kandinsky farbstudie quadrate ii

QCD at nonzero isospin density

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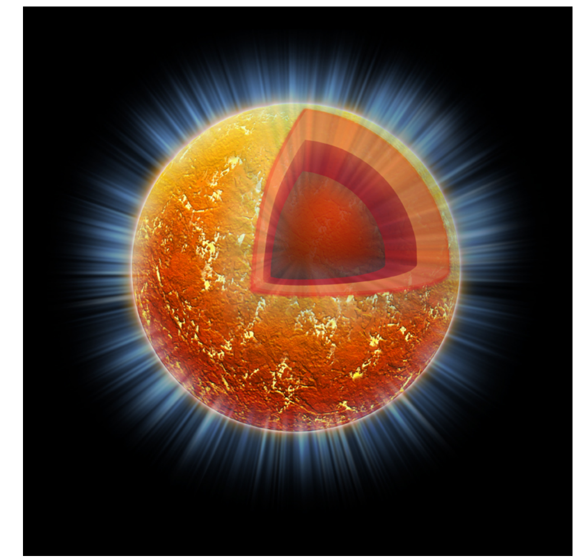
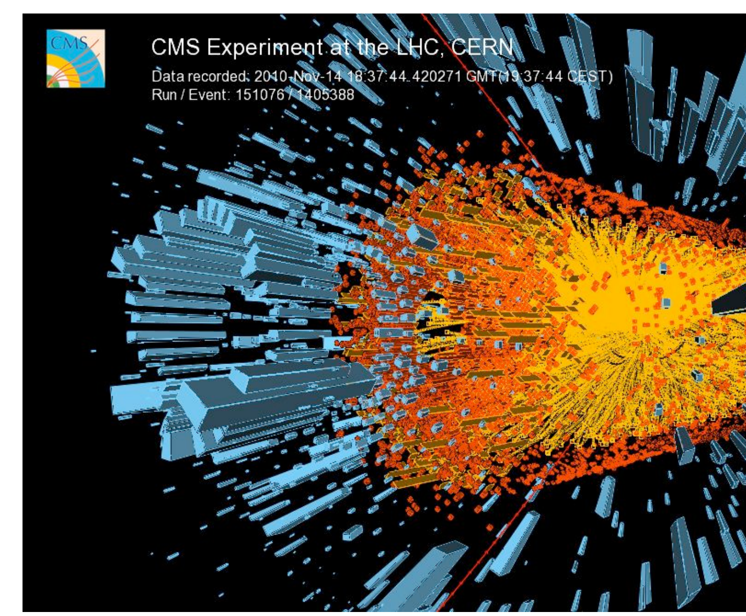
QCD and isospin density

- Why isospin density/chemical potential?
  - Physically occurring dense matter has $\mu_u \neq \mu_d$
    - Neutron matter (n-stars): $N_I \sim N_B/3$
    - Heavy ion collisions (eg Pb-Pb): $N_I \sim N_B/5$
  - Theoretically interesting
    - New phase structures to investigate
    - Relations to other theories at large N_c [Cherman et al. PRL 2011, Hanada et al. PRD2012]
    - Useful test laboratory
  - Computationally possible with current methods





QCD at nonzero μ_I

- Two-flavour QCD with nonzero quark chemical potential

$$\mathcal{L}_{\rm QCD} = \sum_{f=u,d} \overline{\psi}_f D(\mu_f)\psi_f + \mathcal{L}_{\rm YM}$$

$$D(\mu_f) = D_\mu \gamma^\mu + m_f + \mu_f \gamma^0$$

- Isospin chemical potential sets $\mu = \mu_u = - \mu_d$

- After integrating the quark d.o.f, the QCD partition function has positive definite measure (assuming m_u=m_d)

$$\mathcal{Z}_{\rm QCD} = \int \mathcal{D}A \det[D(\mu)] \det[D(-\mu)] e^{-S_{YM}}$$

$$\det[D(\mu)] \det[D(-\mu)] = \det[D(\mu)] \det[D(\mu)^\dagger] = |\det[D(\mu)]|^2$$

- Importance sampling can be used in this theory

- Equivalent to N_f=2 phase quenched QCD

Low energy effective theory

- Effective theory for small μ_I is chiral perturbation theory
  - Constructed by Son & Stephanov [Phys. Rev. Lett. 86, 592 (2001)]

$$\mathcal{L} = \frac{f^2}{8}\left[< D_\mu U D^\mu U^\dagger > + 2\lambda < M^\dagger U + U^\dagger M >\right]$$

$$D_\mu U \;=\; \partial_\mu U + i[\mathbb{V}_\mu, U] \qquad \mathbb{V}_\mu = \mu_I \tfrac{\tau^3}{2}\delta_{\mu,0} \qquad M = m_q + \epsilon\,\tau_2/2$$

  - Minimize effective potential to get ground state (at LO)

$$U_0 = \begin{cases} 1, & |\mu_I| < m_\pi \\ \exp[i\alpha\,\tau^2], & |\mu_I| > m_\pi \end{cases} \qquad \cos\alpha = \frac{m_\pi^2}{\mu_I^2} - \frac{\lambda\epsilon}{\mu_I^2}\cot\alpha$$

$$\langle\overline{\psi}\psi\rangle \;=\; f^2\lambda\cos\alpha$$

$$i\langle\overline{\psi}\tau^2\gamma_5\psi\rangle \;=\; f^2\lambda\sin\alpha$$

$$m_\pi^2 = 2\lambda m$$

  - SU(3), RMT/Epsilon regime [Toublan, Kogut, Splittorff, Verbaarschot ....]
  - Inclusion of baryons [Cohen et al, Bedaque et al.]

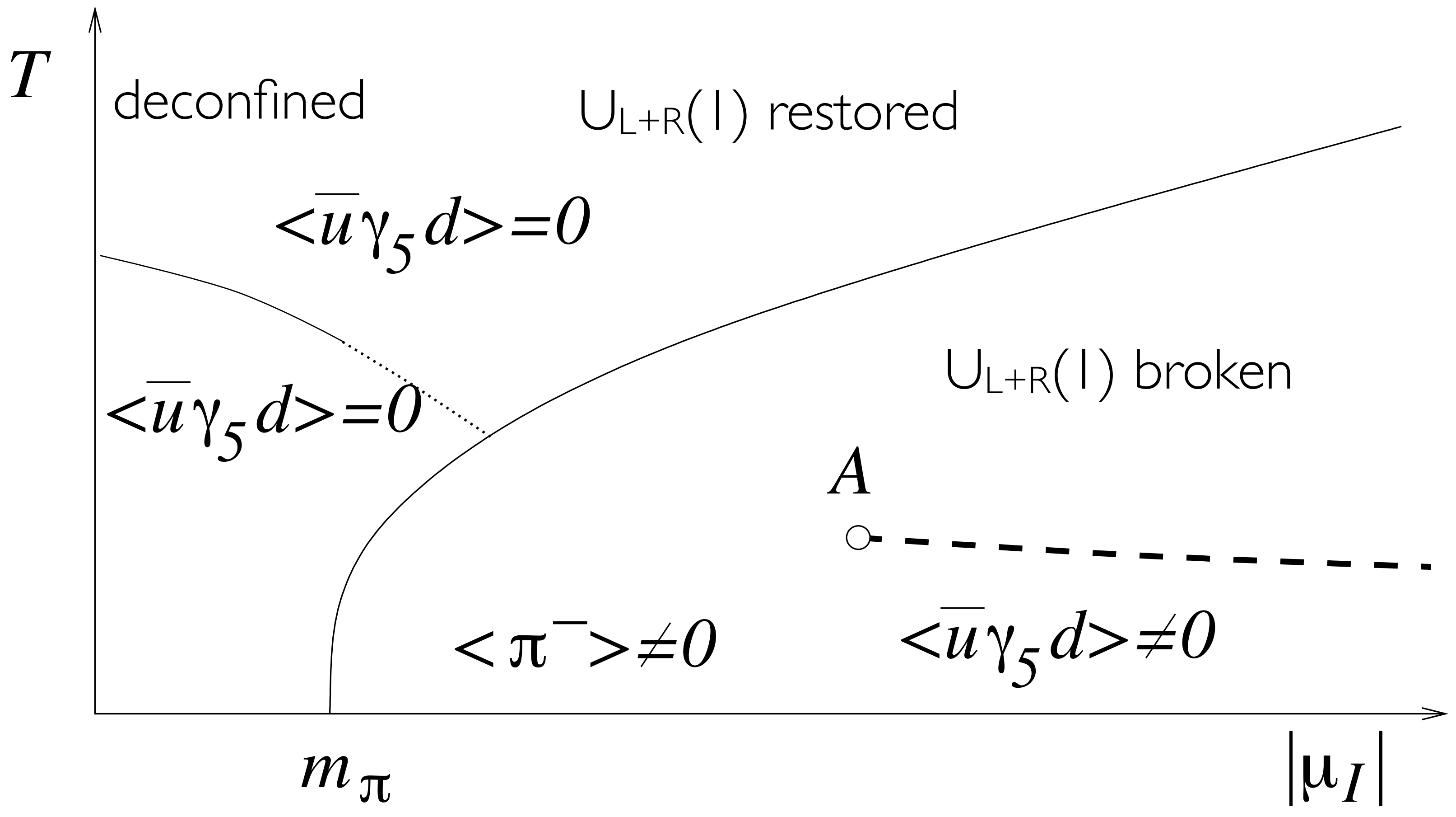
High isospin density

[Son & Stephanov Phys. Rev. Lett. 86, 592 (2001)]

- Asymptotic freedom guarantees weak interactions as chemical potentials becomes large
- Attractive OGE in $\bar{u}\Gamma d$ channels
- Non-perturbative effects favour condensation in $\bar{u}\gamma_5 d$ channel leading to a superconducting (BCS) condensate
- (QCD inequalities require PS channel to condense first)
- Likely that the BEC to BCS transition is a smooth crossover

Phase Diagram

- Conjectured phase diagram [Son & Stephanov]

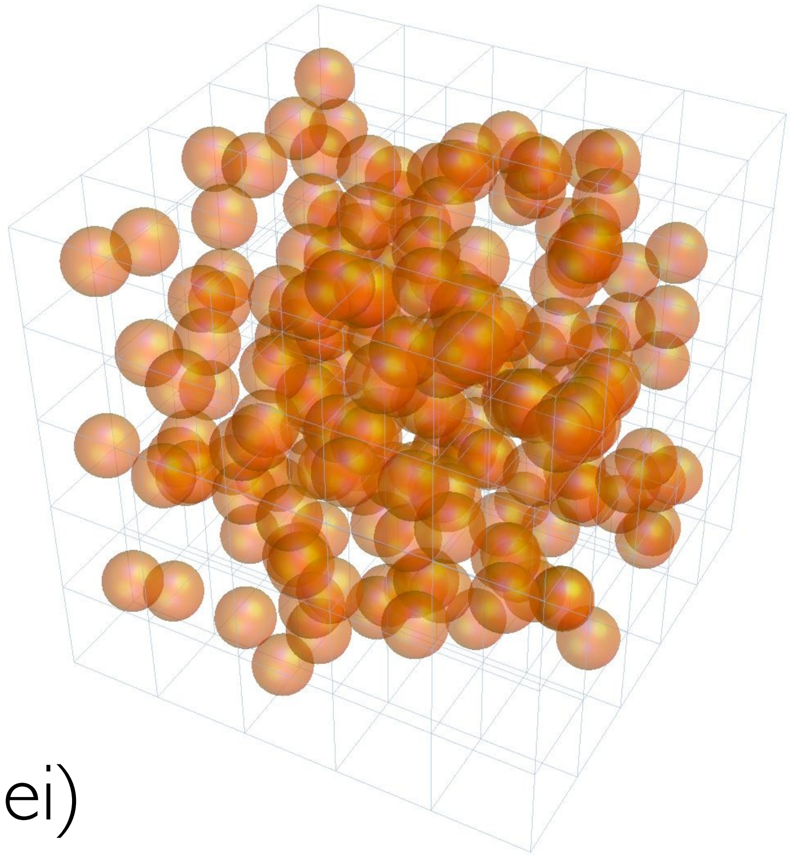


- NB: equivalent to phase quenched QCD

LQCD studies

- Over the years there have been a few studies with isospin chemical potential
    - Kogut & Sinclair [PRD 66 (2002) 014508; PRD 66 (2002) 034505; PRD 70 (2004) 094501; PRD 77 (2008) 114503]
    - de Forcrand, Stephanov & Wenger [PoS LATT2007 237]
    - Cea, Cosmai, d'Elia, Papa & Sanfillipo [Phys. Rev. D85 094512, 2012; PoS LATT12]
    - Nagata, [XQCD13 poster]
- Investigated phase diagram at nonzero-T: evidence for pin condensation
- Tests of extrapolation form imaginary chemical potential
- Studies of Dirac operator spectrum

QCD with explicit isospin charge



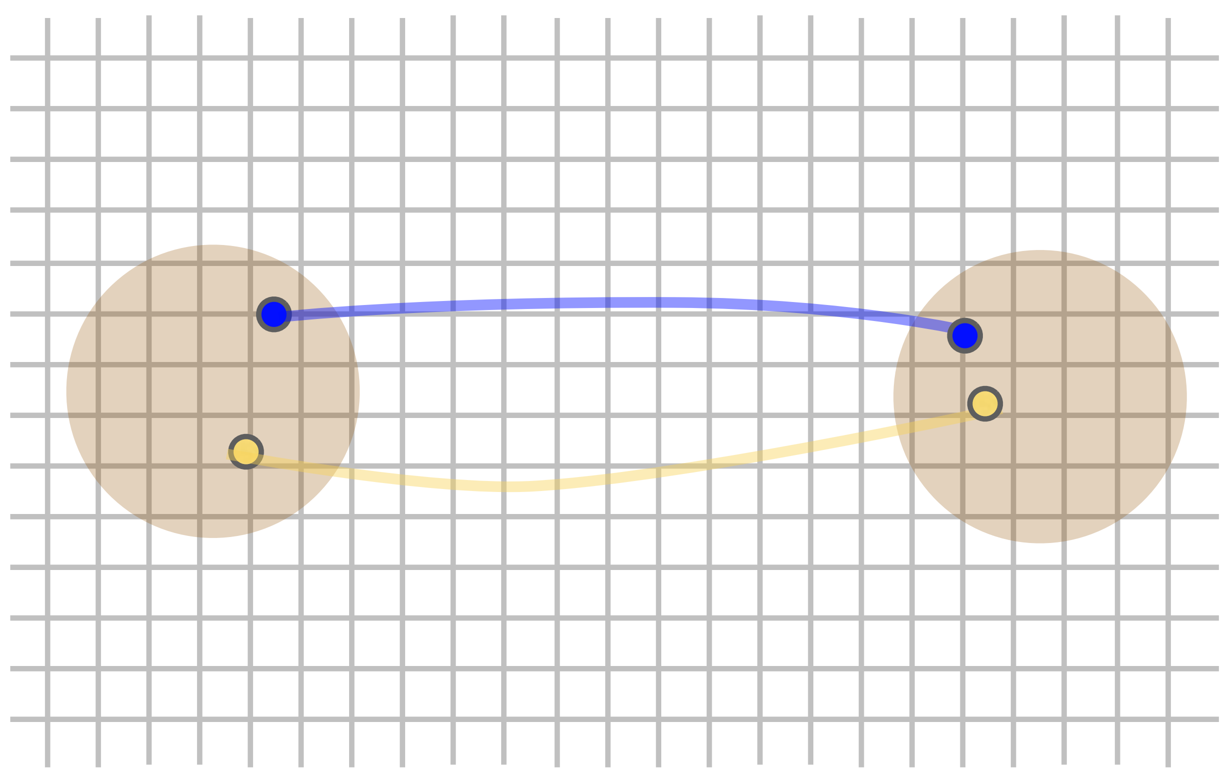
- Another way of probing isospin density is by explicitly adding isospin density to the system
  - Construct correlation functions with "many pions"
    - Wick contractions explode - new techniques necessary (a precursor to nuclei)
- Aim is somewhat different: extract properties of ground state of the system
- Interplay between few body physics (extraction of 2, 3, ... body interactions) and bulk physics

Many mesons in LQCD

- A typical π^{+} correlator ($m_u = m_d$)

$$C \ \ (t) = \left\langle 0 \left| \left[\sum_{\mathbf{x}} \overline{d}\gamma_5 u(\mathbf{x},t)\overline{u}\gamma_5 d(\mathbf{0},0) \right] \ \right| 0 \right\rangle$$

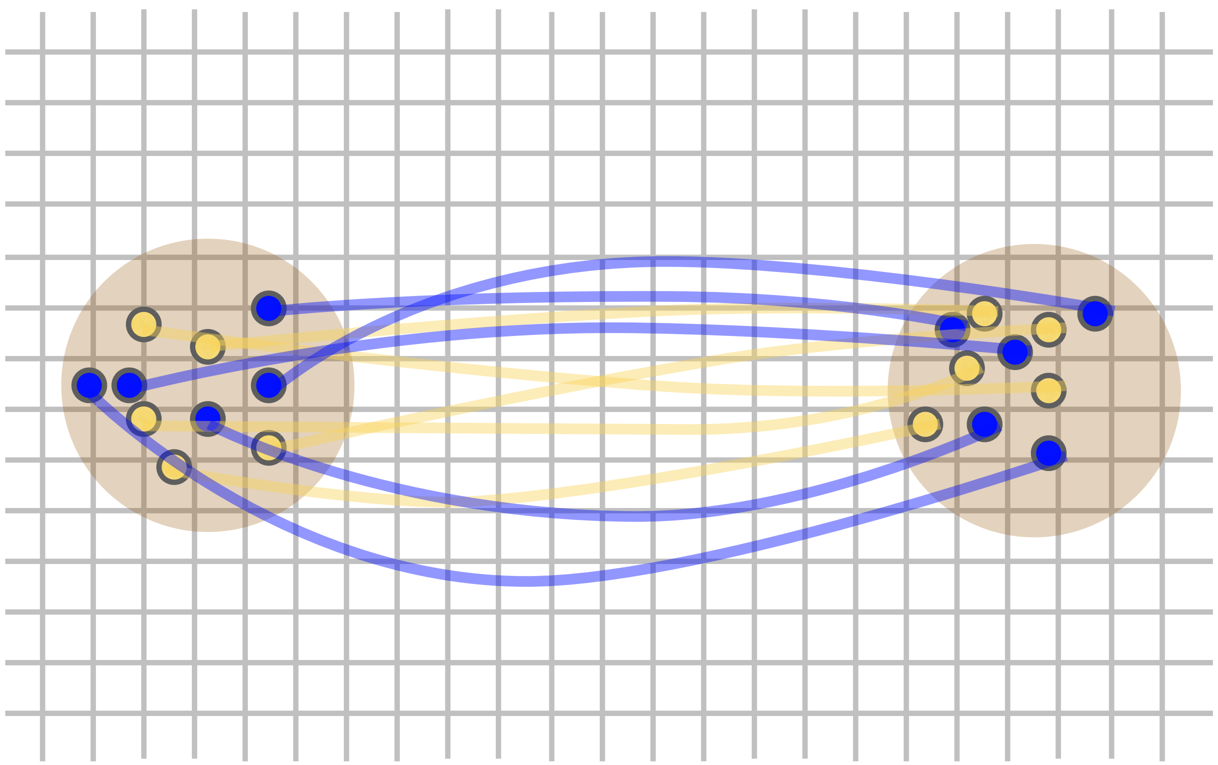
$$\rightarrow A \ e^{-E \ \ t}$$



Many mesons in LQCD

- A typical n $\boldsymbol{\pi}^+$ correlator (m_u=m_d)

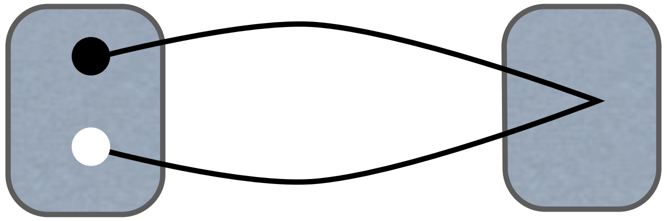
$$C_n(t) = \left\langle 0 \left| \left[\sum_{\mathbf{x}} \overline{d}\gamma_5 u(\mathbf{x},t)\overline{u}\gamma_5 d(\mathbf{0},0)\right]^n \right| 0 \right\rangle$$
$$\rightarrow A\ e^{-E_n t}$$



Contraction Complexity

- Factorial problem with many-meson contractions solved
  - Recursion relations [WD, Savage Phys. Rev. D82, 014501, 2010]
    - Blocks satisfy recursion

$$R_{n+1} = \langle R_n \rangle \; R_1 - n \; R_n \; R_1$$
$$R_1 = \Pi, \qquad R_j = 0, \forall j < 1$$
$$C_n = tr[R_n]$$



  - FFT based methods [WD, K Orginos, Z Shi, PRD 86 (2012) 054507]
    - Allow n-body contractions to be determined $O(n^3)$
    - Limited by lattice artifacts and by precision: quad-double sufficient for 72 pions

Many pion systems

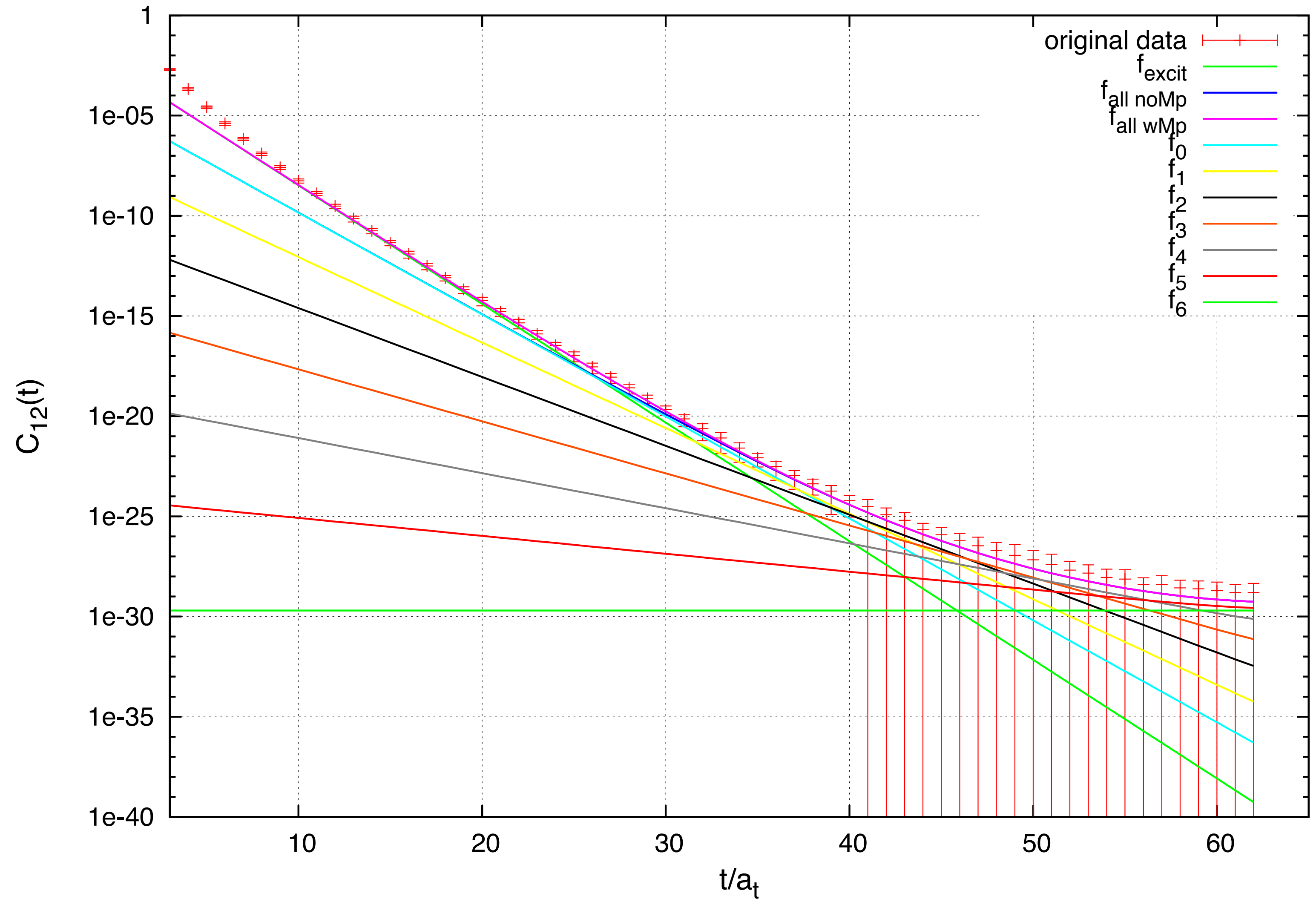
[WD, K Orginos, Zhifeng Shi, PRD 86 (2012) 054507]

- $I_z=n=1,...,72$ pion
- Calculations use anisotropic configs from HSC
  - Clover fermions, Tadpole improved gauge
  - $a_s=0.12$ fm, $a_t=0.04$ fm
- Multiple sources to get to large systems
  - Gauge fixed momentum sources/sinks
- Four volumes: $16^3\times128$, $20^3\times128$, $20^3\times256$, $24^3\times128$
  - Short time extents in two volumes necessitates A+P trick (checked it OK on T=256 data)

Thermal effects

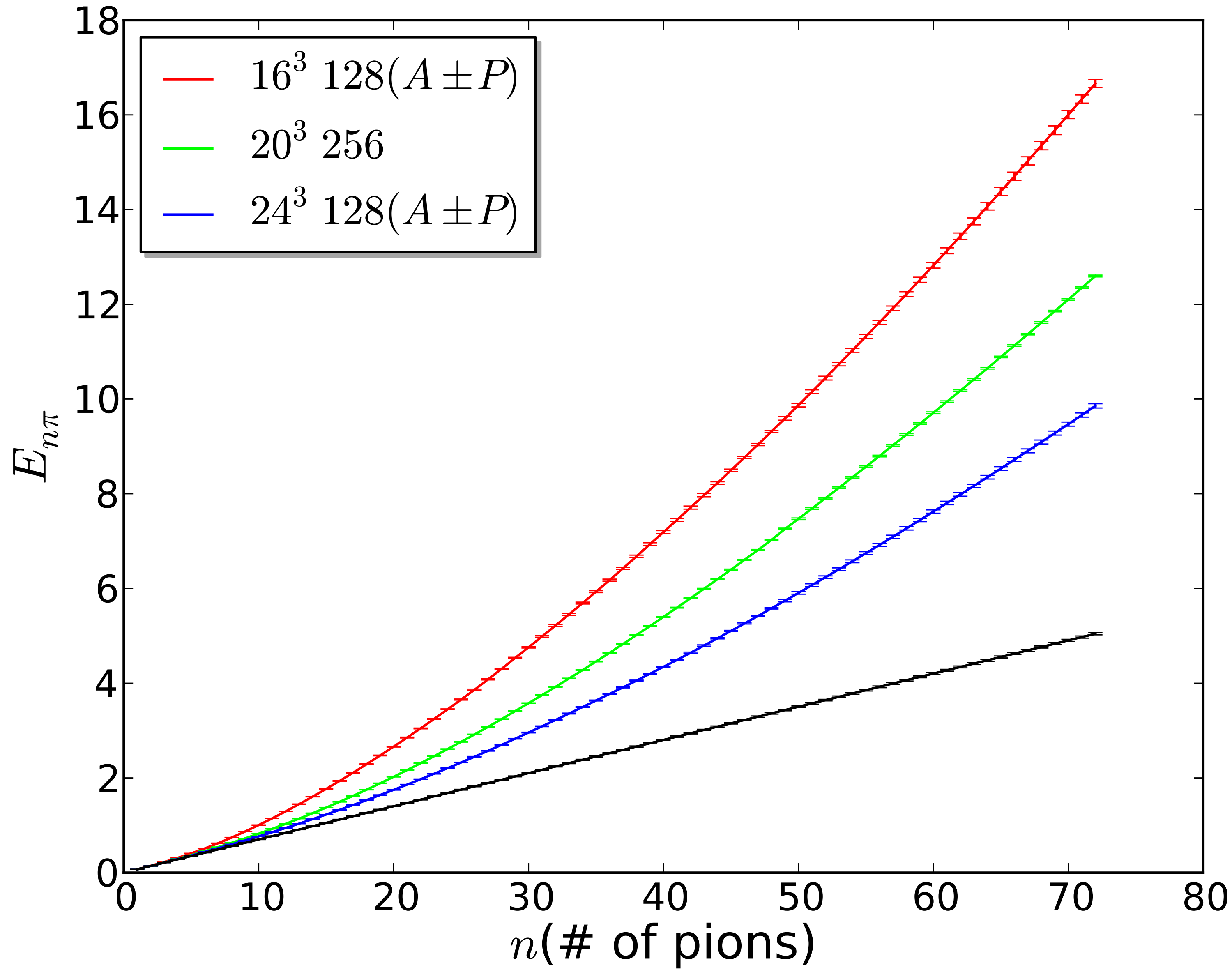
- In lattice of finite temporal extent, contributions where states go around temporal boundary are important

$$C_{n\pi}(t) \; = \; \sum_{m=0}^{\lfloor \frac{n}{2} \rfloor} \binom{n}{m} A^n_m Z^n_m e^{-(E_{n-m}+E_m)T/2} \cosh((E_{n-m} - E_m)(t - T/2)) + \ldots,$$



Energies

- Ground state energy of I_z=n system vs n
- Increasingly repulsive interactions



Effective chemical potential

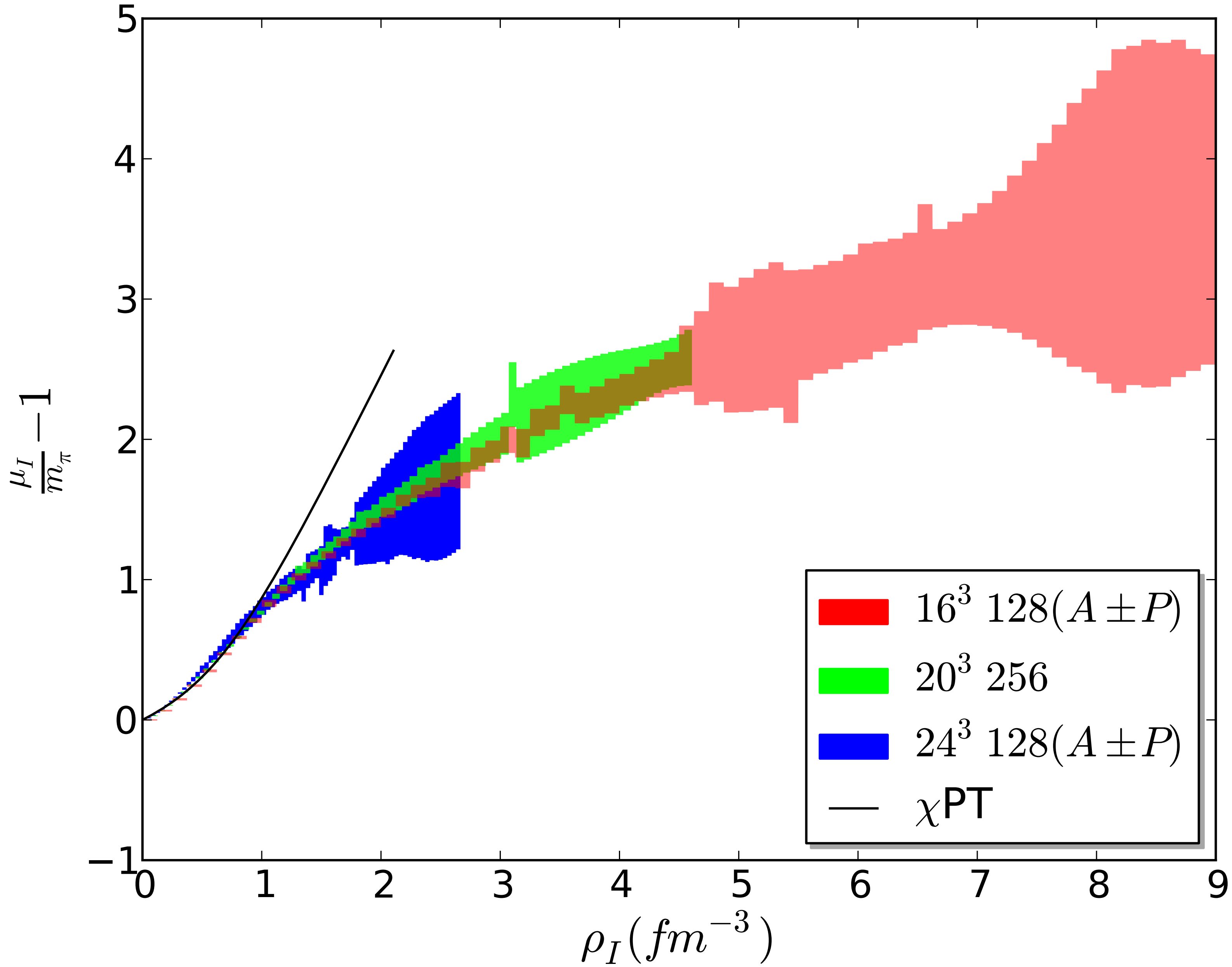
[WD, K Orginos, Zhifeng Shi, PRD 86 (2012) 054507]

- Define "effective chemical potential"

$$\mu_I = \left.\frac{dE}{dn}\right|_V$$

via finite difference

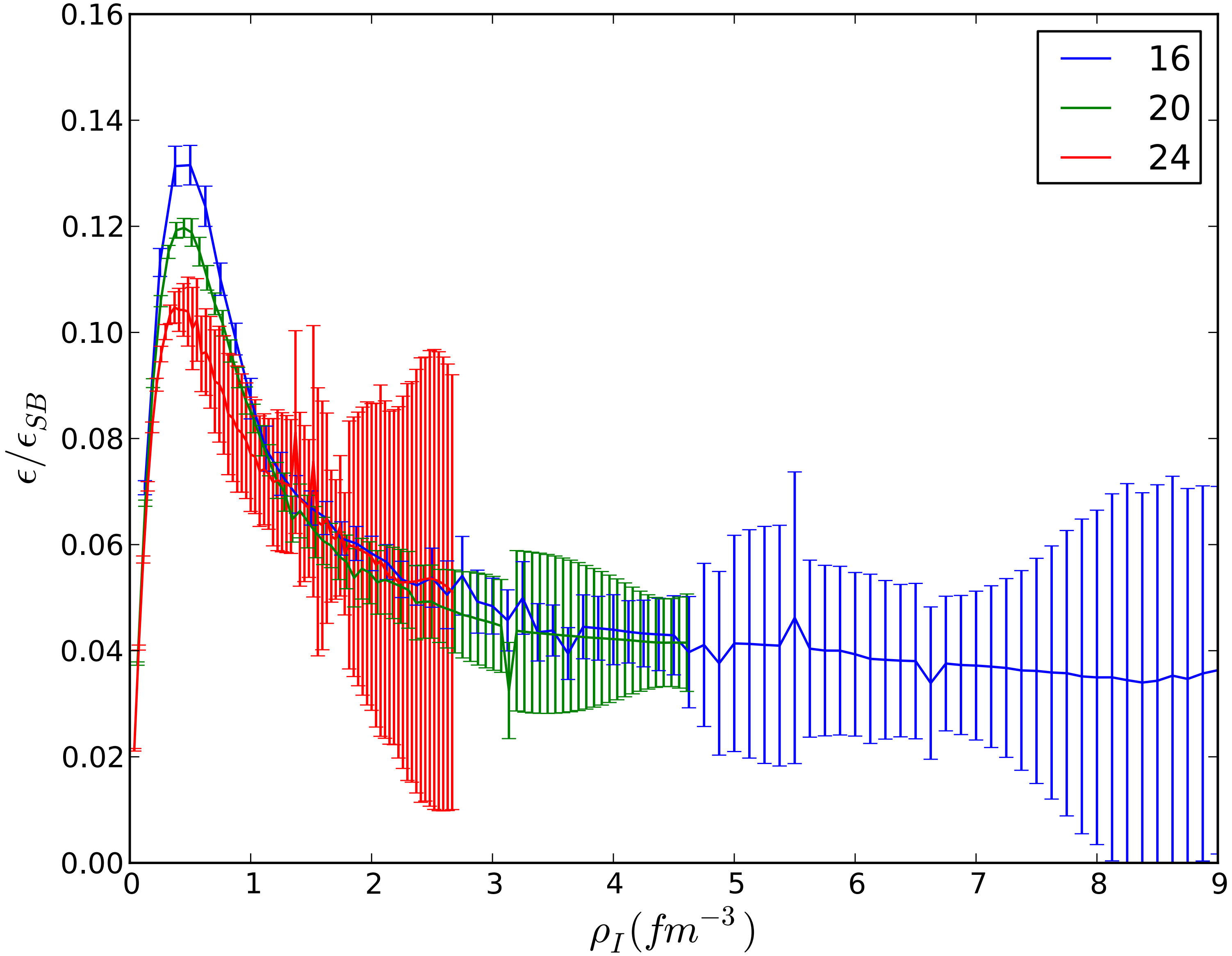
- NB: E is g.s. energy
- Agrees with ChPT expectation at low density but then behaviour changes



Energy density

[WD, K Orginos, Zhifeng Shi, PRD 86 (2012) 054507]

- Energy density c.f. Stefan-Boltzmann expectation
- Peak position corresponds to I~1.3 m_π



Few-body interactions

- Few body systems can be used to extract two- and three-hadron interactions

- For near-threshold systems, Lüscher two-particle quantisation condition generalised to n boson systems [Bogoliubov '47;Huang,Yang '57; Beane, WD, Savage PRD76;074507, 2007; WD+Savage PRD77:057502,2008]

$$\Delta E_n = \frac{4\pi\overline{a}}{M\ L^3}\,{}^nC_2\Big\{1-\left(\frac{\overline{a}}{\pi\ L}\right)\mathcal{I}+\left(\frac{\overline{a}}{\pi\ L}\right)^2\left[\mathcal{I}^2+(2n-5)\mathcal{J}\right]$$
$$-\left(\frac{\overline{a}}{\pi\ L}\right)^3\left[\mathcal{I}^3+(2n-7)\mathcal{I}\mathcal{J}+(5n^2-41n+63)\mathcal{K}\right]\Big\}$$
$$+\,{}^nC_3\,\frac{1}{L^6}\,\hat{\eta}_3^L+\mathcal{O}(L^{-7})$$

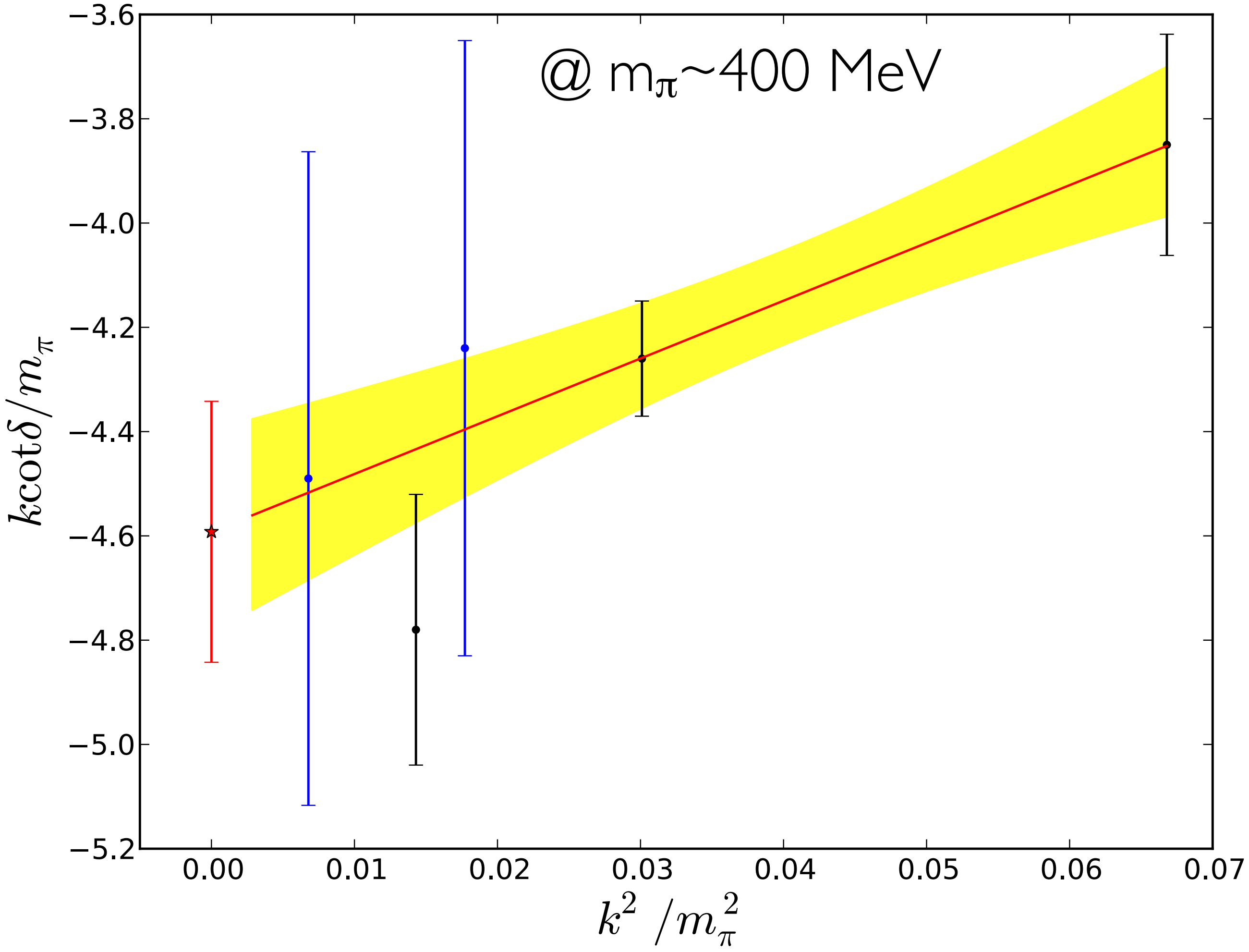
Two-body interaction

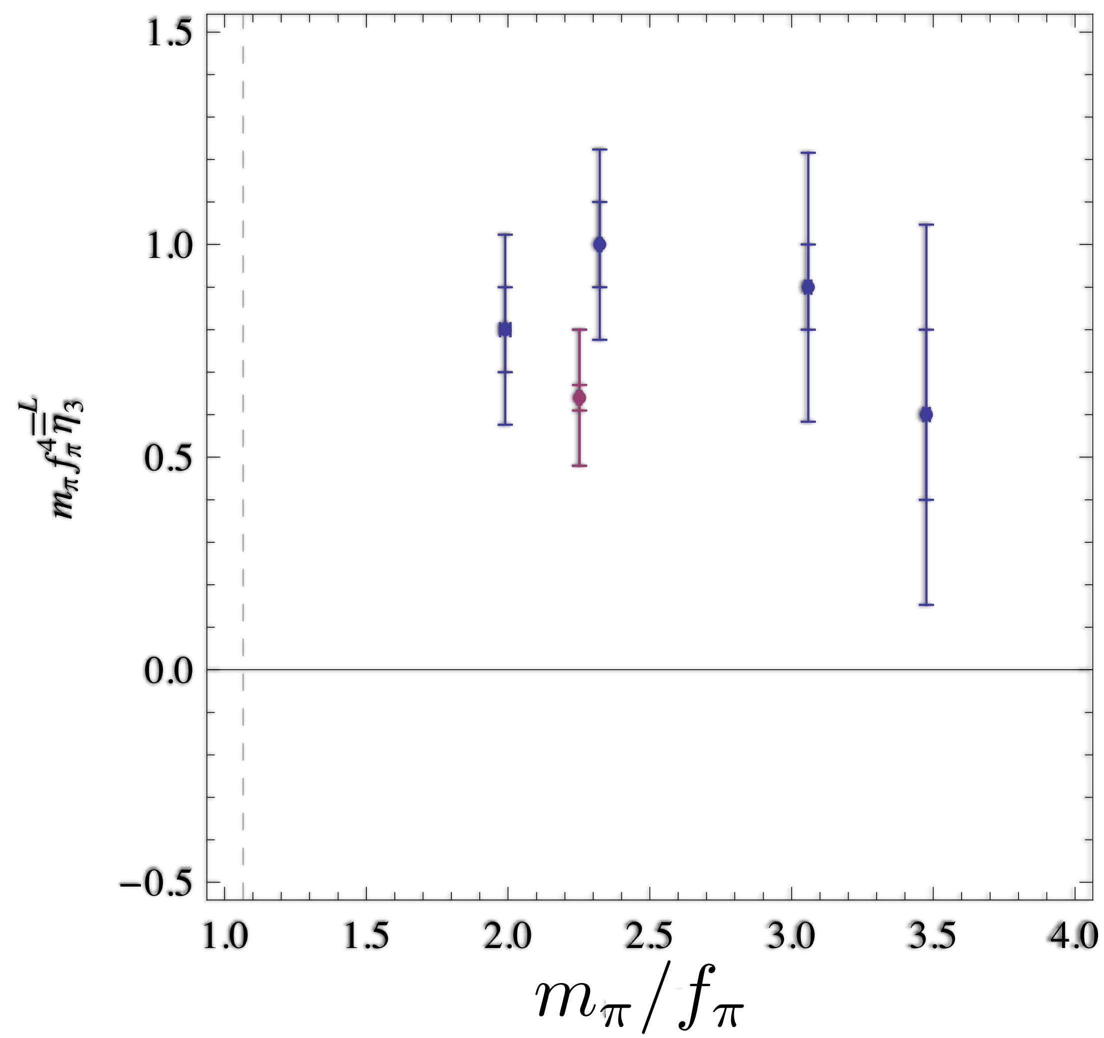
Three-body interaction

- Few body parameters can be extracted from fits to energy shifts

Low energy pion interactions

- Two pion (I=2) and three pion (I=3) interactions





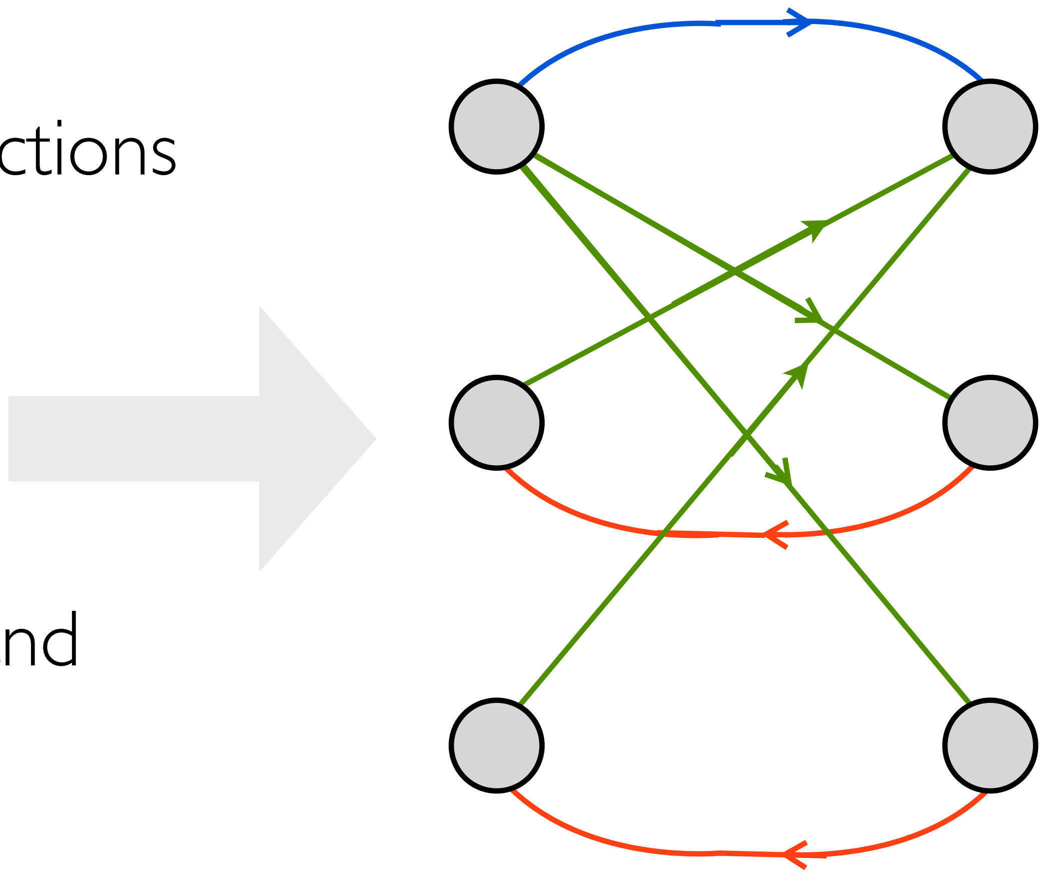
Isospin medium effects

- Medium of fixed isospin density modifies other hadronic properties
- Two examples
    - Baryon masses in medium [with Amy Nicholson]
    - Quarkonium in medium [with Stefan Meinel & Zhifeng Shi]

Baryon masses in medium

[WD, Amy Nicholson, Phys.Rev. D88 (2013) 074501]

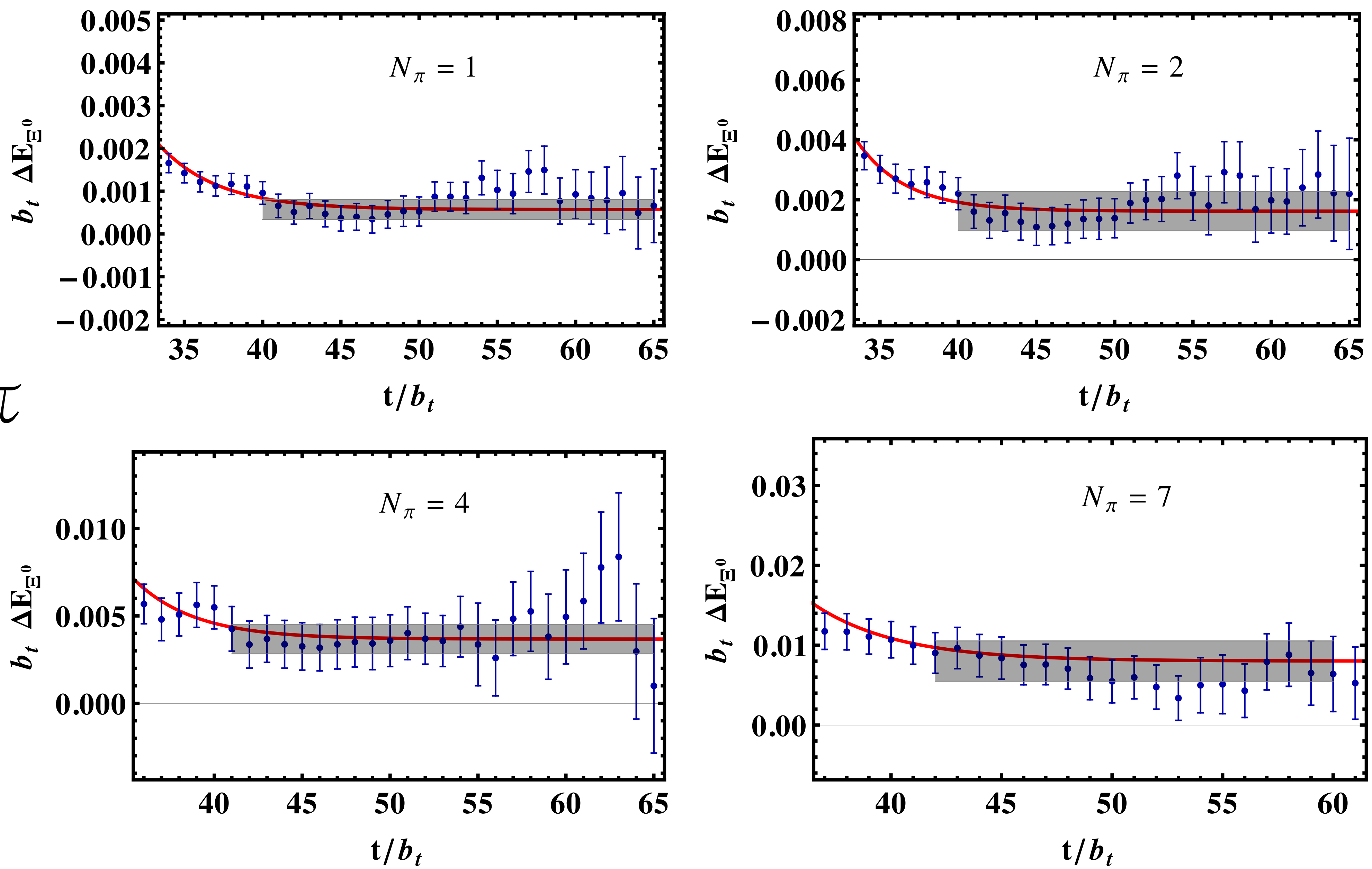
- Systems with quantum numbers of single baryon and many mesons
- Annihilation-less cases: $n(K^+)^N$, $p(K^+)^N$, $\Sigma^+(\pi^+)^n$, $\Xi^0(\pi^+)^n$
- Isospin density dependence of masses: compare with expectations of ChPT
- Extract two- and three- body interactions (MB, MMB)
- Contractions more complicated (require generalised blocks)
- Noisier than many meson systems and thermal effects more problematic



Energy Splittings

$$\Delta M_{\text{eff}}^{(n)}(t) = \ln\left(\frac{C_{B,n}(t)/C_{B,n}(t+1)}{[C_B(t)/C_B(t+1)][C_n(t)/C_n(t+1)]}\right)$$

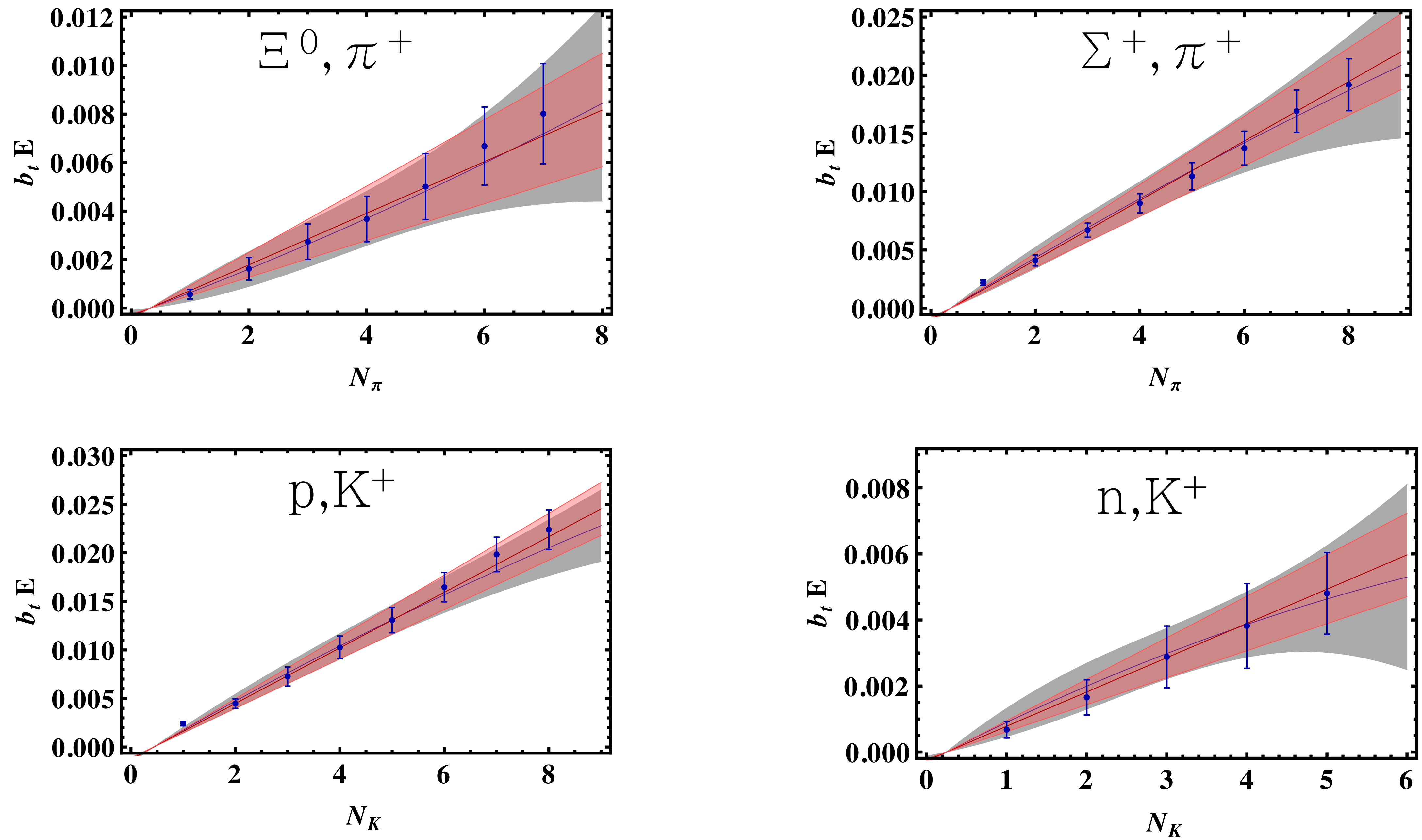
$\Xi^0 n \pi$



Isospin dependence

- Energy shifts vs N_π, N_K and fits to extract ChPT LECs, eg:

$$M_N = M_N^{(0)} - \mu_I \cos\alpha \frac{\tau^3}{2} + 4c_1\left(m_\pi^2\cos\alpha + \lambda\epsilon\sin\alpha\right) + \left(c_2 - \frac{g_A^2}{8M} + c_3\right)\mu_I^2\sin^2\alpha$$



Quarkonium in medium

[WD, Stefan Meinel, Zhifeng Shi, PRD 2013]

- Presence of isospin density modifies the forces binding a quark anti-quark pair together
- Static limit, encapsulated in static quark potential
  - Small screening effect seen [Detmold, Savage PRL 2009]
- Non-static case: modification of quarkonium spectroscopy
  - Study S and P wave states and splittings vs ρ_I
  - NRQCD study of bottomonium [Detmold, Meinel and Shi PRD 2013]
  - RHQ study of charmonium [Z Shi PhD thesis 2013]
  - Extract J/Ψ-π etc interactions

Screening of static potential

- Modification to static quark–anti-quark potential from presence of isospin density

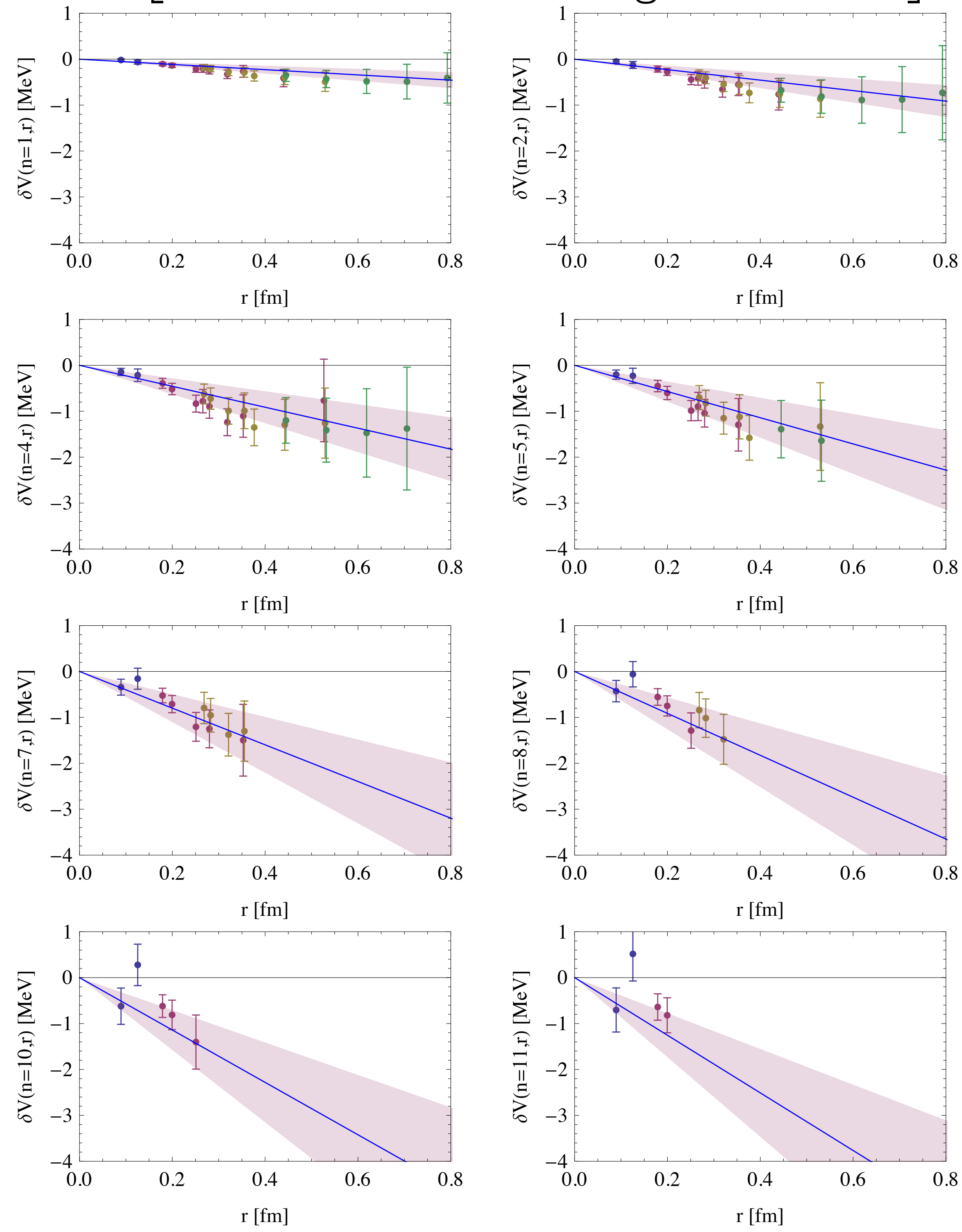
- For relevant distances

$$\delta V(\rho_I, r) = \alpha\ \rho_I\ r,$$
$$\alpha = -8(3)\ \text{MeV fm}^2$$

- Augment Cornell potential by this term and solve for quarkonium states

- Expect larger effects on P wave

[data from WD, M Savage, PRL 2009]



Static potential

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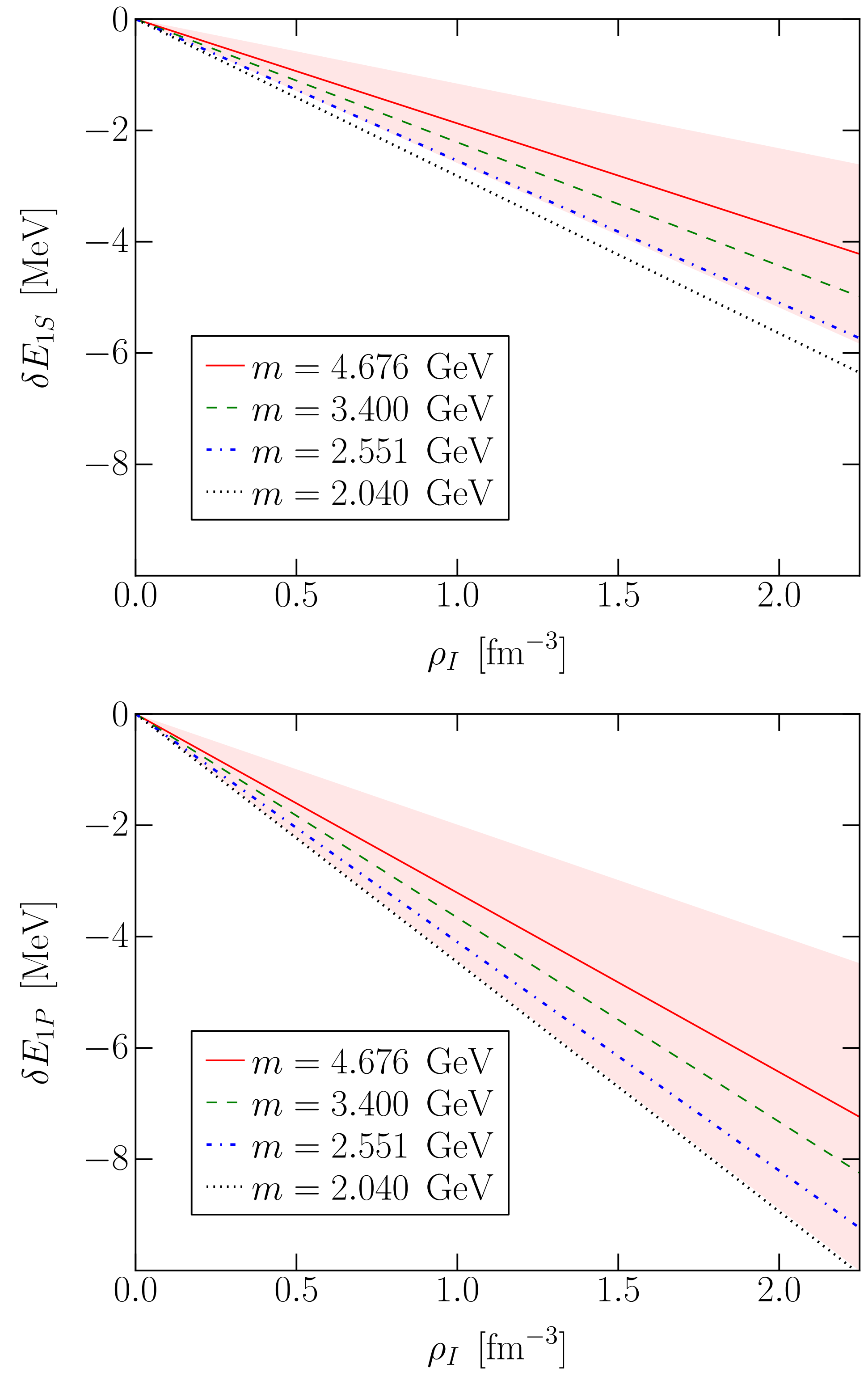


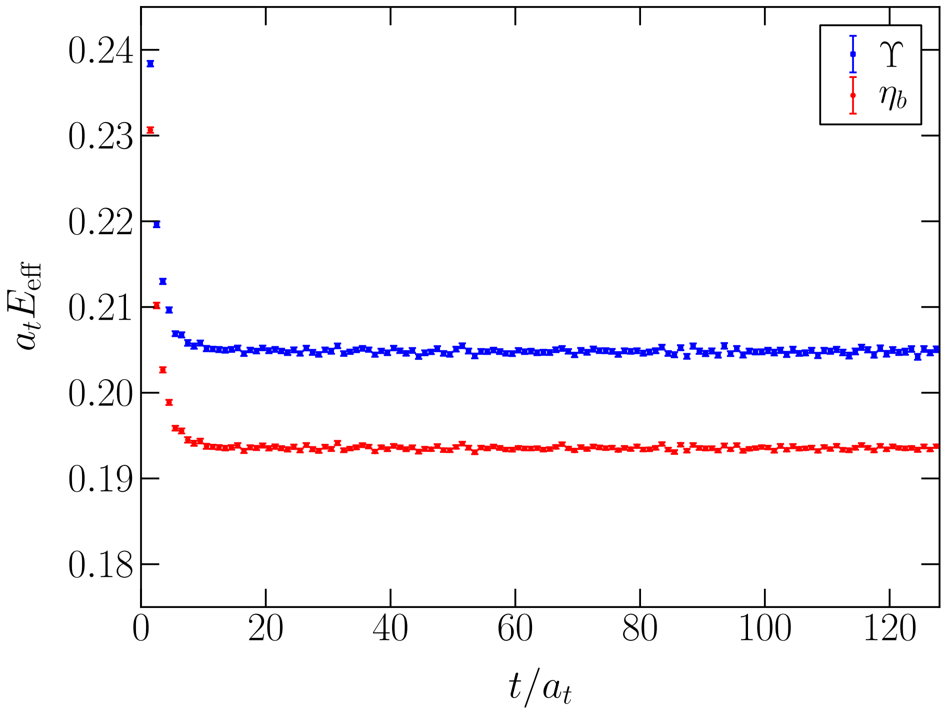
Energy shifts

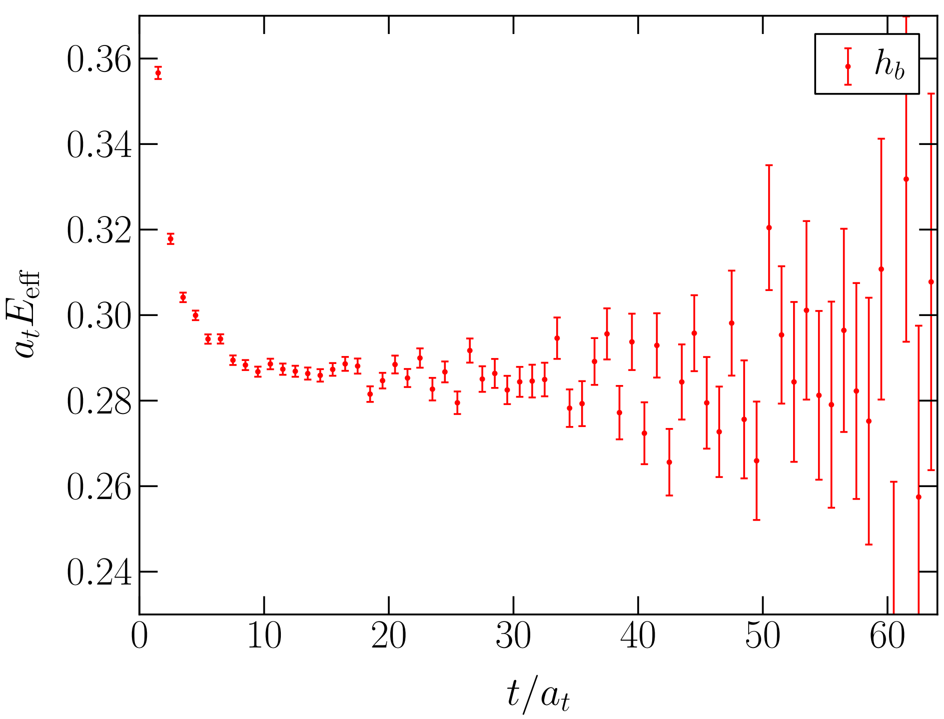
- Use NRQCD for bottom quarks at O(v^6)
- Light quarks as before
- Consider ratios

$$R(n,\bar{b}b;t) = \frac{\langle \mathcal{O}_{\bar{b}b}(t)\mathcal{O}_{n\pi^+}(t)\tilde{\mathcal{O}}^\dagger_{\bar{b}b}(0)\mathcal{O}^\dagger_{n\pi^+}(0)\rangle}{\langle \mathcal{O}_{\bar{b}b}(t)\tilde{\mathcal{O}}^\dagger_{\bar{b}b}(0)\rangle\langle \mathcal{O}_{n\pi^+}(t)\mathcal{O}^\dagger_{n\pi^+}(0)\rangle}$$
$$\longrightarrow Z_{n;\bar{b}b}\exp(-\Delta E_{n;\bar{b}b}t) + \ldots$$

where $\Delta E_{n;\bar{b}b} = E_{n;\bar{b}b} - E_{n\pi^+} - E_{\bar{b}b}$

- Extract energy shift via exponential fits to ratio





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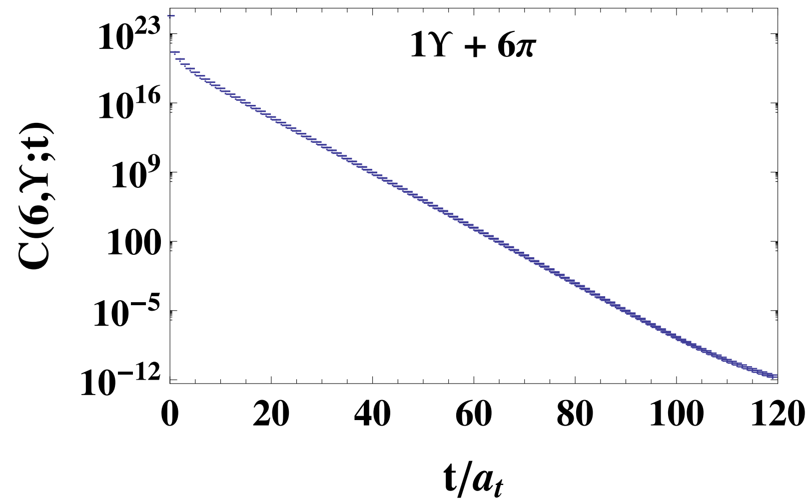

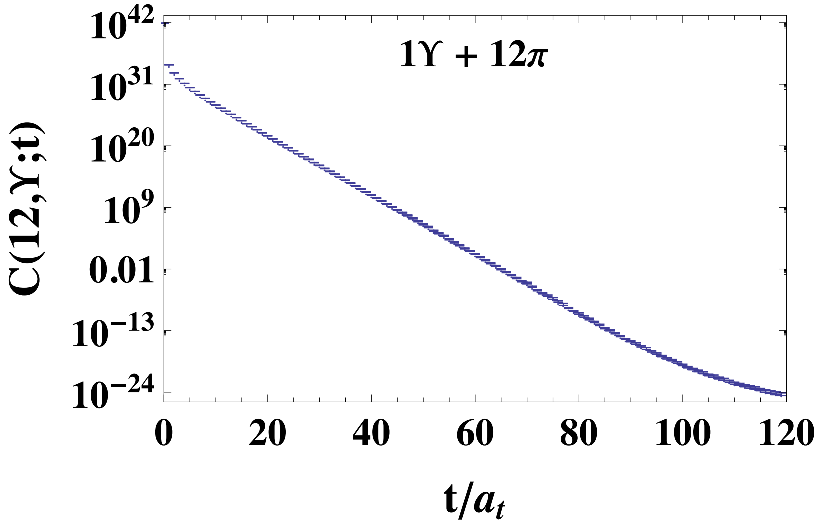

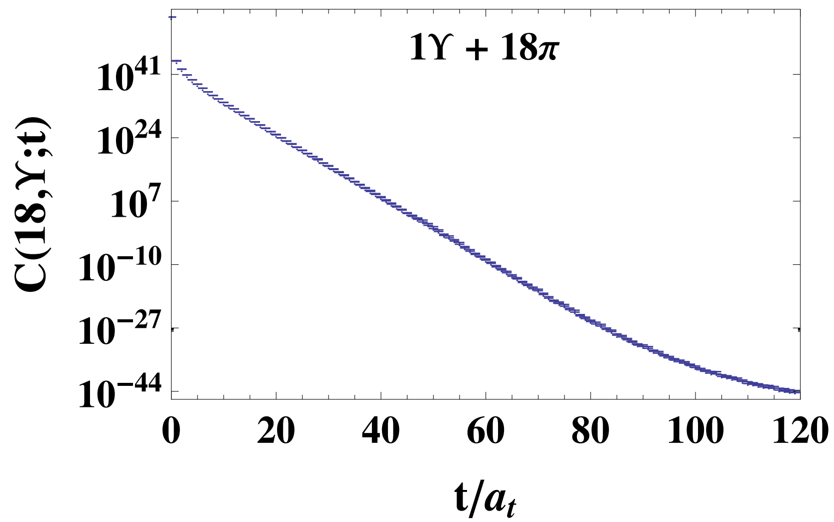

Energy shifts

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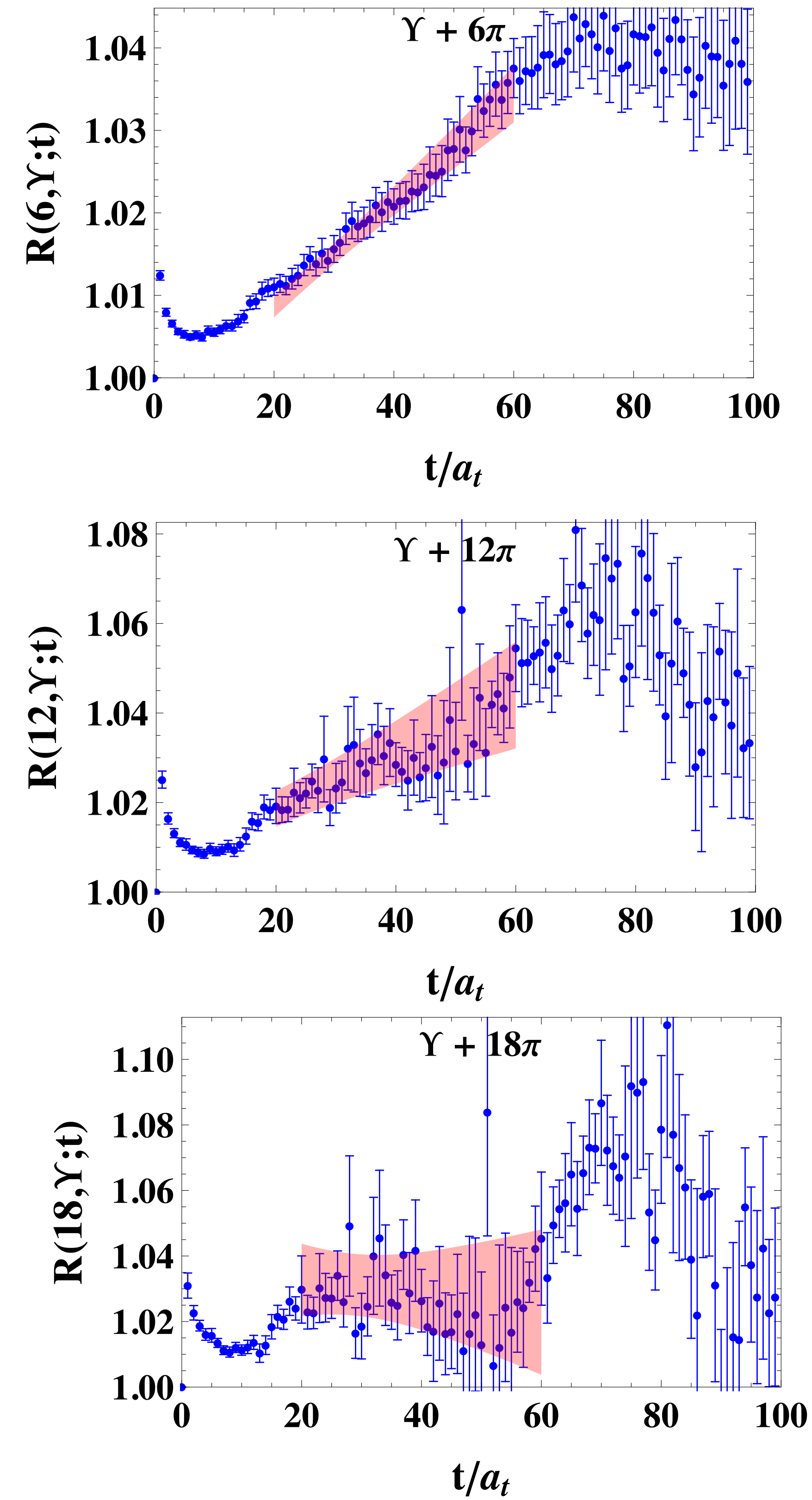
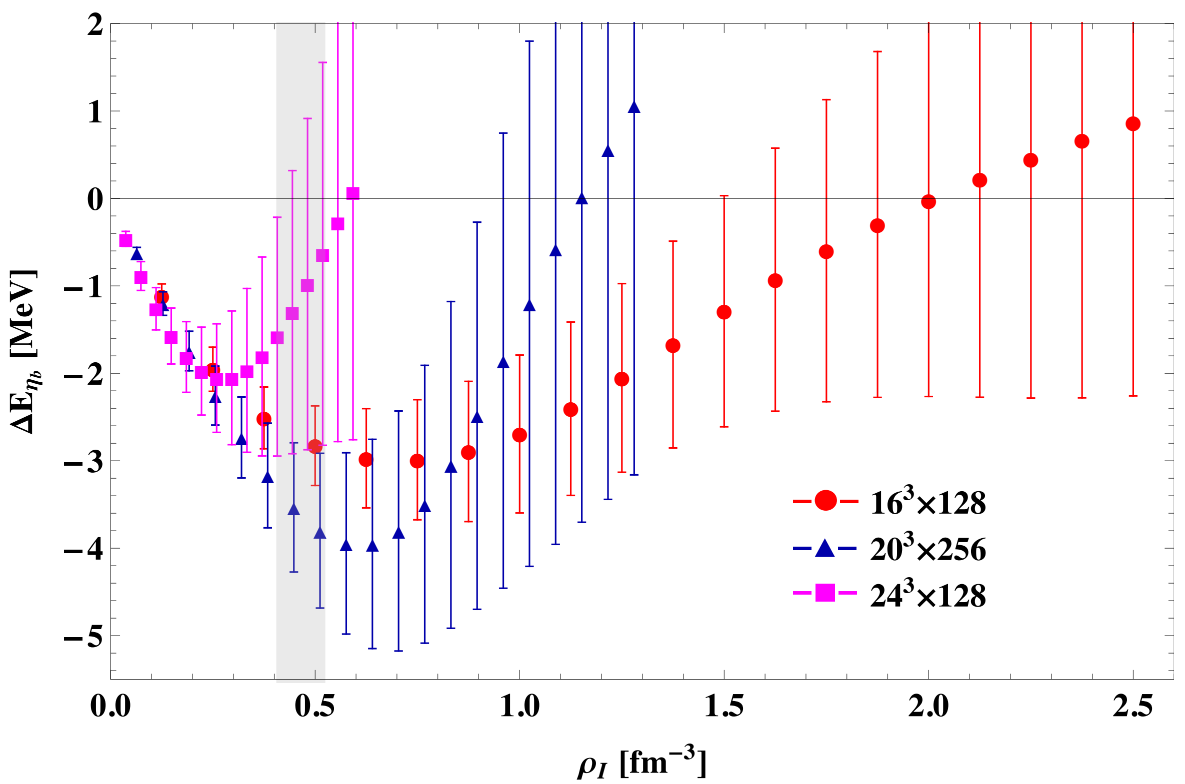
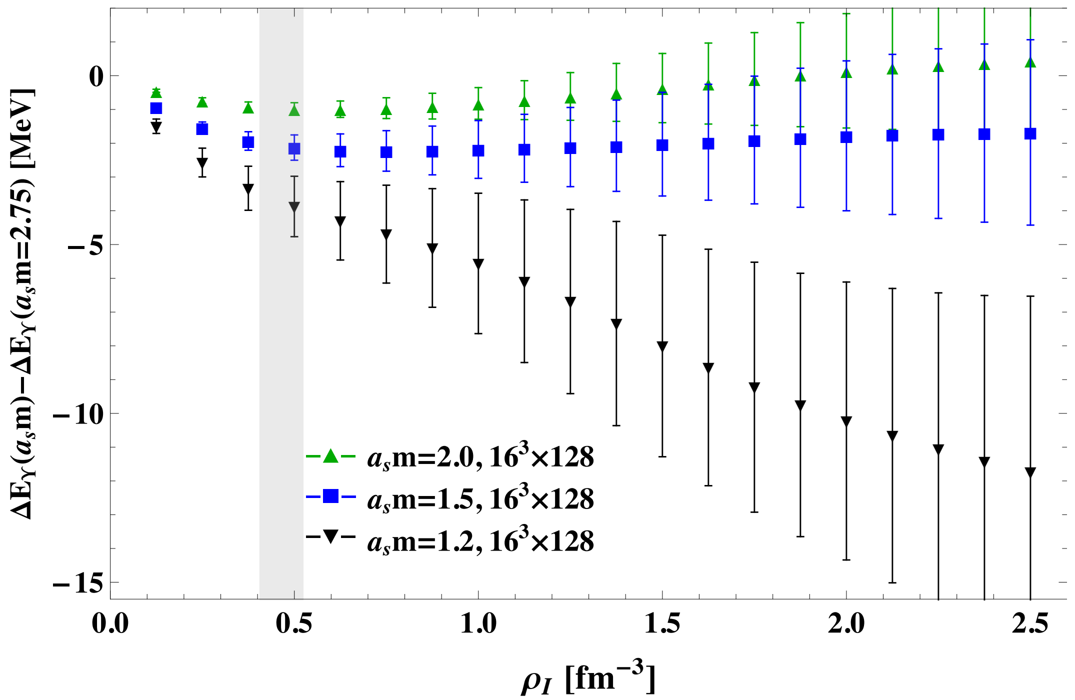
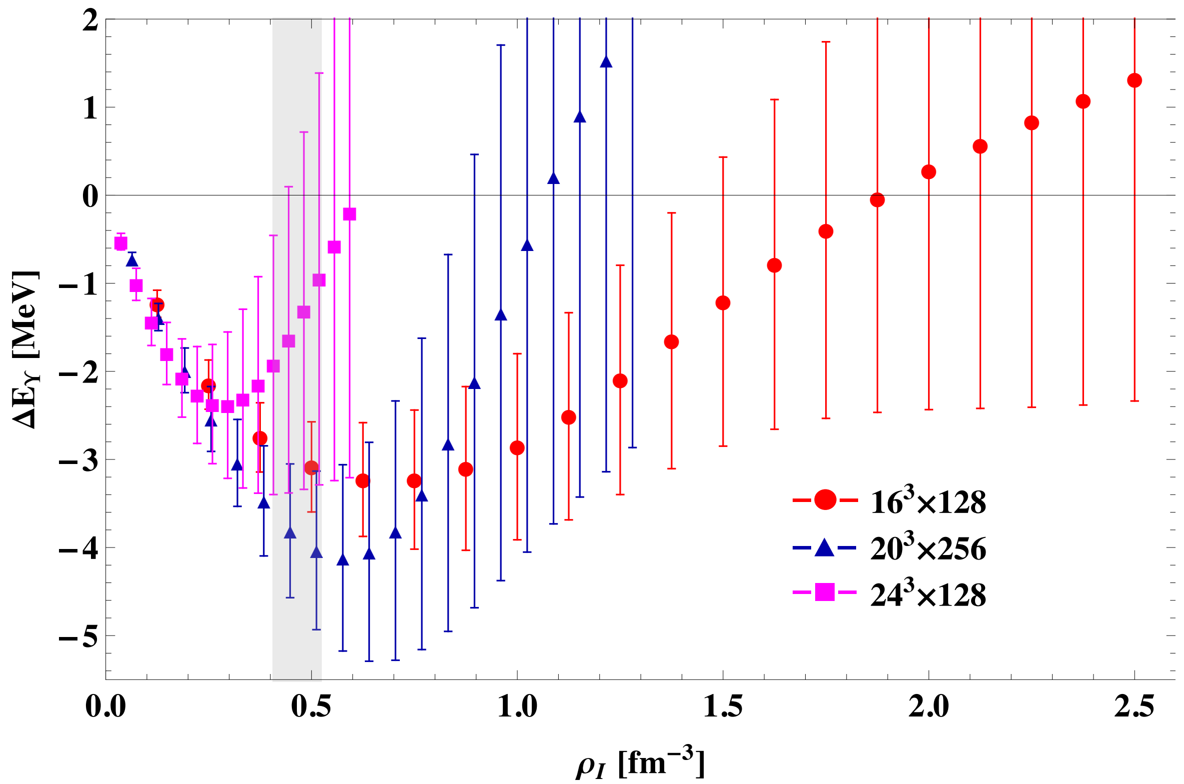


Density dependence

- Dependence on density
- Also investigate for P-wave & hyperfine splitting
- Mass dependence is as expected in potential model

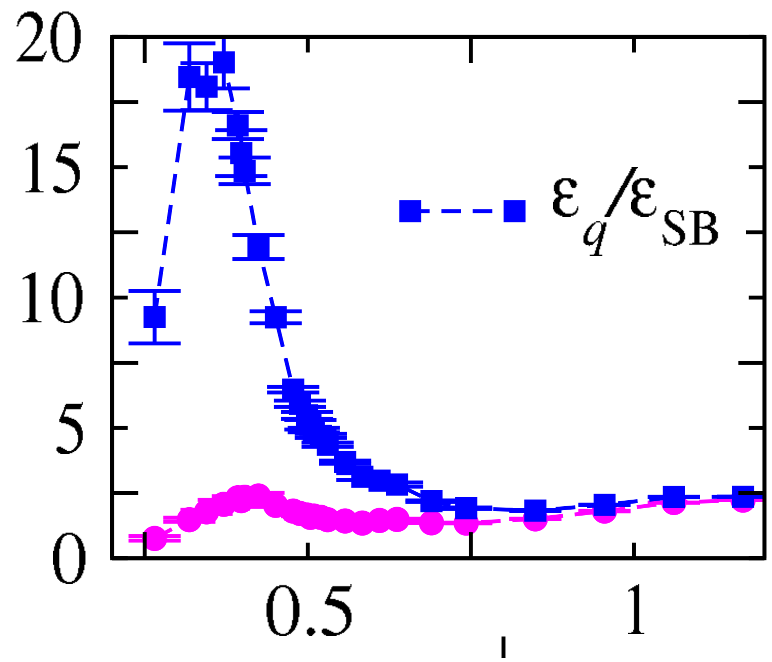


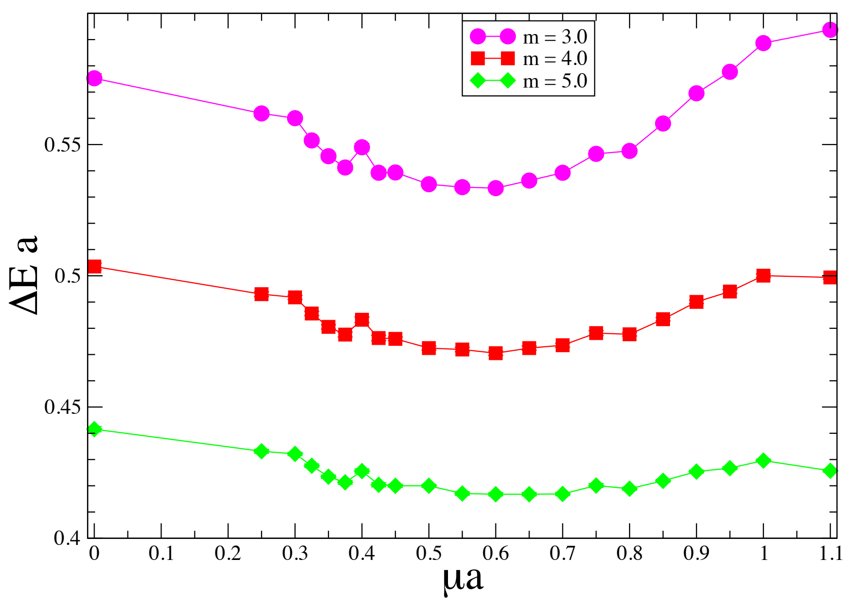




$N_c=N_f=2$

- For N_c=2, quark chemical potential very similar effects seen by Hands, Kim & Skullerud [Phys.Rev. D81 (2010) 091502, Phys.Lett. B711 (2012) 199]





- Beginning studies of many pion approach in $N_c=N_f=2$ Wilson theory for direct comparison

Summary

- Studies of non-zero isospin density enabled by many body contraction techniques
    - two- and three- body interactions
    - bulk properties
    - effects on other hadronic quantities
- Future directions:
    - probe higher density to see BEC-BCS transition

FIN

Baryon masses in medium

- Anisotropic lattices (HSC)
  - clover fermions, tadpole improved gauge
  - $a_s \sim 0.125$ fm, $a_t \sim a_s/3.5$,
  - $m_\pi \sim 390$ MeV, $32^3 \times 256$
  - ~ 200 measurements per configuration
- Noisier than many meson
- Thermal effects more problematic

Many mesons in LQCD

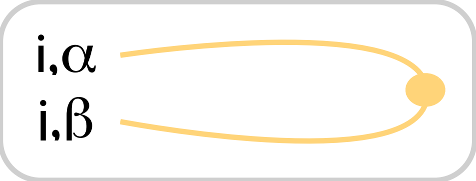
- Consider n π^+ correlator (m_u=m_d)

$$C_n(t) = \left\langle 0 \left| \left[\sum_{\mathbf{x}} \overline{d}\gamma_5 u(\mathbf{x},t)\overline{u}\gamma_5 d(\mathbf{0},0) \right]^n \right| 0 \right\rangle$$
$$\to A\ e^{-E_n t}$$

- $n!^2$ Wick contractions: $(12!)^2 \sim 10^{17}$

- Computable as coefficients in expansion of det=[1+ $\boldsymbol{\lambda\Pi}$]
[WD et al (NLQCD) 2007]

$$C_3(t) = \mathrm{tr}\left[\Pi\right]^3 - 3\ \mathrm{tr}\left[\Pi\right]\mathrm{tr}\left[\Pi^2\right] + 2\ \mathrm{tr}\left[\Pi^3\right]$$
$$\Pi = \sum_{\mathbf{x}} \gamma_5 S(\mathbf{x},t;0)\gamma_5 S^\dagger(\mathbf{x},t;0)$$



- Maximal isospin: only a single quark propagator for small n

- Generalised to multi-species systems [Detmold & Smigielski 2011]

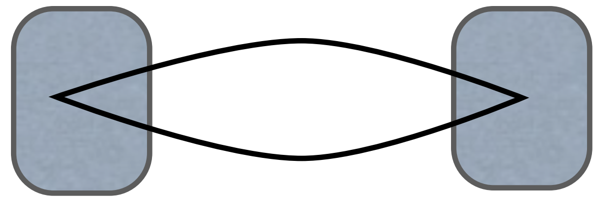
Larger systems

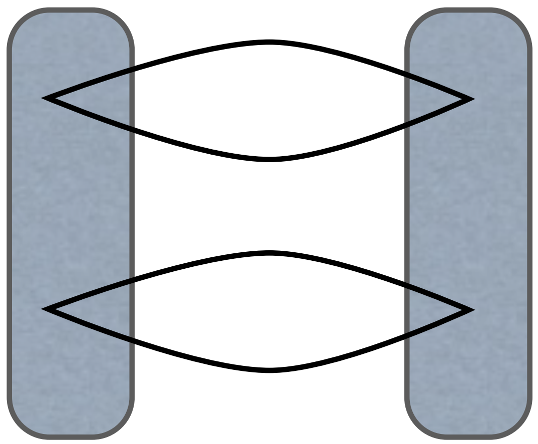
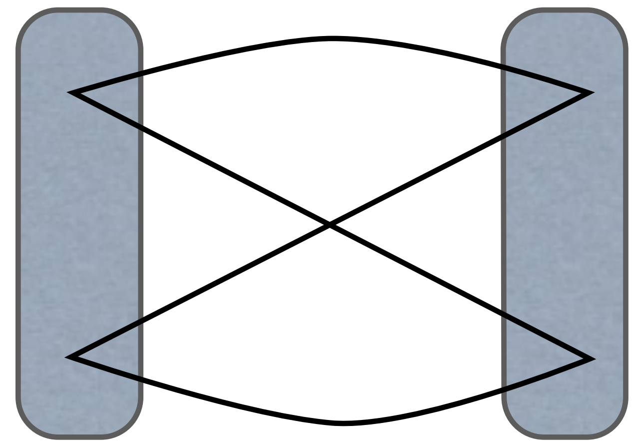
- How do we deal with complexity of contractions?
  - One species: $N_{\text{terms}} \sim e^{\pi\sqrt{2n/3}}/\sqrt{n}$
  - Two-species is harder, more is unfeasible
- How do we go beyond n=12?
  - Previous method fails because of Pauli principle
  - Avoid by using multiple propagator sources but this leads to contraction complexity

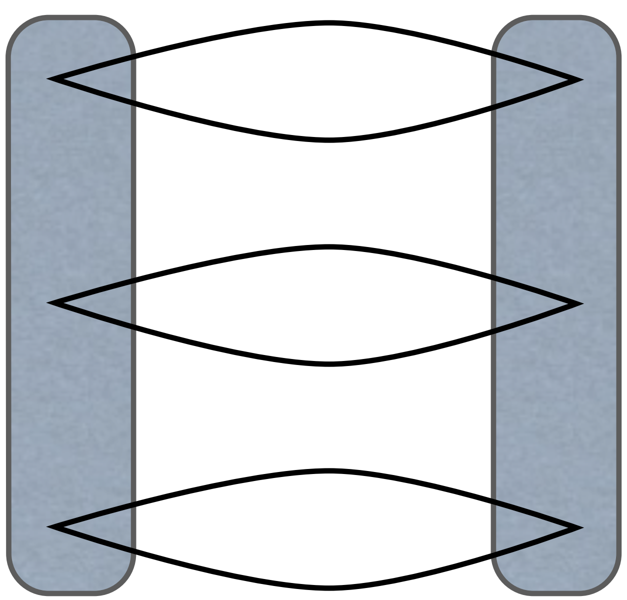
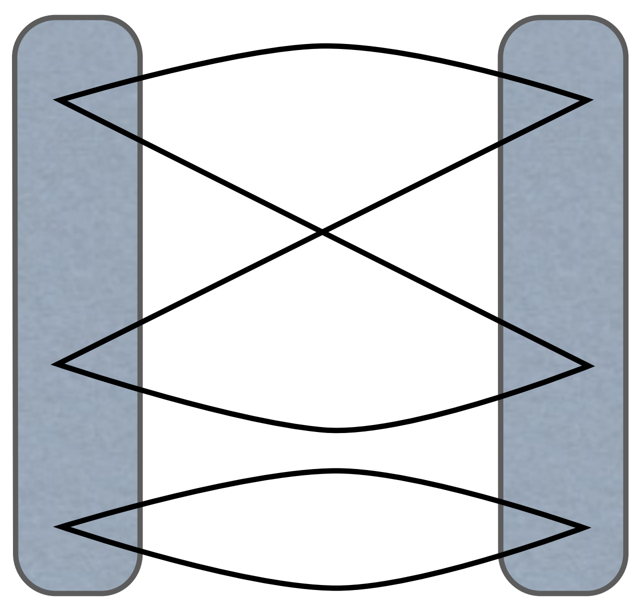
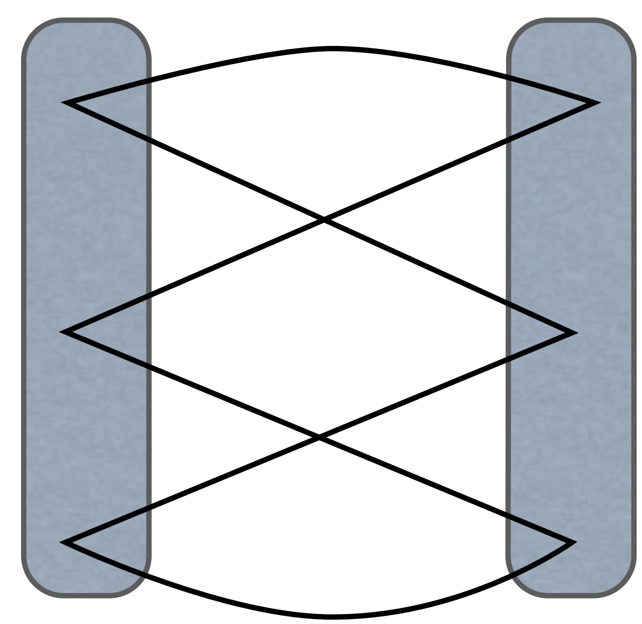
Lattice details

- NPLQCD collaboration [PRL2007,PRD2008,...]
- Calculations use MILC gauge configurations
  - L=2.5 fm, a=0.12 fm, *rooted* staggered
  - also L=3.5 fm and a=0.09 fm
- NPLQCD: domain-wall quark propagators
  - $m_\pi \sim$ 291, 318, 352, 358, 491, 591 MeV
  - 24 propagators / lattice in best case
- I_z=n=1,...,12 pion and (S=n) kaon systems

Few pion contractions

$C_{1\pi}(t) =$ 

$C_{2\pi}(t) =$  $-$ 

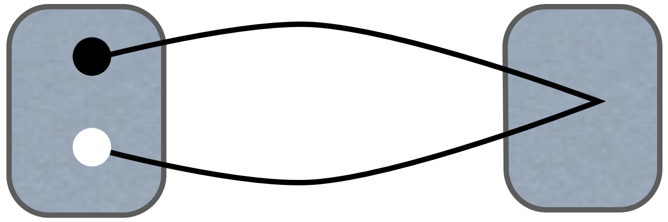
$C_{3\pi}(t) =$  -3  -2 

Blocks

- Define a partly contracted pion correlator

$$\Pi \equiv R_1 = \sum_{\mathbf{x}} S_u(\mathbf{x},t;x_0)\gamma_5 S_d(x_0;\mathbf{x},t)\gamma_5 = \sum_{\mathbf{x}} S_u(\mathbf{x},t;x_0) S_d^\dagger(\mathbf{x},t;x_0)$$

  - Time-dependent 12x12 matrix (spin-colour indices)



- Correlators

$$C_1(t) = \langle \Pi \rangle, \quad C_2(t) = \langle \Pi \rangle^2 - \langle \Pi^2 \rangle, \ldots$$

- Functional definition

$$\Pi_{ij} = \bar{u}_i(x) u_k(x_0) \frac{\delta}{\delta \bar{u}_j(x) \delta u_k(x_0)} C_1(t)$$

- Generalises to

$$(R_n)_{ij} \equiv \bar{u}_i(x) u_k(x_0) \frac{\delta}{\delta \bar{u}_j(x) \delta u_k(x_0)} C_n(t)$$

Recursion relation

[WD, M Savage, Phys. Rev. D82, 014501, 2010]

- The block objects *are* simply related
- Very simple recursion relation

$$R_{n+1} = \langle R_n \rangle \; R_1 - n \; R_n \; R_1$$

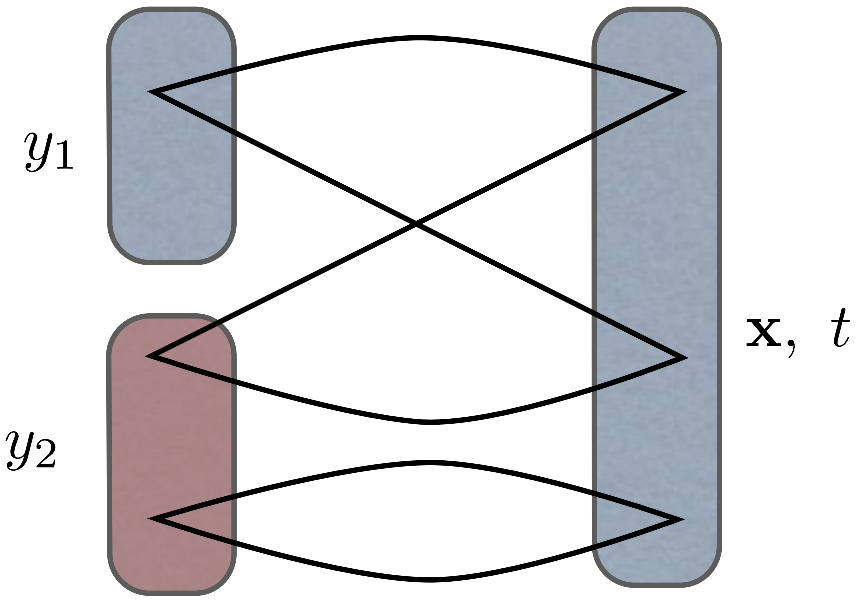
- Initial condition is that $R_1 = \Pi, \qquad R_j = 0, \; \forall j < 1$
- Can also construct a descending recursion as we know that R_{13}=0
- NB: recurrence idea generalised to baryons
  [Doi&Endres 2012; WD & Orginos 2012; Gunther, Toth, Varnhorst Phys.Rev. D87 (2013) 094513]

Multi-source systems

- To get beyond n=12, need to consider multi-source systems
- Consider two sources first

$$C_{(n_1\pi_1^+\ ,\ n_2\pi_2^+)}(t) = \left\langle \left(\sum_{\mathbf{x}} \pi^+(\mathbf{x},t) \right)^{n_1+n_2} \left(\pi^-(\mathbf{y_1},0) \right)^{n_1} \left(\pi^-(\mathbf{y_2},0) \right)^{n_2} \right\rangle$$

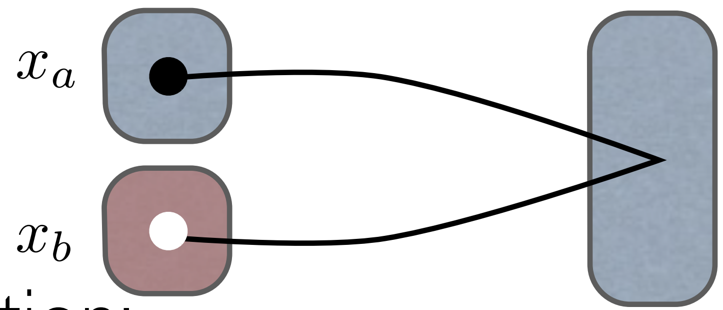
- $C_{(1,2)}(t)$ contains contractions like



Multi-source systems

- Multiple types of blocks needed

$$A_{ab} = \sum_{\mathbf{x}} S_u(\mathbf{x}, t; x_a) S_d^{\dagger}(\mathbf{x}, t; x_b)$$



- Two species case has a simple recursion relation:
  First define

$$P_1 = \left(\begin{array}{c|c} A_{11}(t) & A_{12}(t) \\ \hline 0 & 0 \end{array} \right) \quad , \quad P_2 = \left(\begin{array}{c|c} 0 & 0 \\ \hline A_{21}(t) & A_{22}(t) \end{array} \right)$$

Then the generalisations of the R_n satisfy a recursion

$$Q_{(n_1+1,n_2)} = \langle\, Q_{(n_1,n_2)} \,\rangle\, P_1 \; - \; (n_1+n_2)\, Q_{(n_1,n_2)} \;\; P_1$$
$$+ \langle\, Q_{(n_1+1,n_2-1)} \,\rangle\, P_2 \; - \; (n_1+n_2)\, Q_{(n_1+1,n_2-1)} \;\; P_2$$

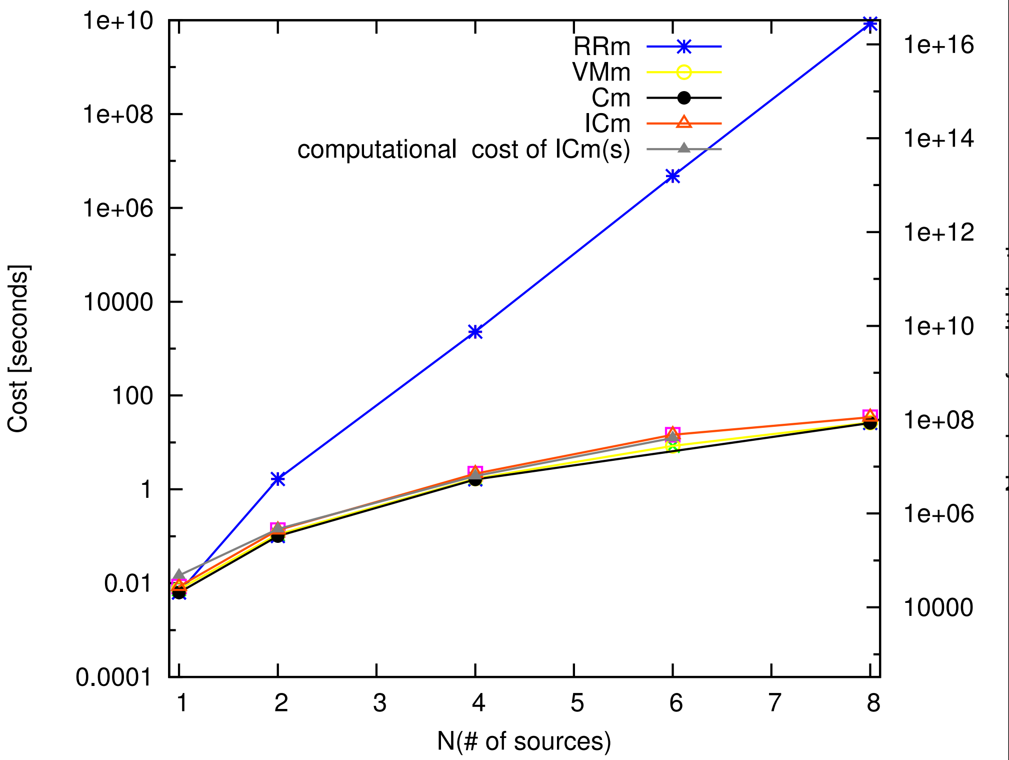
Further algorithms

[WD, K Orginos, Zhifeng Shi, PRD 86 (2012) 054507]

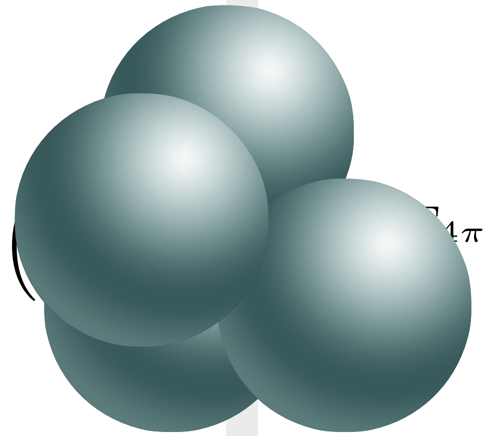
- A number of other ways of performing the contractions
  - Vandermonde matrix method

$$\begin{pmatrix} \frac{\det[1+\lambda_1 A]-1}{\lambda_1} \\ \frac{\det[1+\lambda_2 A]-1}{\lambda_2} \\ \vdots \\ \frac{\det[1+\lambda_{12N} A]-1}{\lambda_{12N}} \end{pmatrix} = \begin{pmatrix} 1 & \lambda_1 & \lambda_1^2 & \dots & \lambda_1^{12N-1} \\ 1 & \lambda_2 & \lambda_2^2 & \dots & \lambda_2^{12N-1} \\ \vdots & & & & \\ 1 & \lambda_n & \lambda_n^2 & \dots & \lambda_n^{12N-1} \end{pmatrix} \cdot \begin{pmatrix} C_{1\pi} \\ C_{2\pi} \\ \vdots \\ C_{12N\pi} \end{pmatrix}$$

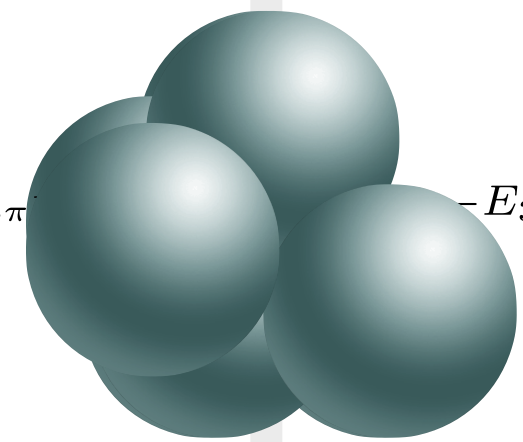
  - Improved recursion method
  - fast Fourier methods
  - eigenvalue method [Anyi Li]
- Scale as N^3 !!



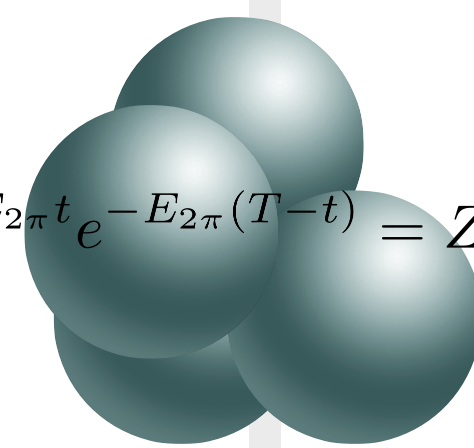
Four pion correlation



$Z_{4\pi} \left(\ldots {}_{4\pi}(T-t) \right)$



$Z_{3/1\pi} \left(e^{-E_{3\pi} \ldots} \ldots e^{-E_{3\pi}(T-t)} e^{-E_{1\pi} t} \right)$



$$Z_{2/2\pi} e^{-E_{2\pi} t} e^{-E_{2\pi}(T-t)} = Z_{2/2\pi} e^{-E_{2\pi} T}$$

t=0

Ratios without correlations

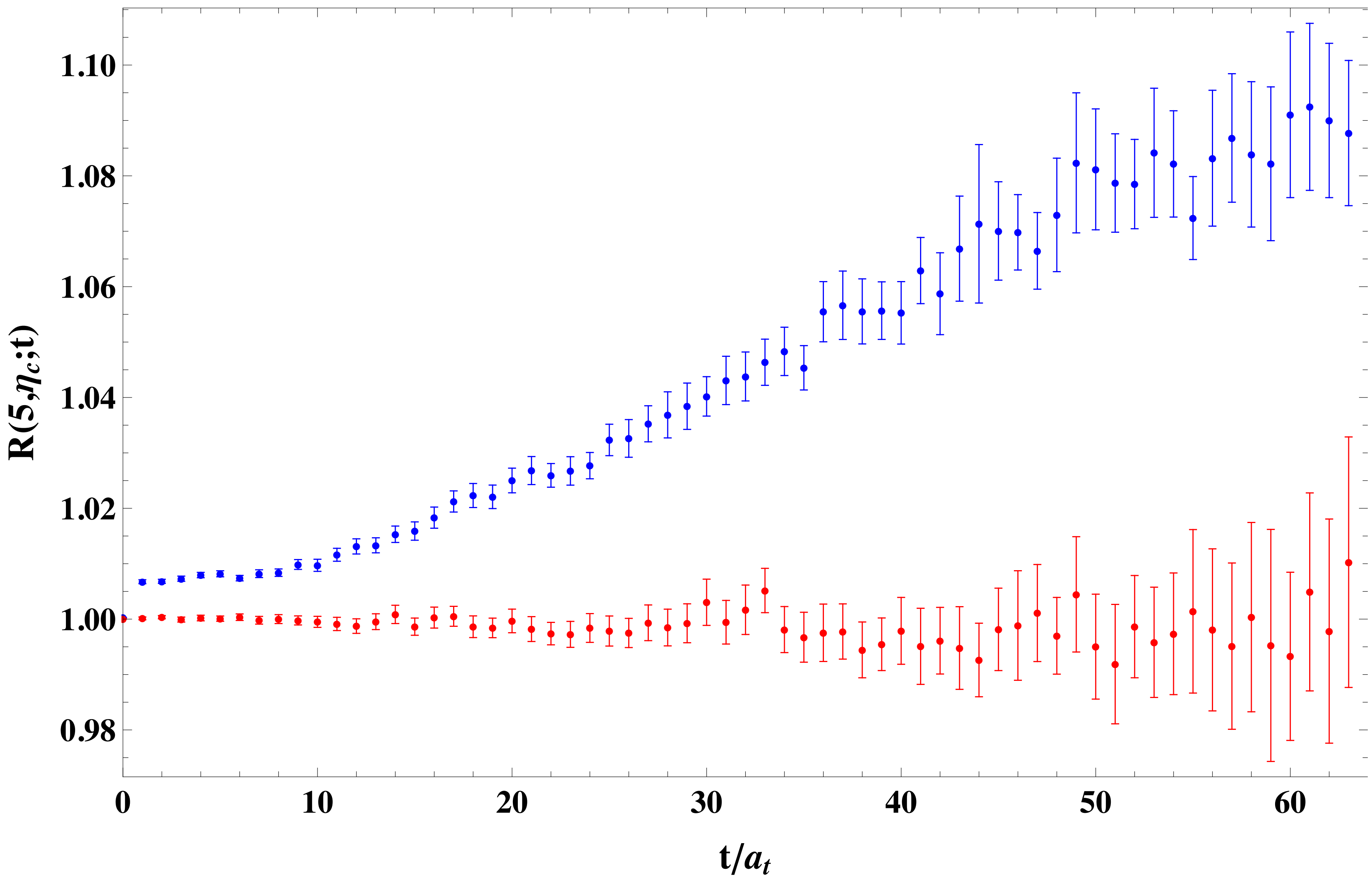
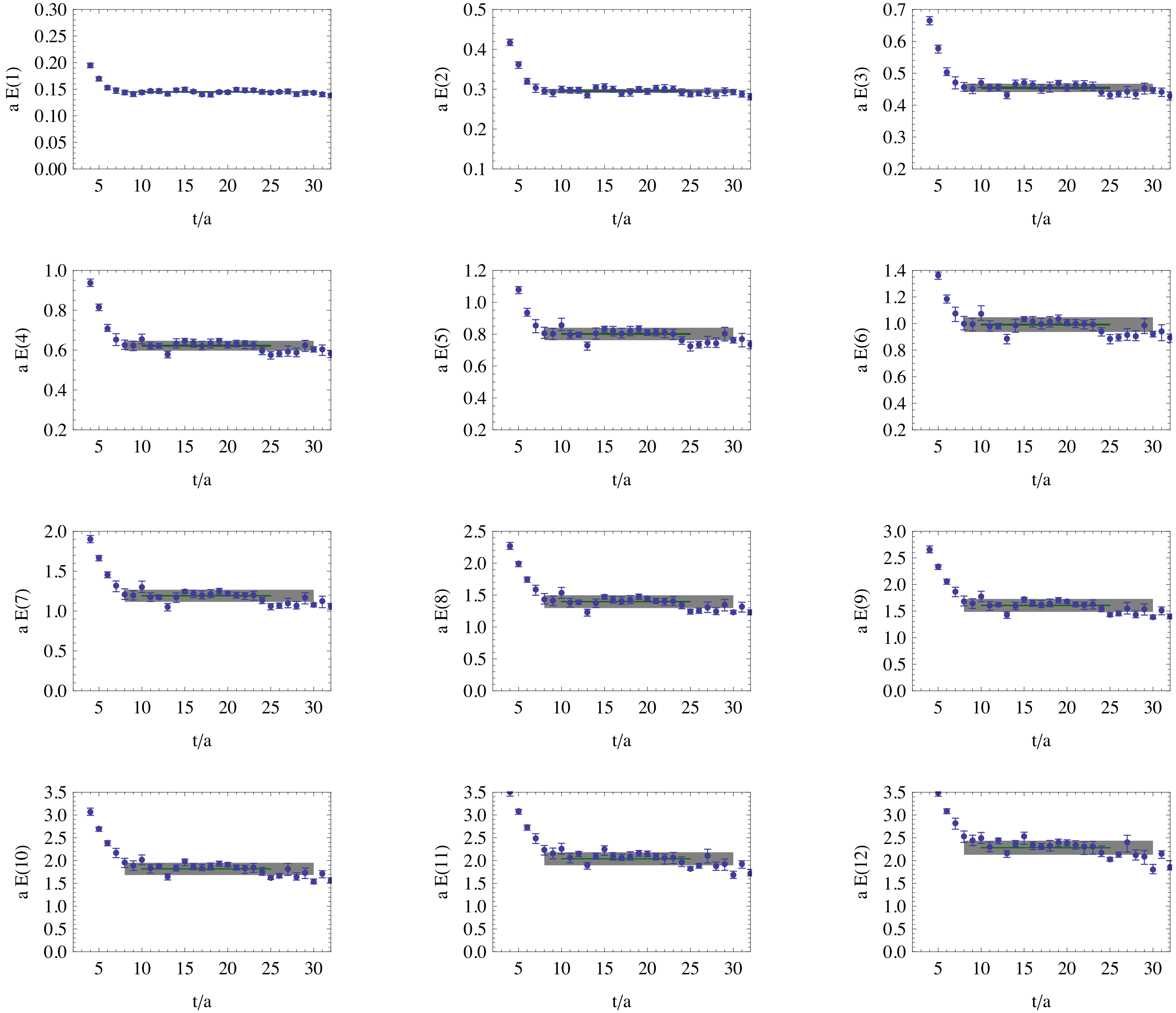


FIG. 5.16: In this figure, correlated contraction and uncorrelated contraction by shifting 50 configurations are compared. When correlations among $C_{\eta_c}(t)$ and $C_{n\pi}(t)$ are taken away, we indeed recover the result for uncorrelated correlation functions such that the ratio is consistent with 1.0.

n-meson energies

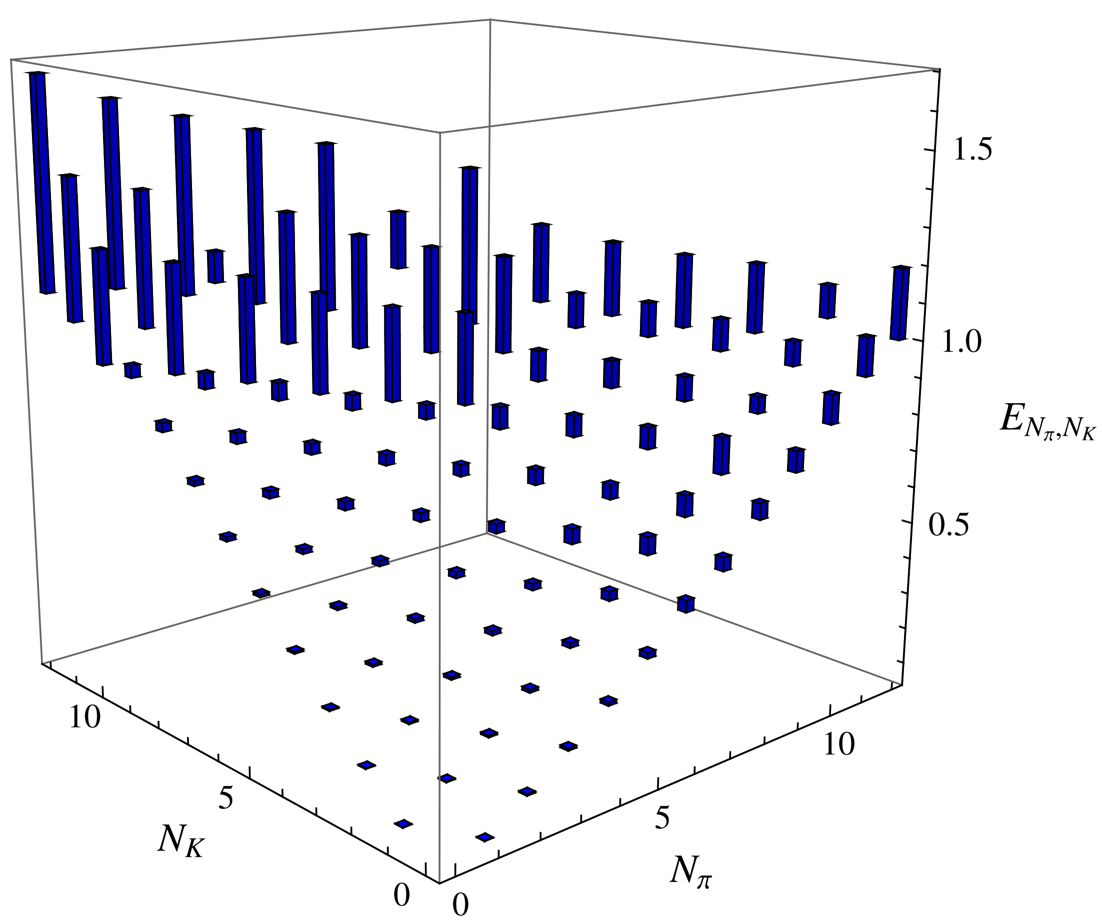
- Effective energy plots: $\log[C_n(t)/C_n(t+1)]$



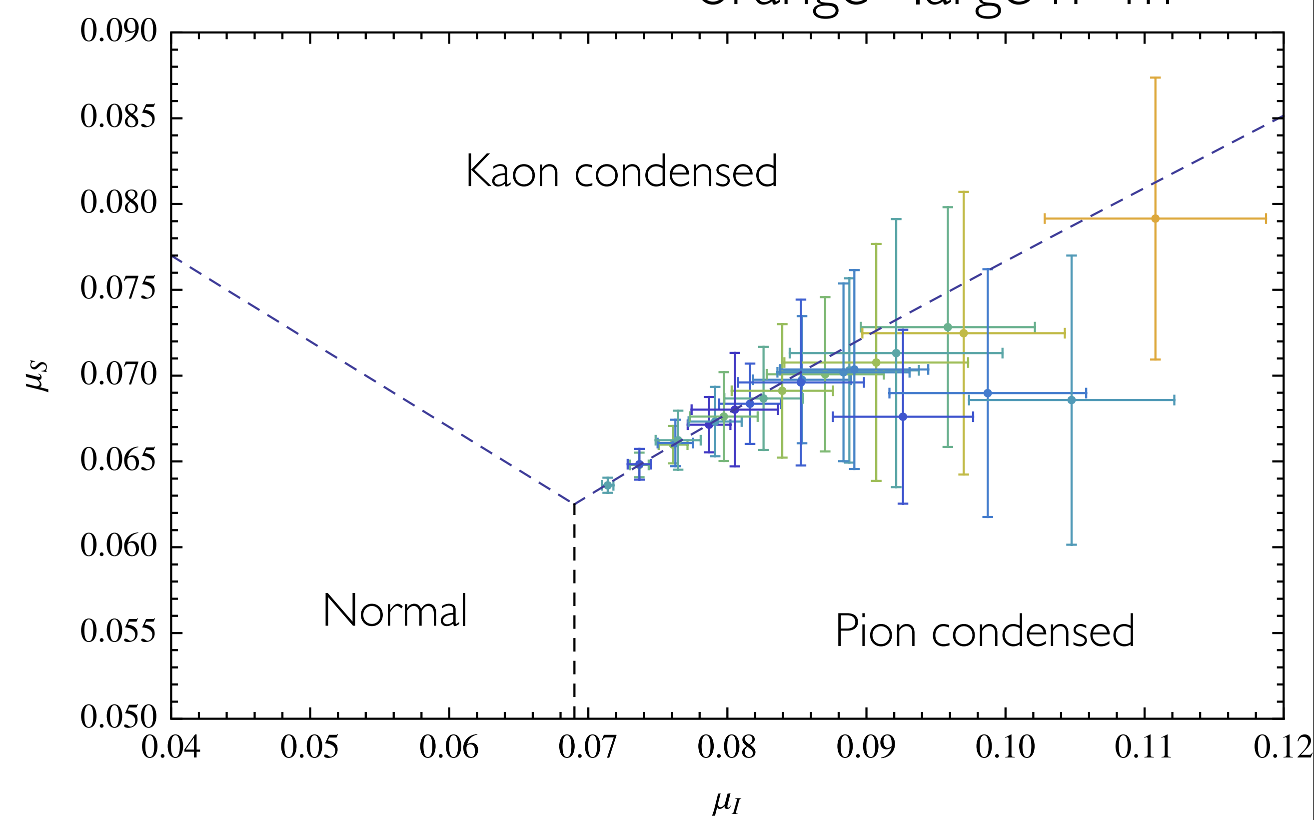
DWF on MILC
m_π = 319 MeV
a=0.09 fm, 28^3x96

Strangeness and Isospin

- LOχPT phase diagram for μ_I, μ_S [Kogut & Toublan, PRD 64, 034007 (2001)]
- Investigate through systems with K^+'s and π^+'s [Detmold & Smigielski, PRD (2011)]
- Contractions and analysis become far more complex
- QCD calculations probe interesting region



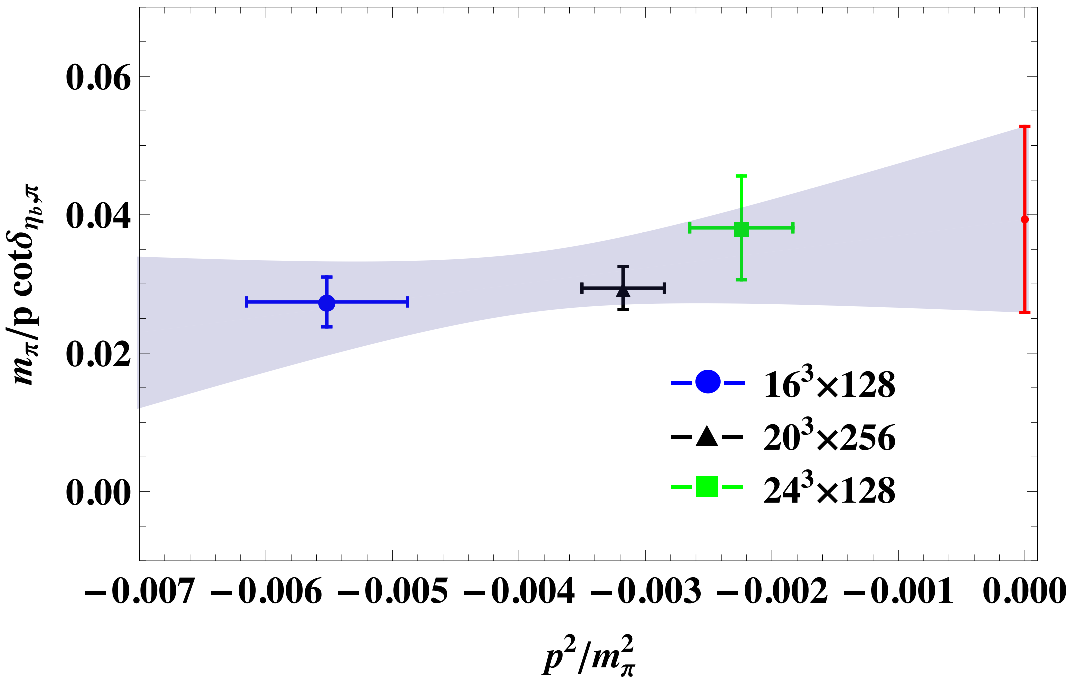
blue=small n+m
orange=large n+m

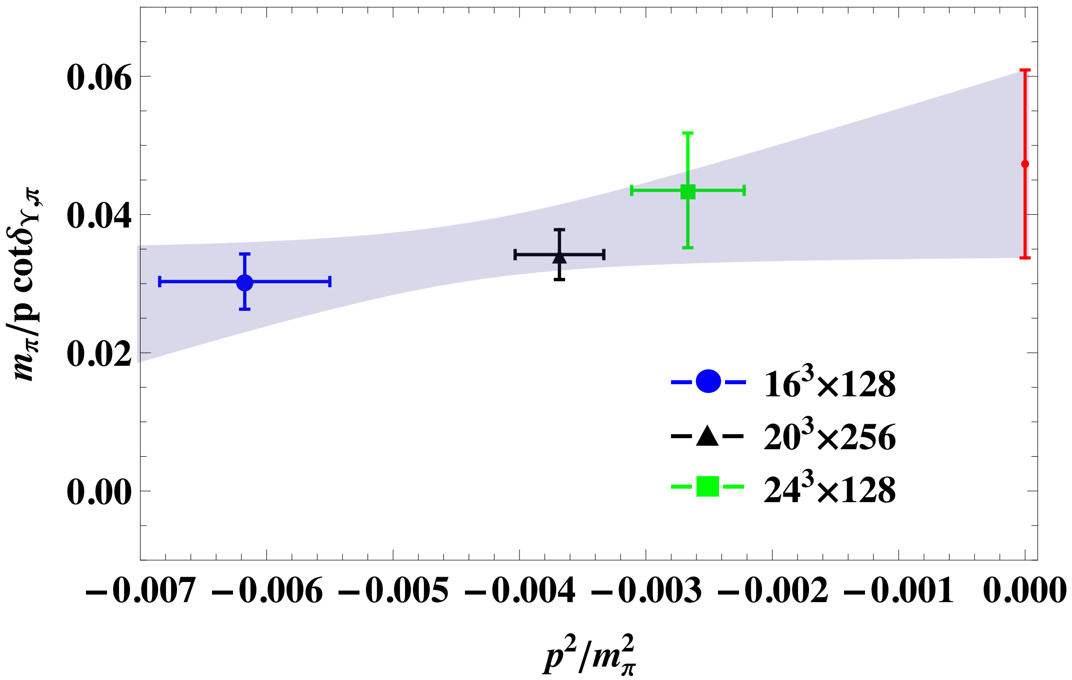


Bottomonium-Pion Interactions

- Bottomonium+Pion System allows extraction of interactions via Lüscher method
- Expectation from Weinberg (I=0 state) and model studies is that the interactions should be small (0 in chiral limit)
- Mass dependence known so can interpolate

$$a^{(\mathrm{phys.})}_{\eta_b,\pi} = 0.0025(8)(6)\ \mathrm{fm}$$
$$a^{(\mathrm{phys.})}_{\Upsilon,\pi} = 0.0030(9)(7)\ \mathrm{fm}$$



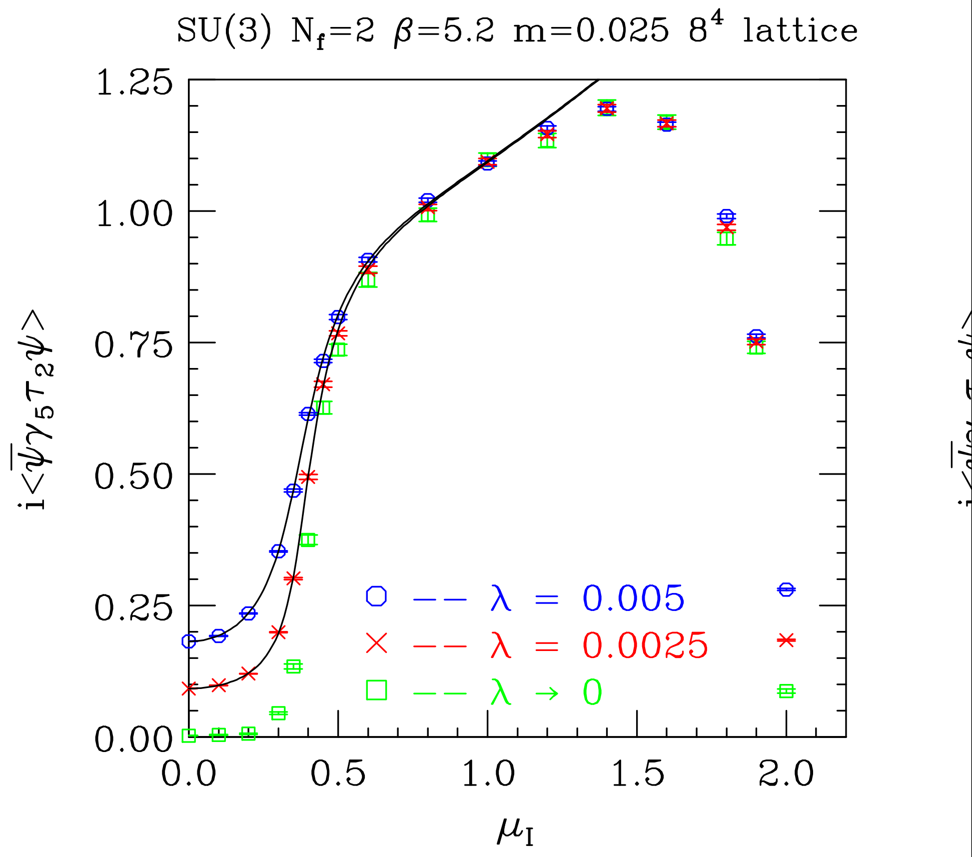


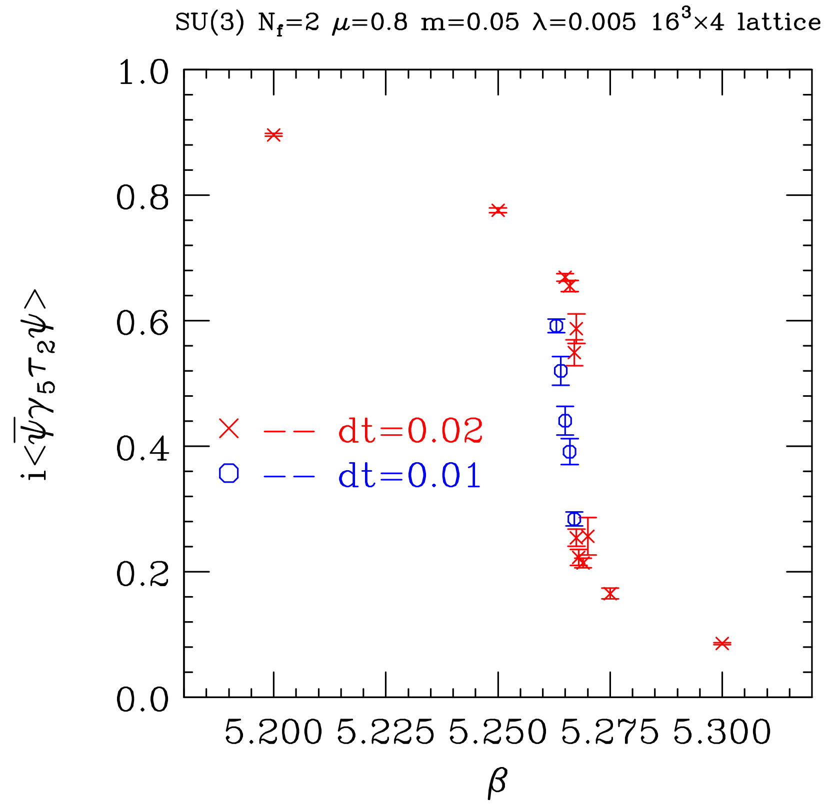
LQCD studies

- Kogut & Sinclair [PRD 66 (2002) 014508; PRD 66 (2002) 034505; PRD 70 (2004) 094501; PRD 77 (2008) 114503]
  - Staggered quarks
  - μ implemented by scaling forward/ backward temporal links [f(x)=e^x]

$$-\frac{1}{2a}\sum_{n\in\Lambda}\Big(f(a\mu)(\mathbb{1}-\gamma_4)_{\alpha\beta}U_4(n)_{ab}\delta_{n+\hat{4},m} +f(a\mu)^{-1}(\mathbb{1}+\gamma_4)_{\alpha\beta}U_4(n-\hat{4})^{\dagger}_{ab}\delta_{n-\hat{4},m}\Big)$$

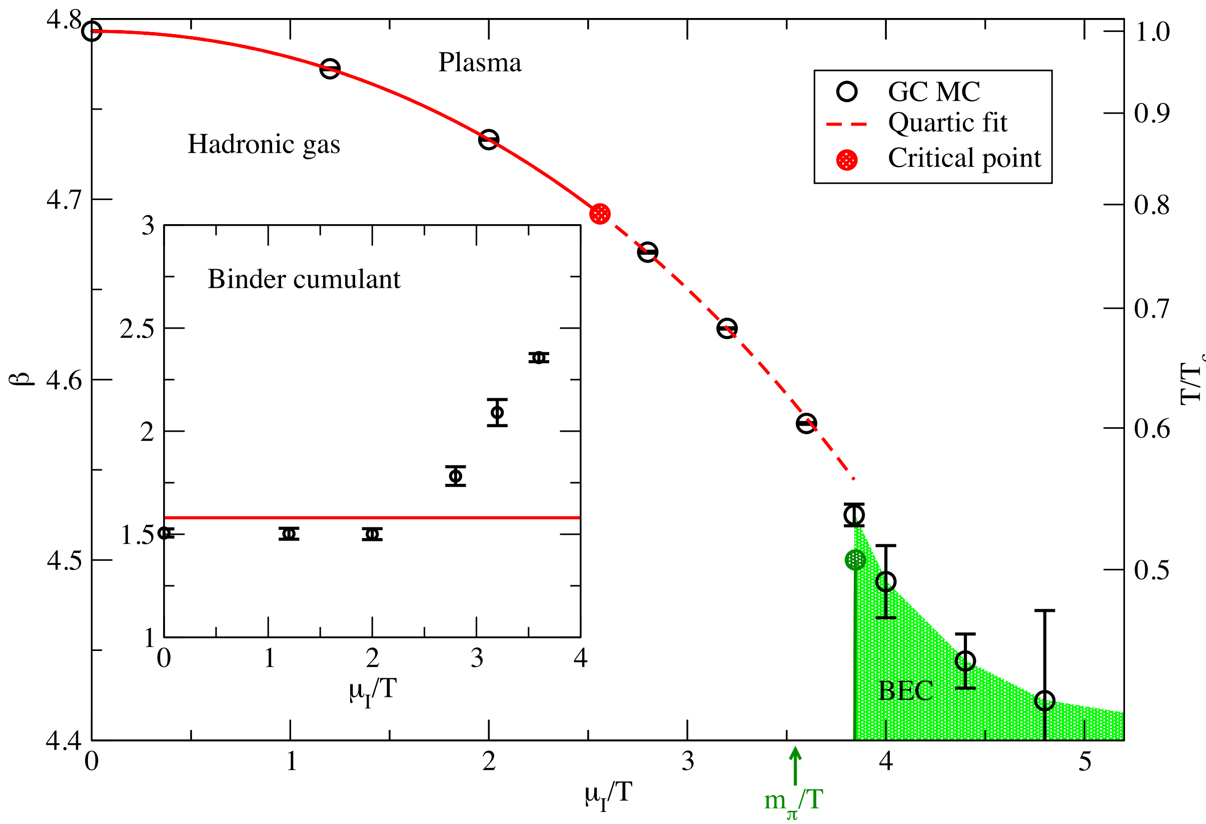
  - Pion condensation consistent with occurring at $\mu_I=m_\pi$
  - Demonstrated existence of phase transition - melting of condensate at high T above $\mu_{I,crit}$

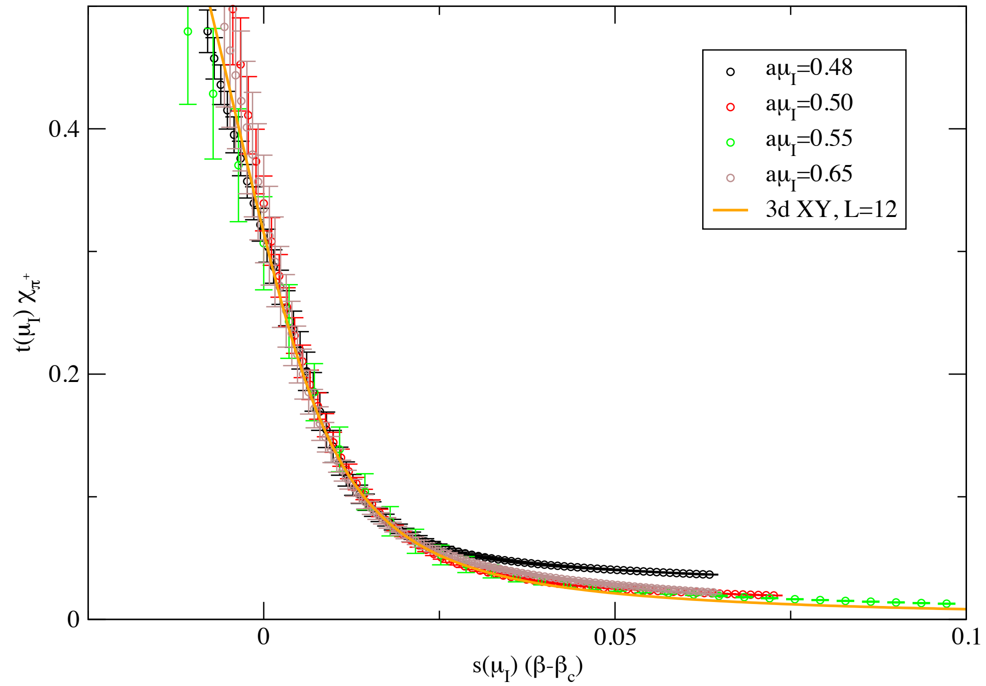




LQCD studies

- de Forcrand, Stephanov & Wenger
  [PoS LATT2007 237]
- Investigated using 2 staggered fermions and re-weighting from 6 values of μ_I to get precise mapping
  $$\frac{1}{2}T_c \lesssim T \lesssim T_c$$
  $$\mu_I/T \lesssim 5$$
- Determine critical μ_I from Maxwell construction in free energy
- Also investigate pion susceptibility – restoration of U(1) going from BEC phase to plasma phase

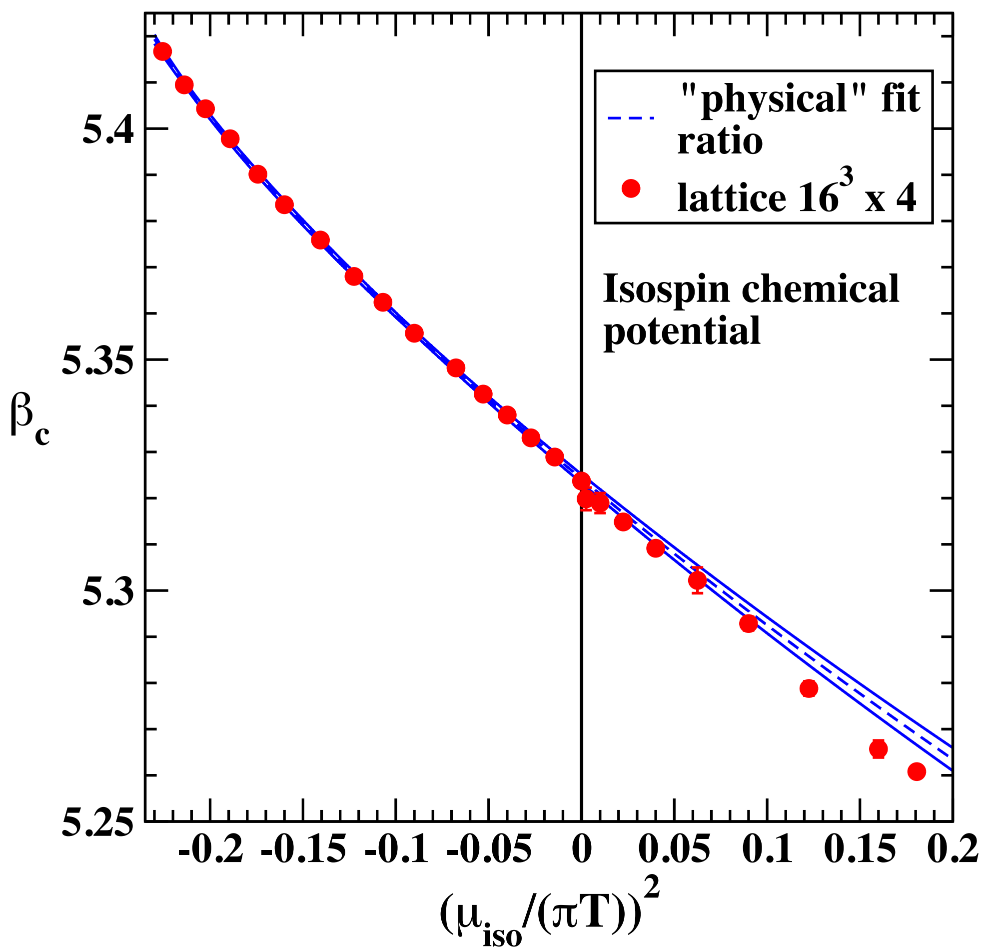


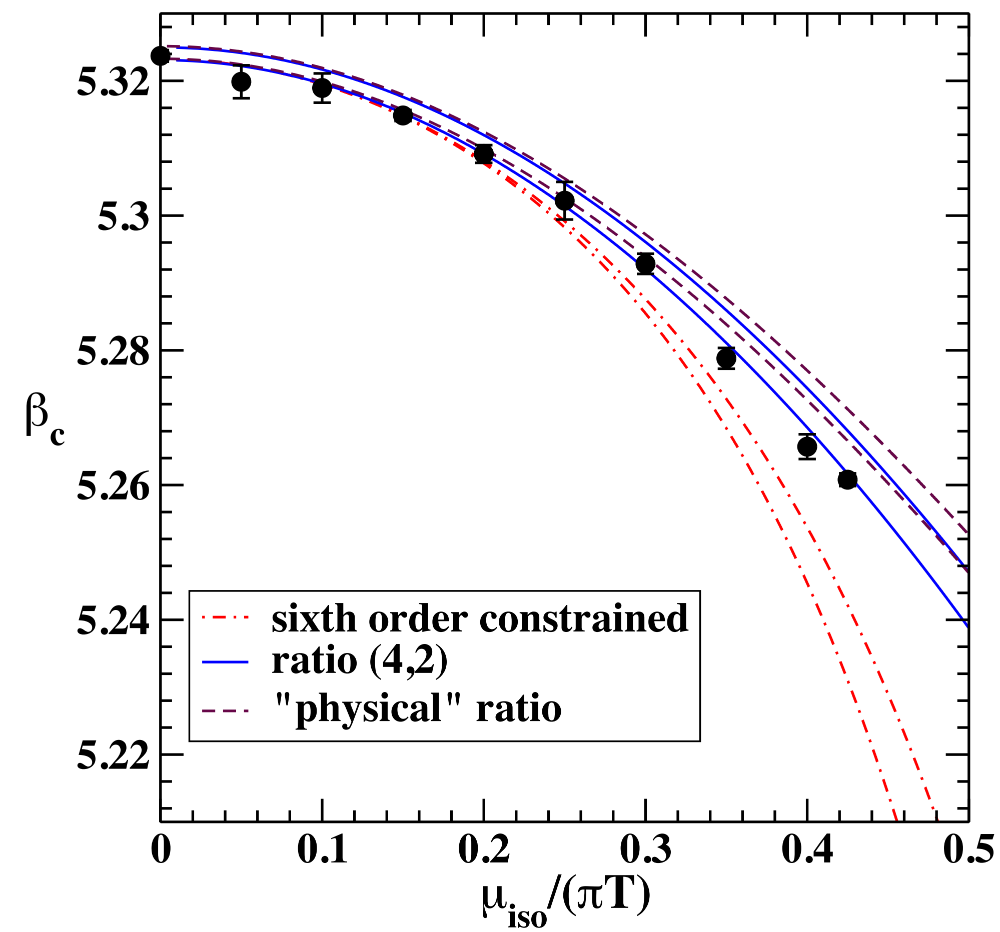


a m = 0.14
$8^3 \times 4$ lattices

LQCD studies

[Cea, Cosmai, d'Elia, Papa & Sanfillipo Phys. Rev. D85 094512, 2012; PoS LATT12]





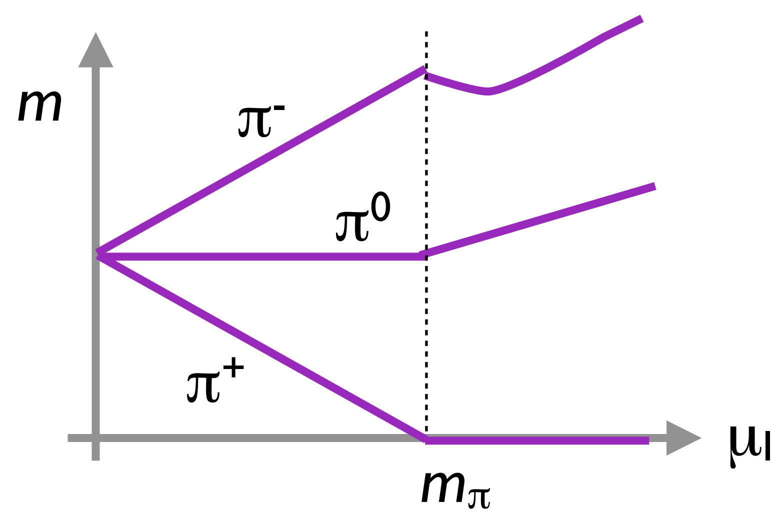
- Isospin used to test convergence of extrapolations for imaginary chemical potential [Cea et al.1210.5896]
- Pseudo-critical coupling from peak of PL susceptibility

Nf=2 staggered
mpi ~ 400 MeV
16^3x4 lattices

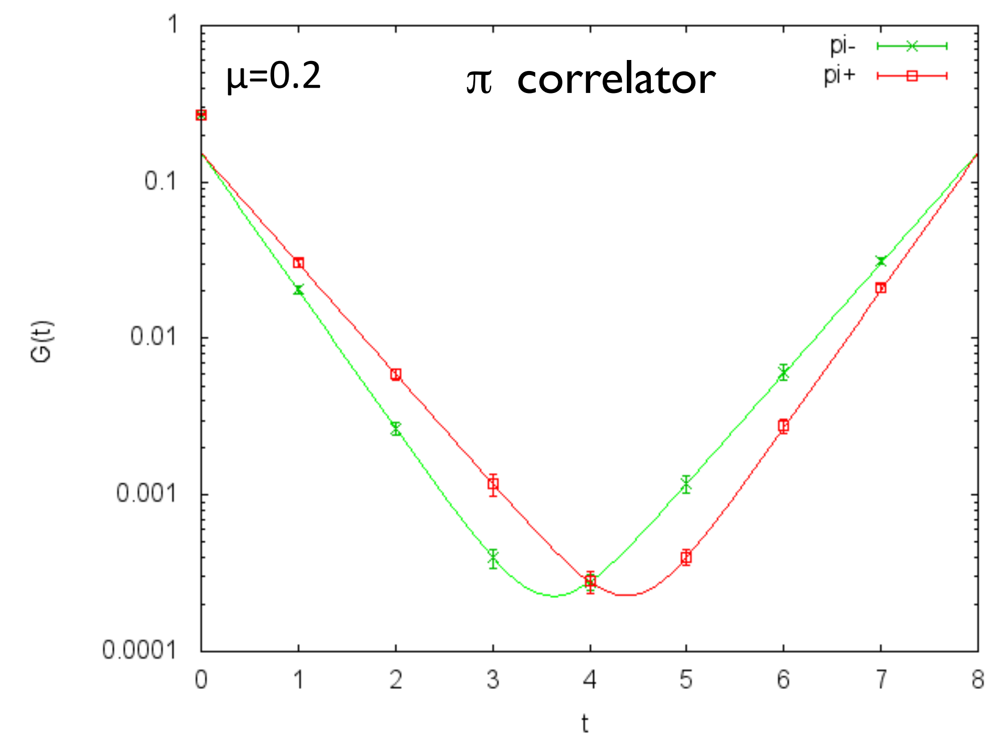
New Study @ Lattice2013

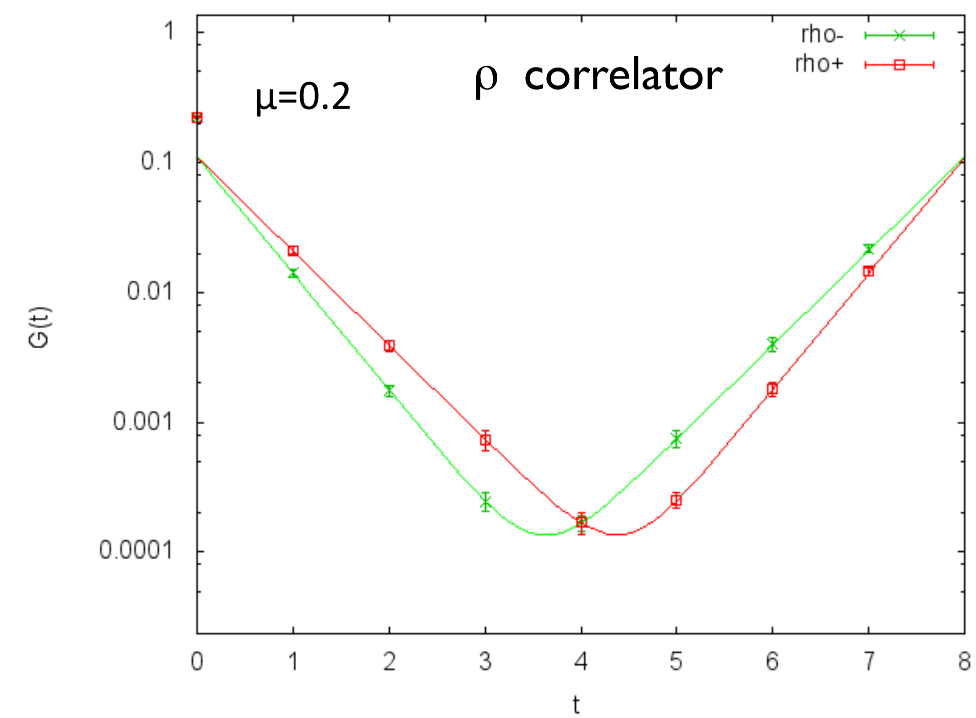
[C Nonaka and M Kondo, Lattice2013]

- Investigated Wilson formulation using explicit source term
  - Relatively small volumes (4^3x8)
- Charged pions split



- Looked for rho condensation, perhaps needs larger μ_I, smaller m_π/m_ρ
  - Explicit rotational breaking?



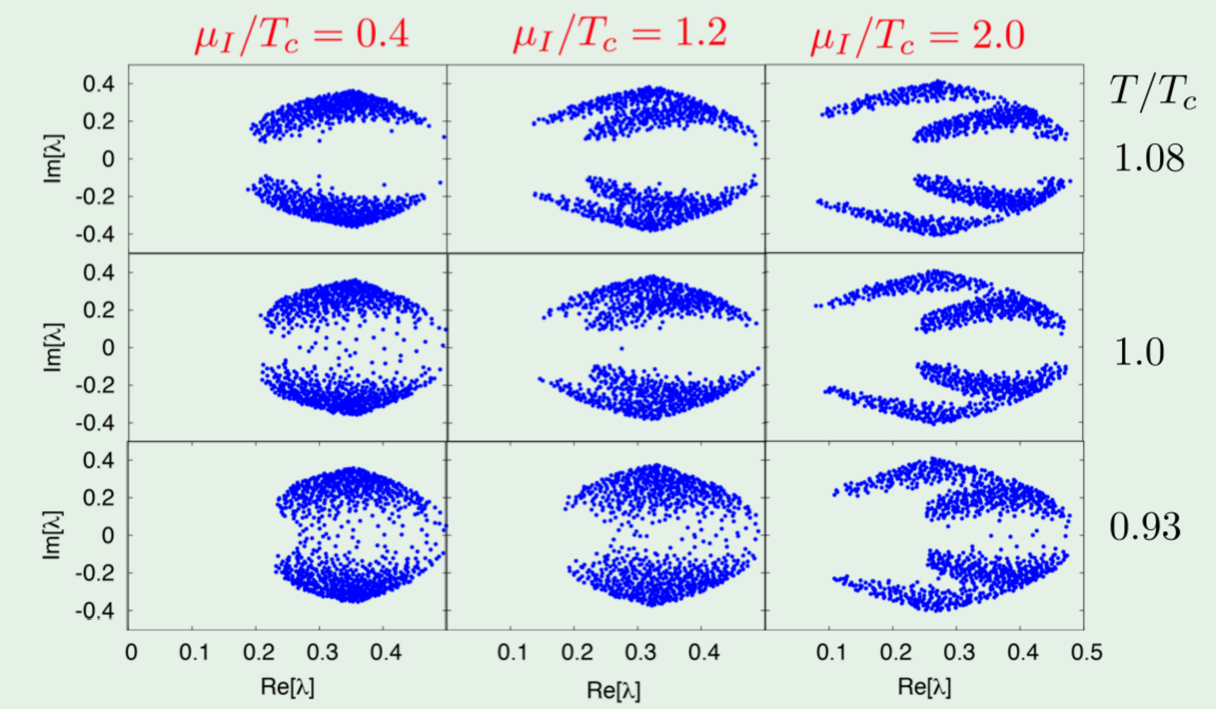


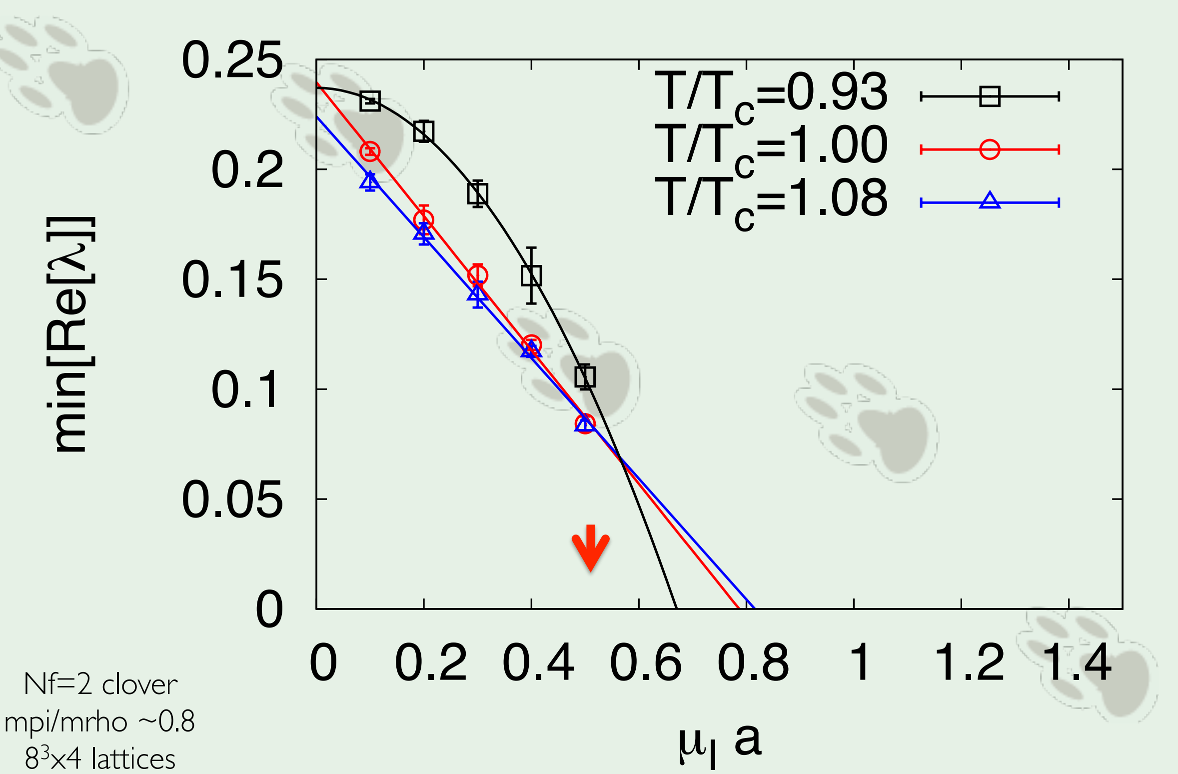
Dirac Operator Spectrum

[Nagata et al, XQCD13 poster]

- Recent study of Dirac operator spectrum at $\mu_I \neq 0$
  - Pion condensation signaled by eigenvalues approaching zero
  - 1000 low eigenmodes extracted
  - Estimate location of phase boundary to be slightly above m_π increasing with T
- Banks-Casher like relation also derived for large μ_I [Kanazawa, Wettig, Yamomoto EPJA 49 (2013) 88]

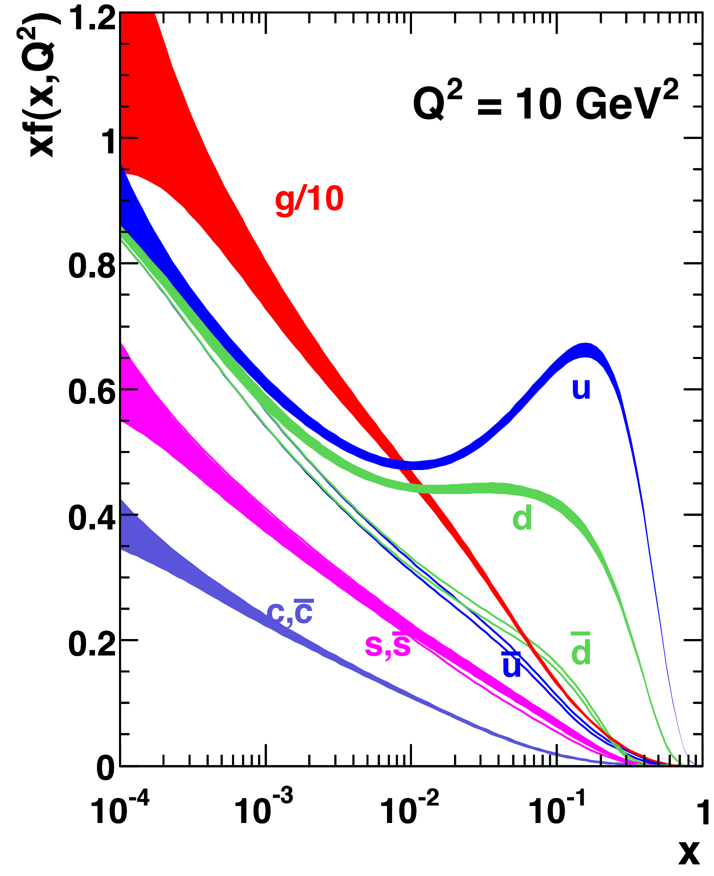
$$\Delta^2 = \frac{2\pi^3}{3N_c}\rho(0)$$





Nf=2 clover
mpi/mrho ~0.8
8^3x4 lattices

Hadron structure in QCD



- DIS probes LC parton distributions $q_H(x)$
- OPE: Mellin moments of PDFs defined by forward matrix elements of local operators

$$\langle x^n \rangle_H = \int_{-1}^{1} dx\, x^n q_H(x)$$

$$\langle H | \overline{\psi} \gamma^{\{\mu_0} D^{\mu_1} \ldots D^{\mu_n\}} | H \rangle = p^{\{\mu_0} \ldots p^{\mu_n\}} \langle x^n \rangle_H$$

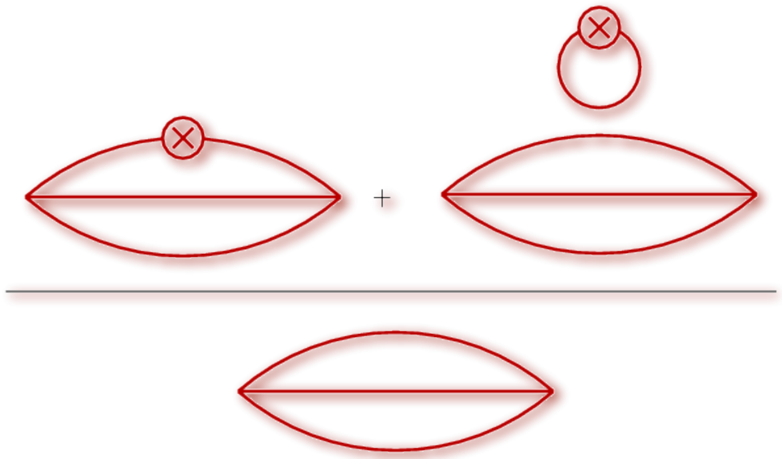
- NB: renormalisation scale dependent
- n=1 corresponds to LC momentum fraction carried by quarks inside H

Hadron structure in QCD

- Intensively studied in QCD using 3-pt functions

$$C_2(t,\mathbf{p}) = \sum_{\mathbf{x}} e^{i\mathbf{p}\cdot\mathbf{x}} \langle 0|\chi_H(0)\chi_H^\dagger(\mathbf{x},t)|0\rangle$$

$$C_3(t,\mathbf{p}) = \sum_{\mathbf{y},\mathbf{x}} e^{i\mathbf{p}\cdot\mathbf{x}} \langle 0|\chi_H(0)\mathcal{O}(\mathbf{y},\tau)\chi_H^\dagger(\mathbf{x},t)|0\rangle$$



$$R = \frac{C_3(t,\mathbf{p})}{C_2(t,\mathbf{p})} \xrightarrow{t\to\infty} \langle H|\mathcal{O}|H\rangle$$

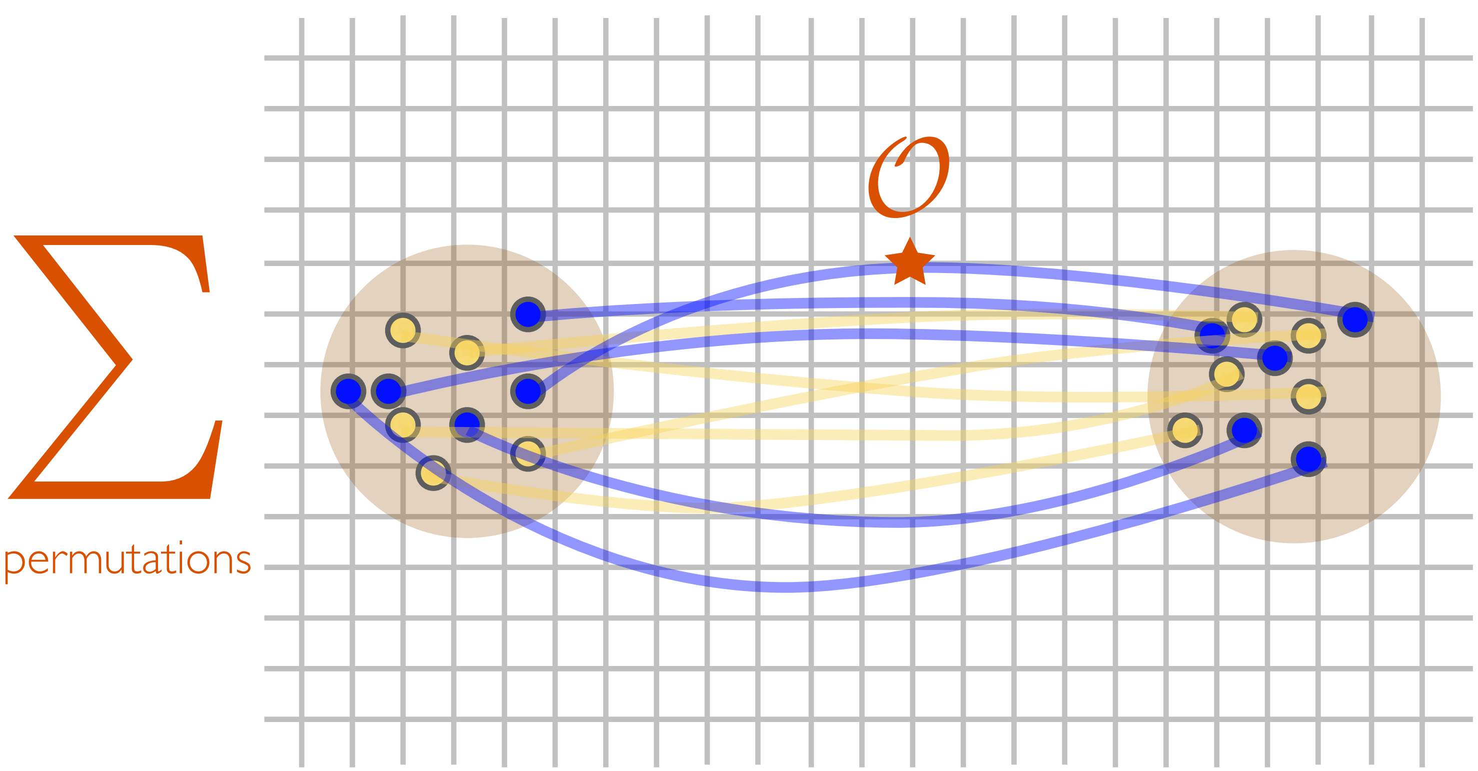
- Limited to low moments by reduced lattice symmetry
- Most studies for nucleon, but also pion, rho, ...
- Generalisations to GPDs ....

Many meson 3-point correlator

- $n\ \boldsymbol{\pi}^+$ 3-point correlator

$$C_3^{(n)}(t;\tau) = \left\langle 0 \left| \left[\sum_{\mathbf{x}} \bar{d}\gamma_5 u(\mathbf{x},t)\bar{u}\gamma_5 d(\mathbf{0},0) \right]^n \sum_{\mathbf{y}} \mathcal{O}(\mathbf{y},\tau) \right| 0 \right\rangle$$

$$\xrightarrow{t \gg \tau \gg 0} A\ e^{-E_n t} \langle n\pi | \mathcal{O} | n\pi \rangle + \ldots$$ Excitations and thermal effects



Many meson 3-point correlator

- $n\ \pi^+$ 3-point correlator

$$C_3^{(n)}(t;\tau) = \left\langle 0 \left| \left[\sum_{\mathbf{x}} \bar{d}\gamma_5 u(\mathbf{x},t)\bar{u}\gamma_5 d(\mathbf{0},0) \right]^n \sum_{\mathbf{y}} \mathcal{O}(\mathbf{y},\tau) \right| 0 \right\rangle$$

$$\xrightarrow{t \gg \tau \gg 0} A\ e^{-E_n t} \langle n\pi | \mathcal{O} | n\pi \rangle + \ldots$$ Excitations and thermal effects

- Contractions performed by treating the struck meson as a separate species

Colour/Dirac structure of operator

$$\Pi = \sum_{\mathbf{x}} \gamma_5 S(\mathbf{x},t;0)\gamma_5 S^\dagger(\mathbf{x},t;0), \qquad \tilde{\Pi}_\tau =_{\mathbf{x},\mathbf{y}} \gamma_5 S(\mathbf{x},t;\mathbf{y},\tau)\Gamma_{\mathcal{O}} S(\mathbf{y},\tau;0)\gamma_5 S^\dagger(\mathbf{x},t;0)$$

- System now looks like (n-1) pions + 1 "kaon"
  - Can be written as products of traces of two matrices
    [WD & B Smigielski, arXiv:1103.4362]

Lattice details

- Calculations use MILC gauge configurations
  - L=2.5 fm, a=0.12 fm, *rooted* staggered
  - also L=3.5 fm and a=0.09 fm
- Domain-wall quark propagators [LHP, NPLQCD]
  - $m_\pi \sim$ 291, 318, 352, 358, 491 MeV
  - few sources / lattice
- Need additional sequential propagators
- Focus on momentum fraction: $\mathcal{O}^{44}$

Double ratio

- Define ratio to extract matrix elements

$$R^{(n)}(t,\tau) = \frac{C_3^{(n)}(t;\tau)}{C_2^{(n)}(t)} \xrightarrow{t\gg\tau} \frac{1}{E_{n\pi}} \langle n\ \pi^+|\mathcal{O}^{44}|n\ \pi^+\rangle$$

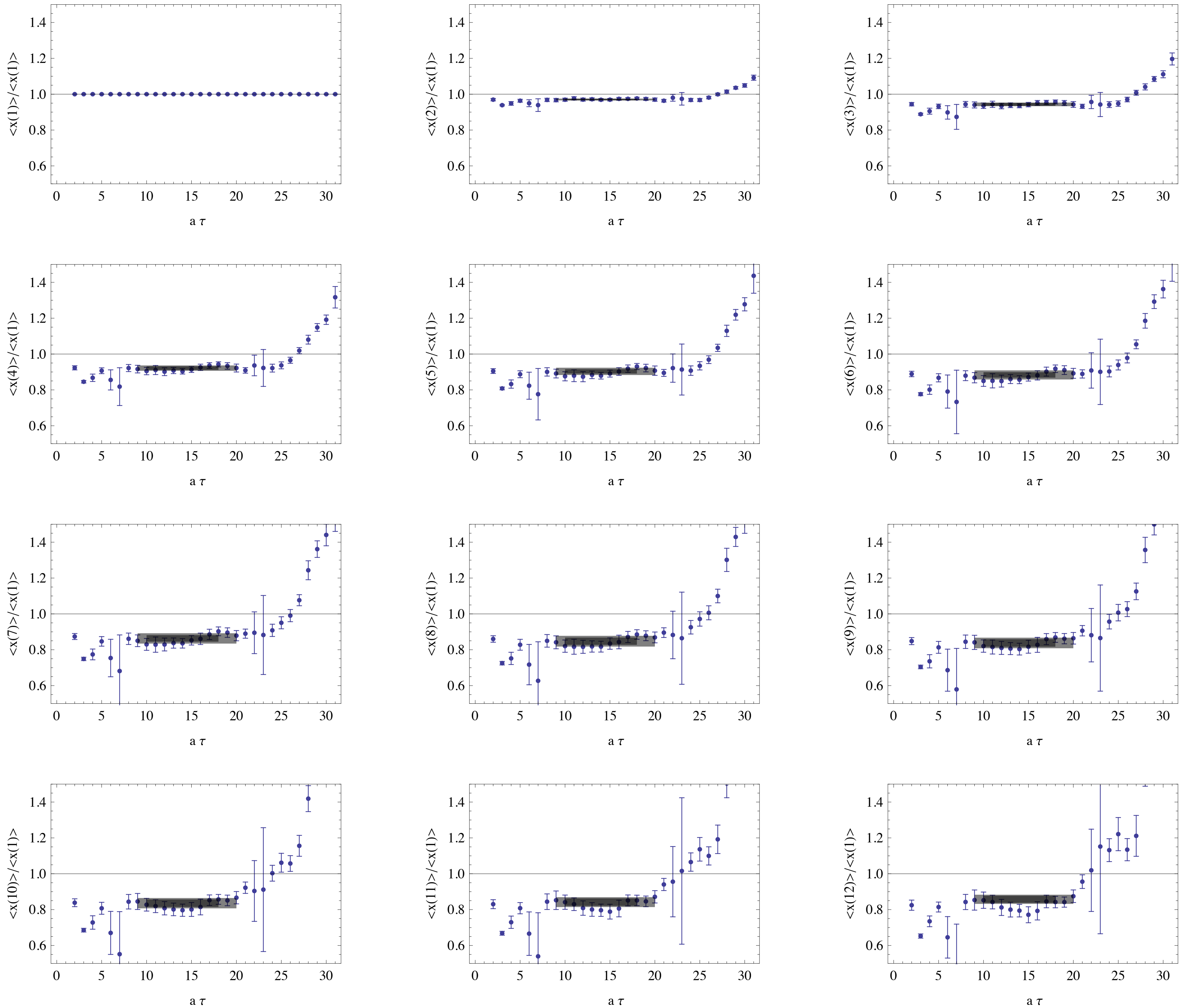
- Double ratio

$$\frac{R^{(n)}(t,\tau)}{R^{(1)}(t,\tau)} \longrightarrow \frac{m_\pi\ \langle n\ \pi^+|\mathcal{O}^{44}|n\ \pi^+\rangle}{E_{n\pi}\ \langle \pi^+|\mathcal{O}^{44}|\pi^+\rangle} \longrightarrow \frac{E_{n\pi}\ \langle x\rangle_{n\pi^+}}{m_\pi \langle x\rangle_{\pi^+}}$$

  - No need to renormalise operator!

- Allows investigation of ratio of moments

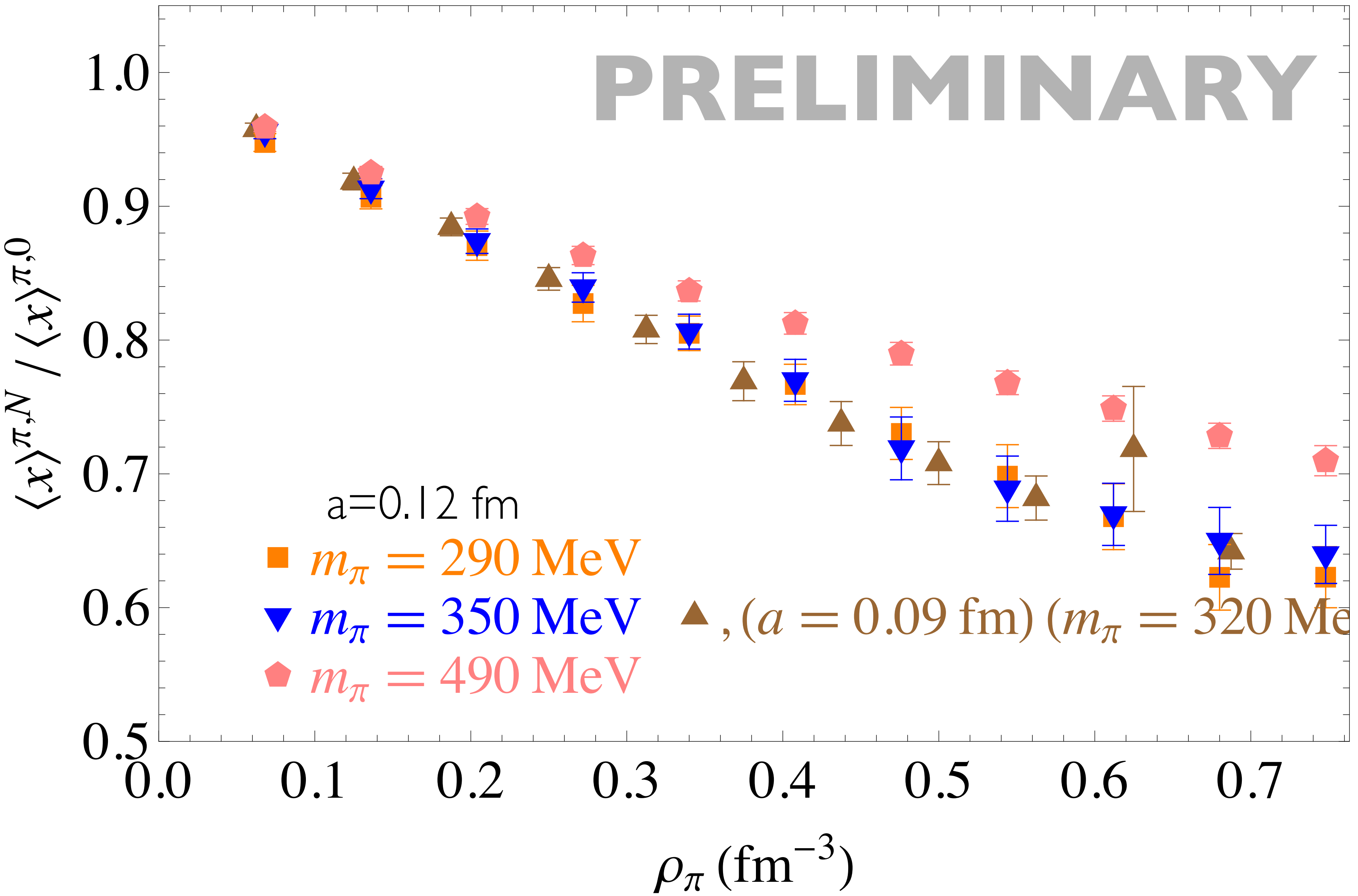
Double ratio



DWF on MILC
m_π = 350 MeV
a=0.12 fm, 20^3x64

Pionic EMC effect

- LC momentum fraction carried by quarks in a pion in a dense medium c.f. in free space



Caveat: Thermal and FV effects not completely sorted out