Towards dense QCD and nuclear matter with an effective lattice theory

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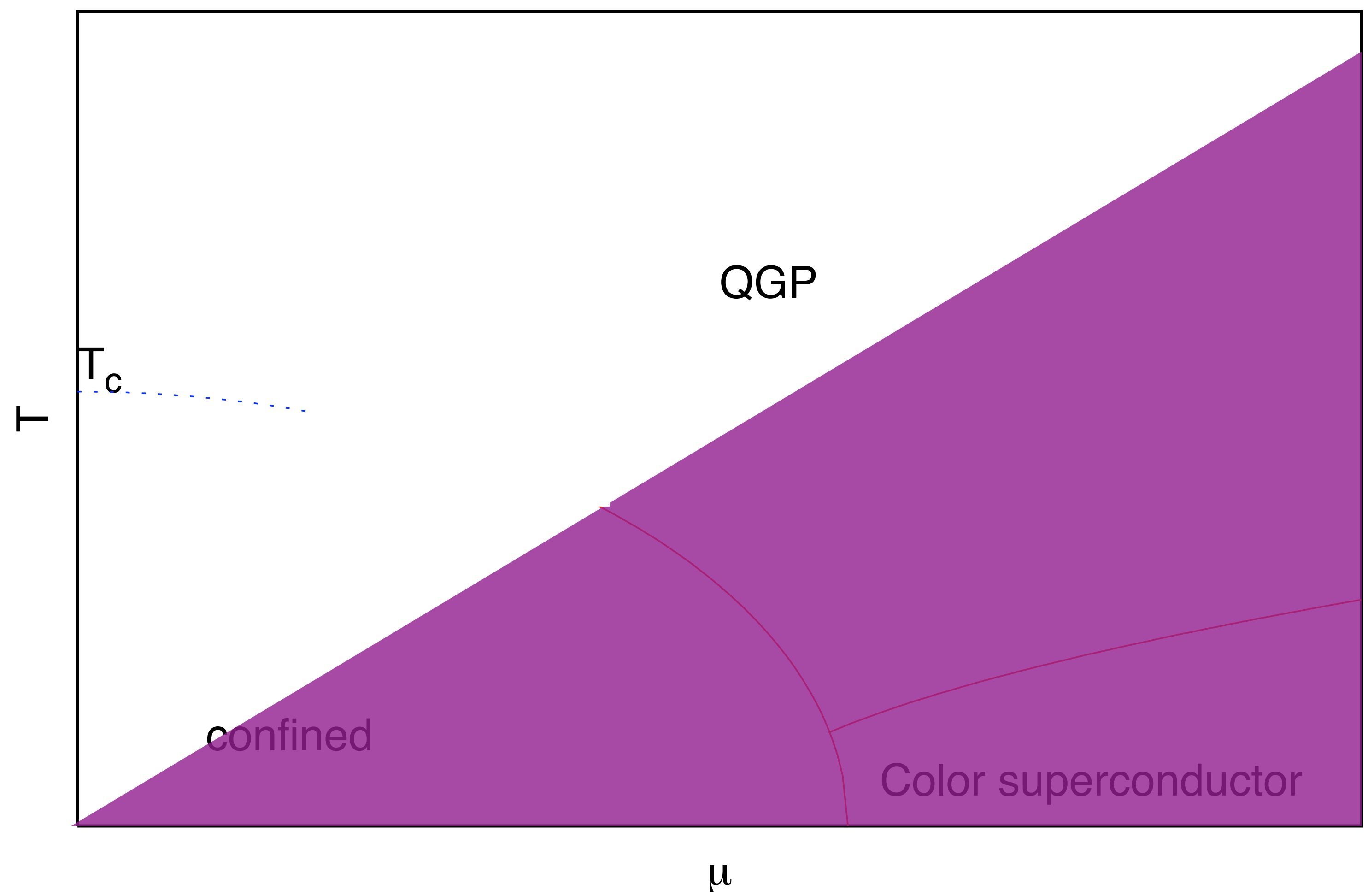
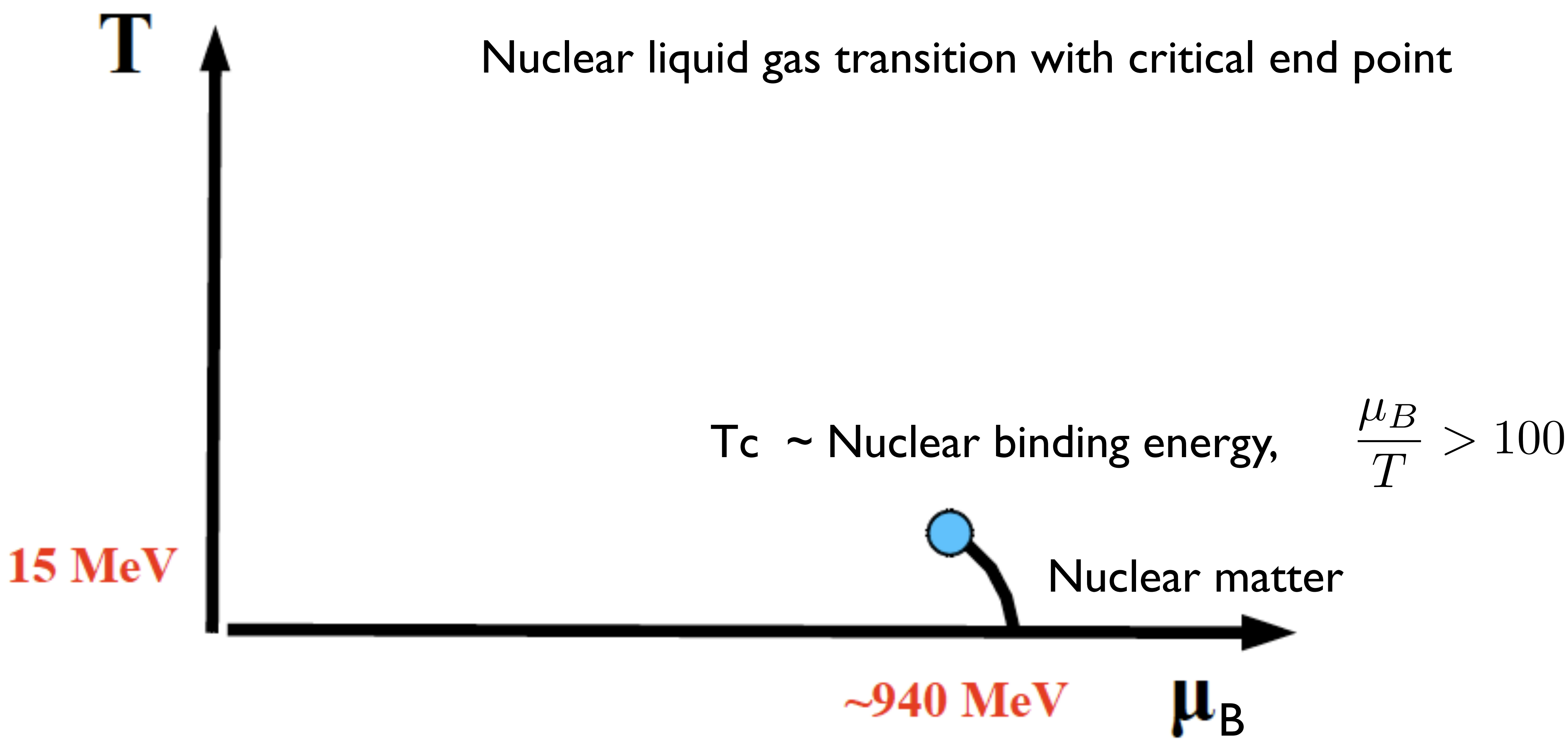
- Motivation for effective theory approach
- The deconfinement transition in QCD with heavy dynamical quarks JHEP 1201 (2012) 042
- Cold and heavy dense QCD: transition to nuclear matter PRL 110 (2013) 122001

The lattice calculable region of the phase diagram



- Sign problem circumvented by approximate methods:
  reweighting, Taylor expansion, imaginary chem. pot., need $\mu/T \lesssim 1 \quad (\mu = \mu_B/3)$
- No critical point in the fully controlled region (where different methods agree)
  Some signals on boundary and beyond; cold and dense region??

Goal: the experimentally established phase diagram



Motivation for an effective theory

- The sign problem, of course...
- Even without the sign problem: cold and dense is difficult!

Requirements: for nuclear liquid gas transition

$$\mu_B \sim m_B \ll a^{-1} \quad \Rightarrow \quad a \sim 0.1\mathrm{fm} \ll 0.2\mathrm{fm}$$

$$T = \frac{1}{aN_\tau} < 10MeV \quad \Rightarrow \quad N_\tau > 200$$

$$L > 3\mathrm{fm} \quad \Rightarrow \quad N_s > 32$$

Need vacuum subtractions with $N_\tau \gg 200$

Need variation in volume for FSS and lattice spacing for continuum limit!

Even without sign problem (or complex Langevin) this will be very hard! Sexty

The effective lattice theory approach

- Two-step treatment:

  I. Calculate effective theory analytically
  II. Simulate effective theory

- Step I.: split temporal and spatial link integrations: cf. Greensite, Langfeld, FRG,...

$$Z = \int DU_0 DU_i \; \det Q \; e^{S_g[U]} \equiv \int DU_0 e^{-S_{eff}[U_0]} = \int DL \; e^{-S_{eff}[L]}$$

  Spatial integration after analytic strong coupling and hopping expansion

- Result: 3d spin model of QCD
- Step II: sign problem milder: Monte Carlo, complex Langevin
- Numerical simulations in 3d without fermion matrix inversion, very cheap!

Wilson's lattice QCD as a 3d SU(3) spin model

Character and hopping parameter expansion: $u(\beta) = \frac{\beta}{18} + \ldots < 1, \quad \kappa = \frac{1}{2am+8}$

In general the model becomes (with $\bar{h}_i(\mu) = h_i(-\mu)$)

$$-S_{\text{eff}} = \sum_i \lambda_i(u,\kappa,N_\tau) S_i^{\text{S}} - 2N_f \sum_i \left[h_i(u,\kappa,\mu,N_\tau) S_i^{\text{A}} + \bar{h}_i(u,\kappa,\mu,N_\tau) S_i^{\dagger\text{A}} \right]$$

Couplings ordered parametrically, keep in order of appearance

Higher powers of loops are resummed into a determinant: fugacity, Pauli principle!

$$Z_{\text{eff}}(\lambda_1, h_1, \bar{h}_1; N_\tau) = \int [\mathrm{d}L] \Big(\prod_{<ij>} [1 + 2\lambda_1 \mathrm{Re} L_i L_j^*] \Big) \; e^{-V[L]}$$

cf. Feo, Seiler, Stamatescu $\longrightarrow$

$$\Big(\prod_x \underbrace{\det[(1 + h_1 W_x)(1 + \bar{h}_1 W_x^\dagger)]^{2N_f}}_{\equiv Q(L_x, L_x^*)^{N_f}} \Big)$$

x corrections

Simulation of the effective theory

Poster of M. Neuman

- Simulation with complex Langevin for real chemical potential
- Successful check of convergence criteria
  Aarts, Seiler, Stamatescu
- Log of fermion determinant has cut on negative real axis $\det = \exp(\mathrm{Tr} \log \det)$
  N.B.: here only static part of the determinant!

  Monitor that crossings of the negative real axis happen
  with probability $< 10^{-4}$ cf. Mollgaard, Splittorff
- Comparison with Monte Carlo on small volumes
- Works for all couplings $\beta = 0....6, \kappa = 0....0.12$

Reliability of effective theory

Talk J. Langelage, poster G. Bergner

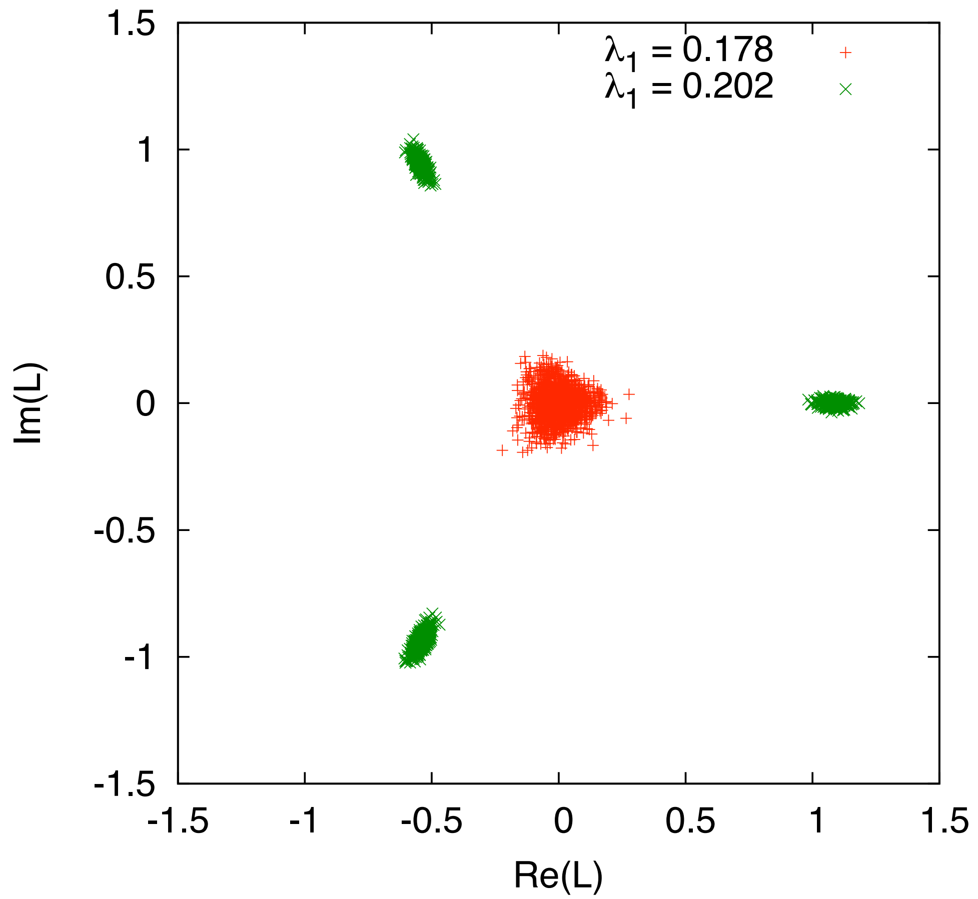
- Approximation with a few couplings not a cure for everything!
- Valid only in convergence region of the expansion
- Valid only for certain quantities:

  Mass spectra, length scales, long range correlations bad

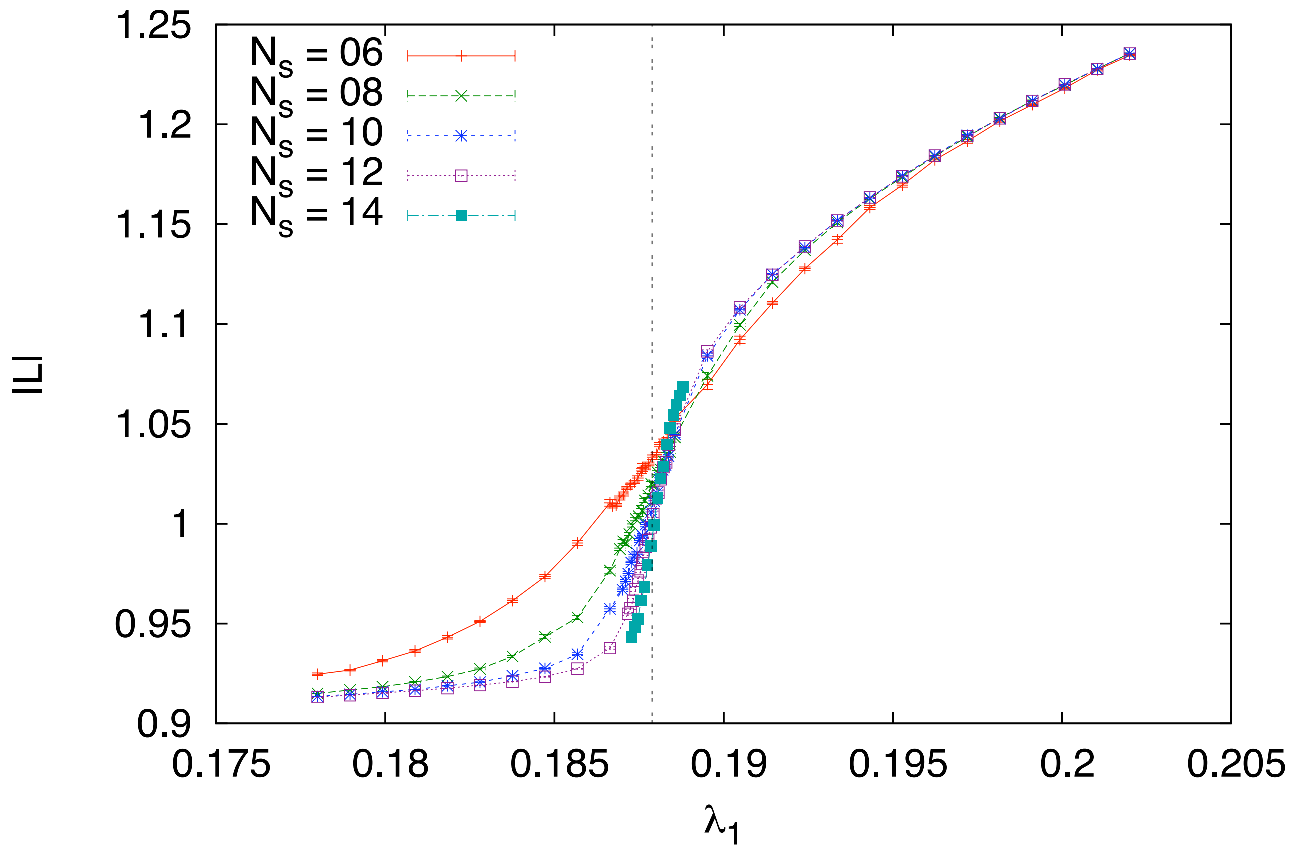
  Bulk thermodynamics, phase structure good

- In the range of validity: weak coupling perturbation theory!

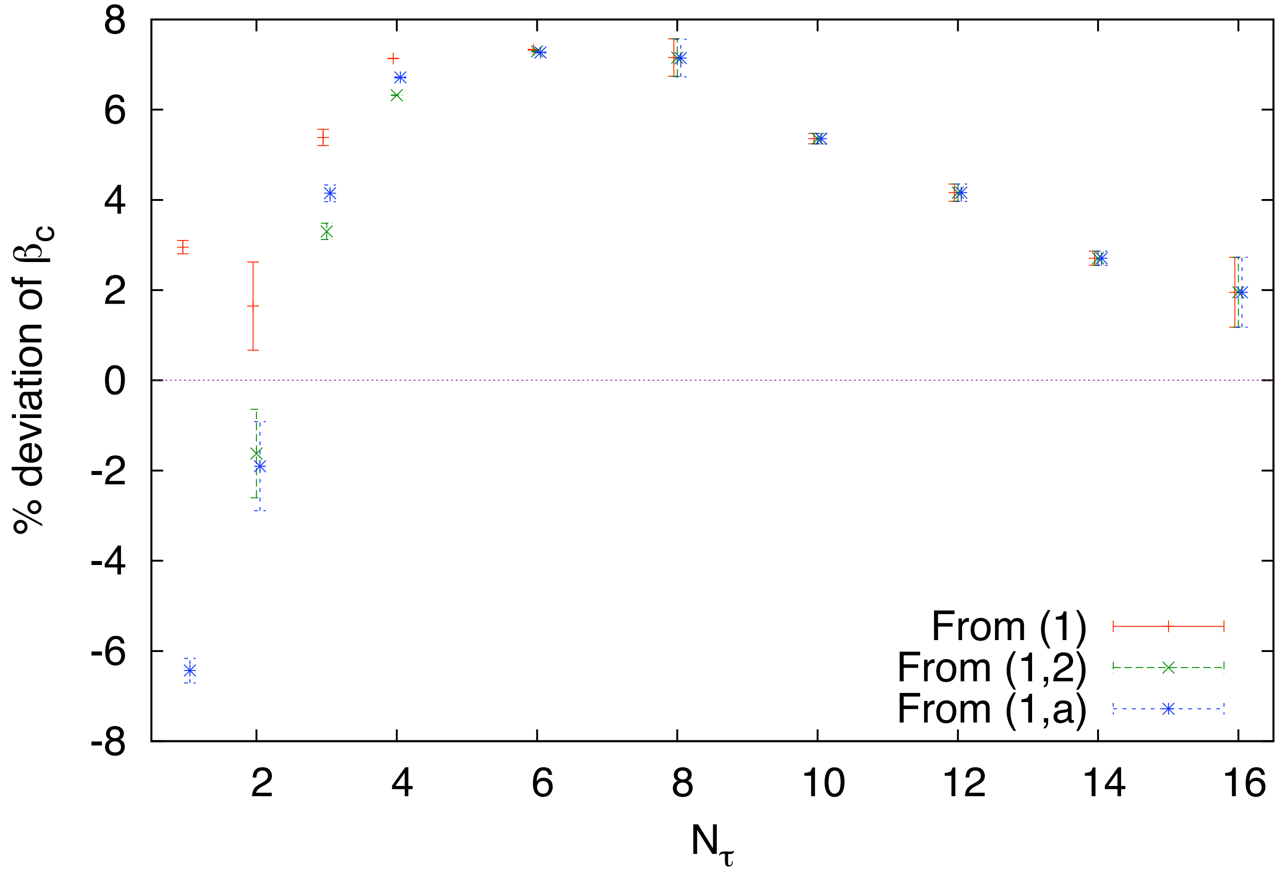
Numerical results for SU(3) Yang-Mills

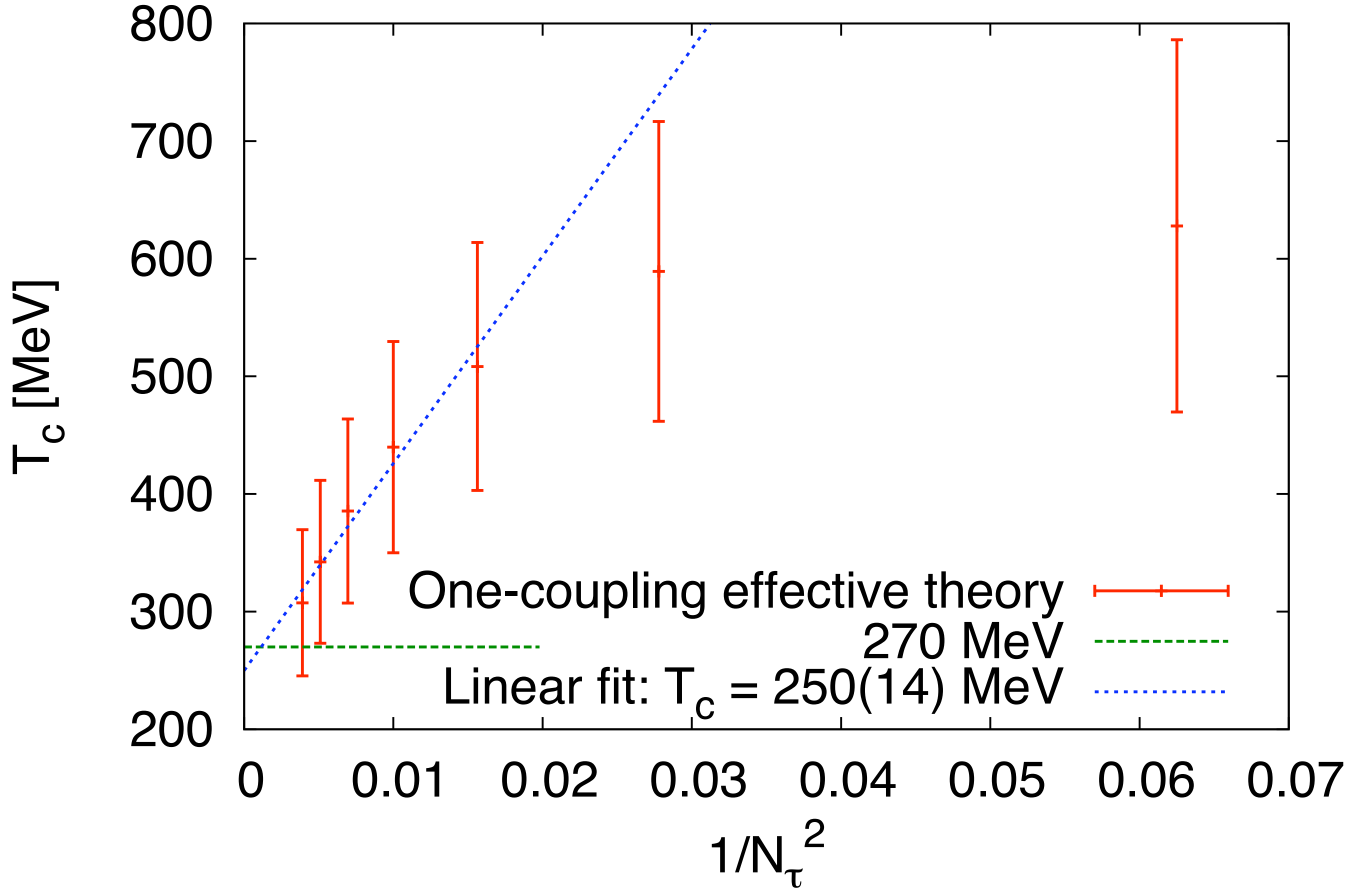


Order-disorder transition



Continuum limit feasible!



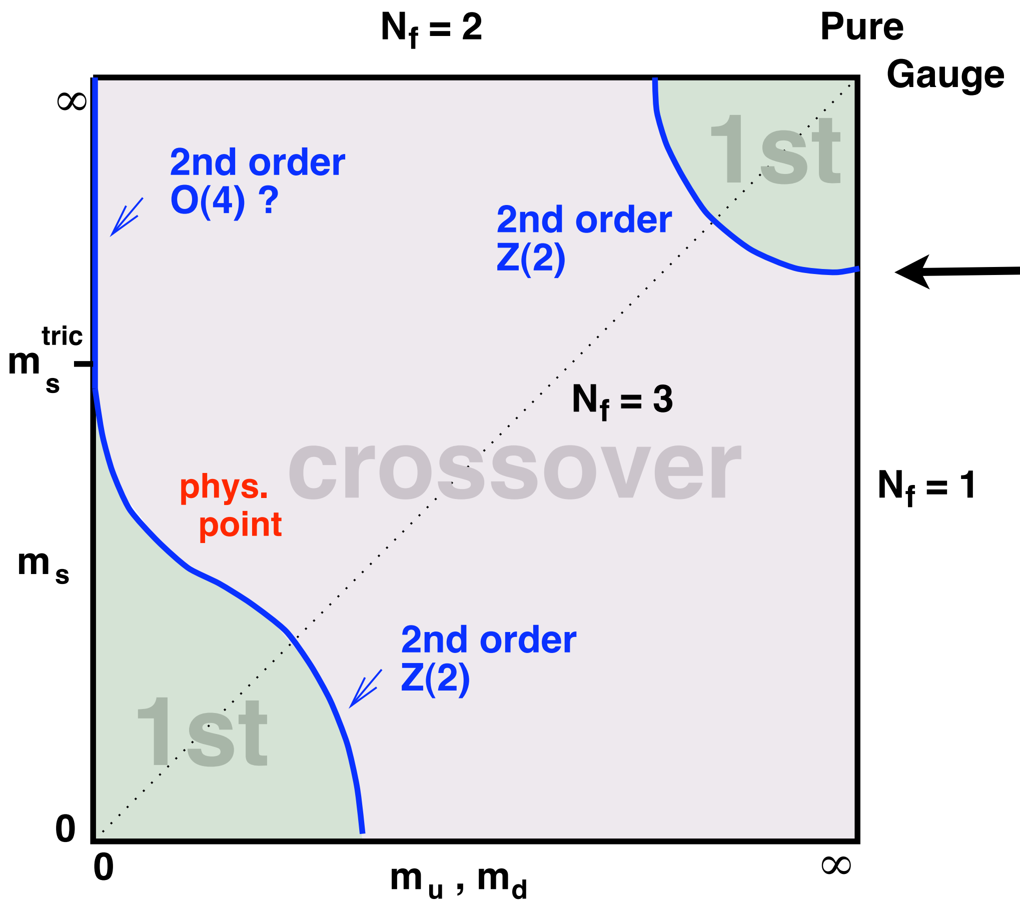


-error bars, right: difference between last two orders in strong coupling

-using non-perturbative beta-function (4d T=0 lattice)

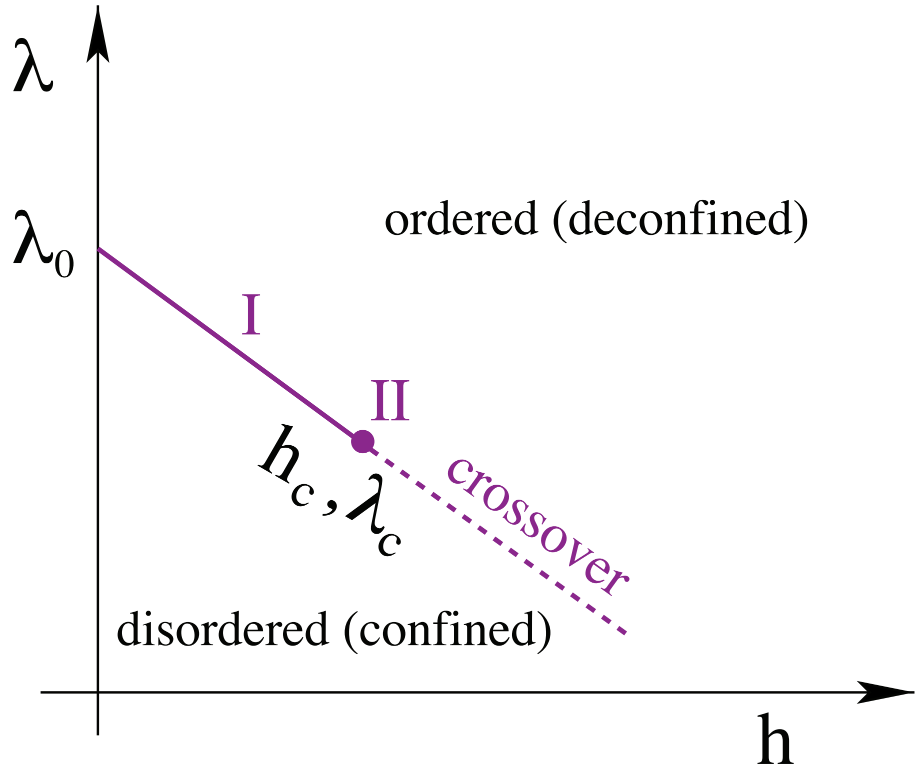
-all data points from one single 3d MC simulation!

QCD: first order deconfinement transition region



deconfinement p.t.:
breaking of global $Z(3)$ symmetry;
explicitly broken by quark masses
transition weakens

Phase diagram in eff. theory:



The critical point

$$\lambda_c = 0.18672(7), h_c = 0.000731(40)$$

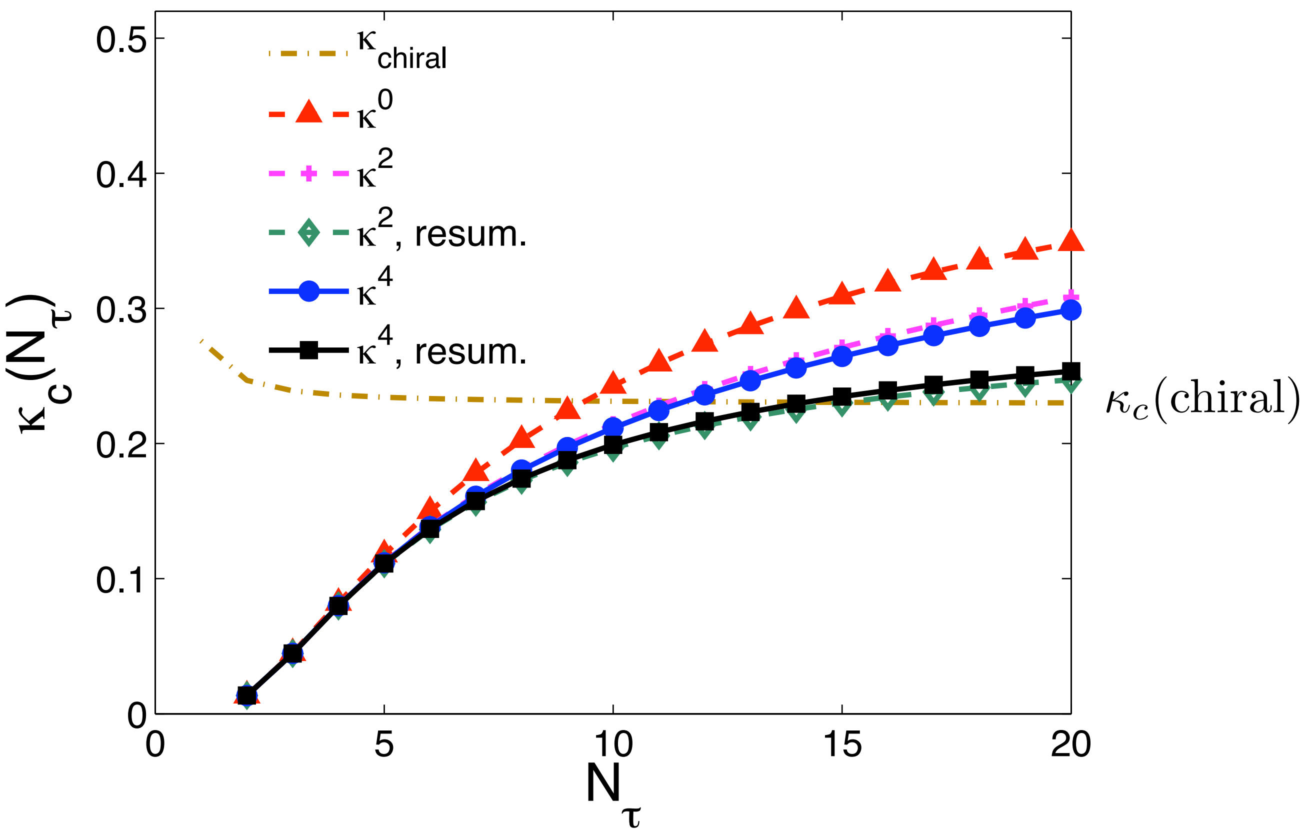
Mapping back to QCD:

		eff. theory	4d MC, WHOT	4d MC, de Forcrand et al
N_f	M_c/T	$\kappa_c(N_\tau = 4)$	$\kappa_c(4)$, Ref. [23]	$\kappa_c(4)$, Ref. [22]
1	7.22(5)	0.0822(11)	0.0783(4)	~ 0.08
2	7.91(5)	0.0691(9)	0.0658(3)	–
3	8.32(5)	0.0625(9)	0.0595(3)	–

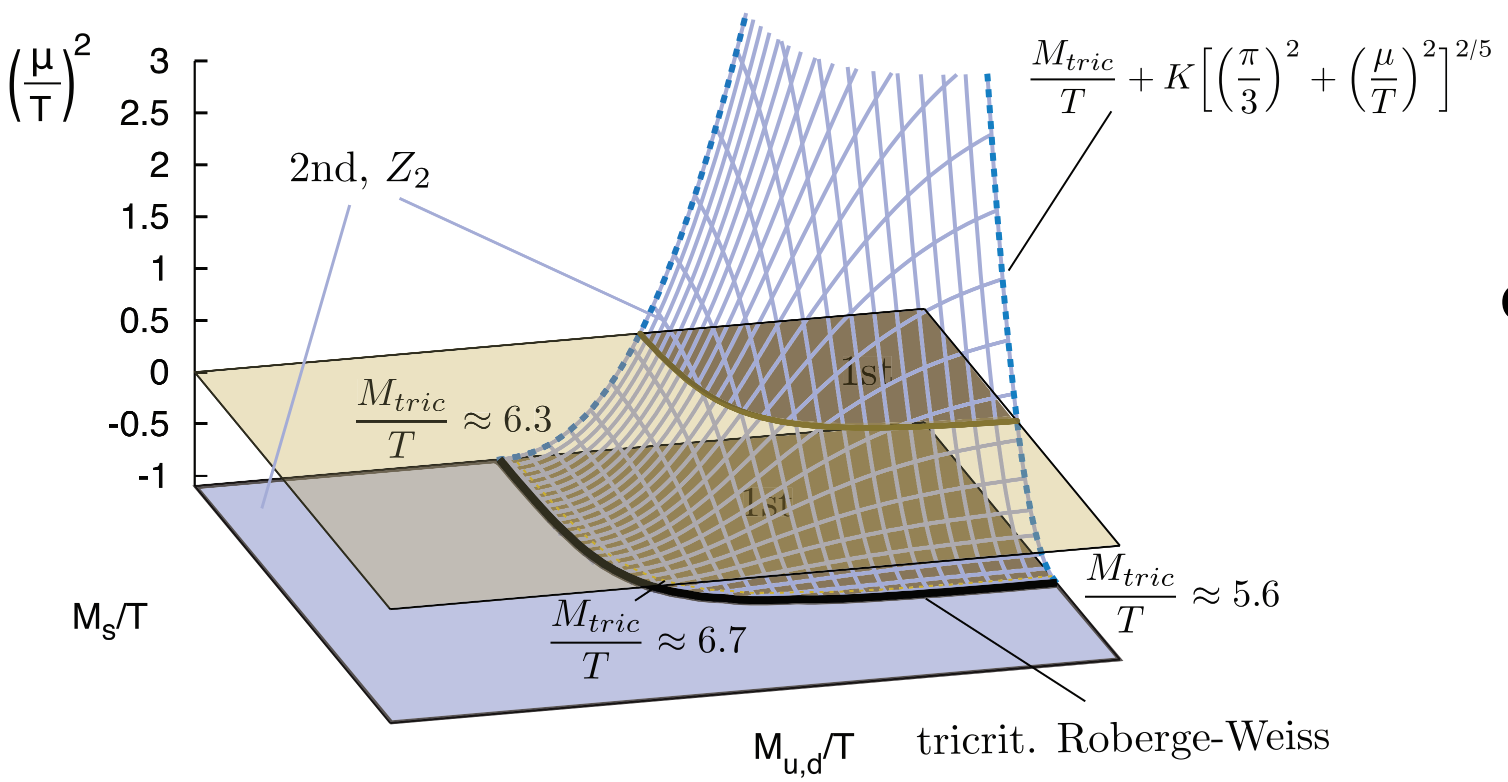
$e^{-M/T} \simeq h/N_f$ [linear approximation in $h \ll 1 \ldots$]

Accuracy ~5%, predictions for Nt=6,8,... available!

Convergence properties:

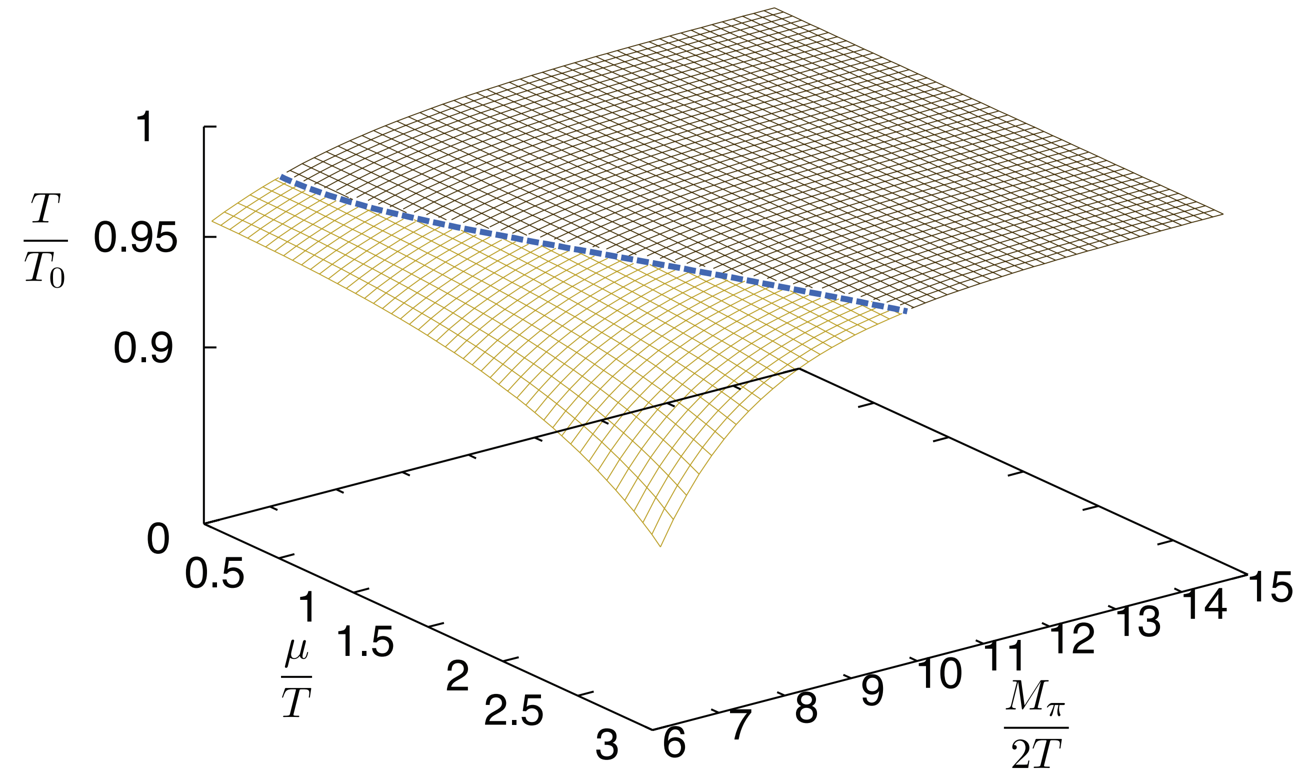


The fully calculated deconfinement transition, all μ



deconfinement critical surface

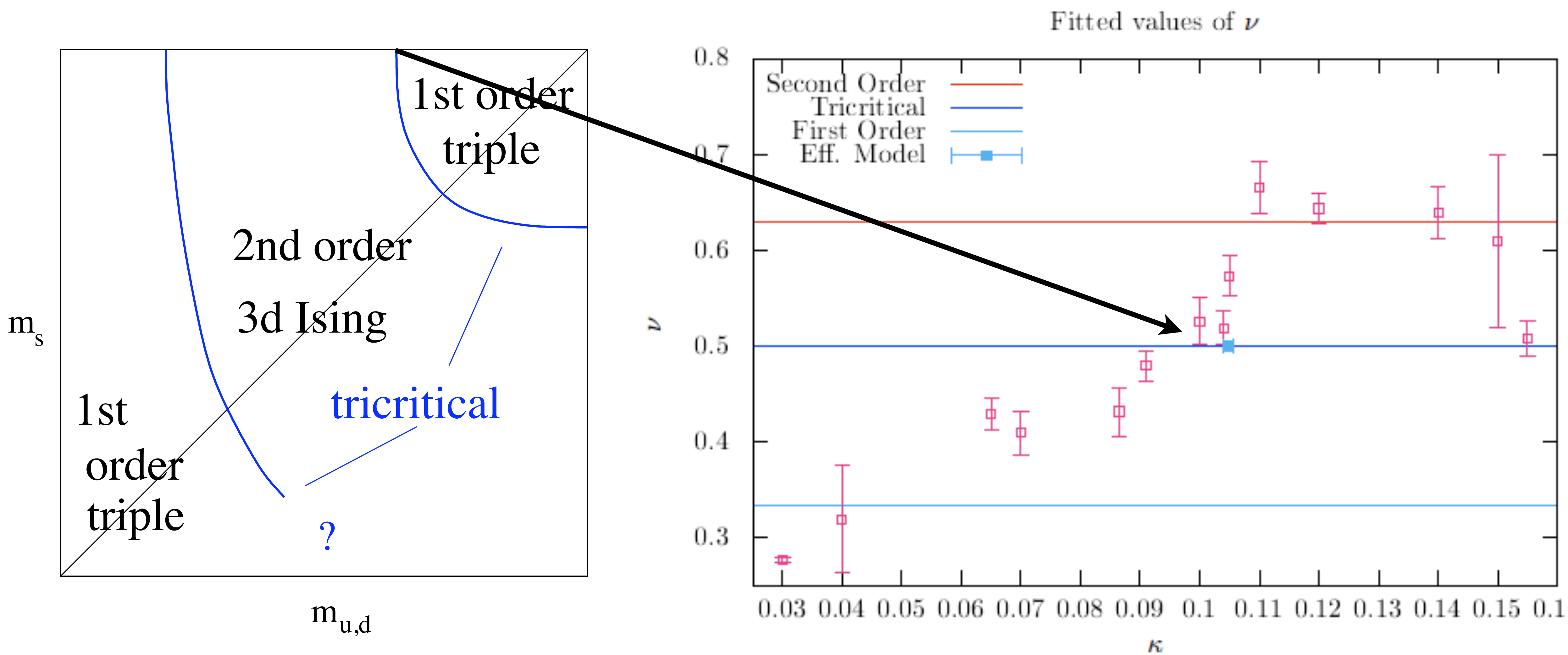
phase diagram for Nf=2, Nt=6



Roberge-Weiss transition, eff. th. against full 4d

Pinke, O.P.

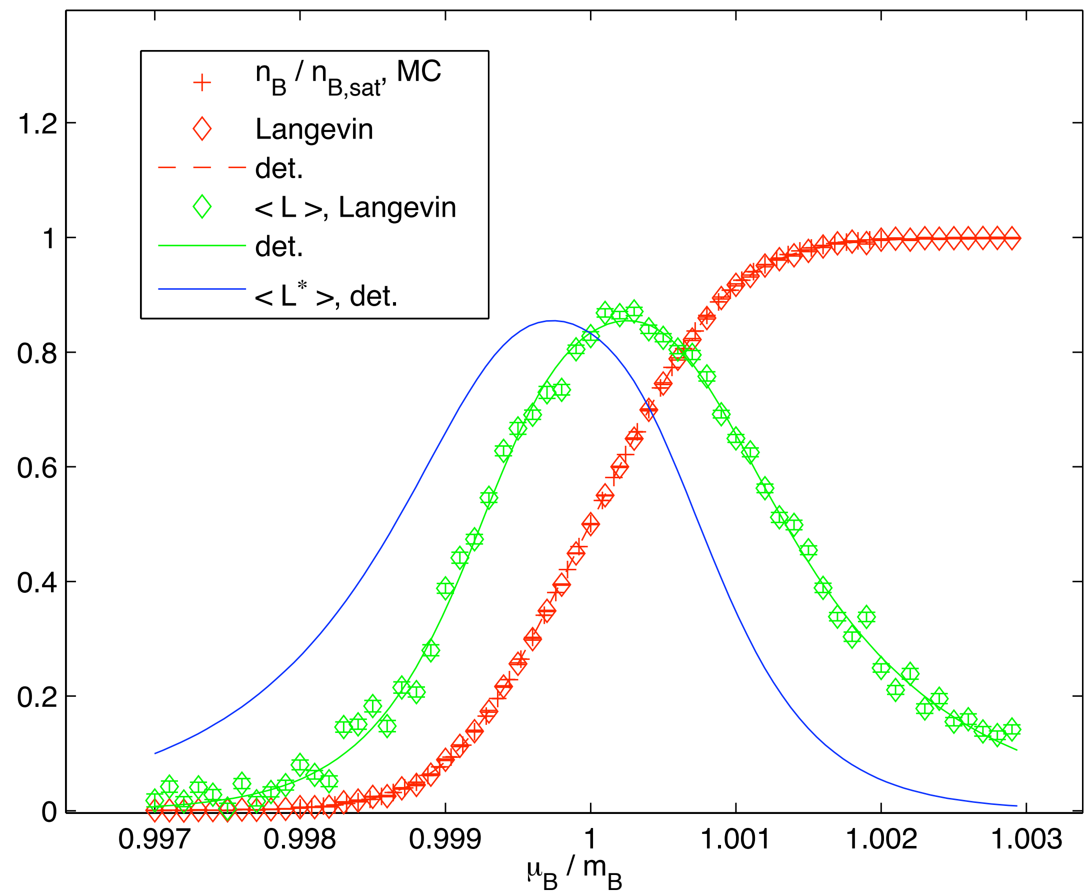
critical exponent distinguishes order of p.t.



$$\mu = i\frac{\pi T}{3}$$

Cold and dense QCD

Z(3) breaking part (fermion determinant), including corrections $\sim \kappa^2$



$m_\pi = 20 \text{ GeV}, T = 10 \text{ MeV}, a = 0.17 \text{ fm}$

$\beta = 5.7, \kappa = 0.0000887, N_\tau = 116$

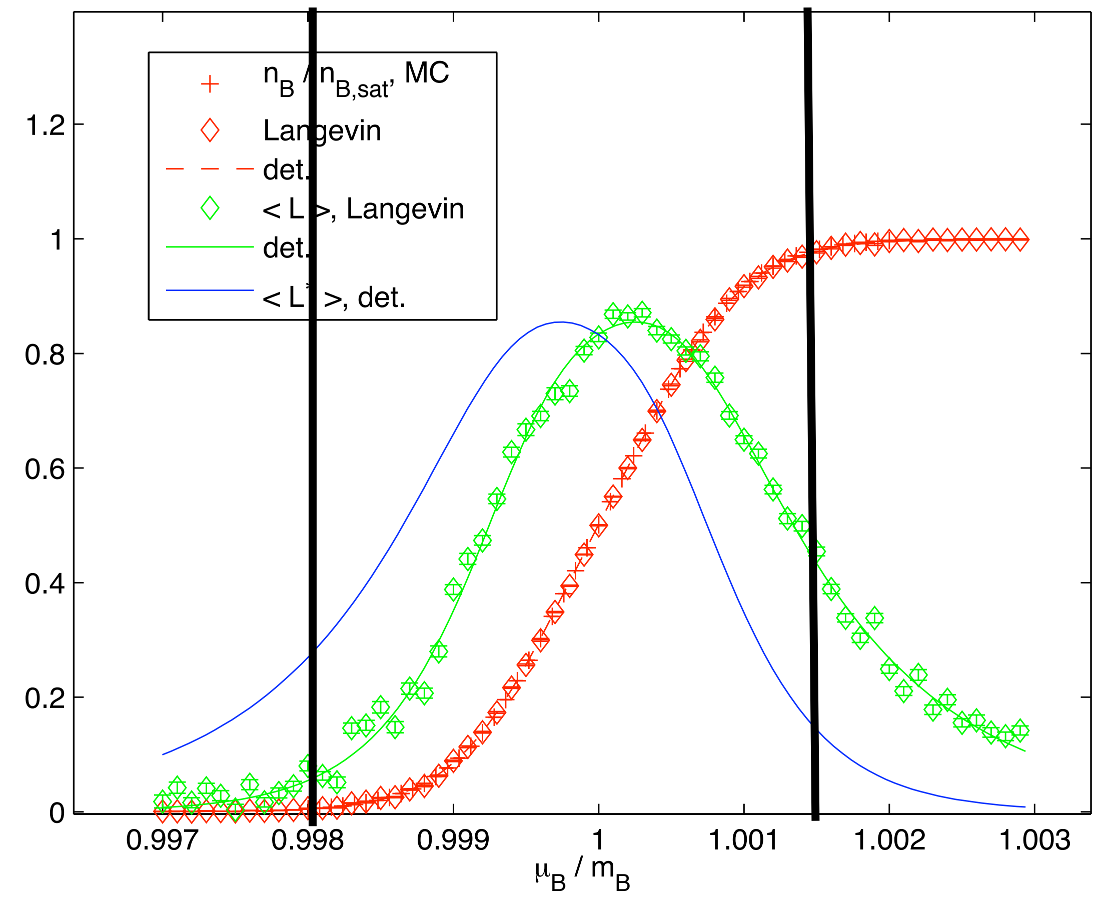
Silver blaze + lattice sautration realised!

Analytic strong coupling soln. valid!

$\lambda(\beta = 5.7, N_\tau = 115) \sim 10^{-27}$

High density difficult with UV cut-off

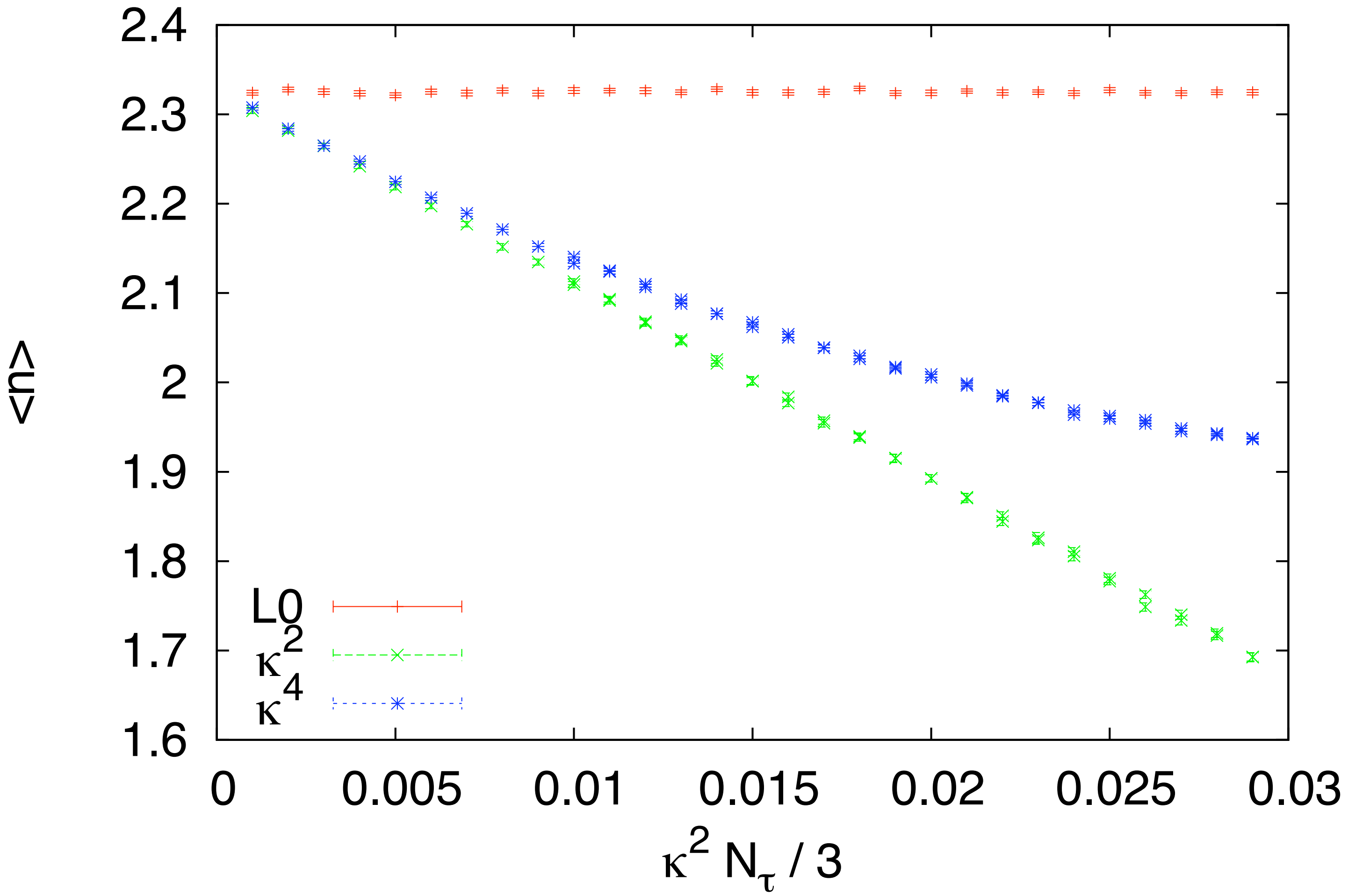
Saturation on the lattice severely limits the accessible densities!



Interesting region very narrow! Higher densities require finer lattices

Convergence region

Quark density at fixed chemical potential near onset

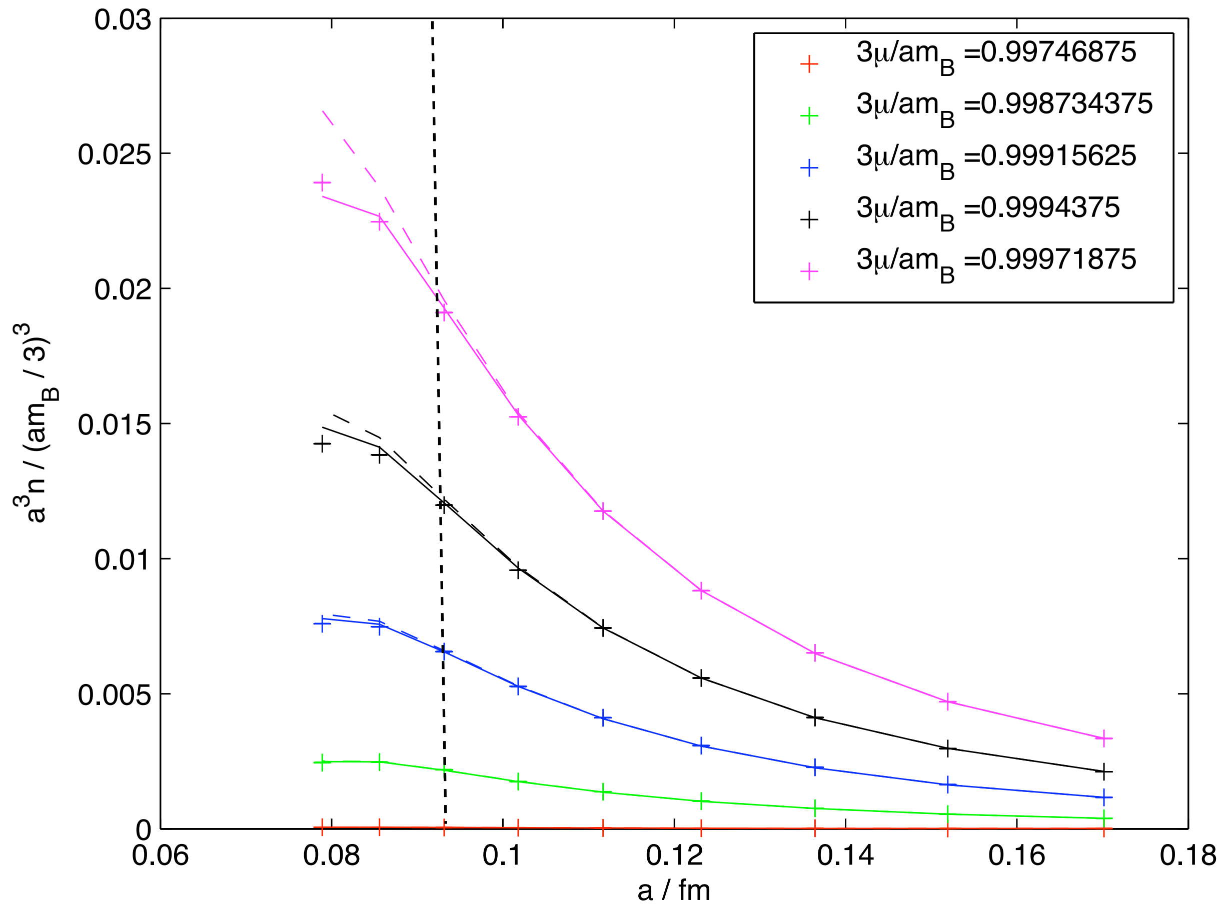


Effective expansion parameter

Continuum extrapolation

Scaling with lattice spacing:

$$\frac{n_{\rm lat}(\mu)}{m_B^3} = \frac{n_{\rm cont}(\mu)}{m_B^3} + A(\mu)a + B(\mu)a^2 + \ldots$$



Solid/dashed lines: analytic strong coupling limit with/without $\mathcal{O}(\kappa^2)$:

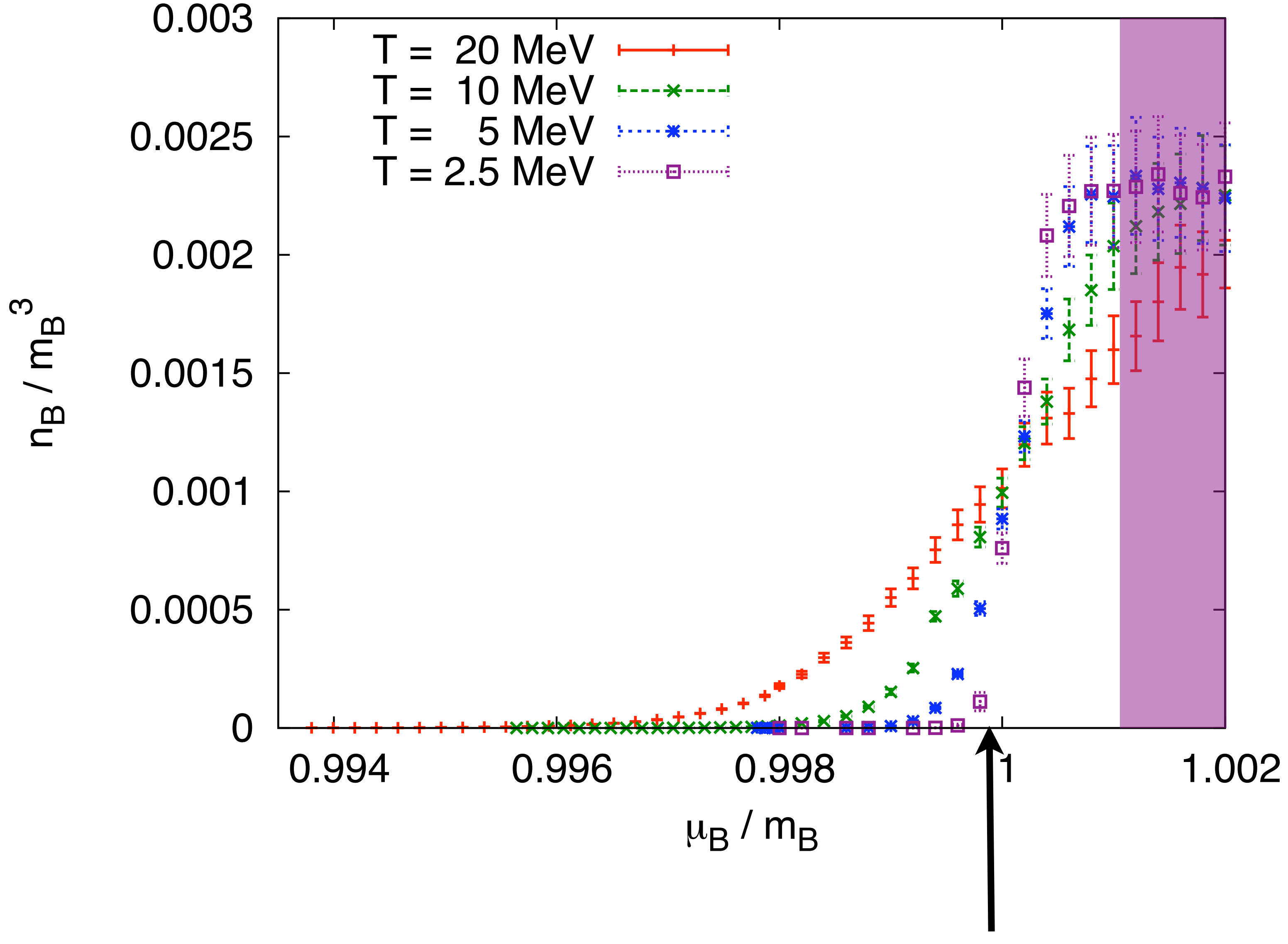
Breakdown of hopping series!

Onset transition to cold nuclear matter

PRL 13

... with very heavy quarks $m_\pi = 20\ \mathrm{GeV}$

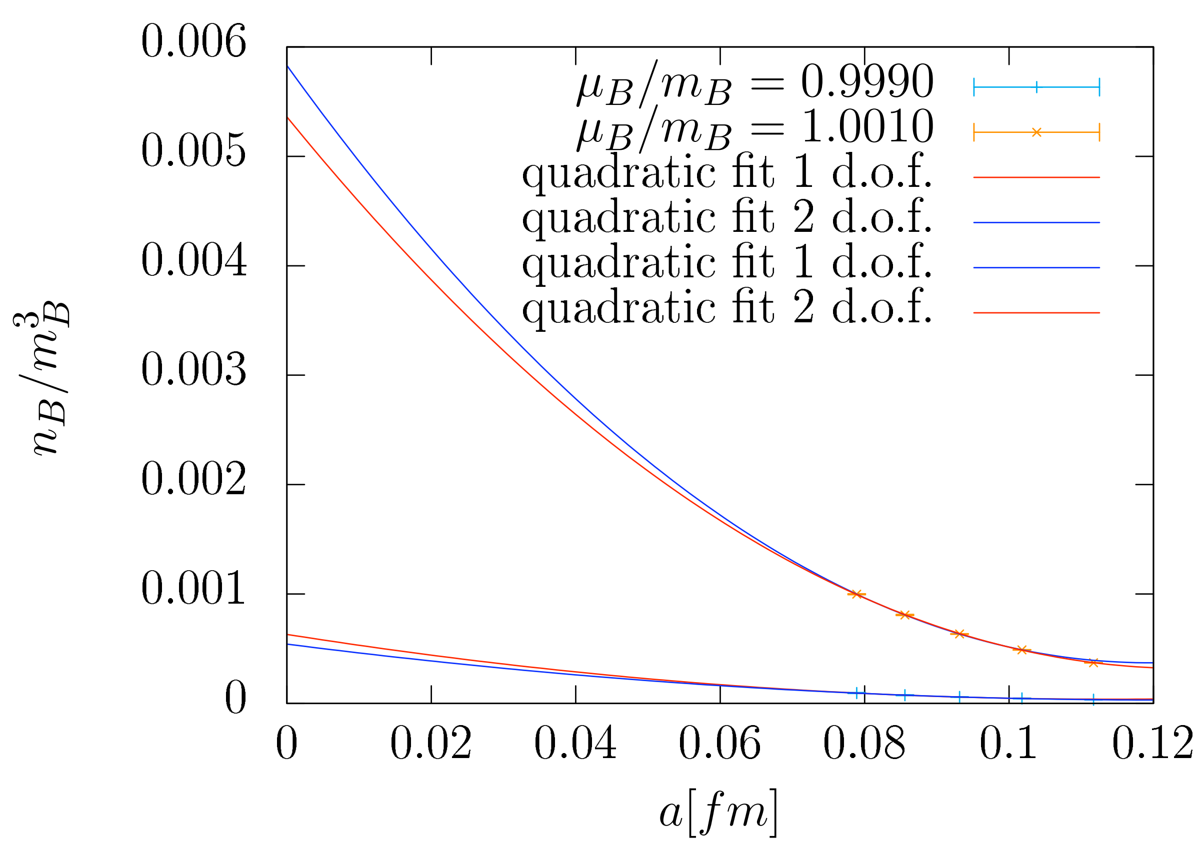
continuum limit with 5-7 lattice spacings per point

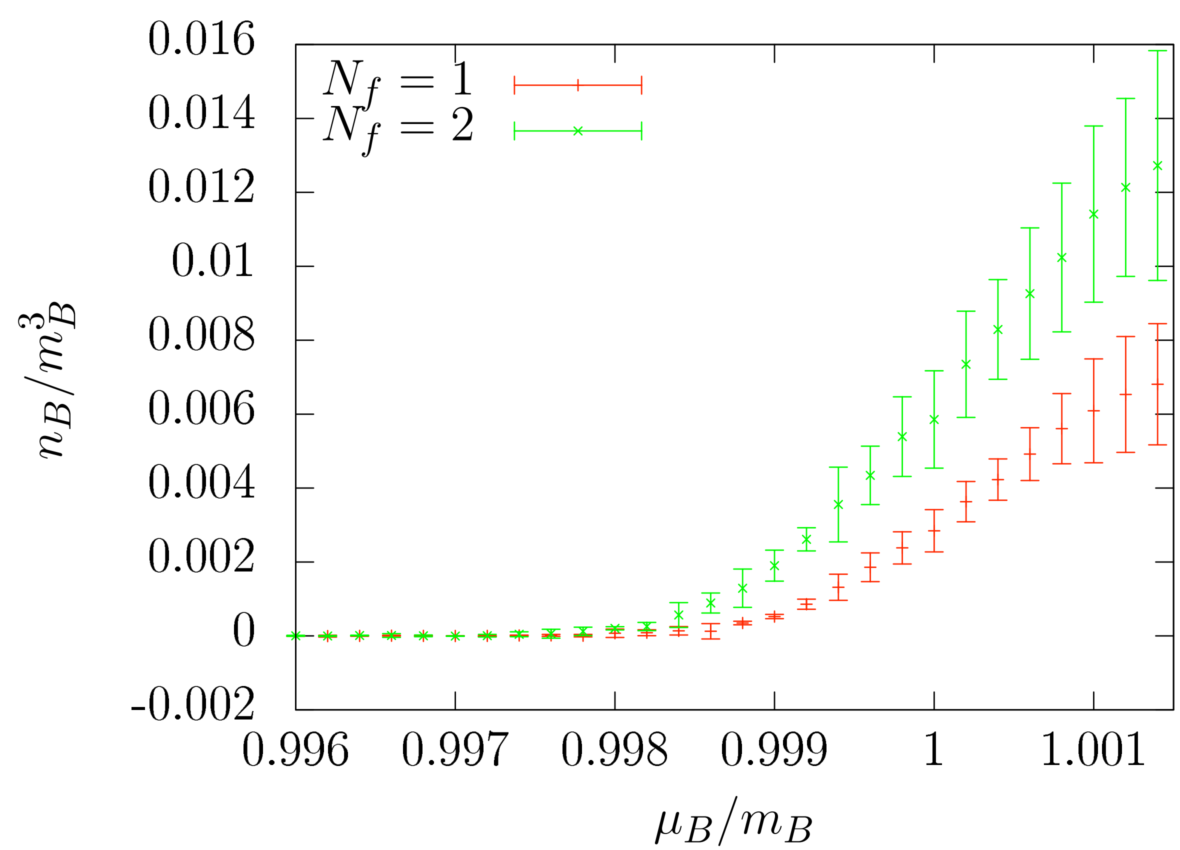


$$\frac{\mu}{T} \sim 4000$$

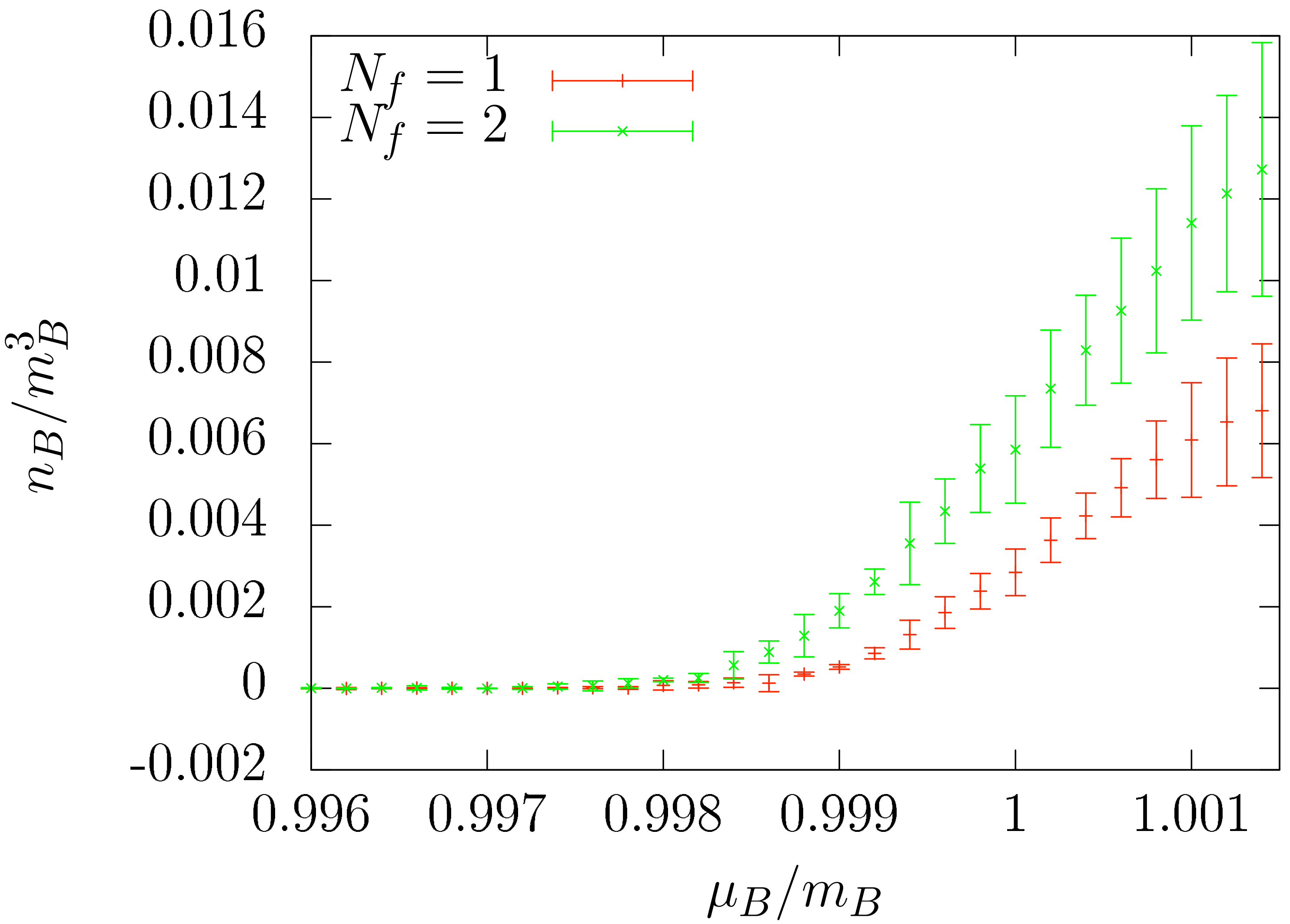
Error budget for the following results $S_{eff} \sim \kappa^n u^m, \quad n+m=4$

- Statistical: negligible
- Systematic: truncation of series;
  include difference between actions $\sim \kappa^2, \kappa^4$
- Systematic: continuum extrapolation;
  include difference between fit ranges, i.e. cut-off effects





The equation of state for nuclear matter



$$S_{eff} \sim \kappa^n u^m, \quad n+m=4$$

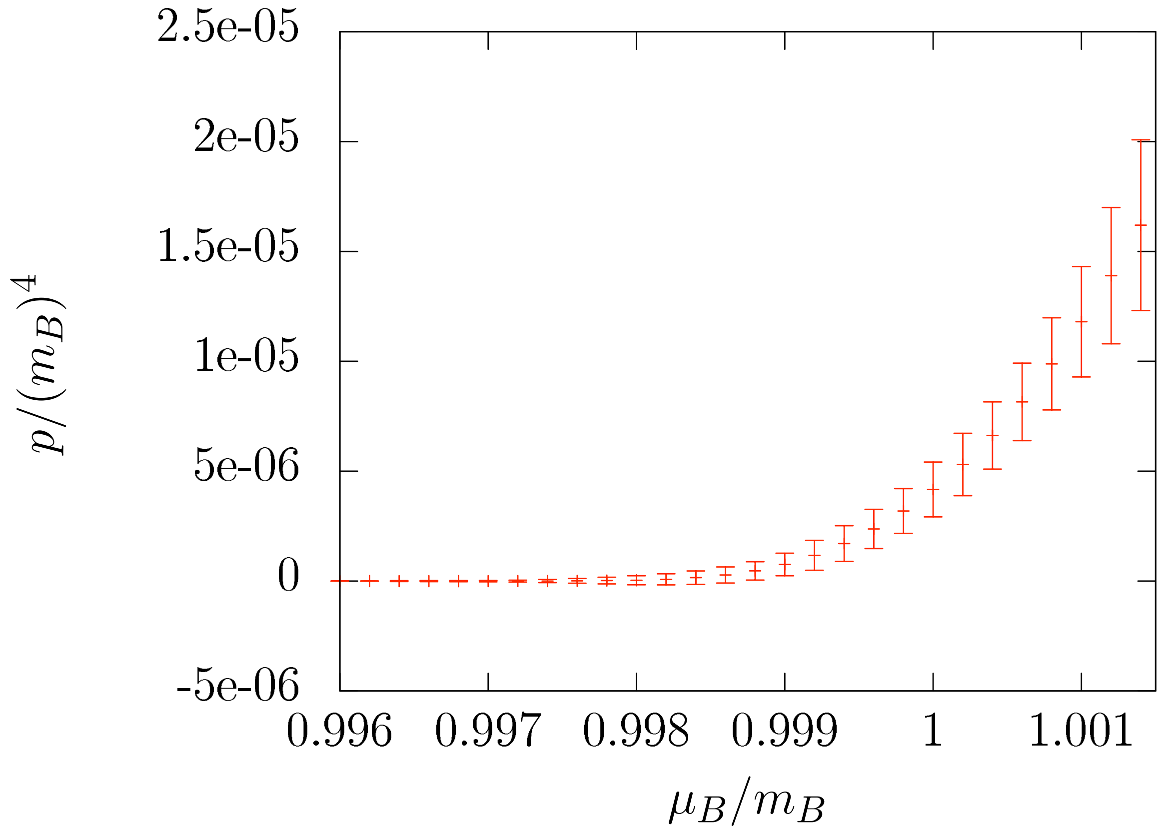
$$m_\pi = 20 \text{ GeV}, T = 10 \text{ MeV}$$

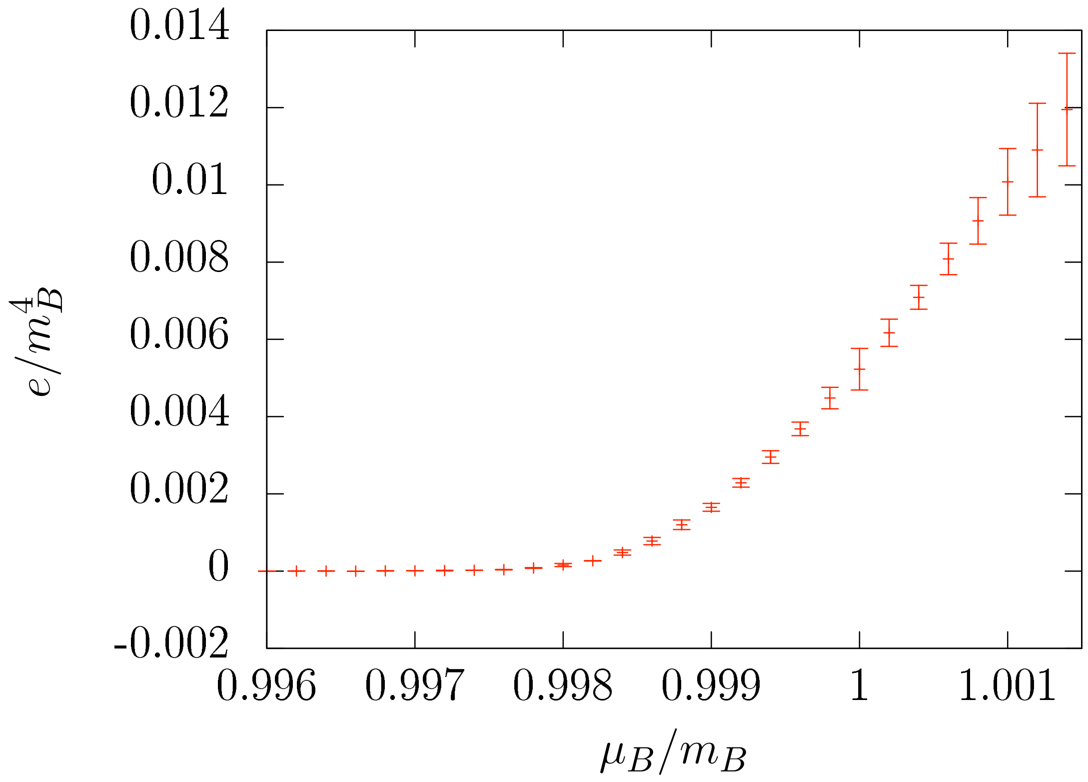
Effect of binding between baryons: $\mu_c < m_B$

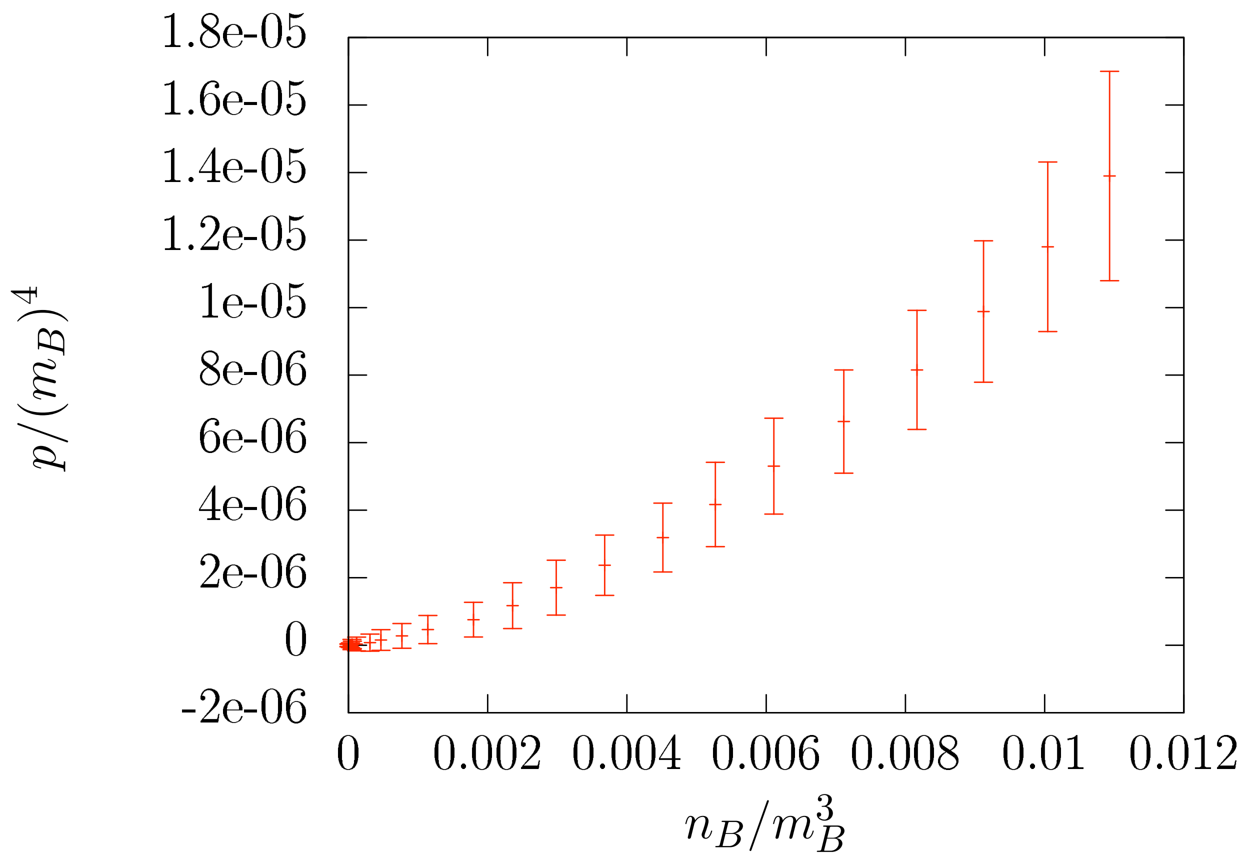
Binding energy per nucleon: $\epsilon = \frac{\mu_c - m_B}{m_B} \sim 10^{-3}$

Transition is smooth crossover: $T > T_c \sim \epsilon \, m_B$

The equation of state for nuclear matter, Nf=2

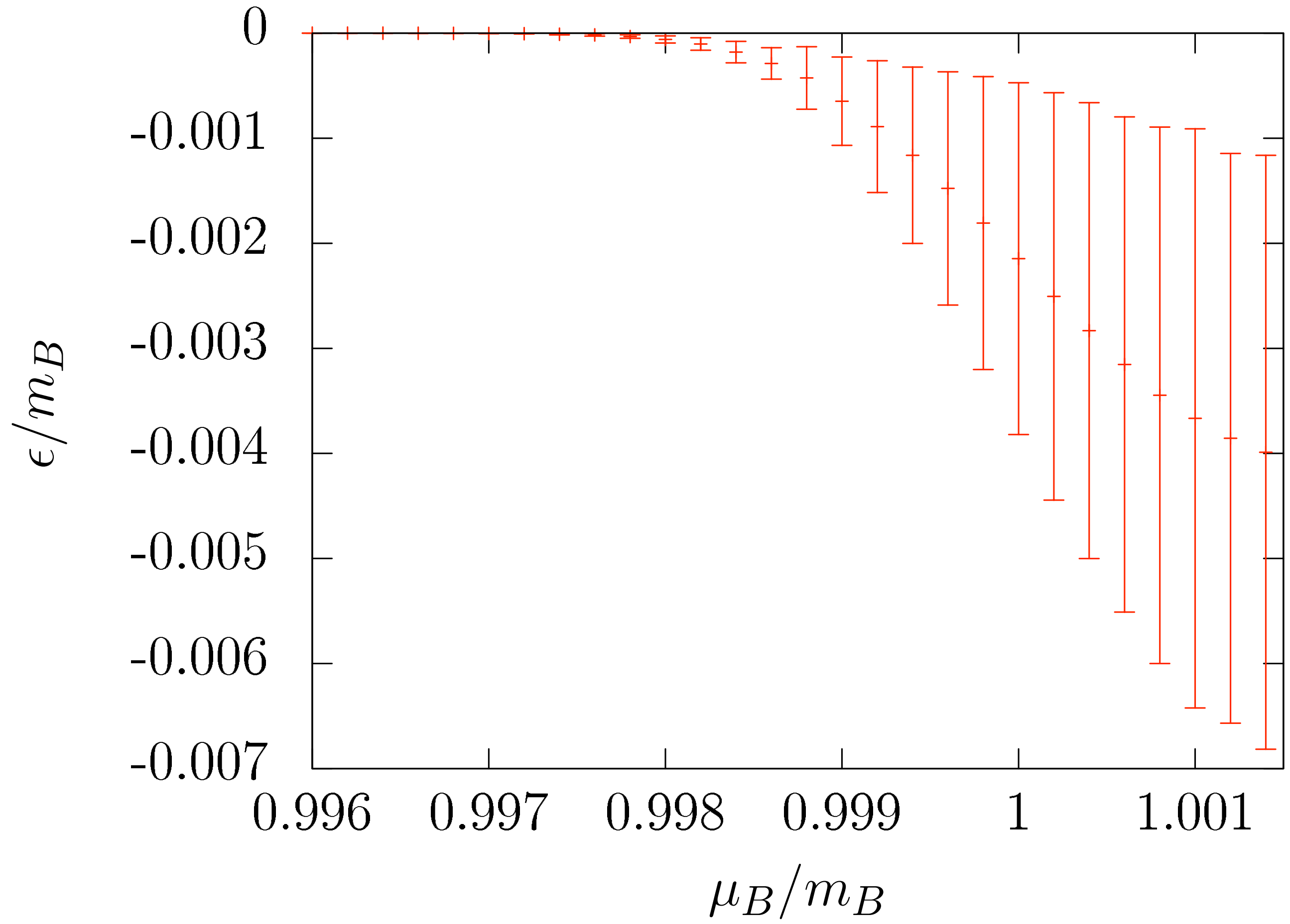






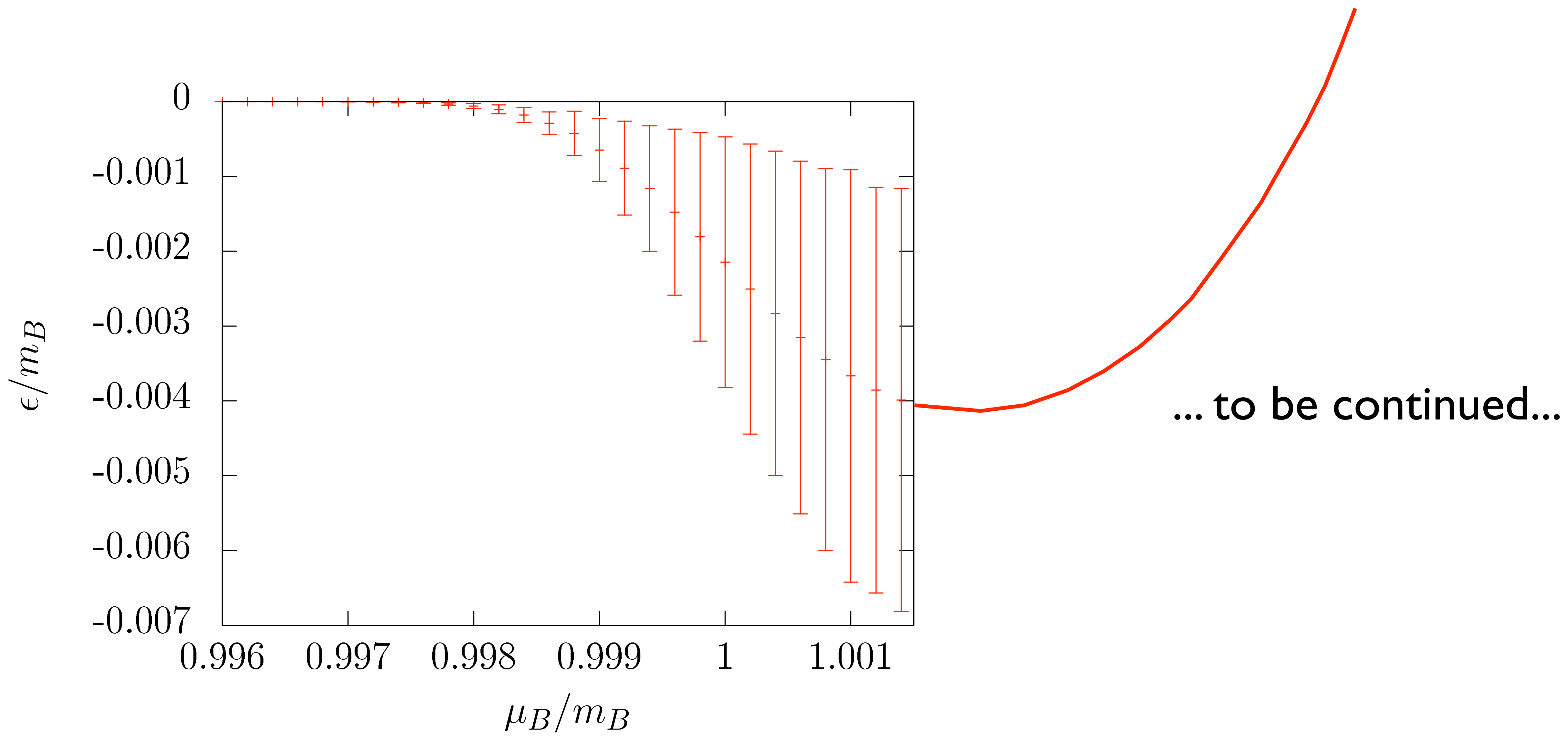
Binding energy per nucleon

$$\epsilon = \frac{e - n_B m_B}{n_B m_B} = \frac{e}{n_B m_B} - 1$$



Binding energy per nucleon

$$\epsilon = \frac{e - n_B m_B}{n_B m_B} = \frac{e}{n_B m_B} - 1$$



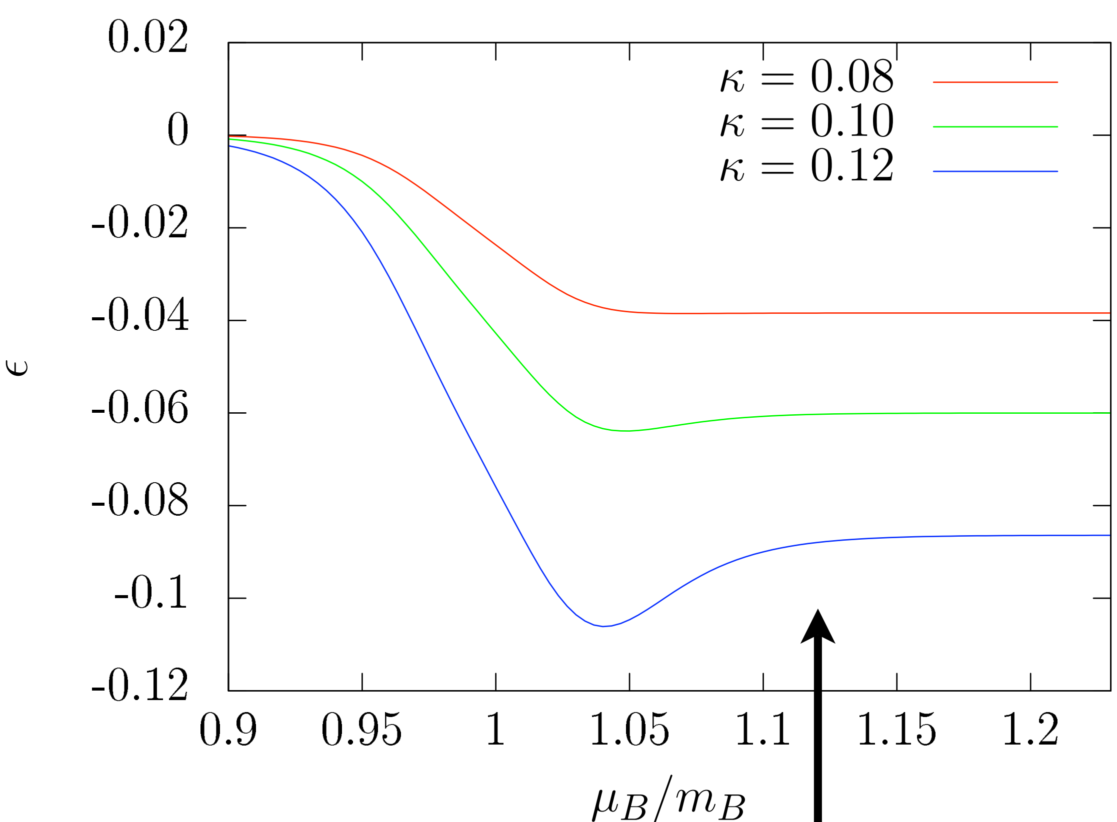
Minimum: access to nucl. binding energy, nucl. saturation density!

$\epsilon \sim 10^{-3}$ consistent with the location of the onset transition

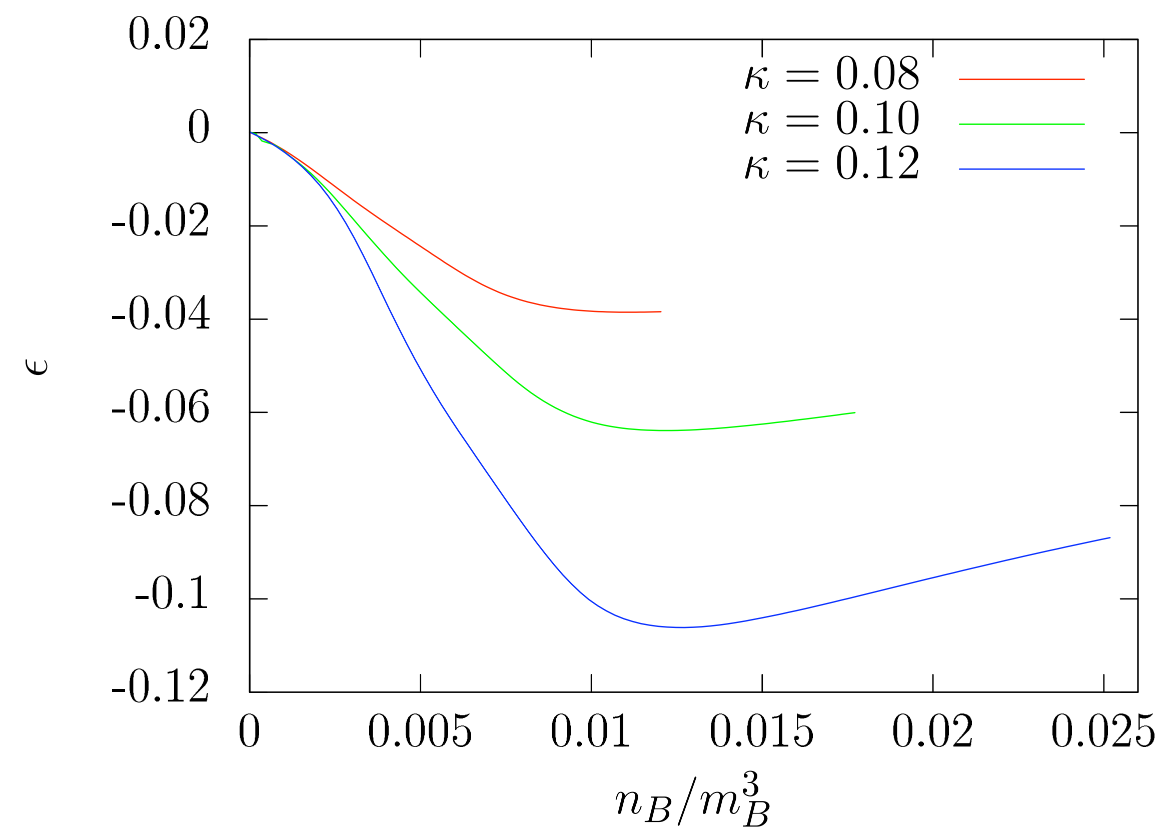
Quark mass dependence of the binding energy:

Expect short range nucl. potential for heavy pions, $V \sim \frac{e^{-m_\pi r}}{r}$

Analytic solution, finite lattice spacing:

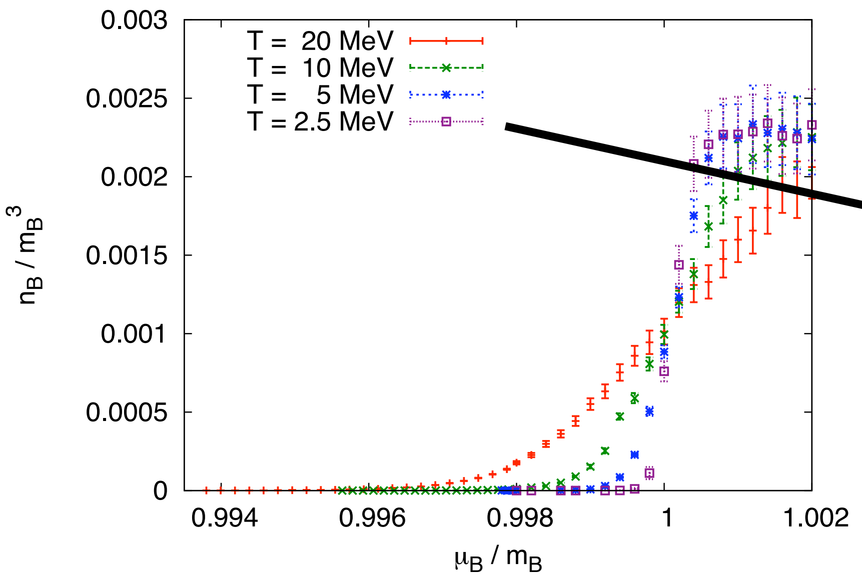


quark mass

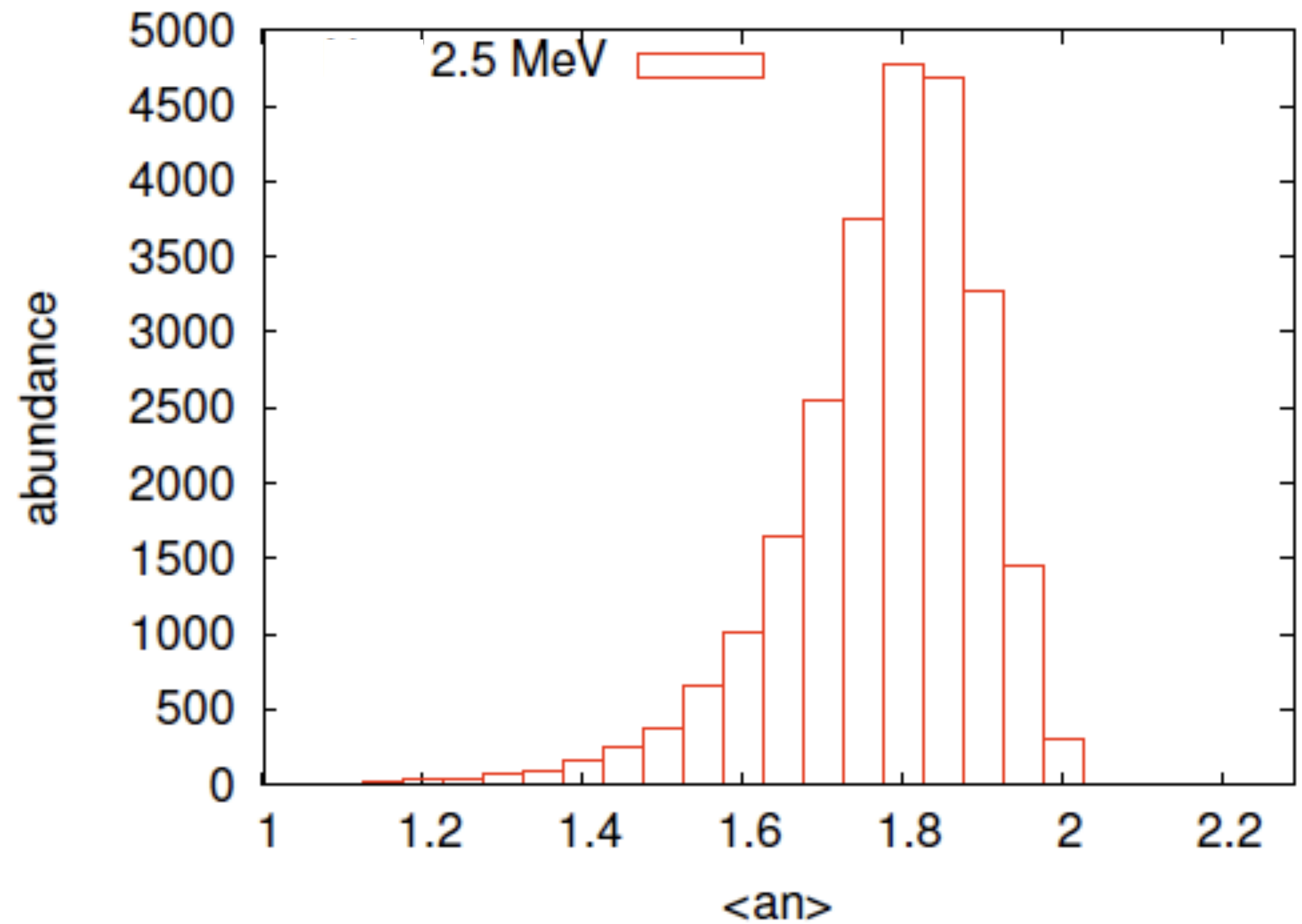


lattice saturation

Order of the onset transition?

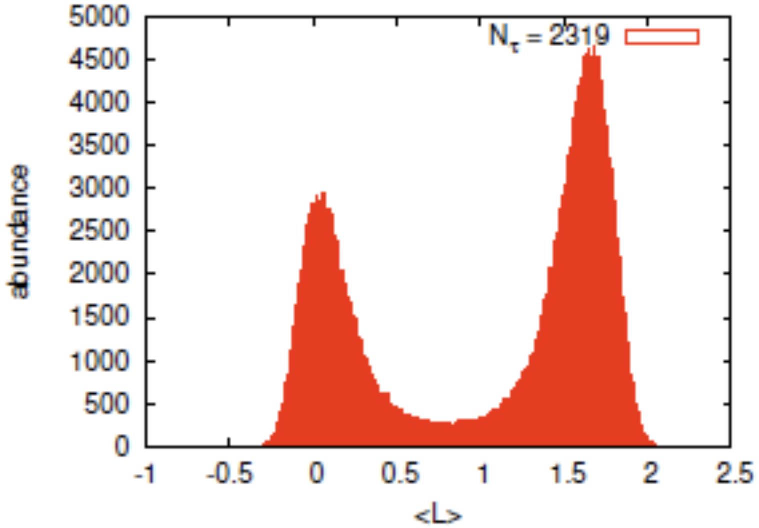


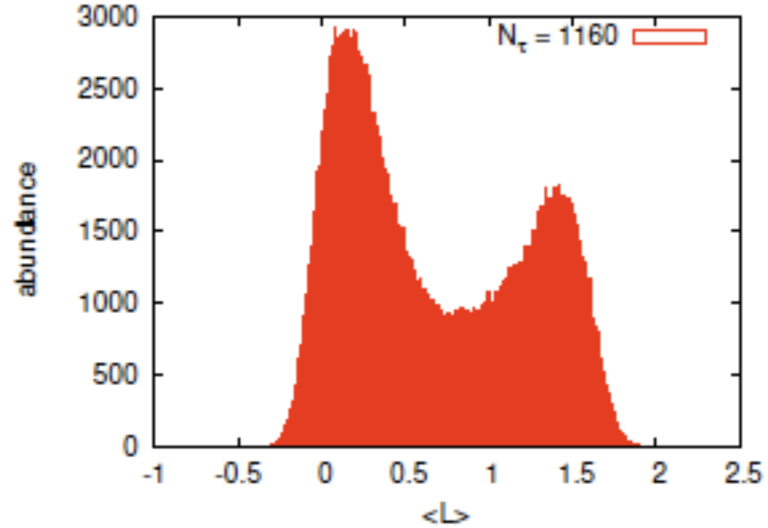
Distribution of fermion density: crossover!?

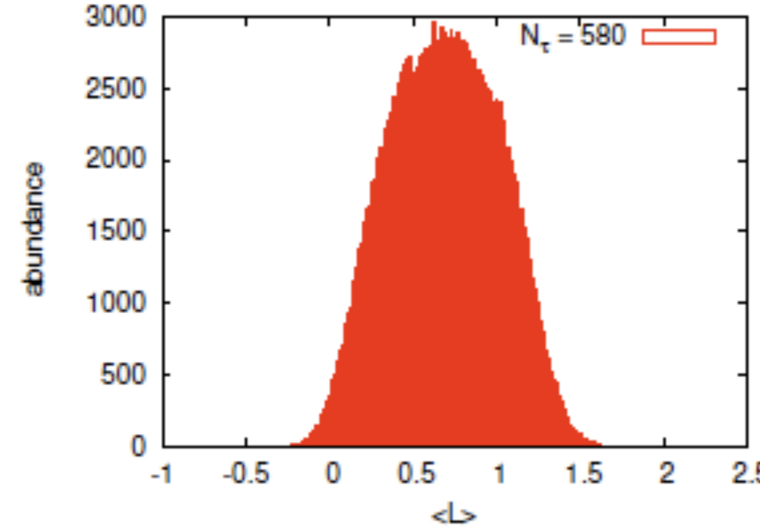


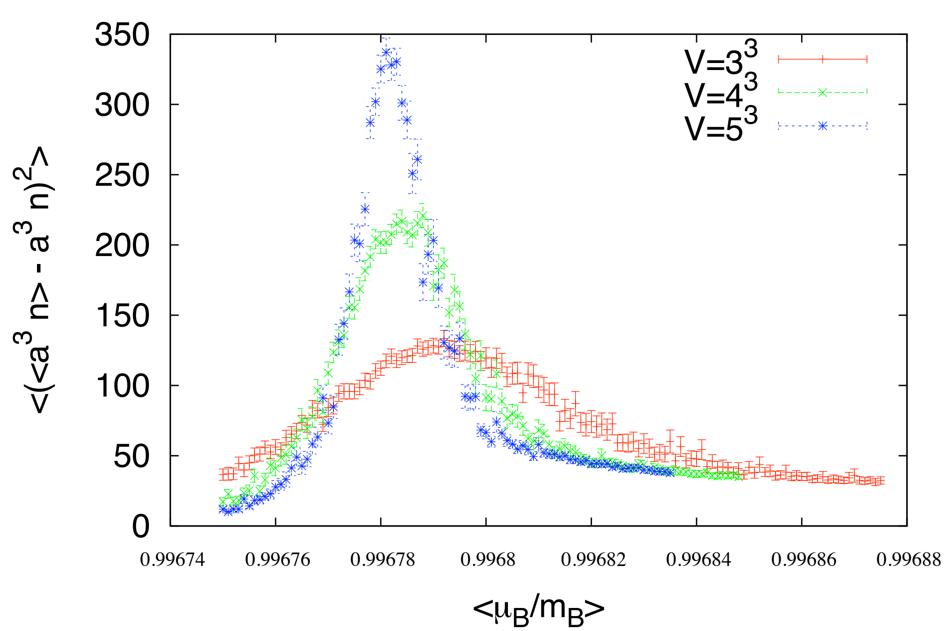
Reason: binding energy suppressed by meson mass

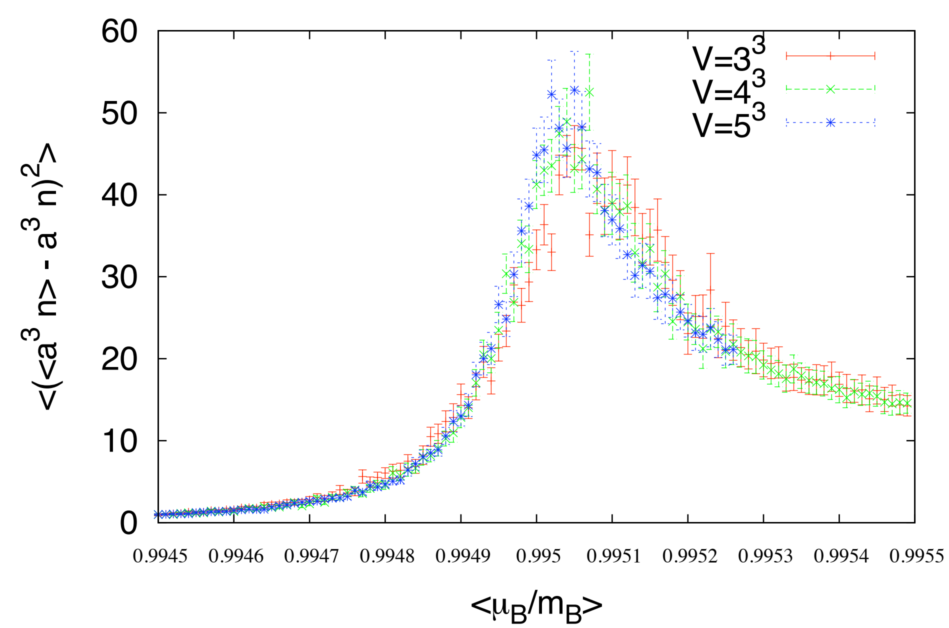
Lighter quarks: First order signal + endpoint!









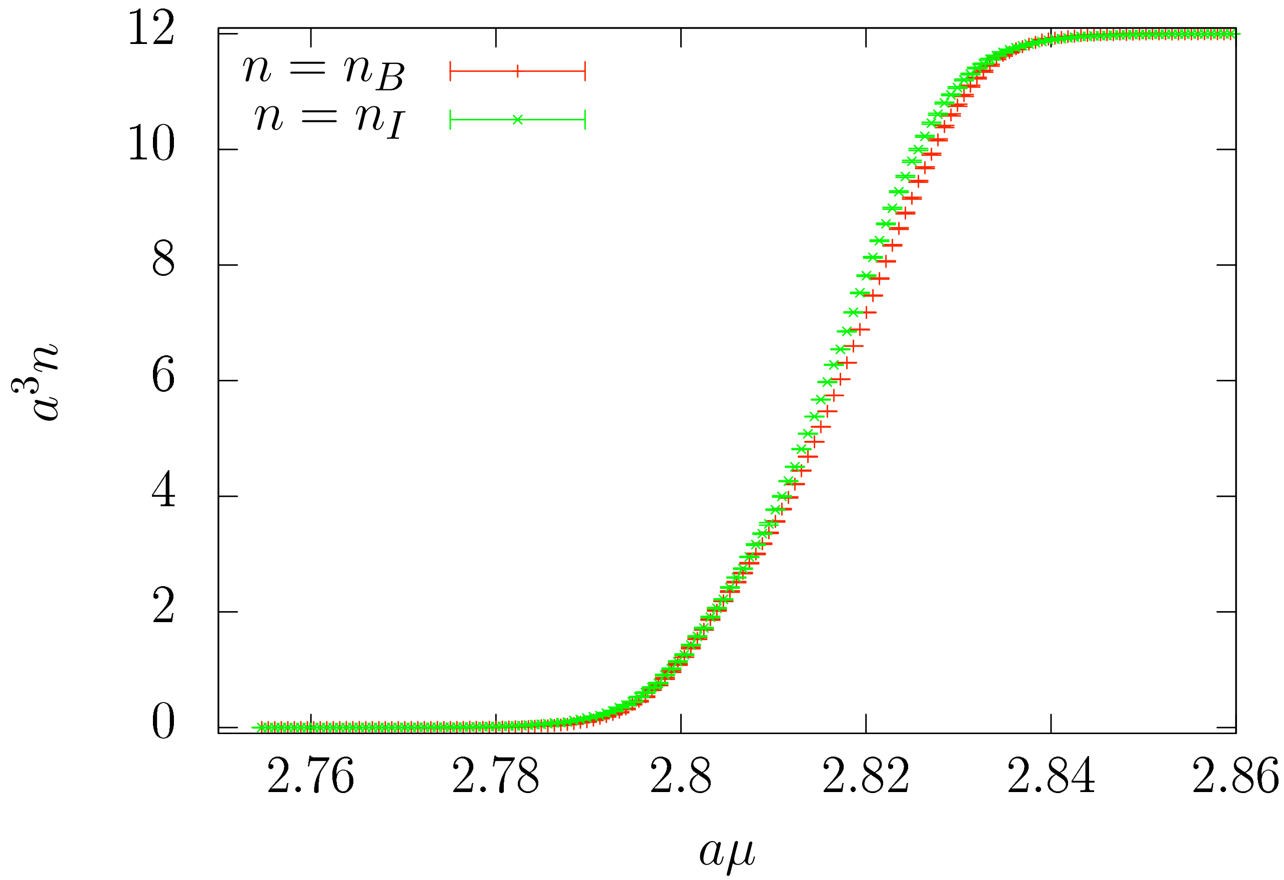


- $O(k^4)$:Stretching the hopping series, $\kappa = 0.12, \beta = 5.6$
- Coexistence of vacuum and finite density phase: 1st order
- If the temperature $T = \frac{1}{aN_\tau}$ or the quark mass is raised this changes to a crossover

attn: no convergence yet!

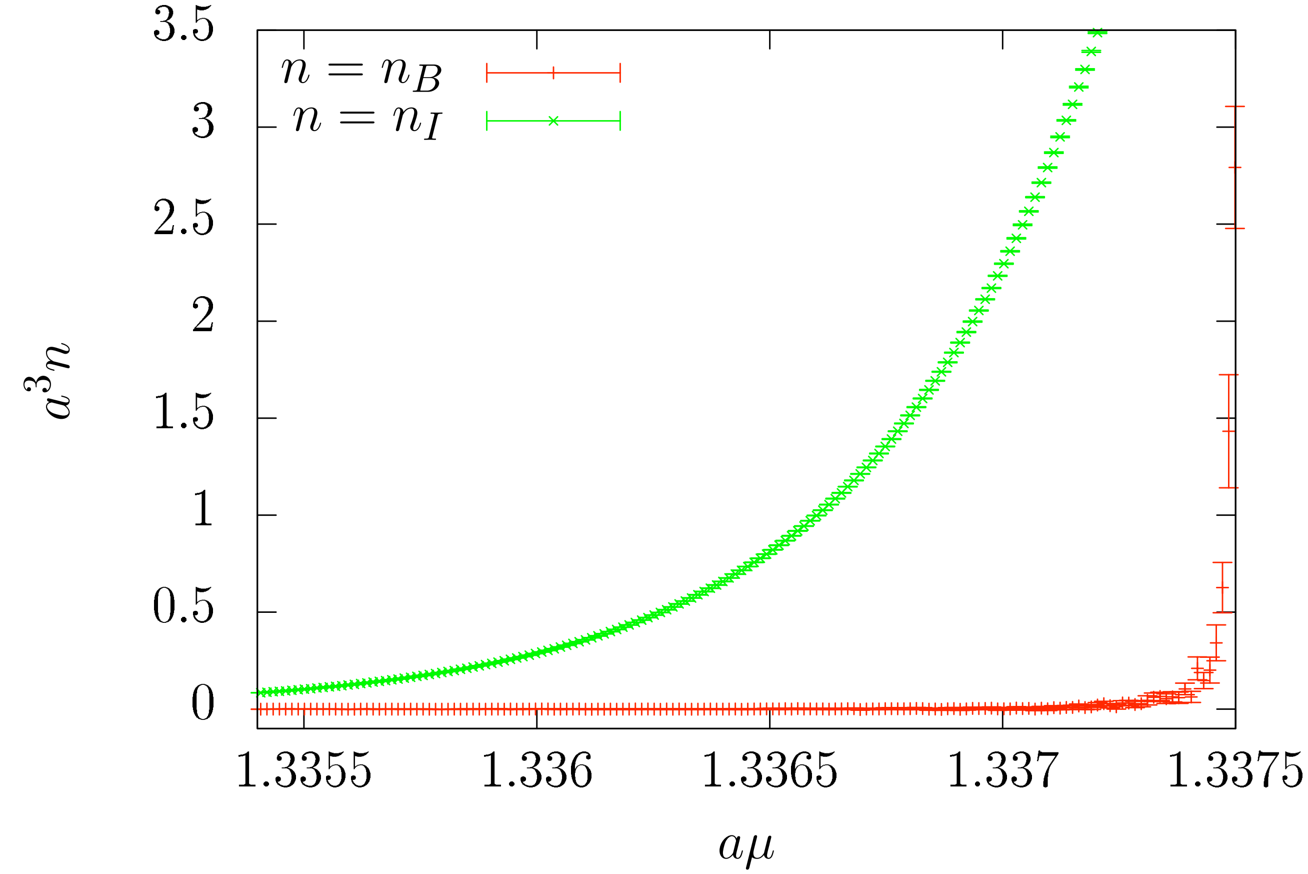
all features of liquid gas transition!!!

Finite isospin vs baryon chemical potential



heavy quarks

$$\frac{m_\pi}{2} \approx \frac{m_B}{3}$$

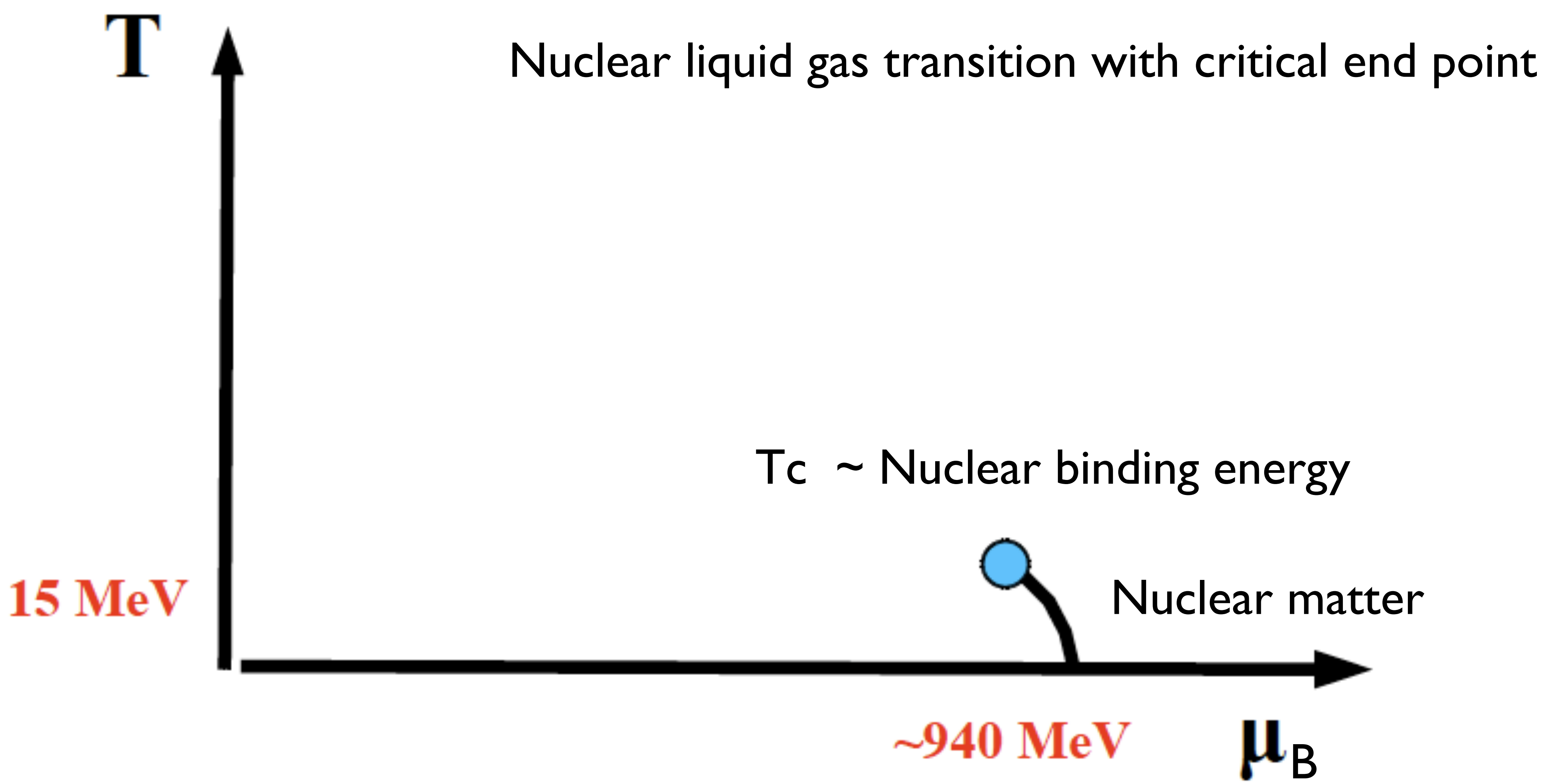


light quarks

$$\frac{m_\pi}{2} < \frac{m_B}{3}$$

onset at smaller chemical potential

Within reach of effective lattice QCD?!



Can we get high enough orders for light quarks?!?

Conclusions

- Effective lattice theory allows to simulate heavy quarks up to nuclear densities
- Onset transition to nuclear matter seen from lattice QCD!
- Success for light quarks not guaranteed, but controllable in all parts
- Next: finite T easier, return to critical endpoint for $\mu_B < 1\,\mathrm{GeV}$

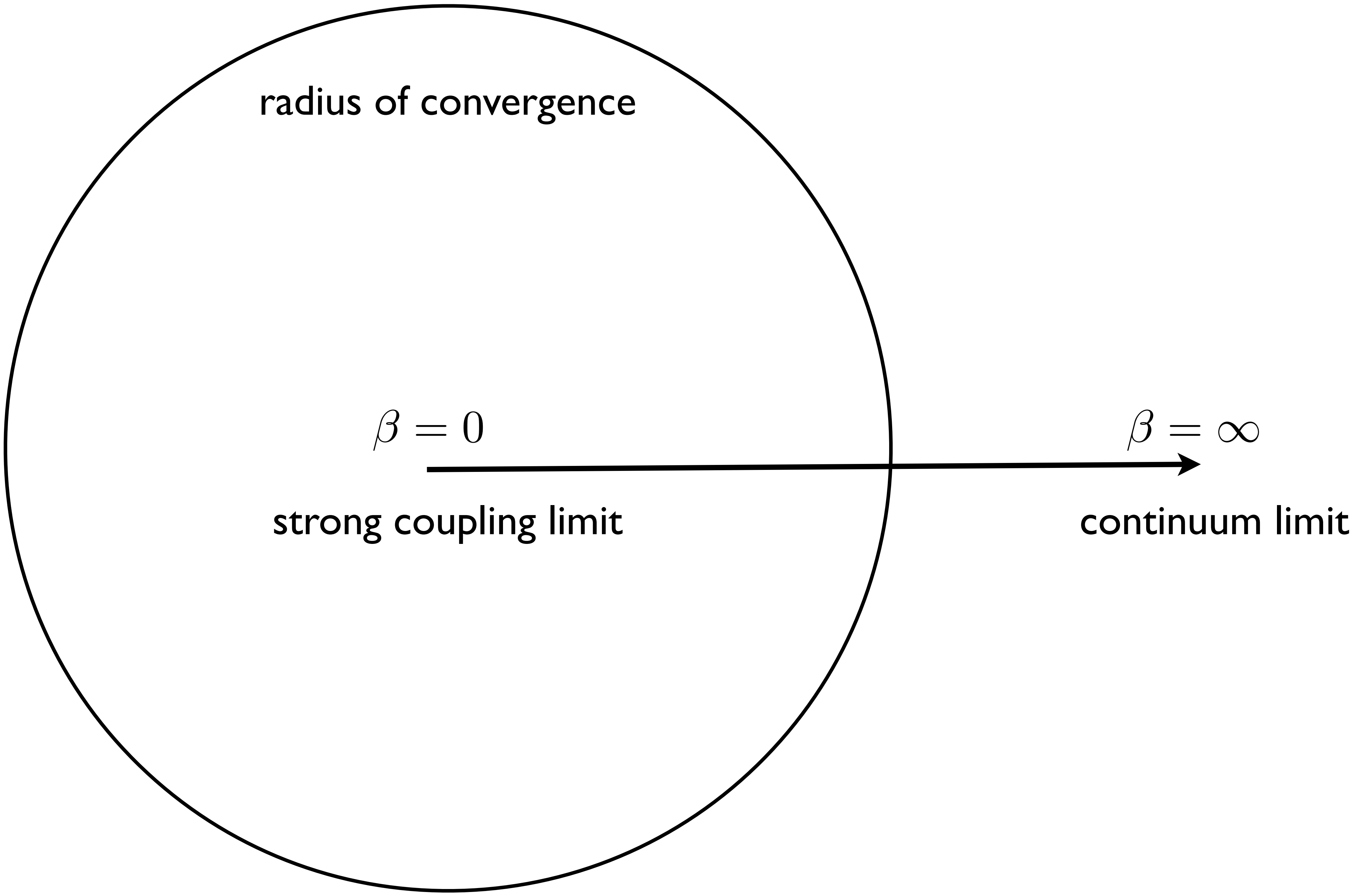
Backup slides

How is this possible?

$\beta = 0$ → $\beta = \infty$

strong coupling limit → continuum limit

How is this possible?



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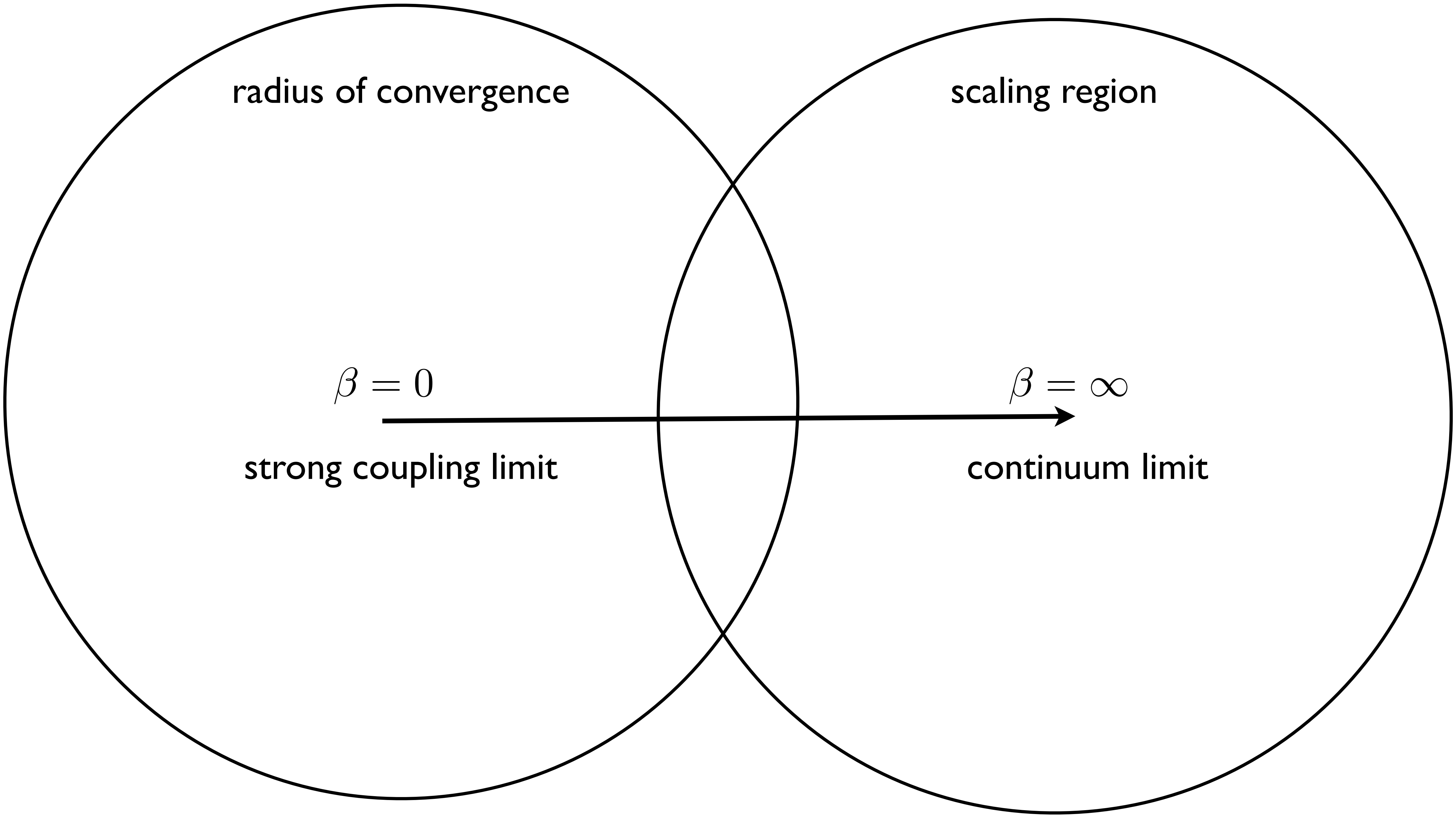

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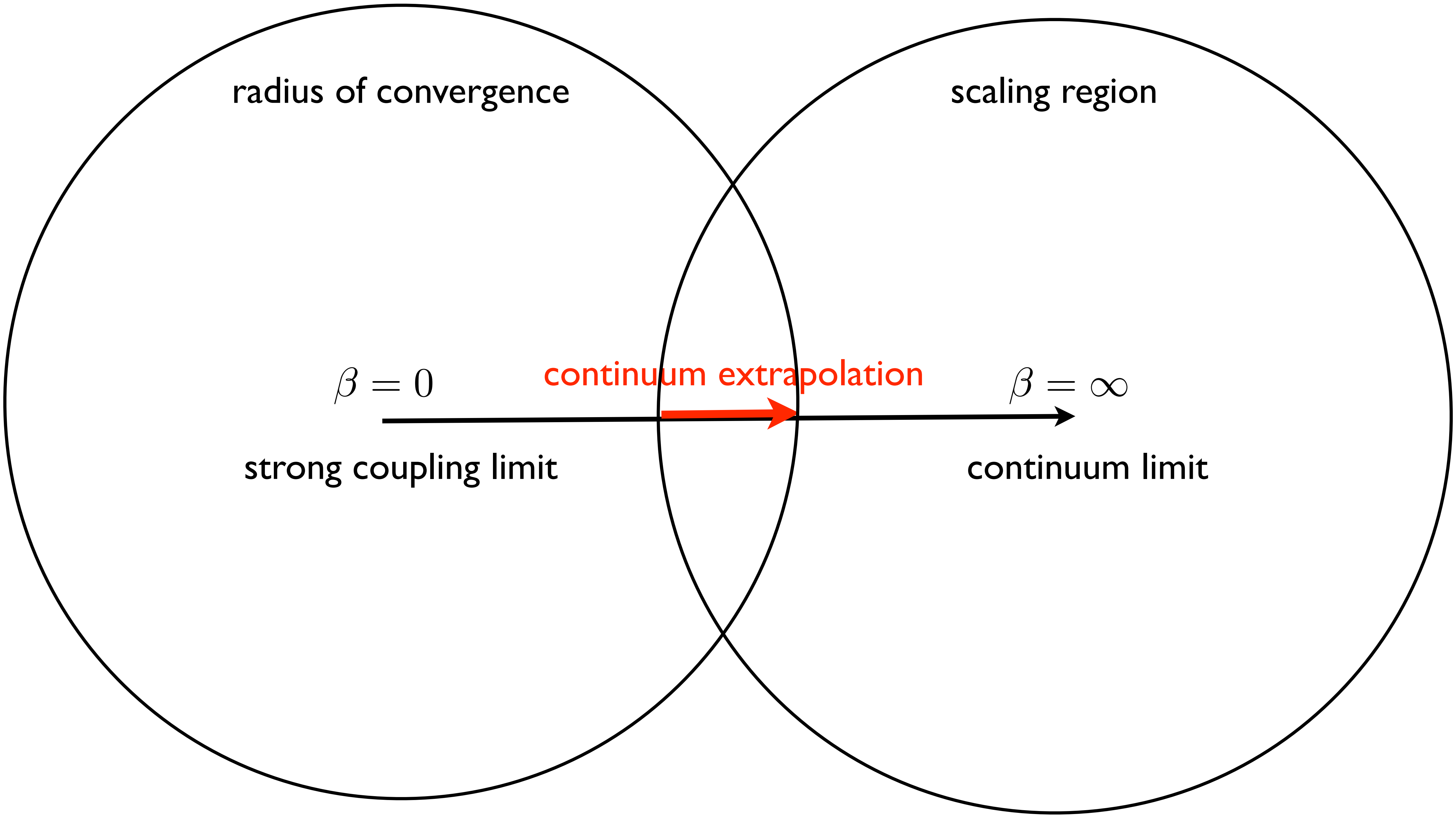

Observable to identify order of p.t.:

$$\delta B_Q = B_4(\delta Q) = \frac{\langle(\delta Q)^4\rangle}{\langle(\delta Q)^2\rangle^2}$$

$$B_4(x) = 1.604 + bL^{1/\nu}(x - x_c) + \ldots$$