Gauge corrections to the Strong Coupling limit: Numerical results

Wolfgang Unger, Uni Frankfurt
with Philippe de Forcrand, Jens Langelage, Owe Philipsen

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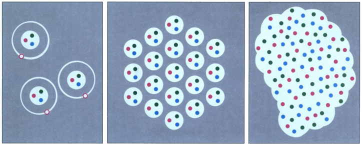


Strong Coupling Partition Function

Exact rewriting after Grassmann integration: Mapping to a MDP representation:

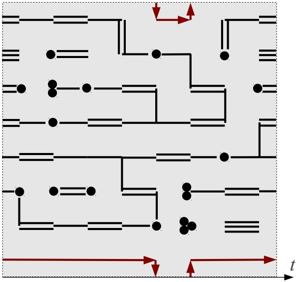
$$Z_F(m_q,\mu,\gamma) = \sum_{\{k,n,\ell\}} \underbrace{\prod_{b=(x,\mu)} \frac{(N_c-k_b)!}{N_c!k_b!}\gamma^{2k_b\delta_{\mu 0}}}_{\text{meson hoppings } M_x M_y} \underbrace{\prod_x \frac{N_c!}{n_x!}(2am_q)^{n_x}}_{\text{chiral condensate } \bar{\chi}\chi} \underbrace{\prod_\ell w(\ell,\mu)}_{\text{baryon hoppings } \bar{B}_x B_y}$$

$$k_b \in \{0,\dots N_c\},\ n_x \in \{0,\dots N_c\},\ \ell_b \in \{0,\pm 1\}$$

- Grassmann constraint:

$$n_x + \sum_{\hat{\mu}=\pm\hat{0},\dots\pm\hat{d}} \left(k_{\hat{\mu}}(x) + \frac{N_c}{2}|\ell_{\hat{\mu}}(x)|\right) = N_c$$

- weight $w(\ell,\mu)$ and sign $\sigma(\ell) \in \{-1,+1\}$ for oriented baryonic loop ℓ depends on loop geometry



finite quark mass

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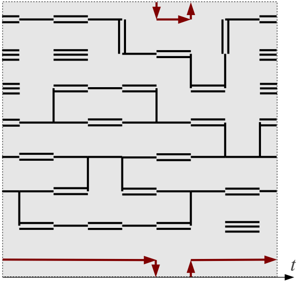

chiral limit: monomers absent

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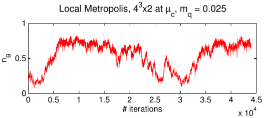
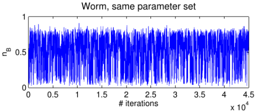
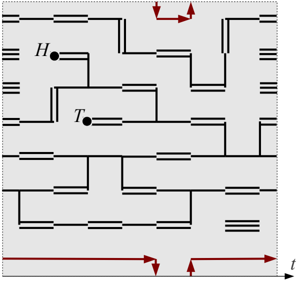
$$k_b \in \{0,\dots N_c\},\ n_x \in \{0,\dots N_c\},\ \ell_b \in \{0,\pm 1\}$$

- Worm algorithm [Prokof'ev & Svistunov 2001]: sampling 2-monomer sector (for U(3): [Adams & Chandrasekharan, 2003])
- SU(3): Worm both in mesonic and baryonic sector
- fast, particularly in the chiral limit







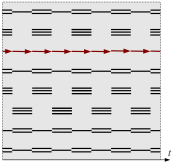
during Worm evolution

Journey through strong coupling phase diagram

Strong Coupling Limit:

- finite temperature chiral transition takes place when **spatial dimers vanish**
- nuclear and chiral transition **coincide**: $\langle \bar{\chi}\chi \rangle$ vanishes as baryonic crystal forms

$\mu = 0,\, T \gg T_c$



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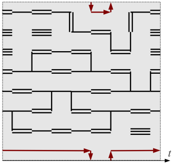
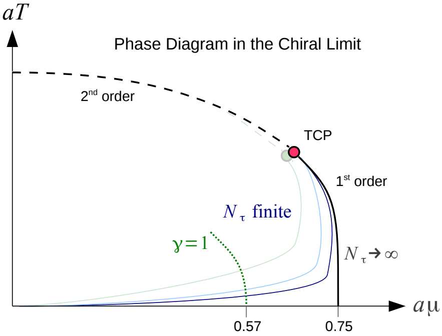
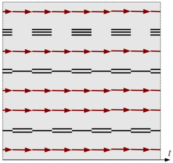
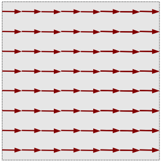




$T \gg T_c,\, \mu > \mu_c$



$T = 0,\, \mu > \mu_c,\, \beta = 0$

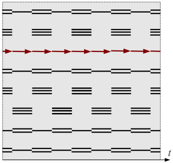


Journey through strong coupling phase diagram

Finite β:

- chiral transition weakens as β increases
- $\langle \bar{\chi}\chi \rangle$ can be non-zero even though baryonic crystal has formed

$\mu = 0$, $T \gg T_c$



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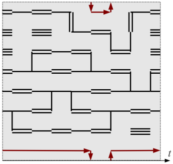
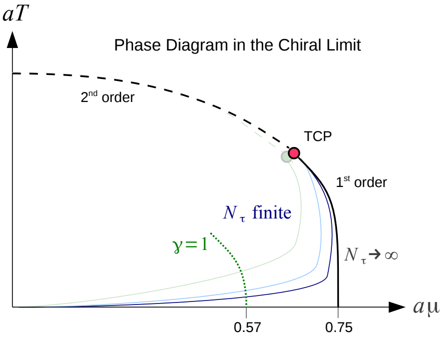
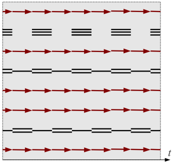




$T \gg T_c$, $\mu > \mu_c$



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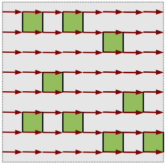

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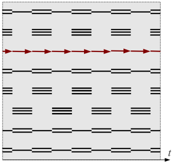

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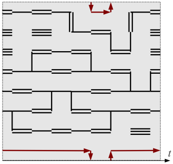

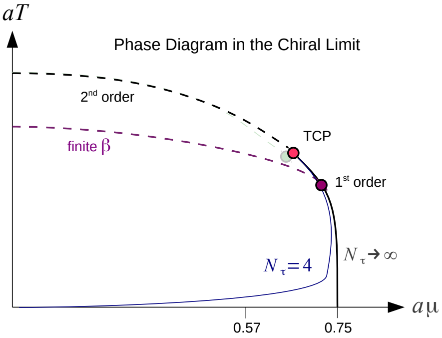

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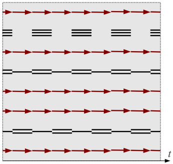

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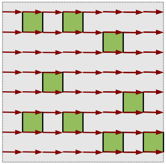

Journey through strong coupling phase diagram

Finite β:

- Want to study variation of phase boundary with β.
- Strategy: scan in polar coordinates $(aT, a\mu) \mapsto (\rho, \phi)$:

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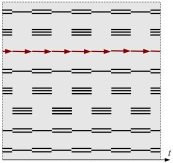

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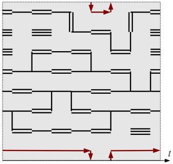

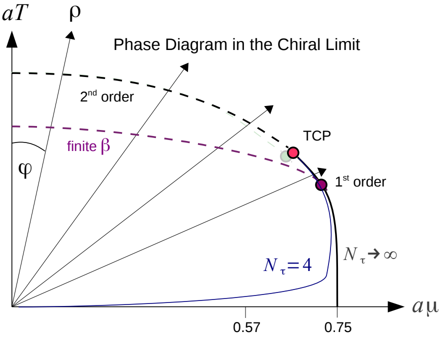

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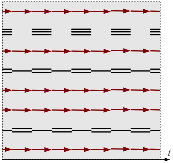

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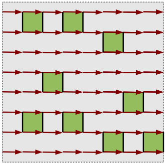

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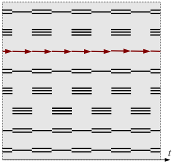

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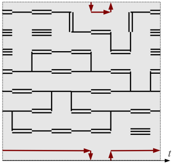

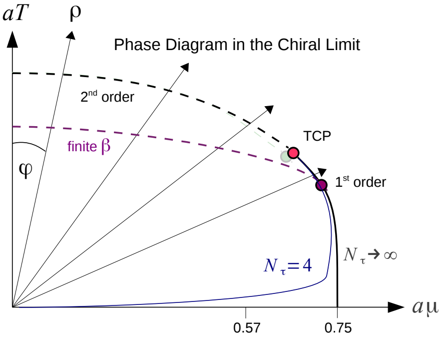

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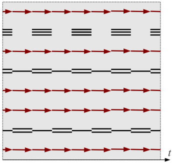

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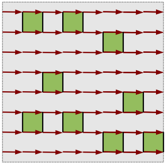
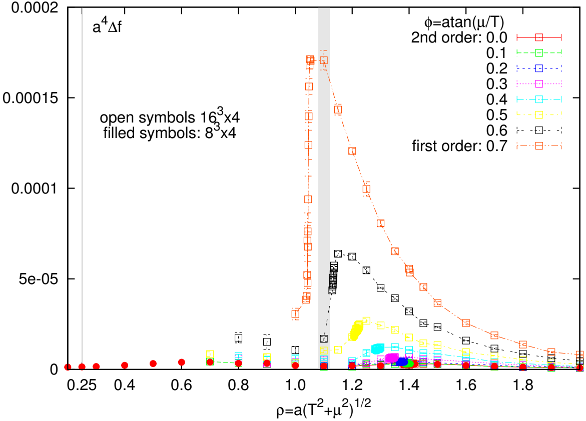


Severity of the Sign Problem

- sign problem is **mild across the 2nd order phase boundary**, large volumes possible
- along 1st order boundary, sign problem gets stronger, smaller volumes suffice

- $\langle \mathrm{sign} \rangle \sim e^{-\frac{V}{T}\Delta f(\mu)}$
- $16^3 \times 4$: $\langle \mathrm{sign} \rangle \approx 0.1$ at tricritical point
- Δf decreases with N_τ, vanishes in continuous time limit



$\mathcal{O}(\beta)$ effective action

- QCD Partition function in terms of systematic expansion in β:

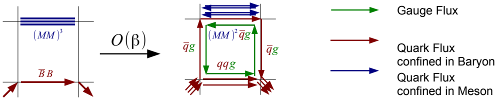
$$Z_{QCD} = \int d\chi d\bar{\chi} dU e^{S_G+S_F} = \int d\chi d\bar{\chi} Z_F \left\langle e^{S_G} \right\rangle_{Z_F}$$

- approximate gauge action by a product of single plaquettes ($\rightarrow$ plaquette-disconnected diagrams)

$$\left\langle e^{S_G} \right\rangle_{Z_F} \approx \prod_P \left\langle 1 + \frac{\beta}{2N_c} \mathrm{tr}[U_P + U_P^\dagger] \right\rangle_{Z_F}$$

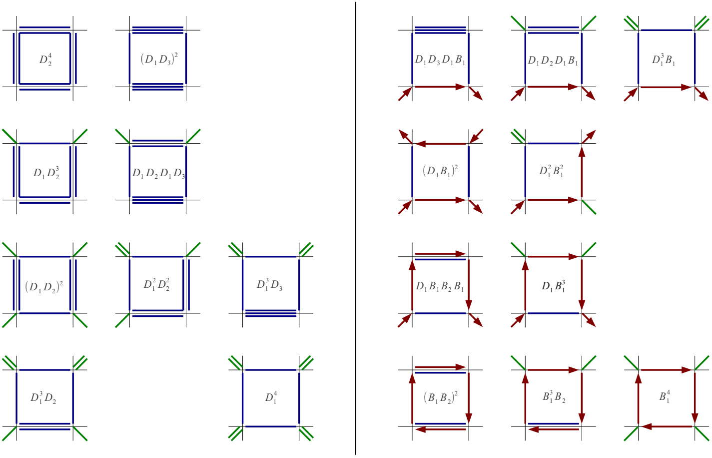
- need to evaluate plaquette expectation value before Grassmann integration:

$$\left\langle \mathrm{tr}[U_P + U_P^\dagger] \right\rangle_{Z_F} = \frac{1}{Z_F} \int dU \mathrm{tr}[U_P + U_P^\dagger] e^{S_F} = \prod_{l \in P} z_l^{-1} \sum_{s=1}^{19} F_P^s(M, B, \bar{B})$$



Classification of $\mathcal{O}(\beta)$ Diagrams

Diagrams classified by external legs (monomers or external dimers)



mesonic sector

baryonic sector

Link Integrations for $\mathcal{O}(\beta)$ diagrams

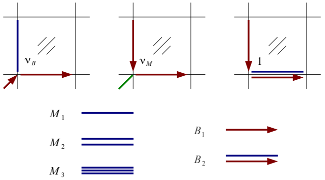
One-Link integrals for links on the edge of an elementary plaquette
(based on techniques from [Creutz 1978], [Azakov & Aliev 1988]):

$$J_{ij} = \underbrace{\sum_{k=1}^{N_c} \frac{(N_c-k)!}{N_c!(k-1)!}(M_\chi M_\varphi)^{k-1}\bar{\chi}_j\varphi_i}_{M_k} - \underbrace{\frac{1}{N_c!(N_c-1)!}\epsilon_{ii_1i_2}\epsilon_{jj_1j_2}\bar{\varphi}_{i_1}\bar{\varphi}_{i_2}\chi_{j_1}\chi_{j_2}}_{B_1} - \underbrace{\frac{1}{N_c}\bar{B}_\varphi B_\chi \bar{\chi}_j\varphi_i}_{B_2}$$

- from this: plaquette link product $P = \mathrm{Tr} J_{ik}J_{kl}J_{lm}J_{mi}$, or Polyakov/Wilson loops
- modifications can be summarized via **site weights ν and link weights ρ:**

$$\nu_M = (N_c-1)!, \quad \nu_B = N_c!$$

$$\rho_{M_k} = k, \quad \rho_{B_1} = \frac{1}{(N_c-1)!}, \quad \rho_{B_2} = (N_c-1)!$$



- B_1 and B_2 represent **color singlets qqg and $B\bar{q}g$**
- introduce a new set of variables, the **plaquette occupation numbers $q_P \in \{0,1\}$**
- $q_P = 1$ enforces site numbers $q_x = 1$ at its 4 corners, and along its 4 edges bond numbers $q_b = 1$ for unoriented bonds and $q_B = 1$ for oriented link states B_1, B_2

Plaquette Partition Function at $\mathcal{O}(\beta)$

- Writing the partition function in integer variables:

$$Z = \int d\chi d\bar{\chi} Z_F \prod_P \left(1 + \frac{1}{g^2} \left(\prod_{l \in P} z_l \right)^{-1} \sum_{s=1}^{19} F_P^s + \dots \right) = \sum_{\{k,n,\ell,q\}} \prod_{b=(x,\mu)} \hat{w}_b \prod_x \hat{w}_x \prod_\ell \hat{w}_\ell \prod_P w_P$$

$$\hat{w}_x = w_x \nu_i^{q_x} \qquad \hat{w}_b = w_b \rho_{M_k}^{q_b}, \qquad \hat{w}_\ell = w_\ell \prod_{B_j \in \ell} \rho_{B_j}^{q_B}, \qquad w_P = \frac{\beta}{2N_c}^{-2q_P}$$

- modified Grassmann constraint:

$$n_x + \sum_{\hat{\mu}=\pm\hat{0},\dots\pm\hat{d}} \left(k_{\hat{\mu}}(x) + \frac{N_c}{2} |\ell_{\hat{\mu}}(x)| \right) = N_c + q_x$$

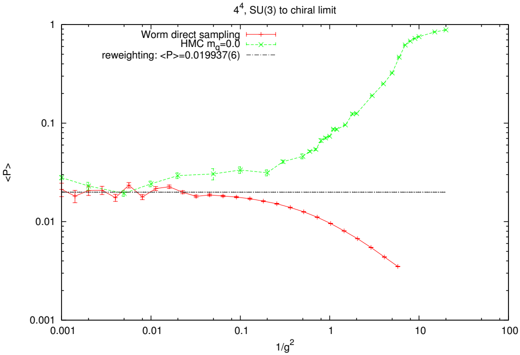
- chiral limit: still keep q_x, only $w_x = 1$ ($n_x = 0$)
- anisotropic lattice: distinguish spatial and temporal plaquettes, $w_P^{(s,t)} = \frac{\beta_{s,t}}{2N_c}^{-2q_P^{(s,t)}}$

Direct Sampling

- Sampling plaquette occupation number at finite β via additional **Metropolis update:** $q_P \to 1 - q_P$

$$\langle P \rangle = \frac{2}{Vd(d-1)} \frac{\partial}{\partial \beta} \log(Z) = \frac{1}{\beta} \langle n_P \rangle \,, \quad n_P = \frac{2}{Vd(d-1)} \sum_P q_P$$

- **saturation expected:** $\langle n_p \rangle \leq \frac{N_c}{2d(d-1)}$ (at most N_c adjacent plaquettes)



- due to missing pure gauge sector:

$$\boxed{\langle P \rangle \to 0 \quad \text{for} \quad \beta \to \infty}$$

- numerical results show indeed saturation of $\langle n_p \rangle$
- direct sampling not optimal (noise, systematic errors)

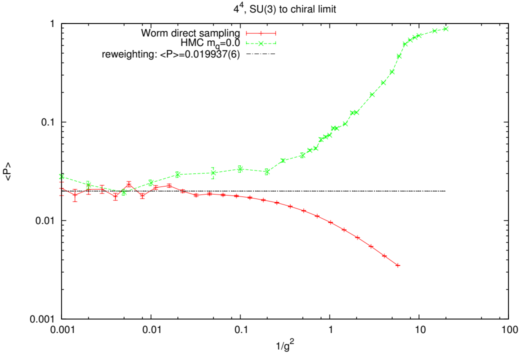
Reweighting

- Reweighting to finite β from the SC-Ensemble:

$$\langle P \rangle |_{\beta=0} = \frac{Z_P}{Z}$$

  with Z_P the one-plaquette sector, $\sum_p q_p = 1$.
- determine weight to update $Z \to Z_P$ with detailed balance satisfied (non-trivial for anisotropic lattice)

- reweighting is much less noisy
- in $\mathcal{O}(\beta)$ truncation scheme: $\langle n_p \rangle \propto \beta \Rightarrow \langle P \rangle = \mathrm{const}$
- extends to various other observables:
  - gauge observables: $\mathcal{O}(\beta^0)$
  - fermionic observables: $\mathcal{O}(\beta)$

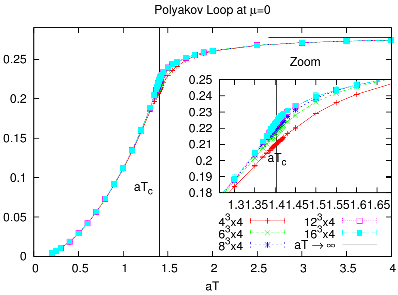


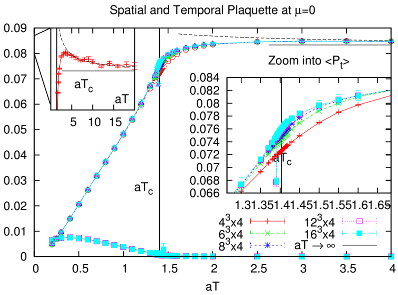
Gauge Observables at Finite Temperature

- Polyakov loop expectation value: ratio of partition function w/o static quark Q, measured via:

$$\langle L\rangle = \frac{\int d\bar{\chi}d\chi\langle L\rangle_{SC}Z_F}{\int d\bar{\chi}d\chi Z_F} = \frac{Z_Q}{Z}, \qquad L(\vec{x}) = \mathrm{Tr}[J_{N_\tau,1}(\vec{x})\prod_{t=1}^{N_\tau} J_{t,t+1}(\vec{x})]$$

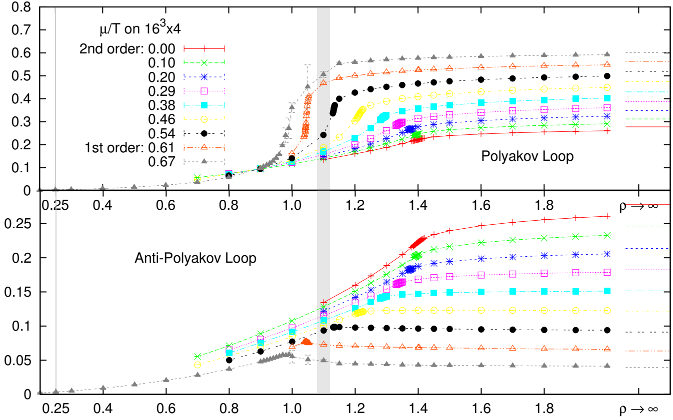
- $\langle L\rangle$ and $\langle P\rangle$ are sensitive to the chiral transition
- $\langle L\rangle$ rises, cusp is imprint of chiral transition rather than deconfinement transition





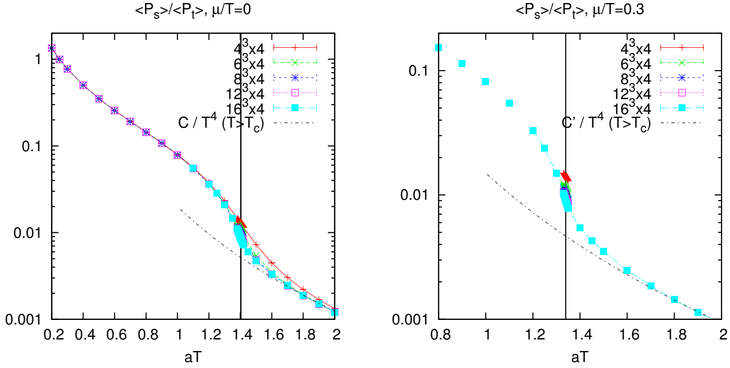
Gauge Observables at Finite Density

- Polyakov loop develops gap and Anti-Polyakov loop develops cusp as the transition turns 1st order
- large μ and/or T limit $\rho \to \infty$ analytically computed



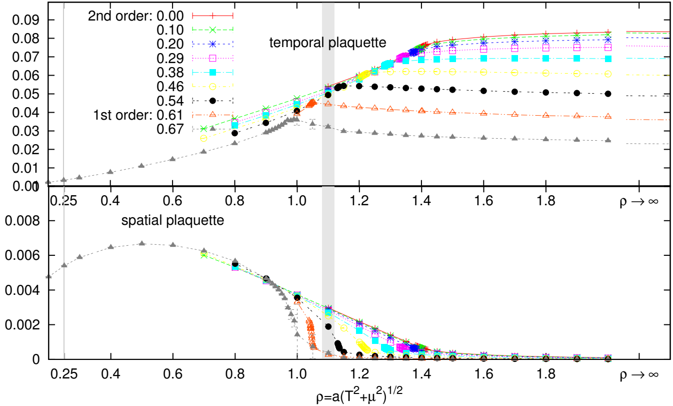
Gauge Observables at Finite Density

- suppression of spatial plaquettes: pairs of parallel spatial dimers are rare at high T
- plaquette weight is nonzero only if non-trivial parallel pair of dimers/flux is present
- additional anisotropy in gauge couplings β_s, β_t
- in strong coupling regime: $\gamma^2 \approx \frac{a_s}{a_t} \approx \sqrt{\frac{\beta_s}{\beta_t}}$



Gauge Observables at Finite Density

- in strong coupling regime, anisotropy in β linked to γ: $\beta_s = \beta\gamma^{-2}, \quad \beta_t = \beta\gamma^2$
- anisotropy can be absorbed into observable: $\beta_s P_s = \beta(\gamma^{-2} P_s)$, $\beta_t P_t = \beta(\gamma^2 P_t)$



Chiral susceptibility in the chiral limit

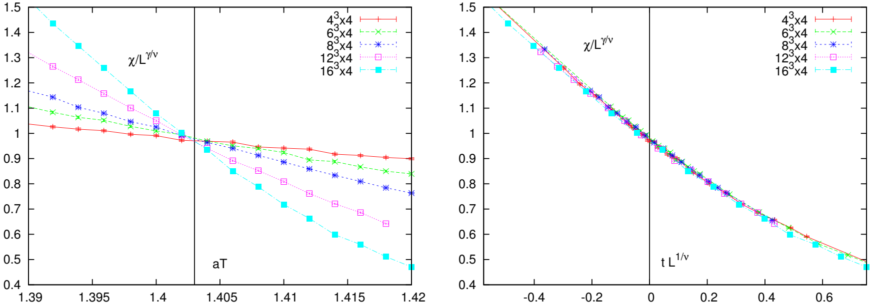
Full chiral susceptibility: $\chi = \frac{1}{V}\frac{\partial^2}{\partial(2am_q)^2}\log Z$ can be expressed in terms of monomers:

$$\chi = \frac{1}{(2am_q)^2 L^3 N_t}\left(\left\langle N_M^2\right\rangle - \langle N_M\rangle^2 - \langle N_M\rangle\right) = \frac{1}{L^3 N_t}\left(\sum_{x_1,x_2} G(x_1,x_2) - \frac{\langle N_M\rangle^2}{(2am_q)^2}\right)$$

In chiral limit:

- $\chi \sim \left\langle(\bar\psi\psi)^2\right\rangle$ is measured with high precision via Worm estimator $G(x_1,x_2)$
- χ has no peak, FSS via: $\chi_L/L^{\gamma/\nu}(t) = A + BtL^{1/\nu}$, $\quad t = \frac{T-T_c}{T_c}$

  with γ, ν **3d O(2) critical exponents**



Taylor Expansion for the Susceptibility

For **fermionic observables**, the first derivative w.r.t β can be measured:

- obtain the slope of the transition temperature from a Taylor coefficient:

$$\chi(\beta) = \chi_0 + \beta c_\chi + \mathcal{O}(\beta^2) \quad \text{with} \quad \chi_0 = \frac{Z_2}{Z},$$

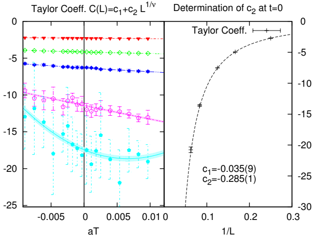
$$c_\chi = \frac{\partial}{\partial\beta} \frac{Z_2(\beta)}{Z(\beta)}\bigg|_{\beta=0} = 3N_s^3 N_t \left(\left\langle (\bar\psi\psi)^2 P \right\rangle - \left\langle (\bar\psi\psi)^2 \right\rangle \langle P \rangle \right)$$

- Z_2: 2-monomer sector sampled by $G(x_1, x_2)$ via Worm algorithm,
- necessary condition: c_χ needs to obey **finite size scaling** to modify aT_c
- in the thermodynamic limit:

$$\frac{c_\chi}{\chi_0} \simeq c_2 L^{1/\nu} \quad \text{in the vicinity of} \quad t = 0$$

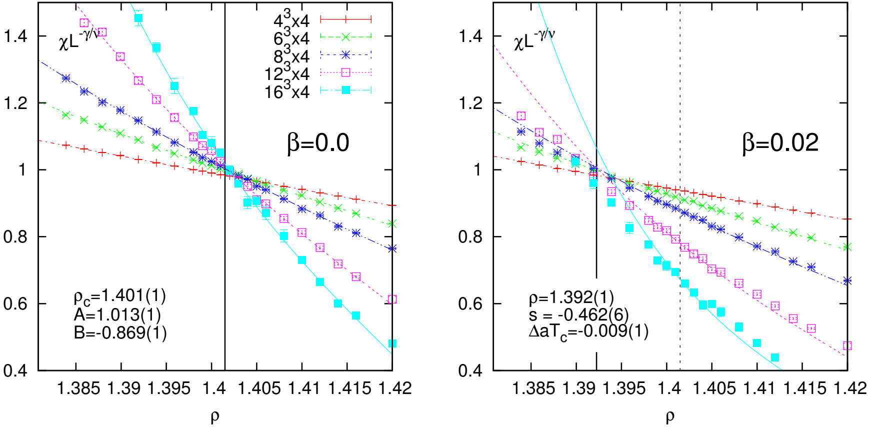
- the **slope of T_c w.r.t β** is related to scaling function parameters A, B and c_2:

$$\frac{\partial}{\partial\beta} aT_c(\beta)|_{\beta=0} = aT_c \frac{A}{B} c_2$$



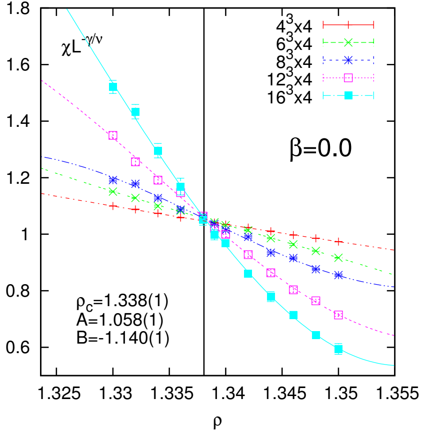
Results on the Slope at Zero and non-Zero Density

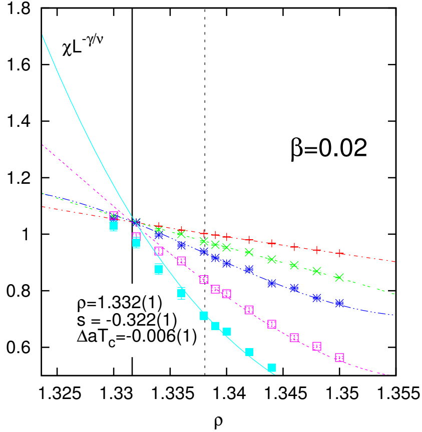
- We obtain for the **slope**: $\frac{\partial}{\partial \beta} aT_c(\beta) \simeq -0.462(6)$ at $\mu = 0$



Results on the Slope at Zero and non-Zero Density

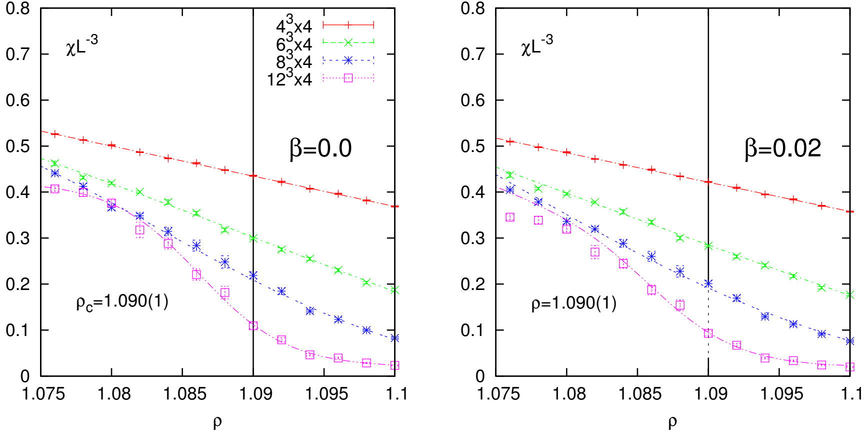
- We obtain for the **slope**: $\frac{\partial}{\partial\beta} aT_c(\beta) \simeq -0.322(1)$ at $\mu/T = 0.29$





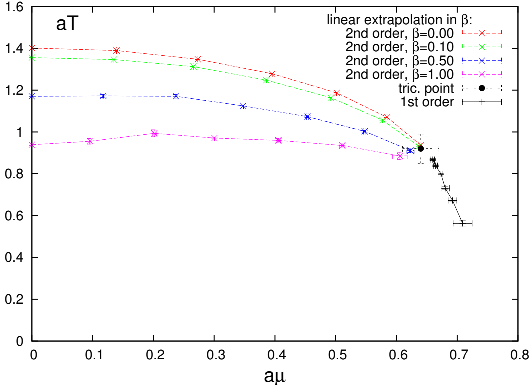
Results on the Slope at Zero and non-Zero Density

- No **slope** at $\mu/T = 0.58$: First order transition, no shift in aT_c (determined via Borgs-Kotecky ansatz)



Corrections to the SC-Phase diagram

Modification of the phase boundary in the $\mu - T$ plane as a function of β:



Ratio $\frac{T_c(\mu=0)}{3\mu_c(T=0)}$:

- too large in the strong coupling limit: $\frac{T_c}{3\mu_c} \approx \frac{1.403}{1.71} = 0.82$
- compare with $m_q = 0$ continuum estimate $\frac{T_c}{3\mu_c} \approx \frac{154\,\mathrm{MeV}}{0.93\,\mathrm{GeV}} = 0.165$
- but: $\frac{T_c}{3\mu_c} \searrow \quad (\beta \nearrow)$

The slope $\frac{\partial}{\partial\beta} aT_c(\beta)|_{\beta=0}$ **vanishes** at $(aT_t, a\mu_t)$ and along the first order line

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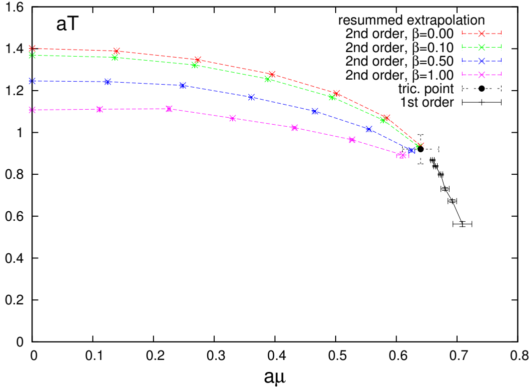

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Resummation: all disconnected plaquette diagrams considered

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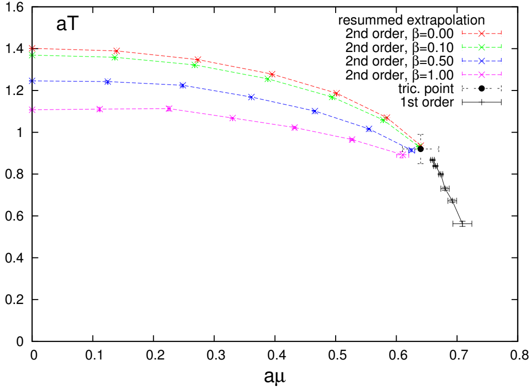

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Open question:
does the tricritical point move downwards along the first order line?

Corrections to the SC-Phase diagram

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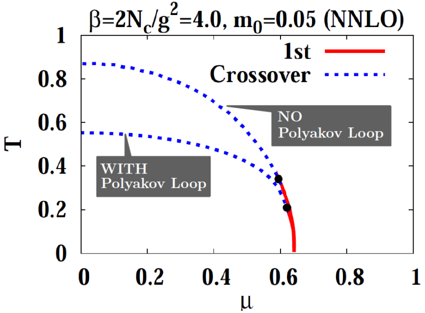

[Miura 2011, PoS (Lattice 2011) 318]

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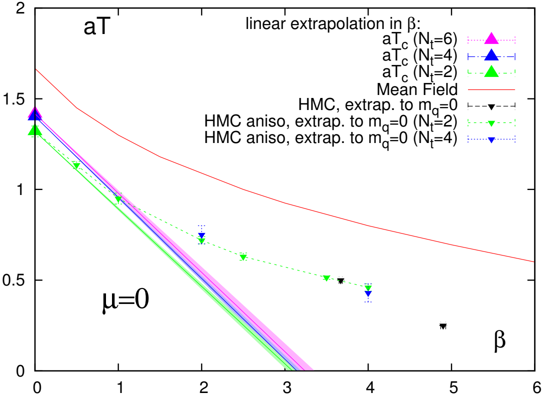
Compare to meanfield:
based on Polyakov effective action for gauge sector: μ_c does not vary

Crosscheck with HMC on anisotropic lattice

- weak N_τ-dependence of the slope $\frac{\partial}{\partial\beta} aT_c(\beta)|_{\beta=0}$
- crosscheck with HMC: simulations on anisotropic lattices, two types of anisotropies: in Dirac couplings: γ, in gauge action: β_s/β_t
- resummation improves result drastically
- comparison with mean field: qualitatively same behaviour



Comparison with mean field results by Miura *et. al*, Phys. Rev. D **80** (2009) 074034 and HMC data: **qualitative agreement after resummation**

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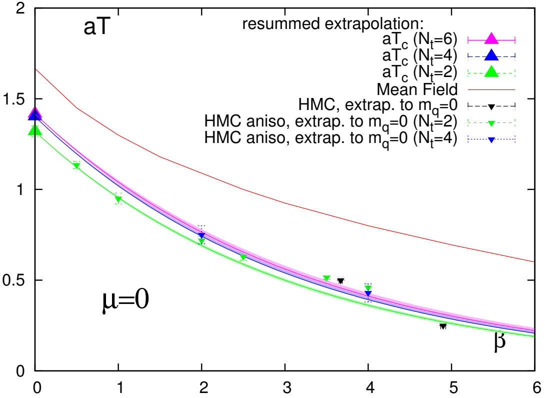

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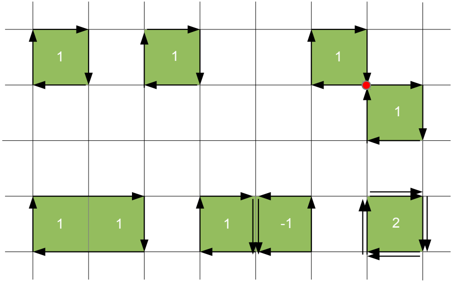
$\mathcal{O}(\beta^2)$ Corrections

At $\mathcal{O}(\beta^2)$, plaquette **orientations** are relevant! Five types of corrections:

1. disconnected plaquettes
2. 2x1 Wilson loops
3. two plaquettes sharing a site
4. two oppositely oriented plaquettes sharing a link
5. doubly occupied plaquette



Link integration possible,
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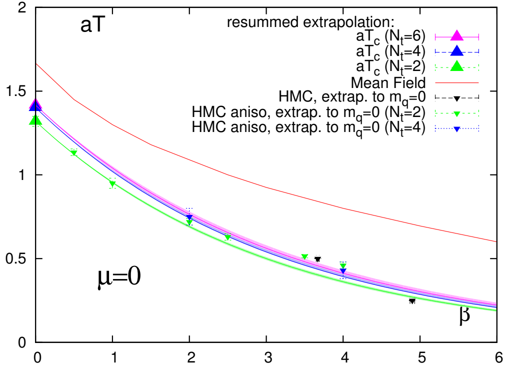

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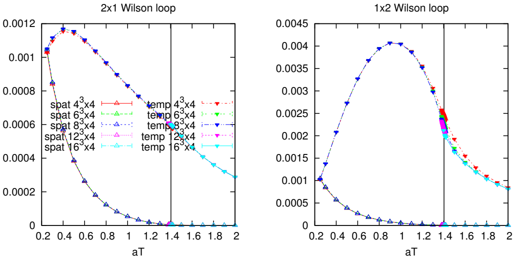

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Conclusions

Achievements:

- correct average plaquette and Polyakov loop at $\beta = 0$, high precision! (crosschecks with HMC performed)
- all measurements **extended to finite μ**
- $\langle L \rangle$ and $\langle P_s \rangle$ are sensitive to the chiral transition
- **slope $\frac{\partial}{\partial\beta} aT_c(\beta)$** determined at finite density up to the tricritical point
- modification of **phase boundary** obtained, aT_c decreases, $a\mu_c$ does not

Goals:

- complete $O(\beta^2)$ corrections to obtain complete curvature of fermionic observables and slope of gauge observables
- needed to obtain gauge corrections of the position of $(aT_t, a\mu_t)$ and first order line

Backup: SC-LQCD at finite temperature

How to vary the temperature?

- $aT = 1/N_\tau$ is discrete with N_τ even
- $aT_c \simeq 1.5$, i.e. ${N_\tau}^c < 2$ $\Rightarrow$ we cannot address the phase transition!

Solution: introduce an **anisotropy** γ in the Dirac couplings:

$$\mathcal{Z}(m_q, \mu, \gamma, N_\tau) = \sum_{\{k,n,l\}} \prod_{b=(x,\mu)} \frac{(3-k_b)!}{3!k_b!} \gamma^{2k_b\delta_{\mu 0}} \prod_x \frac{3!}{n_x!} (2am_q)^{n_x} \prod_l w(\ell, \mu)$$

Should we expect $a/a_\tau = \gamma$, as suggested at weak coupling?

- **No**: meanfield predicts $a/a_\tau = \gamma^2$, since $\gamma_c^2 = N_\tau \frac{(d-1)(N_c+1)(N_c+2)}{6(N_c+3)}$

  $\Rightarrow$ sensible, N_τ-independent definition of the temperature: $aT \simeq \frac{\gamma^2}{N_\tau}$

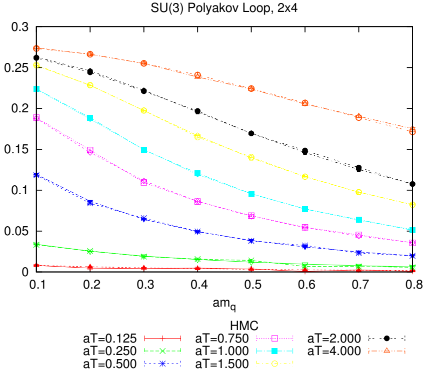
- Moreover, SC-LQCD partition function is a function of γ^2

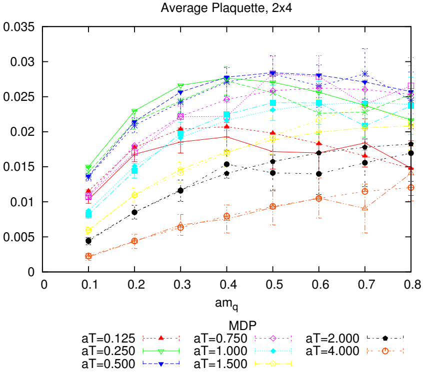
However: **precise correspondence between a/a_τ and γ^2 not known**

Crosschecks at Finite Temperature

Croscheck on small lattices:

- comparison between **HMC** and **MDP** algorithms agrees well
- gauge observables are correctly obtained for **various** am_q, aT:





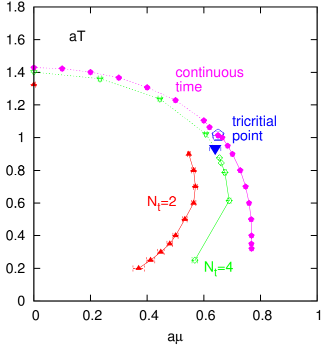
SC-LQCD Phase Diagram

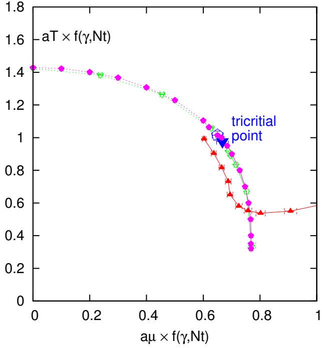
Comparison of phase boundaries for $N_\tau = 2, 4$ and $N_\tau \to \infty$ (continuous time), studied with Worm algorithm [hep-lat/1111.1434]

identifications:

$$aT = \frac{\gamma^2}{N_\tau}$$

$$a\mu = \gamma^2 a_\tau \mu$$





- behavior at low μ qualitatively the same, first order transition shifts to larger μ
- no re-entrance in continuous time
- collapse on unique phase boundary by rescaling with $f(\gamma, Nt)$

μ-Dependence of the Parameters

The slope gets smaller for increasing μ