

Nature of phase transitions in finite temperature and density QCD with many-flavors

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Collaboration with Norikazu Yamada (KEK)

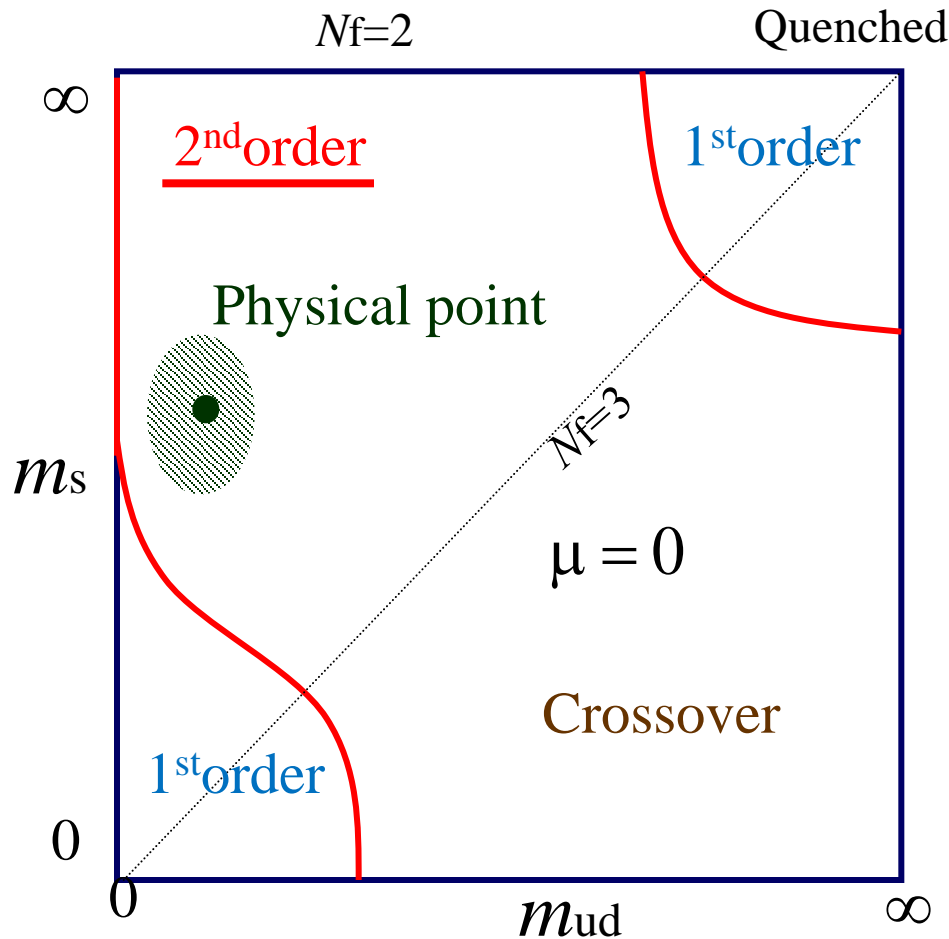
S. Ejiri and N. Yamada, Phys. Rev. Lett. 110, 172001 (2013) [arXiv:1212.5899]

WHOT-QCD(H. Saito, S. Ejiri, S. Aoki, K. Kanaya, Y. Nakagawa, H. Ohno,
K. Okuno, and T. Umeda), arXiv:1309.2445 [hep-lat]

S. Ejiri, Euro. Phys. J. A 49, 86 (2013) [arXiv:1306.0295] (mini-review)

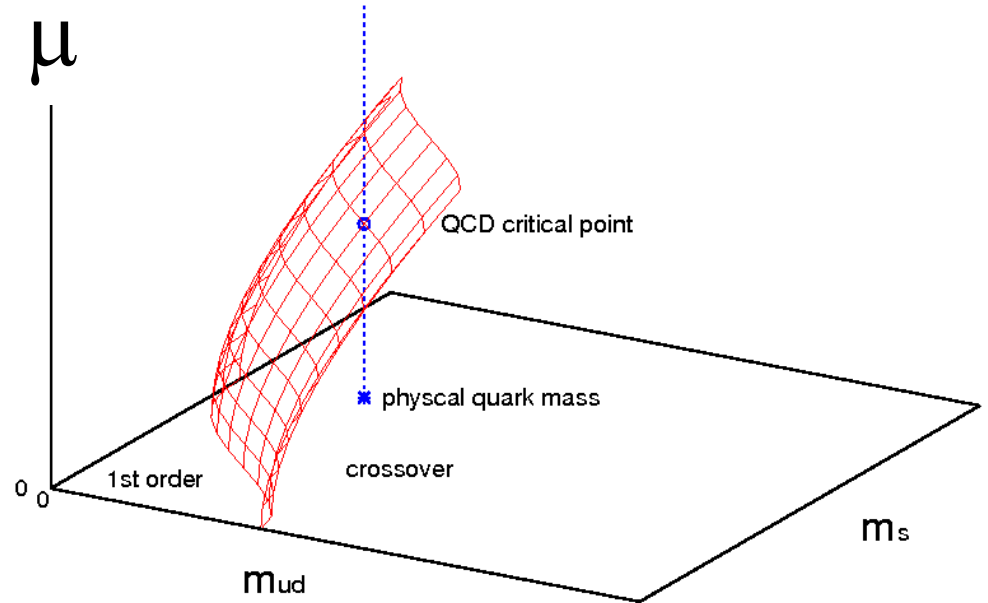
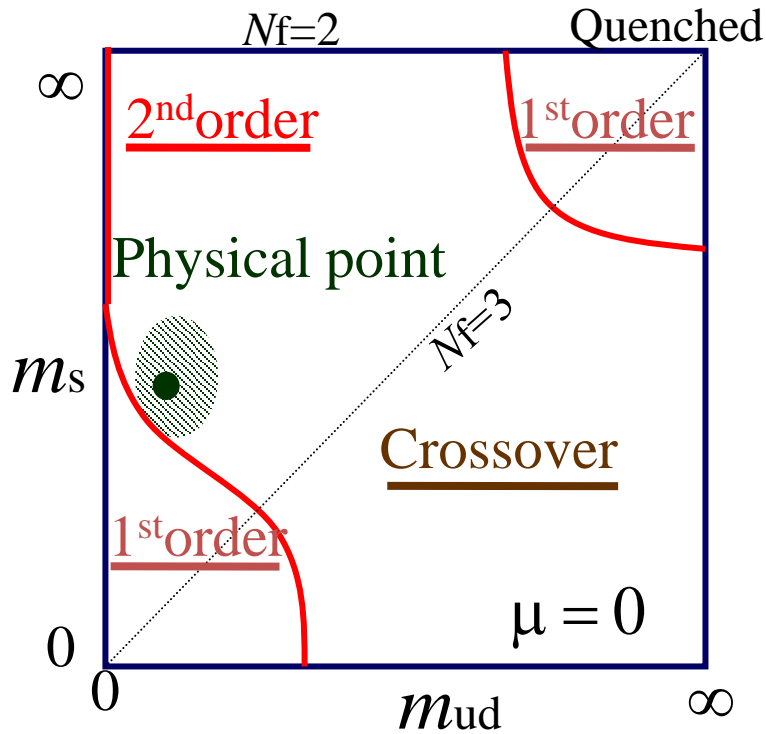
SIGN 2014, GSI, Darmstadt, Germany, Feb. 18 – 21, 2014

Quark Mass dependence of QCD phase transition



- The determination of the boundary of 1st order region: important.
- 2-flavor QCD in the chiral limit
 - O(4) universality class or First order transition
 - UA(1) symmetry restores?

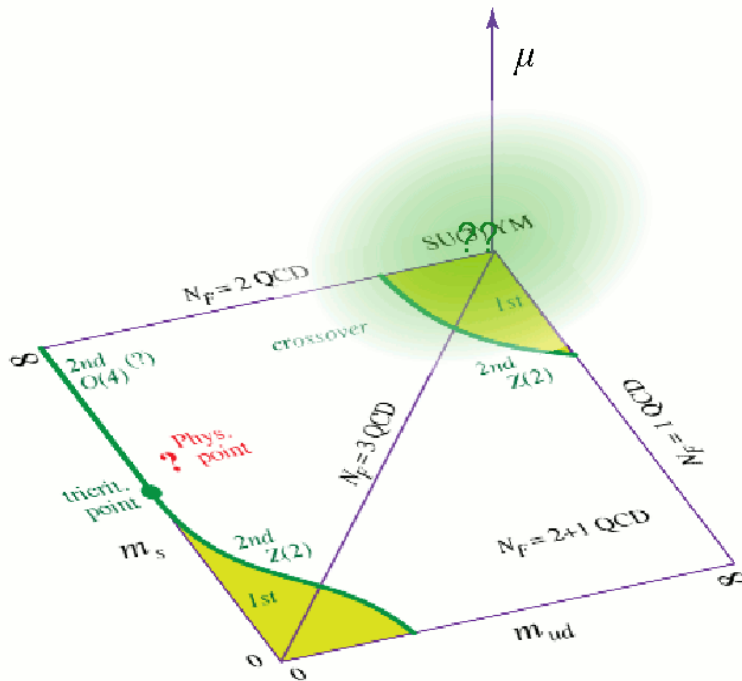
Quark Mass dependence of QCD phase transition



- On the line of physical mass, the crossover at low density \Rightarrow 1st order transition at high density.
- However, the 1st order region is very small, and simulations with very small quark mass are required. \Rightarrow Difficult to study.

Critical surface in the heavy quark region

(WHOT-QCD Collab., Phys.Rev.D84, 054502(2011); arXiv:1309.2445)



Hana Saito's talk

- We determine the **critical surface** in the heavy quark region.
- Performing quenched simulations + Reweighting for dynamical fermions.
- Standard Wilson quark action + plaquette gauge action, $S_g = -6N_{\text{site}}\beta P$
- $24^3 \times 4$ lattice

Hopping parameter expansion

$$\kappa \sim 1/(\text{quark mass})$$

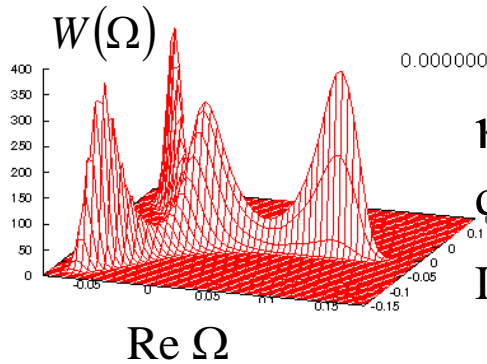
$$N_f \ln \left(\frac{\det M(\kappa, \mu)}{\det M(0,0)} \right) = N_f \left(288 N_{\text{site}} \kappa^4 P + 12 \cdot 2^{N_t} N_s^3 \kappa^{N_t} \left(\cosh(\mu/T) \Omega_R + \underline{\underline{i \sinh(\mu/T) \Omega_I}} \right) + \dots \right)$$

phase

P : plaquette, $\Omega = \Omega_R + i\Omega_I$: Polyakov loop

$$\det M(0,0) = 1 \quad 4$$

Critical surface in the heavy quark region of (2+1)-flavor QCD



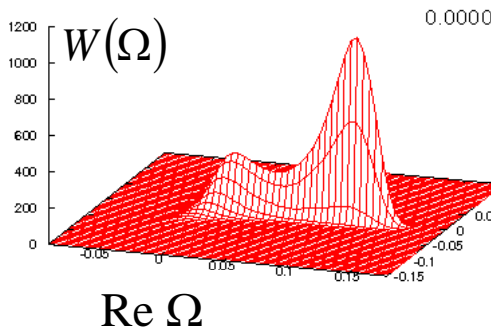
$\kappa=0$: $Z(3)$ symmetric distribution

Im Ω

Re Ω

Critical surface at finite density

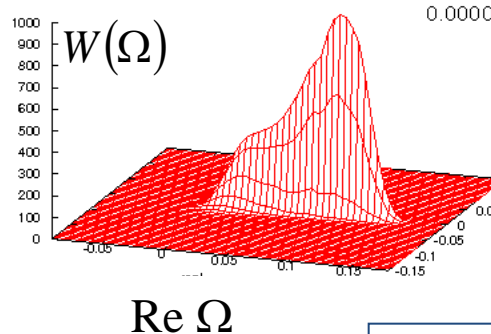
$$2\kappa_{ud\text{cp}}^{N_t} \cosh(\mu_{ud}/T) + \kappa_{s\text{cp}}^{N_t} \cosh(\mu_s/T) \approx 4 \times 10^{-5} \quad (N_t = 4)$$



κ small: two phases coexists (1st order tra.)

Im Ω

Re Ω

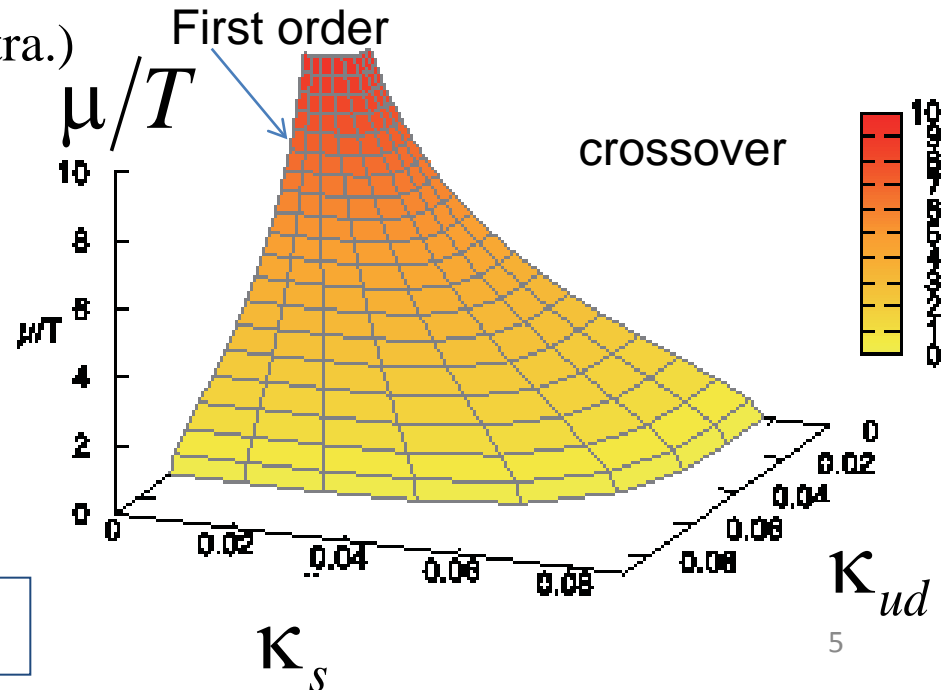


Near the critical κ : becomes one peak

Im Ω

Re Ω

$\kappa \sim 1/(\text{quark mass})$



Phase transitions in many-flavor QCD

We investigate the critical surface

in QCD with 2-light flavors + N_f -massive flavors.

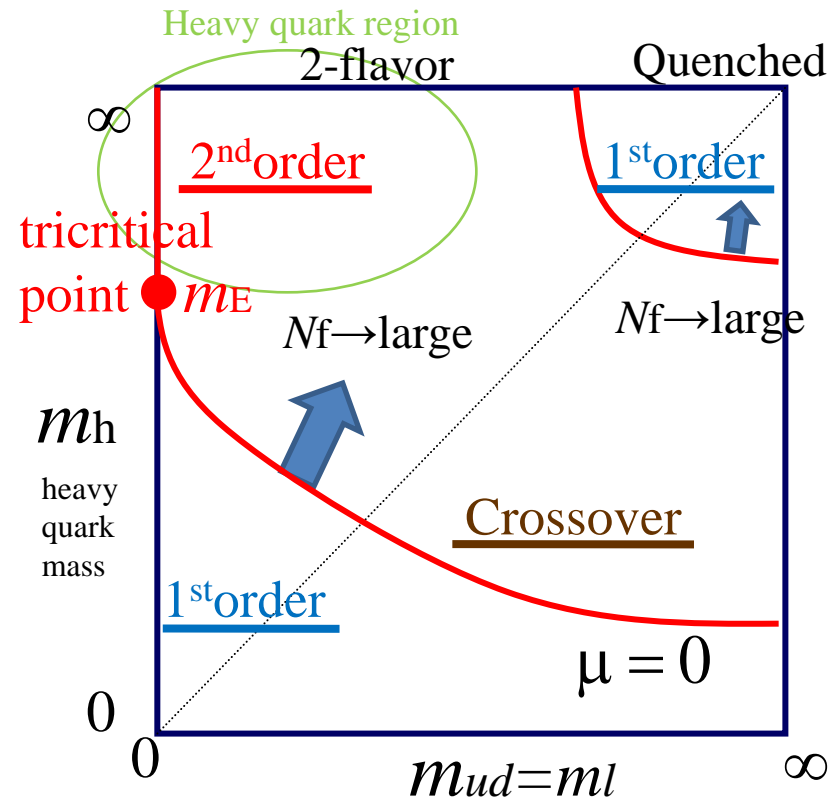
- Good testing ground for (2+1)-flavor QCD
- Electro-weak baryogenesis - Technicolor model
- Plan of this talk
 - Why (2+many)-flavor QCD
 - Histogram method
 - N_f -dependence of the critical heavy quark mass.
 - μ -dependence of the critical curve.
 - Light quark mass-dependence of the critical curve
 - The chiral limit of 2-flavor QCD: 2nd order or 1st order?

Finite T and μ phase transition in (2+many)-flavor QCD

(Cf. Kikukawa, Kohda and Yasuda, Phys.Rev.D77, 015014(2008))

- Technicolor model constructed by many-flavor QCD
- Chiral phase transition of QCD
 - Electroweak phase transition at finite temperature
- Nambu-Goldstone bosons
 - 3 bosons are absorbed into the gauge bosons. (3 massless bosons)
 - The other bosons have not observed yet. (The other bosons: heavy)
 - 2 techni-fermions are massless, and the others are heavy.
- Electro-weak baryogenesis
 - Strong first order transition: required.
 - From the analogy of 2+1-flavor QCD, 1st order at small mass; 2nd order or crossover at large mass.
- It is important to determine the endpoint of the first order region in (2+many)-flavor QCD.

Nature of phase transition of $2+N_f$ -flavor QCD



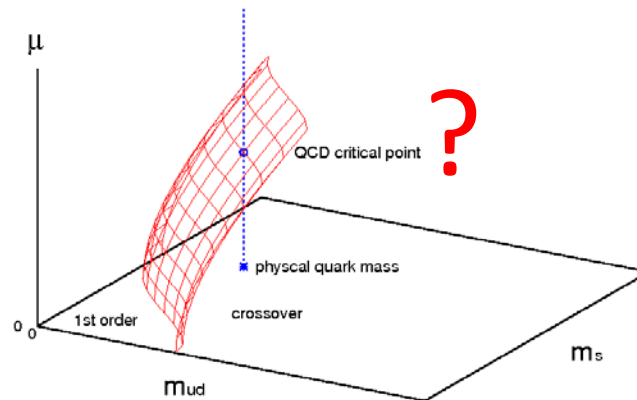
- Assumption: N_f -flavors are heavy.
 - Hopping parameter κ expansion
- Parameter: $\underline{N_f \kappa^{N_t}} \rightarrow 1/m_{h,ct} \sim \kappa_{ct} \propto 1/N_f^{1/N_t}$
- As increasing N_f , critical mass becomes larger.
- Tricritical scaling: the same as (2+1)-flavor QCD

Tricritical point $m_{ud}^c \sim (m_E - m_h)^{5/2}$

m_E : $m_{ud}^c \sim \mu^5$

Good test ground

At finite density?

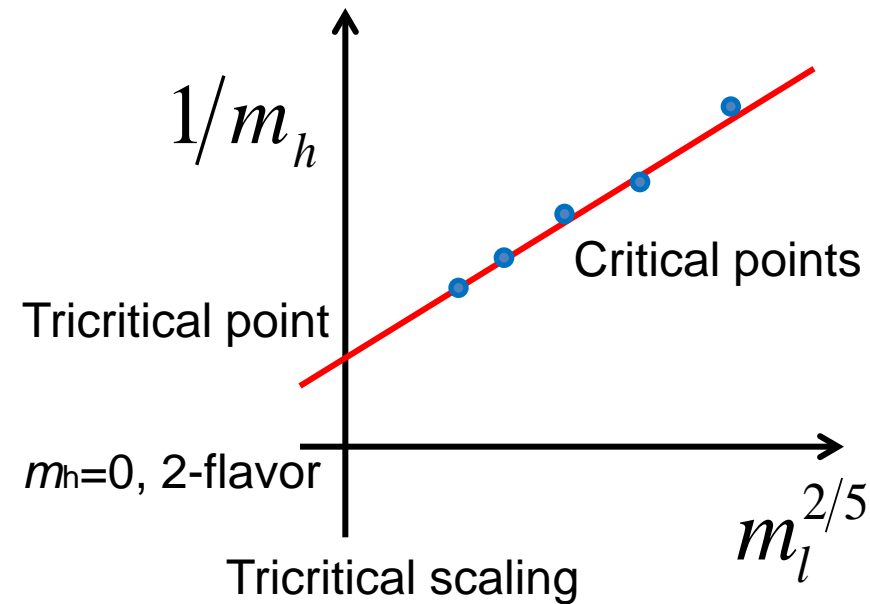


Nature of 2-flavor QCD in the chiral limit

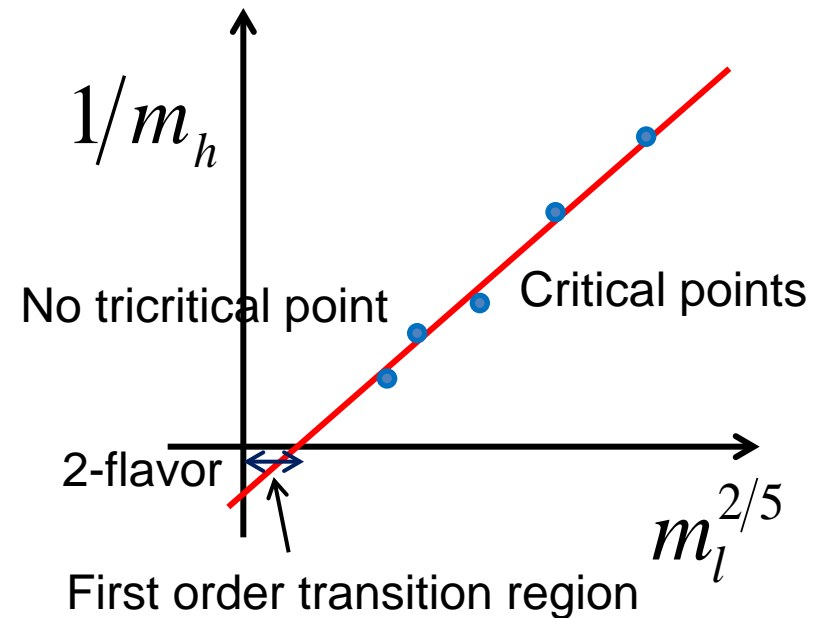
2nd order or 1st order?

Light quark mass (m_l) dependence of the critical line

- Tricritical scaling behavior?
- Is there a first order transition region in 2-flavor QCD?



or



Similar study in QCD with an imaginary chemical potential:

Bonati, D'Elia, de Forcrand, Philipsen, Sanfilippo, arXiv:1311.0473

Histogram method

Distribution function & the effective potential

(S. Ejiri, Phys. Rev. D 77 (2008) 014508)

$$W(X; m, T, \mu) \equiv \int DU \delta(X - \hat{X}) \prod_{f=1}^{N_f} \det M(m_f, \mu_f) e^{-S_g} \quad (\text{Histogram})$$

X : order parameters, total quark number, average plaquette, etc.

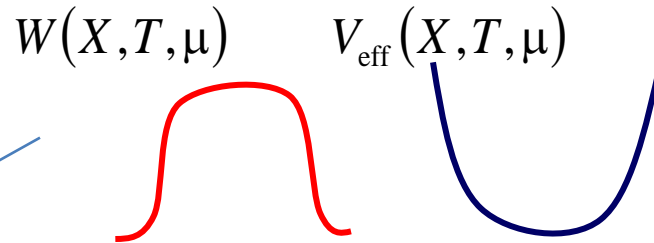
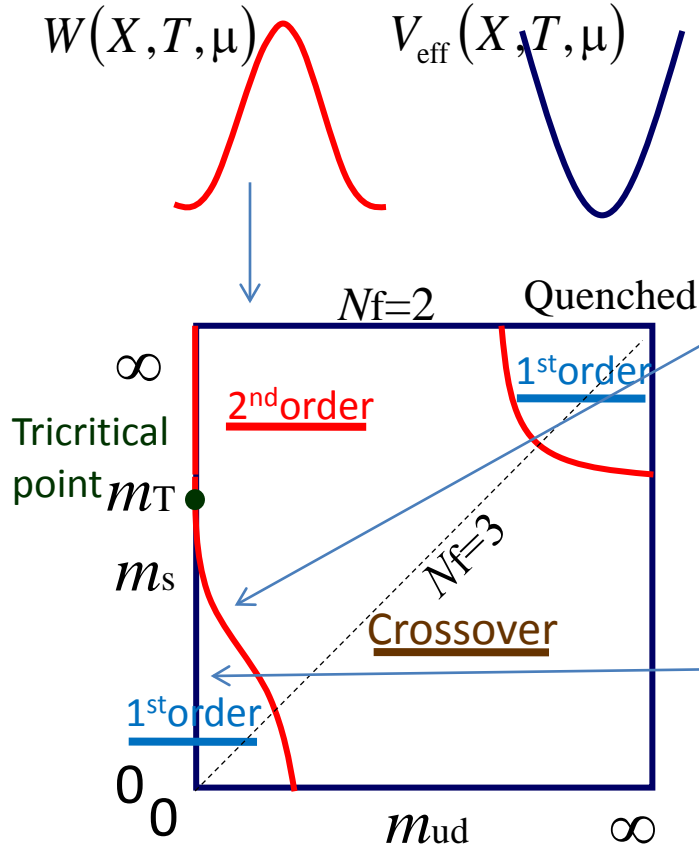
Crossover

$W(X)$: Gaussian function
 $V(X)$: Quadratic function

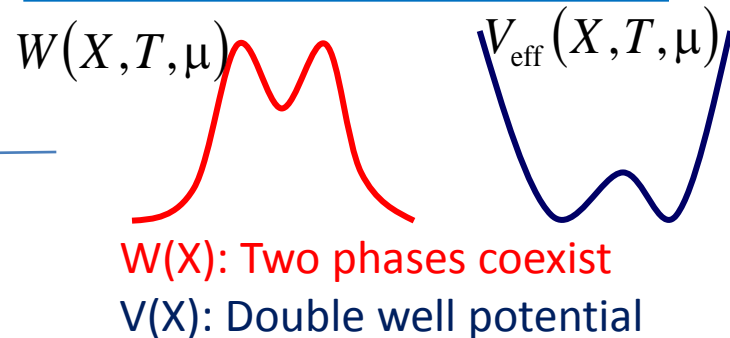
$$V_{\text{eff}}(X) = -\ln W(X)$$

Critical point

$W(X)$: Flat
 $V(X)$: Curvature: Zero



1st order phase transition



Reweighting method for plaquette distribution function

$$W(P, \beta, m, \mu) \equiv \int DU \delta(\hat{P} - P) \prod_{f=1}^{N_f} \det M(m_f, \mu_f) e^{6N_{\text{site}} \beta \hat{P}} \quad S_g = -6N_{\text{site}} \beta \hat{P}$$

$$(\beta = 6/g^2)$$

plaquette P (1x1 Wilson loop for the standard action)

$$R(P, \beta, \beta_0, m, m_0, \mu) \equiv W(P, \beta, m, \mu) / W(P, \beta_0, m_0, 0) \quad \text{(Reweight factor)}$$

$$R(P) = \frac{\left\langle \delta(\hat{P} - P) e^{6N_{\text{site}}(\beta - \beta_0)\hat{P}} \prod_f \frac{\det M(m_f, \mu_f)}{\det M(m_0, 0)} \right\rangle_{(\beta_0, \mu=0)}}{\left\langle \delta(\hat{P} - P) \right\rangle_{(\beta_0, \mu=0)}} \equiv \left\langle e^{6N_{\text{site}}(\beta - \beta_0)\hat{P}} \prod_f \frac{\det M(m_f, \mu_f)}{\det M(m_0, 0)} \right\rangle_{P:\text{fixed}}$$

Effective potential:

$$V_{\text{eff}}(P, \beta, m, \mu) = -\ln[W(P, \beta, m, \mu)] = V_{\text{eff}}(P, \beta_0, m_0, 0) - \ln R(P, \beta, \beta_0, m, m_0, \mu)$$

$$\ln R(P) = \underline{6N_{\text{site}}(\beta - \beta_0)P} + \ln \left\langle \underline{\prod_f \frac{\det M(m_f, \mu_f)}{\det M(m_0, 0)}} \right\rangle_{P:\text{fixed}}$$

First order transition point: two phases coexist

Plaquette distribution function

- Performing simulations of 2-flavor QCD,
- Dynamical effect of N_f -flavors are included by the reweighting.
- We assume N_f -flavors are heavy.
- Hopping parameter (κ) expansion (Wilson quark)

$$N_f \ln \left(\frac{\det M(\kappa, \mu)}{\det M(0,0)} \right) = N_f \left(288 N_{\text{site}} \kappa^4 P + 12 \cdot 2^{N_t} N_s^3 \kappa^{N_t} (\cosh(\mu/T) \Omega_R + i \sinh(\mu/T) \Omega_I) + \dots \right)$$

- Effective potential

$$V_{\text{eff}}(P, \beta, \kappa) = -\ln[R(P, \kappa)W(P, \beta, 0)] = \underbrace{V_{\text{eff}}(P, \beta, 0)}_{\text{2-flavor crossover}} + \underbrace{-\ln[R(P, \kappa)]}_{\text{2+Nf-flavor 1st order transition}} = \underbrace{\text{?}}_{\text{Negative curvature}}$$

$$\ln R(P) = \ln \left\langle e^{6N_{\text{site}}(\beta-\beta_0)\hat{P}} \prod_f \frac{\det M(\kappa_f, \mu_f)}{\det M(\kappa_0, 0)} \right\rangle_{P:\text{fixed}} \approx \ln \left\langle \exp(6hN_s^3 \hat{\Omega}_R) \right\rangle_{P:\text{fixed}} + (\text{linear term of } P)$$

(degenerate mass case at $\mu=0$)

Curvature of the effective potential

$$V_{\text{eff}}(P, \beta, h, \mu) = V_{\text{eff}}(P, \beta_0, 0, 0) - \ln \bar{R}(P, h, \mu) + \text{(linear term of } P)$$

$$\bar{R}(P) = \left\langle \exp(6N_s^3 h \Omega_R) \right\rangle_{P:\text{fixed}} \quad (\text{for the case of } \mu=0)$$

Wilson quark

$$h = 2N_f (2\kappa_h)^{N_t}$$

Staggered quark

$$h = N_f / \left(4(2m_h)^{N_t} \right)$$

- Linear term of P is irrelevant to the curvature
- β -dependence is only in the linear term.
- The curvature is independent of β .

χ_P : plaquette susceptibility

$$\frac{d^2 V_{\text{eff}}(0)}{dP^2} \approx \frac{6N_{\text{site}}}{\chi_P}$$

$$\frac{d^2 V_{\text{eff}}}{dP^2}(P, h, \mu) = \frac{d^2 V_{\text{eff}}}{dP^2}(P, 0, 0) - \frac{d^2 \ln \bar{R}}{dP^2}(P, h, \mu)$$

2-flavor

- If there exists the negative curvature region,



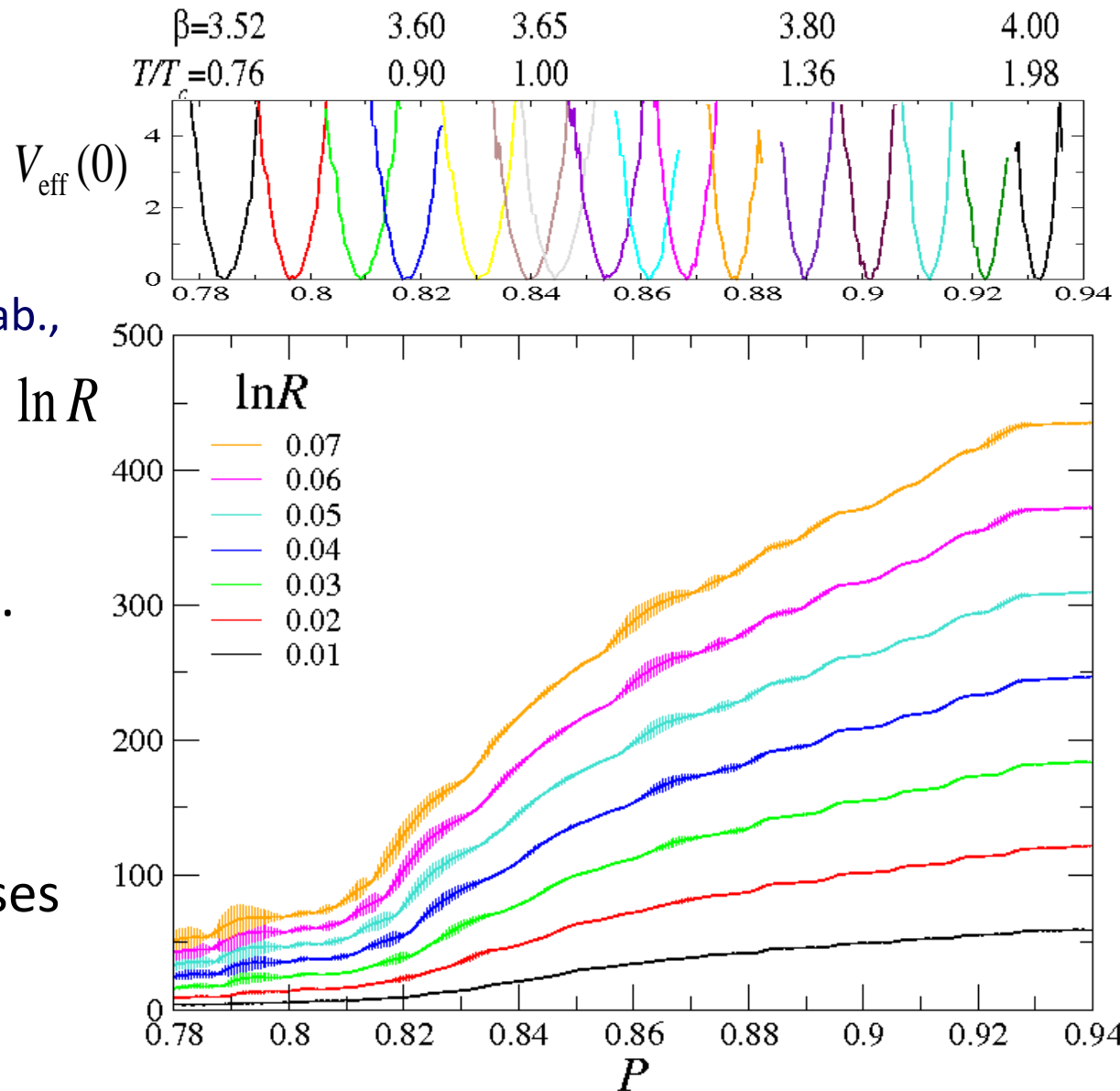
First order transition (double-well potential)

Effective potential at $h \neq 0$ $V_{\text{eff}}(P, \beta, h) = V_{\text{eff}}(P, \beta, 0) - \ln R(P, h)$

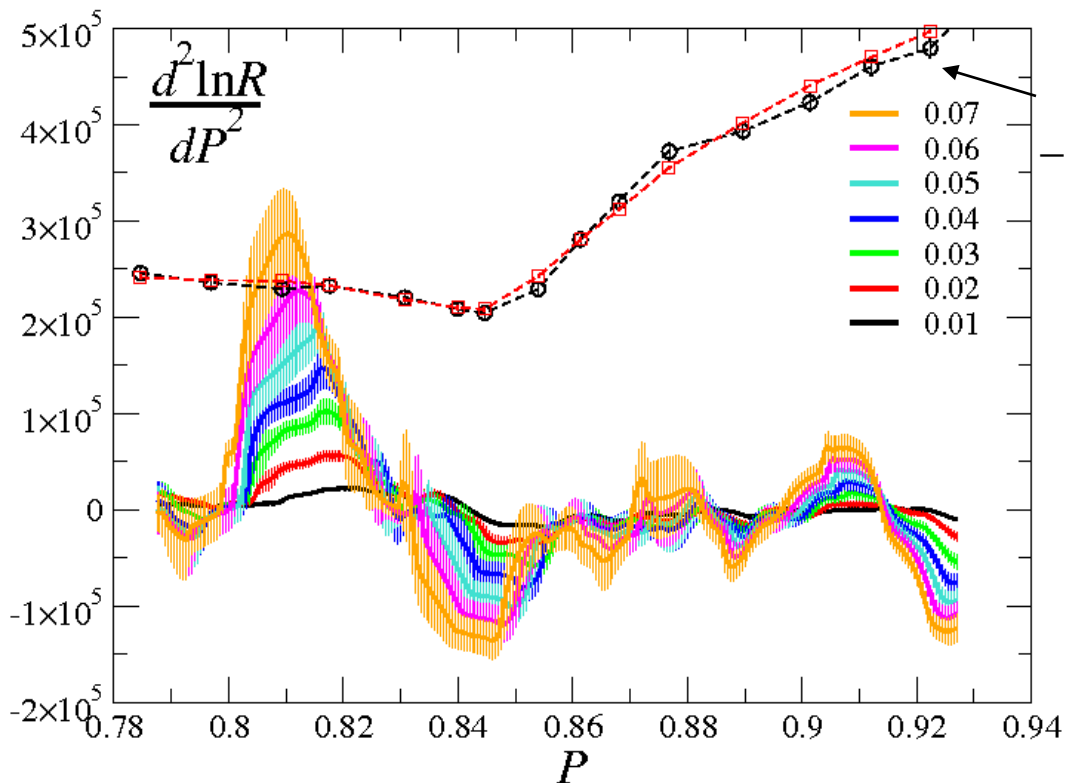
$N_f=2$ p4-staggered,
 $m_\pi/m_\rho \approx 0.7$

data: Beilefeld-Swansea Collab.,
PRD71,054508(2005)

- $\det M$: hopping parameter expansion.
- $\ln R$ increases as increasing h .
- The curvature increases with h .

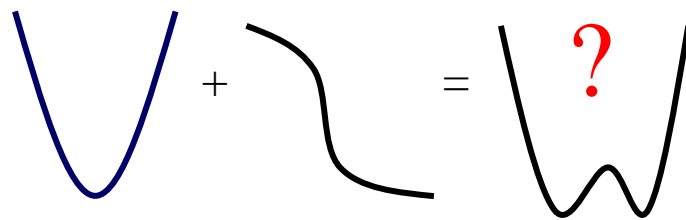
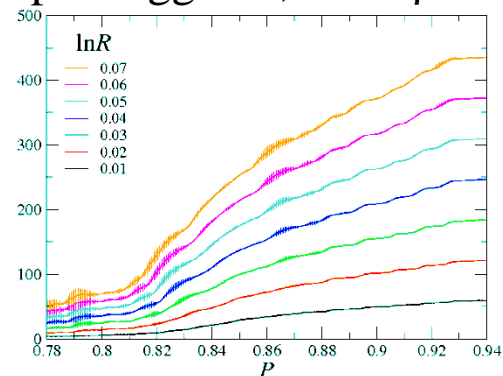


Curvature of the effective potential



$N_f=2$ p4-staggered, $m\pi/m\rho \approx 0.7$

$\frac{d^2 \ln W}{dP^2}$
at $h=0$



First order transition:
$$\frac{d^2 V_{\text{eff}}(P, \beta, h)}{dP^2} = \frac{d^2 V_{\text{eff}}(P, \beta, 0)}{dP^2} - \frac{d^2 \ln \bar{R}(P, h)}{dP^2} < 0$$

$$h = 2N_f (2\kappa_h)^{N_t}$$

(Wilson quarks)



- First order transition for $h > 0.6$

Critical value: $h_c = 0.0614(69)$

N_f -dependence of the critical mass

$$\underline{h_c = 0.0614(69)}$$

- Critical mass increases as N_f increases.

$$h = 2N_f (2\kappa_h)^{N_t} \quad \rightarrow \quad \kappa_h^c = \frac{1}{2} \left(\frac{h_c}{2N_f} \right)^{1/N_t}$$

- When N_f is large, κ is small. Then, the hopping parameter (κ) expansion is good.
- On the hand, when N_f is small, the κ -expansion is bad.
- In a quenched simulation with $N_t=4$, the first and second terms becomes comparable around $\kappa=0.18$.
- For $N_f=10$, $N_t=4$, $h_c = 0.0614(69) \rightarrow \kappa_h^c \approx 0.118$
 - It may be applicable for $N_f \sim 10$.

The effective potential at finite μ

Reweighting factor

$$\ln R(P) = \ln \left\langle \underbrace{\left(\frac{\det M(m, \mu)}{\det M(m, 0)} \right)^2}_{\text{light quarks}} \underbrace{\left(\frac{\det M(h, \mu_h)}{\det M(0, 0)} \right)^{N_f}}_{\text{heavy quarks}} \right\rangle_{P:\text{fixed}}$$

Compare the curvatures

$$\frac{d^2 V_{\text{eff}}}{dP^2}(P, h, \mu) = \frac{d^2 V_{\text{eff}}}{dP^2}(P, 0, 0) - \frac{d^2 \ln R}{dP^2}(P, h, \mu)$$

Evaluation of $\ln R$ at finite μ

- Quark determinant: Taylor expansion up to $O(\mu^6)$

data: Beilefeld-Swansea Collab., PRD71,054508(2005)

$$N_f \ln \det M(\mu) = N_f \sum_{n=0}^N \left[\frac{1}{n!} \left(\frac{\mu}{T} \right)^n \frac{d^n \ln \det M}{d(\mu/T)^n} \right] \quad \begin{array}{l} \theta: \text{complex phase} \\ \theta \equiv \text{Im} \ln \det M \end{array}$$

- Cumulant expansion method (SE, PRD77,014508(2008), WHOT-QCD, PRD82,014508(2010))

$$\langle e^{i\theta} \rangle = \exp \left[\underbrace{i \langle \theta \rangle_c}_{\rightarrow 0} - \frac{1}{2} \langle \theta^2 \rangle_c - \underbrace{\frac{i}{3!} \langle \theta^3 \rangle_c}_{\rightarrow 0} + \frac{1}{4!} \langle \theta^4 \rangle_c + \dots \right]$$

cumulants

$$\langle \theta \rangle_c = \langle \theta \rangle, \quad \langle \theta^2 \rangle_c = \langle \theta^2 \rangle - \langle \theta \rangle^2, \quad \langle \theta^3 \rangle_c = \langle \theta^3 \rangle - 3 \langle \theta^2 \rangle \langle \theta \rangle + 2 \langle \theta \rangle^3, \quad \langle \theta^4 \rangle_c = \dots$$

– Odd terms vanish from a symmetry under $\mu \leftrightarrow -\mu$ ($\theta \leftrightarrow -\theta$)

Source of the complex phase

– If the distribution of θ is Gaussian, $\langle \theta^2 \rangle_c$ term dominates.

– Assuming the Gaussian distribution, we approximate

$$\langle e^{i\theta} \rangle \approx \exp \left[-\frac{1}{2} \langle \theta^2 \rangle_c \right]$$

Curvature of the effective potential at finite μ

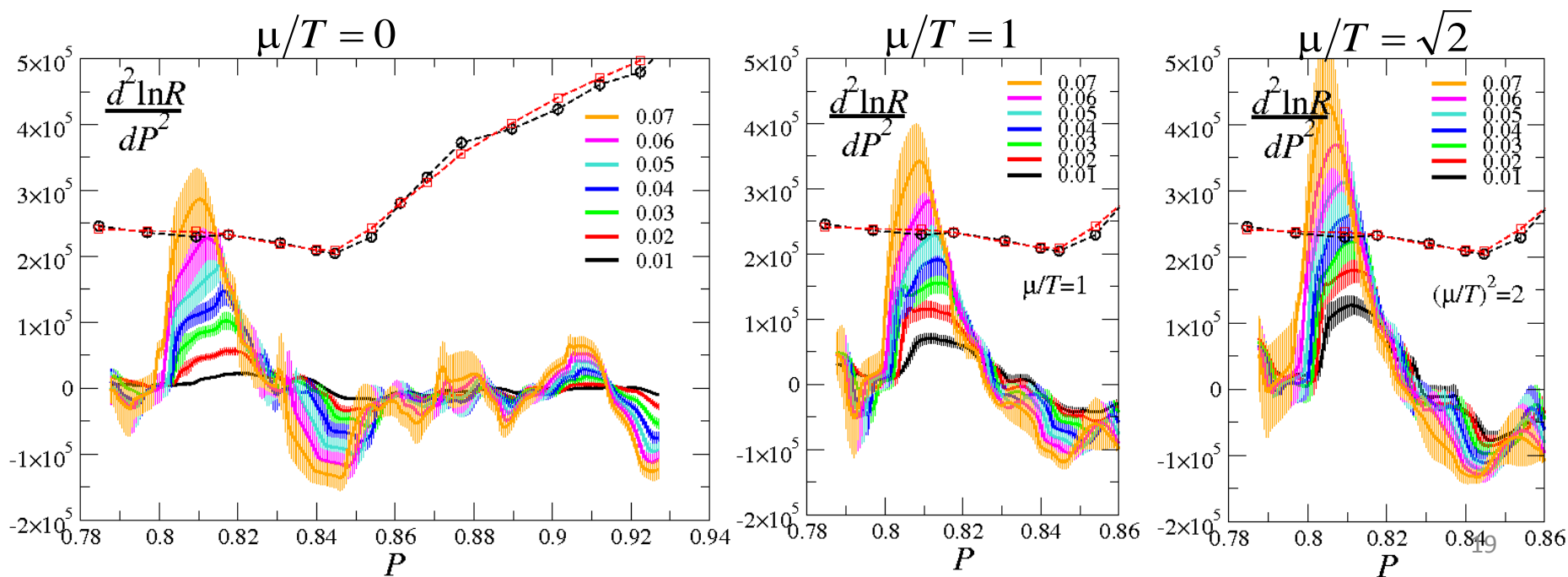
$$\frac{d^2 V_{\text{eff}}}{dP^2}(P, h, \mu) = \frac{d^2 V_{\text{eff}}}{dP^2}(P, 0, 0) - \frac{d^2 \ln R}{dP^2}(P, h, \mu)$$

$$h = 2N_f (2\kappa_h)^{N_t}$$

for Wilson quarks

- Reweighting factor

$$\ln R(P) = \ln \left\langle \underbrace{\left(\frac{\det M(m, \mu)}{\det M(m, 0)} \right)^2}_{\text{green}} \underbrace{\left(\frac{\det M(h, \mu_h)}{\det M(0, 0)} \right)^{N_f}}_{\text{red}} \right\rangle_{P:\text{fixed}} \quad \mu_h/T = 0$$



Critical line at finite density

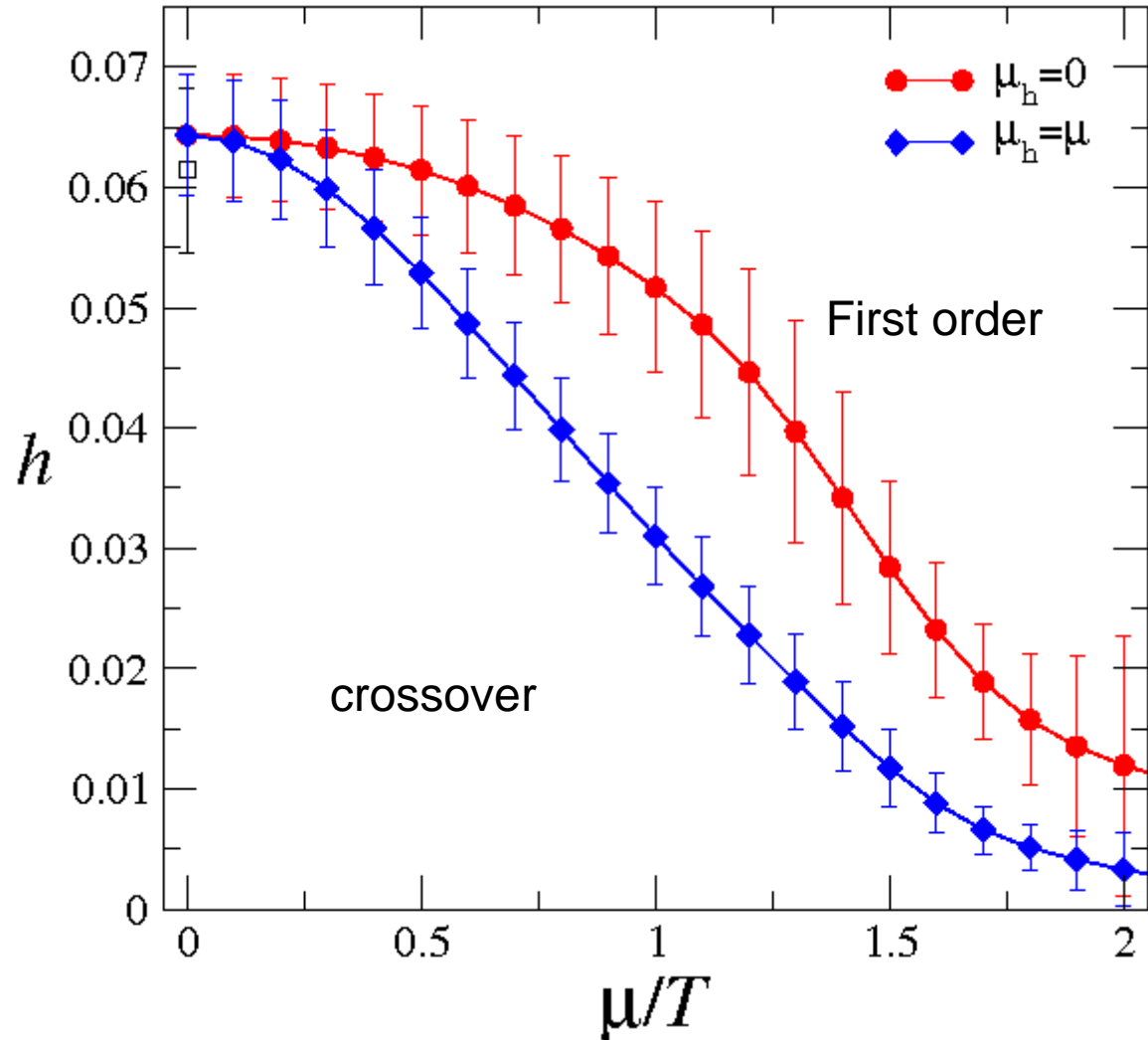
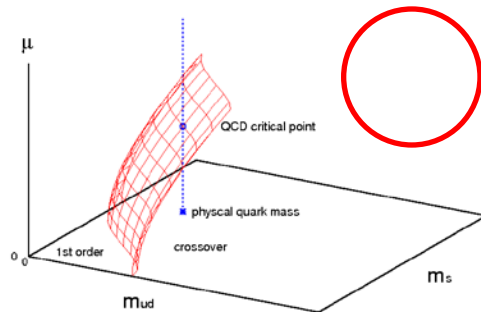
$$h = 2N_f (2\kappa_h)^{N_t}$$

for Wilson quarks

$$h = N_f / (4(2m_h)^{N_t})$$

for staggered quarks

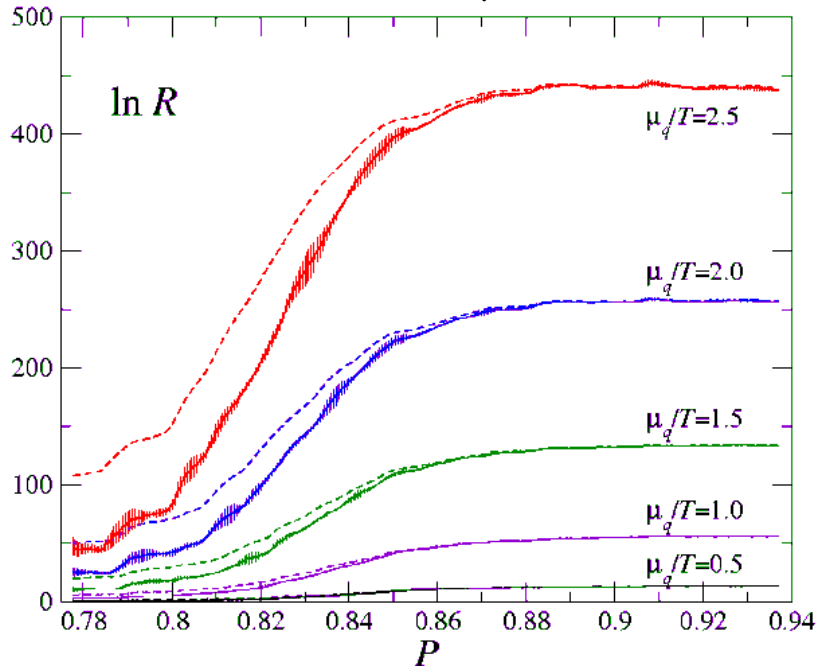
- Calculations of $\det M$: Taylor expansion up to $O(\mu^6)$
- Distribution function of the complex phase of $\det M$: approximated by a Gaussian function



Reweighting factors at $h \neq 0$ $\mu \neq 0$

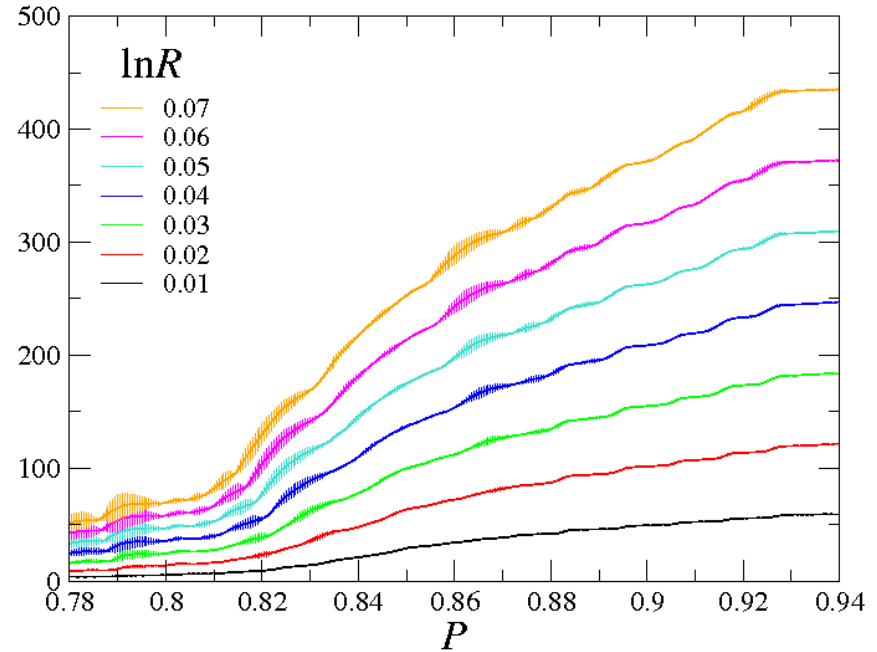
$$\ln R(P; h, \mu) = \ln \left\langle \left(\frac{\det M(m, \mu)}{\det M(m, 0)} \right)^2 \left(\frac{\det M(h, 0)}{\det M(0, 0)} \right)^{N_f} \right\rangle_{P:\text{fixed}} \approx \ln R(P; 0, \mu) + \ln R(P; h, 0)$$

$\ln R(P; 0, \mu)$



(S. Ejiri, Phys. Rev. D 77 (2008) 014508)

$\ln R(P; h, 0)$



$N_f=2$ p4-staggared, $m_\pi/m_\rho \approx 0.7$ [data in PRD71,054508(2005)]

- The curvatures of $\ln R(P; \mu, 0)$ and $\ln R(P; 0, h)$ are large at the same P .



The curvature of $\ln R(P; \mu, h)$ is enhanced.

Phase structure of (2+many)-flavor QCD using Wilson quark action

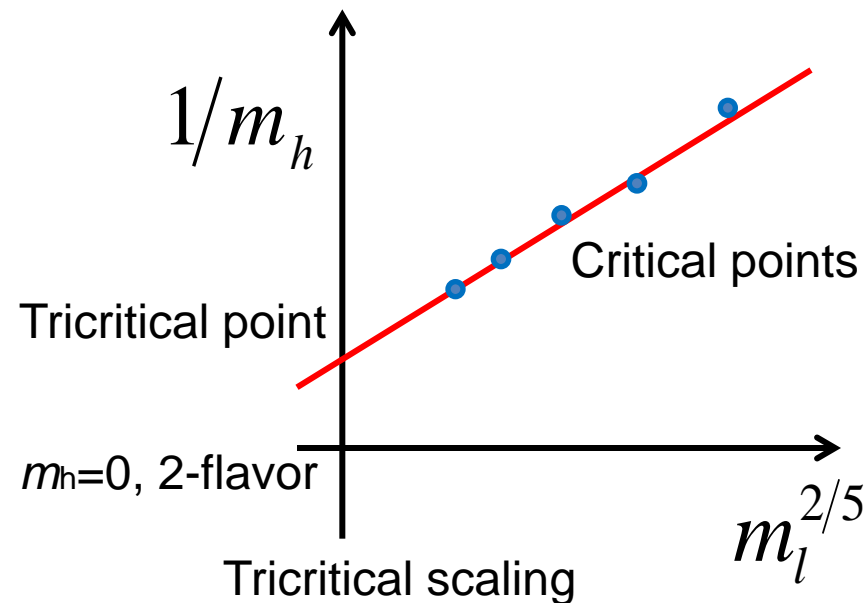
- Light quark mass dependence
- (Chemical potential dependence: in progress)

Phase structure of (2+many)-flavor QCD using Wilson quark action

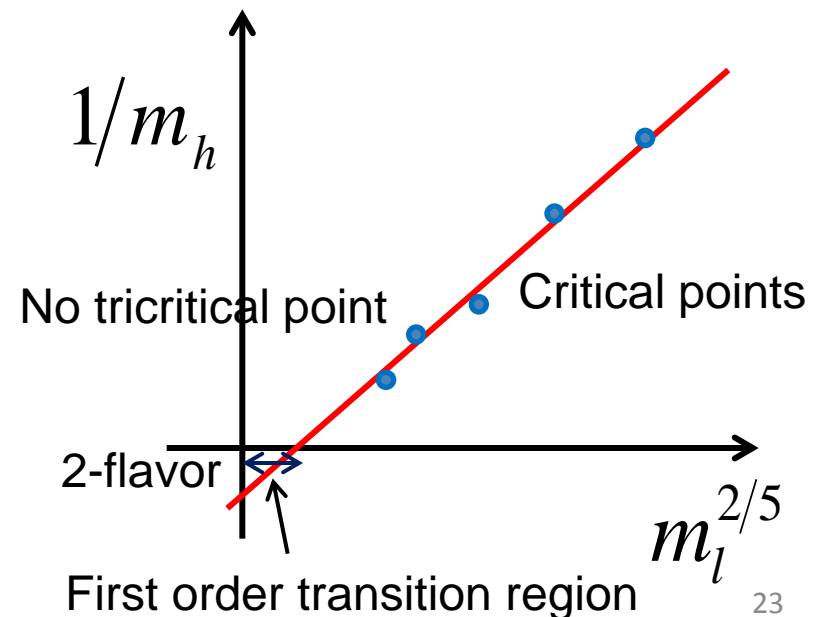
2-flavor QCD simulations + reweighting

Light quark mass dependence of the critical line

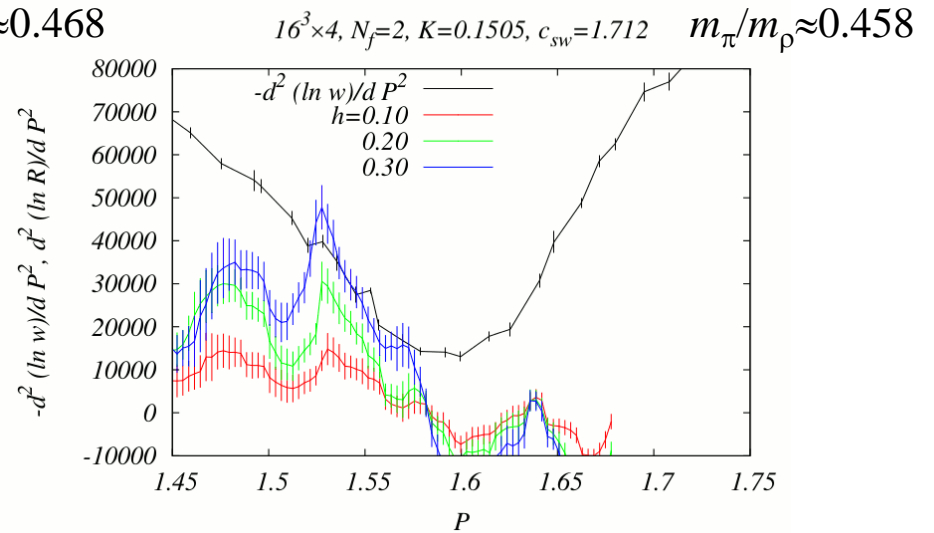
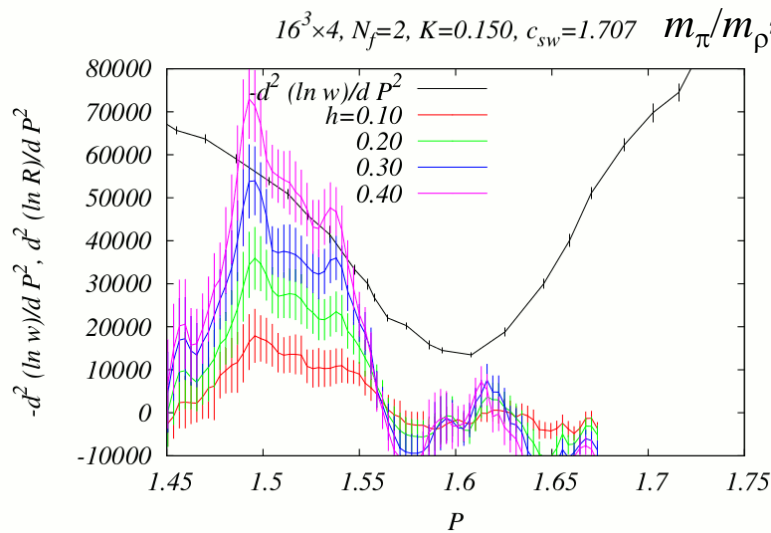
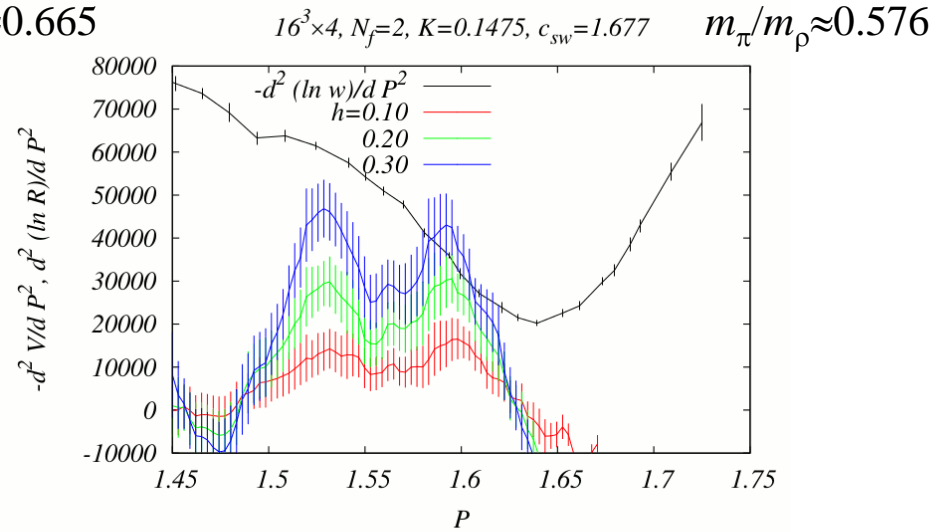
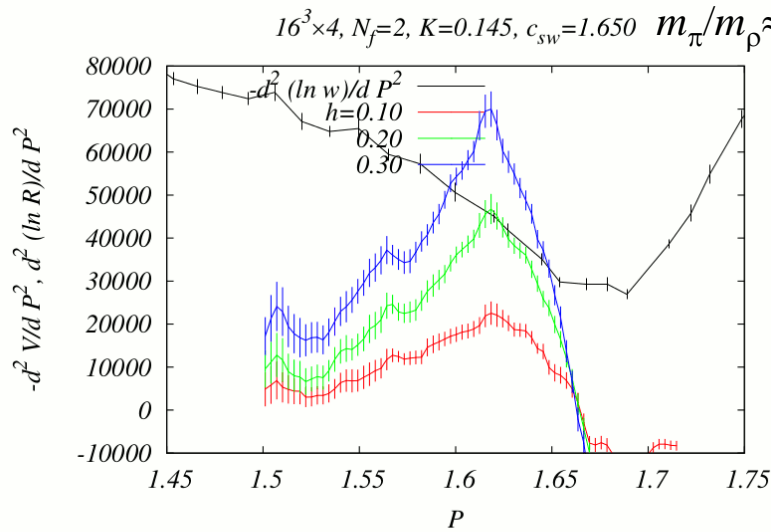
- Tricritical scaling behavior?
- Is there a first order transition region in 2-flavor QCD?



or



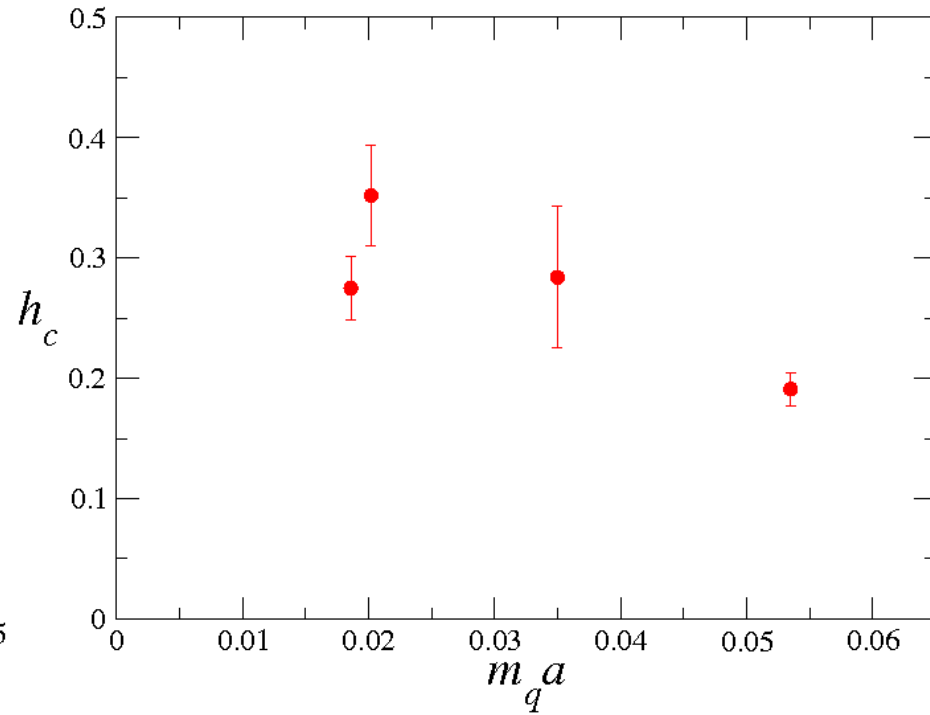
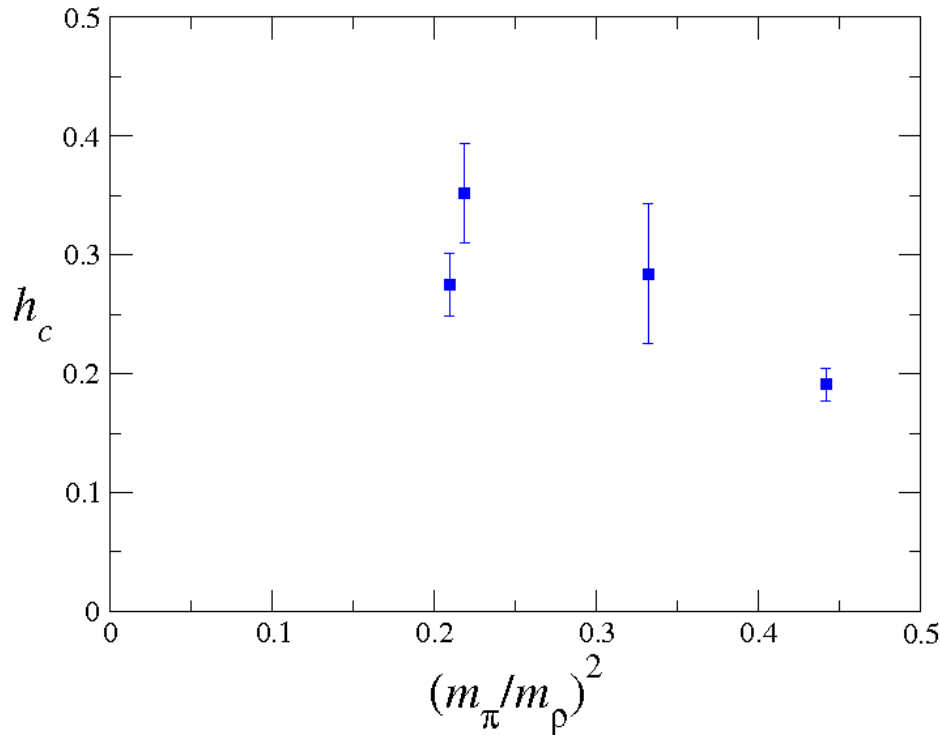
Light quark mass dependence (preliminary)



- Critical point: light quark mass dependence is small in this region.

Light quark mass dependence (preliminary)

$$h = 2N_f (2\kappa_h)^{N_t} \text{ for Wilson quarks}$$



- Critical point: light quark mass dependence is small in the region we investigated.
- The first order transition in the massless 2-flavor QCD is not suggested.

Summary

- We investigated the **phase structure of (2+N_f)-flavor QCD**.
S. Ejiri & N. Yamada, Phys. Rev. Lett. 110, 172001 (2013) [arXiv:1212.5899]
 - This model is interesting for the feasibility study of the **electroweak baryogenesis** in the **technicolor scenario**.
 - An appearance of a first order phase transition at finite temperature is required for the baryogenesis.
- Applying the reweighting method, we determine the critical mass of heavy flavors terminating the first order region.
 - The critical mass becomes larger with N_f .
 - The first order region becomes wider as increasing μ . (p4-staggered)
 - The light quark mass dependence the critical heavy quark mass is small in the region we investigated. (clover-Wilson)
 - The first order transition in 2-flavor QCD is not suggested.
- This may be a good test for the determination of boundary of the first order region in (2+1)-flavor QCD at finite density.