Isospin chemical potential and magnetic fields

Gergely Endrődi

University of Regensburg

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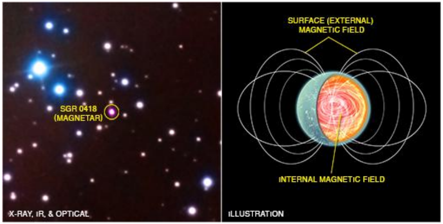
Outline

- introduction
  - isospin density and magnetic fields
- QCD at $\mu_I > 0$
  - lattice implementation
  - results
- QCD at $\mu_I > 0$ and $B > 0$
  - lattice implementation
  - results
- summary

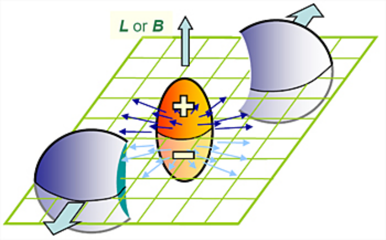
Introduction

- isospin chemical potential $\mu_I = \mu_u = -\mu_d$, conjugate to isospin density n_I (excess of neutrons over protons)
  - inner core of neutron stars
  - heavy-ion collisions
- direct lattice simulation is possible
  - analogy to baryon chemical potential: similar phenomena (Silver Blaze, condensation)
- study interplay between isospin density and magnetic field B
  - B distinguishes quarks due to electric charges $q_u \neq q_d$
  - relevant for fast-spinning neutron stars (magnetars)
  - relevant for non-central heavy-ion collisions

Examples



[Rea et al. '13]



[STAR collaboration, '10]

Fermion action

- 2-flavor QCD with fermion matrix

$$M = \not{D}(\tau_3\mu_I) + m\mathbb{1} + i\lambda\gamma_5\tau_2$$

- seeking spontaneous symmetry breaking realized as
  - $\langle\bar{\psi}\psi\rangle = \langle\bar{u}u\rangle + \langle\bar{d}d\rangle \neq 0$
  - $i\langle\bar{\psi}\gamma_5\tau_2\psi\rangle = \langle\bar{u}\gamma_5 d\rangle - \langle\bar{d}\gamma_5 u\rangle \neq 0$
- introducing explicit breaking terms (necessary in finite volume)
  - m to break chiral symmetry (creates gap at $\mu_I = 0$)
  - λ to form a pion condensate (creates gap at $\mu_I \neq 0$)
- extrapolation $\lambda \to 0$ necessary at end

Fermion action with staggered quarks

- staggered fermion matrix with $\eta_5 = (-1)^{n_x+n_y+n_z+n_t}$

$$M = \begin{pmatrix} \not{D}_{\mu} + m & \lambda\eta_5 \\ -\lambda\eta_5 & \not{D}_{-\mu} + m \end{pmatrix}$$

- γ_5-hermiticity

$$\eta_5 \not{D}_{\mu} \eta_5 = \not{D}^{\dagger}_{-\mu}$$

- determinant is real

$$(\det M)^* = \det(M^\dagger) = \det(\tau_1\eta_5 M \eta_5 \tau_1) = \det M$$

- determinant is positive

$$B = \begin{pmatrix} 1 & 0 \\ 0 & \eta_5 \end{pmatrix} \quad \det M = \det(BMB) = \det|\not{D}_{\mu} + m|^2 + \lambda^2 > 0$$

Simulation algorithm

- to obtain a positive definite matrix

$$M^\dagger M = \begin{pmatrix} [\not{D}_{\mu} + m]^\dagger [\not{D}_{\mu} + m] + \lambda^2 & 0 \\ 0 & [\not{D}_{-\mu} + m]^\dagger [\not{D}_{-\mu} + m] + \lambda^2 \end{pmatrix}$$

- two blocks have same determinant since:

$$M' = AMB \qquad A = \begin{pmatrix} 1 & 0 \\ 0 & -\eta_5 \end{pmatrix} \qquad B = \begin{pmatrix} 1 & 0 \\ 0 & \eta_5 \end{pmatrix}$$

$$M'^\dagger M' = \begin{pmatrix} [\not{D}_{\mu} + m]^\dagger [\not{D}_{\mu} + m] + \lambda^2 & 0 \\ 0 & [\not{D}_{\mu} + m]^\dagger [\not{D}_{\mu} + m] + \lambda^2 \end{pmatrix}$$

- therefore

$$\det M = \det(M') = \sqrt{\det(M'^\dagger M')} = \det([\not{D}_{\mu} + m]^\dagger [\not{D}_{\mu} + m] + \lambda^2)$$

Simulation algorithm

- pseudofermionic integral

$$\det M = \int \mathcal{D}\phi^\dagger \mathcal{D}\phi \, \exp \left[\phi^\dagger \left([\not{D}_\mu + m]^\dagger [\not{D}_\mu + m] + \lambda^2 \right)^{-1} \phi \right]$$

Simulation algorithm

- pseudofermionic integral with rooting

$$\det M^{1/4} = \int \mathcal{D}\phi^\dagger \mathcal{D}\phi \, \exp\left[\phi^\dagger \left([\not{D}_\mu + m]^\dagger [\not{D}_\mu + m] + \lambda^2 \right)^{-1/4} \phi \right]$$

Simulation setup

- exploratory study, 8^4 lattice
- gauge action
  - plaquette action
  - $\beta = 5.2$
- fermionic action
  - 2 flavor, naive staggered action, no smearing, $\sqrt[4]{\ }$
  - $am = 0.025$
  - $a\lambda = 0.0075 \ldots 0.0025$
- same setup as in [Kogut, Sinclair '02] to check algorithm+code
- pion propagator gives $am_\pi = 0.402(5)$, $af_\pi = 0.382(4)$
- Wilson flow w_0 to set scale [Borsányi et al '12]: $a = 0.299(1)$ fm
  $\Rightarrow m_\pi = 0.265(4)$ GeV, $f_\pi = 0.252(3)$ GeV

Observables

- partition function

$$\mathcal{Z} = \int \mathcal{D}U \, e^{-\beta S_g} \, (\det M)^{1/4}$$

- scalar condensate

$$\langle \bar\psi\psi \rangle = \frac{\partial \log \mathcal{Z}}{\partial m} = \frac{1}{4} \left\langle \mathrm{tr} M^{-1} \right\rangle$$

- pion condensate $\Gamma = i\gamma_5\tau_2$

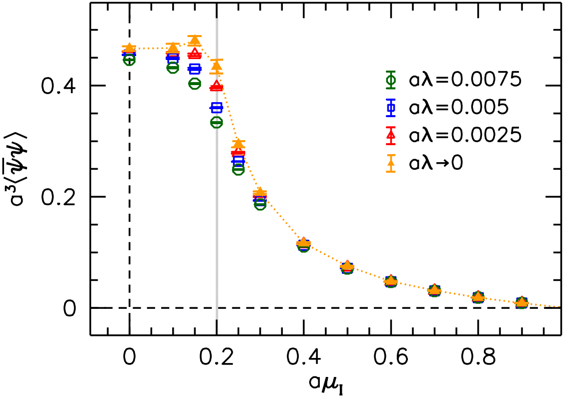
$$\langle \bar\psi\Gamma\psi \rangle = \frac{\partial \log \mathcal{Z}}{\partial \lambda} = \frac{1}{4} \left\langle \mathrm{tr} M^{-1}\Gamma \right\rangle$$

- isospin density ($' = \partial/\partial\mu_I$)

$$\langle n_I \rangle = \frac{\partial \log \mathcal{Z}}{\partial \mu_I} = \frac{1}{4} \left\langle \mathrm{tr} M^{-1} M' \right\rangle$$

Observables

- scalar condensate (compare [Kogut, Sinclair '02])
  note Silver Blaze up to $\mu_I = m_\pi/2$



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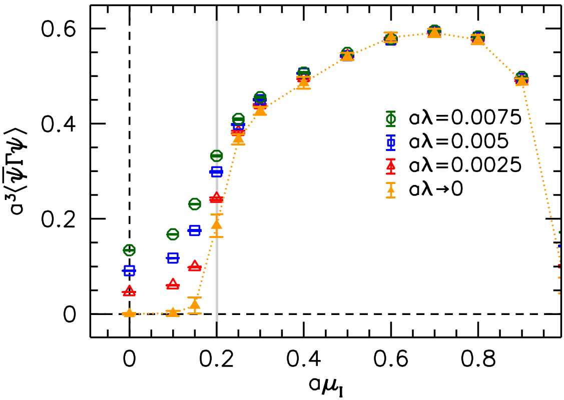

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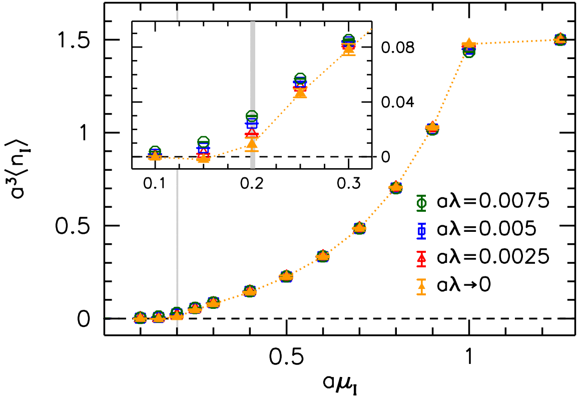

Fermion action at nonzero magnetic field

- 2-flavor QCD with fermion matrix

$$M = \begin{pmatrix} \not{D}_{\mu}(q_u) + m & \lambda\eta_5 \\ -\lambda\eta_5 & \not{D}_{-\mu}(q_d) + m \end{pmatrix}$$

- U(1) vector potential $A_y = Bx$, $\Rightarrow$ multiply y-links by $e^{ia^2 qBx}$
- γ_5-hermiticity is lost unless $q_u = q_d$

$$\eta_5 \not{D}_{\mu}(q_u)\eta_5 = \not{D}^{\dagger}_{-\mu}(q_u)$$

- fermion action becomes complex

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- fermion action becomes complex

- use direct simulation at $\mu_I > 0$ and Taylor-expansion in B

Taylor-expansion

- magnetization ($\dot{} = \partial/\partial(eB)$)

$$\langle \mathcal{M}_B \rangle = \left.\frac{\partial \log \mathcal{Z}}{\partial(eB)}\right|_{B=0} = \frac{1}{4}\left\langle \mathrm{tr} M^{-1}\dot{M} \right\rangle = 0$$

- magnetic susceptibility

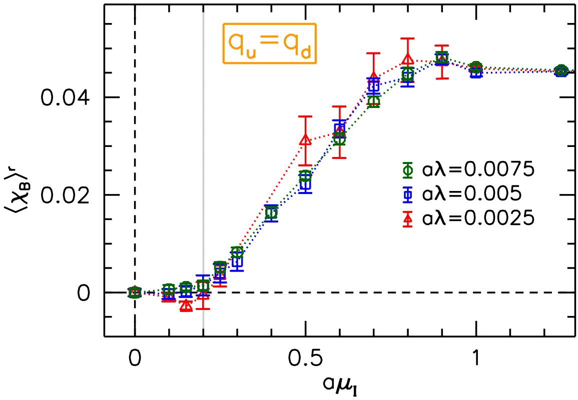
$$\langle \chi_B \rangle = \left.\frac{\partial^2 \log \mathcal{Z}}{\partial(eB)^2}\right|_{B=0} = \frac{1}{4}\left\langle \mathrm{tr} M^{-1}\ddot{M} \right\rangle + \ldots$$

- derivative with respect to magnetic field B
  - magnetic flux is quantized → ill-defined derivative
  - instead: consider half-half setup [DeTar et al '13] with zero flux and continuous field-dependence
- additive renormalization (inherited from charge renorm.)

$$\langle \chi_B \rangle^r = \langle \chi_B \rangle - \langle \chi_B \rangle_{\mu_I=0}$$

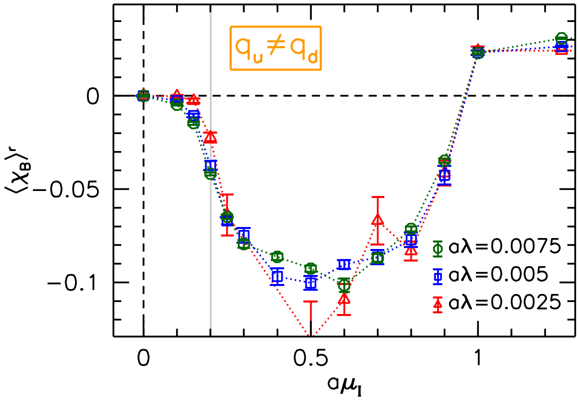
Magnetic susceptibility in a hypothetical world

- equal quark charges: neutral pions condense



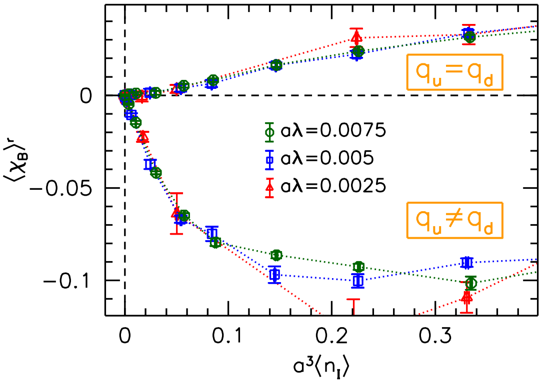
Magnetic susceptibility in nature

- different quark charges: charged pions condense



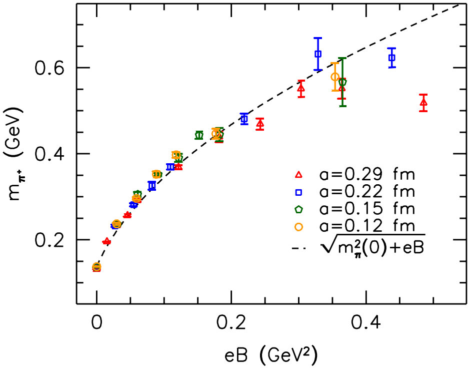
Magnetic susceptibility

- neutral pions don't couple to B: ground state $\sim$ as at $\mu_I = 0$ where $\langle \chi_B \rangle^r > 0$ [Bali et al '13, D'Elia et al '13, DeTar et al '13]
- dilute gas of charged pions: angular momentum works against magnetic field $\rightarrow$ diamagnetism $\langle \chi_B \rangle^r < 0$



Pion mass in magnetic field

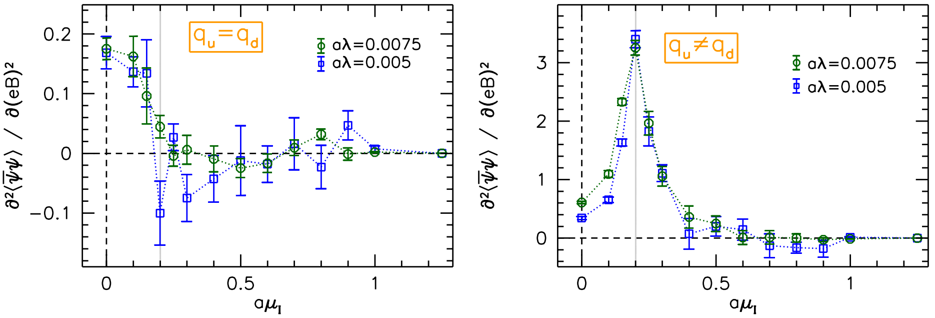
- point-like charged pion: $m_{\pi^\pm} = \sqrt{m_\pi^2(0) + eB}$
- point-like neutral pion $m_{\pi^0} = m_\pi(0)$



[Bali et al '11]

Scalar condensate – Taylor coefficient

- $\left\langle \mathrm{tr} M^{-1} \ddot{M} M^{-1} \right\rangle, \ldots$
- note 'magnetic catalysis' at $\mu_I = 0$



Scalar condensate at nonzero B

- $\langle\bar\psi\psi\rangle_B = \langle\bar\psi\psi\rangle_{B=0} + \frac{1}{2}\frac{\partial^2\langle\bar\psi\psi\rangle}{\partial(eB)^2}\cdot(eB)^2$
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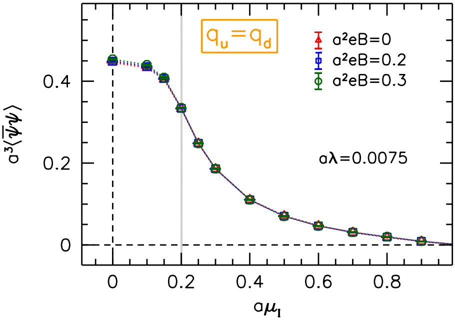
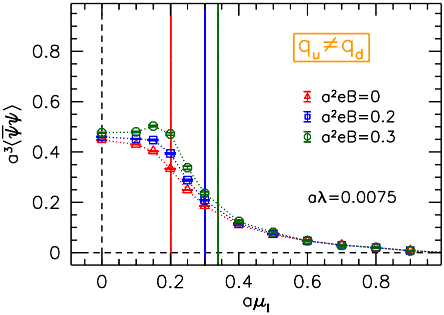
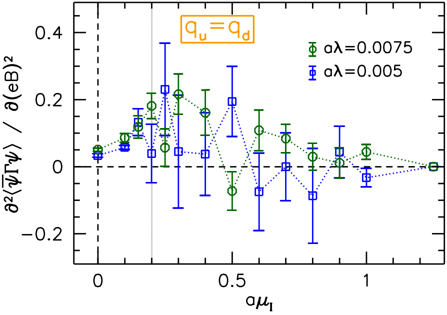
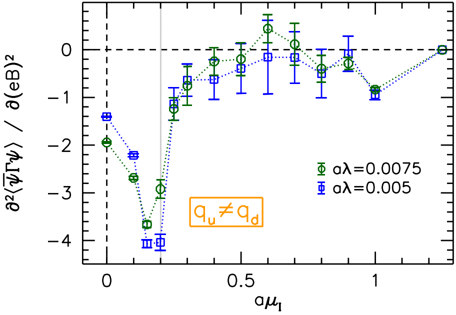




Pion condensate – Taylor coefficient

- $\left\langle \mathrm{tr} M^{-1} \ddot{M} M^{-1} \Gamma \right\rangle$, ...





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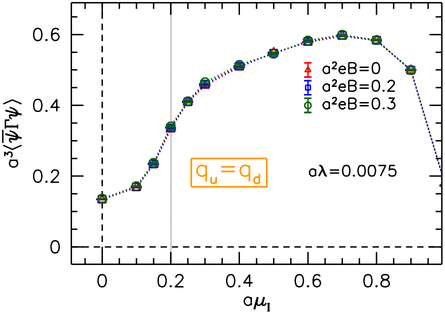
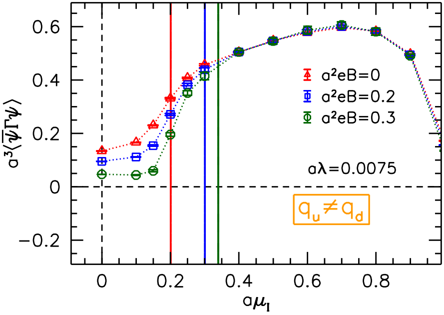
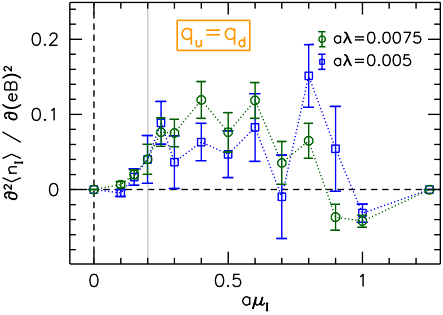
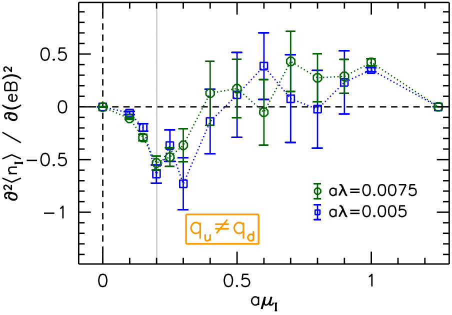




Isospin density – Taylor coefficient

- $\left\langle \mathrm{tr} M^{-1} \ddot{M} M^{-1} M' \right\rangle$, ...





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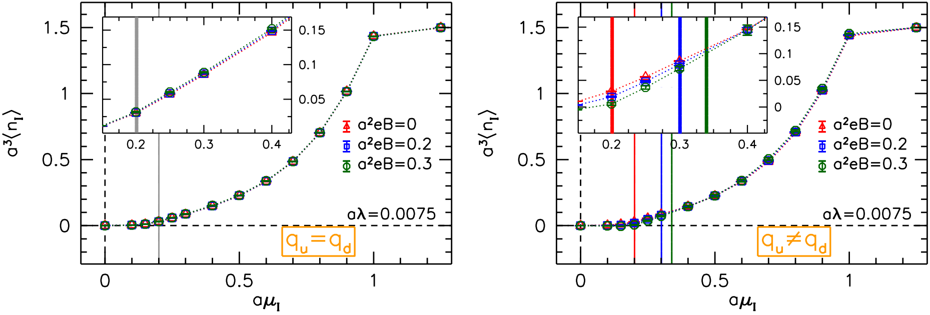
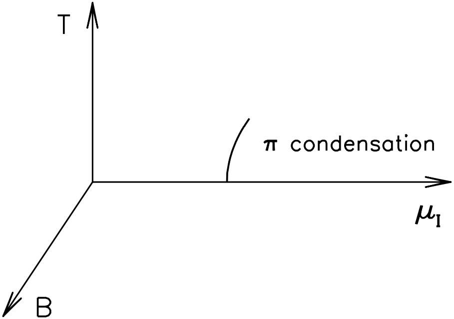


Summary

- condensation of $\pi^\pm$ for $\mu_I > m_{\pi^\pm}(B)/2 = \sqrt{m_\pi^2 + eB}/2$
  - note: $q_u = q_d$ would imply condensation of neutral pions



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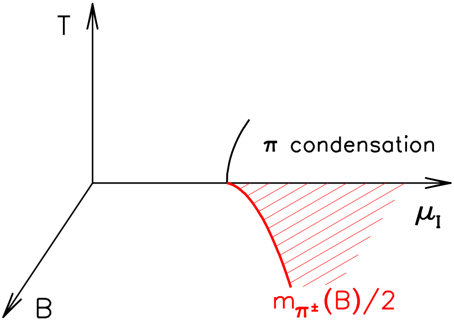

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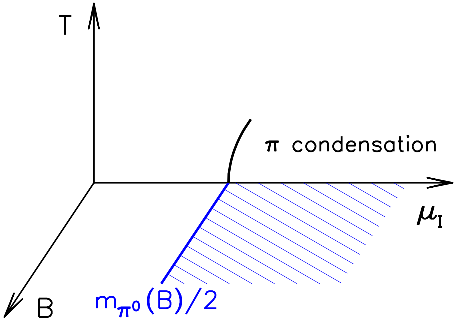

Summary

- dilute gas of charged pions is *diamagnetic* $\langle\chi_B\rangle^r(\mu_I) < 0$ in the condensation phase
  - this can be understood from the pressure $p = \int \mathrm{d}\mu_I n(\mu_I)$