F_{LU}^{\sin\phi_h} + S_L \left[\sqrt{2\varepsilon(1+\varepsilon)} \sin\phi_h F_{UL}^{\sin\phi_h} + \varepsilon \sin(2\phi_h) F_{UL}^{\sin 2\phi_h} \right] \\
 &+ S_L \lambda_e \left[\sqrt{1-\varepsilon^2} F_{LL} + \sqrt{2\varepsilon(1-\varepsilon)} \cos\phi_h F_{LL}^{\cos\phi_h} \right] \\
 &+ S_T \left[\sin(\phi_h - \phi_S) \left(F_{UT,T}^{\sin(\phi_h - \phi_S)} + \varepsilon F_{UT,L}^{\sin(\phi_h - \phi_S)} \right) + \varepsilon \sin(\phi_h + \phi_S) F_{UT}^{\sin(\phi_h + \phi_S)} \right. \\
 &+ \varepsilon \sin(3\phi_h - \phi_S) F_{UT}^{\sin(3\phi_h - \phi_S)} + \sqrt{2\varepsilon(1+\varepsilon)} \sin\phi_S F_{UT}^{\sin\phi_S} \\
 &+ \left. \left. \sqrt{2\varepsilon(1+\varepsilon)} \sin(2\phi_h - \phi_S) F_{UT}^{\sin(2\phi_h - \phi_S)} \right] + S_T \lambda_e \left[\sqrt{1-\varepsilon^2} \cos(\phi_h - \phi_S) F_{LT}^{\cos(\phi_h - \phi_S)} \right. \right. \\
 &+ \left. \left. \sqrt{2\varepsilon(1-\varepsilon)} \cos\phi_S F_{LT}^{\cos\phi_S} + \sqrt{2\varepsilon(1-\varepsilon)} \cos(2\phi_h - \phi_S) F_{LT}^{\cos(2\phi_h - \phi_S)} \right] \right\}
 \end{aligned}$$

UNPOLARIZED SIDIS

$$\frac{d\sigma}{dx\,dy\,d\phi_S\,dz\,d\phi_h\,dP_{h\perp}^2} = \frac{\alpha^2}{x\,y\,Q^2} \frac{y^2}{2(1-\varepsilon)} \left\{ F_{UU,T} + \varepsilon F_{UU,L} + \sqrt{2\varepsilon(1+\varepsilon)} \cos\phi_h F_{UU}^{\cos\phi_h} + \varepsilon \cos(2\phi_h) F_{UU}^{\cos 2\phi_h} \right. \\ \left. + \lambda_e \sqrt{2\varepsilon(1-\varepsilon)} \sin\phi_h F_{LU}^{\sin\phi_h} \right\}$$

UNPOLARIZED SIDIS

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Note: this expression can be applied to any process $l + P \rightarrow l' + h + X$, including DVMP and even DVCS, if h is replaced by a photon.

UNPOLARIZED SIDIS

$$\frac{d\sigma}{dx dy d\phi_S dz d\phi_h dP_{h\perp}^2} = \frac{\alpha^2}{x y Q^2} \frac{y^2}{2(1-\varepsilon)} \left\{ F_{UU,T} + \varepsilon F_{UU,L} + \sqrt{2\varepsilon(1+\varepsilon)} \cos \phi_h F_{UU}^{\cos \phi_h} + \varepsilon \cos(2\phi_h) F_{UU}^{\cos 2\phi_h} \right. \\ \left. + \lambda_e \sqrt{2\varepsilon(1-\varepsilon)} \sin \phi_h F_{LU}^{\sin \phi_h} \right\}$$

Note: this expression can be applied to any process $l + P \rightarrow l' + h + X$, including DVMP and even DVCS, if h is replaced by a photon.

Usually, the exclusive expressions are given in terms of t instead of $P_{h\perp}$

$$t = -\frac{P_{h\perp}^2}{z} - z \left(M_h^2 + \frac{1-z}{z} Q^2 \right) + \dots, \quad dt = -\frac{dP_{h\perp}^2}{z}$$

INCLUSIVE DIS

$$\frac{d\sigma}{dx\,dy} = \frac{4\pi\alpha^2}{x\,y\,Q^2} \frac{y^2}{2\,(1-\varepsilon)} \left\{ F_{UU,T} + \varepsilon F_{UU,L} \right\}$$

INCLUSIVE DIS

$$\frac{d\sigma}{dx\,dy} = \frac{4\pi\alpha^2}{x\,y\,Q^2} \frac{y^2}{2\,(1-\varepsilon)} \left\{ \textcolor{red}{F}_{UU,T} + \varepsilon \textcolor{red}{F}_{UU,L} \right\}$$

$$F_{UU,T} = 2x_B F_1,$$

$$F_{UU,L} = (1 + \gamma^2) F_2 - 2x_B F_1$$

INCLUSIVE DIS

$$\frac{d\sigma}{dx dy} = \frac{4\pi\alpha^2}{x y Q^2} \frac{y^2}{2(1-\varepsilon)} \left\{ F_{UU,T} + \varepsilon F_{UU,L} \right\}$$

$$F_{UU,T} = 2x_B F_1,$$

$$F_{UU,L} = (1 + \gamma^2) F_2 - 2x_B F_1$$

Note: this expression can be applied to any process $l + P \rightarrow l' + X$, including elastic scattering.

HELICITY STRUCTURE FUNCTIONS

Bacchetta, Diehl, Goeke, Metz, Mulders, Schlegel, [hep-ph/0611265](https://arxiv.org/abs/hep-ph/0611265)

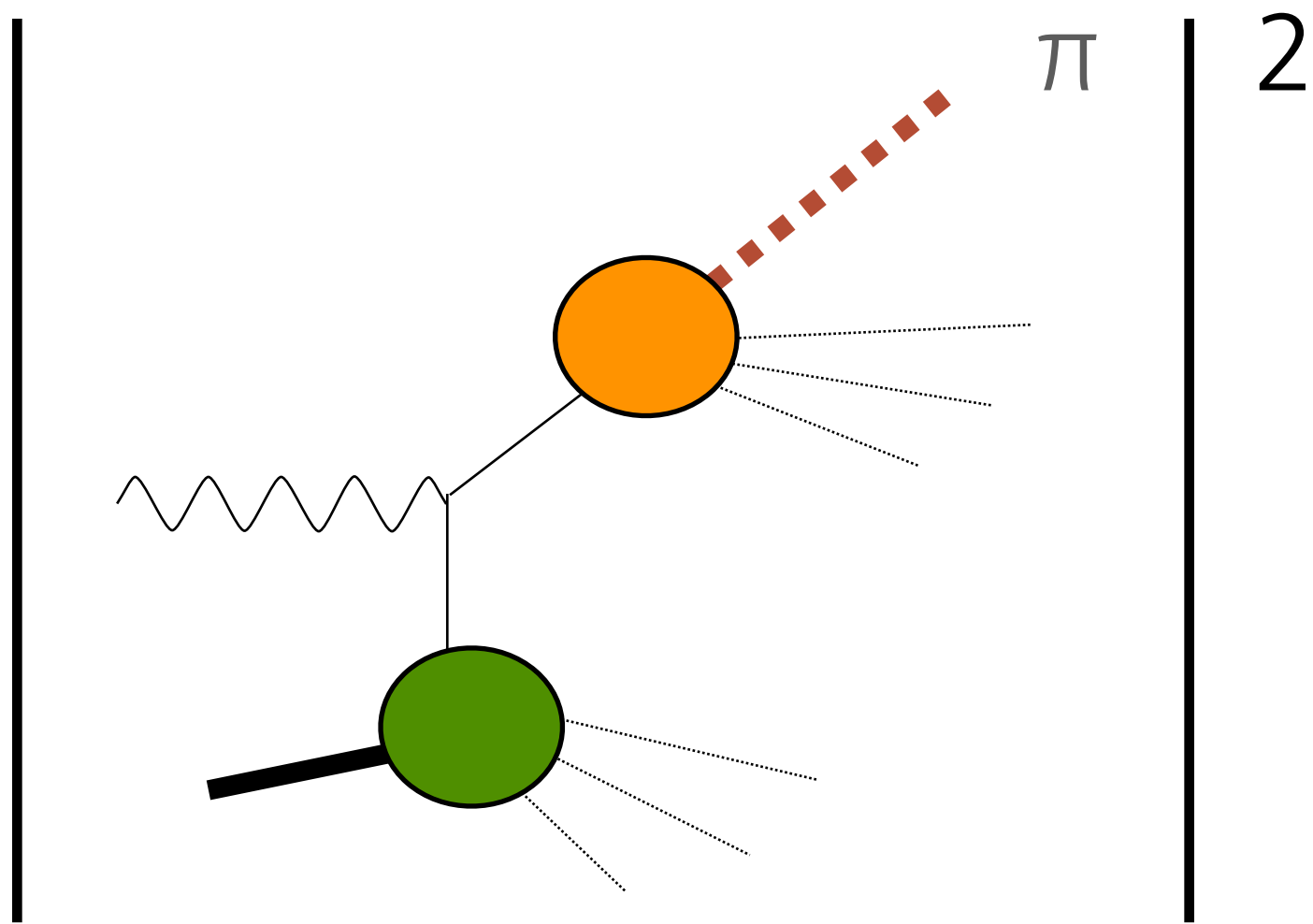
see also Diehl, Sapeta, [hep-ph/0503023](https://arxiv.org/abs/hep-ph/0503023)

Helicity structure functions are often used instead of the previously defined ones

$$\begin{aligned} F_{UU,T} &= \frac{1}{2} (F_{++}^{++} + F_{++}^{--}), & F_{UU,L} &= F_{00}^{++}, \\ F_{UU}^{\cos \phi_h} &= -\frac{1}{\sqrt{2}} \operatorname{Re} (F_{+0}^{++} + F_{+0}^{--}), & F_{UU}^{\cos 2\phi_h} &= -\operatorname{Re} F_{+-}^{++}, \end{aligned} \tag{A.2}$$

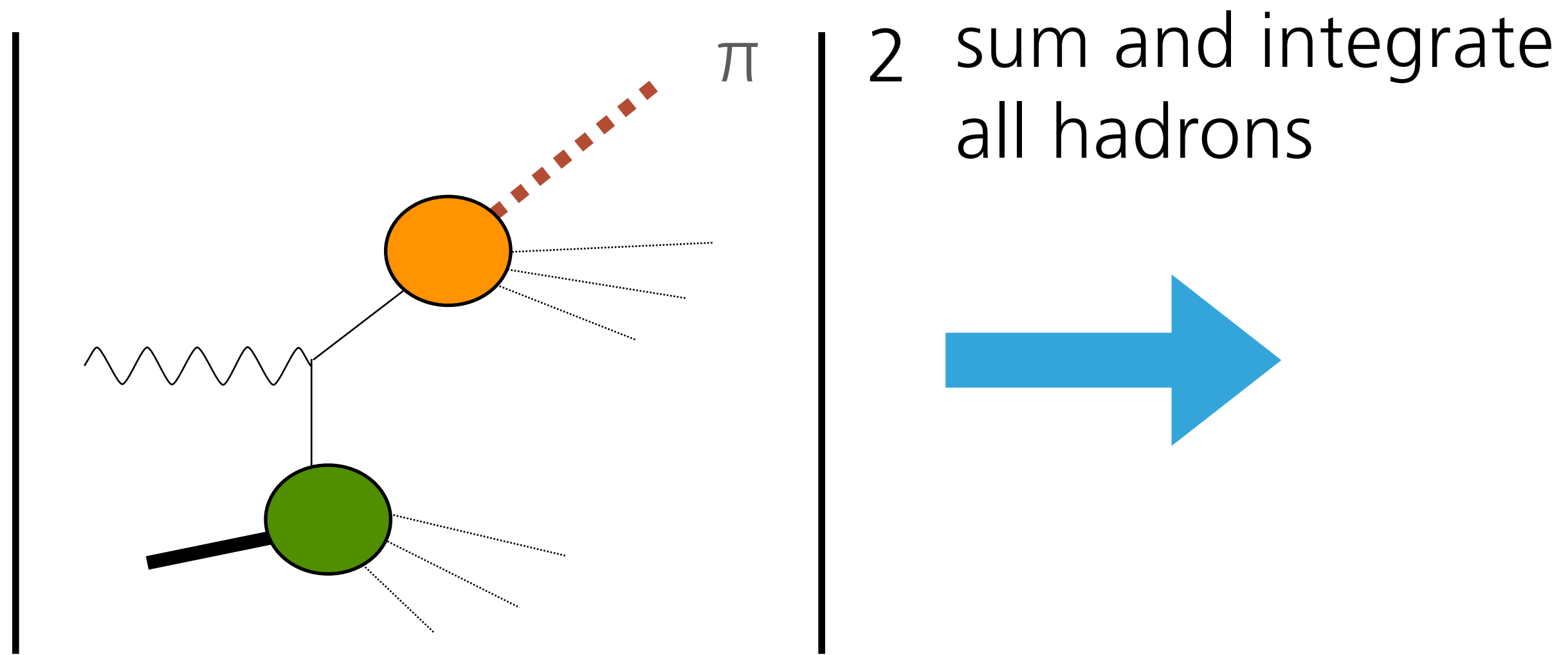
$$F_{LU}^{\sin \phi_h} = -\frac{1}{\sqrt{2}} \operatorname{Im} (F_{+0}^{++} + F_{+0}^{--}), \tag{A.3}$$

FROM SEMI-INCLUSIVE TO INCLUSIVE



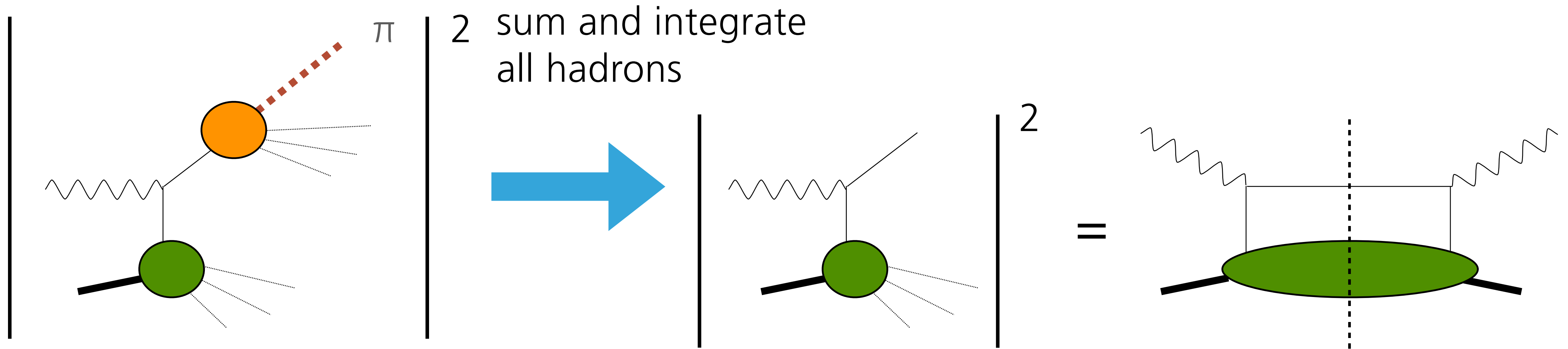
leading-order and leading-twist
contribution to SIDIS

FROM SEMI-INCLUSIVE TO INCLUSIVE



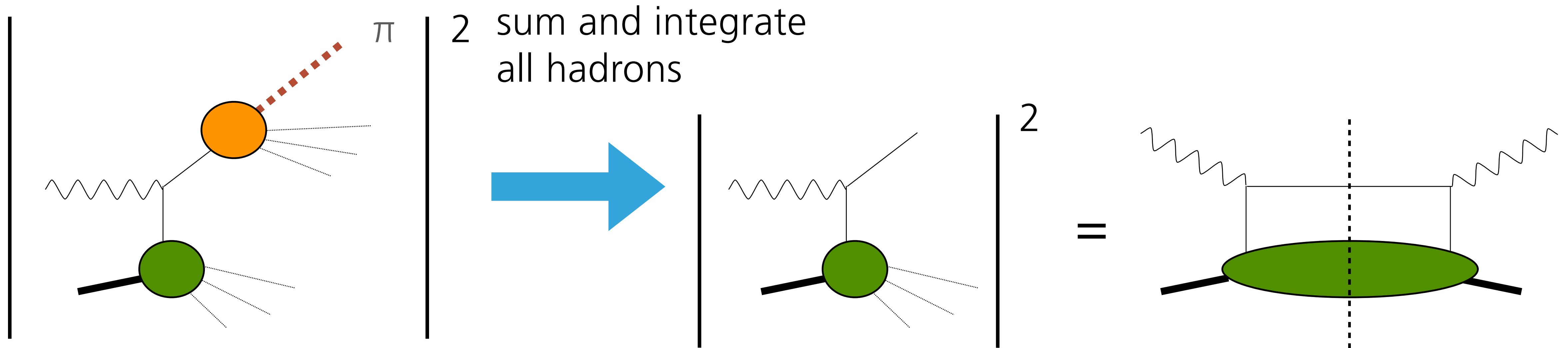
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FROM SEMI-INCLUSIVE TO INCLUSIVE



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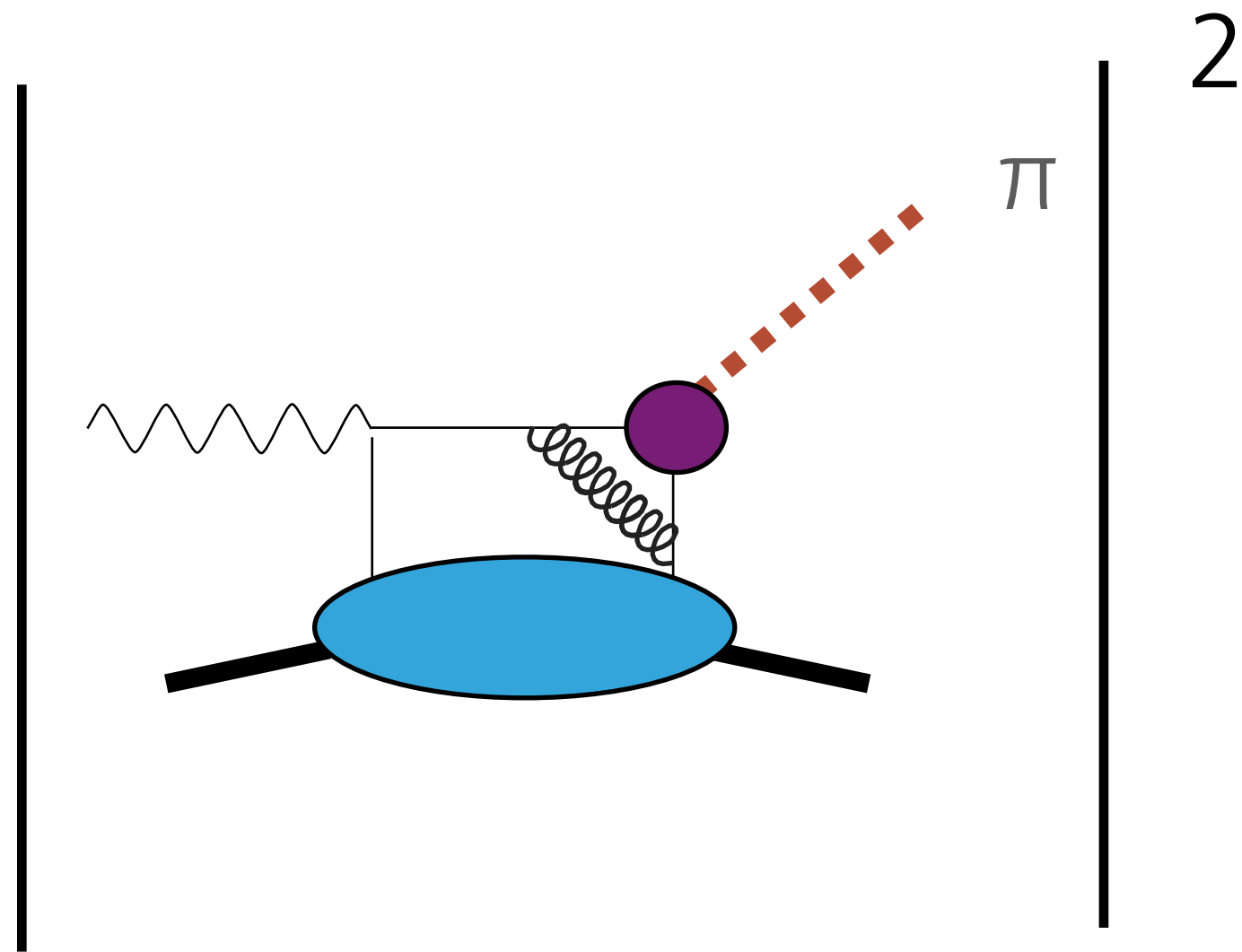
FROM SEMI-INCLUSIVE TO INCLUSIVE



leading-order and leading-twist
contribution to SIDIS

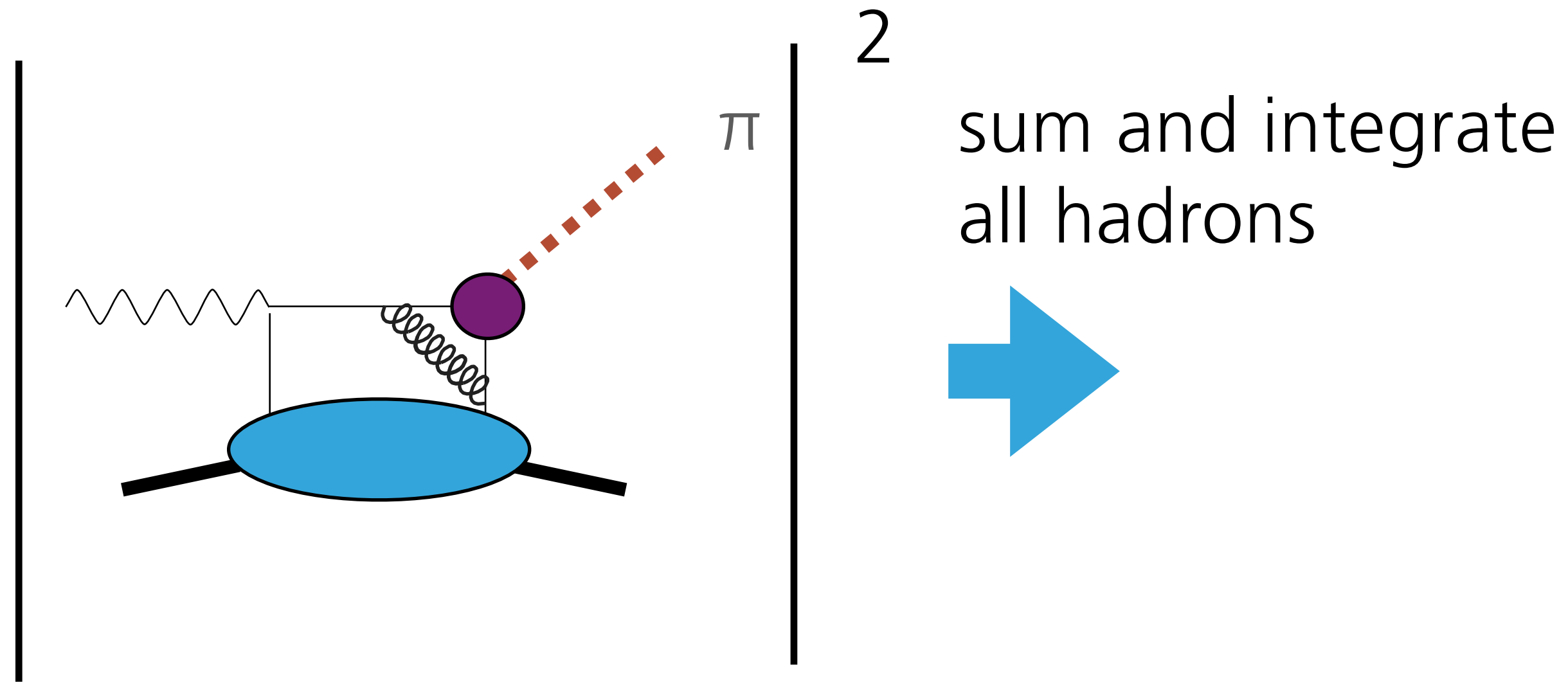
we should recover
the leading-order, leading-twist part of
INCLUSIVE DIS

EXCLUSIVE “CONTAMINATIONS”



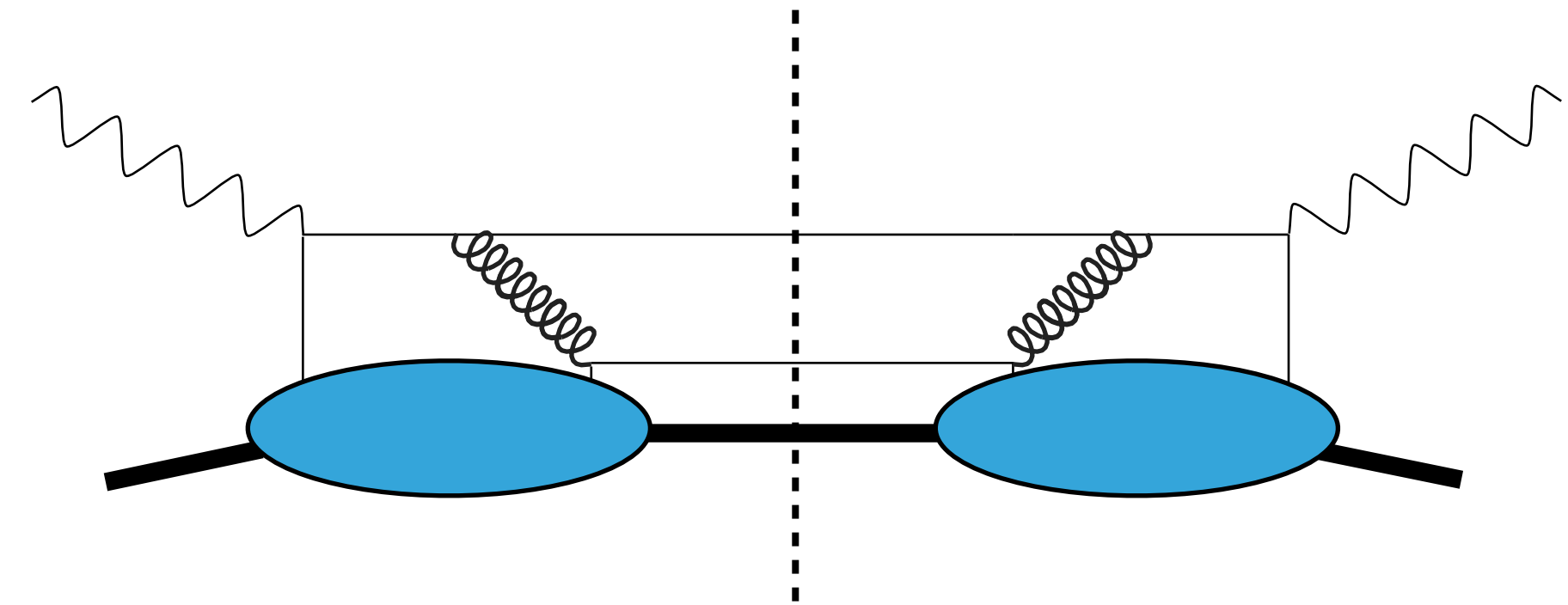
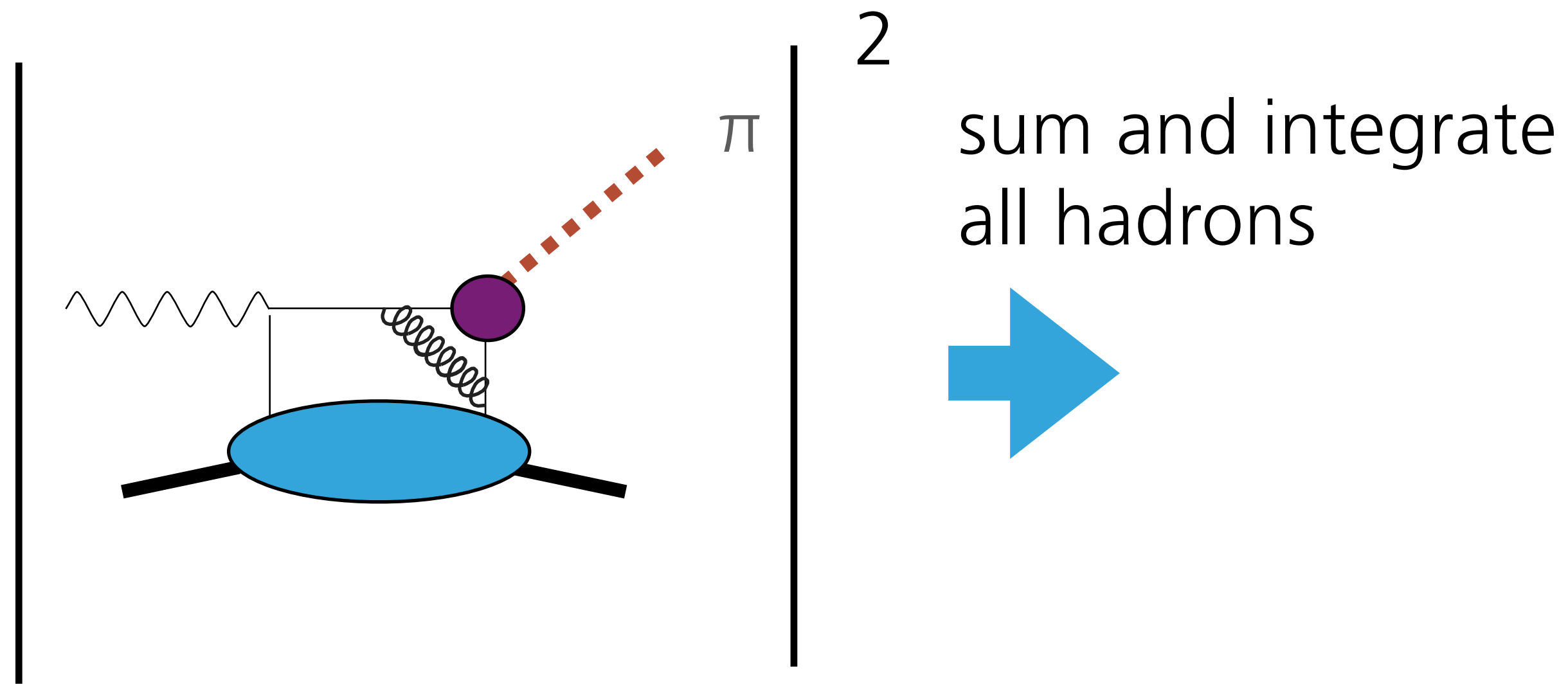
leading-order and leading-twist
contribution to DVMP

EXCLUSIVE “CONTAMINATIONS”



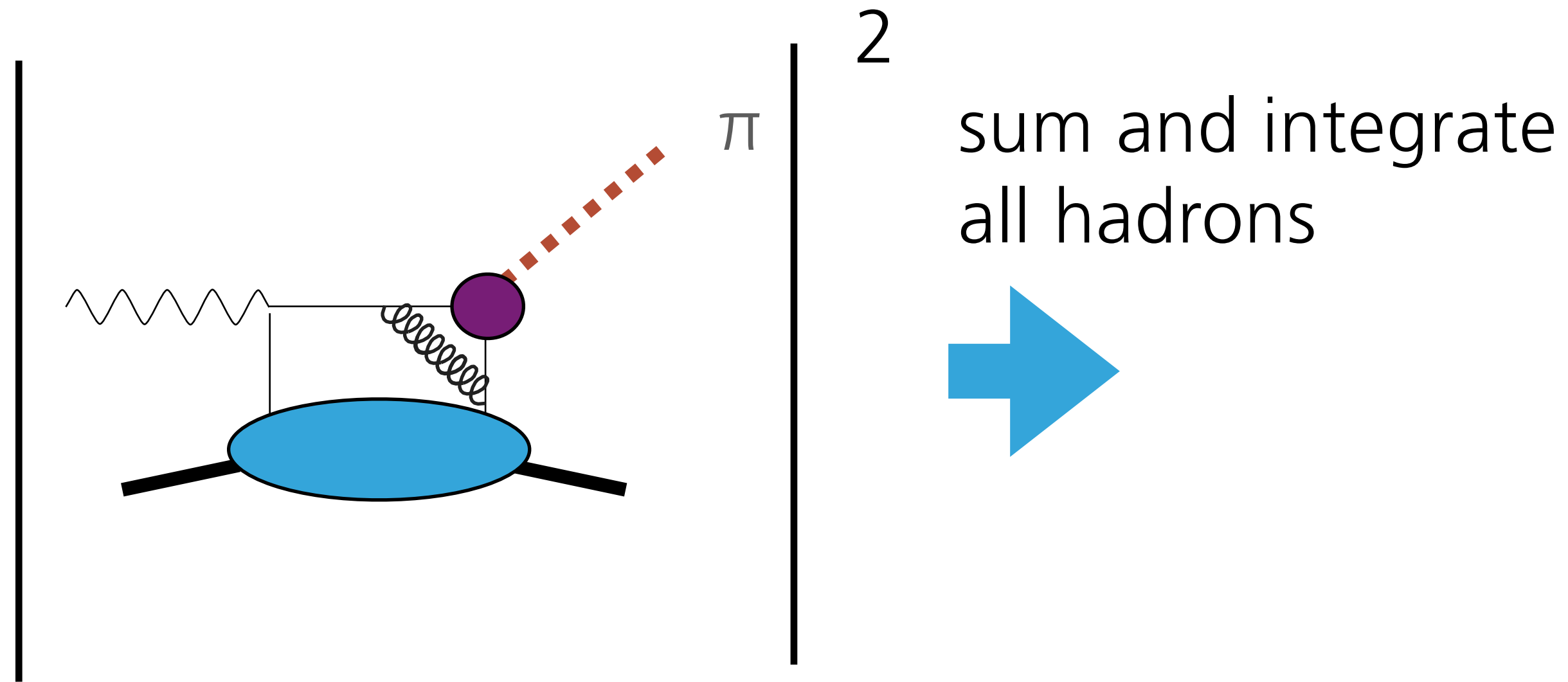
leading-order and leading-twist
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EXCLUSIVE “CONTAMINATIONS”

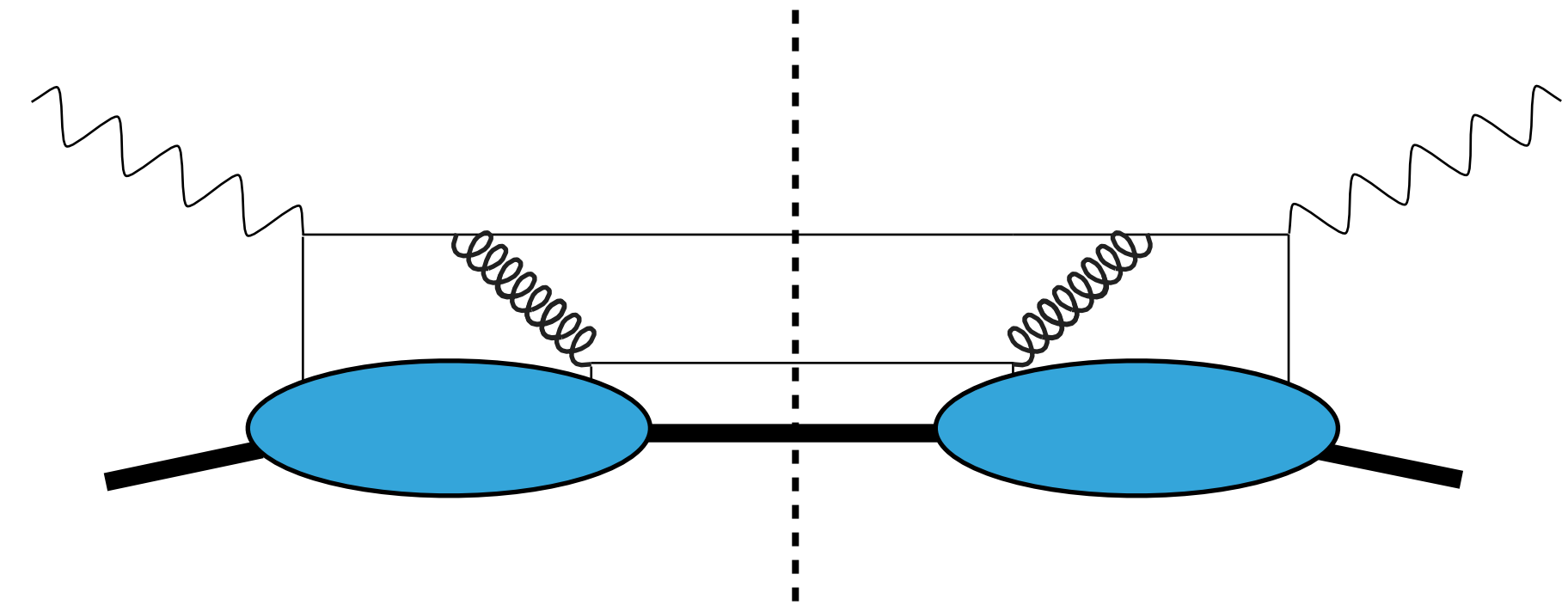


leading-order and leading-twist
contribution to DVMP

EXCLUSIVE “CONTAMINATIONS”

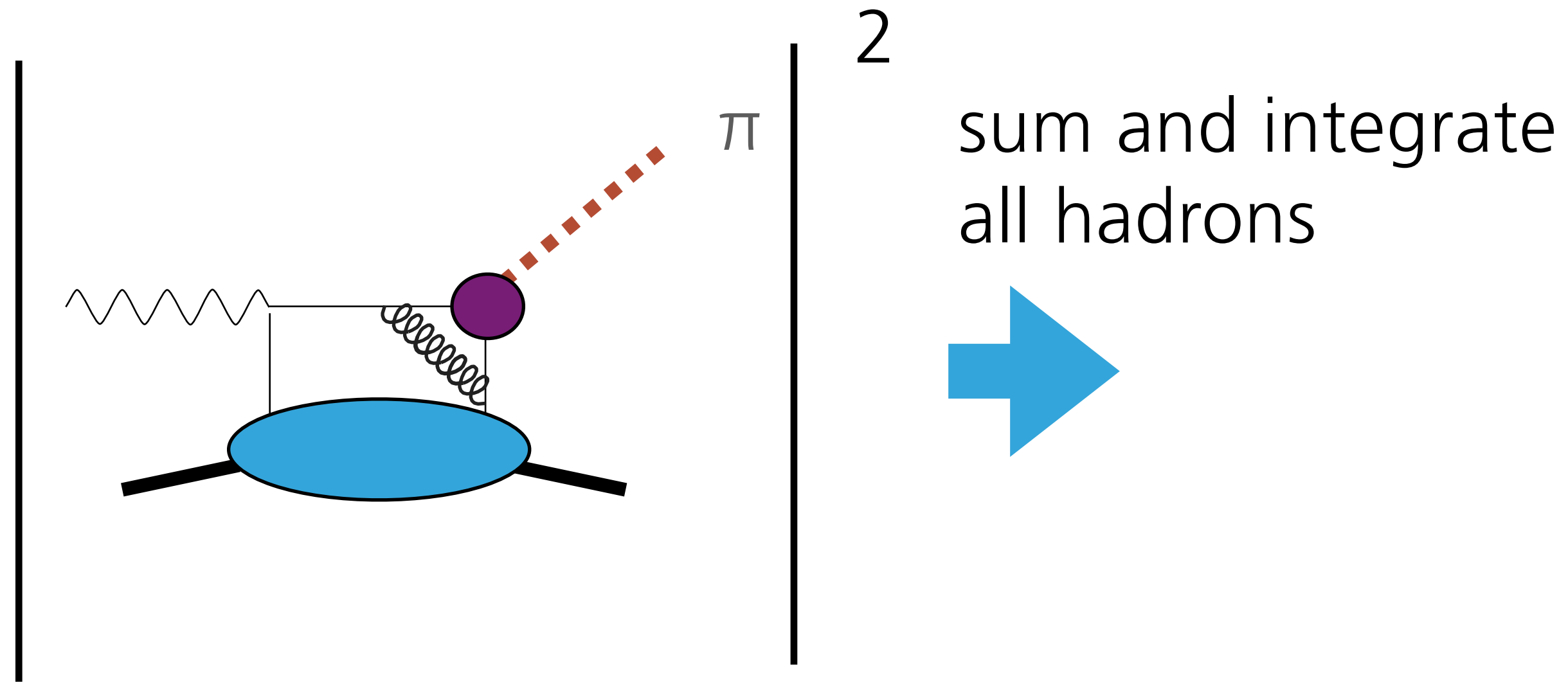


leading-order and leading-twist
contribution to DVMP

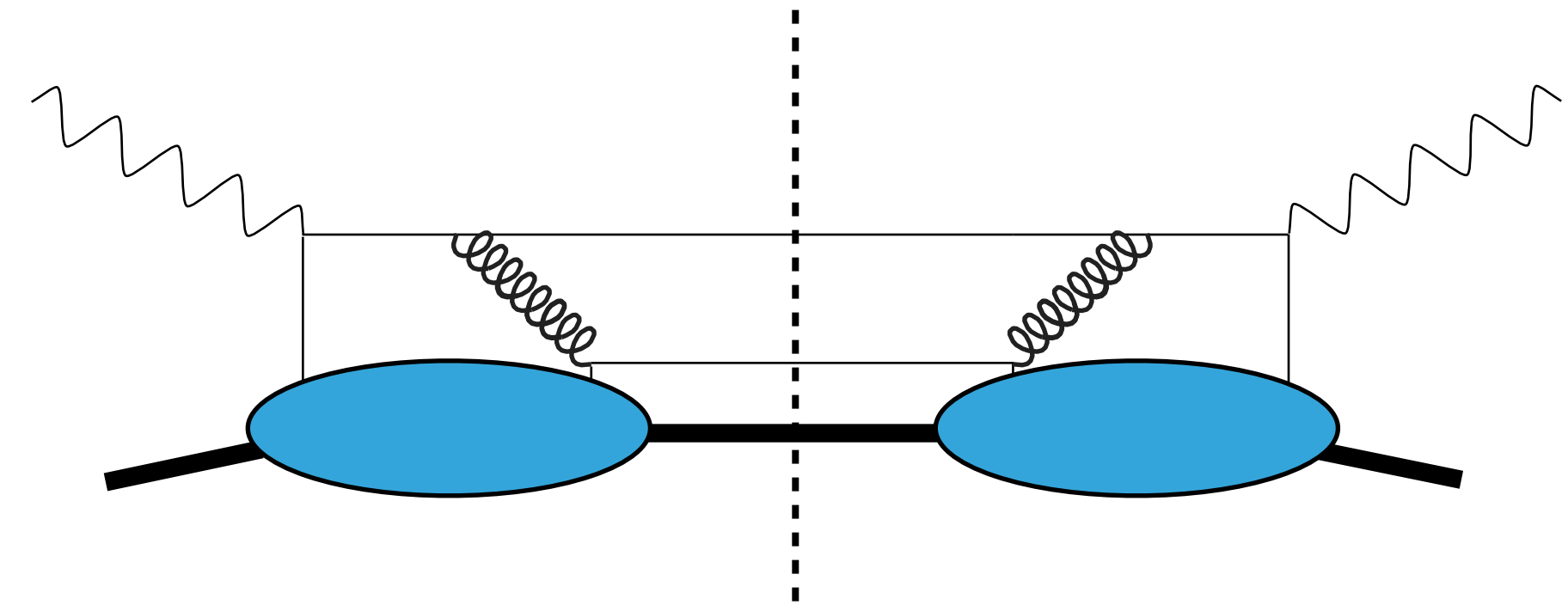


This is a suppressed contribution to
INCLUSIVE DIS

EXCLUSIVE “CONTAMINATIONS”



leading-order and leading-twist
contribution to DVMP



This is a suppressed contribution to
INCLUSIVE DIS

There are other contributions,
most of them not from exclusive processes

CONTRIBUTIONS TO INCLUSIVE DIS

$$F_{UU,T} = x f_1 + \alpha_s \left[C_T \otimes f_1 \right]$$

$$F_{UU,L} = \alpha_s \left[C_L \otimes f_1 \right]$$

CONTRIBUTIONS TO INCLUSIVE DIS

$$F_{UU,T} = x f_1 + \alpha_s \left[C_T \otimes f_1 \right]$$

$$F_{UU,T}^{\text{excl}} = \mathcal{O}\left(\frac{M^6}{Q^6}\right)$$

$$F_{UU,L} = \alpha_s \left[C_L \otimes f_1 \right]$$

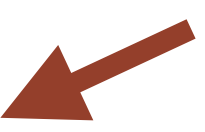
$$F_{UU,L}^{\text{excl}} \propto \frac{\alpha_s}{Q^4} \left[C \otimes \tilde{H} \right]$$

CONTRIBUTIONS TO INCLUSIVE DIS

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GPDs

CONTRIBUTIONS TO INCLUSIVE DIS

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$$F_{UU,L}^{\text{excl}} \propto \frac{\alpha_s}{Q^4} \left[C \otimes \tilde{H} \right] \swarrow \text{GPDs}$$

There are other contributions,
most of them not from exclusive processes

CONTRIBUTIONS TO INCLUSIVE DIS

Even if this is in principle highly suppressed,
it has been found to be large in experiments

$$F_{UU,T} = x f_1 + \alpha_s \left[C_T \otimes f_1 \right]$$

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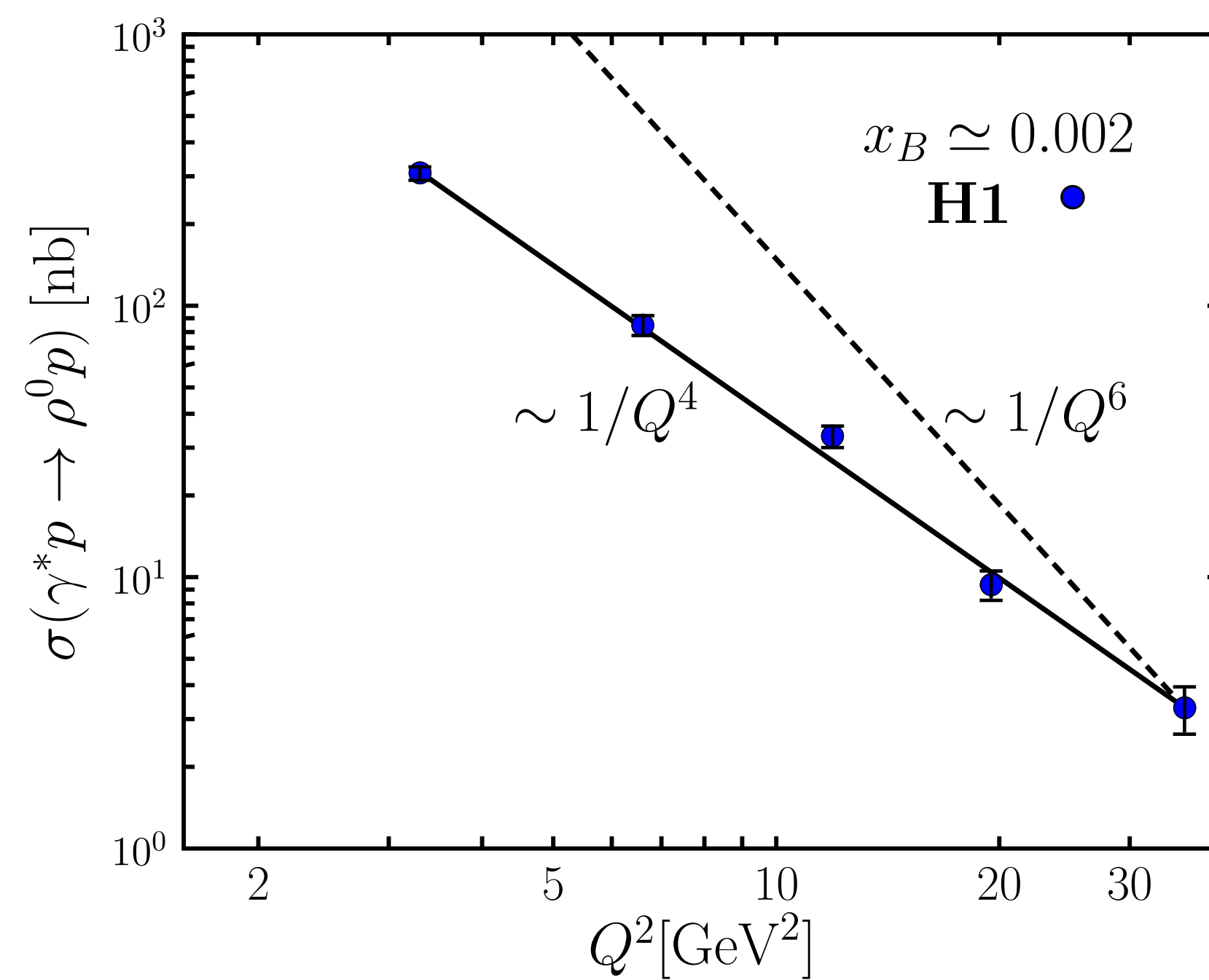
GPDs



There are other contributions,
most of them not from exclusive processes

EXPERIMENTAL MEASUREMENTS FOR EXCLUSIVE ρ^0

from L. Favart, M. Guidal, T. Horn, P. Kroll, [arXiv:511.04535](https://arxiv.org/abs/511.04535)



SIDIS STRUCTURE FUNCTIONS IN DIFFERENT REGIMES

(schematically only)

Bacchetta, Boer, Diehl, Mulders, [arXiv:0803.0227](#)

Low transverse momentum

High transverse momentum

Integrated

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High transverse momentum

Integrated

$$F_{UU,T} = \mathcal{C} \left[f_1 D_1 \right]$$

Twist 2

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Twist 2

High transverse momentum

$$F_{UU,T} = \frac{\alpha_s}{P_{h\perp}^2} \left[f_1 \otimes D_1 \right]_A$$

Twist 2

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Twist 2

Integrated

$$F_{UU,T} = x f_1 D_1 + \alpha_s \left[f_1 \otimes D_1 \right]_C$$

Twist 2

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Twist 2

Integrated

$$F_{UU,T} = x f_1 D_1 + \alpha_s \left[f_1 \otimes D_1 \right]_C$$

Twist 2

$$F_{UU,L} = \mathcal{O} \left(\frac{M^2}{Q^2}, \frac{P_{h\perp}^2}{Q^2} \right)$$

Twist 4

see talk by L. Gamberg

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Twist 4

see talk by L. Gamberg

High transverse momentum

$$F_{UU,T} = \frac{\alpha_s}{P_{h\perp}^2} \left[f_1 \otimes D_1 \right]_A$$

Twist 2

$$\begin{aligned} F_{UU,L} &= \frac{\alpha_s}{P_{h\perp}^2} \frac{P_{h\perp}^2}{Q^2} \left[f_1 \otimes D_1 \right]_B \\ &= 2 F_{UU}^{\cos 2\phi_h} \end{aligned}$$

Twist 2

Integrated

$$F_{UU,T} = x f_1 D_1 + \alpha_s \left[f_1 \otimes D_1 \right]_C$$

Twist 2

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Twist 4

see talk by L. Gamberg

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$$= 2 F_{UU}^{\cos 2\phi_h}$$

Twist 2

$$F_{UU,L} = \alpha_s \left[f_1 \otimes D_1 \right]_D$$

Twist 2

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(schematically only)

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Twist 2

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Twist 2

Integrated

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Twist 2

$$F_{UU,L} = \mathcal{O} \left(\frac{M^2}{Q^2}, \frac{P_{h\perp}^2}{Q^2} \right)$$

Twist 4

see talk by L. Gamberg

$$\begin{aligned} F_{UU,L} &= \frac{\alpha_s}{P_{h\perp}^2} \frac{P_{h\perp}^2}{Q^2} \left[f_1 \otimes D_1 \right]_B \\ &= 2 F_{UU}^{\cos 2\phi_h} \end{aligned}$$

Twist 2

$$F_{UU,L} = \alpha_s \left[f_1 \otimes D_1 \right]_D$$

Twist 2

All of them can contain corrections

$$\mathcal{O} \left(\frac{M^2}{Q^2}, \frac{P_{h\perp}^2}{Q^2} \right)$$

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see talk by L. Gamberg

$$\begin{aligned} F_{UU,L} &= \frac{\alpha_s}{P_{h\perp}^2} \frac{P_{h\perp}^2}{Q^2} \left[f_1 \otimes D_1 \right]_B \\ &= 2 F_{UU}^{\cos 2\phi_h} \end{aligned}$$

Twist 2

$$F_{UU,L} = \alpha_s \left[f_1 \otimes D_1 \right]_D$$

Twist 2

All of them can contain corrections

$$\mathcal{O} \left(\frac{M^2}{Q^2}, \frac{P_{h\perp}^2}{Q^2} \right)$$

Nice property of $F_{UU,L}$:
at low transverse momentum
is a clean twist-4 signal

LOW AND HIGH P_T

low
 $P_{h\perp}$

high
 $P_{h\perp}$

	observable	twist	twist
<i>“SIDIS F_T”</i>	$F_{UU,T}$	2	2
<i>“SIDIS F_L”</i>	$F_{UU,L}$	4	2
<i>“Cahn” - $f^\perp$</i>	$F_{UU}^{\cos \phi_h}$	3	2
<i>“Boer-Mulders”</i>	$F_{UU}^{\cos 2\phi_h}$	2	2
<i>$e, g^\perp$ and friends</i>	$F_{LU}^{\sin \phi_h}$	3	2

LOW AND HIGH P_T

low
 $P_{h\perp}$ high
 $P_{h\perp}$

There are several possibilities:

	observable	twist	twist
<i>“SIDIS F_T”</i>	$F_{UU,T}$	2	2
<i>“SIDIS F_L”</i>	$F_{UU,L}$	4	2
<i>“Cahn” - $f^\perp$</i>	$F_{UU}^{\cos \phi_h}$	3	2
<i>“Boer-Mulders”</i>	$F_{UU}^{\cos 2\phi_h}$	2	2
<i>$e, g^\perp$ and friends</i>	$F_{LU}^{\sin \phi_h}$	3	2

LOW AND HIGH P_T

	low $P_{h\perp}$	high $P_{h\perp}$
observable	twist	twist
<i>“SIDIS F_T”</i> $F_{UU,T}$	2	2
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<i>“Boer-Mulders”</i> $F_{UU}^{\cos 2\phi_h}$	2	2
<i>$e, g^\perp$ and friends</i> $F_{LU}^{\sin \phi_h}$	3	2

There are several possibilities:

- Twist 2 TMD matching twist 2 PDF

LOW AND HIGH P_T

low
 $P_{h\perp}$ high
 $P_{h\perp}$

There are several possibilities:

- Twist 2 TMD matching twist 2 PDF
- Twist 3 TMD matching twist 2 PDF

“SIDIS F_T ”
“SIDIS F_L ”
“Cahn” - $f^\perp$
“Boer-Mulders”
 $e, g^\perp$ and friends

observable	twist	twist
$F_{UU,T}$	2	2
$F_{UU,L}$	4	2
$F_{UU}^{\cos \phi_h}$	3	2
$F_{UU}^{\cos 2\phi_h}$	2	2
$F_{LU}^{\sin \phi_h}$	3	2

LOW AND HIGH P_T

	low $P_{h\perp}$	high $P_{h\perp}$
observable	twist	twist
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<i>“SIDIS F_L”</i> $F_{UU,L}$	4	2
<i>“Cahn” - $f^\perp$</i> $F_{UU}^{\cos \phi_h}$	3	2
<i>“Boer-Mulders”</i> $F_{UU}^{\cos 2\phi_h}$	2	2
<i>$e, g^\perp$ and friends</i> $F_{LU}^{\sin \phi_h}$	3	2

There are several possibilities:

- Twist 2 TMD matching twist 2 PDF
- Twist 3 TMD matching twist 2 PDF
- Twist 2 TMD matching twist 3 PDF

LOW AND HIGH P_T

		low $P_{h\perp}$	high $P_{h\perp}$
	observable	twist	twist
<i>“SIDIS F_T”</i>	$F_{UU,T}$	2	2
<i>“SIDIS F_L”</i>	$F_{UU,L}$	4	2
<i>“Cahn” - $f^\perp$</i>	$F_{UU}^{\cos \phi_h}$	3	2
<i>“Boer-Mulders”</i>	$F_{UU}^{\cos 2\phi_h}$	2	2
<i>$e, g^\perp$ and friends</i>	$F_{LU}^{\sin \phi_h}$	3	2

There are several possibilities:

- Twist 2 TMD matching twist 2 PDF
- Twist 3 TMD matching twist 2 PDF
- Twist 2 TMD matching twist 3 PDF
- Expected mismatch

LOW AND HIGH P_T

	low $P_{h\perp}$	high $P_{h\perp}$
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- Twist 2 TMD matching twist 2 PDF
- Twist 3 TMD matching twist 2 PDF
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- Twist 4 TMD matching twist 2 PDF?

POSSIBLE TERMS AT LOW TRANSVERSE MOMENTUM

see, e.g., Wei, Song, Chen, Liang, [arxiv:1611.08688](https://arxiv.org/abs/1611.08688)

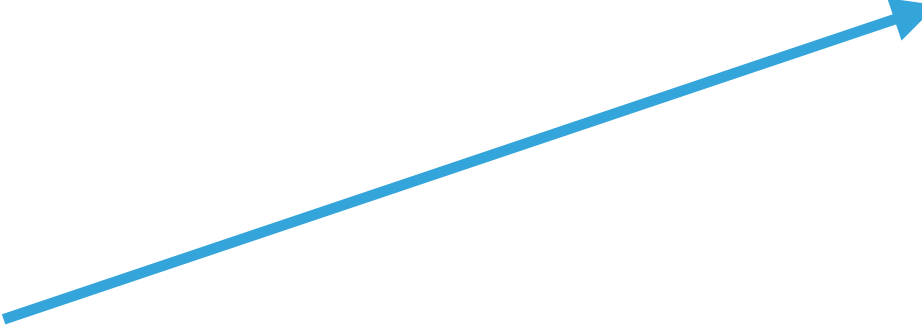
$$F_{UU,L} = \frac{M^2}{Q^2} \mathcal{C} \left[\frac{4k_T^2}{M^2} f_1 D_1 + \frac{m^2}{M^2} f_1 D_1 + \tilde{f}_2 D_1 + f_1 \tilde{D}_2 + \dots \right]$$

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kinematic twist 4
(à la Wandzura-Wilczek)



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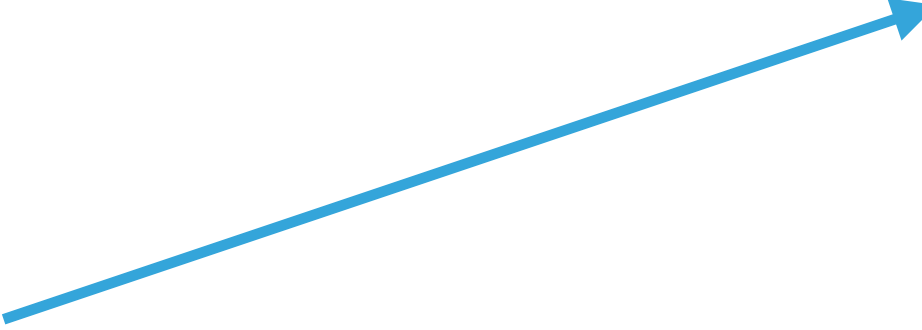
mass corrections

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mass corrections



dynamic twist 4



sometimes denoted with

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mass corrections

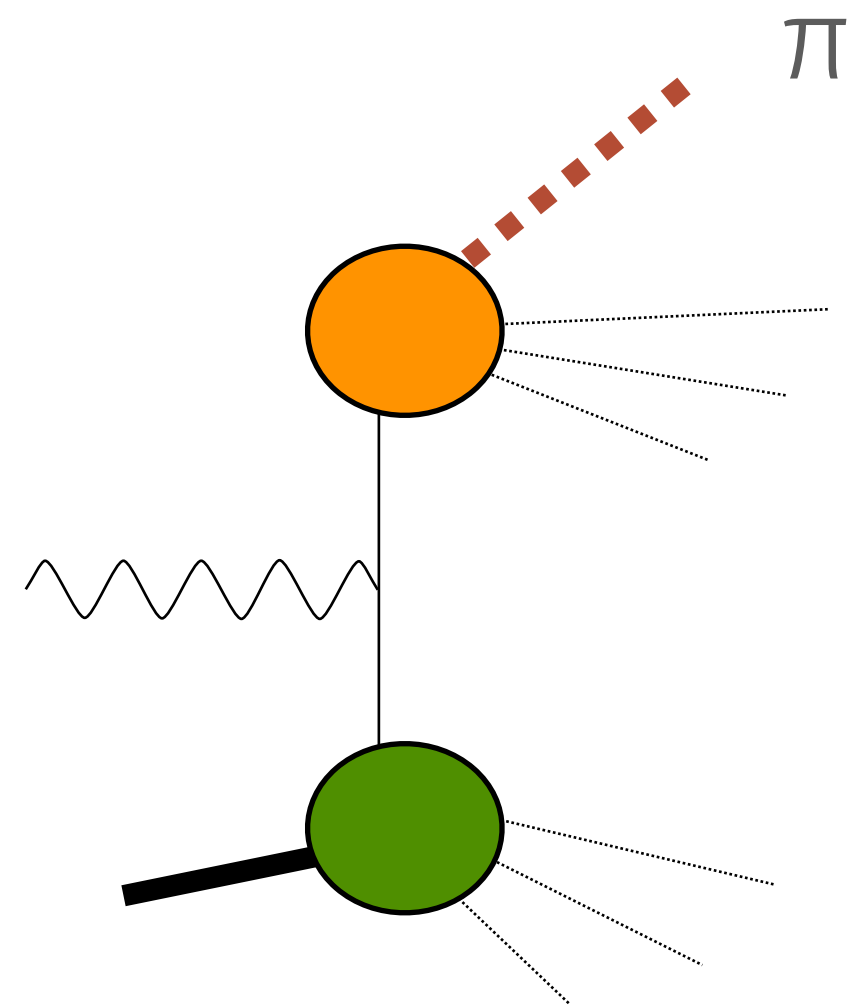
dynamic twist 4

factorization breaking terms?

sometimes denoted with

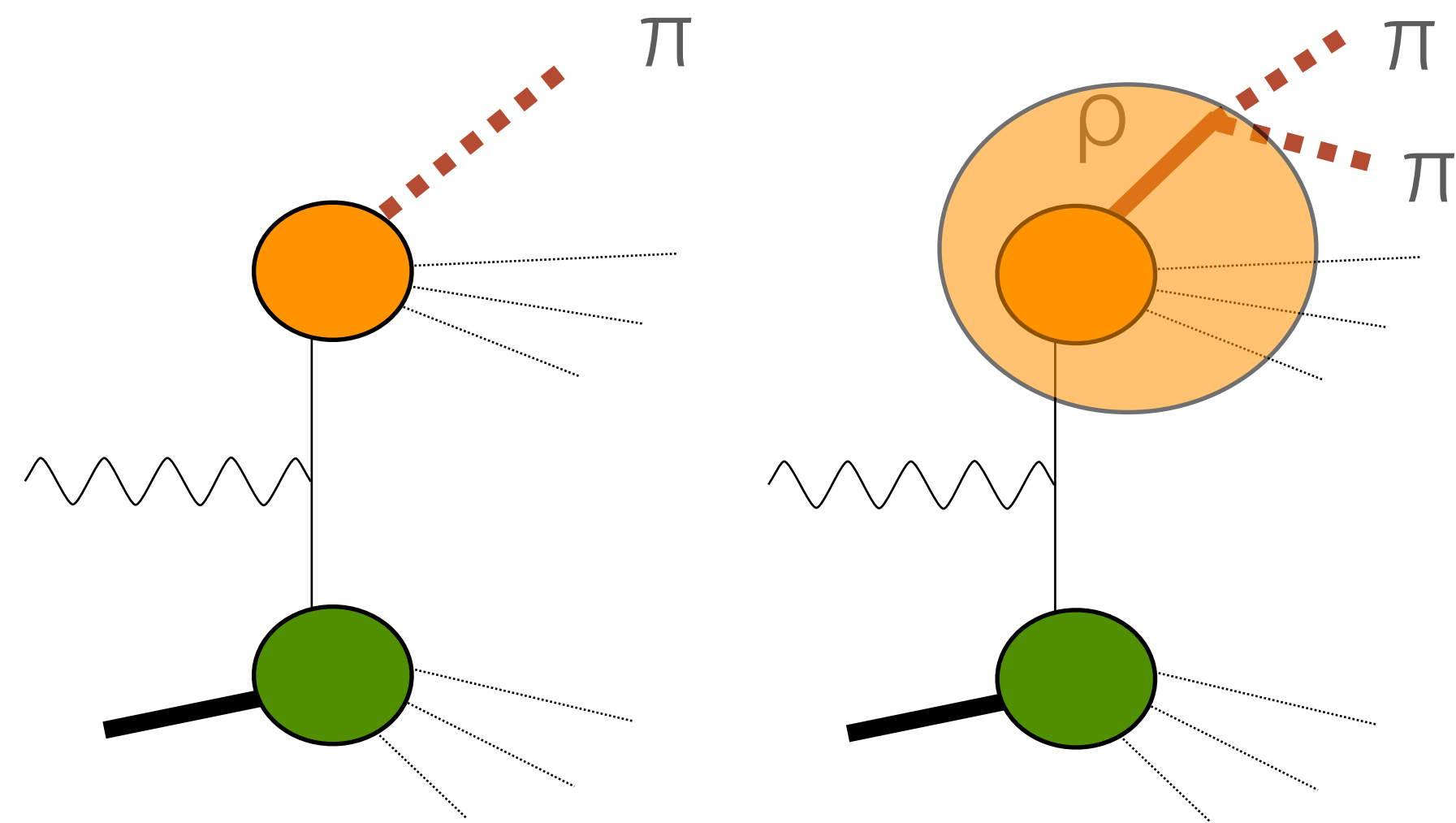
f_3

EXCLUSIVE “CONTAMINATIONS” IN SEMI-INCLUSIVE DIS



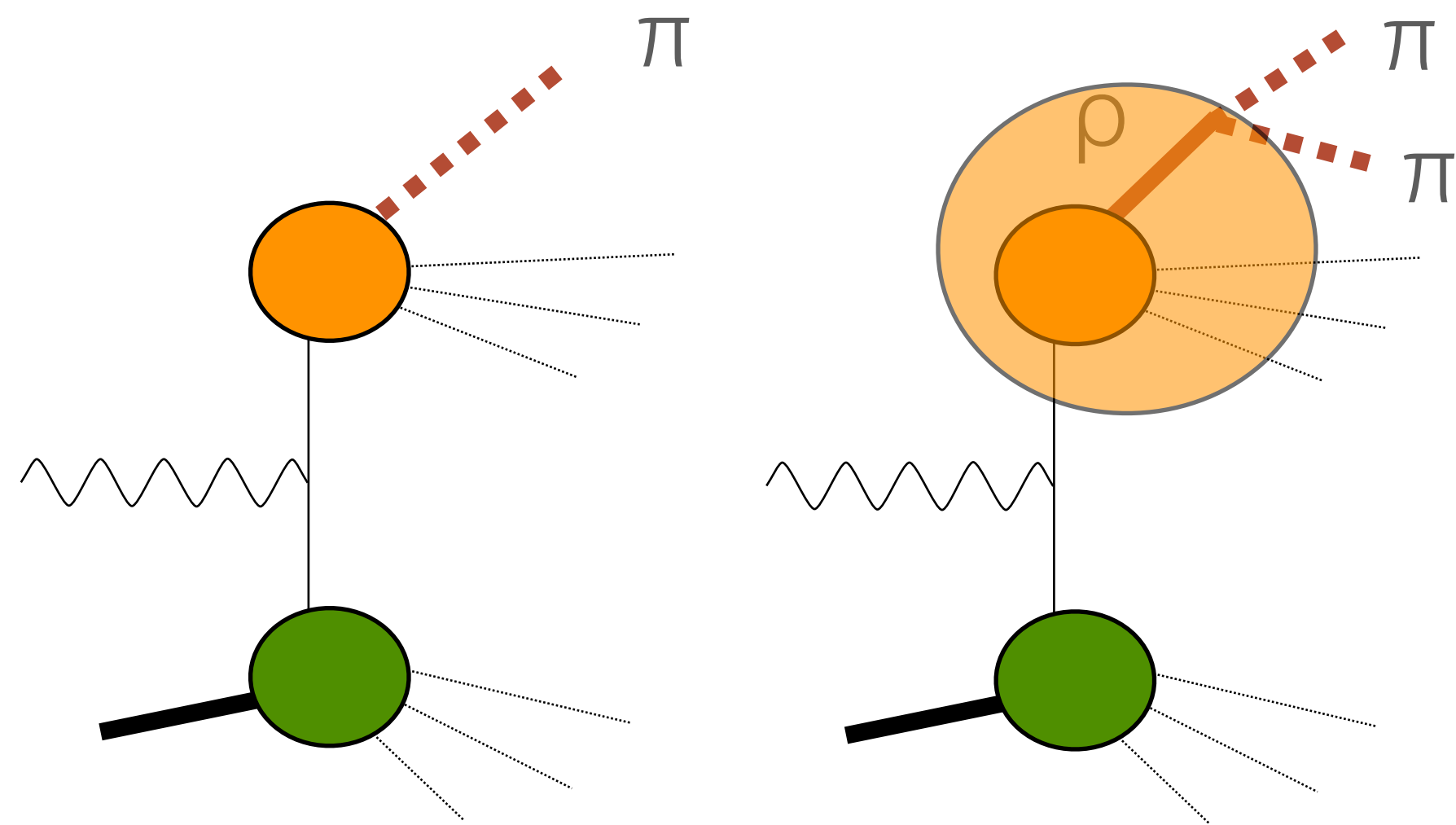
Semi-inclusive

EXCLUSIVE “CONTAMINATIONS” IN SEMI-INCLUSIVE DIS

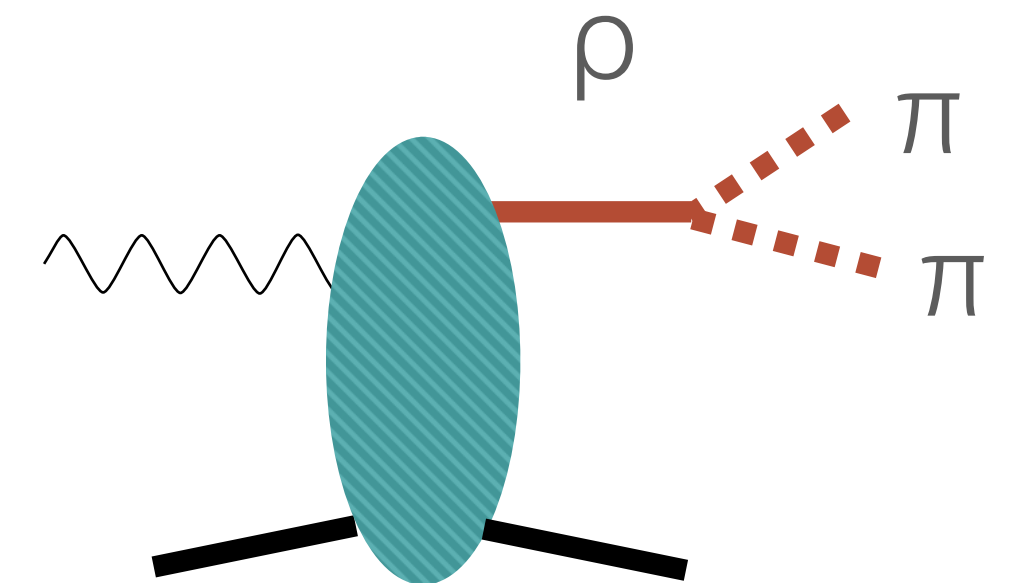
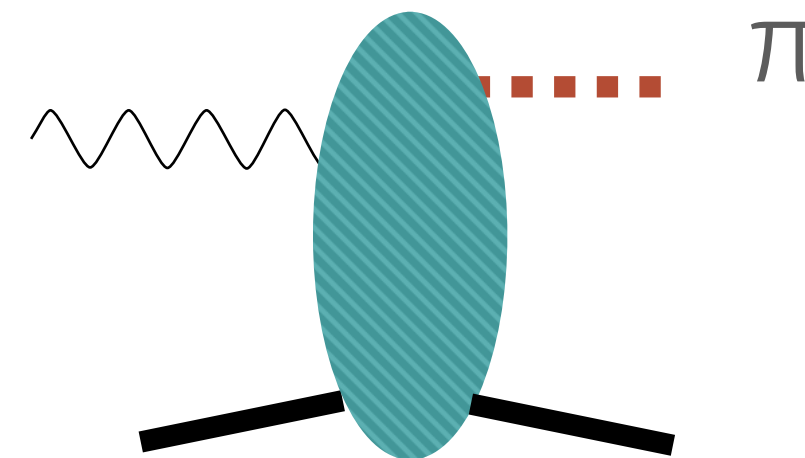


Semi-inclusive

EXCLUSIVE “CONTAMINATIONS” IN SEMI-INCLUSIVE DIS



Semi-inclusive



Exclusive

EXCLUSIVE “CONTAMINATIONS” IN SEMI-INCLUSIVE DIS

*see, e.g., Diehl, Sapeta, [hep-ph/0503023](#)
Diehl, Kugler, Schäfer, Weiss, [hep-ph/0506171](#)*

Integrated over transverse momentum

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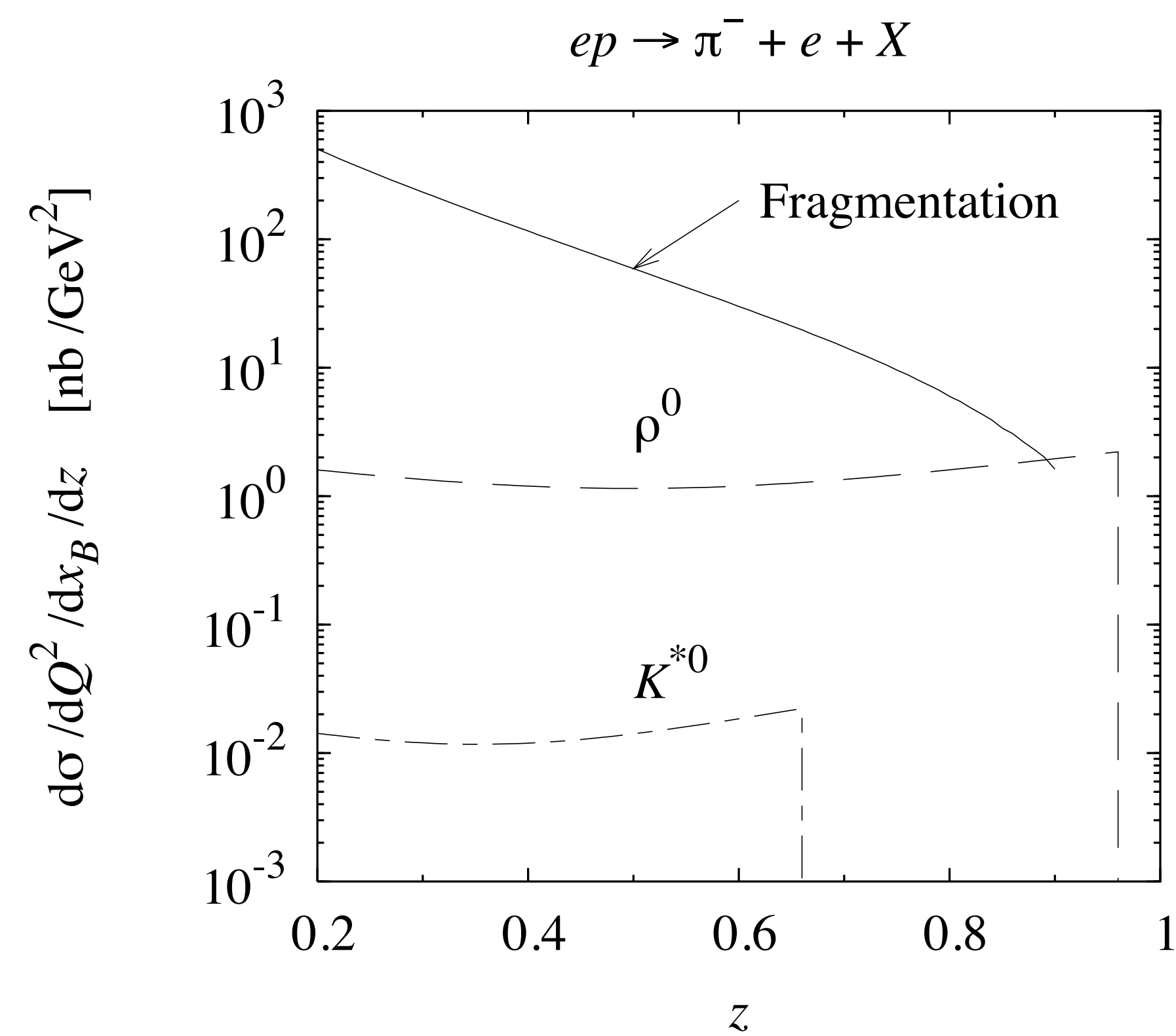
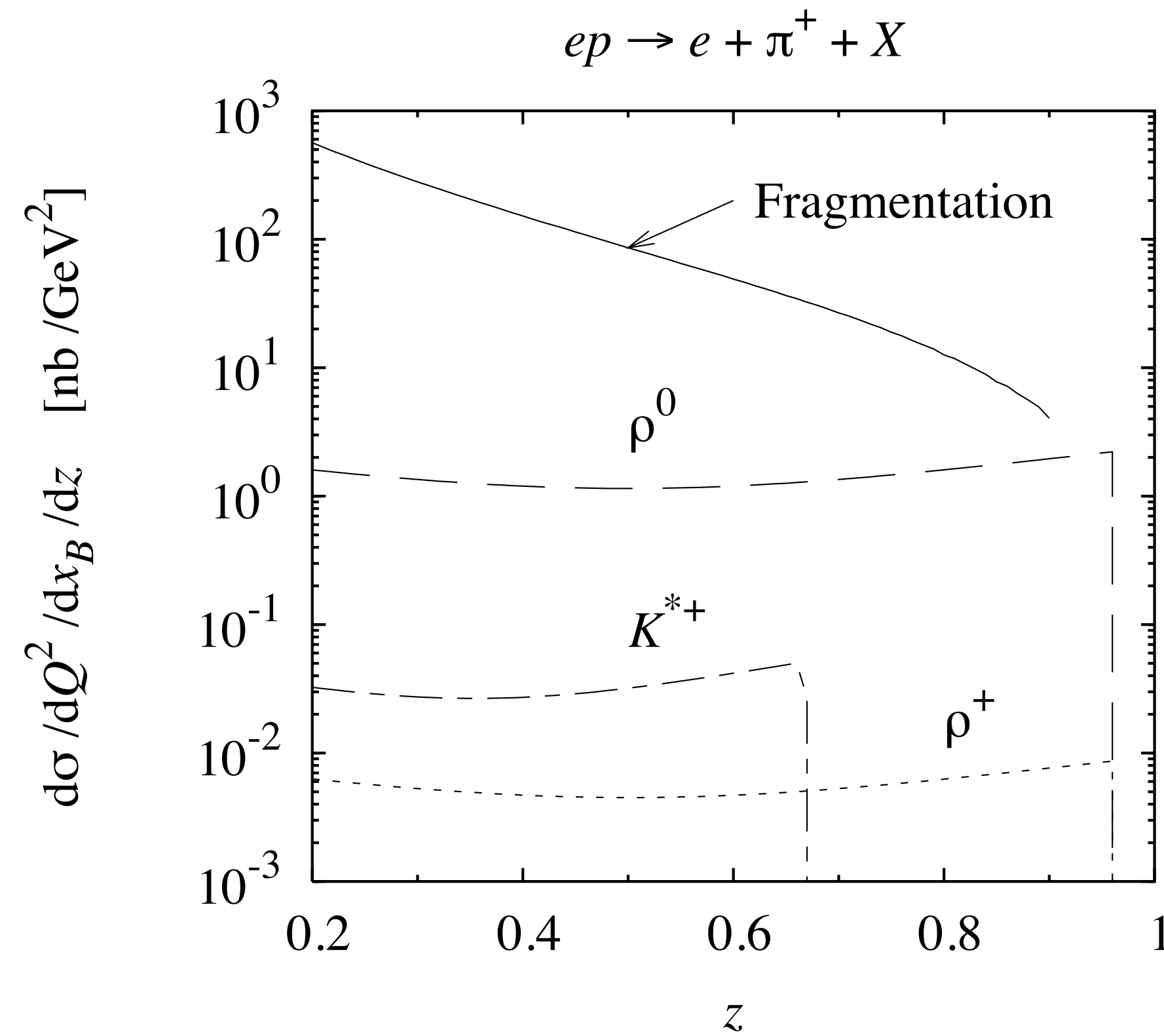
$$F_{UU,L}^{\text{excl}} \propto \frac{\alpha_s}{Q^4} \left[\phi_\pi \otimes \tilde{H} + D_{\rho \rightarrow \pi^+ \pi^-} \phi_\rho \otimes H \right]$$

GPDs

meson distribution amplitudes

EXAMPLE OF THEORY ESTIMATES

Diehl, Kugler, Schäfer, Weiss, [hep-ph/0506171](https://arxiv.org/abs/hep-ph/0506171)



I am not aware of similar calculations for $P_{h\perp}$ dependence

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