The critical surface of QCD in the heavy quark region

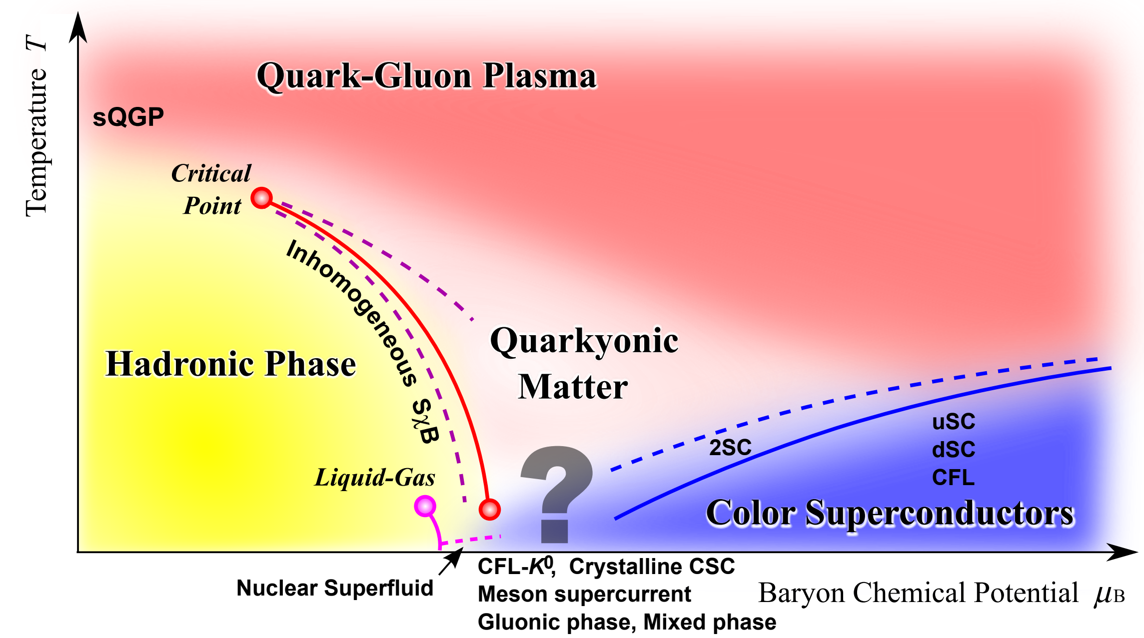
H. Saito for WHOT-QCD collaboration

DESY NIC University of Tsukuba

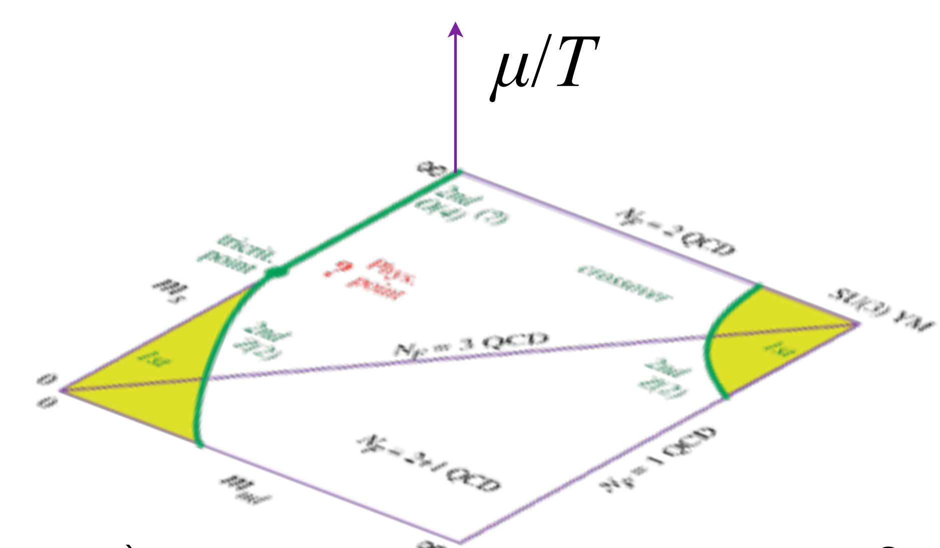
H. Saito *et al* for whot-qcd PRD84 054502, arXiv:1106.0974, 1011.4747, 1309.2445
S. Ejiri *et al* for whot-qcd arXiv:1212.0762

QCD at finite density

- Phase diagram of finite density QCD
  - Conjectures
  - Qualitative study with non-perturbative approaches (like lattice QCD sim.)
  - Estimate of critical point
- Lattice QCD : Critical surface in Colombia plot



Fukushima, Hatsuda, Rept. Prog. Phys. 74, 014001 (2011), arXiv:1005.4814



Lattice QCD at finite density

- **Sign Problem**
  - quark determinant to be complex at finite μ

    $$[\det M(\mu)]^* \neq \det M(\mu)$$

  - complex distribution spoils Monte Carlo sim.

- **This study**
  - histogram method + reweighting + cumulant exp.
  - critical surface in the heavy quark region

Action

- Plaquette action for gauge

$$S_g = -\frac{\beta}{3}\sum_n \sum_{\mu<\nu} P_{\mu\nu} = -6\beta N_{\rm site} P$$

$\beta = 6/g^2$: coupling

$$P_{\mu\nu} = \frac{1}{18}\mathrm{Retr}\left[U_{n,\mu}U_{n+\hat\mu,\nu}U^\dagger_{n+\hat\mu+\hat\nu,\mu}U^\dagger_{n,\nu}\right]$$

$N_{\rm site} \equiv N_s^3 \times N_t$: the number of sites

- Standard Wilson fermion

Quark chemical potential (μ) dependence :

$$\boxed{U_{n,4} \to e^{\mu a}U_{n,4} \quad U^\dagger_{n,4} \to e^{-\mu a}U^\dagger_{n,4}}$$

$$\begin{aligned} S_q &= \sum_n \bar\psi_n\psi_n - \kappa\sum_n \bar\psi_n \Big\{ \sum_{\mu=0}^{3}\left[(1-\gamma_\mu)U_{n,\mu}\psi_{n+\hat\mu} + (1+\gamma_\mu)U^\dagger_{n-\hat\mu,\mu}\psi_{n-\hat\mu}\right] \\ &\qquad + e^{\mu a}(1-\gamma_4)U_{n,4}\psi_{n+\hat 4} + e^{-\mu a}(1+\gamma_4)U^\dagger_{n-\hat 4,4}\psi_{n-\hat 4}\Big\} \\ &\equiv \sum \bar\psi M(\kappa,\mu)\psi \\ &\to \det M(\kappa,\mu) \end{aligned}$$

$\kappa \sim m^{-1}$ in heavy quark region : hopping parameter

$[\det M(\mu)]^* \neq \det M(\mu)$: ***sign problem*** at real μ

histogram methods + reweighting

Probability distribution

- probability distribution func.

$$w(X;\beta,\vec{\kappa},\vec{\mu}) = \int \mathcal{D}U \underbrace{\prod_i \delta(X_i - \hat{X}_i)}_{\text{delta func.}} \underbrace{e^{-S_g} \prod_{f=1}^{N_\mathrm{f}} \det M(\kappa_f,\mu_f)}_{\text{from action}}$$

$X = (X_1, X_2, \ldots)$: (one or more) physical quantities,

$\vec{\kappa} = (\kappa_1, \cdots, \kappa_{N_\mathrm{f}})$: hopping parameter, $\vec{\mu} = (\mu_1, \cdots, \mu_{N_\mathrm{f}})$: chemical potential

- At various points with reweighting

to calculate an expectation value at a certain simulation point (μ, κ)
by using configurations at different simulation point $(\mu = 0, \kappa_0)$

$$\frac{w(X;\beta,\vec{\kappa},\vec{\mu})}{w(X;\beta_0,\vec{\kappa}_0,\vec{0})} = \left\langle e^{6(\beta-\beta_0)N_{\mathrm{site}}\hat{P}} \prod_{f=1}^{N_\mathrm{f}} \left[\frac{\det M(\kappa_f,\mu_f)}{\det M(\kappa_{0f},0)}\right] \right\rangle_{X;\beta_0,\vec{\kappa}_0,\vec{0}}$$

where $\langle\cdots\rangle_{X;\beta_0,\vec{\kappa}_0,\vec{0}} \equiv \left\langle \cdots \Pi_i \delta(X_i - \hat{X}_i)\right\rangle_{\beta_0,\vec{\kappa}_0,\vec{0}} \Big/ \left\langle \Pi_i \delta(X_i - \hat{X}_i)\right\rangle_{\beta_0,\vec{\kappa}_0,\vec{0}}$

<u>In this study</u> : focusing on the heavy quark region $\vec{\kappa}_0 = \vec{0}$

Reweighting
(in the heavy quark region)

- Hopping parameter expansion

Imaginary part of Polyakov loop

$$\prod_{f=1}^{N_f}\left[\frac{\det M(\kappa_f,\mu_f)}{\det M(0,0)}\right] = \exp\left[288 N_{\rm site}\sum_{f=1}^{N_f}\kappa_f^4 \hat{P} + 3\times 2^{N_t+2} N_s^3 \left\{\sum_{f=1}^{N_f}\kappa_f^{N_t}\cosh\left(\frac{\mu_f}{T}\right)\hat{\Omega}_{\rm R} + i\sum_{f=1}^{N_f}\kappa_f^{N_t}\sinh\left(\frac{\mu_f}{T}\right)\hat{\Omega}_{\rm I}\right\} + \cdots\right]$$

Reweighting param.

(β, κ_f, μ_f) → β for turning to the transition point → Only (κ_f, μ_f) ⇒ h.p.e. ⇒ Essentially, $({\kappa_f}^{Nt}\cosh(\mu_f/T), {\kappa_f}^{Nt}\sinh(\mu_f/T))$

Lattice setup

- Heat bath
  $\det M$ is evaluated by using a hopping parameter expansion.
- β & Number of Configurations

β	# of Confs.
5.68	100,000
5.685	430,000
5.69	500,000
5.6925	670,000
5.7	100,000

1st order transition point in the heavy quark limit

- Lattice size: 24^3 x 4

At zero μ

Plaquette potential

- distribution func. of plaquette

$$w(P', \beta, \kappa) = \int \mathcal{D}U \mathcal{D}\psi \mathcal{D}\bar{\psi} \delta(P(U) - P') e^{-S(\beta,\kappa)}$$

$P(U) = \frac{1}{N_{\rm site}} \sum_n \sum_{\mu<\nu} P_{\mu\nu}$: average of plaquette

where $P_{\mu\nu} = \mathrm{Re\ tr}\left[U_{n,\mu} U_{n+\hat{\mu},\nu} U^\dagger_{n+\hat{\nu},\mu} U^\dagger_{n,\nu}\right]$

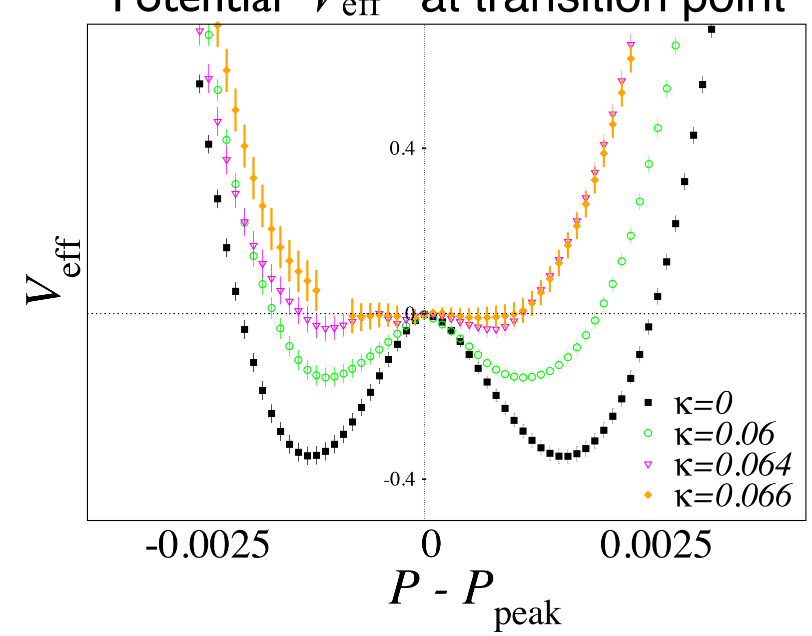
$\beta = 6/g^2$: coupling , κ : hopping parameter

- Reweighting in direction κ
- Potential: $V_{\rm eff}(P, \beta, \kappa) = -\ln w(P, \beta, \kappa)$
- The order of the transition from shape of the potential:

double well ⇔ **single well**

At a transition point of 1st order transition — On the other points

Potential $V_{\rm eff}$ at transition point



The order of the phase transition in the heavy quark region

- Critical point in the heavy quark region for $N_f = 2$:

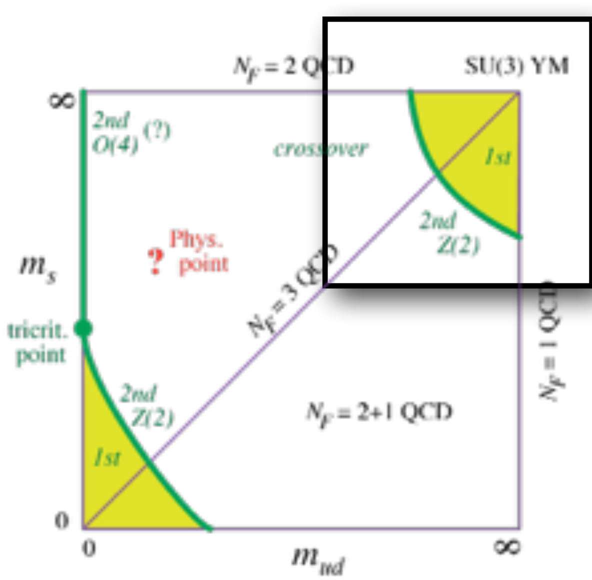
(hopping parameter $\kappa \sim m^{-1}$ in the heavy quark region)

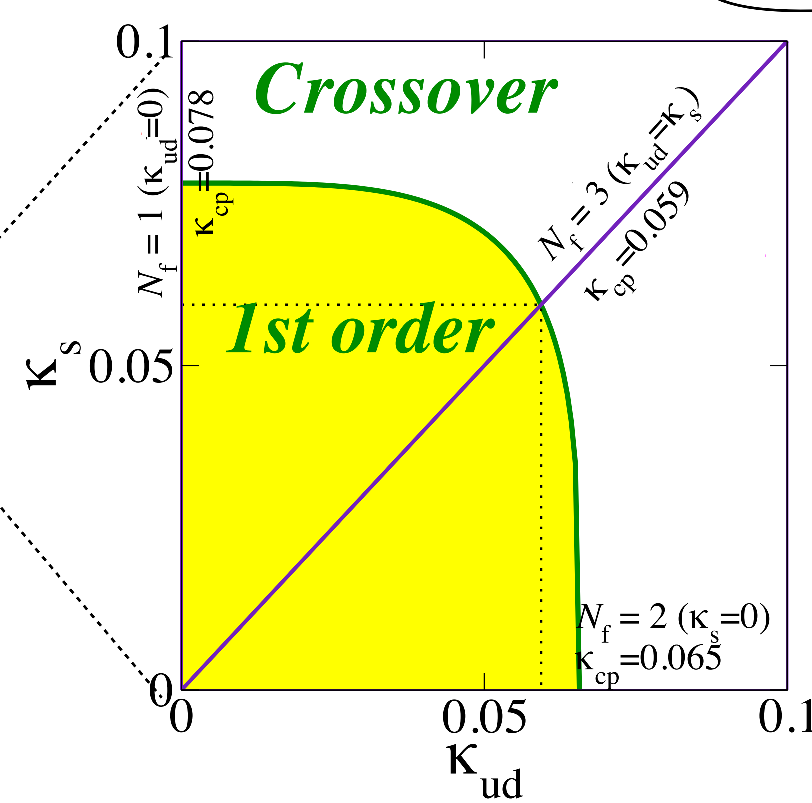
$$\kappa_{cp} = 0.0658(3)$$

- Extent the result of $N_f = 2$ to $N_f = 2+1$ case

degenerate light 2-flavor & heavy 1-flavor

right top corner



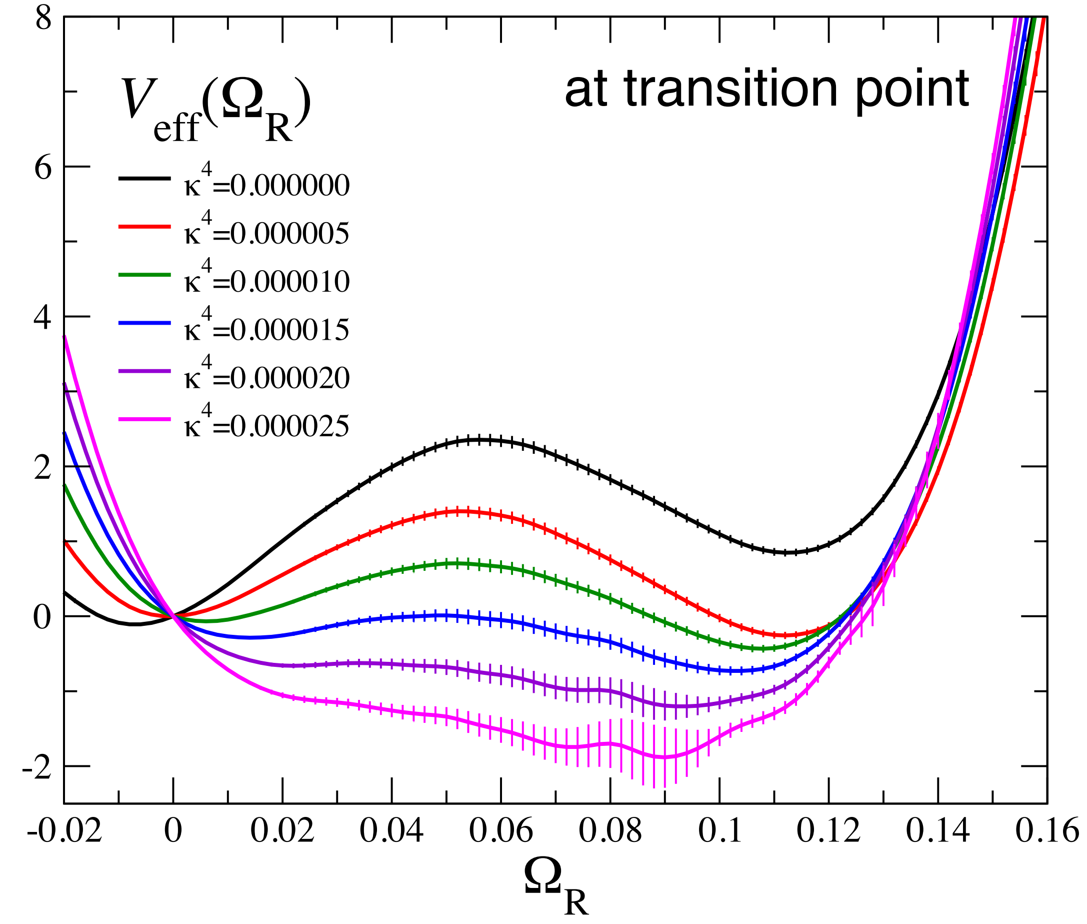


Polyakov loop potential

- Polyakov loop : order parameter of pure gauge
- Polyakov loop potential:

$$V_{\text{eff}}(\Omega_R;\beta,\vec{\kappa}) = -\ln w(\Omega_R;\beta,\vec{\kappa})$$

  - ✦ Reweighting in direction κ
  - ✦ consistent result with plaquette potential



At finite μ

Phase quenched case

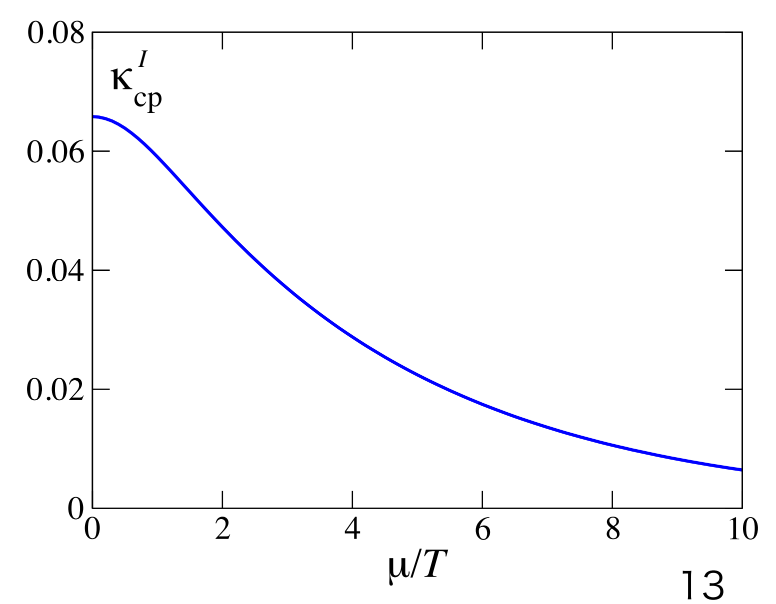
- Ex. degenerate $N_{\mathrm{f}} = 2 \Rightarrow$ isospin chemical potential
- Hopping parameter exp.:

$$\prod_{f=1}^{N_{\mathrm{f}}} \left[\frac{\det M(\kappa_f,\mu_f)}{\det M(0,0)}\right] = \exp\left[288 N_{\mathrm{site}} \sum_{f=1}^{N_{\mathrm{f}}} \kappa_f^4 \hat{P} + 3\times 2^{N_t+2} N_s^3 \left\{ \sum_{f=1}^{N_{\mathrm{f}}} \kappa_f^{N_t} \cosh\left(\frac{\mu_f}{T}\right)\hat{\Omega}_{\mathrm{R}} + \cancel{i \sum_{f=1}^{N_{\mathrm{f}}} \kappa_f^{N_t} \sinh\left(\frac{\mu_f}{T}\right)\hat{\Omega}_{\mathrm{I}}} \right\} + \cdots \right]$$

Imaginary part of Polyakov loop

<u>Reweighting param.</u> : only $\sum_f \kappa_f^{N_t} \cosh(\mu/T)$

- Note : common form to $\mu = 0$ except for the param., shape of potential to be common with cnst. $\sum_f \kappa_f^{N_t} \cosh(\mu/T)$
- Critical line: $\kappa_{\mathrm{cp}}(\mu) = \dfrac{\kappa_{\mathrm{cp}}(0)}{[\cosh(\mu/T)]^{1/N_t}}$



Polyakov loop potential at finite μ (1)

- Hopping parameter exp. :

$$\prod_{f=1}^{N_f}\left[\frac{\det M(\kappa_f,\mu_f)}{\det M(0,0)}\right] = \exp\left[288 N_{\rm site}\sum_{f=1}^{N_f}\kappa_f^4 \hat{P} + 3\times 2^{N_t+2} N_s^3 \left\{\sum_{f=1}^{N_f}\kappa_f^{N_t}\cosh\left(\frac{\mu_f}{T}\right)\hat{\Omega}_{\rm R} + i\underline{\sum_{f=1}^{N_f}\kappa_f^{N_t}\sinh\left(\frac{\mu_f}{T}\right)\hat{\Omega}_{\rm I}}\right\} + \cdots\right]$$

<u>im. Polyakov loop</u> **complex phase**

- Polyakov loop potential
- When computing pot., complex phase $\left\langle e^{i\hat{\theta}}\right\rangle_{\Omega_R;\beta,\vec{0},\vec{0}}$

where $\hat{\theta} = 3\times 2^{N_t+2}N_s^3\sum_{f=1}^{N_f}\kappa_f^{N_t}\sinh\left(\frac{\mu_f}{T}\right)\hat{\Omega}_I$

  - ✴ Calculated via Ω_I
  - ✴ Cumulant exp.

expansion to compute expectation value of exponential func. with exponential func. of function of expectation values

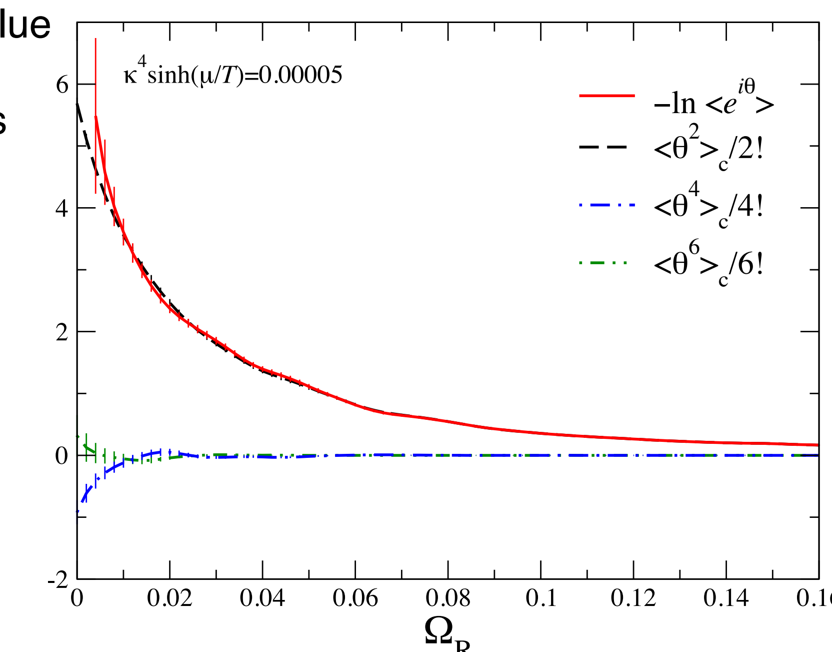
S. Ejiri et al. PRD82, 014508, PRD77, 014508

Odd terms $\langle\theta^n\rangle_c = 0$ (for n odd) due to time reflection sym.

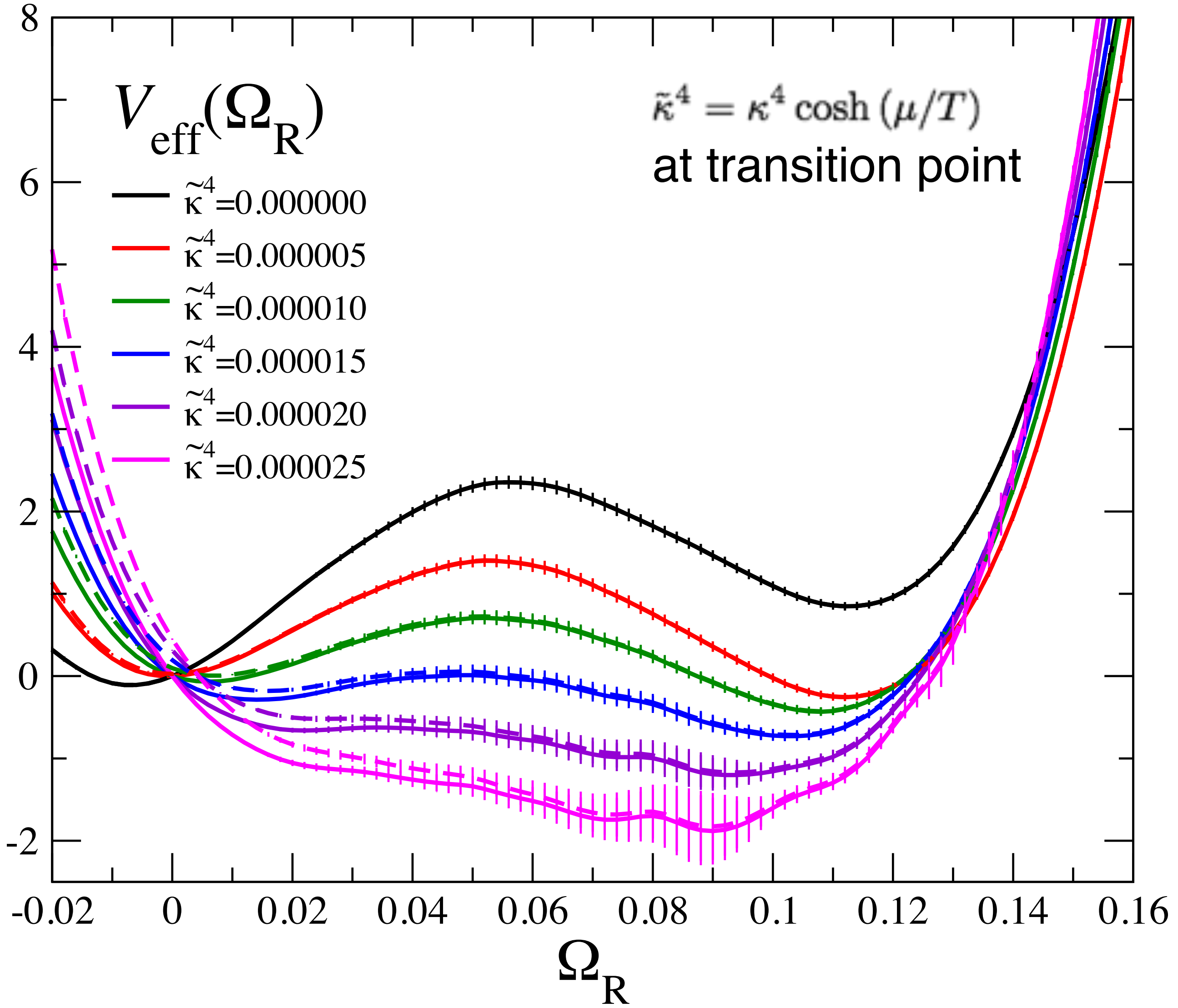
$$\left\langle e^{i\hat{\theta}}\right\rangle_{\Omega_R;\beta,\vec{0},\vec{0}} = \exp\left[-\frac{\left\langle\hat{\theta}\right\rangle_c^2}{2} + \cancel{\frac{\left\langle\hat{\theta}\right\rangle_c^4}{4!}} - \cancel{\frac{\left\langle\hat{\theta}\right\rangle_c^6}{6!}} + \cdots\right]$$

where $\left\langle\hat{\theta}\right\rangle_c^2 \equiv \left\langle\hat{\theta}^2\right\rangle_{\Omega_R;\beta,\vec{0},\vec{0}}$ $\quad \left\langle\hat{\theta}^4\right\rangle_c \equiv \left\langle\hat{\theta}^4\right\rangle_{\Omega_R;\beta,\vec{0},\vec{0}} - 3\left\langle\hat{\theta}\right\rangle^2_{\Omega_R;\beta,\vec{0},\vec{0}}$

$$\left\langle\hat{\theta}^6\right\rangle_c \equiv \left\langle\hat{\theta}^6\right\rangle_{\Omega_R;\beta,\vec{0},\vec{0}} - 15\left\langle\hat{\theta}^4\right\rangle_{\Omega_R;\beta,\vec{0},\vec{0}}\left\langle\hat{\theta}^2\right\rangle_{\Omega_R;\beta,\vec{0},\vec{0}} + 30\left\langle\hat{\theta}^2\right\rangle^3_{\Omega_R;\beta,\vec{0},\vec{0}}$$



Polyakov loop potential at finite μ (2)



- solid line : w/o the complex phase
- broken line (beside each solid line): w complex phase within 2nd order of cumulant exp., the μ-dep. of the phase $\sinh(\mu/T)$ is replaced with $\cosh(\mu/T)$ because upper bound of $\sinh(\mu/T)$ is given by $\cosh(\mu/T)$

Critical surface for $N_f = 2+1$

- Effect of the complex phase on the potential is so small to be neglected for determination of the critical point.

- Critical surface for $N_f = 2+1$

  Condition of critical surface from assumption that same values of reweighing param. give same shape of potential :

$$2\left[\kappa_{ud}(\boldsymbol{\mu})\right]^{N_t}\cosh\left(\frac{\mu_{ud}}{T}\right)+\left[\kappa_s(\boldsymbol{\mu})\right]^{N_t}\cosh\left(\frac{\mu_s}{T}\right)=2\left[\kappa_{ud}(0)\right]^{N_t}$$

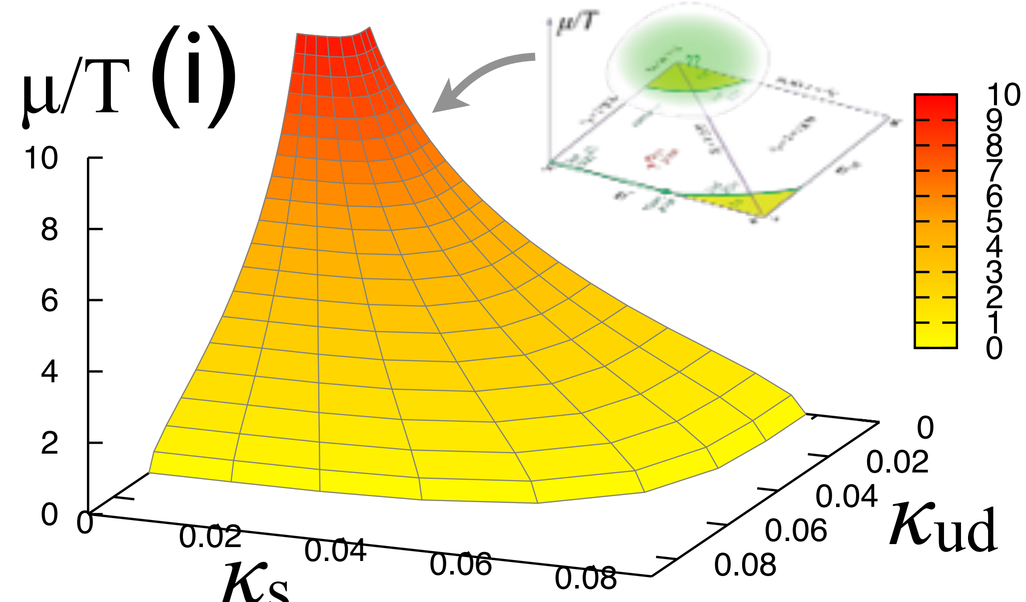
where $\boldsymbol{\mu} = (\mu_{ud}, \mu_s)$

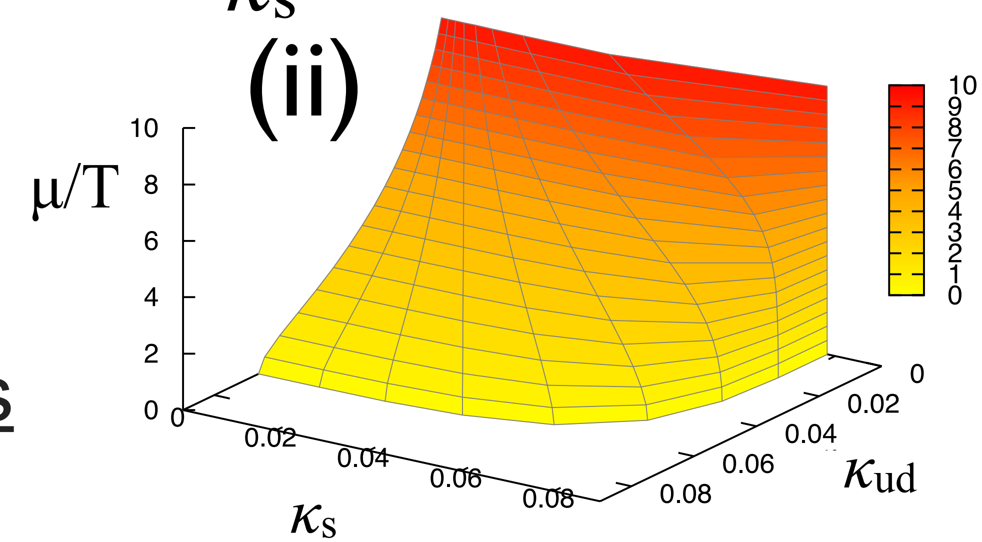
(i) $\mu_{ud} = \mu_s = \mu$

(ii) $\mu_{ud} = \mu$, $\mu_s = 0$ (Heavy ion collision)

μ/T increases

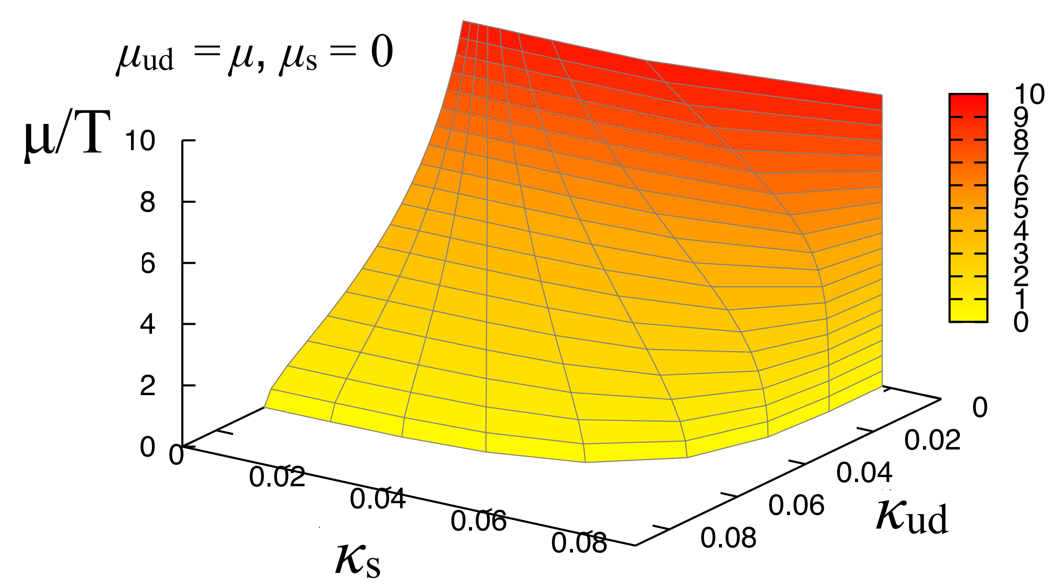
⇒ Region of the 1st order transition shrinks





Conclusion

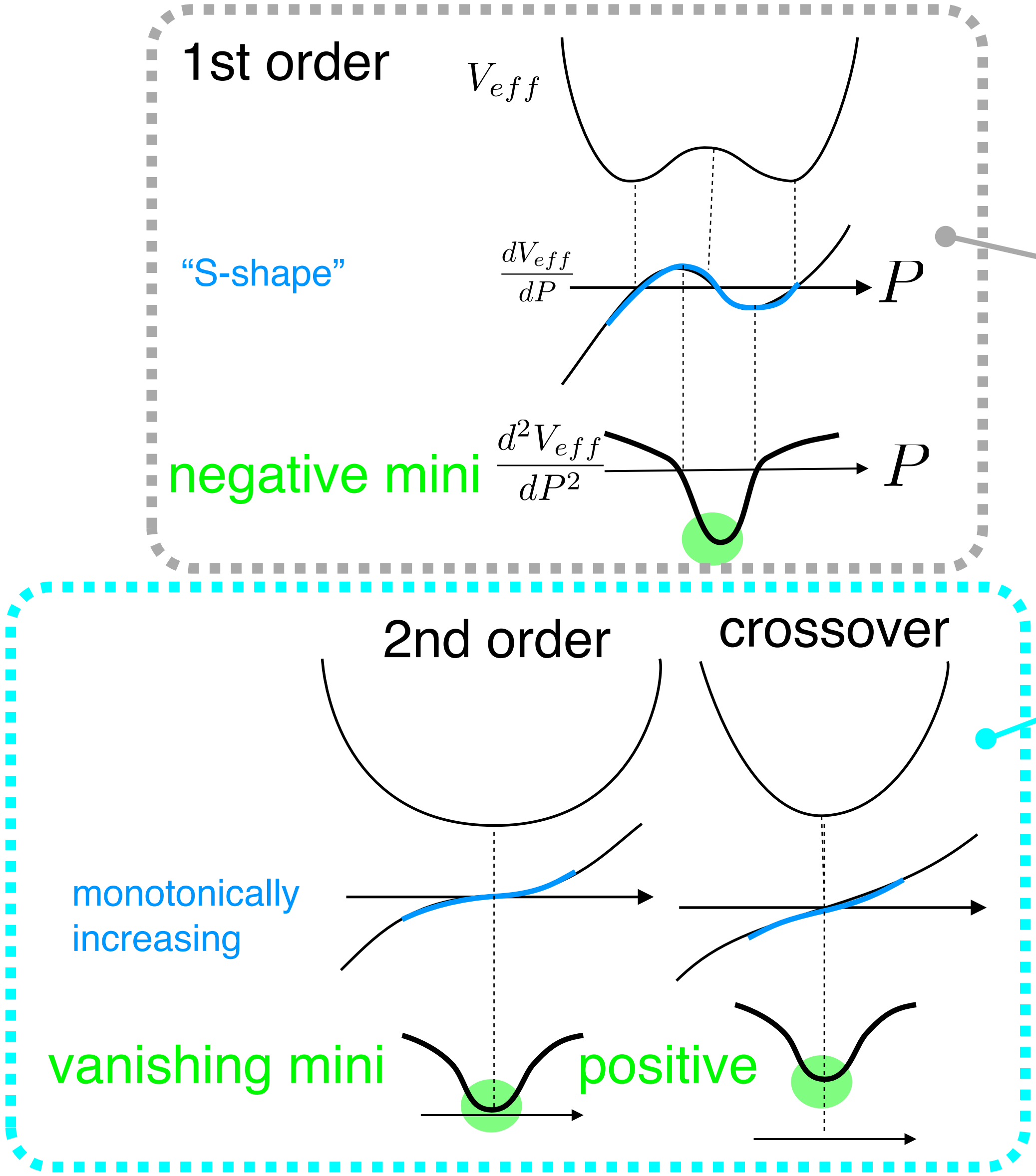
- To identify the critical surface in heavy quark region, we employ the histogram method combining with reweighting technique.
- The effect of the complex phase on the potential for investigation of critical point in the heavy quark region : ***small !!***
- Critical surface at finite μ
- Region of the 1st order transition shrinks as μ/T increases
- Future prospect : application to the light quark region

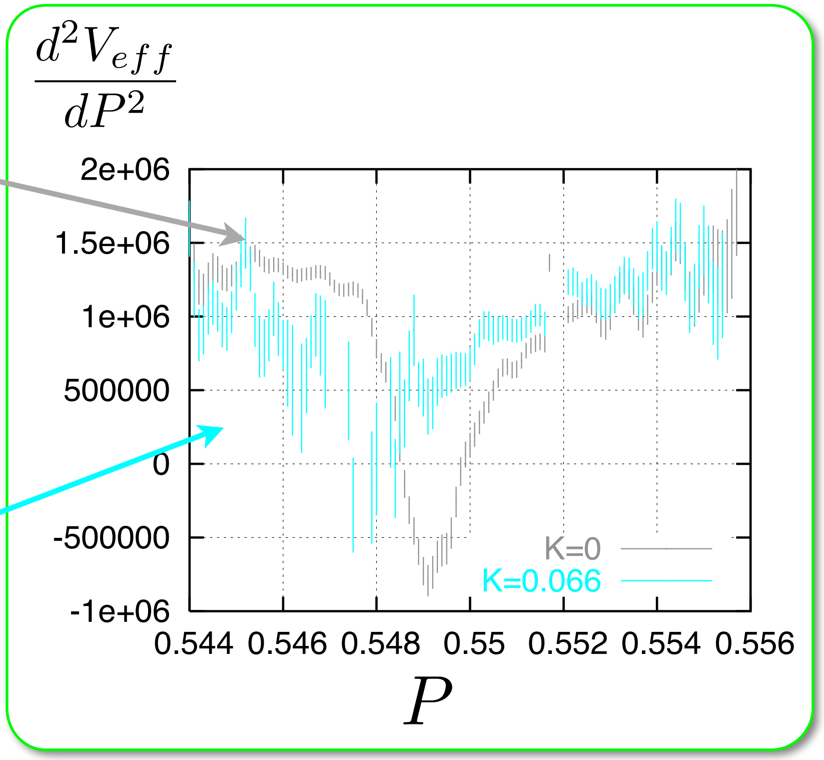


Backup slide

Our previous study at $\mu = 0$

- ***Advantage***
  : easy to identify the order of the transition





discriminate between them
⇒ a critical point

Our previous study at $\mu = 0$

Latent heat as a thermal observables

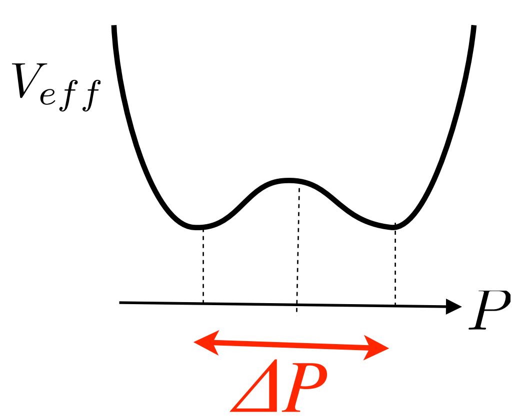
- ✦ Formulation : $$\frac{\Delta\epsilon}{T^4} \approx -6N_t^4 a \frac{\partial\beta}{\partial a} \langle \Delta P \rangle$$

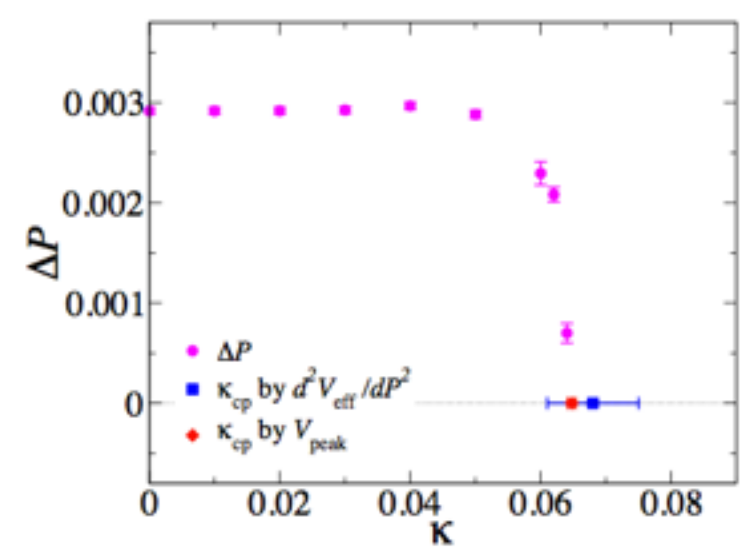
  $\beta = 6/g^2$: coupling, a : lattice spacing

  $a\dfrac{\partial\beta}{\partial a}$: beta function of coupling

✳ **beta function**
✳ **gap of plaquette**

- ✦ Gap ΔP : distance between two minima of $V_{\mathrm{eff}}(P)$



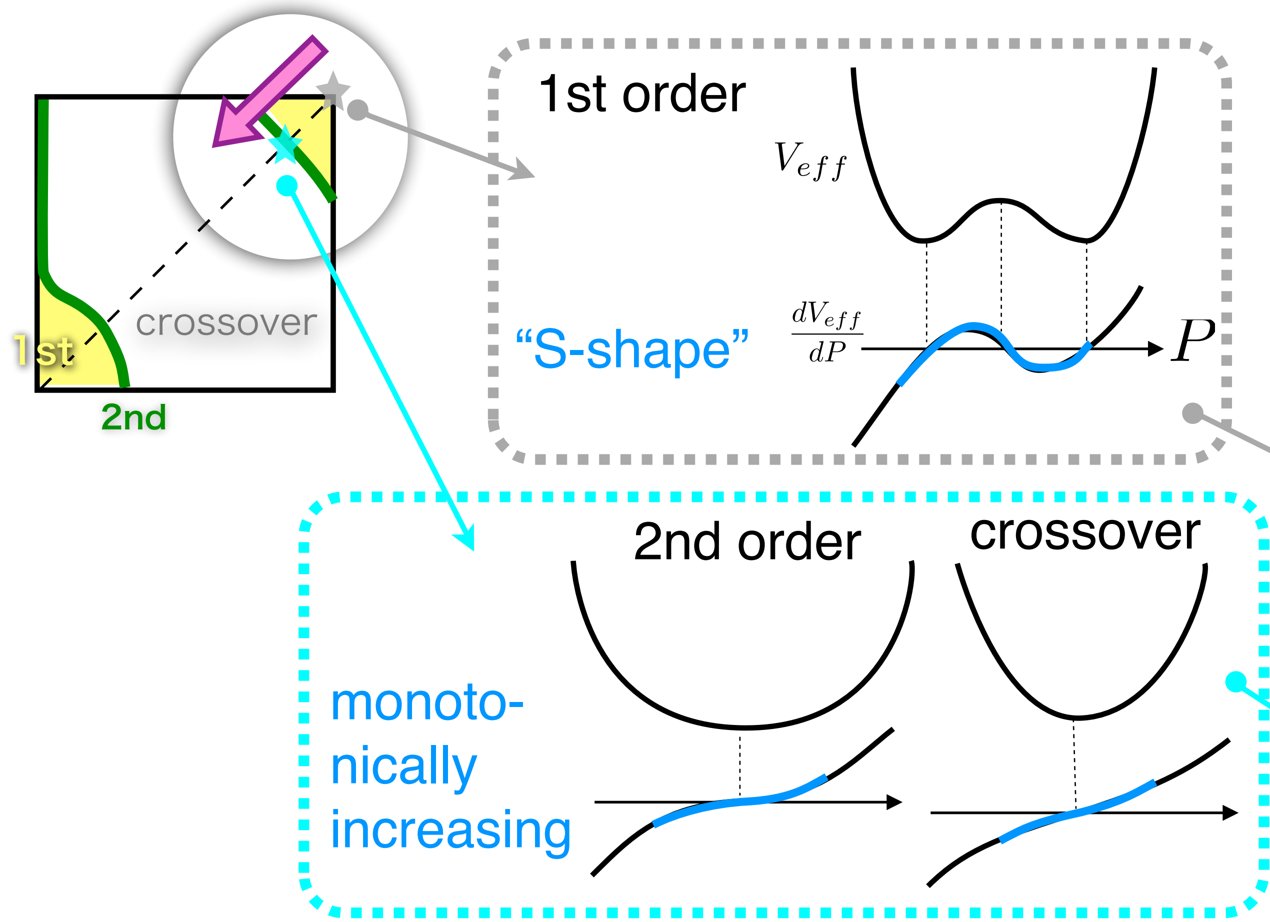


<u>ΔP vanishes</u>
<u>⇒ The latent heat vanishes</u>

Our previous study at $\mu = 0$

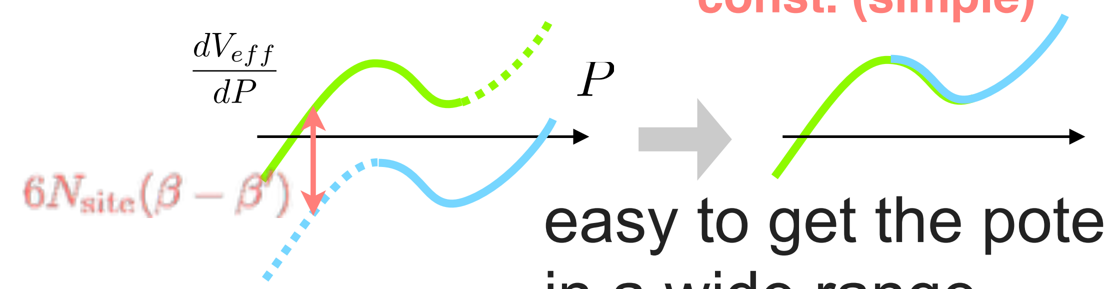
Advantage : easy to change values of parameters

Ex.) Derivative of the potential

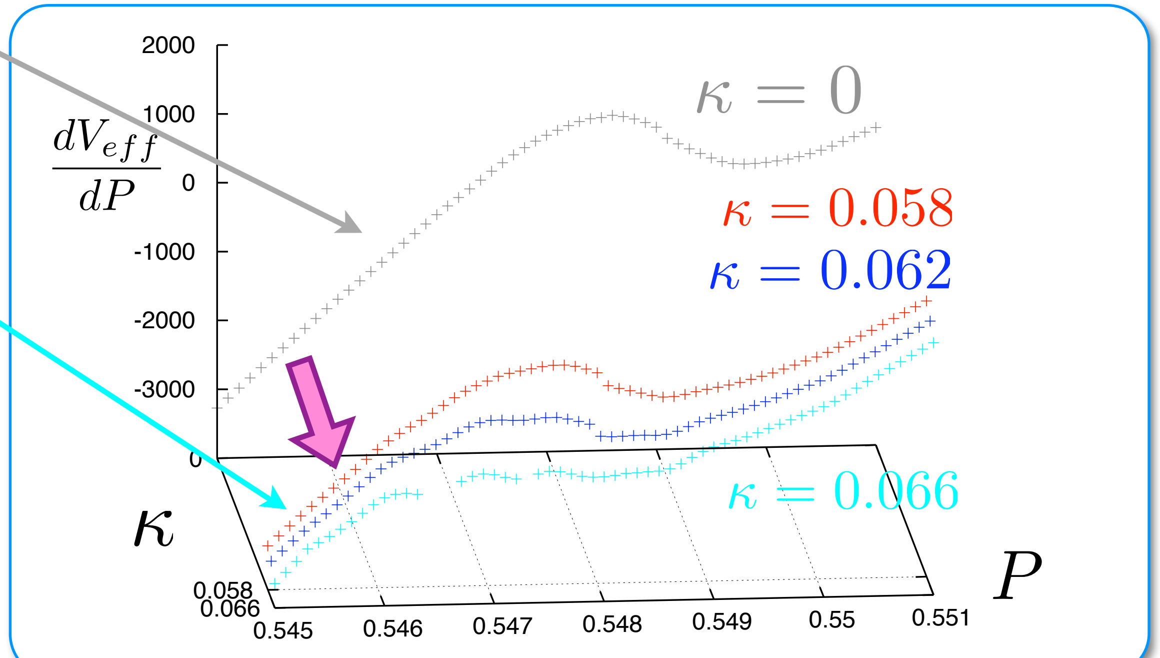


$$\frac{dV_0(P,\beta)}{dP} = \frac{dV_0(P,\beta')}{dP} - 6N_{\rm site}(\beta-\beta')$$

const. (simple)

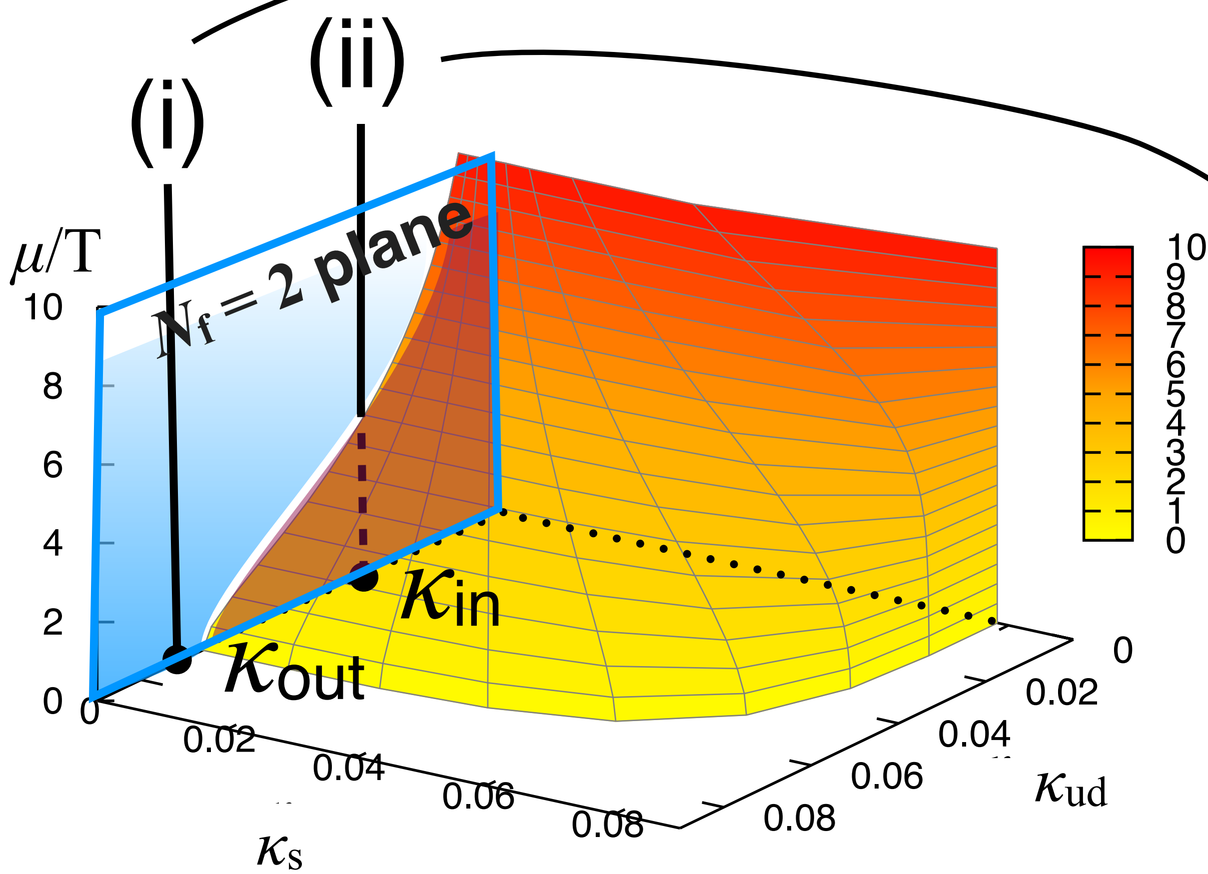


easy to get the potential in a wide range

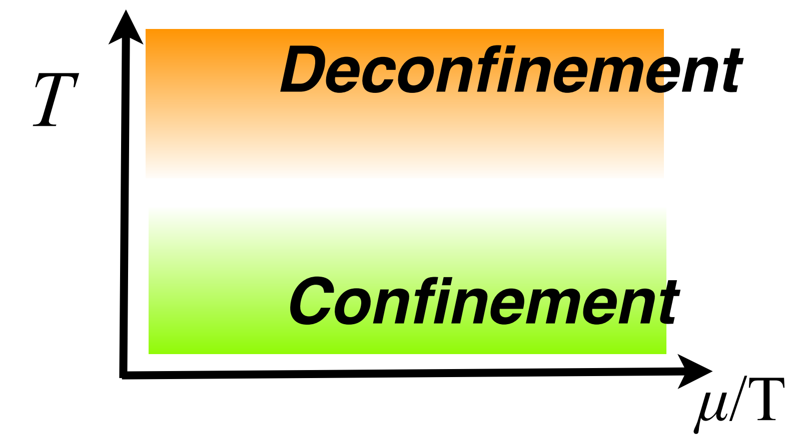


Phase structure in the heavy quark region

- Ex. Two phase structures for $N_f = 2$

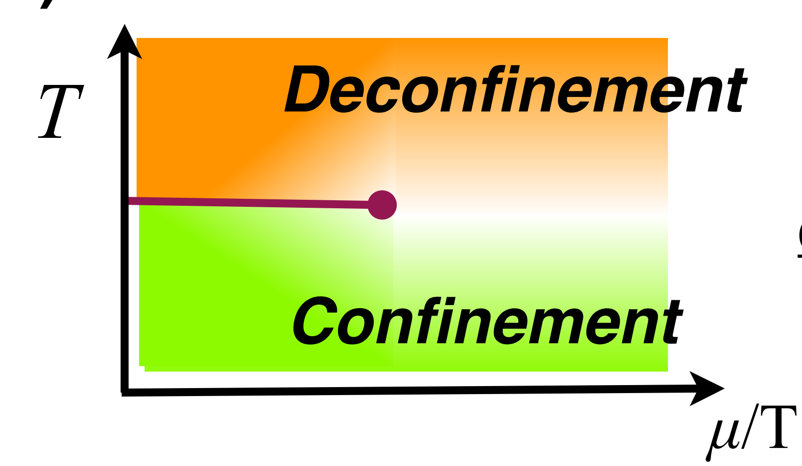


(i) $\kappa = \kappa_{out}$



no 1st order transition

(ii) $\kappa = \kappa_{in}$



1st order transition disappears at non-zero μ/T