Atomic Quantum Simulation of Abelian and non-Abelian Gauge Theories

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Outline

Quantum Simulation to Address Severe Sign Problems

Wilson's Lattice Gauge Theory versus Quantum Link Models

Atomic Quantum Simulator for $U(1)$ Gauge Theory Coupled to Fermionic Matter

Atomic Quantum Simulator for $U(N)$ and $SU(N)$ Non-Abelian Gauge Theories

Conclusions

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Conclusions

Some sign problems are completely solvable:

Meron-cluster algorithm analytically identifies cancellations by an improved estimator and samples only the positive uncanceled configurations, thus completely solving several severe sign problems.

• 2-d $O(3)$ model at non-zero θ or μ:
W. Bietenholz, A. Pochinsky, UJW, PRL 75 (1995) 4524.
S. Chandrasekharan, B. Scarlet, UJW, CPC 147 (2002) 388.
M. Bögli, F. Niedermayer, M. Pepe, UJW, JHEP 1204 (2012) 117.
P. de Forcrand, M. Pepe, UJW, PRD 86 (2012) 075006.

• 2-d $\mathbb{C}P(N-1)$ models at $\theta = \pi$:
B. B. Beard, M. Pepe, S. Riederer, UJW, PRL 94 (2005) 010603.

• 3-d $\mathbb{Z}(3)$ Polyakov loop model for dense QCD:
M. Alford, S. Chandrasekharan, J. Cox, UJW, NPB 602 (2001) 61.

• Strongly correlated fermions with a severe sign problem:
S. Chandrasekharan, UJW, PRL 83 (1999) 3116.
S. Chandrasekharan, J. Cox, K. Holland, UJW, NPB 576 (2000) 481.
S. Chandrasekharan, J. Osborn, PRB 66 (2002) 045113.

Some sign problems are harder:

• Geometrically frustrated quantum magnets:
M. Nyfeler, F.-J. Jiang, F. Kämpfer, UJW, PRL 100 (2008) 247206.

• Real-time evolution driven by measurements:
D. Banerjee, F.-J. Jiang, M. Kon, UJW, to be published.

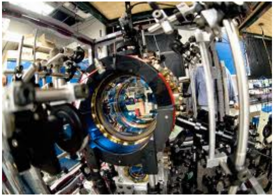
Some sign problems are too hard:

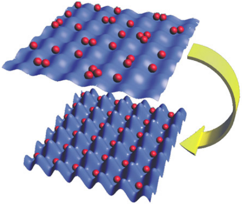
• As hard as the NP-complete traveling salesman problem:
M. Troyer, UJW, PRL 94 (2005) 170201.

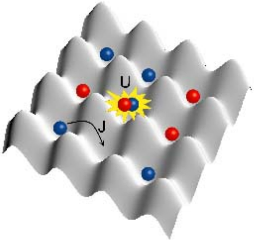
Feynman's vision: Int. J. Theor. Phys. 21 (1982) 467.

"I'm not happy with all the analyses that go with just the classical theory, because nature isn't classical, dammit, and if you want to make a simulation of nature, you'd better make it quantum mechanical, and by golly it's a wonderful problem, because it doesn't look so easy."

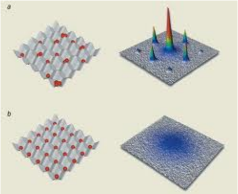
Ultra-cold atoms in optical lattices as analog quantum simulators







Superfluid-Mott insulator transition in the bosonic Hubbard model



M. Greiner, O. Mandel, T. Esslinger, T. Hänsch, I. Bloch,
Nature 415 (2002) 39.

Outline

Hamiltonian formulation of $U(1)$ lattice gauge theory

$$U = \exp(i\varphi), \ U^\dagger = \exp(-i\varphi) \in U(1)$$

Electric field operator E

$$E = -i\partial_\varphi, \ [E, U] = U, \ [E, U^\dagger] = -U^\dagger, \ [U, U^\dagger] = 0$$

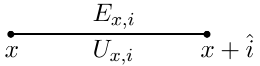
Generator of $U(1)$ gauge transformations

$$G_x = \sum_i (E_{x-\hat{i},i} - E_{x,i}), \quad [H, G_x] = 0$$

$U(1)$ gauge invariant Hamiltonian

$$H = \frac{g^2}{2} \sum_{x,i} E_{x,i}^2 - \frac{1}{2g^2} \sum_{x,i\neq j} (U_{x,i} U_{x+\hat{i},j} U^\dagger_{x+\hat{j},i} U^\dagger_{x,j} + \text{h.c.})$$

operates in an infinite-dimensional Hilbert space per link



$U(1)$ quantum link model

$$U = S_1 + iS_2 = S_+, \; U^\dagger = S_1 - iS_2 = S_-$$

Electric flux operator E

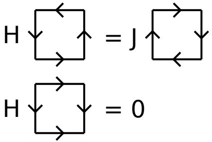
$$E = S_3, \; [E, U] = U, \; [E, U^\dagger] = -U^\dagger, \; [U, U^\dagger] = 2E$$

Generator of $U(1)$ gauge transformations

$$G_x = \sum_i (E_{x-\hat{i},i} - E_{x,i}), \quad [H, G_x] = 0$$

Gauge invariant Hamiltonian for $S = \frac{1}{2}$

$$H = -J \sum_\square (U_\square + U_\square^\dagger)$$



defines a gauge theory with a 2-d Hilbert space per link

D. Horn, PLB B100 (1981) 149.

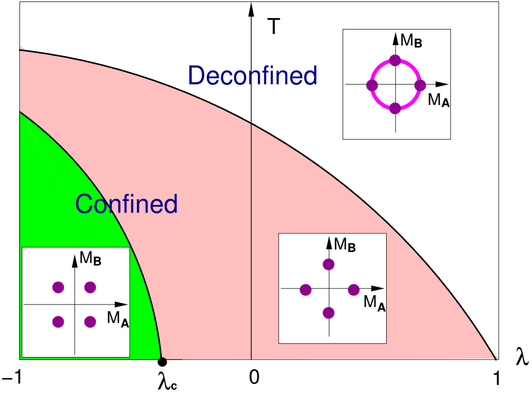
P. Orland, D. Rohrlich, NPB338 (1990) 647.

S. Chandrasekharan, UJW, NPB 492 (1997) 455.

Hamiltonian with Rokhsar-Kivelson term

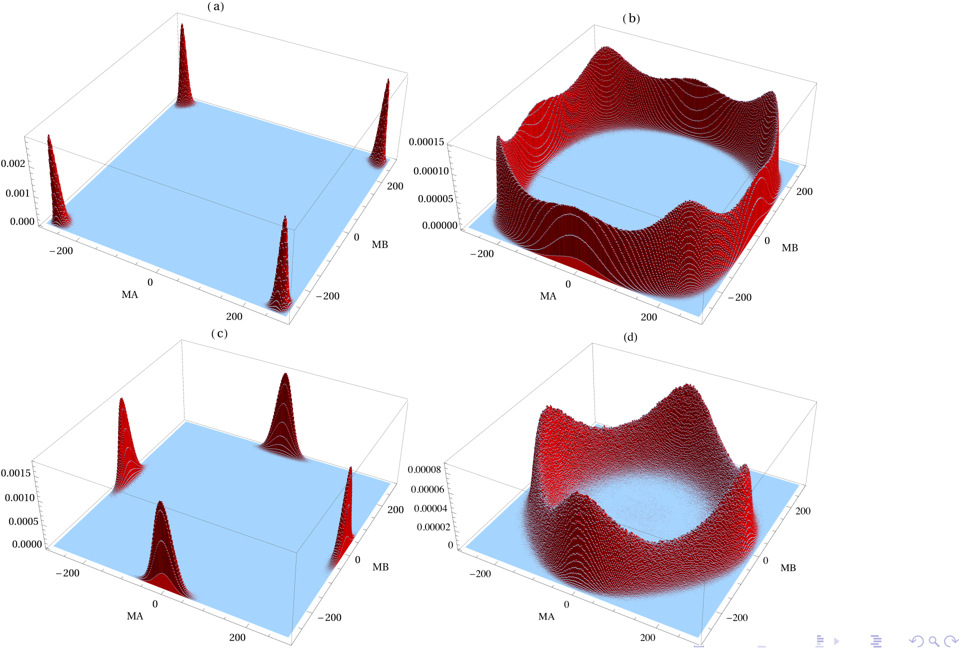
$$H = -J \left[\sum_{\Box} (U_{\Box} + U_{\Box}^{\dagger}) - \lambda \sum_{\Box} (U_{\Box} + U_{\Box}^{\dagger})^2 \right]$$

Phase diagram

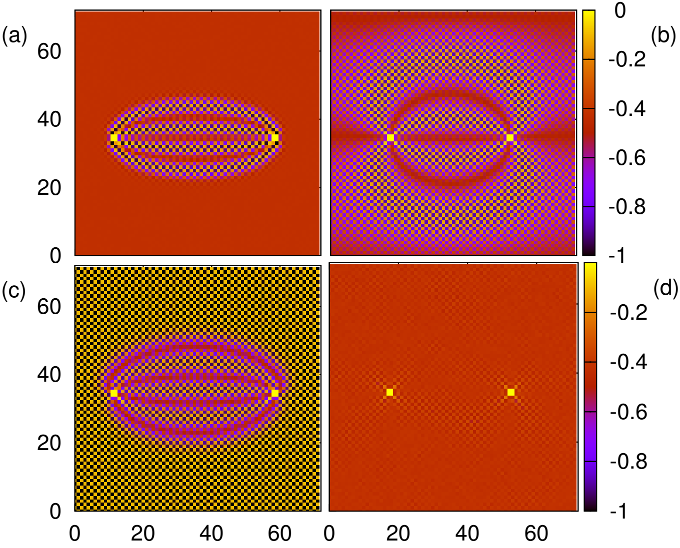


D. Banerjee, F.-J. Jiang, P. Widmer, UJW, arXiv:1303.6858, JSTAT (2013) P12010.

Probability Distribution of the Order Parameters



Energy density of charge-anti-charge pair $Q = \pm 2$



M. Müller, I. Lesanovsky, H. Weimer, H. P. Büchler, P. Zoller, Phys. Rev. Lett. 102 (2009) 170502.
H. Weimer, M. Müller, I. Lesanovsky, P. Zoller, H. P. Büchler, Nat. Phys. 6 (2010) 382.
L. Tagliacozzo, A. Celi, P. Orland, M. Lewenstein, Nature Communications 4 (2013) 2615.
L. Tagliacozzo, A. Celi, A. Zamora, M. Lewenstein, Ann. Phys. 330 (2013) 160.

Outline

Hamiltonian for staggered fermions and $U(1)$ quantum links

$$H = -t \sum_x \left[\psi_x^\dagger U_{x,x+1} \psi_{x+1} + \mathrm{h.c.}\right] + m \sum_x (-1)^x \psi_x^\dagger \psi_x + \frac{g^2}{2} \sum_x E_{x,x+1}^2$$

Bosonic rishon representation of the quantum links

$$U_{x,x+1} = b_x b_{x+1}^\dagger, \quad E_{x,x+1} = \frac{1}{2}\left(b_{x+1}^\dagger b_{x+1} - b_x^\dagger b_x\right)$$

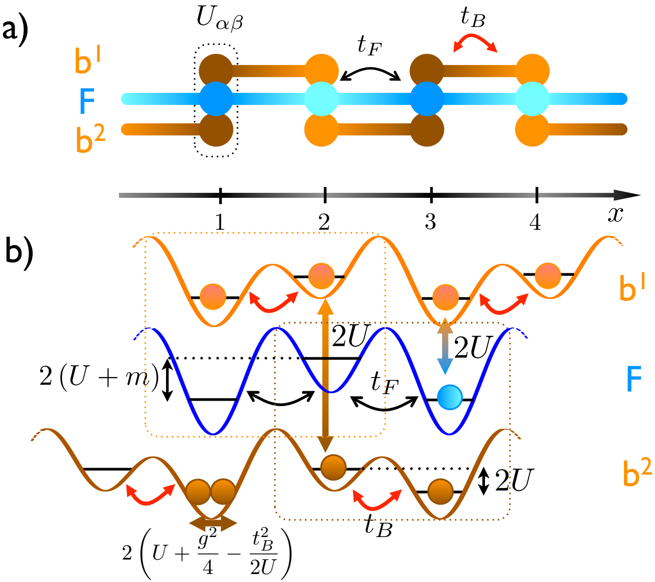
Gauge generator

$$\widetilde{G}_x = n_x^F + n_x^1 + n_x^2 - 2S + \frac{1}{2}\left[(-1)^x - 1\right]$$

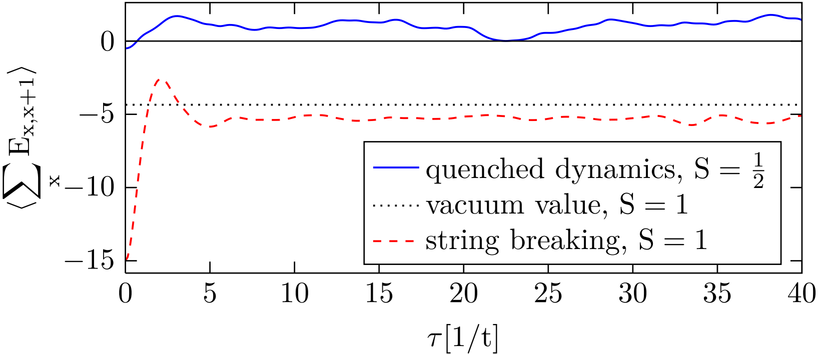
Microscopic Hubbard model Hamiltonian

$$\begin{aligned}
\widetilde{H} &= \sum_x h_{x,x+1}^B + \sum_x h_{x,x+1}^F + m \sum_x (-1)^x n_x^F + U \sum_x \widetilde{G}_x^2 \\
&= -t_B \sum_{x \text{ odd}} b_x^{1\dagger} b_{x+1}^1 - t_B \sum_{x \text{ even}} b_x^{2\dagger} b_{x+1}^2 - t_F \sum_x \psi_x^\dagger \psi_{x+1} + \mathrm{h.c.} \\
&+ \sum_{x,\alpha,\beta} n_x^\alpha U_{\alpha\beta} n_x^\beta + \sum_{x,\alpha} (-1)^x U_\alpha n_x^\alpha
\end{aligned}$$

Optical lattice with Bose-Fermi mixture of ultra-cold atoms



Quantum simulation of the real-time evolution of string breaking



D. Banerjee, M. Dalmonte, M. Müller, E. Rico, P. Stebler, UJW, P. Zoller, Phys. Rev. Lett. 109 (2012) 175302.

Outline

$U(N)$ quantum link operators

$$U^{ij} = S_1^{ij} + iS_2^{ij}, \ U^{ij\dagger} = S_1^{ij} - iS_2^{ij}, \ i,j \in \{1,2,\dots,N\}, \ [U^{ij}, (U^\dagger)^{kl}] \neq 0$$

$SU(N)_L \times SU(N)_R$ gauge transformations of a quantum link

$$[L^a, L^b] = if_{abc} L^c, \ [R^a, R^b] = if_{abc} R^c, \ a,b,c \in \{1,2,\dots,N^2-1\}$$

$$[L^a, R^b] = [L^a, E] = [R^a, E] = 0$$

Infinitesimal gauge transformations of a quantum link

$$[L^a, U] = -\lambda^a U, \ [R^a, U] = U\lambda^a, \ [E, U] = U$$

Algebraic structures of different quantum link models

$U(N)$: $U^{ij}, \ L^a, \ R^a, E, \ 2N^2 + 2(N^2-1) + 1 = 4N^2 - 1$ $SU(2N)$ generators

$SO(N)$: $O^{ij}, \ L^a, \ R^a, \ N^2 + 2\dfrac{N(N-1)}{2} = N(2N-1)$ $SO(2N)$ generators

$Sp(N)$: $U^{ij}, \ L^a, \ R^a, \ 4N^2 + 2N(2N+1) = 2N(4N+1)$ $Sp(2N)$ generators

R. Brower, S. Chandrasekharan, UJW, Phys. Rev. D60 (1999) 094502

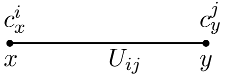
d-dimensional $SU(N)$ gauge theory with staggered fermions

$$
\begin{aligned}
H &= -t \sum_{\langle xy \rangle} \left(s_{xy} \psi_x^{i\dagger} U_{xy}^{ij} \psi_y^j + \mathrm{h.c.} \right) + m \sum_x (-1)^x \psi_x^{i\dagger} \psi_x^i \\
&+ \frac{g^2}{2} \sum_{\langle xy \rangle} \left(L_{xy}^a L_{xy}^a + R_{xy}^a R_{xy}^a \right) + \frac{g'^2}{2} \sum_{\langle xy \rangle} E_{xy}^2 \\
&- \frac{1}{4g^2} \sum_{\langle wxyz \rangle} \left(U_{wx} U_{xy} U_{yz} U_{zw} + \mathrm{h.c.} \right) - \gamma \sum_{\langle xy \rangle} \left(\det U_{xy} + \mathrm{h.c.} \right)
\end{aligned}
$$

Fermionic rishons at the two ends of a link

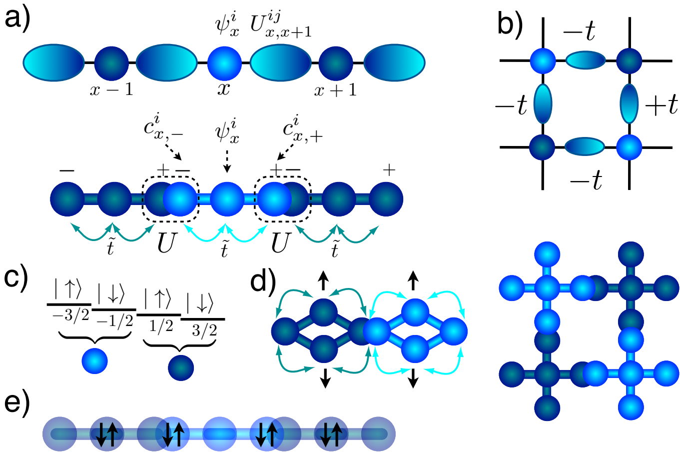
$$
\{c_x^i, c_y^{j\dagger}\} = \delta_{xy} \delta_{ij}, \ \{c_x^i, c_y^j\} = \{c_x^{i\dagger}, c_y^{j\dagger}\} = 0
$$

Rishon representation of link algebra



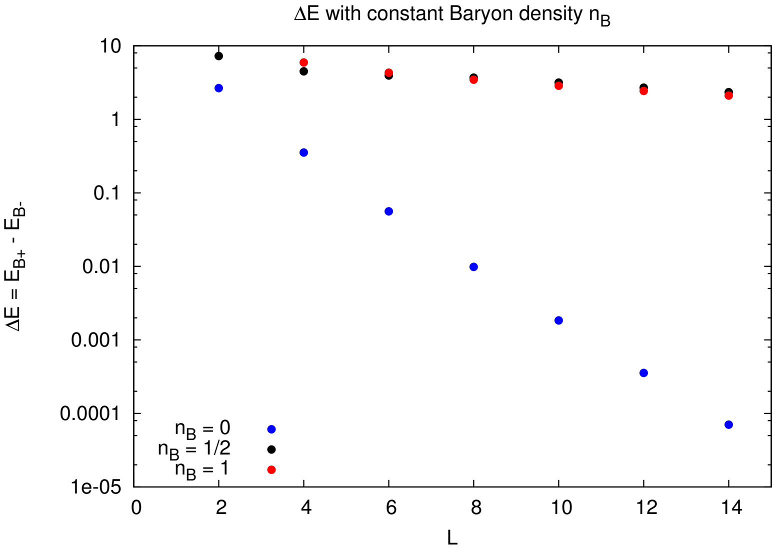
$$
U_{xy}^{ij} = c_x^i c_y^{j\dagger}, \ L_{xy}^a = c_x^{i\dagger} \lambda_{ij}^a c_x^j, \ R_{xy}^a = c_y^{i\dagger} \lambda_{ij}^a c_y^j, \ E_{xy} = \frac{1}{2} (c_y^{i\dagger} c_y^i - c_x^{i\dagger} c_x^i)
$$

Optical lattice with ultra-cold alkaline-earth atoms (^{87}Sr or ^{173}Yb) with color encoded in nuclear spin

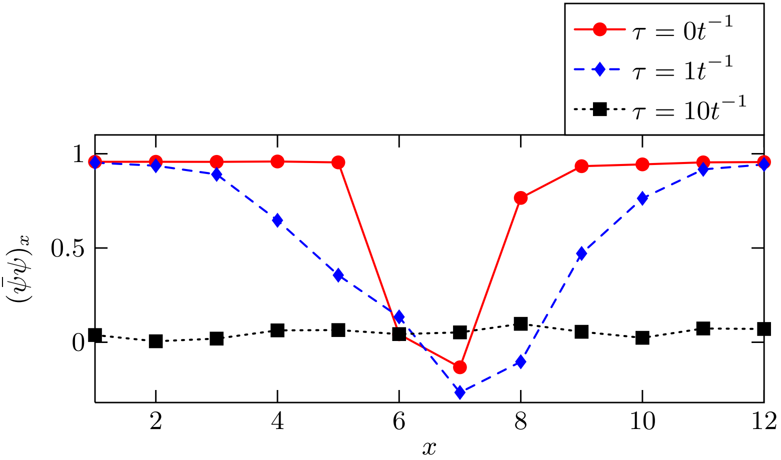


D. Banerjee, M. Bögli, M. Dalmonte, E. Rico, P. Stebler, UJW, P. Zoller, Phys. Rev. Lett. 110 (2013) 125303

Restoration of chiral symmetry at baryon density $n_B \geq \frac{1}{2}$



Expansion of a "fireball" mimicking a hot quark-gluon plasma



Outline

Conclusions

- If quantum link models can be implemented with ultra-cold atoms, such systems can be used as quantum simulators for dynamical Abelian and non-Abelian gauge theories, which can be validated in efficient classical cluster algorithm simulations, at least in the Abelian case.

- Quantum simulator constructions have already been presented for the $U(1)$ quantum link model as well as for $U(N)$ and $SU(N)$ quantum link models with fermionic matter.

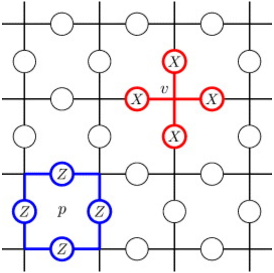
- This would allow the quantum simulation of the real-time evolution of string breaking as well as the quantum simulation of "nuclear" physics and dense "quark" matter, at least in qualitative toy models for QCD.

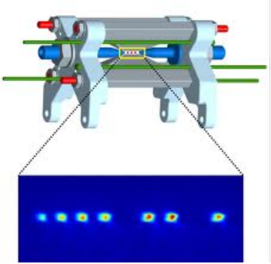
- Accessible effects may include chiral symmetry restoration, baryon superfluidity, or color superconductivity at high baryon density, as well as the quantum simulation of "nuclear" collisions.

- The path towards quantum simulation of QCD will be a long one. However, with a lot of interesting physics along the way.

- INT workshop on Quantum Simulation from March 23 to May 8, 2015, organized in collaboration with Peter Zoller and other atomic physicists.

Digital quantum simulation of Kitaev's toric code (a $\mathbb{Z}(2)$ lattice gauge theory) with trapped ions





• Precisely controllable many-body quantum device, which can execute a prescribed sequence of quantum gate operations.

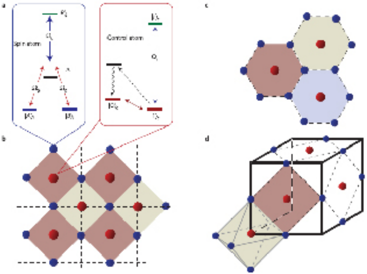
• State of simulated system encoded as quantum information.

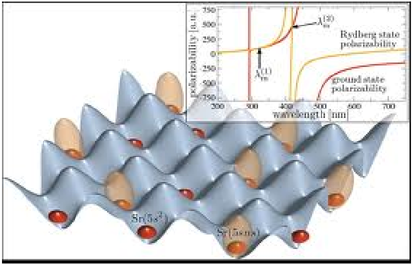
• Dynamics is represented by a sequence of quantum gates, following a stroboscopic Trotter decomposition.

A. Y. Kitaev, Ann. Phys. 303 (2003) 2.

B. P. Lanyon, C. Hempel, D. Nigg, M. Müller, R. Gerritsma, F. Zähringer, P. Schindler, J. T. Barreiro, M. Rambach, G. Kirchmair, M. Hennrich, P. Zoller, R. Blatt, C. F. Roos, Science 334 (2011) 6052.

Quantum spin liquids ($U(1)$ gauge theories) to be simulated with Rydberg atoms in an optical lattice





- Lasers can excite atoms to high-lying Rydberg states.
- Rydberg atoms are large and have collective interactions.
- Ensemble Rydberg atoms represent qubits at link centers.
- Control atoms at lattice sites ensure the Gauss' law.

M. Müller, I. Lesanovsky, H. Weimer, H. P. Büchler, P. Zoller, Phys. Rev. Lett. 102 (2009) 170502.
H. Weimer, M. Müller, I. Lesanovsky, P. Zoller, H. P. Büchler, Nat. Phys. 6 (2010) 382.
L. Tagliacozzo, A. Celi, P. Orland, M. Lewenstein, Nature Communications 4 (2013) 2615.
L. Tagliacozzo, A. Celi, A. Zamora, M. Lewenstein, Ann. Phys. 330 (2013) 160.

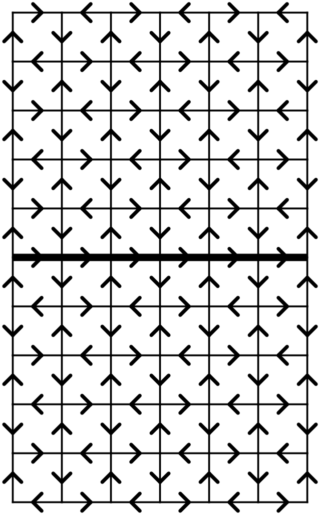
Analog quantum simulators

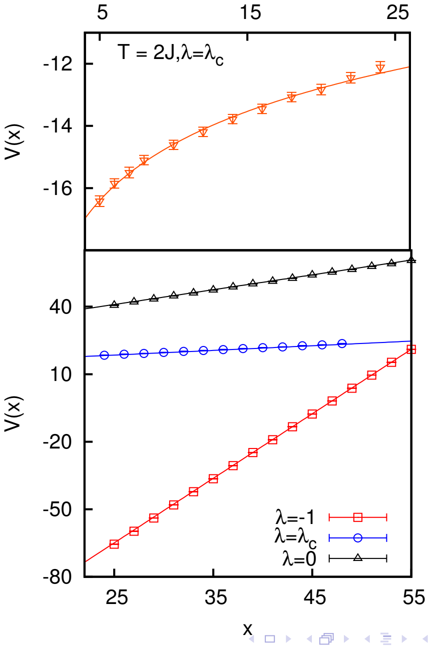
- Time evolution proceeds continuously, not using discrete quantum gates.
- Limited to simpler interactions, but more easily scalable.

Proposals for analog quantum simulators for Abelian and non-Abelian gauge theories with and without matter

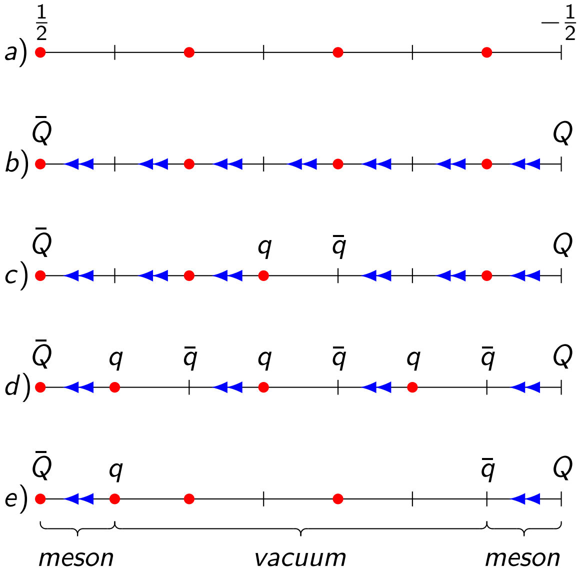
H. P. Büchler, M. Hermele, S. D. Huber, M. P. A. Fisher, P. Zoller, Phys. Rev. Lett. 95 (2005) 040402.
E. Kapit, E. Mueller, Phys. Rev. A83 (2011) 033625.
E. Zohar, B. Reznik, Phys. Rev. Lett. 107 (2011) 275301.
E. Zohar, J. Cirac, B. Reznik, Phys. Rev. Lett. 109 (2012) 125302.
E. Zohar, J. Cirac, B. Reznik, Phys. Rev. Lett. 110 (2013) 055302.
E. Zohar, J. Cirac, B. Reznik, Phys. Rev. Lett. 110 (2013) 125304.
E. Zohar, J. Cirac, B. Reznik, Phys. Rev. A 88 (2013) 023617.
D. Banerjee, M. Dalmote, M. Müller, E. Rico, P. Stebler, UJW, P. Zoller, Phys. Rev. Lett. 109 (2012) 175302.
D. Banerjee, M. Bögli, M. Dalmote, E. Rico, P. Stebler, UJW, P. Zoller, Phys. Rev. Lett. 110 (2013) 125303.
UJW, Annalen der Physik 525 (2013) 777, arXiv:1305.1602.

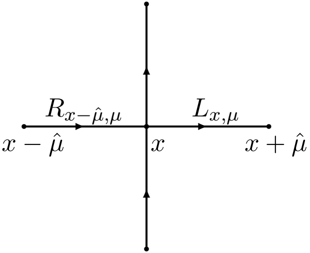
Confinement versus Deconfinement





Schematic illustration of string breaking in real time in the 1-d $S = 1$ $U(1)$ quantum link model





Generator of $SU(N)$ gauge transformations

$$G_x^a = \sum_\mu (R^a_{x-\hat{\mu},\mu} + L^a_{x,\mu})$$

$U(N)$-invariant Hamiltonian "action" operator

$$H = -J \sum_{x,\mu<\nu} \mathrm{Tr}(U_{x,\mu} U_{x+\hat{\mu},\nu} U^\dagger_{x+\hat{\nu},\mu} U^\dagger_{x,\nu} + \mathrm{h.c.}), \;\; [H, G_x^a] = 0$$

Functional integral of a quantum link model

$$Z = \mathrm{Tr} \exp(-\beta H)$$

defines a quantum field theory using discrete variables

Low-energy effective action of a quantum link model

$$S[G_\mu] = \int_0^\beta dx_5 \int d^4x \, \frac{1}{2e^2} \left(\mathrm{Tr} \; G_{\mu\nu} G_{\mu\nu} + \frac{1}{c^2} \mathrm{Tr} \; \partial_5 G_\mu \partial_5 G_\mu \right) , \; G_5 = 0$$

undergoes dimensional reduction from $4+1$ to 4 dimensions

$$S[G_\mu] \to \int d^4x \, \frac{1}{2g^2} \mathrm{Tr} \; G_{\mu\nu} G_{\mu\nu}, \; \frac{1}{g^2} = \frac{\beta}{e^2}, \; \frac{1}{m} \sim \exp\left(\frac{24\pi^2\beta}{11Ne^2}\right)$$