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Surprising Phase Structures for Wilson and Twisted Mass Fermions

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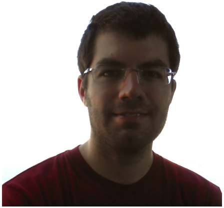
My Collaborators



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Outline

- Chiral Lagrangian in the Microscopic Limit
- Phase Diagram of Wilson Fermions without twist
- Phase Diagram of Wilson Fermions at maximal twist
- Summary and Outlook

Chiral Lagrangian

in the Microscopic Limit

Kenneth G. Wilson

The setting: Wilson Dirac Operator

$$D_{\mathrm{W}} = \gamma^{\alpha} D_{\alpha} + a D_{\alpha} D^{\alpha} + m \cos\omega + \imath m \sin\omega \gamma_5 \tau_3 + \mu \gamma_0 \tau_3$$

- Dirac matrices: γ_{α}
- two flavours $\rightarrow$ Pauli matrices: τ_i
- covariant derivative: D_{α}
- degenerate quark mass: m
- isospin chemical potential: μ
- lattice spacing: a
- twisting angle: $\omega \in [0, \pi/2]$ (Frezzotti, Rossi (2000))

$N_{\mathrm{f}} = 2$, $T = \mu_{\mathrm{B}} = 0$, and $\mu_{\mathrm{I}}, a \neq 0$

The symmetries: Wilson Dirac Operator

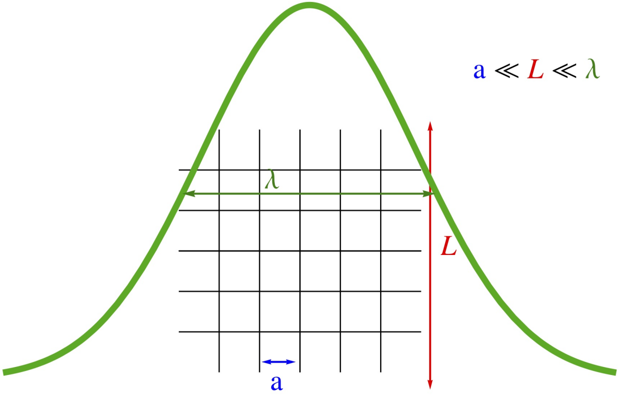
$$D_{\mathrm{W}} = \gamma^{\alpha} D_{\alpha} + a D_{\alpha} D^{\alpha} + m \cos \omega + \imath m \sin \omega \gamma_5 \tau_3 + \mu \gamma_0 \tau_3$$

- μ real: $D_{\mathrm{W}}^{\dagger} = \gamma_5 \tau_1 D_{\mathrm{W}}^{\dagger} \gamma_5 \tau_1 = \gamma_5 \tau_2 D_{\mathrm{W}}^{\dagger} \gamma_5 \tau_2 \to D_{5;1/2} = \gamma_5 \tau_{1/2} D_{\mathrm{W}}$
- μ imaginary and $\omega = 0$: $D_{\mathrm{W}}^{\dagger} = \gamma_5 D_{\mathrm{W}}^{\dagger} \gamma_5 \to D_5 = \gamma_5 D_{\mathrm{W}}$

Microscopic Limit

- infrared limit of QCD
- large Compton wavelength of Goldstone bosons $\gg$ box size $V^{1/4} = L$
- lattice volume (space-time volume) $V \to \infty$
- lattice spacing $a^2 V$, quark masses mV, momenta $p^4 V$, and chemical potential $\mu^2 V$ fixed

! slightly different counting scheme in contrast to Aoki et al. (2003/11)



Chiral Lagrangian (two flavours)

$$\mathcal{L}_\chi = \frac{\Sigma m V}{2}\left[\cos\omega\,\mathrm{tr}\,(U+U^{-1}) + \imath\sin\omega\,\mathrm{tr}\,\tau_3(U-U^{-1})\right]$$

$$-F^2\mu^2 V\,\mathrm{tr}\,U\tau_3U^{-1}\tau_3 - \overbrace{(W_6+W_8/2)}^{C_2/16} a^2 V\,\mathrm{tr}^2(U+U^{-1})$$

with $U \in \mathrm{SU}(2)$

- part of continuum QCD: $\widehat{m} = \Sigma m V$
- part of isospin chemical potential: $\widehat{\mu}^2 = F^2\mu^2 V$
- part of discretization: $\widehat{a}^2 = (W_6+W_8/2)a^2 V = C_2 a^2/16 V$
- low energy constants (to be fixed): $\Sigma,\ F,\ W_{6/8}$

- $\mu = \omega = 0$: Sharpe, Singleton (1998); Bär, Rupak, Shoresh (2004); Sharpe (2006); Bär, Necco, Schaefer (2009);
- $\mu = 0$: Sharpe, Wu (2004) (Note a slightly different counting scheme)

Thermodynamic limit

- $\widehat{m}, \widehat{\mu}^2, \widehat{a}^2 \to \infty$ (all parameters of the same order)
- for $\widehat{\mu} = 0$: Sharpe, Wu (2004)

$\Rightarrow$ saddlepoint approximation

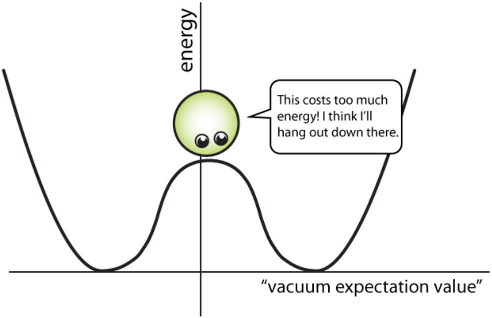


image from www.quantumdiaries.org

Parametrization

$$\mathcal{L}_\chi = \frac{\widehat{m}}{2}\left[\cos\omega\,\mathrm{tr}\,(U+U^{-1}) + \imath\sin\omega\,\mathrm{tr}\,\tau_3(U-U^{-1})\right] - \widehat{\mu}^2\mathrm{tr}\,U\tau_3U^{-1}\tau_3 - \widehat{a}^2\mathrm{tr}^2(U+U^{-1})$$

with

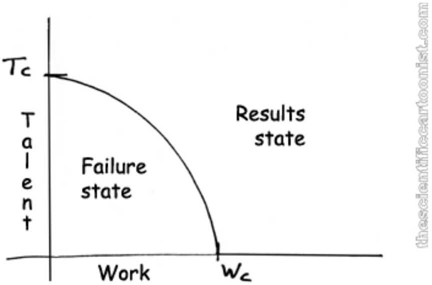
$$U = \cos\varphi\,\mathbb{1}_2 + \imath\sin\varphi\begin{bmatrix}\cos\vartheta_1 & e^{\imath\vartheta_2}\sin\vartheta_1\\ e^{-\imath\vartheta_2}\sin\vartheta_1 & -\cos\vartheta_1\end{bmatrix}$$

$\varphi, \vartheta_1 \in [0,\pi]$ and $\vartheta_2 \in [0, 2\pi]$

Good quantities:

- chiral condensate: $\Sigma \propto \langle\bar{\psi}\psi\rangle \leftrightarrow \cos\varphi$
- π_0 condensate: $\langle\bar{\psi}\gamma_5\tau_3\psi\rangle \leftrightarrow \sin\varphi\cos\vartheta_1$
- isospin charge density: $\partial\mathcal{L}/\partial\widehat{\mu} \propto \langle\bar{\psi}\gamma_0\tau_3\psi\rangle \leftrightarrow \widehat{\mu}(2\sin^2\varphi\sin^2\vartheta_1 - 1)$
- one can also consider $\pi^\pm$ condensate: $\langle\bar{\psi}\gamma_5\tau_{1/2}\psi\rangle \leftrightarrow \sin\varphi\sin\vartheta_1(e^{\imath\vartheta_2} \pm e^{-\imath\vartheta_2})$

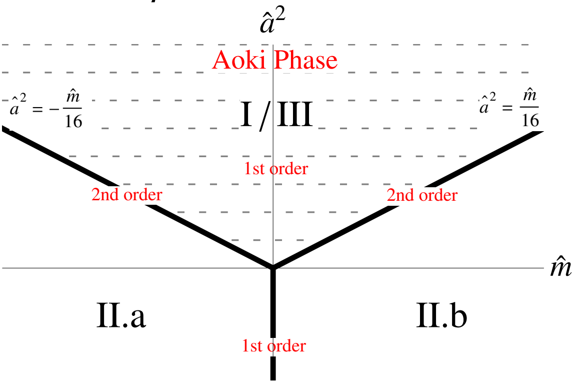
Phase Diagram of Wilson Fermions without twist ($\omega = 0$)



Phase diagram of the scientific productivity.

image from thescientificcartoonist.com

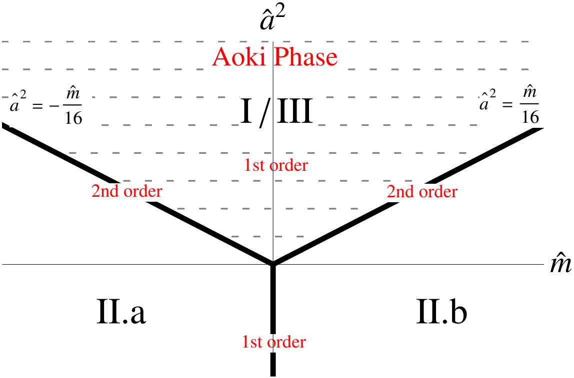
$\hat{\mu}^2$=0, No Twist



II.a/II.b: $U = \text{sign}\, \widehat{m} \mathbb{1}_2$

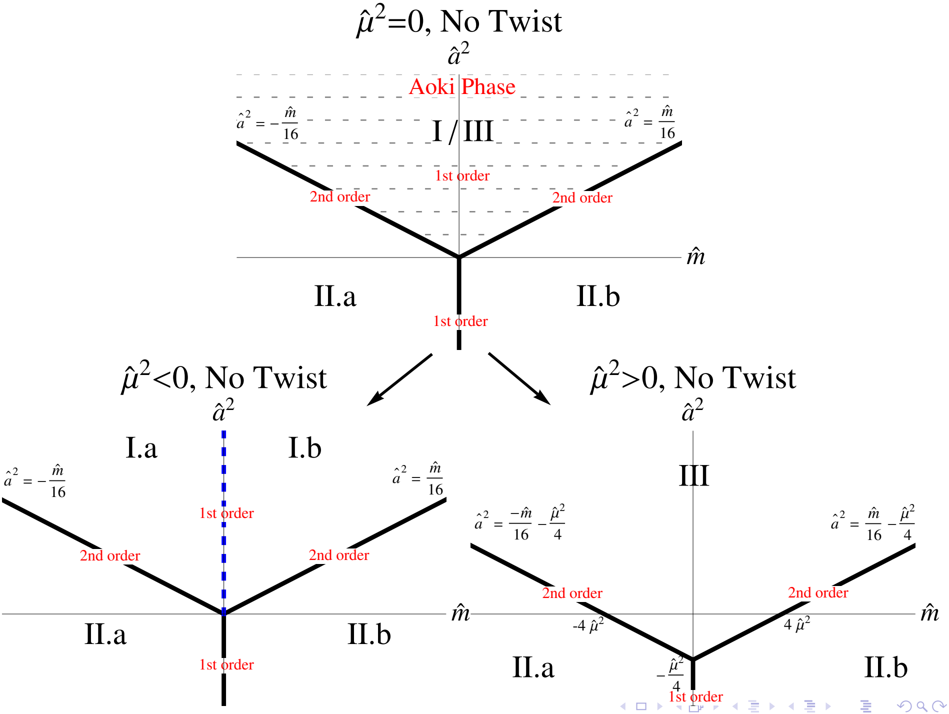
- $\langle \bar{\psi}\psi \rangle \propto \text{sign}\, \widehat{m}$
- $\langle \bar{\psi}\gamma_5 \tau_{1/2/3} \psi \rangle = 0$
- $\langle \bar{\psi}\gamma_0 \tau_3 \psi \rangle = 0$
- **Sharpe-Singleton scenario for $\widehat{a}^2 < 0$**

$\hat{\mu}^2$=0, No Twist

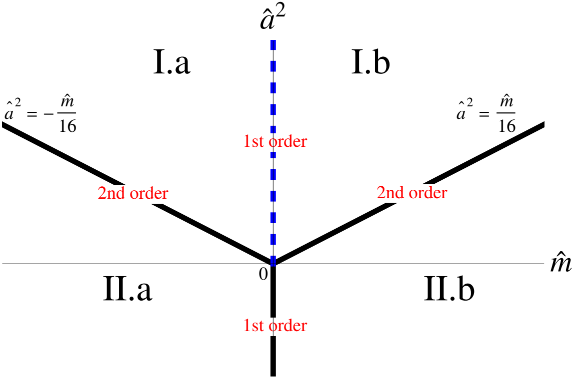


I/III: $$U = \frac{\hat{m}}{16\hat{a}^2}\mathbb{1}_2 + \imath\sqrt{1-\left(\frac{\hat{m}}{16\hat{a}^2}\right)^2}\begin{pmatrix} \cos\vartheta_1 & e^{\imath\vartheta_2}\sin\vartheta_1 \\ e^{-\imath\vartheta_2}\sin\vartheta_1 & -\cos\vartheta_1 \end{pmatrix}$$

- $\langle\bar{\psi}\psi\rangle \propto \hat{m}/16\hat{a}^2$
- $\langle\bar{\psi}\gamma_5\tau_{1/2/3}\psi\rangle = 0$, $\langle(\bar{\psi}\gamma_5\tau_{1/2/3}\psi)^2\rangle \propto 1-(\hat{m}/16\hat{a}^2)^2$
- $\langle\bar{\psi}\gamma_0\tau_3\psi\rangle = 0$
- **Aoki phase is phase boundary of a first order phase transition!**


$\hat{\mu}^2$=0, No Twist
$\hat{a}^2$
Aoki Phase
$\hat{a}^2 = -\frac{\hat{m}}{16}$
I / III
$\hat{a}^2 = \frac{\hat{m}}{16}$
1st order
2nd order
2nd order
$\hat{m}$
II.a
II.b
1st order
$\hat{\mu}^2$<0, No Twist
$\hat{a}^2$
I.a
I.b
$\hat{a}^2 = -\frac{\hat{m}}{16}$
$\hat{a}^2 = \frac{\hat{m}}{16}$
1st order
2nd order
2nd order
$\hat{m}$
II.a
II.b
1st order
$\hat{\mu}^2$>0, No Twist
$\hat{a}^2$
III
$\hat{a}^2 = \frac{-\hat{m}}{16} - \frac{\hat{\mu}^2}{4}$
$\hat{a}^2 = \frac{\hat{m}}{16} - \frac{\hat{\mu}^2}{4}$
2nd order
2nd order
$\hat{m}$
-4 $\hat{\mu}^2$
4 $\hat{\mu}^2$
II.a
$-\frac{\hat{\mu}^2}{4}$
II.b
1st order

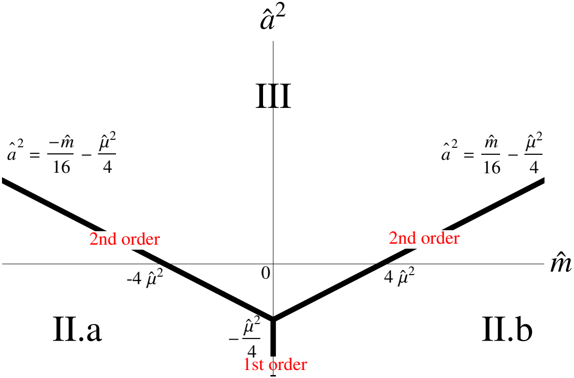
$\hat{\mu}^2 < 0$, No Twist



I.a/I.b: $$U = \frac{\hat{m}}{16\hat{a}^2}\mathbb{1}_2 - \imath\,\mathrm{sign}\,\hat{m}\sqrt{1-\left(\frac{\hat{m}}{16\hat{a}^2}\right)^2}\tau_3$$

- $\langle\bar{\psi}\psi\rangle \propto \hat{m}/16\hat{a}^2$
- spontaneous symmetry breaking with a small twisted mass: $\langle\bar{\psi}\gamma_5\tau_{1/2}\psi\rangle = 0$, $\langle\bar{\psi}\gamma_5\tau_3\psi\rangle \propto -\mathrm{sign}\,\hat{m}\sqrt{1-(\hat{m}/16\hat{a}^2)^2}$
- $\langle\bar{\psi}\gamma_0\tau_3\psi\rangle \propto \hat{\mu}$

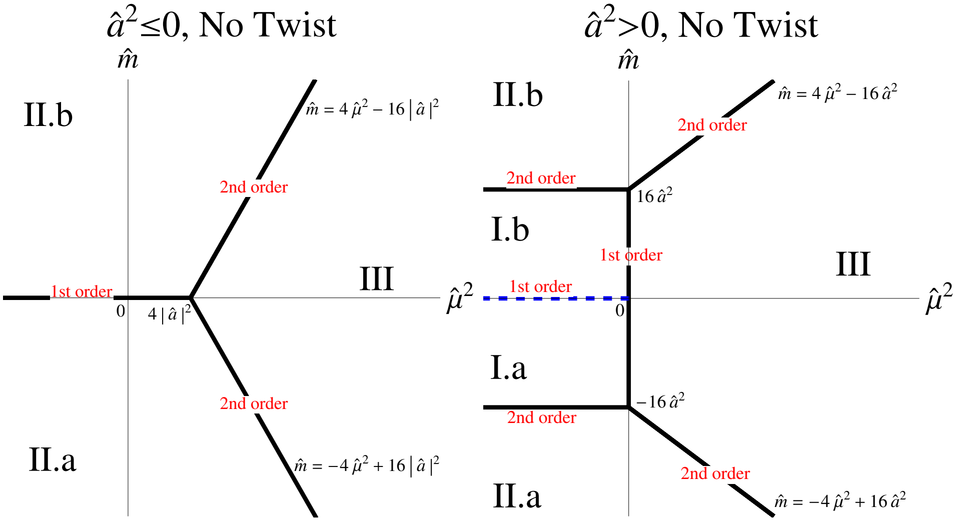
$\hat{\mu}^2 > 0$, No Twist



III:

$$U = \frac{\widehat{m}}{16\widehat{a}^2 + 4\widehat{\mu}^2} \mathbb{1}_2 + \imath \sqrt{1 - \left(\frac{\widehat{m}}{16\widehat{a}^2 + 4\widehat{\mu}^2} \right)^2} \begin{pmatrix} 0 & e^{\imath\vartheta_2} \\ e^{-\imath\vartheta_2} & 0 \end{pmatrix}$$

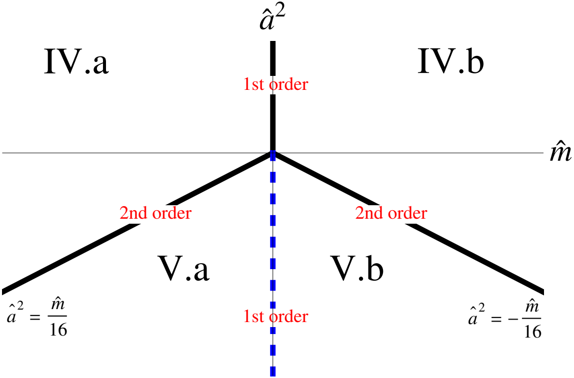
- $\langle \bar{\psi}\psi \rangle \propto \widehat{m}/(16\widehat{a}^2 + 4\widehat{\mu}^2)$
- $\langle \bar{\psi}\gamma_5\tau_3\psi \rangle = 0$
- $\langle \bar{\psi}\gamma_5\tau_{1/2}\psi \rangle = 0$, $\langle (\bar{\psi}\gamma_5\tau_{1/2}\psi)^2 \rangle \propto 1 - (\widehat{m}/(16\widehat{a}^2 + 4\widehat{\mu}^2))^2$
- $\langle \bar{\psi}\gamma_0\tau_3\psi \rangle \propto \widehat{\mu} - 2\widehat{\mu}\widehat{m}^2/(16\widehat{a}^2 + 4\widehat{\mu}^2)^2$



- Aoki phase = 1st order phase transition (Ia/Ib $\rightarrow$ III)
- for $\hat{a} = 0$: Son, Stephanov (2001, χPT), Kogut, Sinclair (2002, lattice sim.)

Phase Diagram of Wilson Fermions at maximal twist ($\omega = \pi/2$)

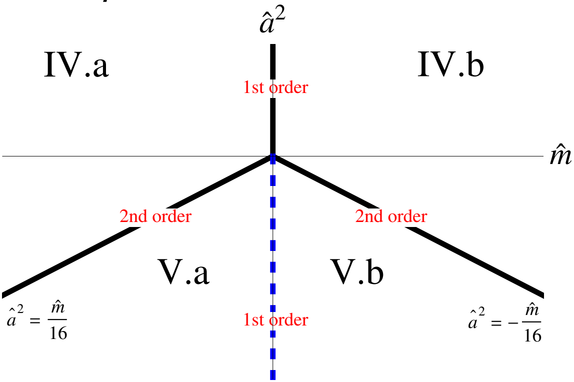
$\hat{\mu}^2 \leq 0$, Maximal Twist



IV.a/IV.b: $U = -\imath \operatorname{sign} \widehat{m}\, \tau_3$

- $\langle \bar{\psi}\gamma_5\tau_3\psi \rangle \propto -\operatorname{sign} \widehat{m}$
- $\langle \bar{\psi}\psi \rangle = \langle \bar{\psi}\gamma_5\tau_{1/2}\psi \rangle = 0$
- $\langle \bar{\psi}\gamma_0\tau_3\psi \rangle \propto \widehat{\mu}$

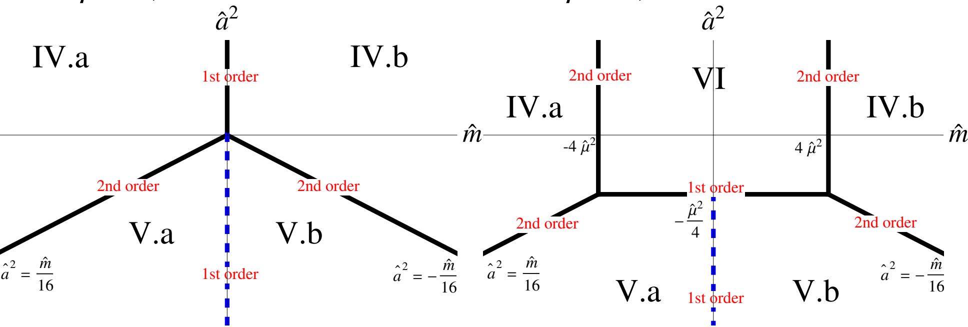
$\hat{\mu}^2 \leq 0$, Maximal Twist



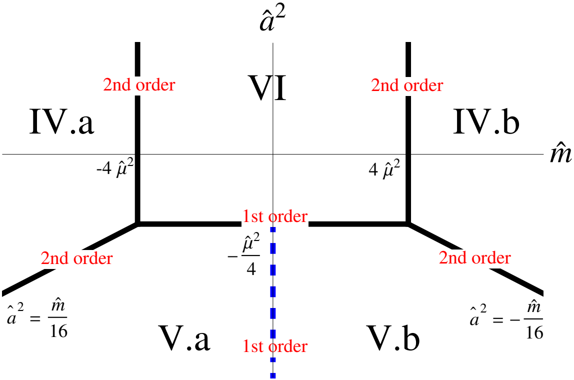
V.a/V.b: $U = \mathrm{sign}\,\hat{m}\sqrt{1 - \left(\frac{\hat{m}}{16\hat{a}^2}\right)^2}\,\mathbb{1}_2 - \imath\frac{\hat{m}}{16|\hat{a}|^2}\tau_3$

- spontaneous symmetry breaking with a small untwisted mass: $\langle\bar{\psi}\psi\rangle = \mathrm{sign}\,\hat{m}\sqrt{1 - (\hat{m}/16\hat{a}^2)^2}$
- $\langle\bar{\psi}\gamma_5\tau_{1/2}\psi\rangle = 0$, $\langle\bar{\psi}\gamma_5\tau_3\psi\rangle \propto -\hat{m}/16|\hat{a}|^2$
- $\langle\bar{\psi}\gamma_0\tau_3\psi\rangle \propto \hat{\mu}$

$\hat{\mu}^2 \leq 0$, Maximal Twist $\longrightarrow$ $\hat{\mu}^2 > 0$, Maximal Twist



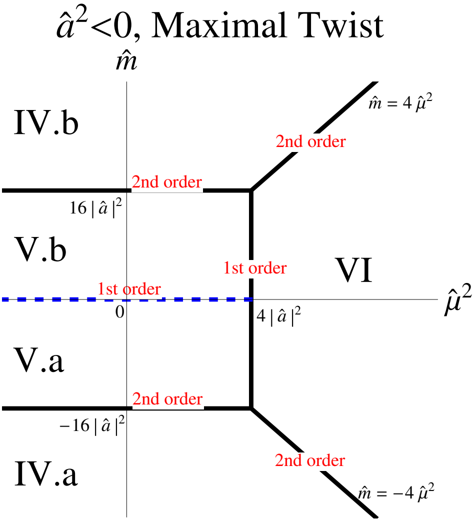
$\hat{\mu}^2 > 0$, Maximal Twist

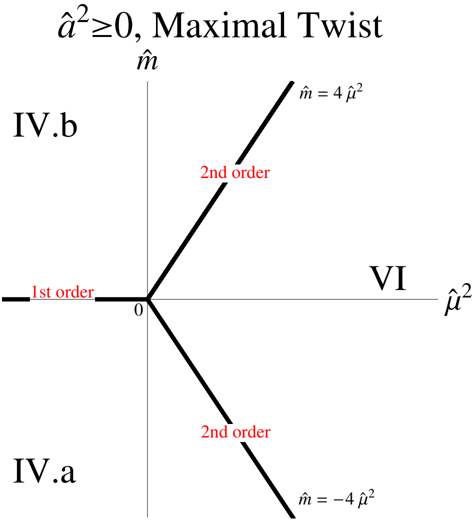


<u>VI:</u>

$$U = \imath \begin{pmatrix} -\widehat{m}/4\widehat{\mu}^2 & e^{\imath\vartheta_2}\sqrt{1-(\widehat{m}/4\widehat{\mu}^2)^2} \\ e^{-\imath\vartheta_2}\sqrt{1-(\widehat{m}/4\widehat{\mu}^2)^2} & \widehat{m}/4\widehat{\mu}^2 \end{pmatrix}$$

- $\langle\bar{\psi}\psi\rangle = 0$
- $\langle\bar{\psi}\gamma_5\tau_3\psi\rangle \propto -\widehat{m}/4\widehat{\mu}^2$
- $\langle\bar{\psi}\gamma_5\tau_{1/2}\psi\rangle = 0$, $\langle(\bar{\psi}\gamma_5\tau_{1/2}\psi)^2\rangle \propto 1-(\widehat{m}/4\widehat{\mu}^2)^2$
- $\langle\bar{\psi}\gamma_0\tau_3\psi\rangle \propto \widehat{\mu} - \widehat{m}^2/8\widehat{\mu}^3$





- physical situation ($\hat{\mu}$ real) already consists of a rich phase space structure

Summary

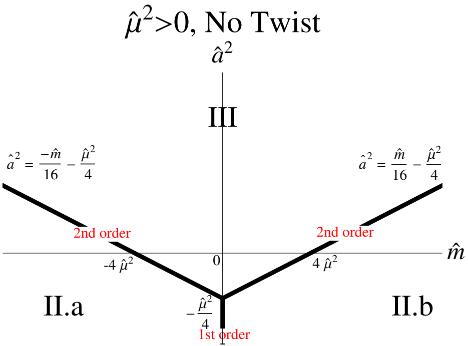
and

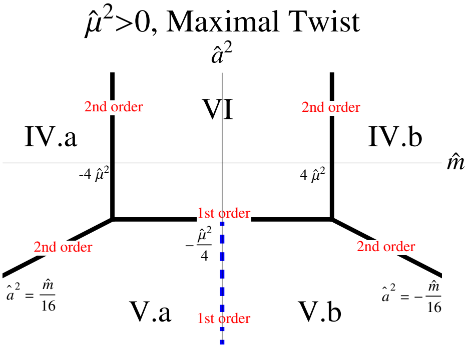
Outlook

image from libertyscientist.com

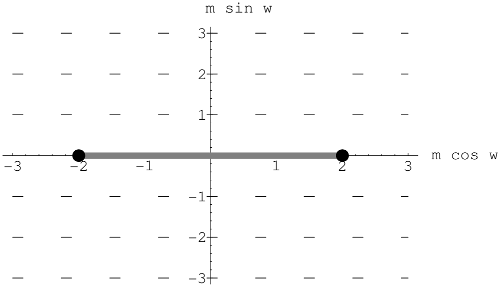
Summary

- Aoki phase is a phase boundary (1st order phase transition)
- rich phase space structure (5 phases for no twist as well as maximal twist)
- drastic change of phase diagram under twist (also in the physical situation $\widehat{\mu}^2 > 0$)





Discussion by Sharpe and Wu ($\widehat{\mu} = 0$; 2004)



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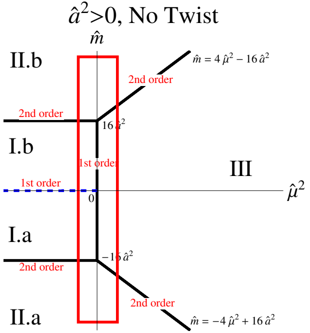
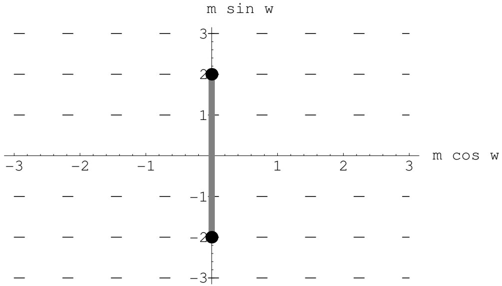
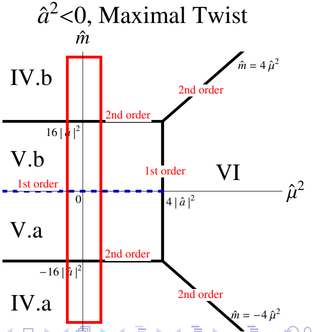


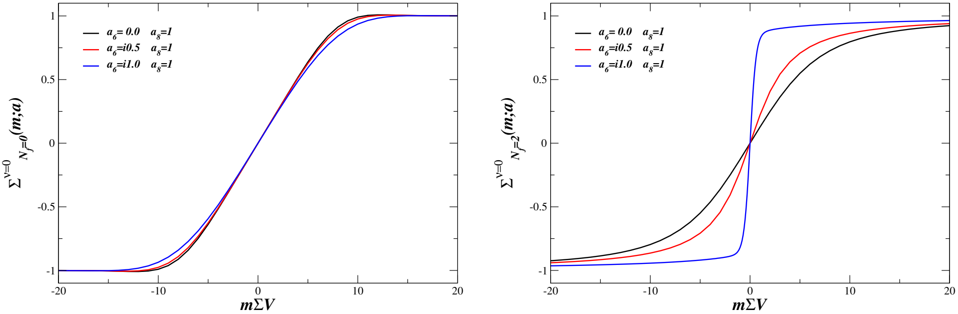


$\rightarrow$



Outlook

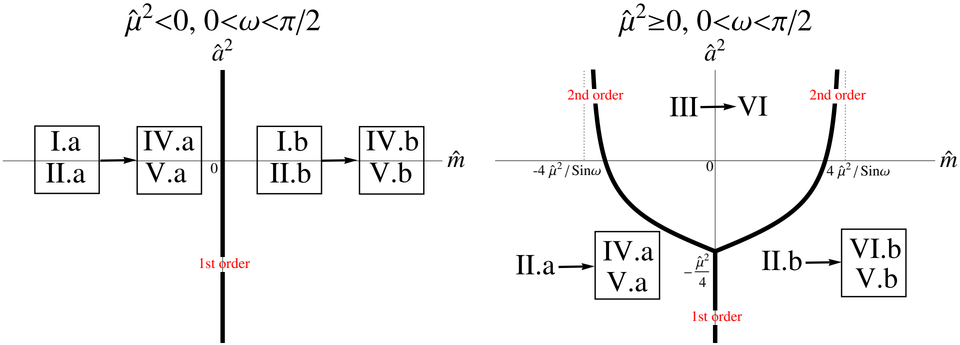
- impact of phases on the spectral density
- quenched QCD (**Remember, Sharpe-Singleton scenario is not realized in this setting!**)



Kieburg, Splittorff, Verbaarschot (2012)

Outlook

- impact of phases on the spectral density
- quenched QCD (**Remember, Sharpe-Singleton scenario is not realized in this setting!**)
- phases at arbitrary twist (in work)



There is still much work to do!

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Thank you for your attention!