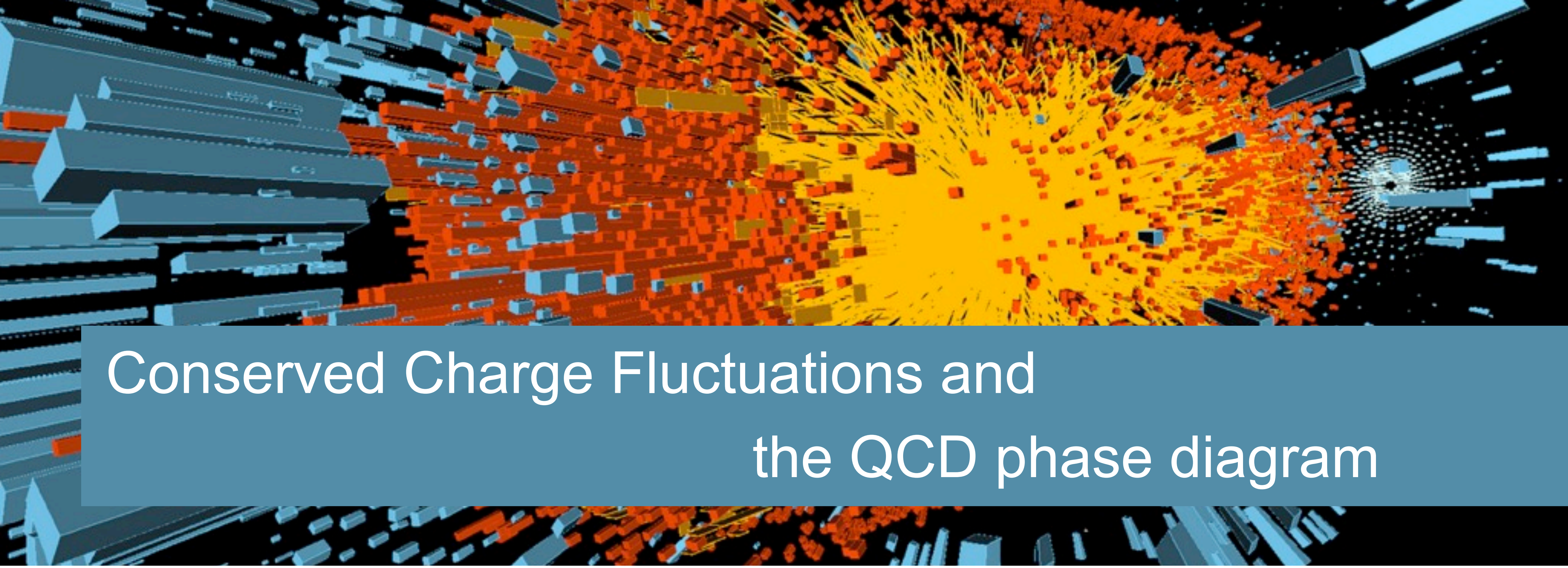

# Conserved Charge Fluctuations and the QCD phase diagram

Christian Schmidt

Universität Bielefeld

# Critical Point and Onset of Deconfinement

# CPOD 2014

## University of Bielefeld, Germany
## November 17-21, 2014

**Scientific Topics:**

- Critical Point
- Phase transitions in hot and dense matter
- Deconfinement and chiral symmetry restoration
- Hadronization and chemical freeze-out
- Compact Stars
- Future facilities, detectors and methods

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# Motivation: Extending lattice QCD to $\mu_B \neq 0$

**Expected phase diagram of QCD:**

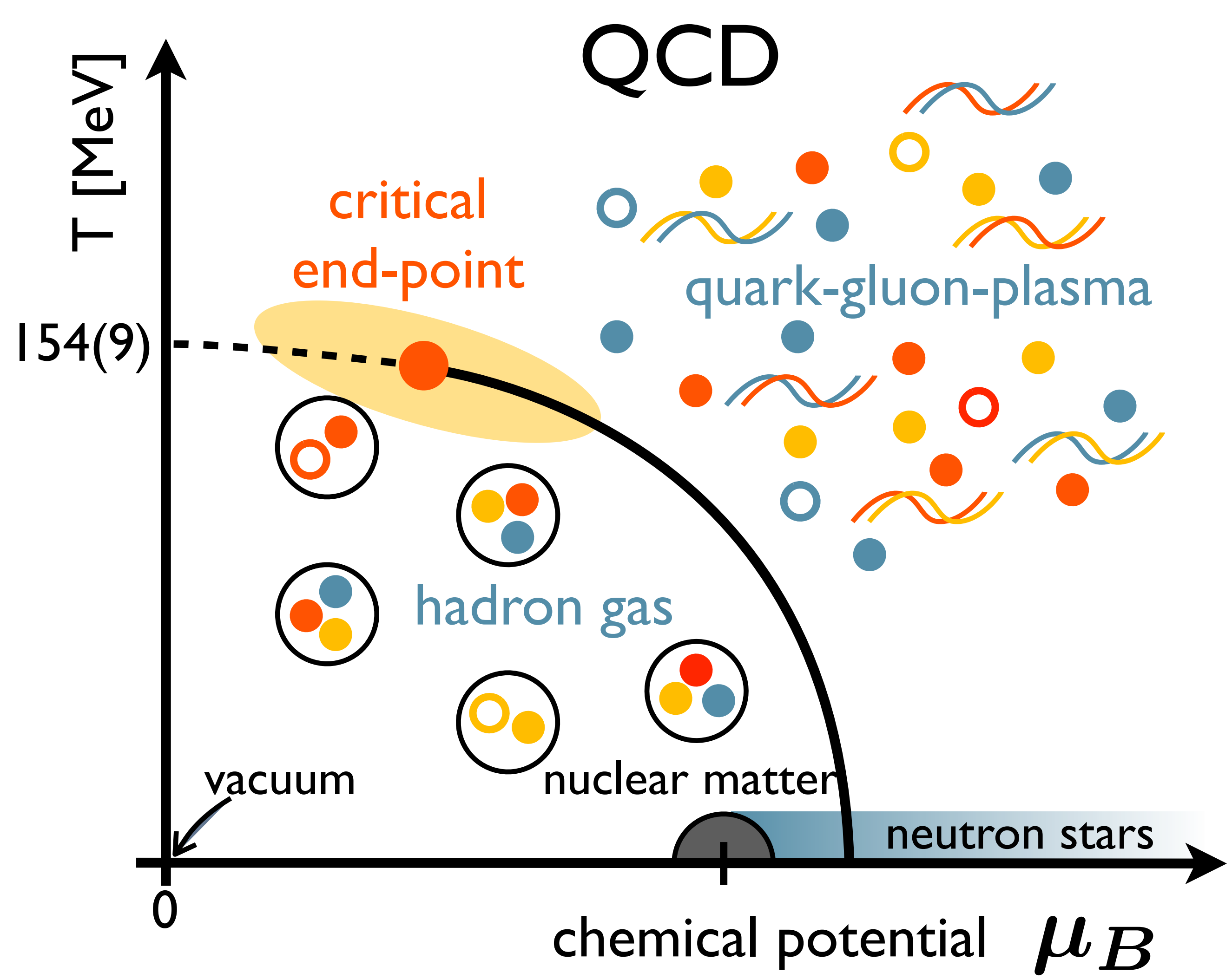
**What are the Phases of QCD ?**

1) hadronic states at low T, low densities
2) quasi-free quarks and gluons at high T and/or high densities

**What are the underlying mechanisms ?**

1) spontaneous chiral symmetry breaking
2) (de-)confinement

**Is there a critical end-point?**



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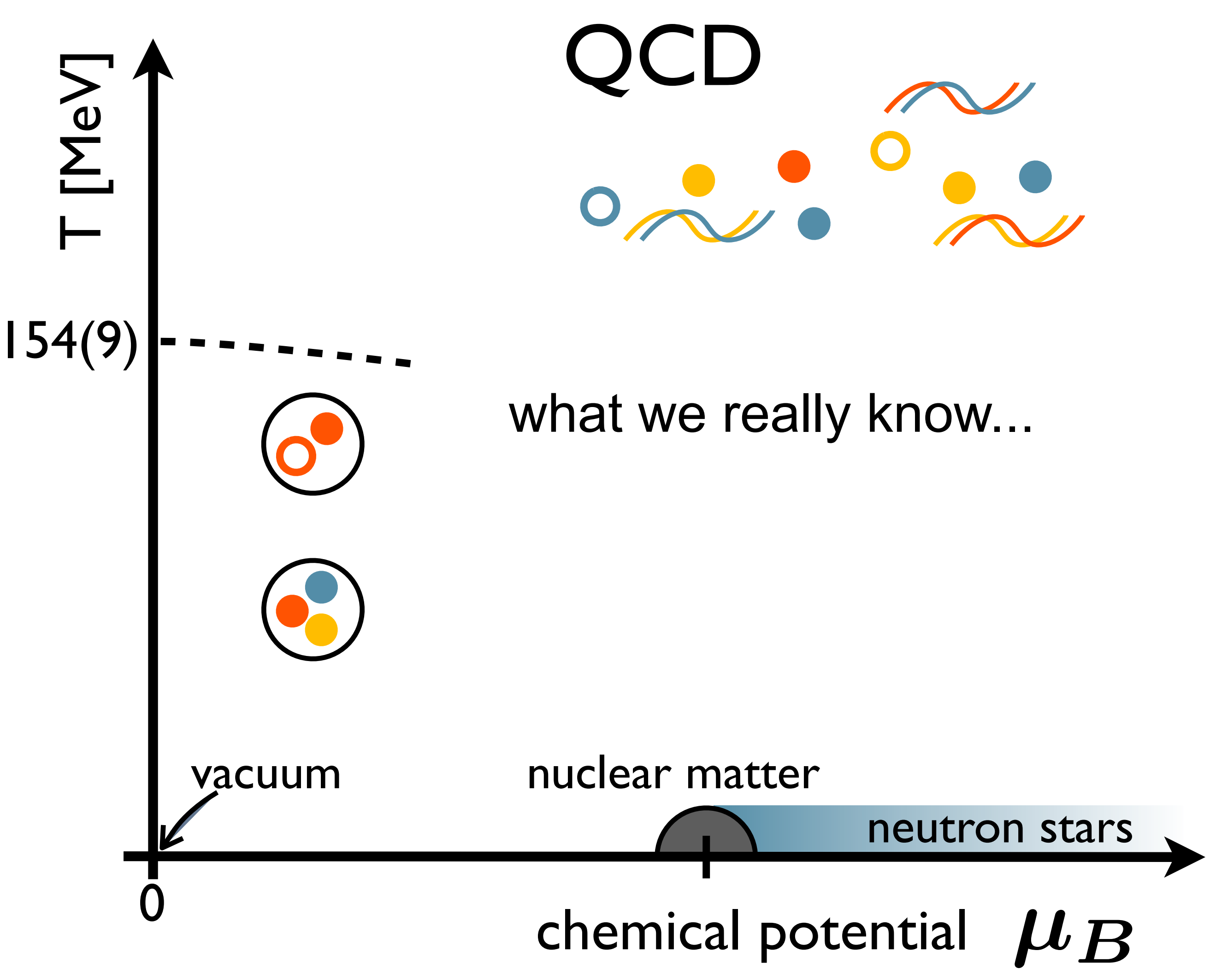

# Problem: The complex determinant

- Integrating out the fermion fields

$$Z(V,T,\mu) = \int \mathcal{D}A\mathcal{D}\psi\mathcal{D}\bar{\psi}\ \exp\{S_F(A,\psi,\bar{\psi}) - \beta S_G(A)\}$$

$$= \int \mathcal{D}A\ \det[M](A,\mu)\exp\{-\beta S_G(A)\}$$

complex for $\mu > 0$

no probabilistic interpretation!
no MC with importance sampling!

we find: $[\det M(\mu)]^* = \det M(-\mu^*)$

$\longrightarrow$ determinant is real only for

$\mu = 0$ or $\mu = i\mu_I$

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→ new variables ?

→ new algorithms ?

# (Some) Approaches to nonzero density QCD

**Taylor Expansion**
Bielefeld-Swansea, 2002

**Complex Langevin**
Karsch, Wyld 1985
Aarts *et al.*, 2009
Sexty 2013

**imaginary chem. pot.**
deForcarnd, Philipsen, 2002
D'Elia, Lomardo 2003

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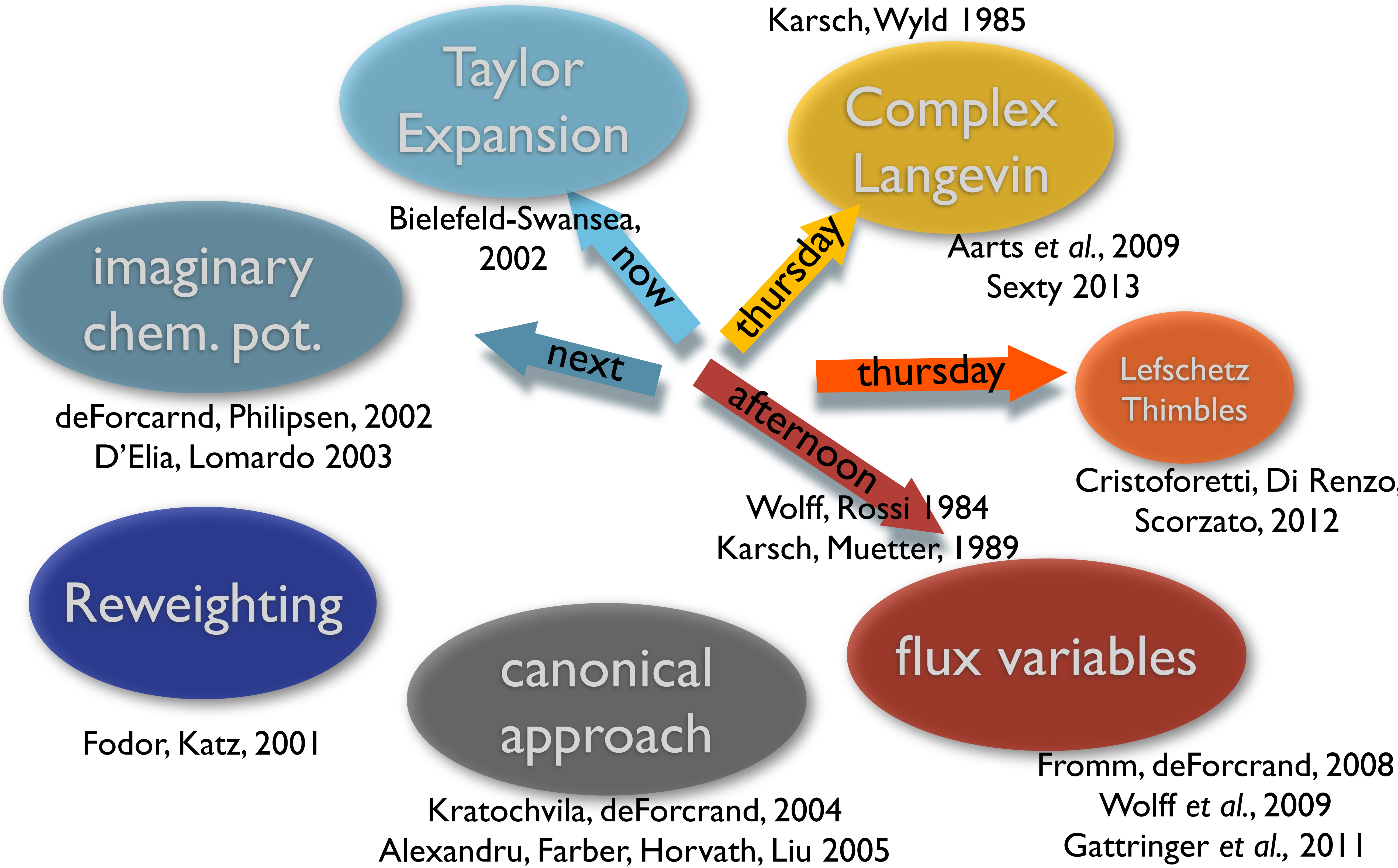
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now
thursday
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# Outline

## 1) Methodology
- The Taylor Expansion
- Definitions of cumulants and correlations

## 2) The lattice setup and results
- HISQ action, parameter, numerical methods, ...
- results: cumulants and correlations up to 6th order

## 3) QCD critical behavior
- critical points in the QCD phase diagram
- sensitivity of cumulants to universal scaling

## 4) Summary

**BNL-Bielefeld Collaboration:**
A. Bazavov, H.-T. Ding, P. Hegde, O. Kaczmarek, F. Karsch, E. Laermann,
S. Mukherjee, P. Petreczky, C. Schmidt, S. Sharma, W. Soeldner, M. Wagner

# Generalized Susceptibilites vs Cumulants

- derivatives of ***lnZ*** with respect to $\mu_{B,Q,S}$ can also be studied in **heavy ion collisions**

Expansion of the pressure:

$$\frac{p}{T^4} = \sum_{i,j,k=0}^{\infty} \frac{1}{i!j!k!} \chi_{ijk,0}^{BQS} \left(\frac{\mu_B}{T}\right)^i \left(\frac{\mu_Q}{T}\right)^j \left(\frac{\mu_S}{T}\right)^k$$

$X = B, Q, S$: conserved charges

**Lattice**

$$\chi_{n,0}^{X} = \frac{1}{VT^3} \frac{\partial^n \ln Z}{\partial(\mu_X/T)^n}\bigg|_{\mu_X=0}$$

generalized susceptibilities

$\Rightarrow$ only at $\mu_X = 0$ !

**Experiment**

$$\begin{aligned}
VT^3\,\chi_2^X &= \left\langle (\delta N_X)^2 \right\rangle \\
VT^3\,\chi_4^X &= \left\langle (\delta N_X)^4 \right\rangle - 3\left\langle (\delta N_X)^2 \right\rangle^2 \\
VT^3\,\chi_6^X &= \left\langle (\delta N_X)^6 \right\rangle \\
&\quad -15\left\langle (\delta N_X)^4 \right\rangle \left\langle (\delta N_X)^2 \right\rangle \\
&\quad +30\left\langle (\delta N_X)^2 \right\rangle^3
\end{aligned}$$

cumulants of net-charge fluctuations

$$\delta N_X \equiv N_X - \langle N_X \rangle$$

$\Rightarrow$ only at freeze-out $(\mu_f(\sqrt{s}), T_f(\sqrt{s}))$!

# Important Applications

**1) Explore the QCD phase diagram**

- Analyze higher order cumulants and test universal scaling behavior

  $\longrightarrow$ make prediction on the radius of convergence and possible experimental observables

(this talk)

**2) Analyze freeze-out conditions**

- Match various cumulant ratios of measured fluctuations to QCD

  $\longrightarrow$ determine freeze-out parameter

see BNL-Bielefeld, PRL 109 (2012) 192302.

**3) Identify the relevant degrees of freedom**

- Compare (lattice) QCD fluctuations to various hadronic/quasiparticle models:

  $\longrightarrow$ deconfinement vs. chiral transition

  $\longrightarrow$ dissolution of boundstates

$\longrightarrow$ talk by Sayantan Sharma

see BNL-Bielefeld, PRL 111 (2013) 082301.

# The Lattice Setup

**Action:** highly improved staggered quarks (HISQ)

**Lattice size:** $24^3 \times 6,\ 32^3 \times 8,\ 48^3 \times 12$

**Mass:** $m_q = m_s/20 \longrightarrow m_\pi \approx 160\ \mathrm{MeV}$

**Statistics:** $O(10^3)$ - $O(10^4)$

**Observables:** traces of combinations of $M$ and $M' = \partial M/\partial\mu$

$$\frac{\partial \ln Z}{\partial \mu} = \frac{1}{Z}\int \mathcal{D}U\ \mathrm{Tr}\left[M^{-1}M'\right]\ e^{\mathrm{Tr}\ \ln M} e^{-\beta S_G}$$

$$= \left\langle \mathrm{Tr}\left[M^{-1}M'\right]\right\rangle$$

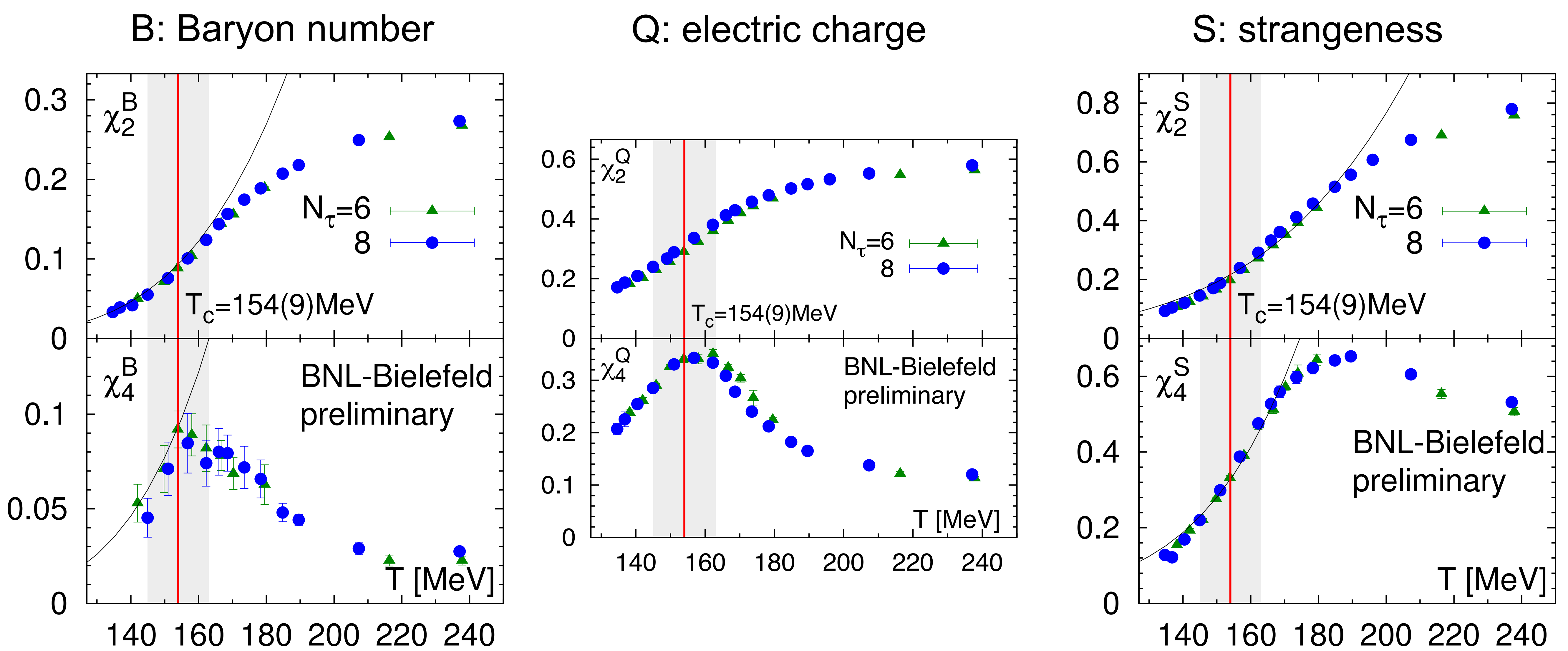
$$\frac{\partial^2 \ln Z}{\partial \mu^2} = \left\langle \mathrm{Tr}\left[M^{-1}M''\right]\right\rangle - \left\langle \mathrm{Tr}\left[M^{-1}M'M^{-1}\right]\right\rangle + \left\langle \mathrm{Tr}\left[M^{-1}M'\right]^2\right\rangle$$

**Method:** stochastic estimators with $N = 1500$ random vectors

$$\mathrm{Tr}\left[Q\right] \approx \frac{1}{N}\sum_{i=1}^{N} \eta_i^\dagger Q \eta_i \quad \text{with} \quad \lim_{N\to\infty} \frac{1}{N}\sum_{i=1}^{N} \eta_{i,x}^\dagger \eta_{i,y} = \delta_{x,y}$$

# Results: fluctuations up to 4th order

**BQS diagonal cumulants**

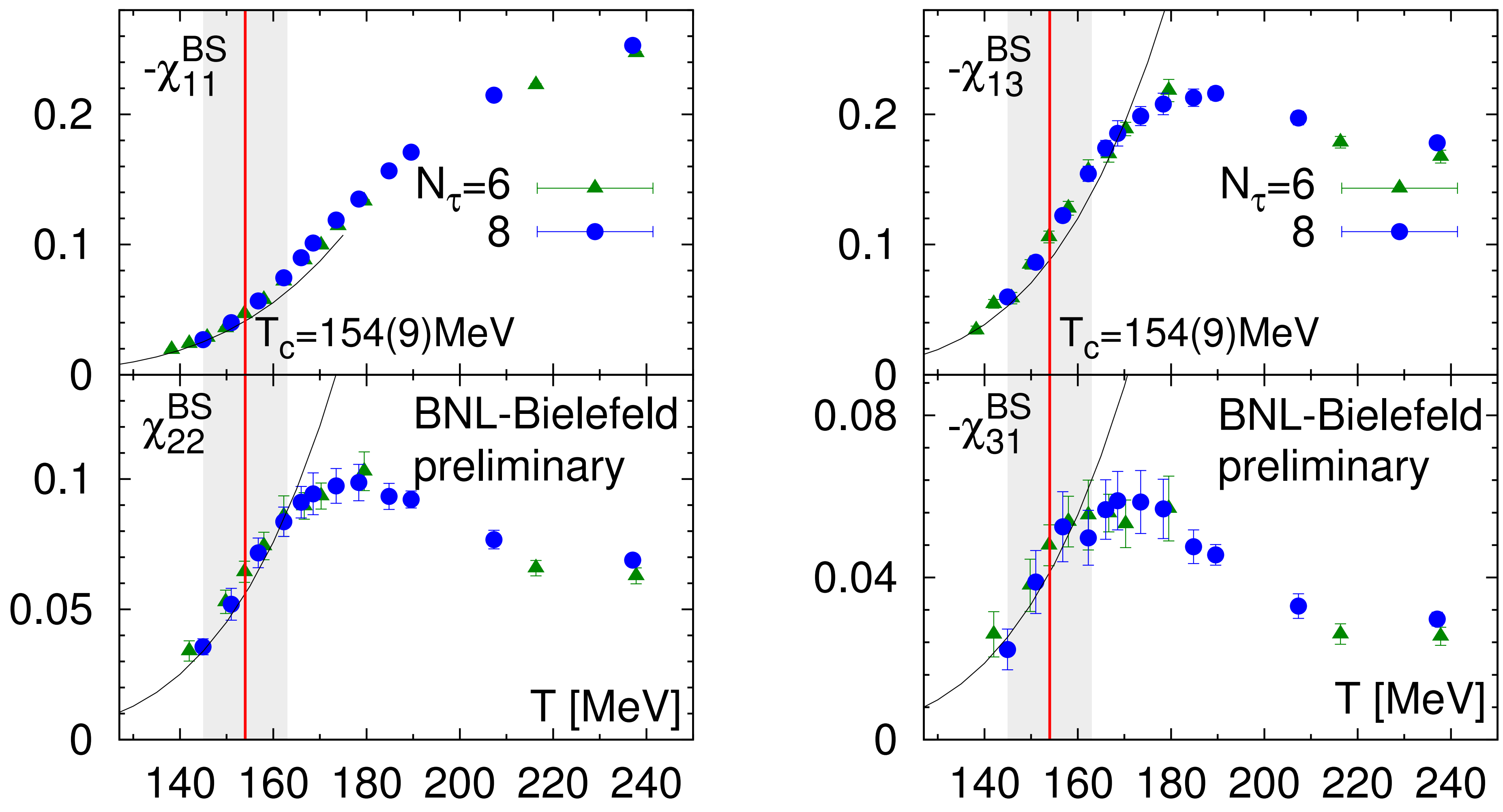



⇒ statistical and systematical errors are under control

⇒ find good agreement with HRG model for T<150 MeV
(electric charge fluctuations are more tricky, due to contributions from the light pions)

# Results: fluctuations up to 4th order

**BQS off-diagonal cumulants**




⇒ statistical and systematical errors are under control

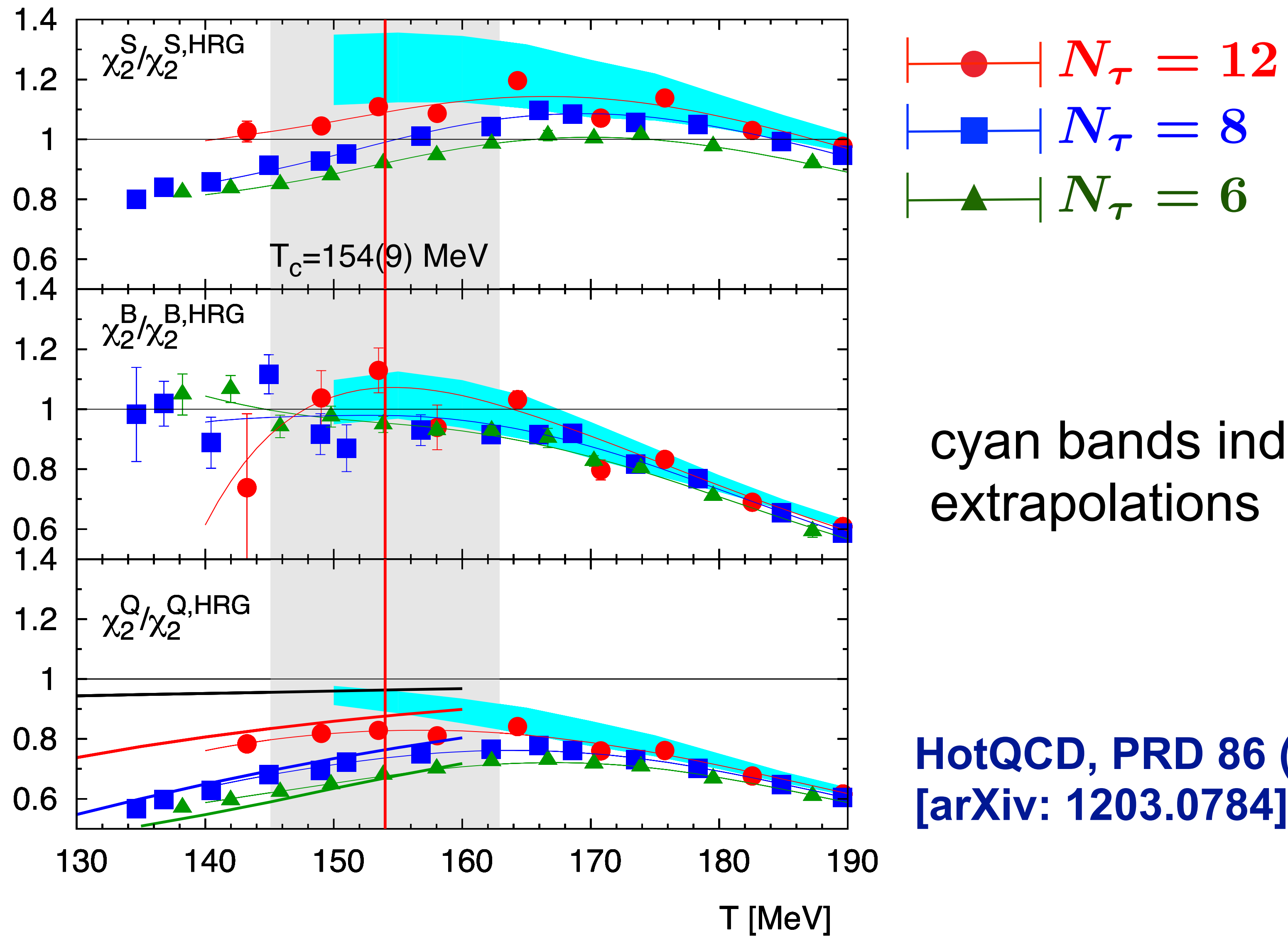
⇒ find good agreement with HRG model for T<150 MeV

# Results: aproach to the HRG

**HRG:**

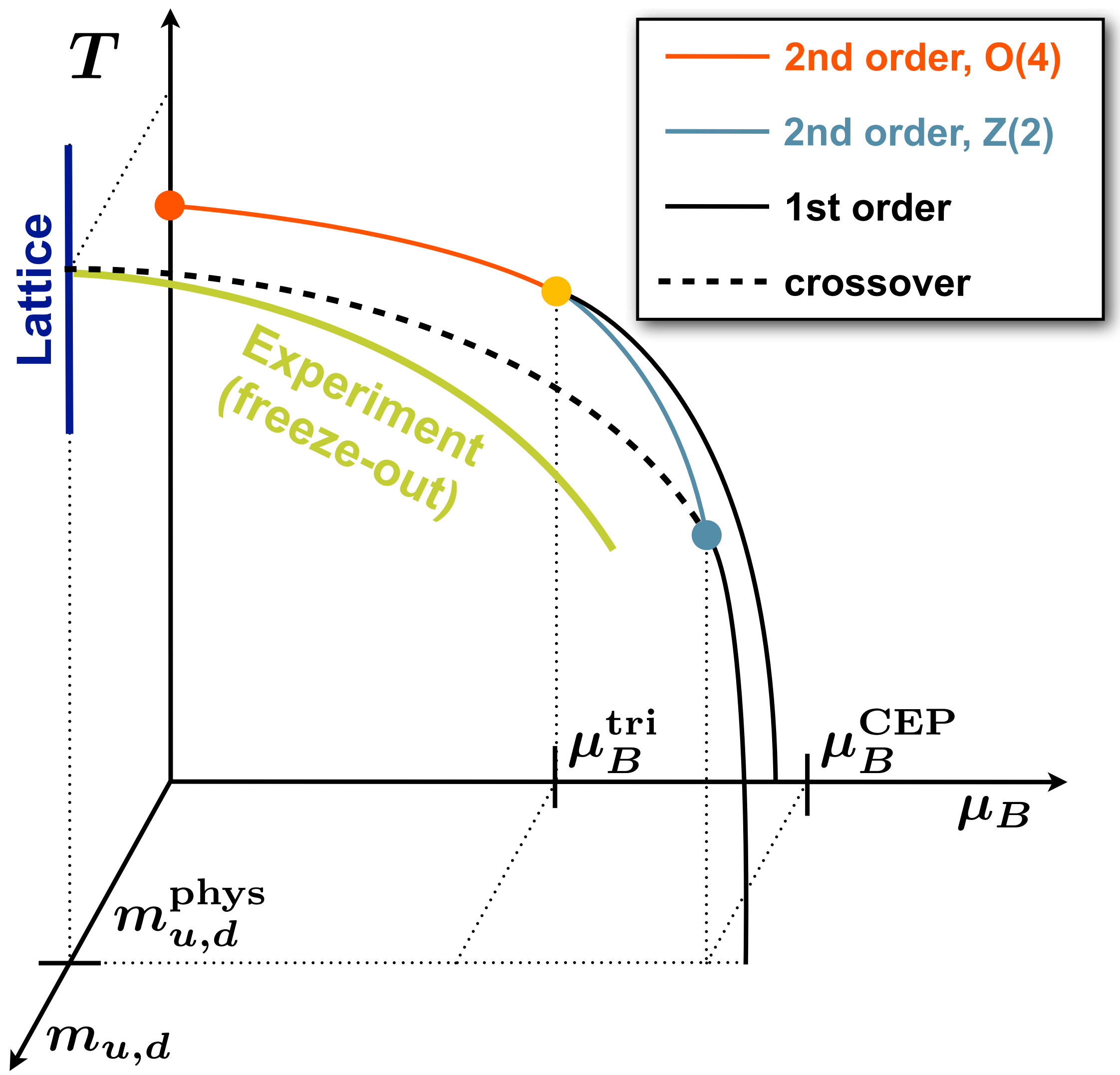
$$\frac{p^{HRG}}{T^4} = \frac{1}{VT^3} \sum_{i \in\, mesons} \ln \mathcal{Z}^M_{m_i}(T,V,\mu_S) + \frac{1}{VT^3} \sum_{i \in\, baryons} \ln \mathcal{Z}^B_{m_i}(T,V,\mu_B,\mu_S)$$

hadron massed from PDG up to 2.5 GeV




cyan bands indicate continuum extrapolations

**HotQCD, PRD 86 (2012) 034509 [arXiv: 1203.0784]**




assume scaling hypothesis for the free energy:

$$f = f_s(t,h) + \text{regular}$$

in the chiral limit (h=0):

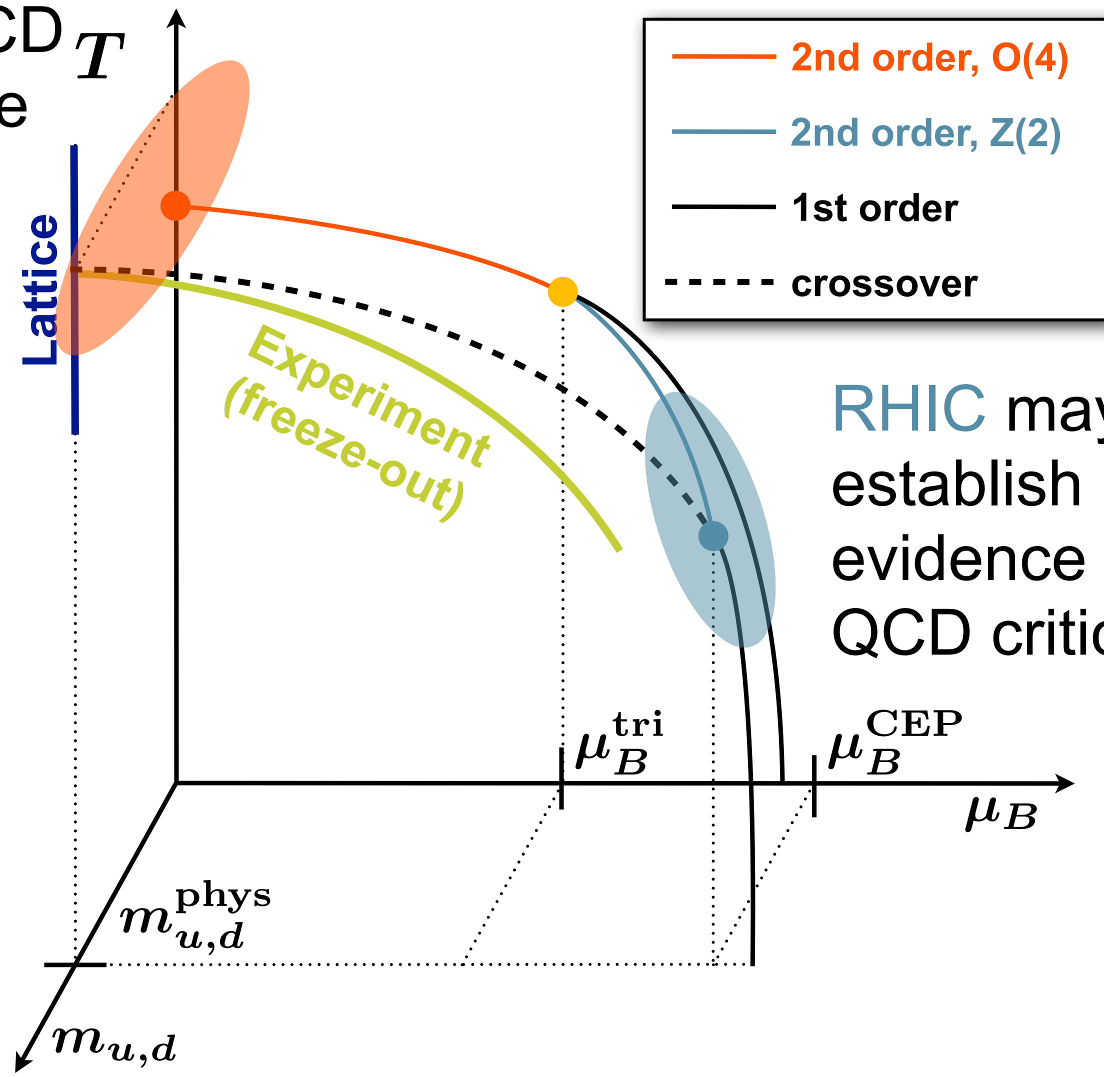
$$f \sim A_{\pm} |t|^{2-\alpha} + \text{regular}$$

| | $\alpha$ |
|---|---|
| O(4) | -0.213 |
| Z(2) | +0.107 |

$\Rightarrow$ at $\mu_B = 0$, 4th order cumulants develop a cusp, 6th oder cumulants diverge

# QCD critical behavior

LHC may establish contact with the QCD chiral phase transition




RHIC may establish evidence for a QCD critical point

assume scaling hypothesis for the free energy:

$$f = f_s(t,h) + \text{regular}$$

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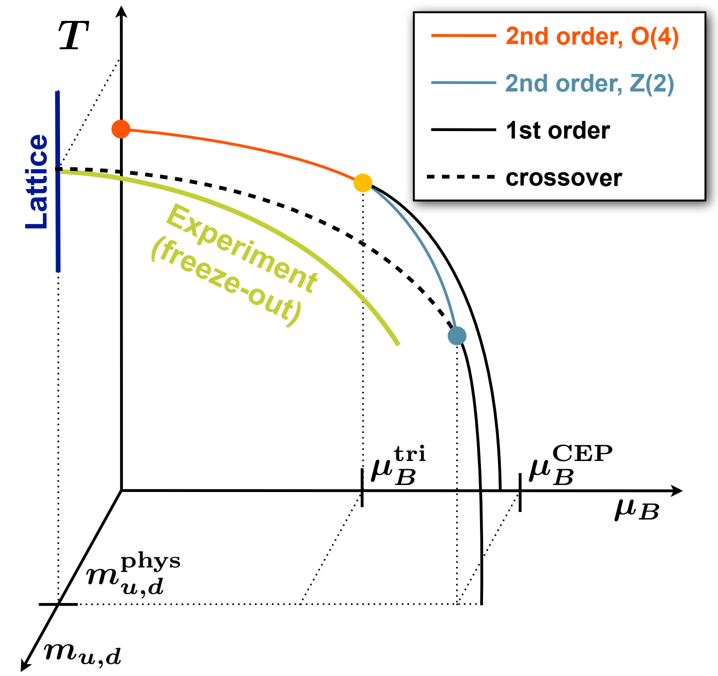
$\Rightarrow$ analyze universal scaling behavior

# QCD critical behavior




matching scaling fields to QCD at $\mu_B = 0$:

$$h = \frac{m_q}{h_0} \qquad t = \frac{1}{t_0}\left(\left(\frac{T-T_c}{T_c}\right) + \kappa\left(\frac{\mu_B}{T}\right)^2\right)$$

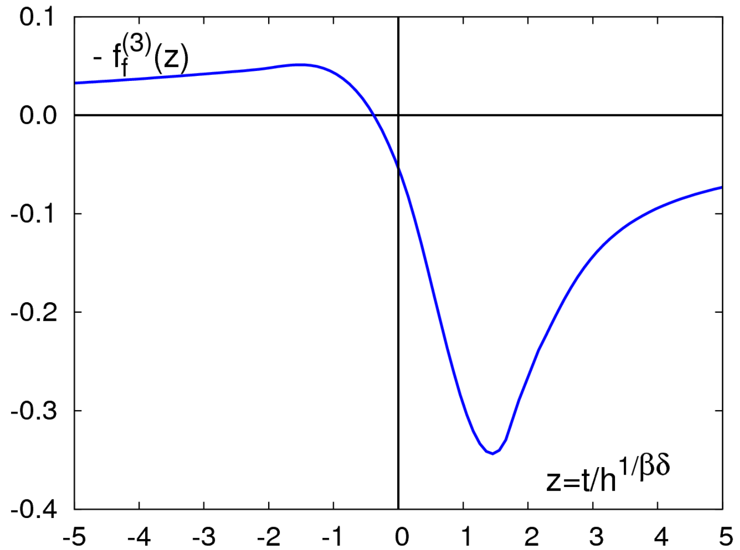
controlled by non-universal normalization constants $t_0, h_0, \kappa$

$$\frac{p}{T^4} = -h^{(2-\alpha)/\beta\delta} f_f(t/h^{1/\beta\delta}) - f_r(V,T,\vec{\mu})$$

(universal scaling function) (regular part)

$\Rightarrow$ critical behavior of cumulants:

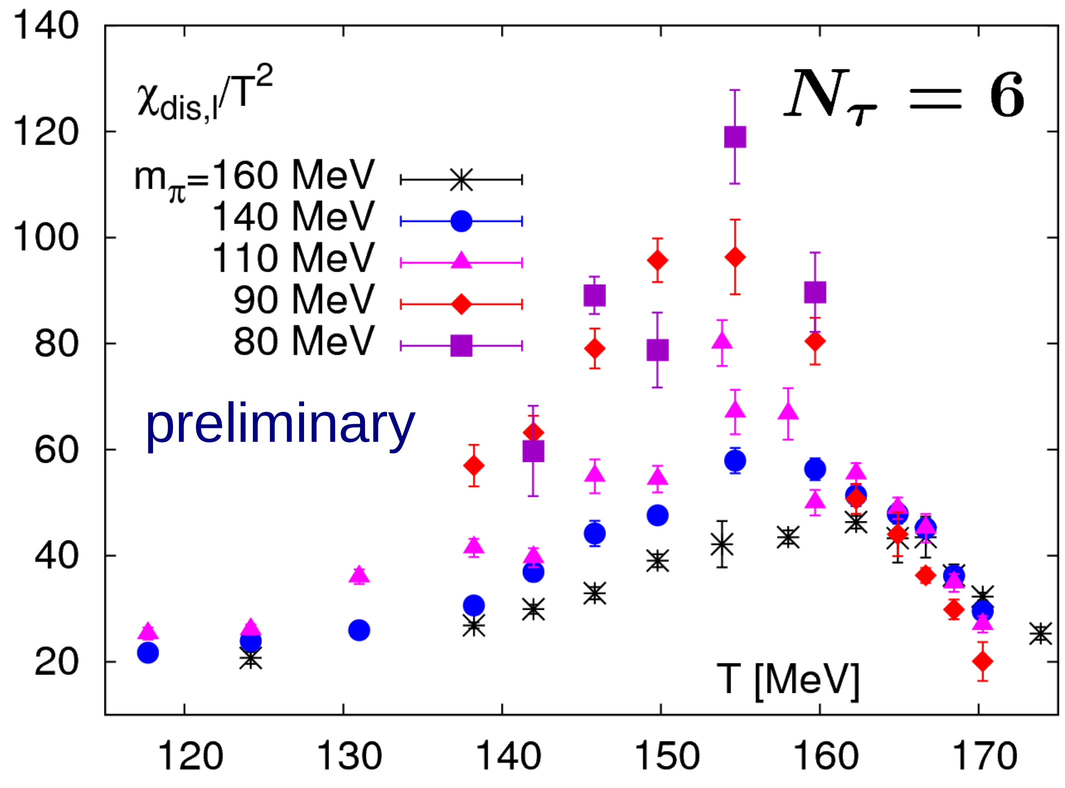
$$\chi_B^{(n)} \sim m_q^{(2-\alpha-n/2)/\beta\delta} f_f^{(n/2)}(t/h^{1/\beta\delta})$$



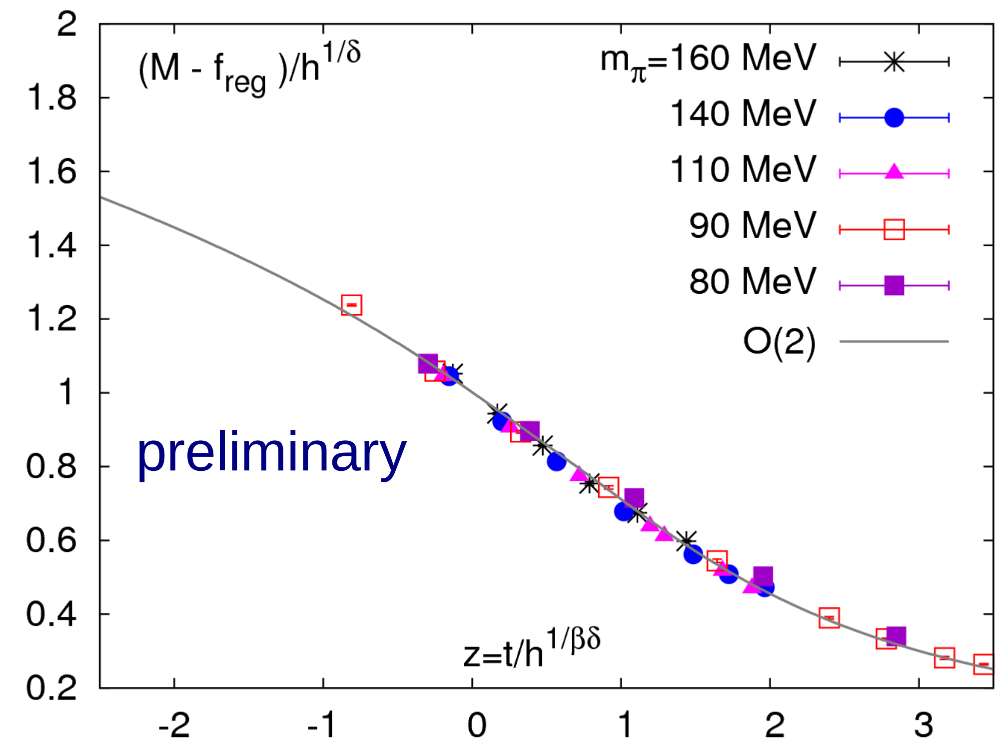

Karsch, Engels, PRD 85 (2012) 094506

# QCD critical behavior

evidence for universal scaling behavior with HISQ from chiral condensate and chiral susceptibility (H.-T. Ding et al. Lattice 2013)




$$\chi_{dis}/T^2 \sim m_\pi^{2(1/\delta-1)}$$

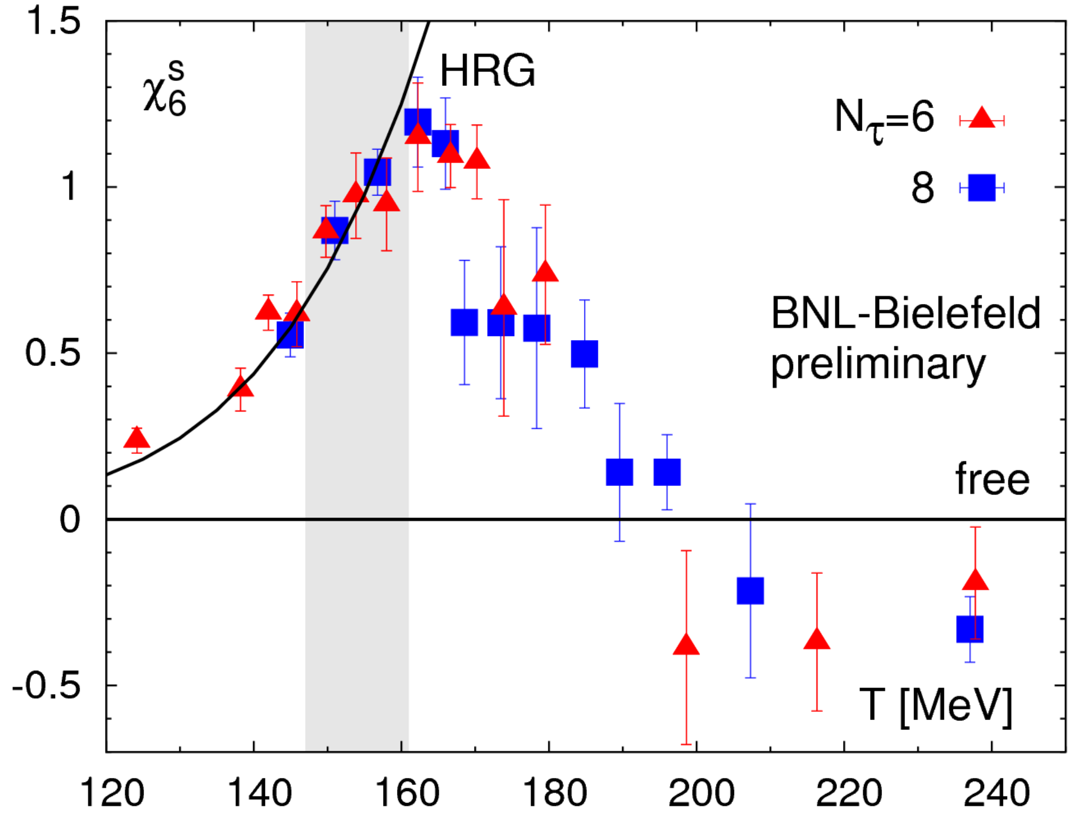



$$M = m_s \langle \bar{\psi}\psi \rangle / T^4$$

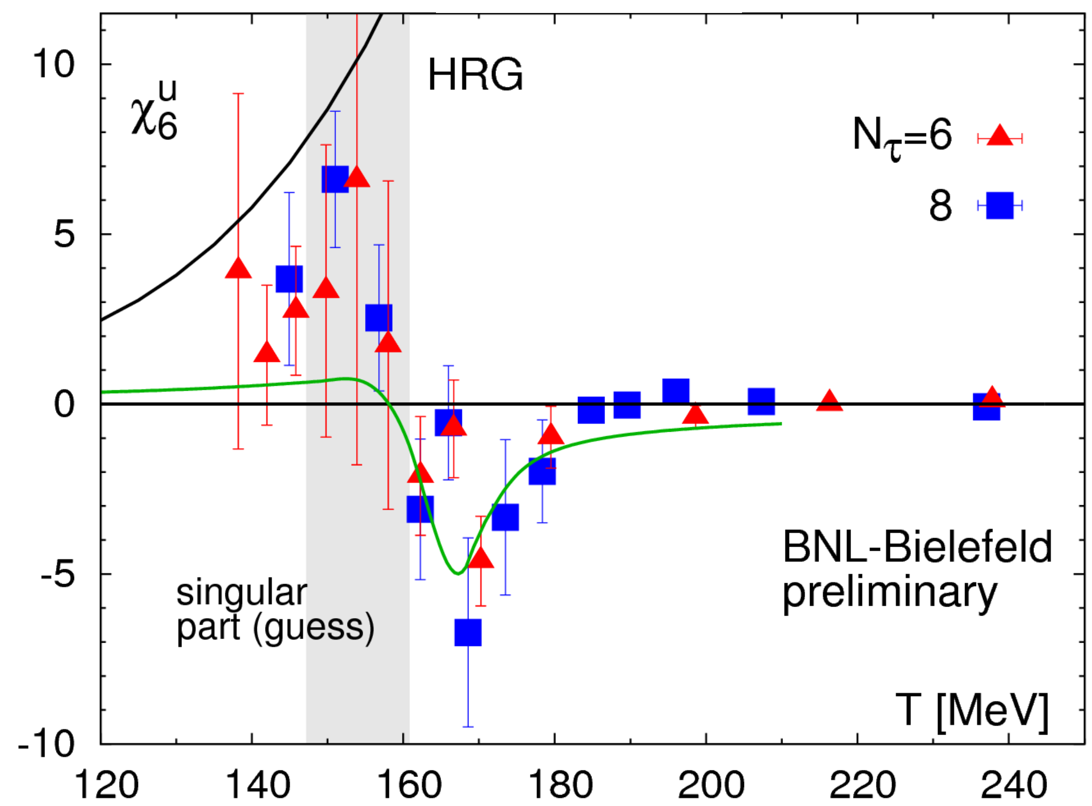
$\Rightarrow$ scaling region extents to physical pion mass

# QCD critical behavior

strange




light




⇒ no evidence for typical O(4) singular structure

⇒ clear evidence for typical O(4) singular structure

⇒ regular contribution dominates

⇒ regular and singular contribution

# QCD critical behavior

some universal numbers:

width of the transition region (as seen by $\chi_6^B$):

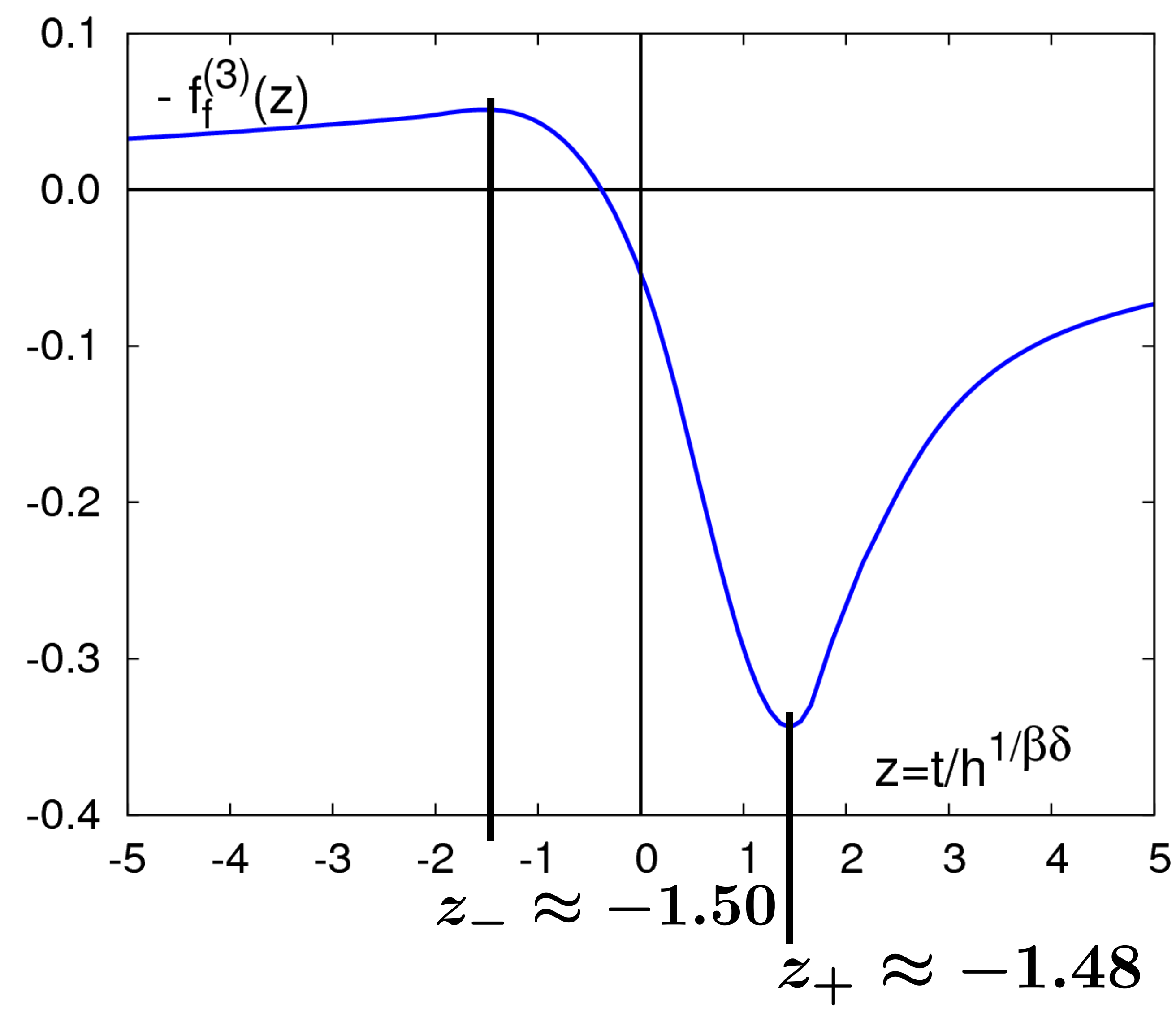
$$\Delta z = z_+ - z_- \approx 3$$

$$\Rightarrow$$

$$T_+ - T_- = \frac{1}{\Delta z} \frac{t_0 T_c}{h_0^{1/\beta\delta}} \left( \frac{m_l}{m_s} \right)^{1/\beta\delta}$$

at the physical point:

$$T_+ - T_- \approx 0.2 T_c$$

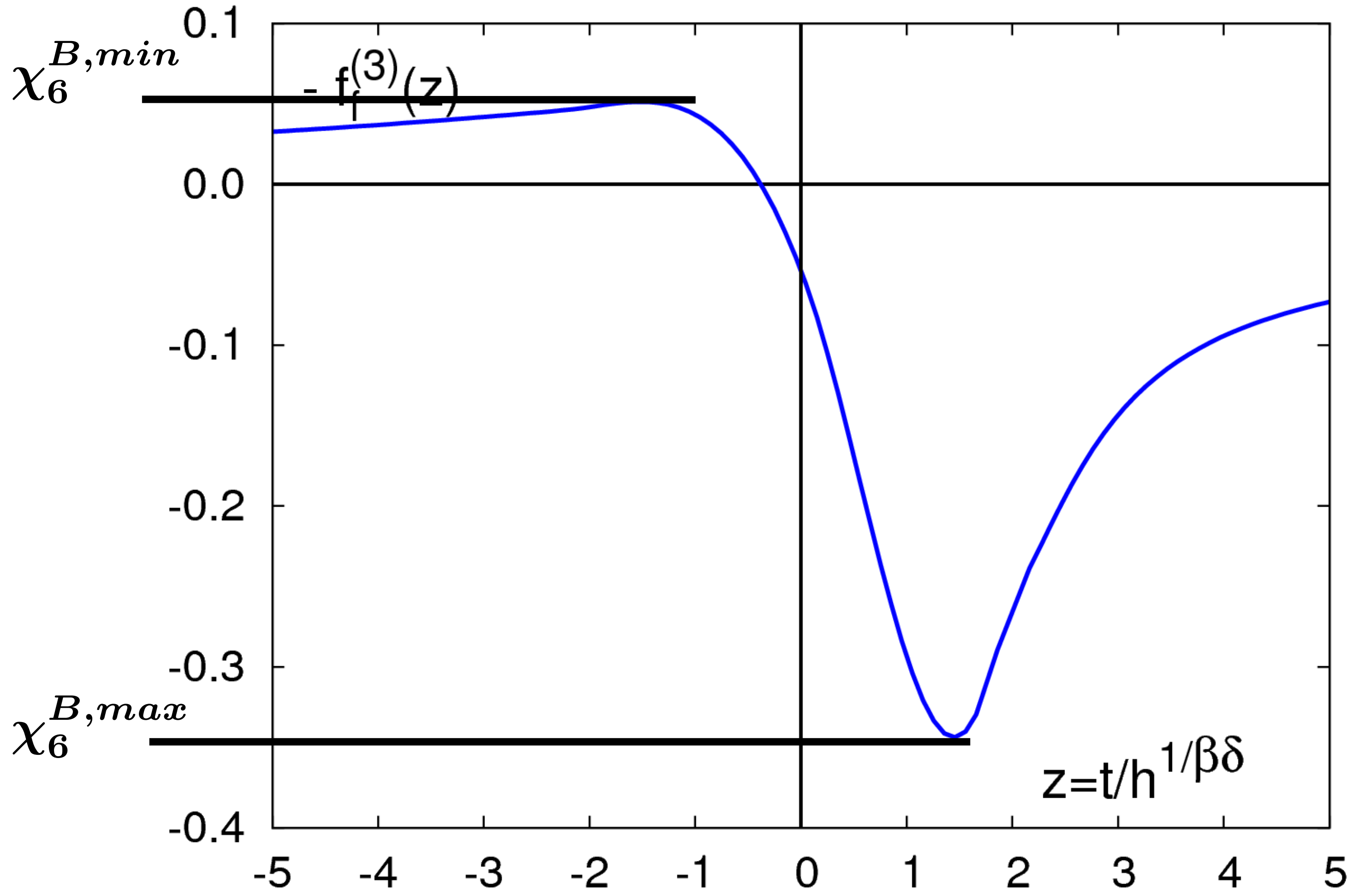


some universal numbers:

ratio of minimum to maximum:

$$\chi_6^{B,min}/\chi_6^{B,max} \approx -6.7$$

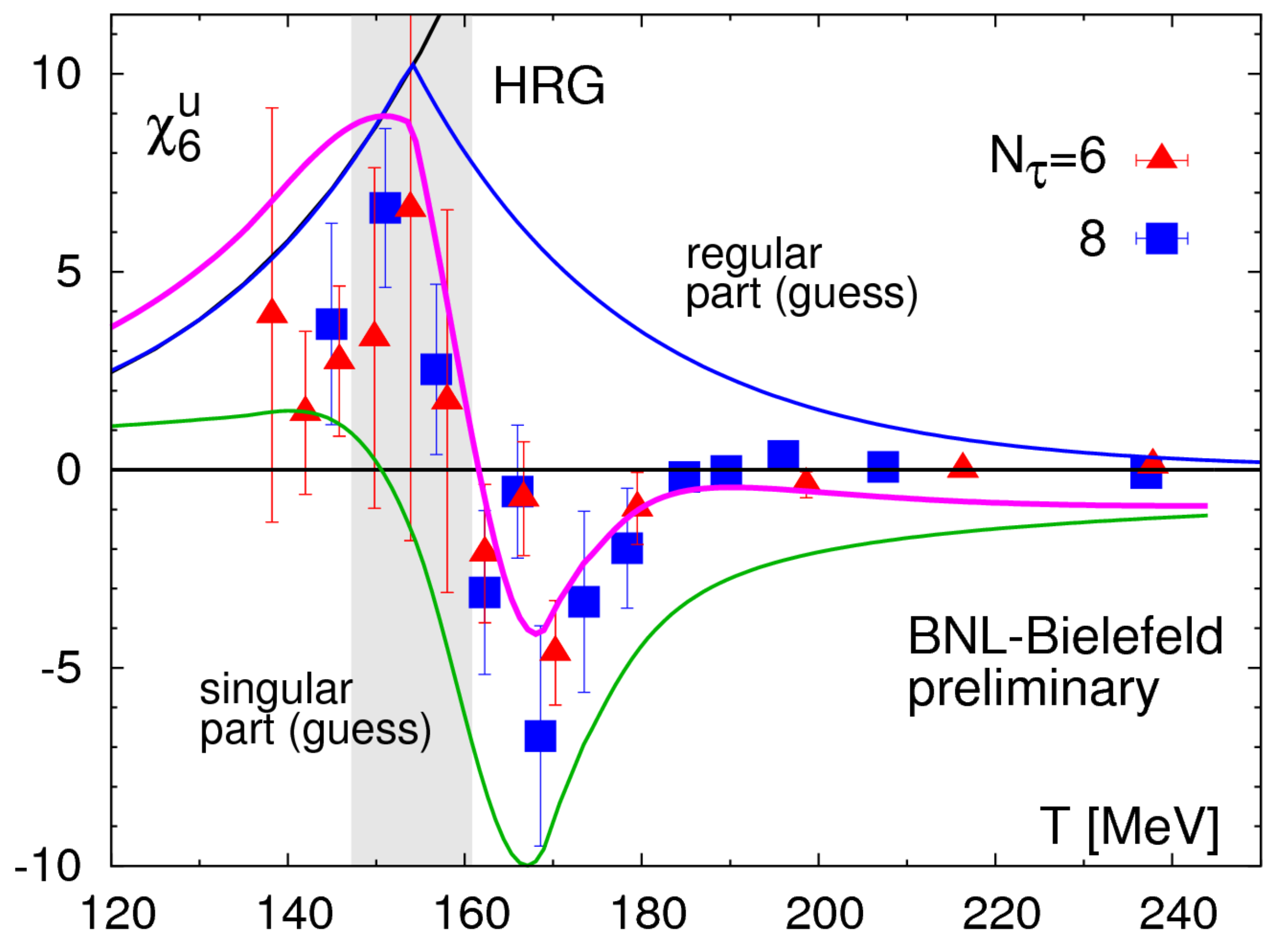
$\Rightarrow$ depth of minimum at high T fixes maximal singular contribution at high T



on the interplay of regular and singular contributions (so far a guess and not a fit)

F. Karsch, NFQCD 2013

regular part dominated by HRG at low T



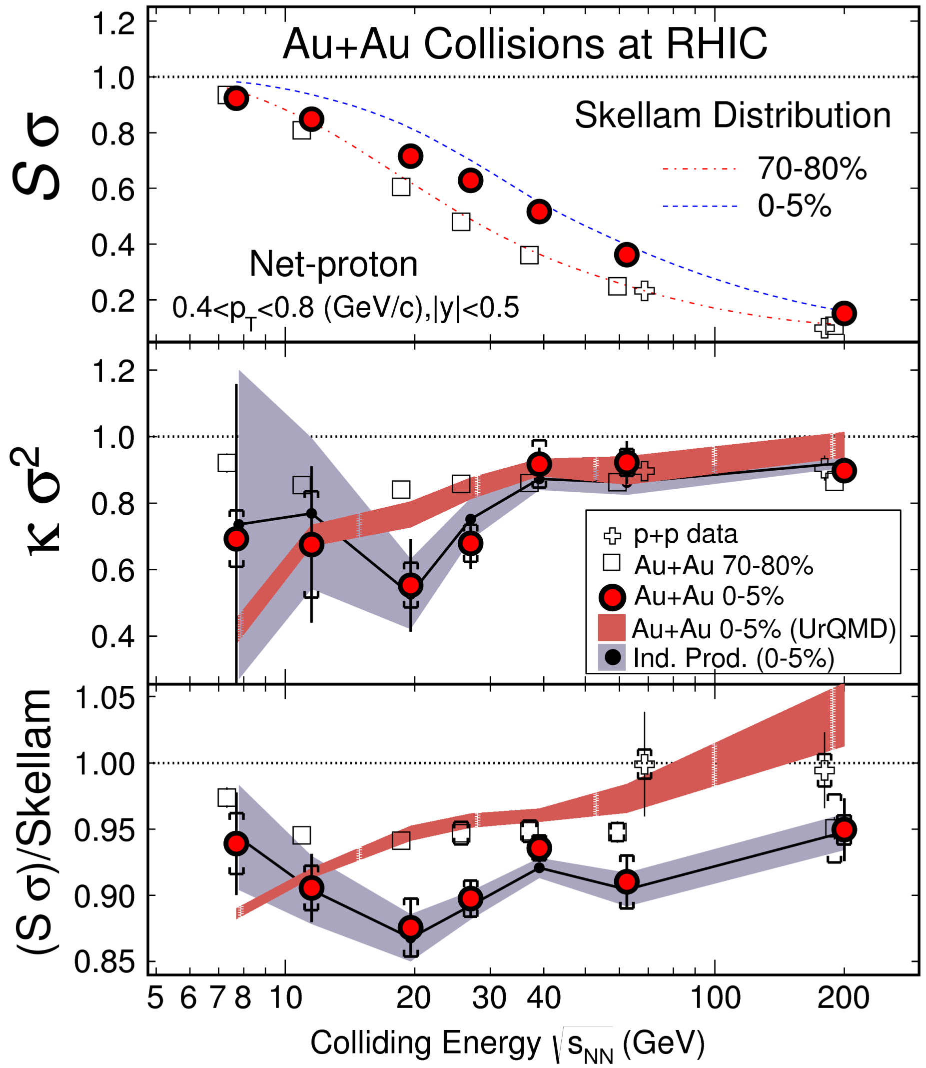

singular part generates dip at high T

total = singular + regular

one may expect an overshooting of HRG by at most a factor of 2 ?

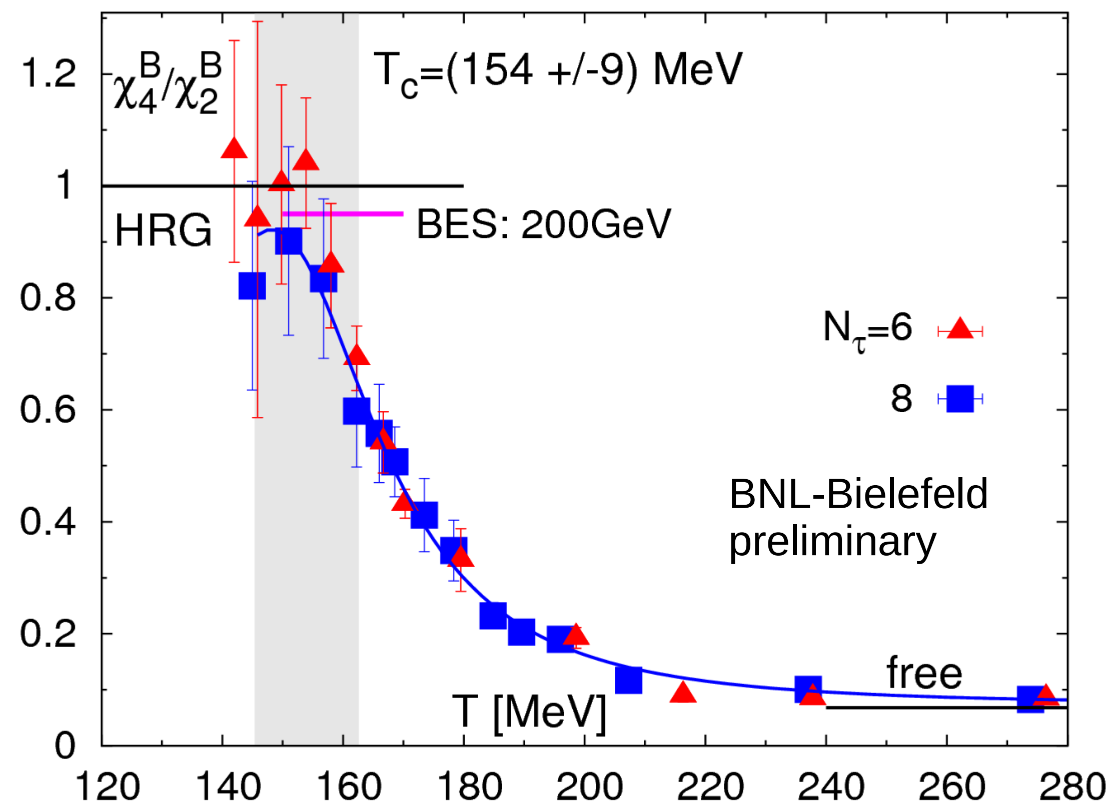
# Critical point search

proton number fluctuations




STAR Collaboration, arXiv: 1309.5681

relative strength of the NLO correction to the pressure is controlled by $\chi_4^B/\chi_2^B$




$$T < T_c :$$

$$0.8 \le \chi_4^B/\chi_2^B \le 1.0$$

$P_4/P_2 < 1$ for $\mu_B \lesssim 3.5$

# Critical point search by radius of convergence

$$\frac{p}{T^4} = \frac{1}{VT^3}\ln Z(V,T,\mu_B) = \sum_{n,even}\frac{1}{n!}\chi_n^B\left(\frac{\mu_B}{T}\right)^n$$

$\Rightarrow$ consider radius of convergence

$$\left(\frac{\mu_B}{T}\right)_{crit} = \lim_{n\to\infty}\sqrt{\left|\frac{(n+2)!\chi_n^B}{n!\chi_{n+2}^B}\right|}$$

$\Rightarrow$ basic quantities

$\chi_n^B/\chi_{n+2}^B$

=1 for HRG

need to deviate from HRG like $n^2$

to obtain finite radius of convergence

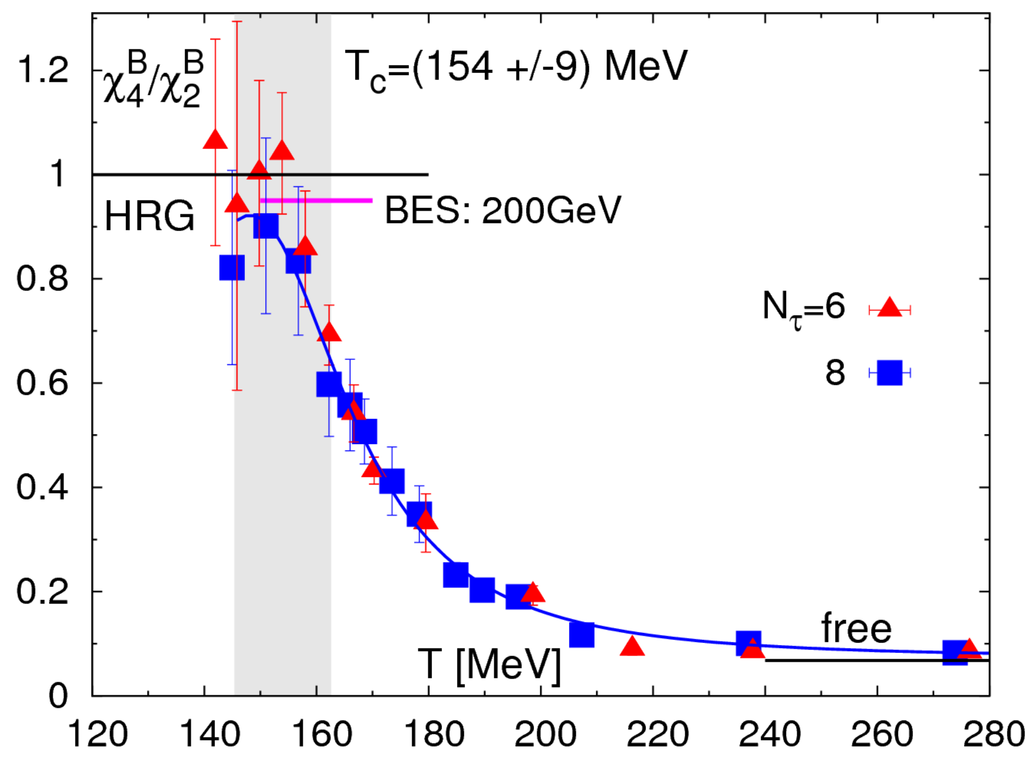
$\Rightarrow$ singularity on the real axis **only if**

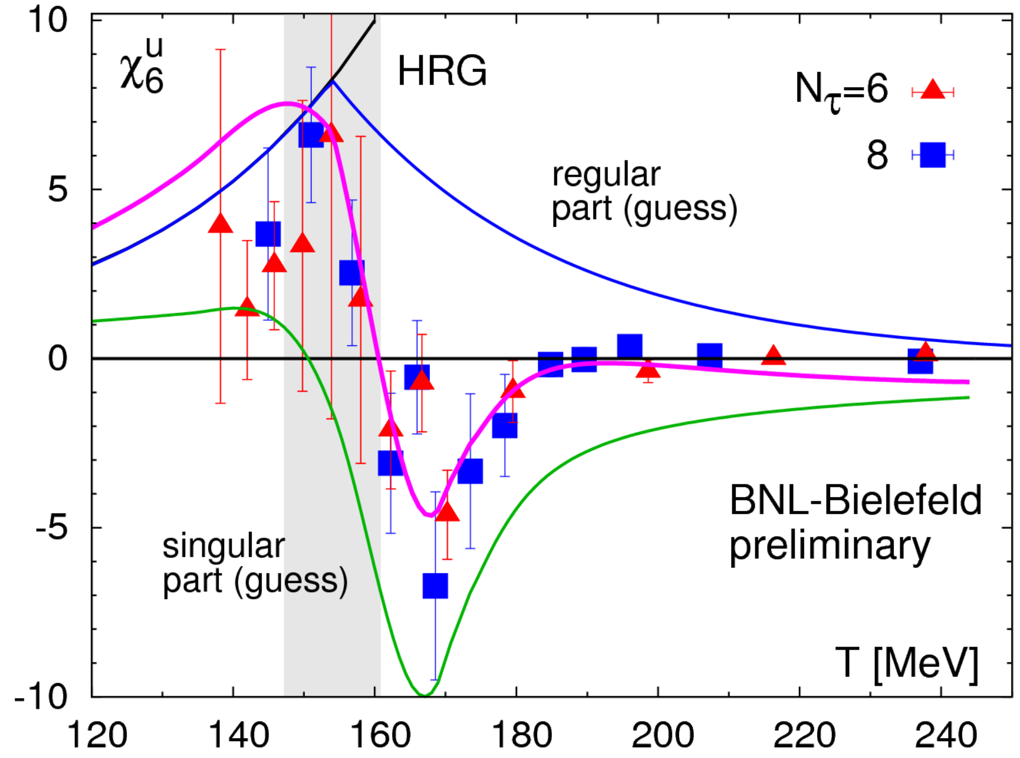
$\chi_n^B > 0$ for all $n > n_0$

# Critical point search by radius of convergence

However, so far no evidence for large enhancement over HRG for $T < T_c$

...remember:




this suggests a large $\mu_B^{crit}$

# Summary

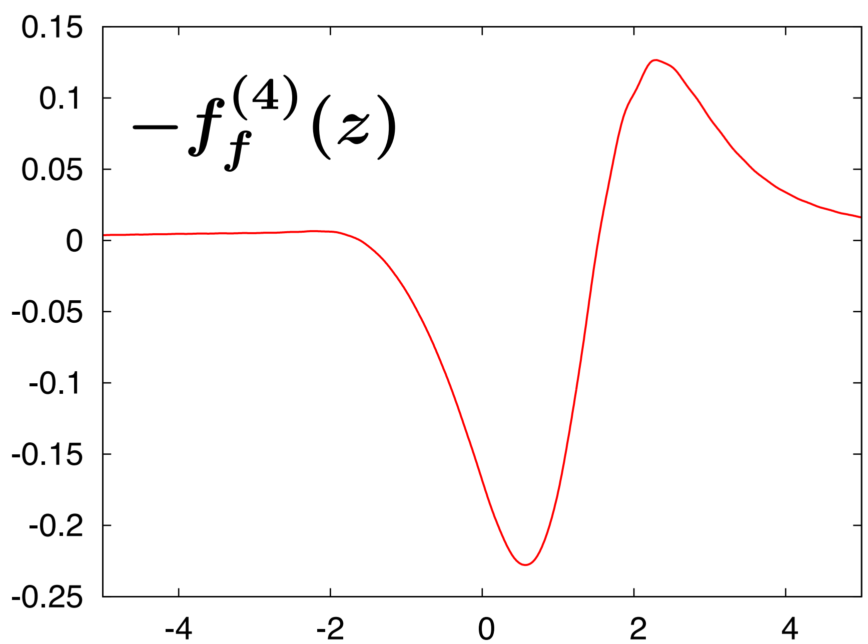
- Approximate agreement with HRG model calculations at freeze-out and sensitivity to O(4) singular behavior are not inconsistent with each other

- 6th order cumulants are sensitive to O(4) scaling but will pick up only a small singular contribution at low T. This favors estimates for the location of a critical end point at large baryon chemical potentials

# QCD critical behavior at $\mu_B > 0$

$$\mu_B > 0: \quad \chi^B_{4,\mu} = -3(2\kappa_B t_0^{-1})^2 h^{-\alpha/\Delta} f_f^{(2)}(z) -6(2\kappa_B t_0^{-1})^3 (\hat{\mu}_B^c)^2 h^{-(1+\alpha)/\Delta} f_f^{(3)}(z) -(2\kappa_B t_0^{-1} \hat{\mu}_B^c)^4 h^{-(2+\alpha)/\Delta} f_f^{(4)}(z) \text{ + regular}$$

dominates in the chiral limit or if $\hat{\mu}_B^c > 0 \gtrsim 1$




$\Rightarrow$ close to $T_c$:

$$\chi_4^B(\mu_B) < 0$$

B.Friman, FK, K.Redlich, V.Skokov, Eur. Phys. J. C71, 1694 (2011)

$\Rightarrow$ mapping of scaling variables non trivial

M. Stephanov, PRL 107 (2011) 052301

**Does deconfinement take place above the chiral crossover temperature?**

**The chiral crossover line:**

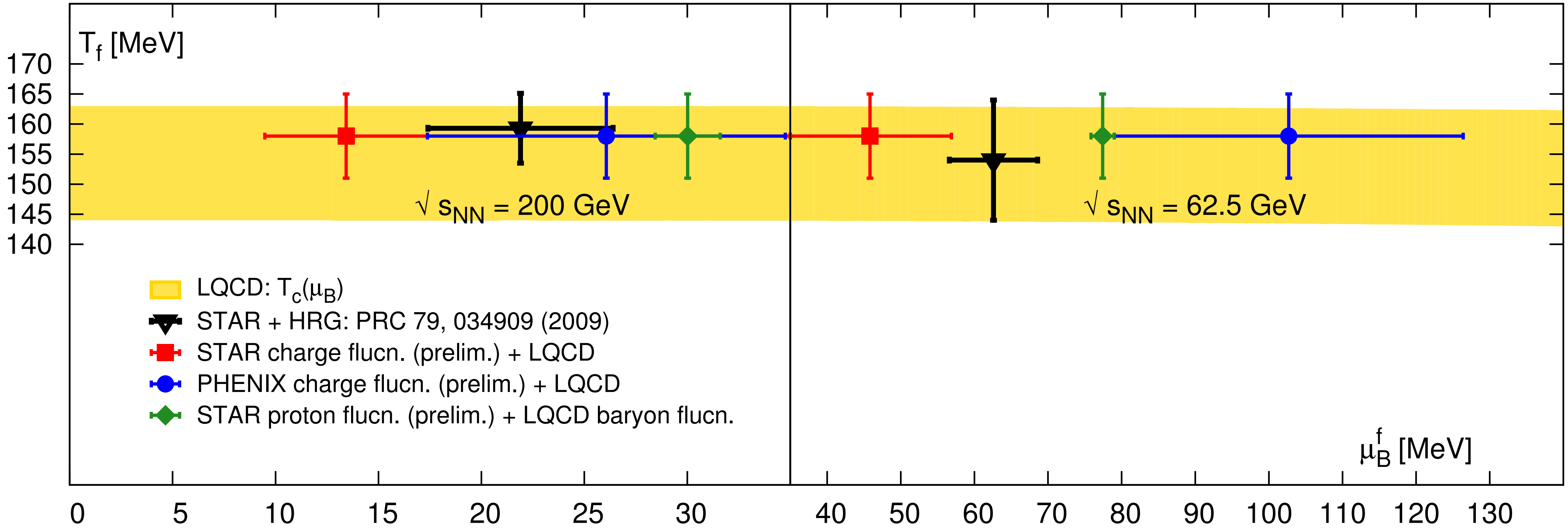
$$T_c = 154(9)\ \mathrm{MeV}$$

HotQCD, PRD 85 (2012) 054503

$$T_c(\mu_B) = T_c(0)\left[1 - 0.0066(7)\mu_B^2\right]$$

BNL-BI, PRD 83 (2011) 014504

Karsch, CPOD 2013, arXiv:1307.3978




⇒ freeze-out points are in agreement with the chiral crossover line

⇒ apparent discrepancies among the freeze-out points that need to be resolved