SFB TRR55

BERGISCHE UNIVERSITÄT WUPPERTAL

JÜLICH FORSCHUNGSZENTRUM

Fluctuations of conserved charges

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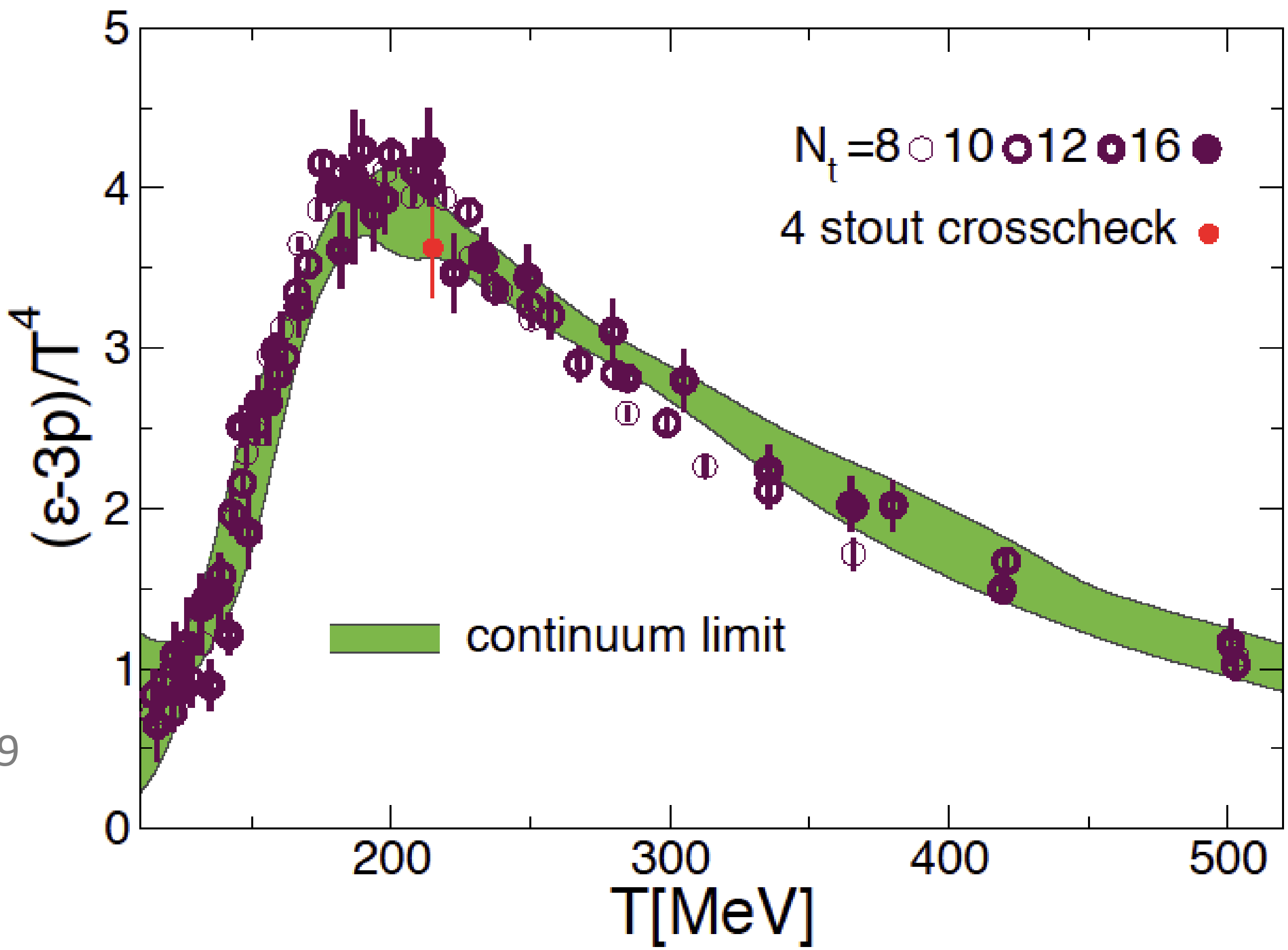
(Wuppertal-Budapest collaboration)

18.02.2014| Stefan Krieg

Overview

1. Updates and crosschecks
2. **Fluctuations**
3. **Thermometer/Baryometer**
4. Outlook

Updates and crosschecks: full N_f=2+1 EoS



PLB **730**, 99
1309.5258

Updates and crosschecks: full N_f=2+1 EoS

- We work at M_π = phys $\rightarrow$ no chiral extrapolation (→ 2d spline)
- No finite volume effects found in 2010/2011, 2013: larger vol

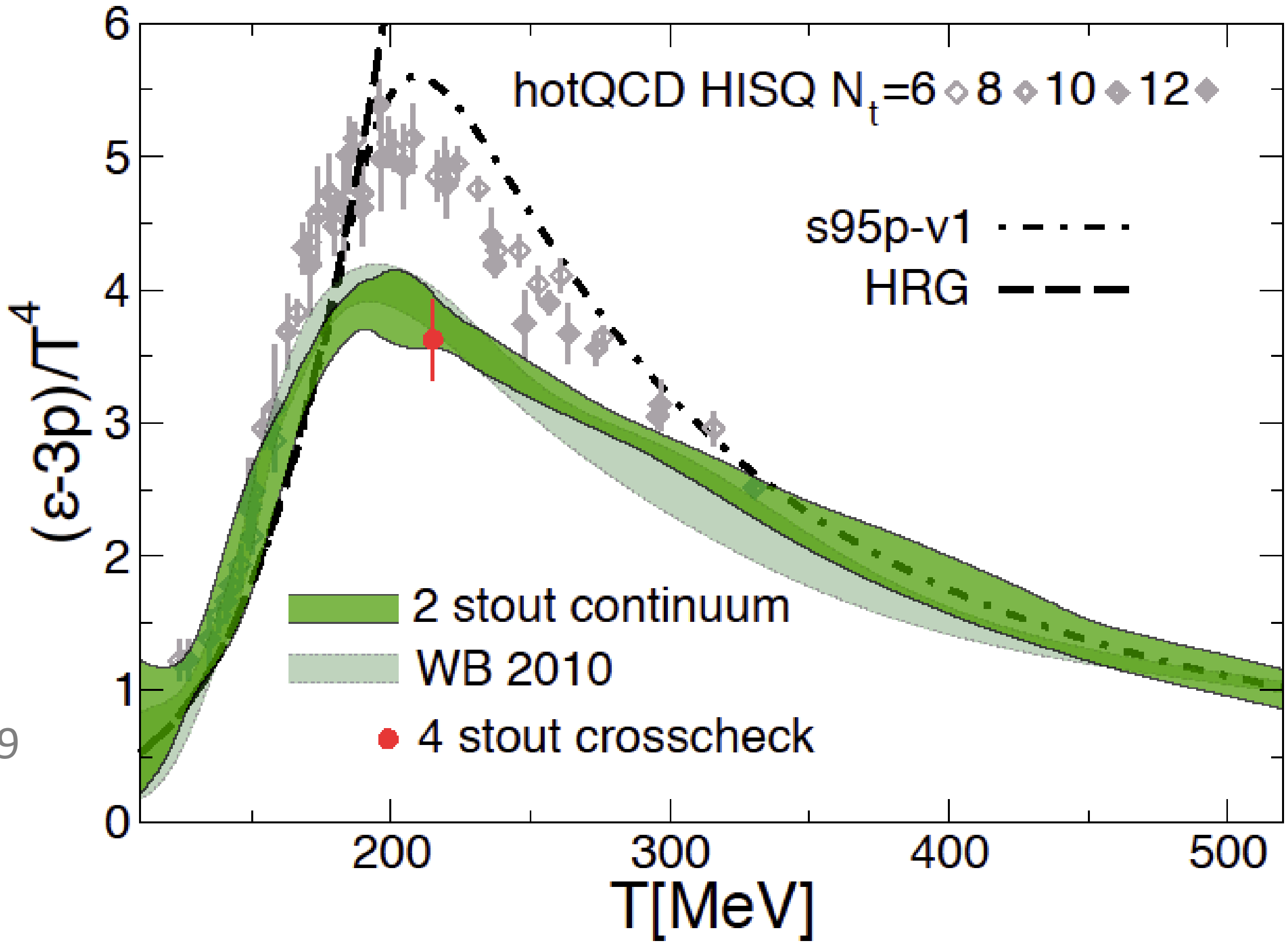
Systematics: histogram method

- vacuum fits, 7 different fit models (incl. direct subtr. w. interp.)
- continuum extrapolation
  - Vary node points (8 different sets)
  - Include or leave out leave N_t=6
  - With or without improvement factors
  - We use two different scale settings (f_k vs. w_0)
  - Fit includes a^2 or a^2 and a^4 terms

→ This results in $7\times8\times2\times2\times2\times2$ = 896 different fits

- Weighting: we consider AICc, Q, or unweighted histograms

Updates and crosschecks: full N_f=2+1 EoS



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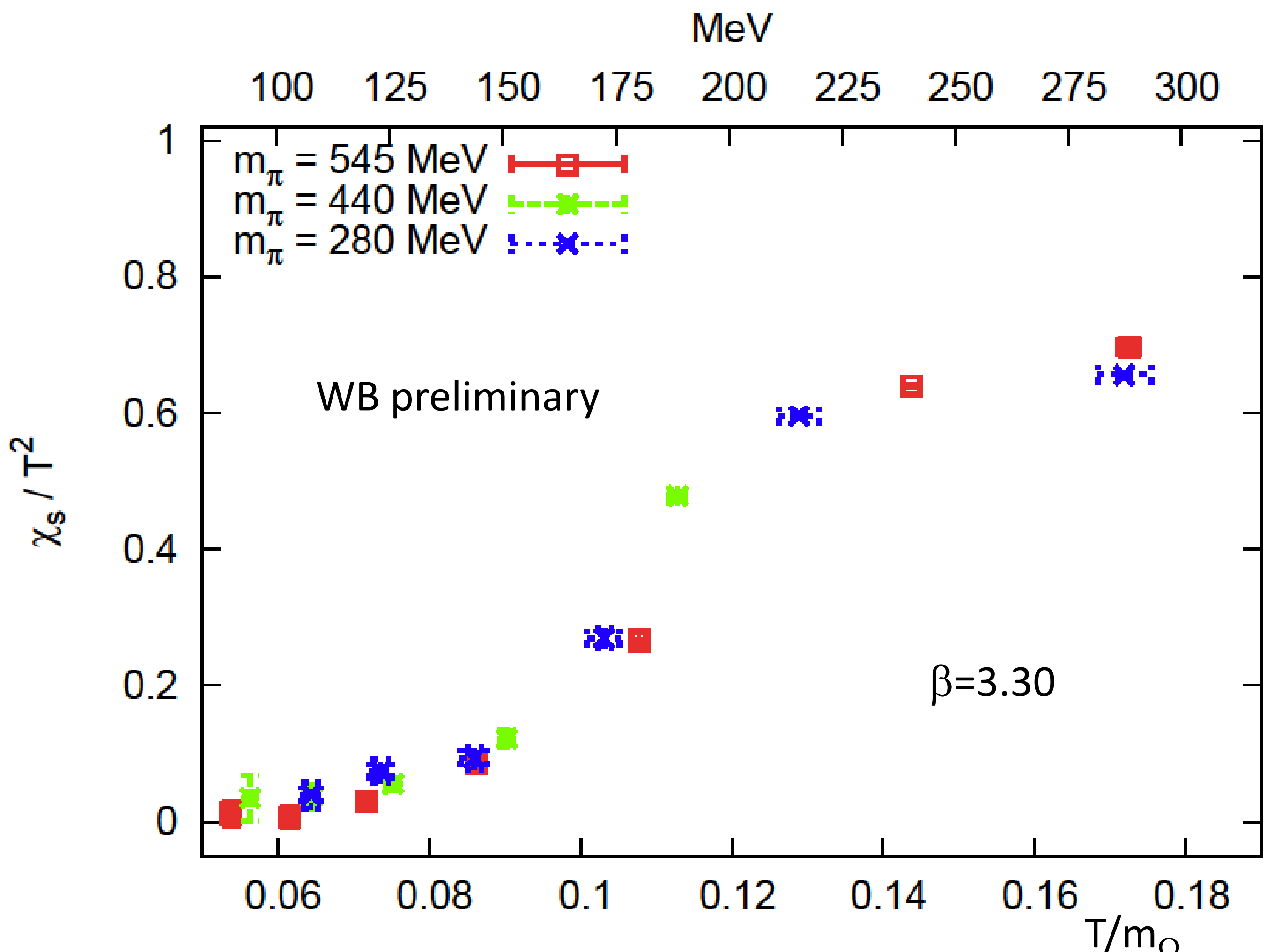
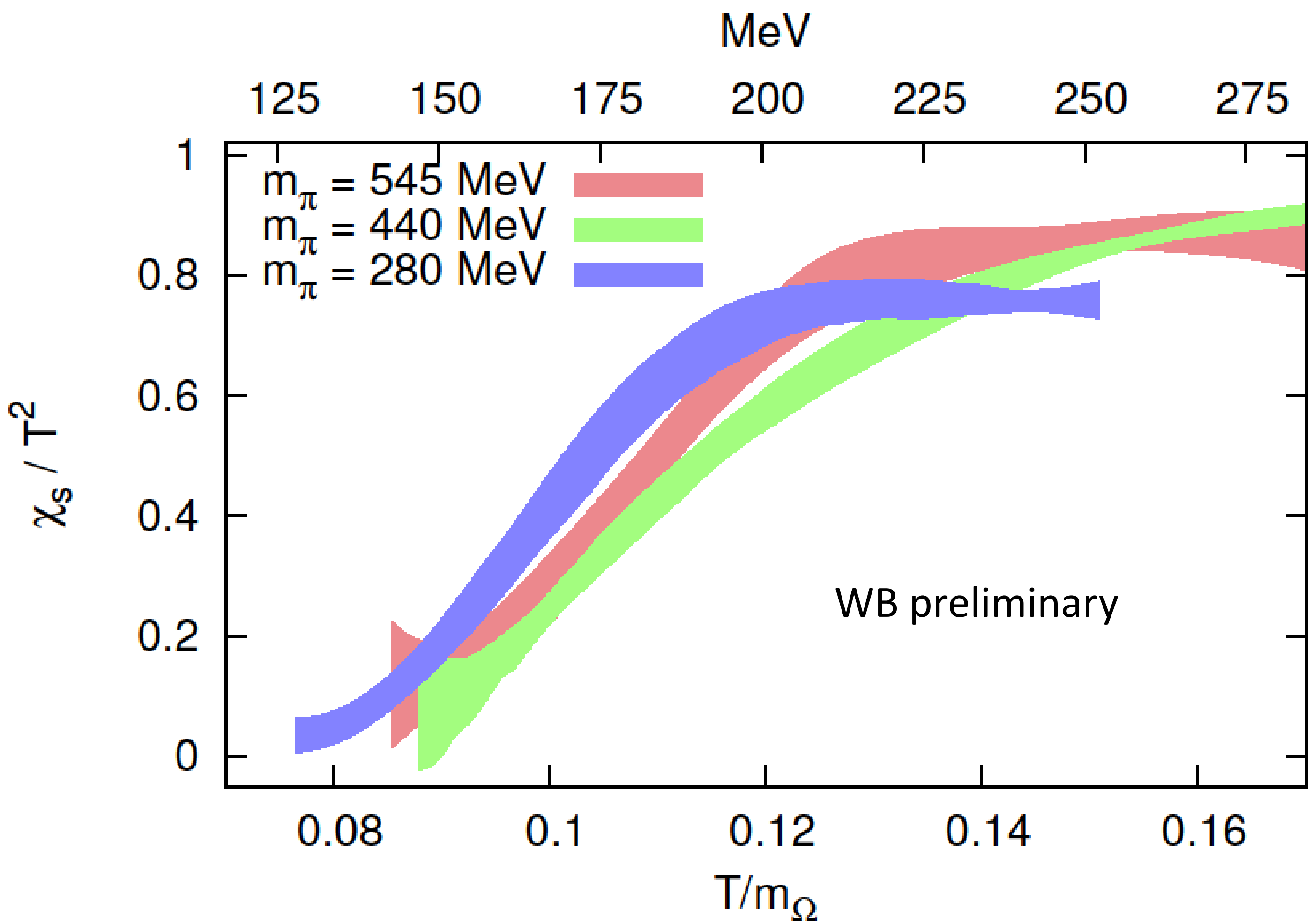
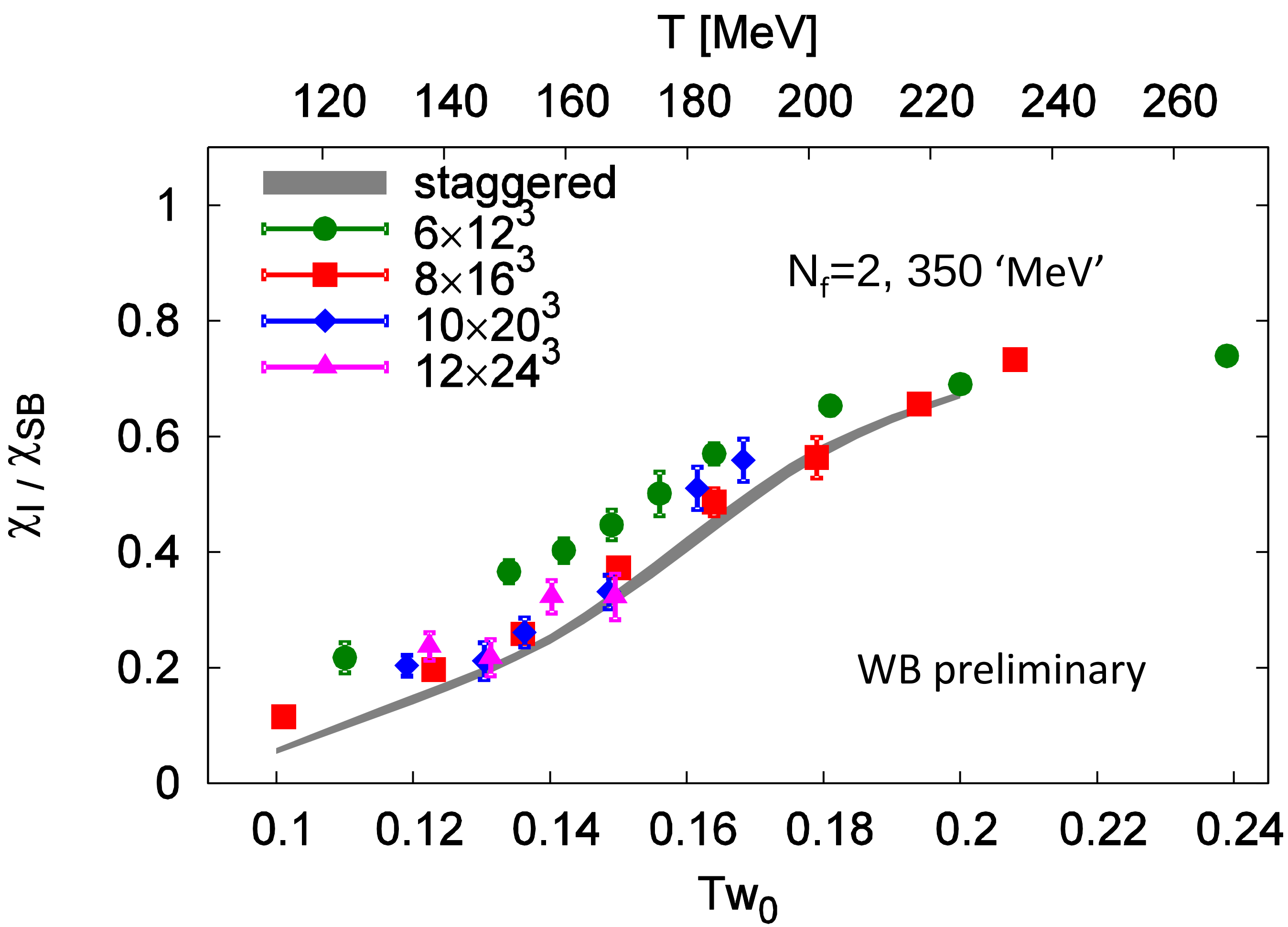
Updates and crosschecks: Wilson thermo



Updates and crosschecks: Wilson thermo



Updates and crosschecks: Overlap thermo



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Fluctuations

- signal the proximity of the of the critical point
- can be calculated in theory and compared directly to experiment
- HRG results may be incorrect at $T_{\text{freeze-out}}$
- can be calculated in LQCD (µVT, → data cuts)
- can then be used to extract the freeze-out parameters

However: fluctuations in proton number (experiment) have to be matched to fluctuations in baryon number (theory)

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Fluctuations

- Net yields are given by $\langle N_X \rangle = -T \frac{\partial \log Z}{\partial \mu_X}$ $\langle N_X^2 \rangle - \langle N_X \rangle^2 = \frac{\partial^2 \log Z}{(\partial \mu_X / T)^2}$
- Defining: $\hat{\chi}_2^X = \frac{1}{VT^3} \frac{\partial^2 \log Z}{(\partial \mu_X / T)^2}$ $\hat{\chi}_4^X = \frac{1}{VT^3} \frac{\partial^4 \log Z}{(\partial \mu_X / T)^4}$
- Computable at finite μ through:

$$\frac{p}{T^4} \equiv \frac{1}{VT^3} \ln Z(T, \mu_q) = \sum_{n=0}^{\infty} c_n(T, m_q) \left(\frac{\mu_q}{T} \right)^n \qquad c_n(T, m_q) = \frac{1}{n!} \frac{1}{VT^3} \frac{\partial^n \ln Z(T, \mu_q)}{\partial (\mu_q / T)^n} \bigg|_{\mu_q = 0}$$

- χ_2 requires (finite μ_B): $\hat{\chi}_{22}^{XB} = \frac{1}{VT^3} \frac{\partial^4 \log Z}{(\partial \mu_X / T)^2 (\partial \mu_B / T)^2}$

Challenges:

- **Derivatives come with volume penalty in the statistics**
- **Electric charge is pion-dominated $\rightarrow$ fine lattices and taste-improvement.**
- **Baryon number noisy $\rightarrow$ large statistics.**

Fluctuations: use as thermo-/baryometer

- Freeze-out parameters directly from QCD:
  - Ratios of cumulants (Karsch 1202.4173), volume cancels

$$\frac{\sigma_B^2}{M_B} = \frac{\chi_{2,\mu}^B}{\chi_{1,\mu}^B}, \qquad S_B\sigma_B = \frac{\chi_{3,\mu}^B}{\chi_{2,\mu}^B}, \qquad \kappa_B\sigma_B^2 = \frac{\chi_{4,\mu}^B}{\chi_{2,\mu}^B}, \qquad \frac{S_B\sigma_B^3}{M} = \frac{\chi_{3,\mu}^B}{\chi_{1,\mu}^B}$$

$$\frac{\sigma_B^2}{M_B} \equiv \frac{\chi_{2,\mu}^B}{\chi_{1,\mu}^B} = \frac{T}{\mu_B}\left[\frac{1+\frac{1}{2}\frac{\chi_4^B}{\chi_2^B}(\mu_B/T)^2+\ldots}{1+\frac{1}{6}\frac{\chi_4^B}{\chi_2^B}(\mu_B/T)^2+\ldots}\right]$$

$$\kappa_B\sigma_B^2 \equiv \frac{\chi_{4,\mu}^B}{\chi_{2,\mu}^B} = \frac{\chi_4^B(T)}{\chi_2^B(T)}\left[\frac{1+\frac{1}{2}\frac{\chi_6^B(T)}{\chi_4^B(T)}(\mu_B/T)^2+\ldots}{1+\frac{1}{2}\frac{\chi_4^B(T)}{\chi_2^B(T)}(\mu_B/T)^2+\ldots}\right]$$

  - Alternative: Q instead of B

Fluctuations: use as thermo-/baryometer

- Freeze-out parameters directly from LQCD:
  - Experimental setting (Bazavov *et al*. 1208.1220):

$$M_S \equiv 0\ , \quad M_Q = r M_B, \quad r \approx 0.4 \text{ lead}$$

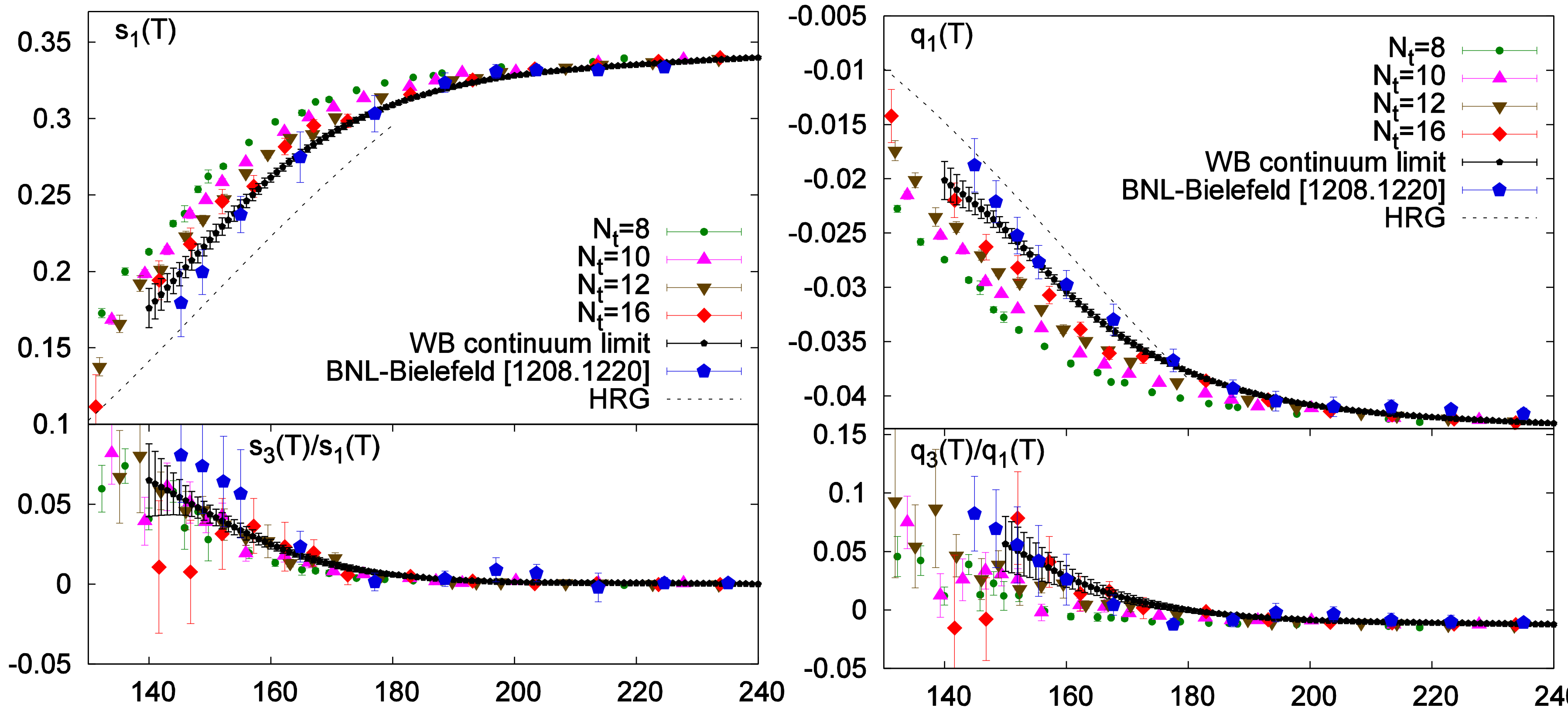
$$\hat{\mu}_Q = q_1\ \hat{\mu}_B + q_3\ \hat{\mu}_B^3\ , \quad \hat{\mu}_S = s_1\ \hat{\mu}_B + s_3\ \hat{\mu}_B^3\ , \quad \hat{\mu}_X = \mu_X/T$$

$$q_1 = \frac{r\left(\chi_2^B\chi_2^S - \chi_{11}^{BS}\chi_{11}^{BS}\right) - \left(\chi_{11}^{BQ}\chi_2^S - \chi_{11}^{BS}\chi_{11}^{QS}\right)}{\left(\chi_2^Q\chi_2^S - \chi_{11}^{QS}\chi_{11}^{QS}\right) - r\left(\chi_{11}^{BQ}\chi_2^S - \chi_{11}^{BS}\chi_{11}^{QS}\right)}\ , \quad s_1 = -\frac{\chi_{11}^{BS}}{\chi_2^S} - \frac{\chi_{11}^{QS}}{\chi_2^S}\ q_1$$

$$R_{12}^X \equiv \frac{M_X}{\sigma_X^2} = \hat{\mu}_B\left(R_{12}^{X,1} + R_{12}^{X,3}\ \hat{\mu}_B^2 + \mathcal{O}(\hat{\mu}_B^4)\right)$$

$$R_{12}^{X,1} = \frac{\chi_{11}^{BX}}{\chi_2^X} + q_1\frac{\chi_{11}^{XQ}}{\chi_2^X} + s_1\frac{\chi_{11}^{XS}}{\chi_2^X} \quad R_{31}^Q = \frac{\chi_3^Q}{\chi_1^Q} = \frac{\chi_{31}^{QB} + \chi_4^Q q_1 + \chi_{31}^{QS} s_1}{\chi_{11}^{QB} + \chi_2^Q q_1 + \chi_{11}^{QS} s_1} + \mathcal{O}(\hat{\mu}_B^4)$$

Fluctuations: expansion parameters



WB: PRL 111, 062005, 1305.5161

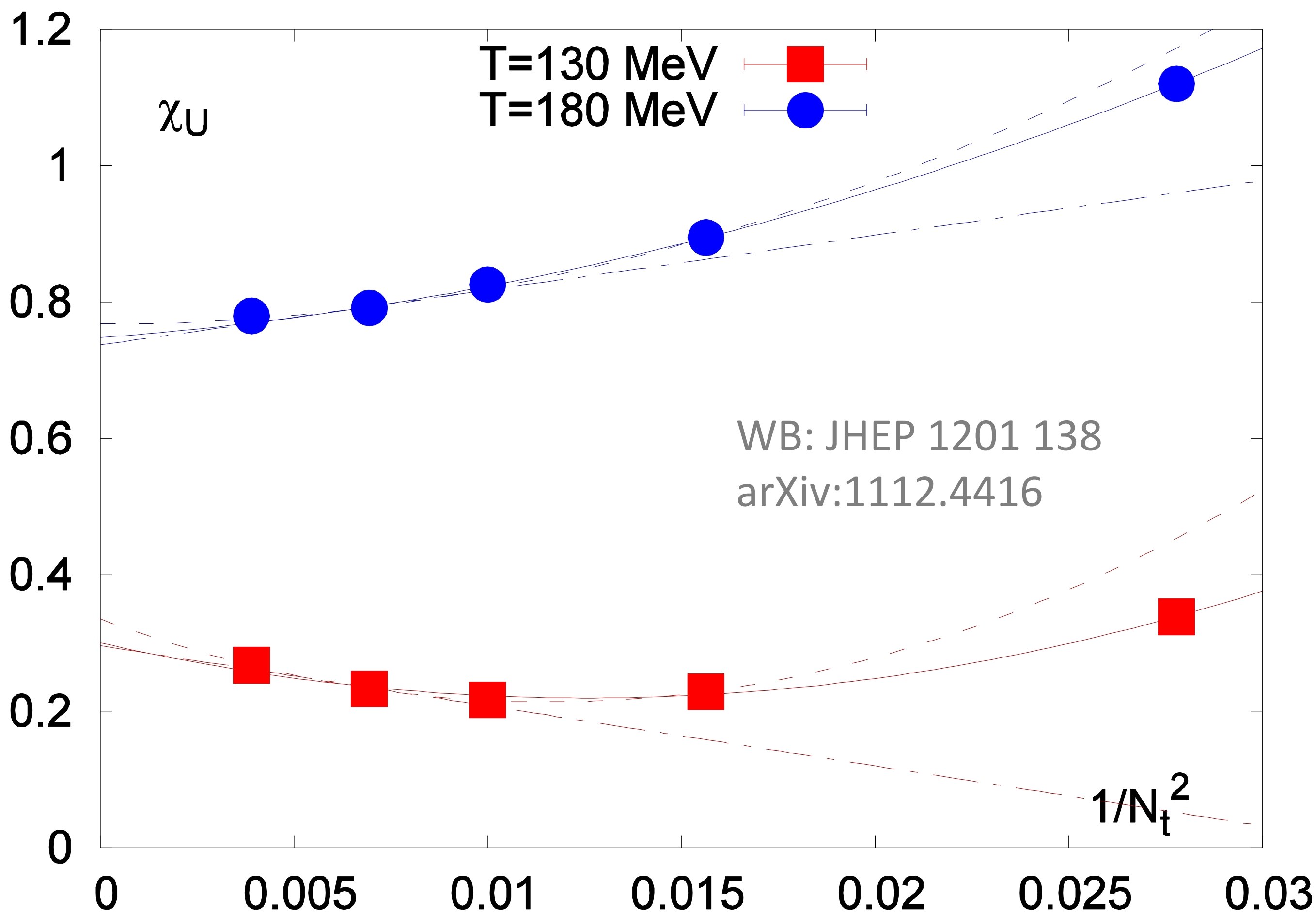
Fluctuations: error estimation

We follow our histogram ansatz as in EoS calculation

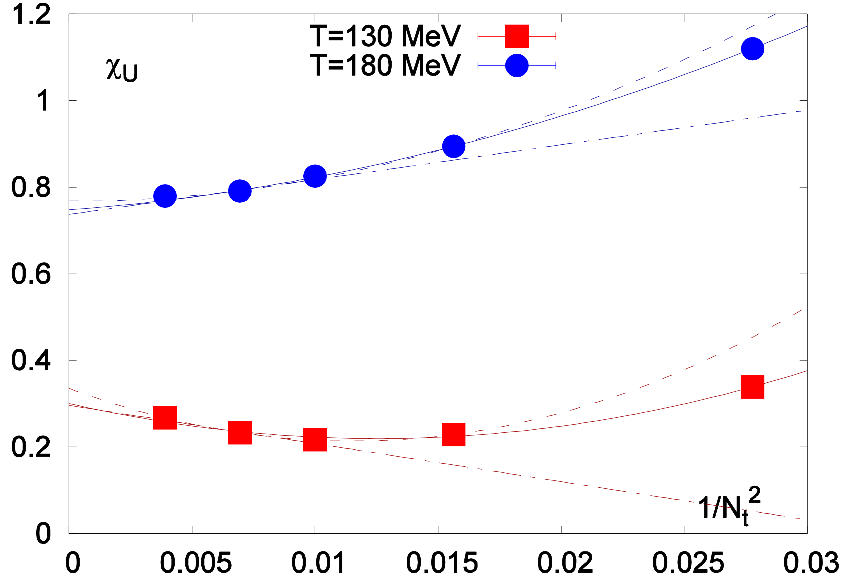
- Continuum extrapolation:
  - Multiple sets of nodepoints for spline interpolation
  - Extrapolate χ and $1/\chi$
  - Use different extrapolation formulas as in EoS
- Use jacknife estimate of deviation form mean to estimate statistical uncertainty

Systematic uncertainty is dominant

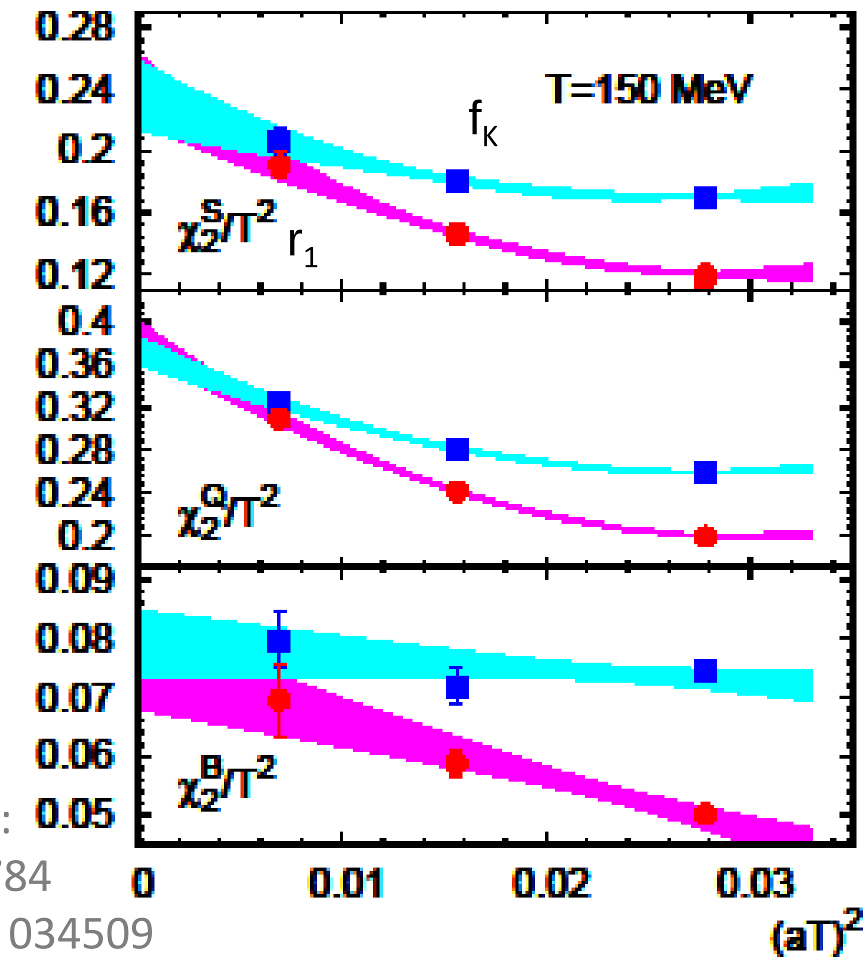
Fluctuations: continuum extrapolation



Fluctuations: continuum extrapolation

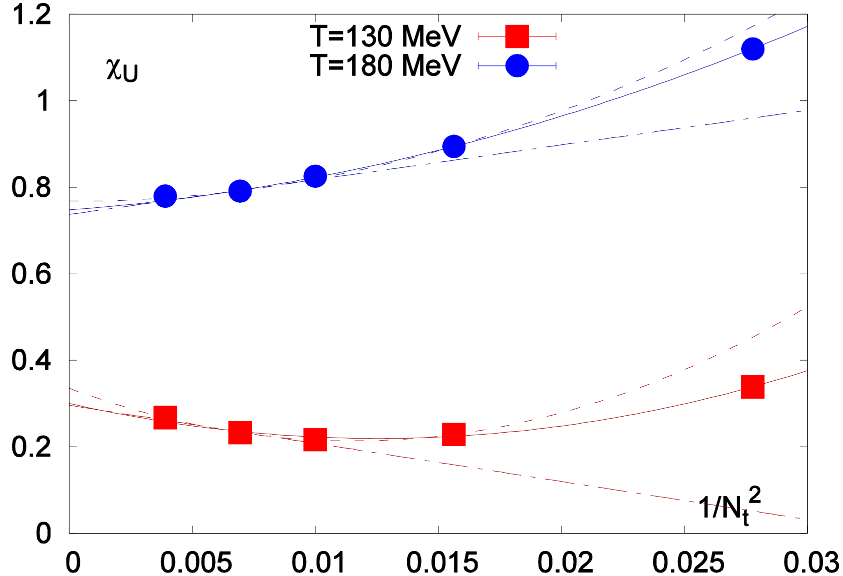


WB: JHEP 1201 138
arXiv:1112.4416



hotQCD:
1203.0784
PRD 86, 034509

Fluctuations: continuum extrapolation



WB: JHEP 1201 138
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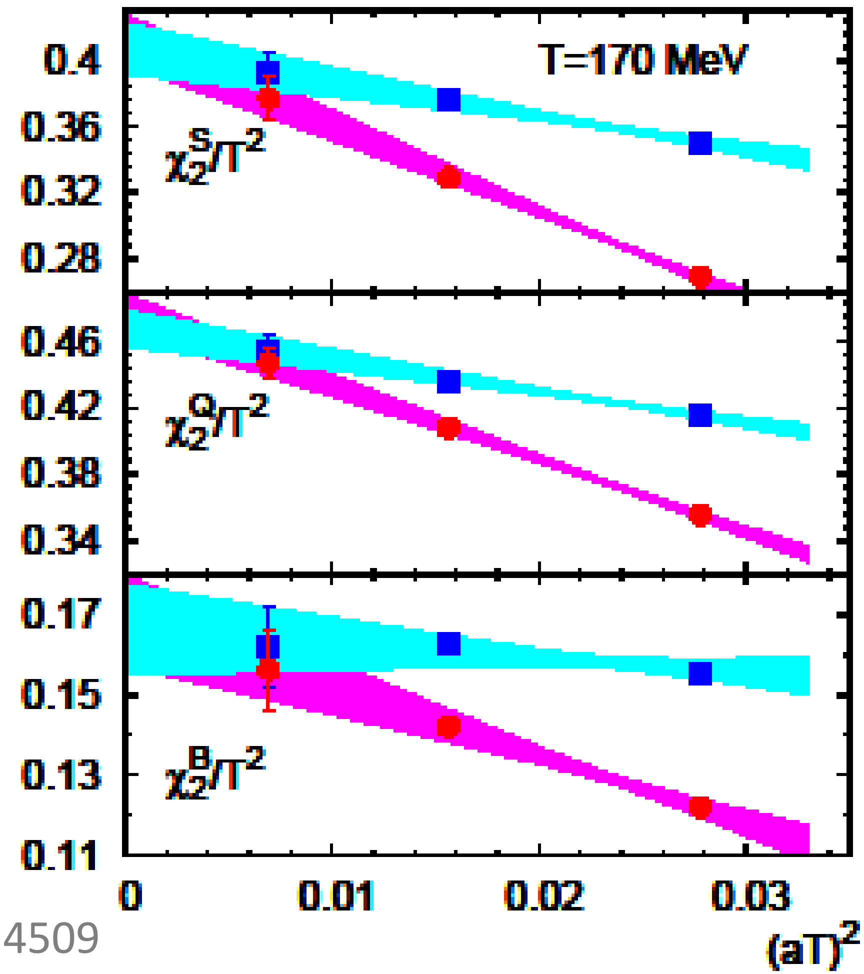
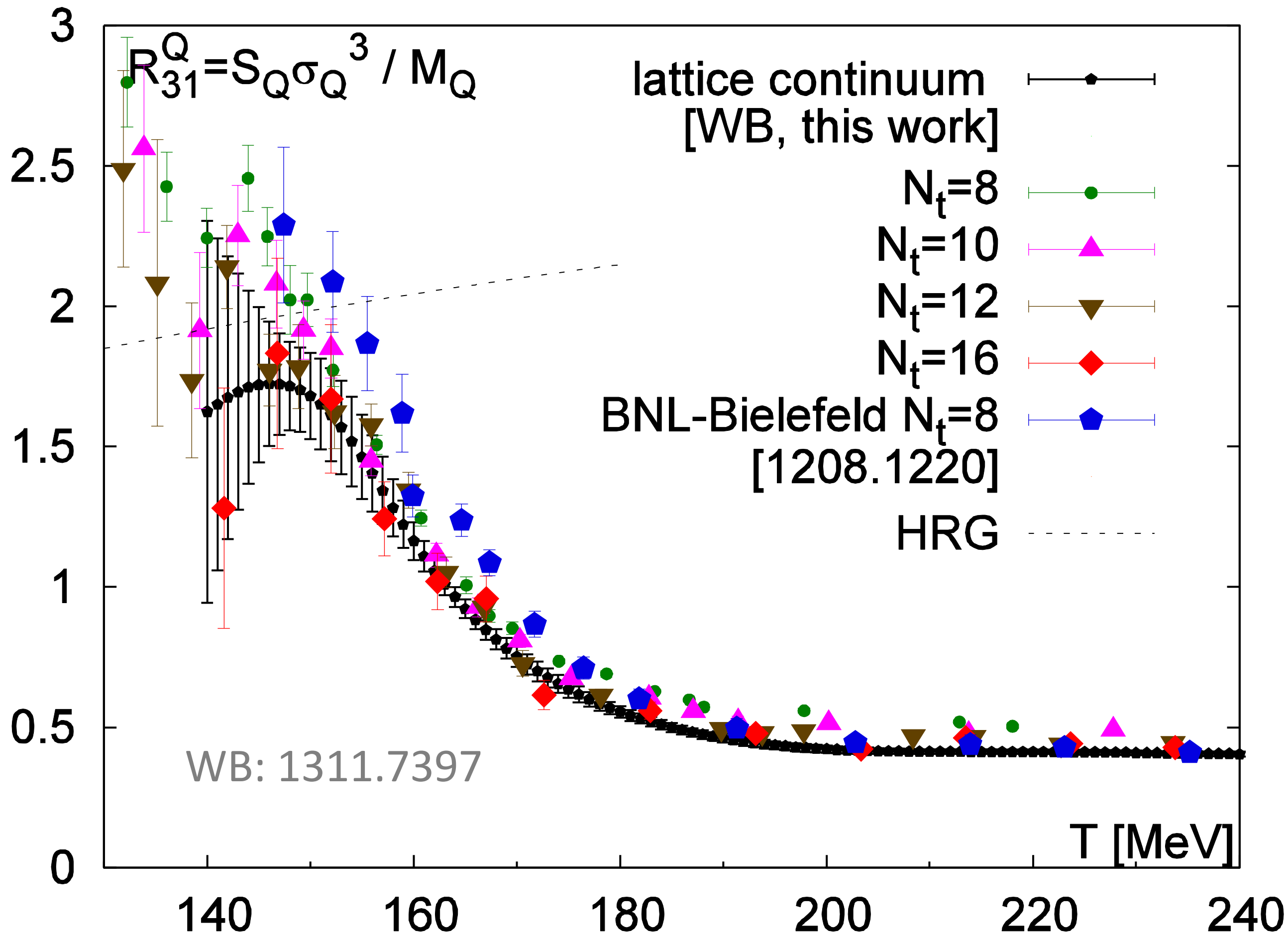
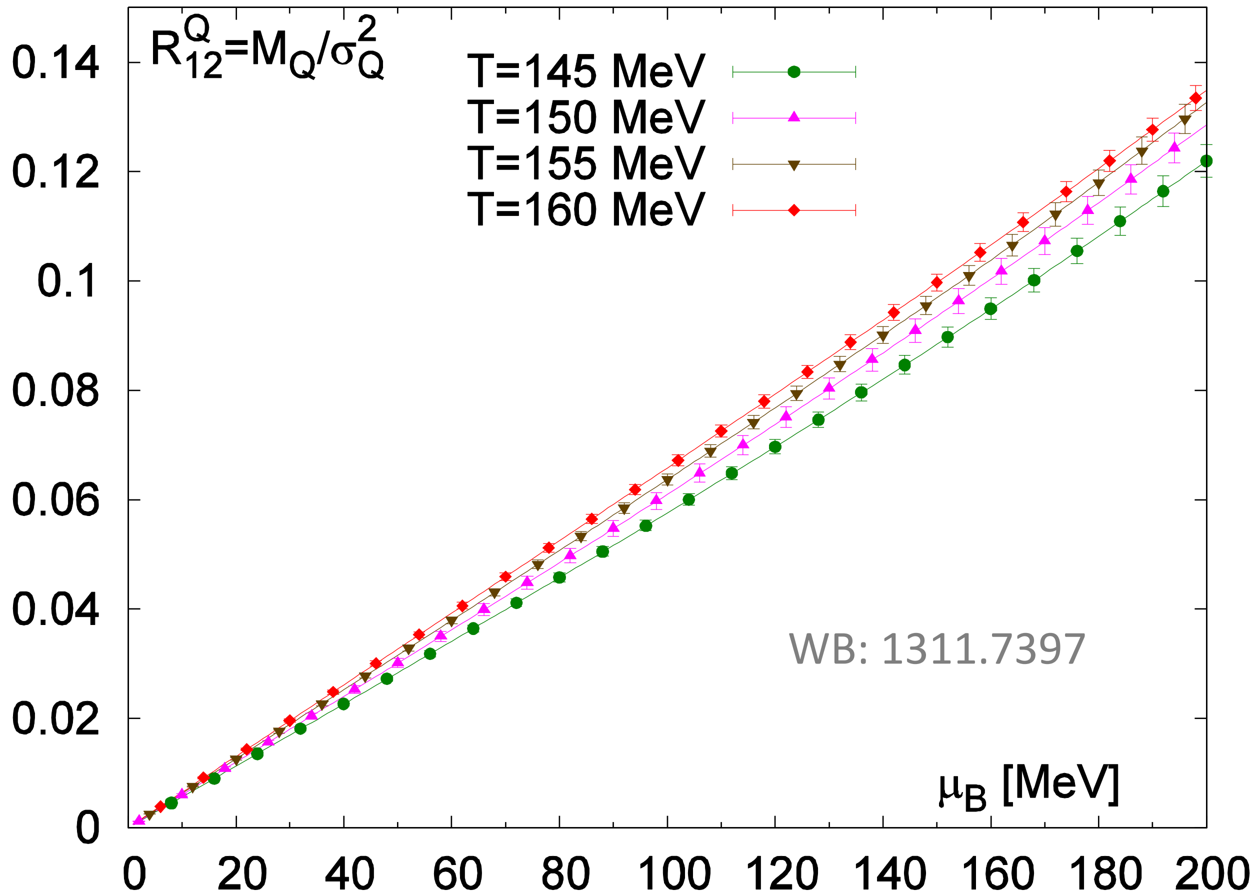


Mitglied der Helmholtz-Gemeinschaft

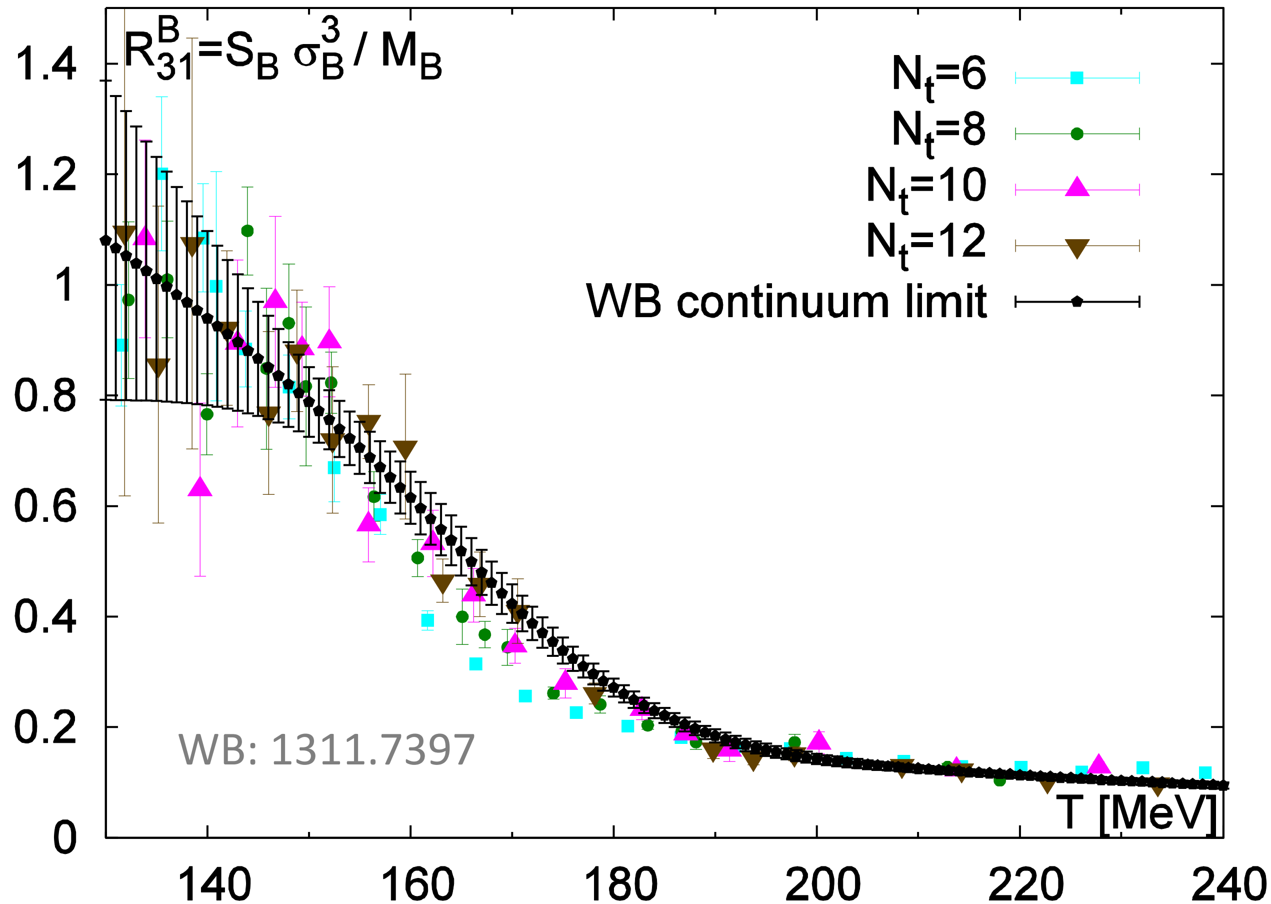
Thermometer: charge fluctuations



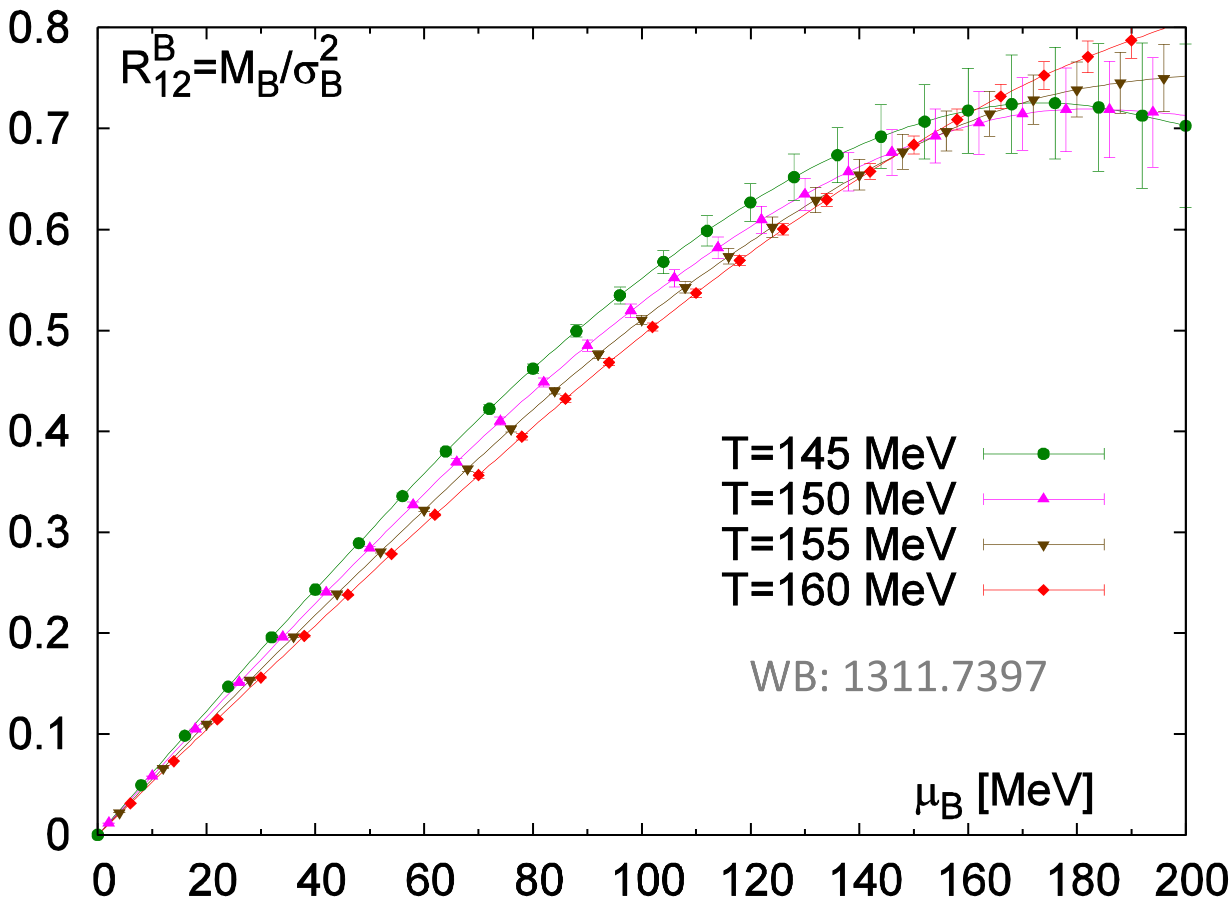
Thermometer: charge fluctuations



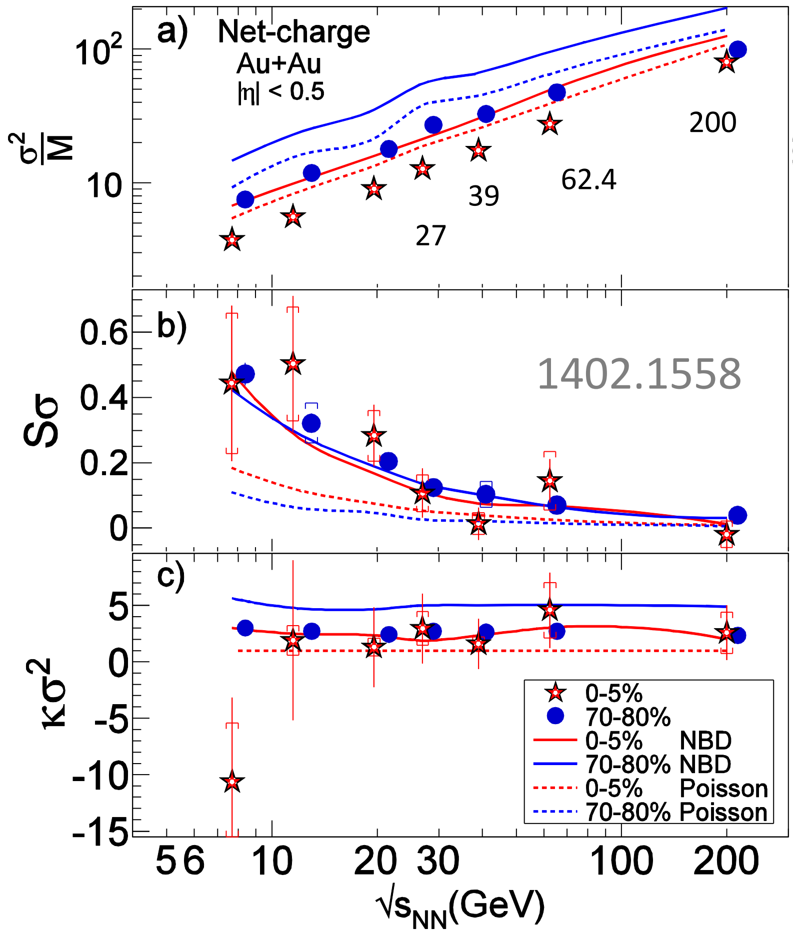
Thermometer: baryon number fluctuations

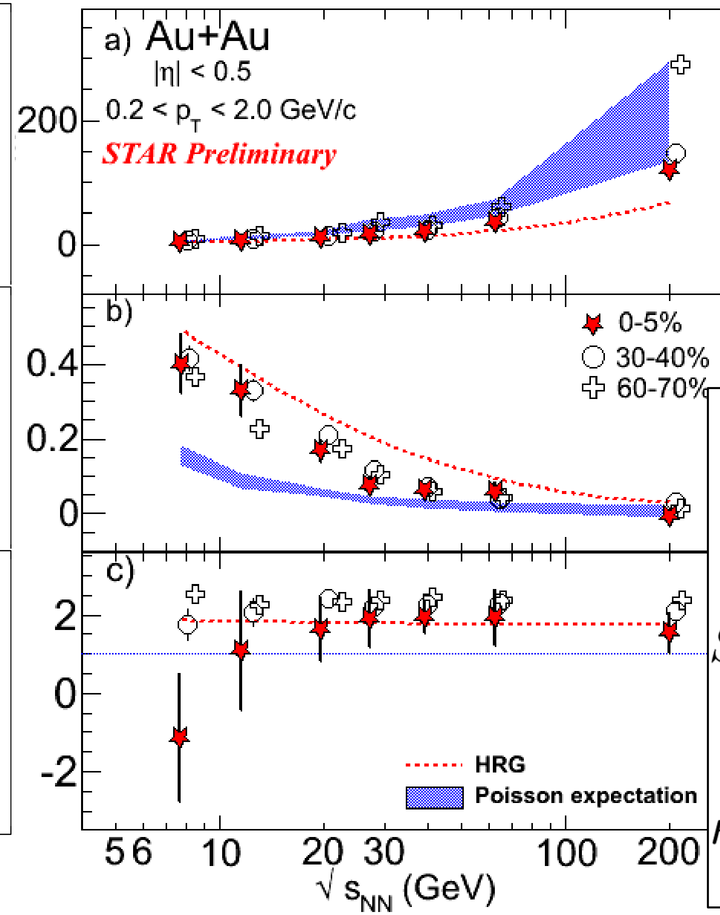


Thermometer: charge fluctuations



Thermometer: new results from STAR





1212.3892

$$\frac{\sigma_B^2}{M_B} = \frac{\chi_{2,\mu}^B}{\chi_{1,\mu}^B}$$

$$S_B\sigma_B = \frac{\chi_{3,\mu}^B}{\chi_{2,\mu}^B}$$

$$\kappa_B\sigma_B^2 = \frac{\chi_{4,\mu}^B}{\chi_{2,\mu}^B}$$

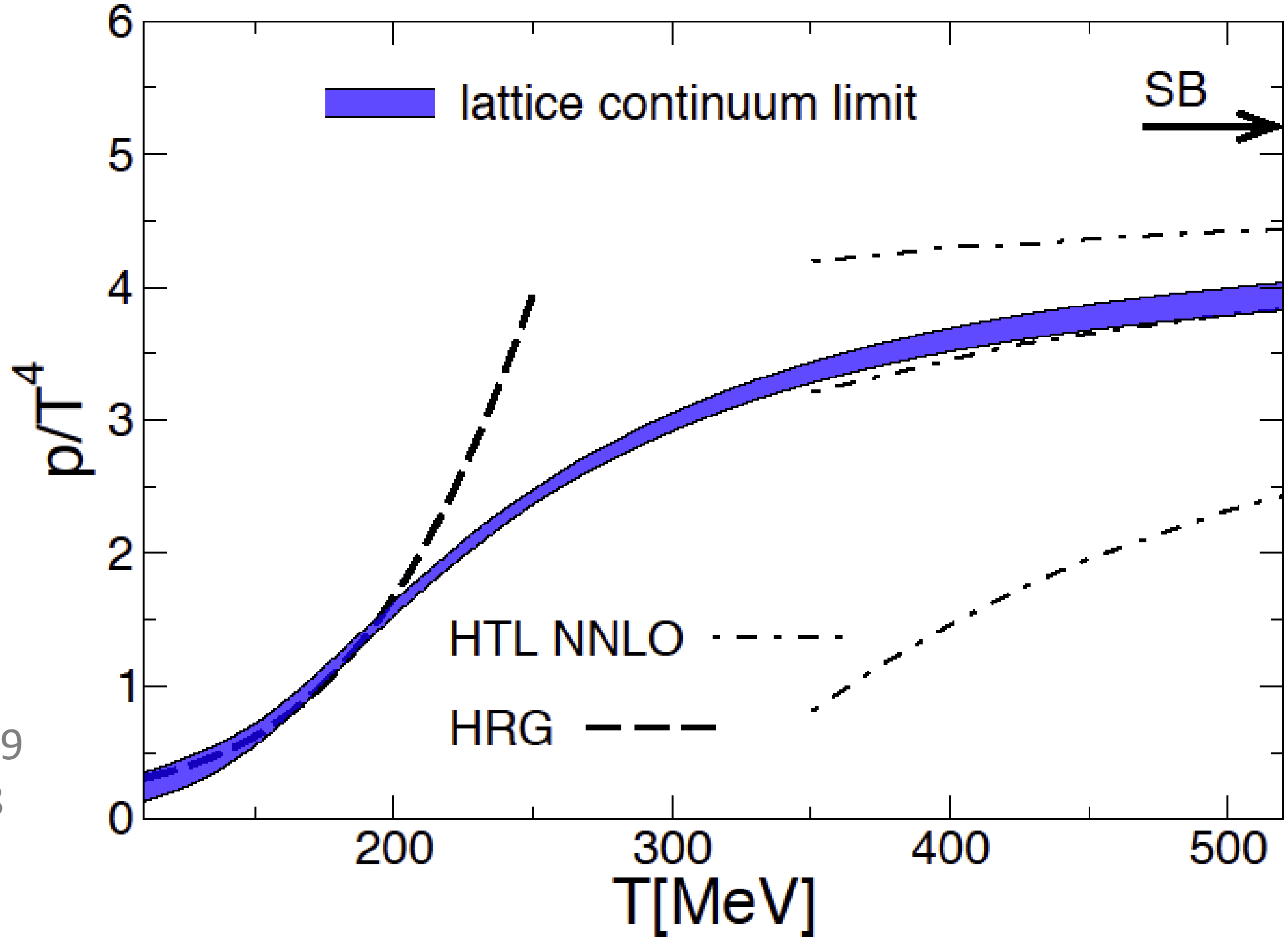
Conclusion and outlook

- Precision of LQCD results in thermodynamics greatly improved
- Crosschecks using ‘other discretizations’ are becoming available
- Fluctuations can be extracted up to higher orders (χ_6 very difficult)
- First LQCD based results on freeze-out conditions available
- New data from STAR (1402.1558)
  - Update of our results (PRL 111, 062005) pending
  - Check compatibility of μ_B based on B and Q

Thank You.

Backup

EoS: pressure et al.



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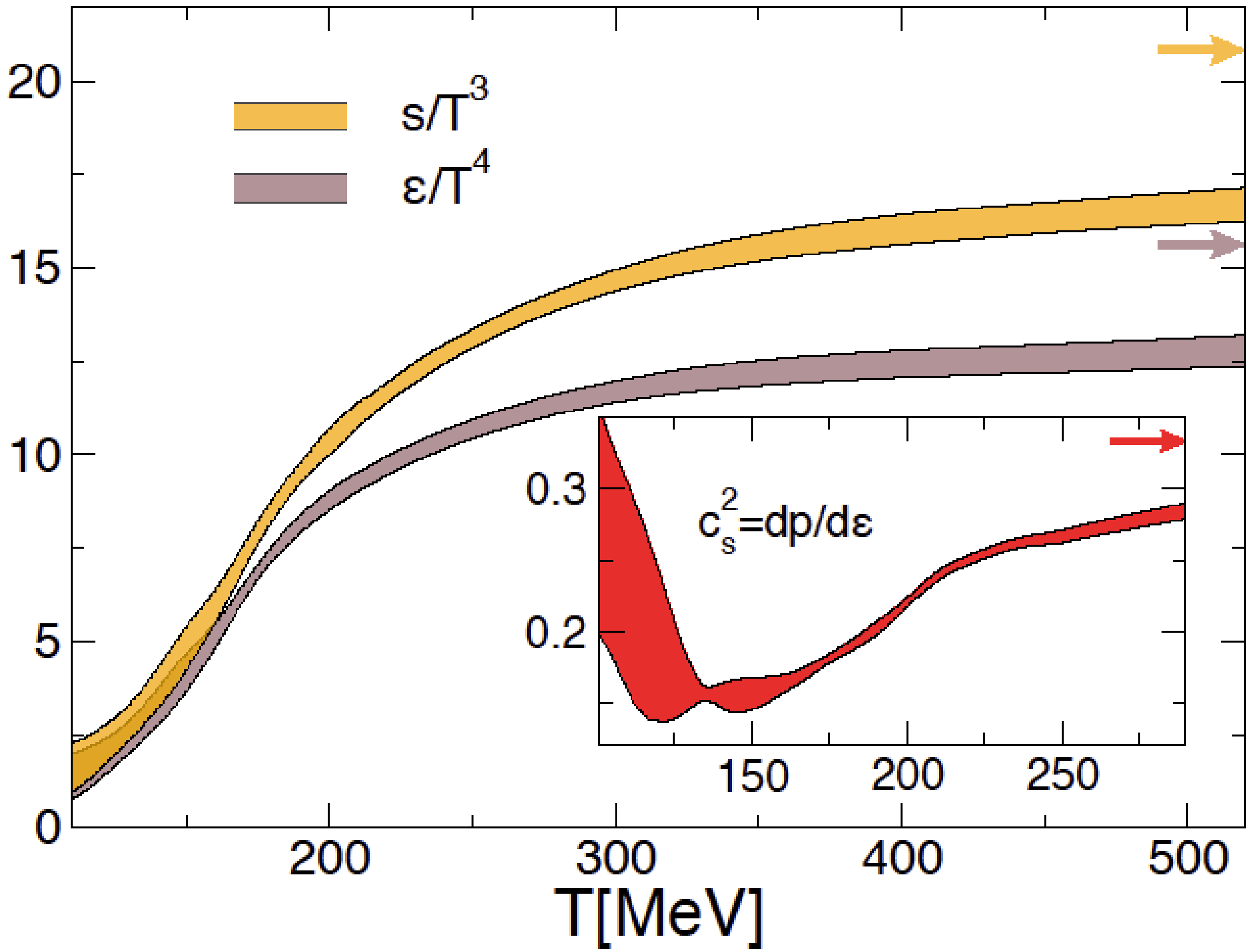
EoS: pressure et al.



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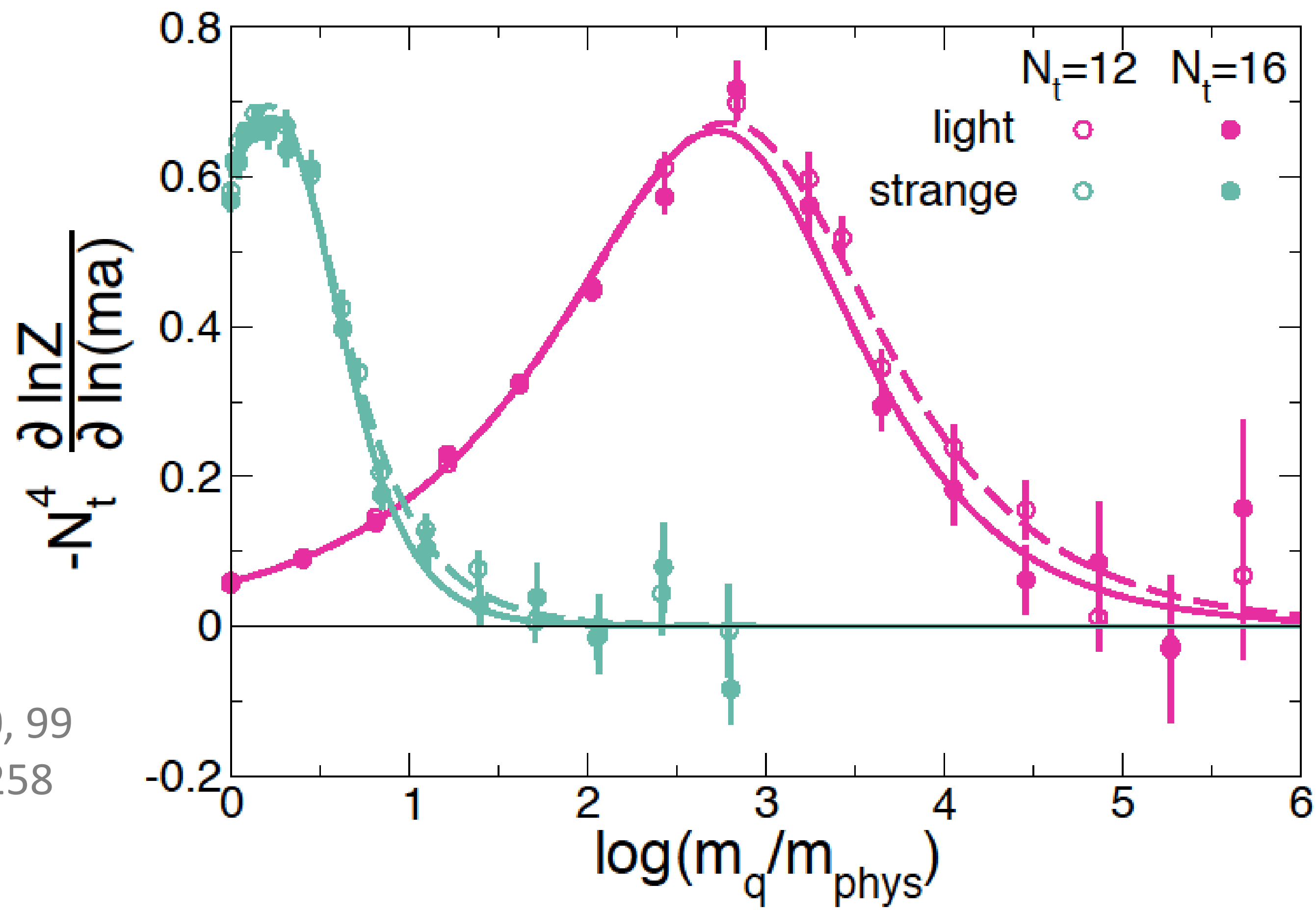
EoS: Normalization



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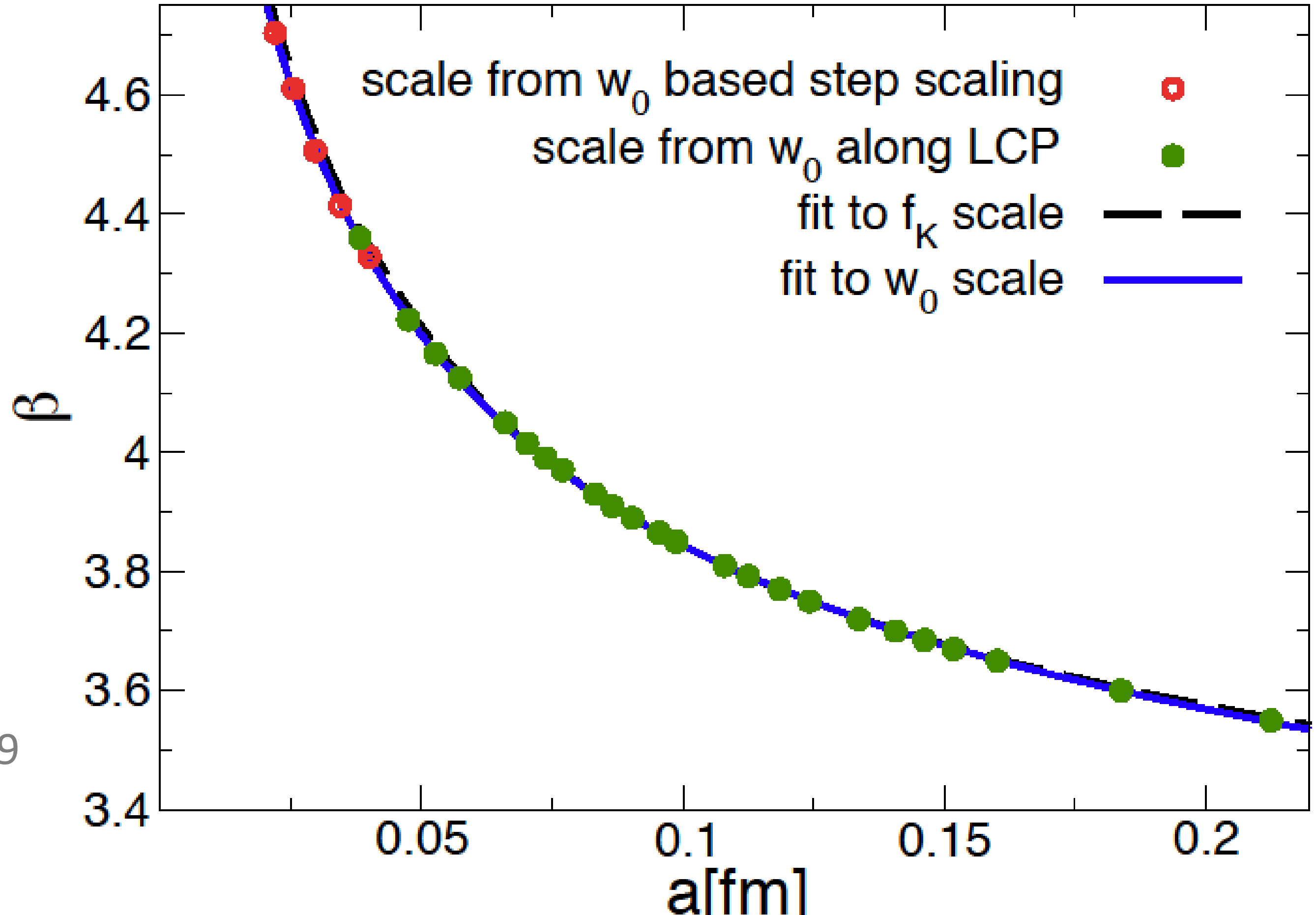
EoS: Scale setting



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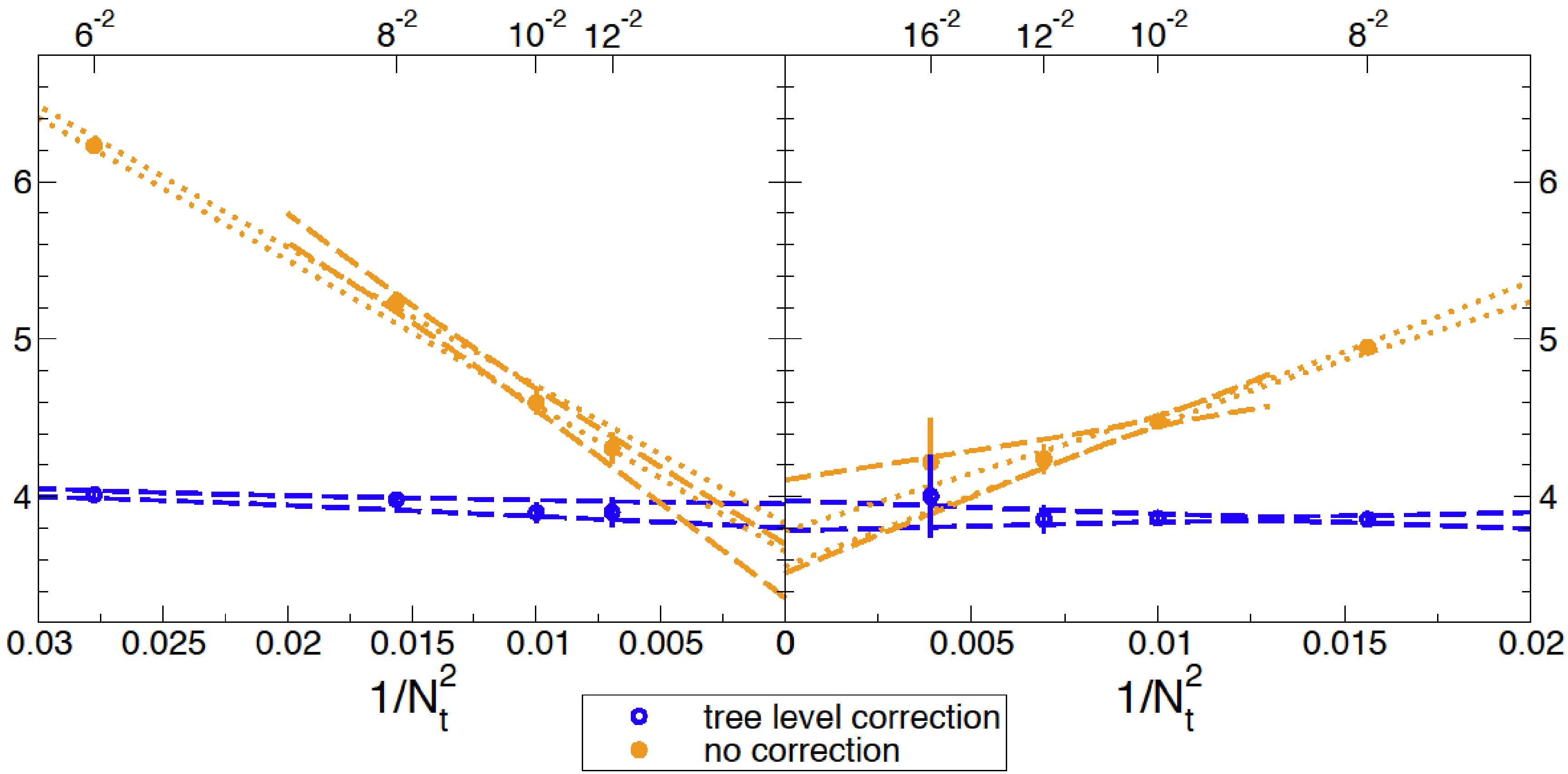
EoS: 214 MeV crosscheck



Fluctuations: quark mass basis

$$\mu_u = \frac{1}{3}\mu_B + \frac{2}{3}\mu_Q$$
$$\mu_d = \frac{1}{3}\mu_B - \frac{1}{3}\mu_Q$$
$$\mu_s = \frac{1}{3}\mu_B - \frac{1}{3}\mu_Q - \mu_S$$

- LQCD action is in ‘quark mass basis’
- express the chemical potentials in terms of flavor chemical potentials

$$\frac{d}{d\mu_B} = \frac{1}{3}\partial_u + \frac{1}{3}\partial_d + \frac{1}{3}\partial_s$$
$$\frac{d}{d\mu_Q} = \frac{2}{3}\partial_u - \frac{1}{3}\partial_d - \frac{1}{3}\partial_s$$
$$\frac{d}{d\mu_I} = \frac{1}{2}\partial_u - \frac{1}{2}\partial_d$$
$$\frac{d}{d\mu_S} = -\partial_s$$

- The interesting fluctuations require derivatives w.r.t. ‘rotated’ basis
- use above map to ‘rotate‘ derivatives appropriately

Fluctuations: derivatives

$$\chi^{us}_{i,j} = \frac{T}{V} \frac{\partial^{i+j} \log Z}{(\partial \mu_u)^i (\partial \mu_s)^j}$$

$$\partial_i \log Z = \frac{1}{Z} \int \mathcal{D}U \; \partial_i e^{-S_{\text{eff}}} = \langle A_i \rangle \qquad (=0 \text{ at } \mu = 0)$$

$$\begin{aligned} \partial_j \langle X \rangle &= -\langle X \rangle (\partial_j \log Z) + \left\langle X \partial_j e^{-S_{\text{eff}}} \right\rangle + \langle \partial_j X \rangle \\ &= \langle X A_j \rangle - \langle X \rangle \langle A_j \rangle + \langle \partial_j X \rangle \end{aligned}$$

$$\begin{aligned} \partial_i \partial_j \log Z &= \langle A_i A_j \rangle - \langle A_i \rangle \langle A_j \rangle + \delta_{ij} \langle B_i \rangle \\ \partial_i^4 \log Z &= \langle A_i^4 \rangle - 3 \langle A_i^2 \rangle^2 + 3 \left(\langle B_i^2 \rangle - \langle B_i \rangle^2 \right) \\ &\quad +6 \left(\langle A_i^2 B_i \rangle - \langle A_i^2 \rangle \langle B_i \rangle \right) + 4 \langle A_i C_i \rangle + \langle D_i \rangle \end{aligned}$$

Fluctuations: lattice techniques

- Use SET to estimate traces (Gottlieb et al. PRL 59 (**1987**) 2247)

$$\texttt{tr}(A) \approx \frac{1}{s}\sum_{k=1}^{s} v_k^\top A v_k$$

- Disconnected diagrams $\rightarrow$ use different sets of random vectors ($c_4 \rightarrow 4\times$ inversions)
- diagrams (e.g. $\langle A_i^2\rangle$ and $\langle A_i^2 \cdot B\rangle$) allow reuse of inversions $\rightarrow$ increase statistics
- We use up to 1000 sources to estimate the trace or O(10k) inversions