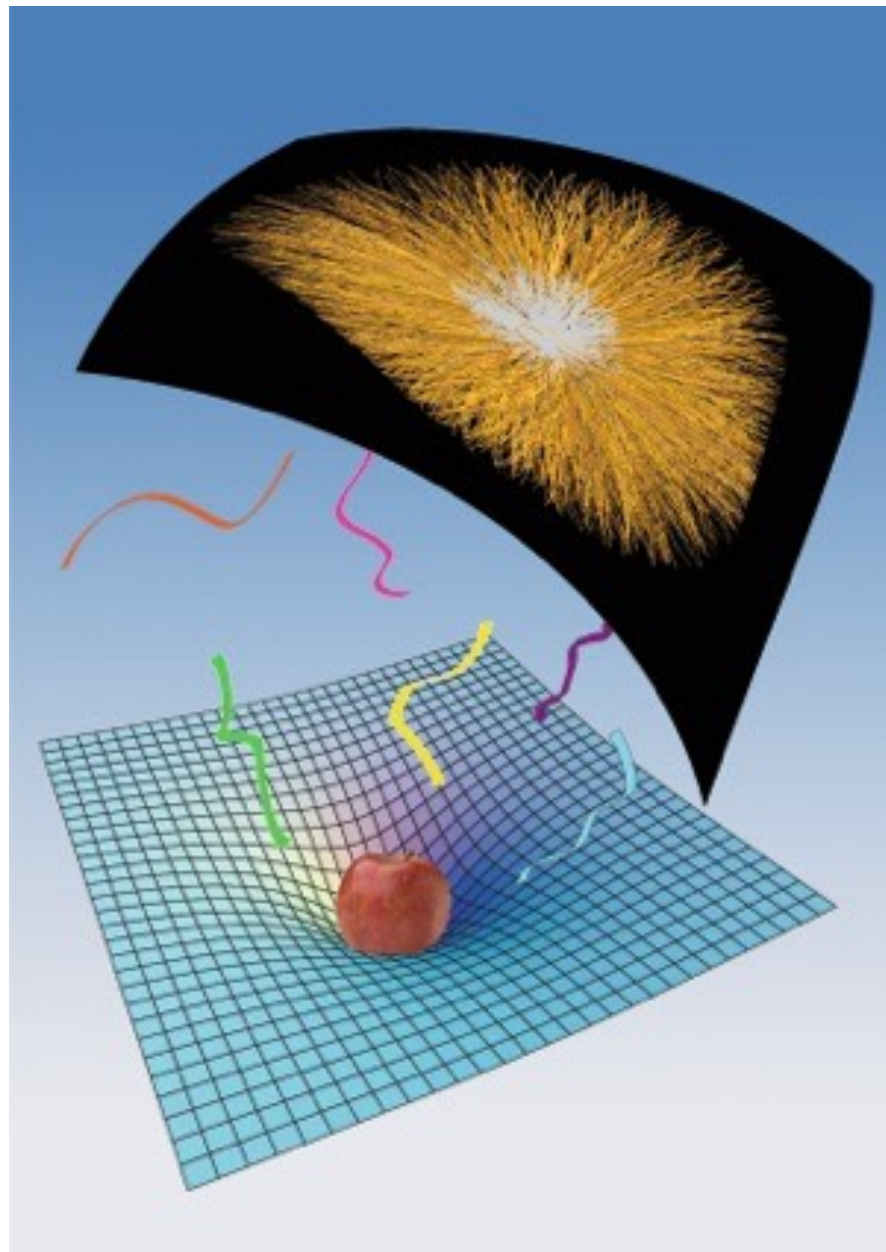


Holographic view on heavy quarks in hot and dense plasma



Carlo Ewerz

EMMI / GSI
& Heidelberg University

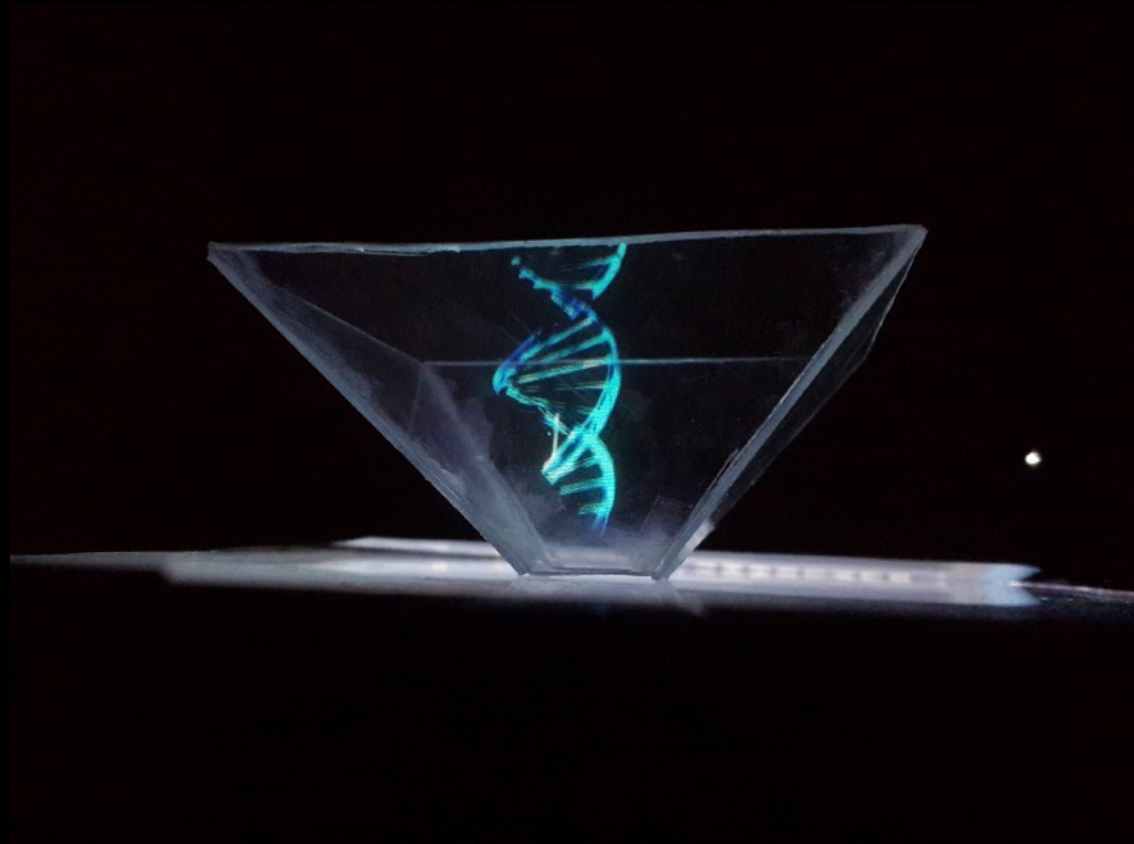
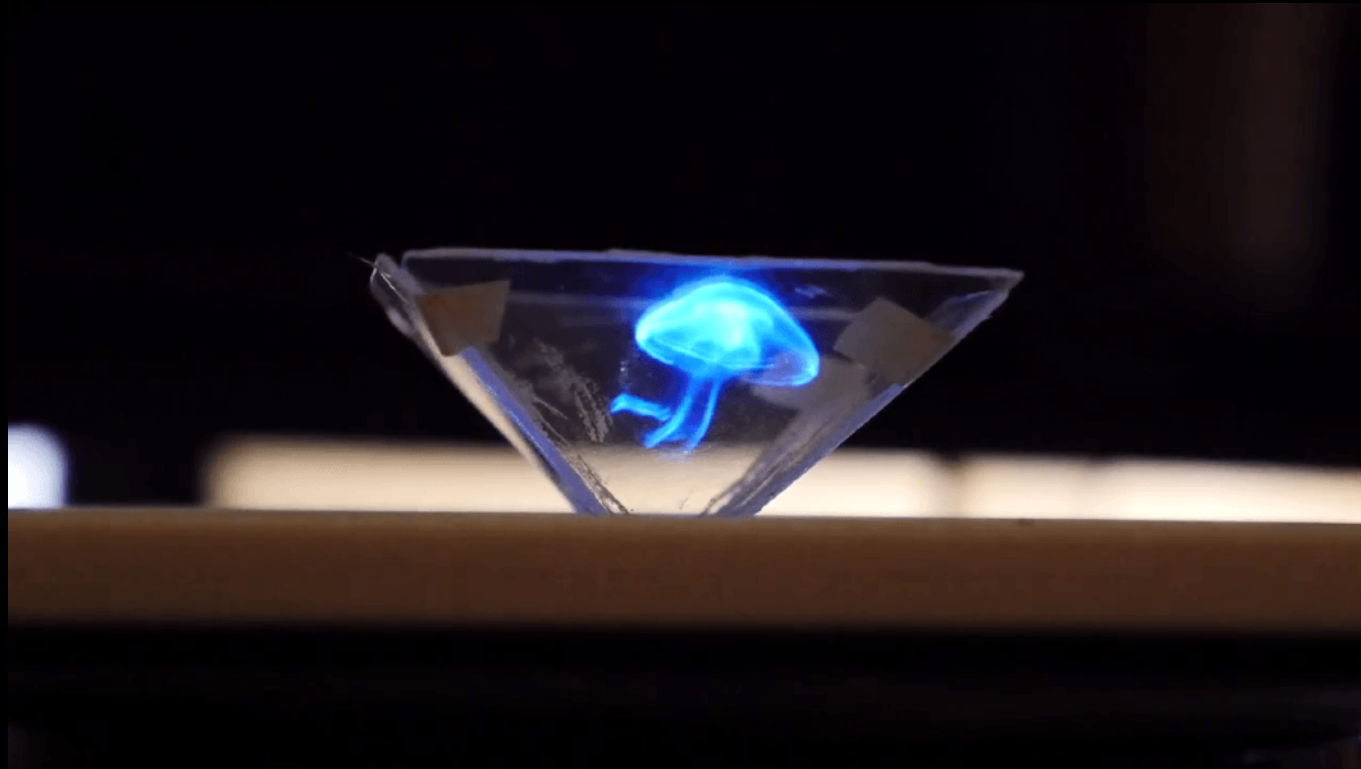


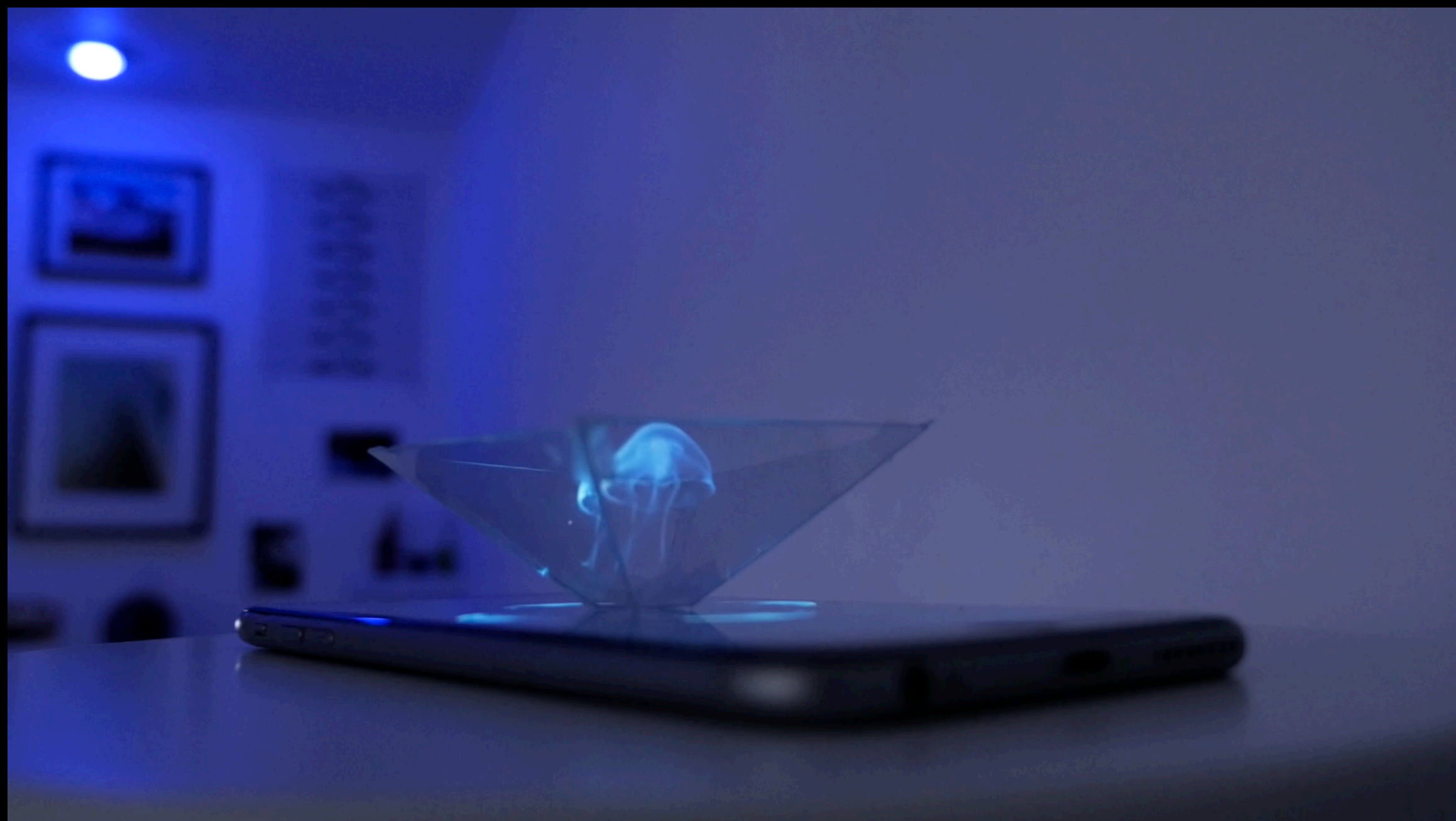
Unlocking the QCD phase diagram
Darmstadt
September 8, 2026

Holography

Holograms

Google:

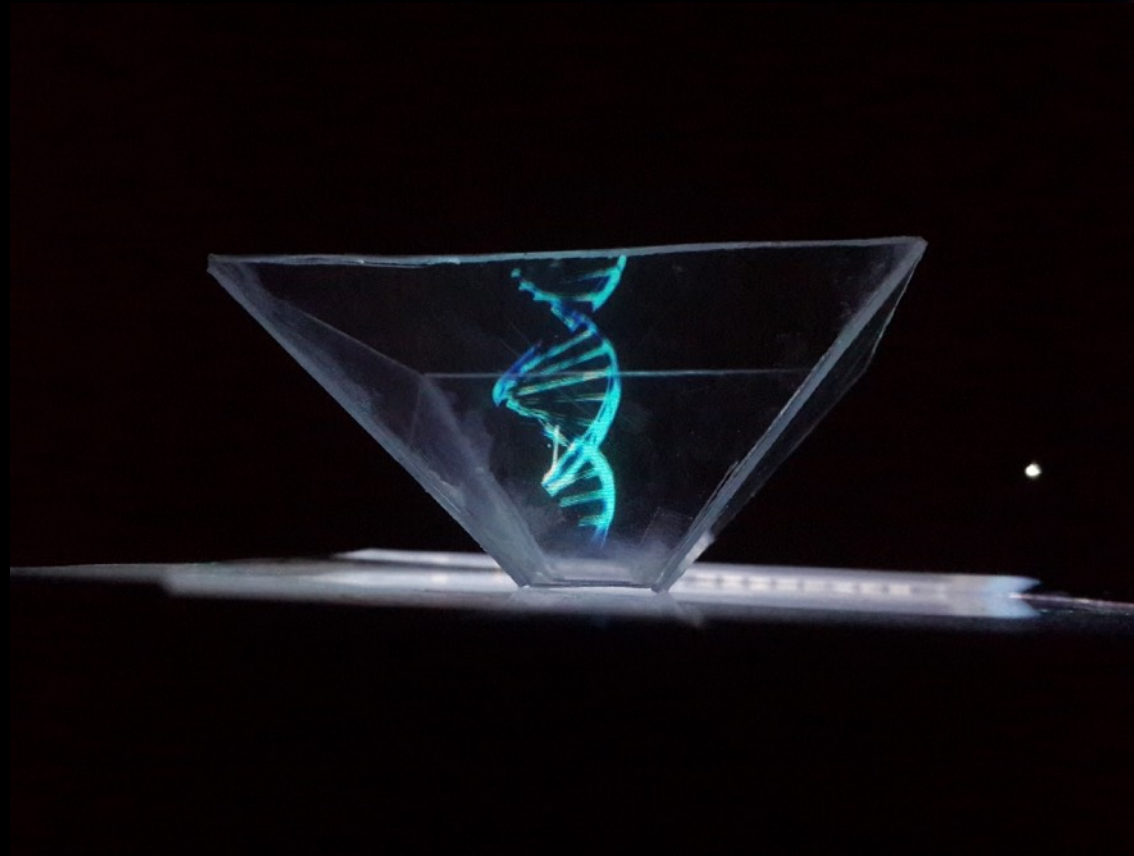
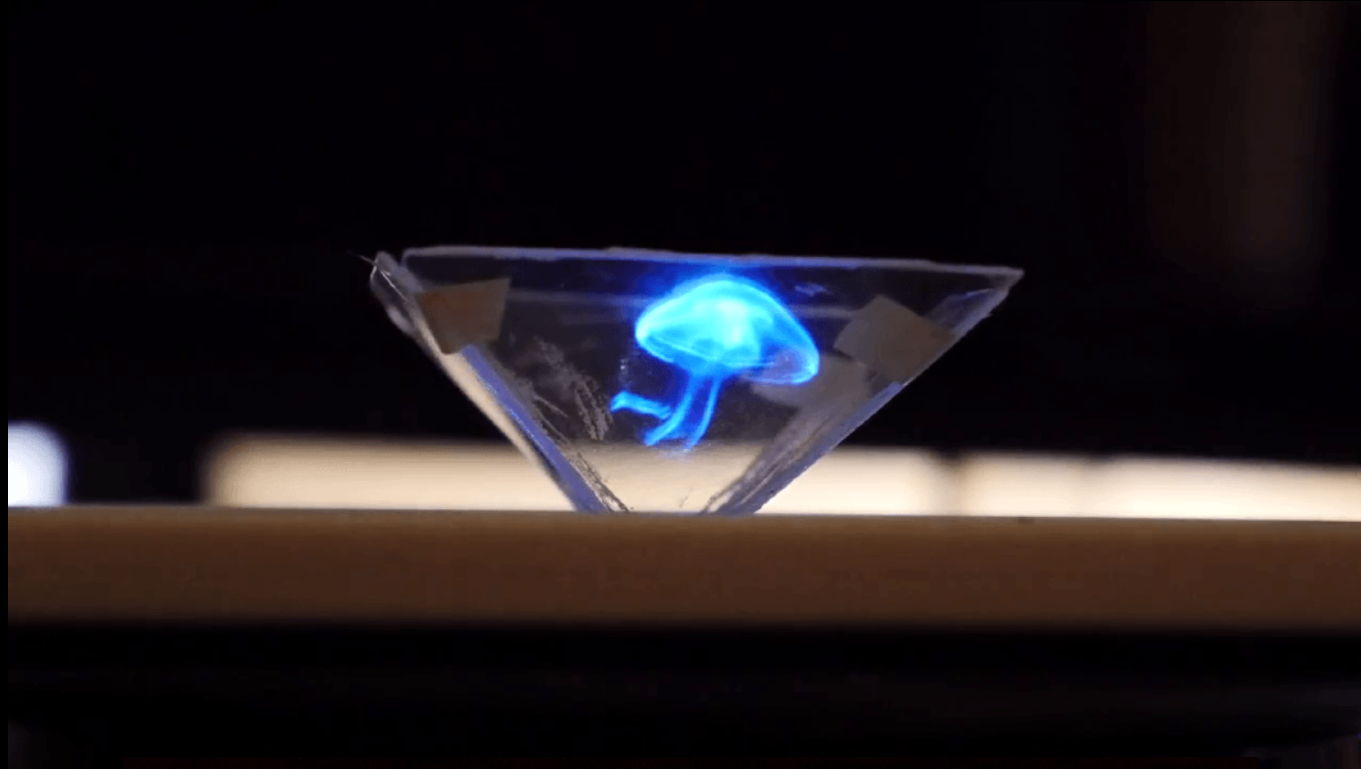




~~Holograms~~

Pepper's Ghost

Google:



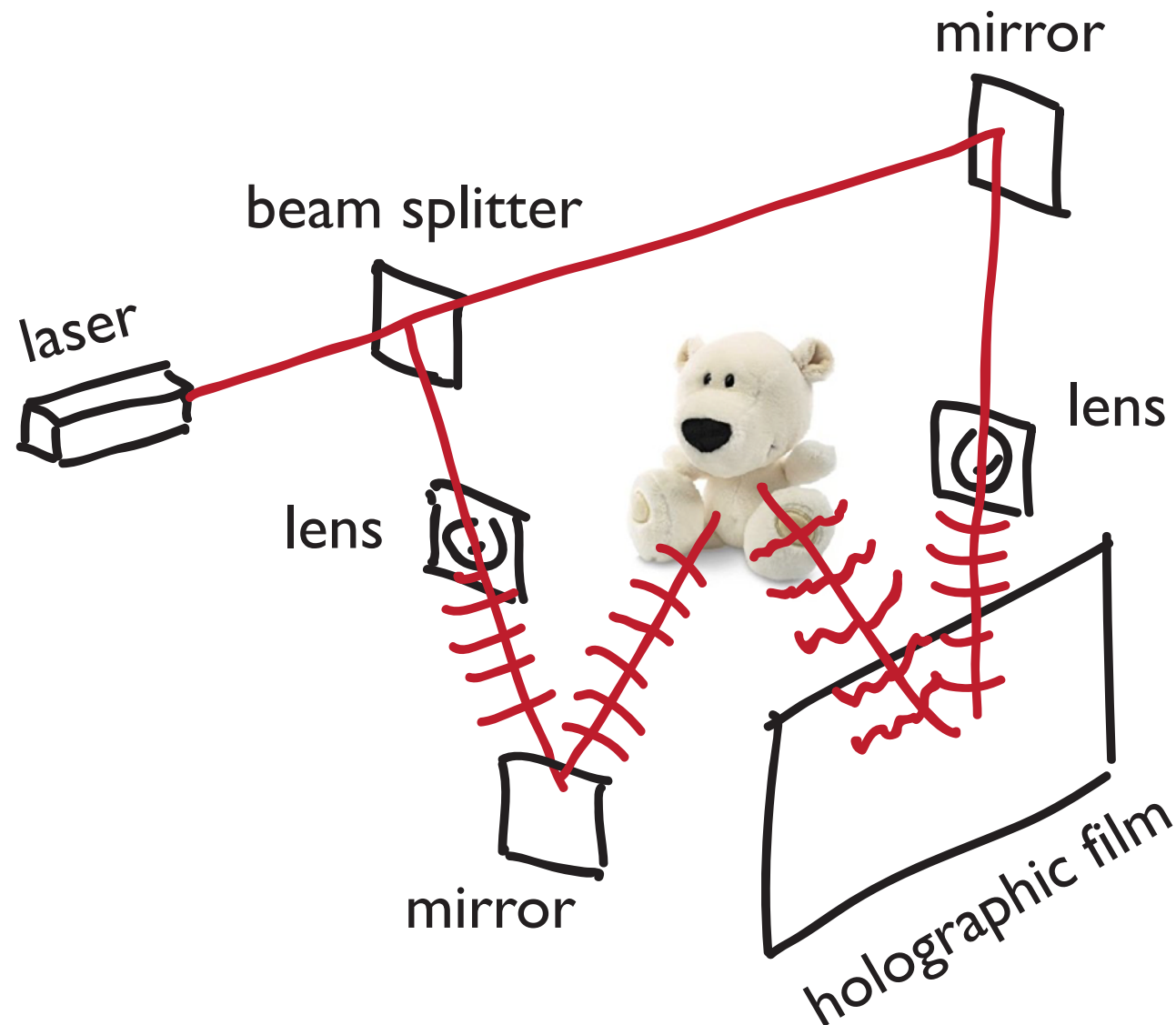
Pepper's Ghost

Reflection and illusion effect:

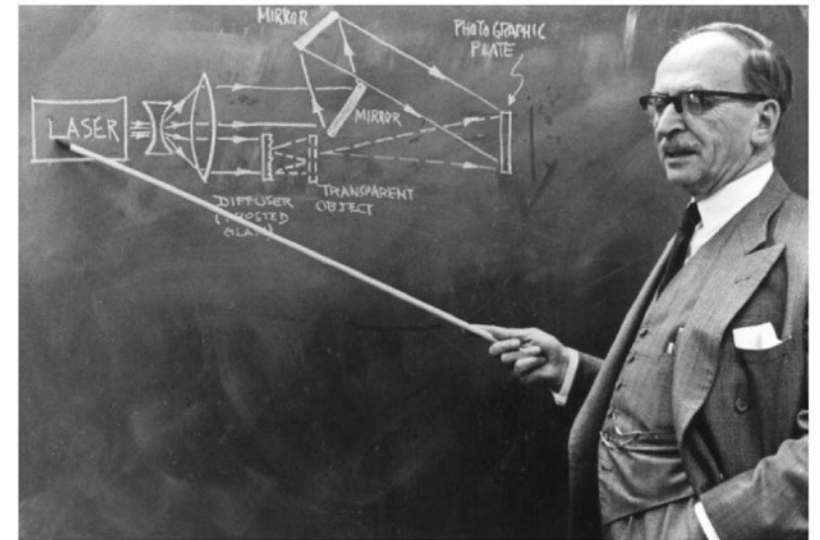


Holography

Interference with reference wave!



2d picture contains full phase information, and thus 3d information!



Dennis Gabor

potato variety, but no method is known by which it can spread in Nature. The case of paracrinkle is often quoted as evidence of the heterogenesis of viruses by those who hold this view. The third item was the analysis, for the first time, of a plant virus complex by differential methods of transmission, and the isolation of the two potato viruses now universally known as X and Y.

In 1931 the virus of tomato spotted wilt was discovered for the first time in Europe; it was found in an ornamental plant sent to Cambridge from Cardiff. Before this it had not been seen outside Australia. Since then the distribution of the virus has become world-wide, and in Great Britain it is one of the major problems of the tomato grower with 'mixed houses'.

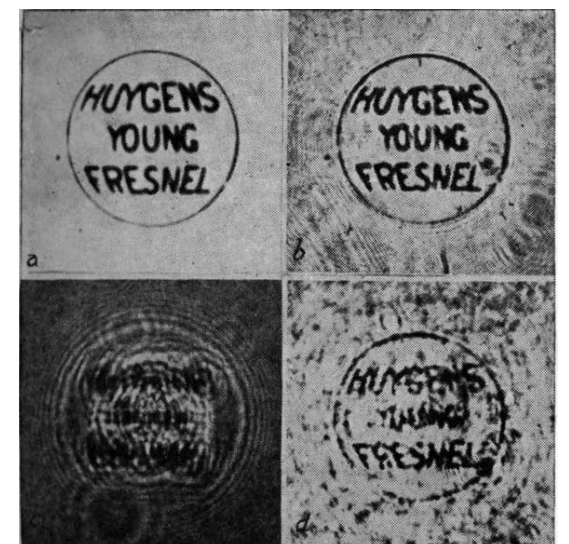
The viruses of tomato bushy stunt and tobacco necrosis, both described for the first time in Cambridge, have proved of great scientific interest. The virus of tomato bushy stunt, about which more is known than of most viruses, was the first to be isolated in a three-dimensional crystalline form, and this was accomplished by Bawden and Pirie, after the former had left Cambridge. Shortly after this the virus of tobacco necrosis was isolated as thin crystalline plates. About this time, also, the com-

A NEW MICROSCOPIC PRINCIPLE

By DR. D. GABOR
Research Laboratory, British Thomson-Houston Co., Ltd.,
Rugby

IT is known that the spherical aberration of electron lenses sets a limit to the resolving power of electron microscopes at about 5 Å. Suggestions for the correction of objectives have been made; but these are difficult in themselves, and the prospects of improvement are further aggravated by the fact that the resolution limit is proportional to the fourth root of the spherical aberration. Thus an improvement of the resolution by one decimal would require a correction of the objective to four decimals, a practically hopeless task.

The new microscopic principle described below offers a way around this difficulty, as it allows one to dispense altogether with electron objectives. Micrographs are obtained in a two-step process, by electronic analysis, followed by optical synthesis, as in Sir Lawrence Bragg's 'X-ray microscope'. But



Holographic Duality

AdS/CFT

AdS/CFT Correspondence

Juan Maldacena, 1997

Discovery made with help of string theory

Duality between AdS and CFT:

Anti-de-Sitter space and **Conformal Field Theory**

AdS/CFT Correspondence

quantum field theory without gravity
in $3+1$ dimensions (CFT)

dual to

gravity theory in
 $4+1$ dimensions (AdS)

- not yet strictly proven, but successfully tested in many special cases
- independent of question whether string theory is fundamental theory of nature

AdS/CFT - a Strong-Weak Duality

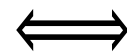
AdS/CFT correspondence maps **strong** coupling in one theory to **weak** coupling in the other, and vice versa.

Difficult problems are mapped to simple problems:
weakly coupled gravity describes strongly coupled gauge theory!

AdS/CFT - the Full Conjecture

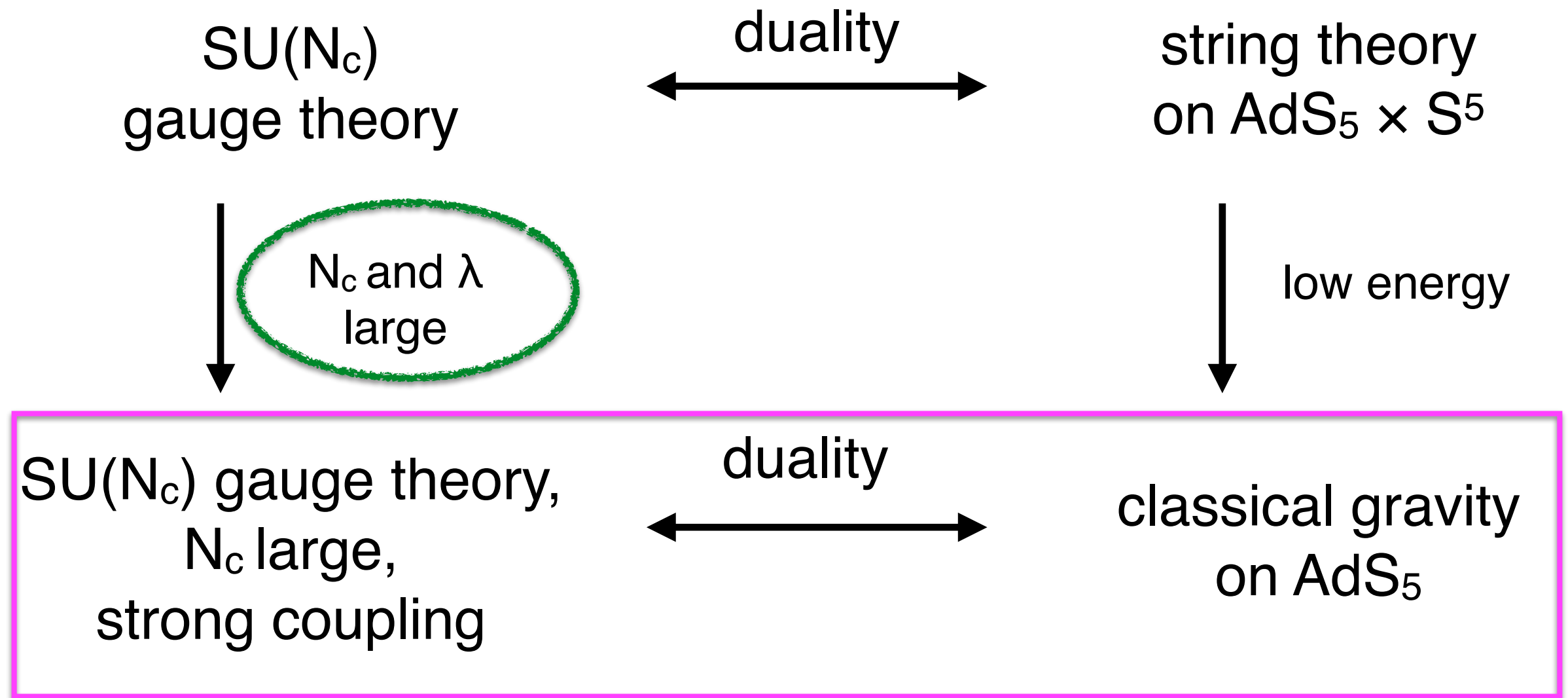
Maldacena

$\mathcal{N} = 4$ super Yang-Mills $SU(N)$ theory
in $3+1$ dimensions



Typ IIB string theory on $AdS_5 \times S^5$

Simplification in Limits



number of colors N_c , 't Hooft coupling $\lambda = g^2 N_c$

Classical gravity describes quantized gauge theory!

AdS Space, Temperature and Black Holes

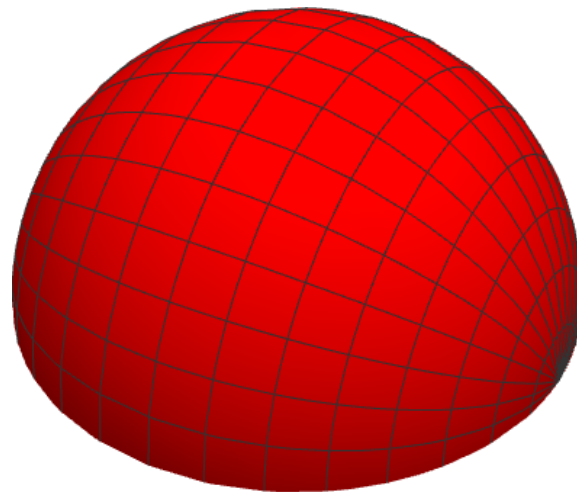
Anti-de-Sitter Space AdS₅

space-time with 4+1 dimensions with constant negative curvature

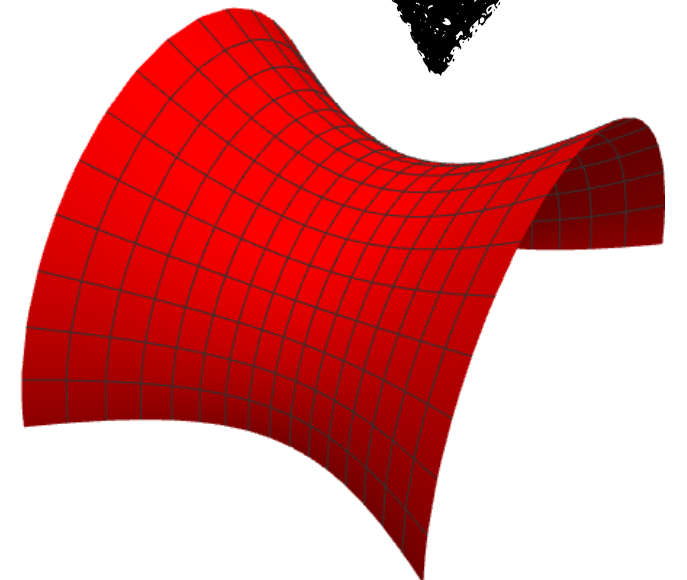
solution of Einstein equations of general relativity with negative cosmological constant

$$S = \frac{1}{16\pi G_N^{(D)}} \int d^D x \sqrt{-g} (\mathcal{R} - 2\Lambda)$$

$$2\Lambda = -\frac{12}{L_{\text{AdS}}}$$



positive curvature



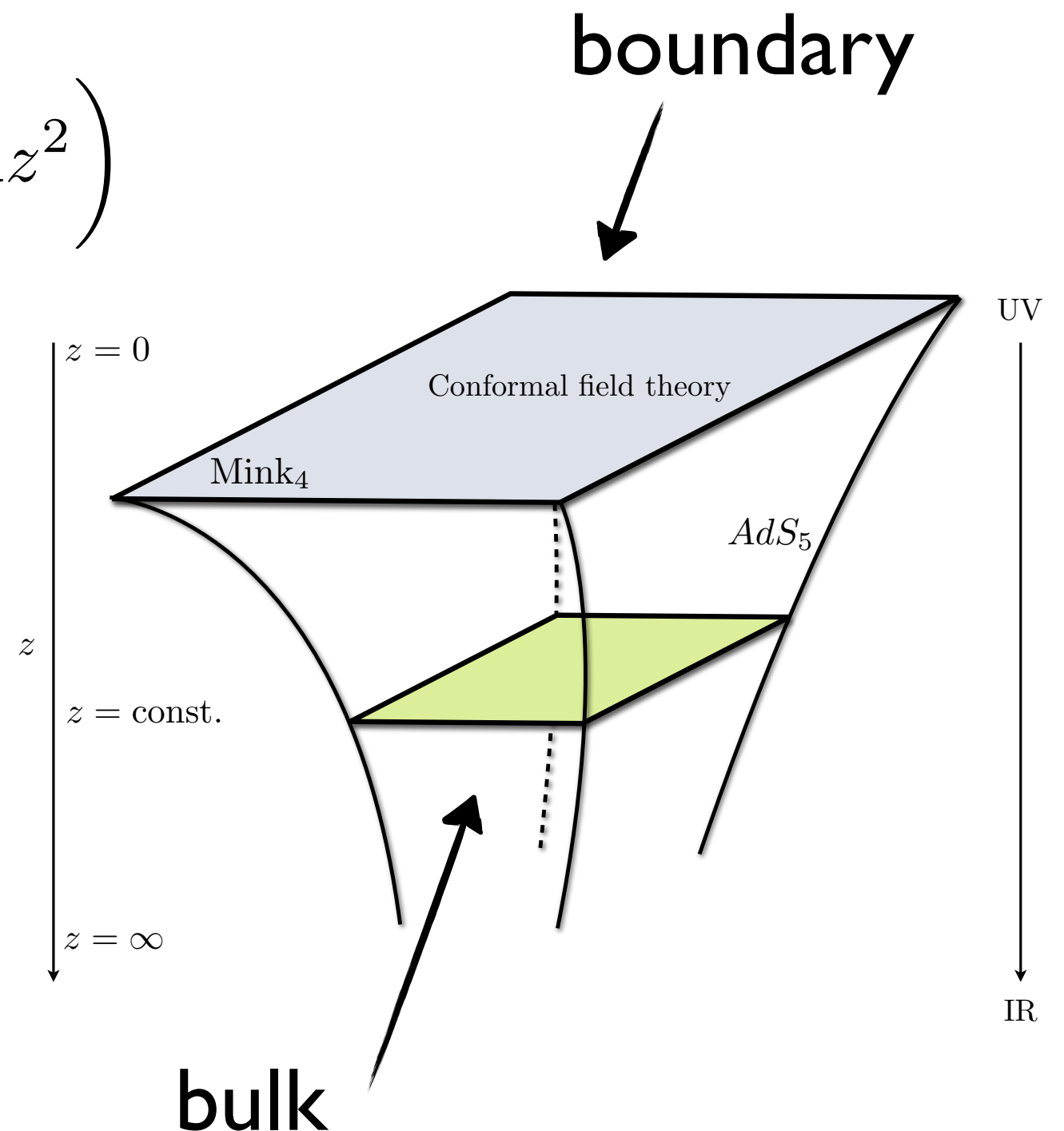
negative curvature

Anti-de-Sitter Space AdS_5

AdS_5 metric

$$ds^2 = \frac{L^2}{z^2} \left(-dt^2 + d\vec{x}^2 + dz^2 \right)$$

Minkowski space
for any fixed z



Holographic Dictionary

Gubser, Klebanov, Polyakov
Witten

1-to-1 translation of AdS- to CFT quantities

examples:

observables in CFT as boundary values of
fields in AdS space

→ “Gauge theory lives on boundary of AdS space”

Finite Temperature

CFT with temperature in AdS/CFT mapped
to AdS space with black hole,
more precisely: black brane

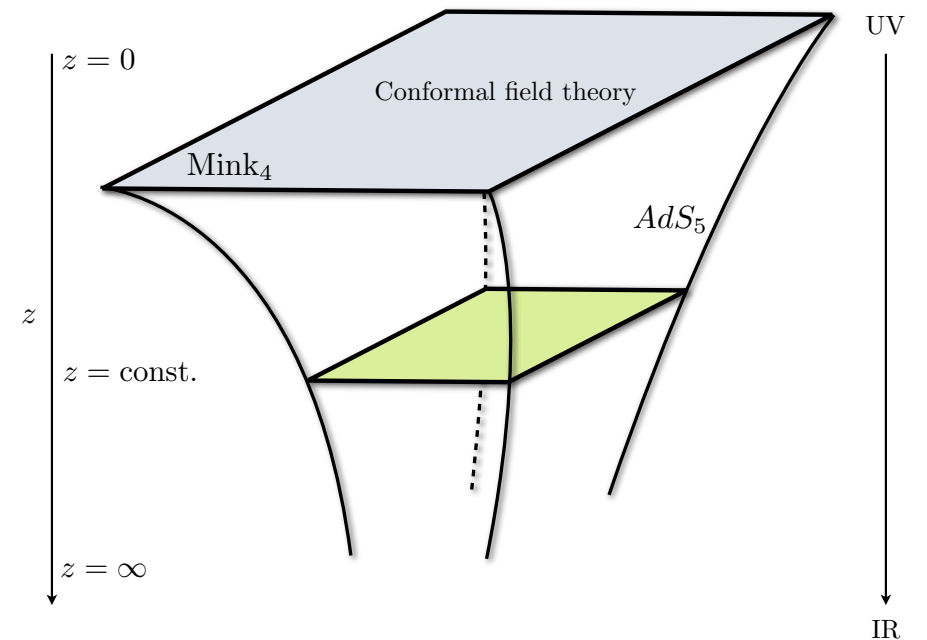
Physics of thermal systems completely encoded
in physics of black holes in higher dimensions!

Temperature of field theory equals
Hawking temperature of black hole.

Finite Temperature

zero temperature: AdS_5

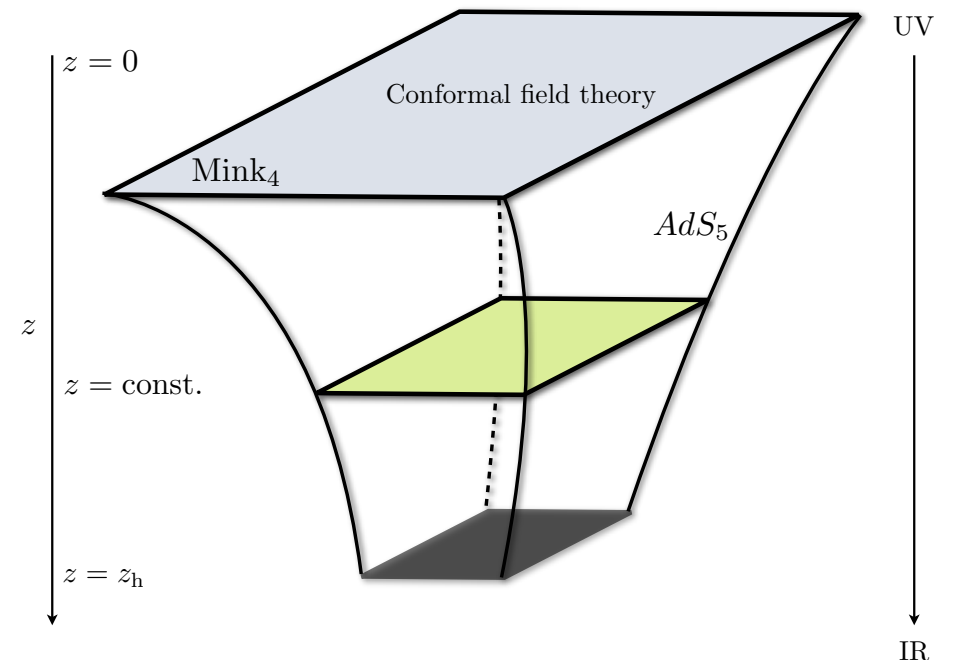
$$ds^2 = \frac{L^2}{z^2} \left(-dt^2 + d\vec{x}^2 + dz^2 \right)$$



finite temperature T :
 AdS_5 with **black hole**

$$ds^2 = \frac{L^2}{z^2} \left(-h(z)dt^2 + d\vec{x}^2 + \frac{dz^2}{h(z)} \right)$$

$$h(z) = 1 - \frac{z^4}{z_h^4} \qquad T = \frac{1}{\pi z_h}$$



Gauge-/Gravity Duality or Holography

Extensions of the Correspondence

CFT in AdS/CFT is supersymmetric and not realistic.

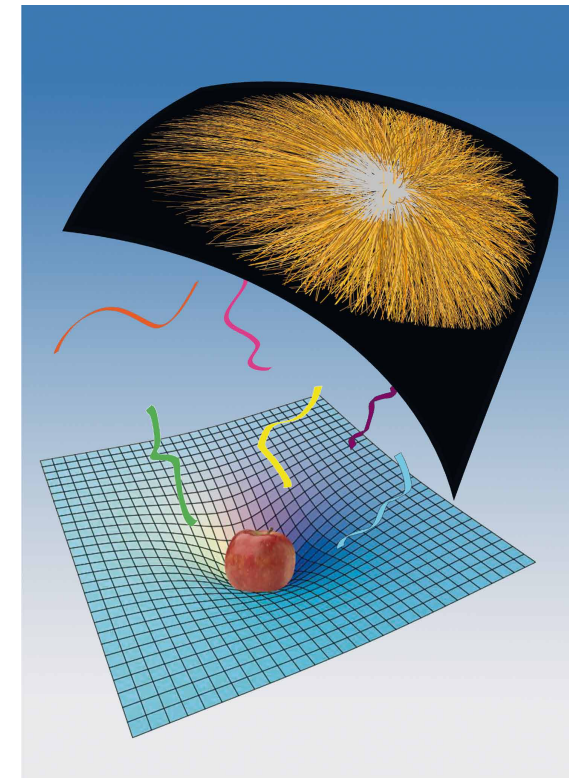
Many extensions of the correspondence:

- non-conformal
- spatially anisotropic
- with chemical potential
 - electrically charged black hole!
- non-relativistic
- other number of dimensions
 - Gauge-/Gravity Duality or Holography

Applications of Holography

Aspects of the Applications

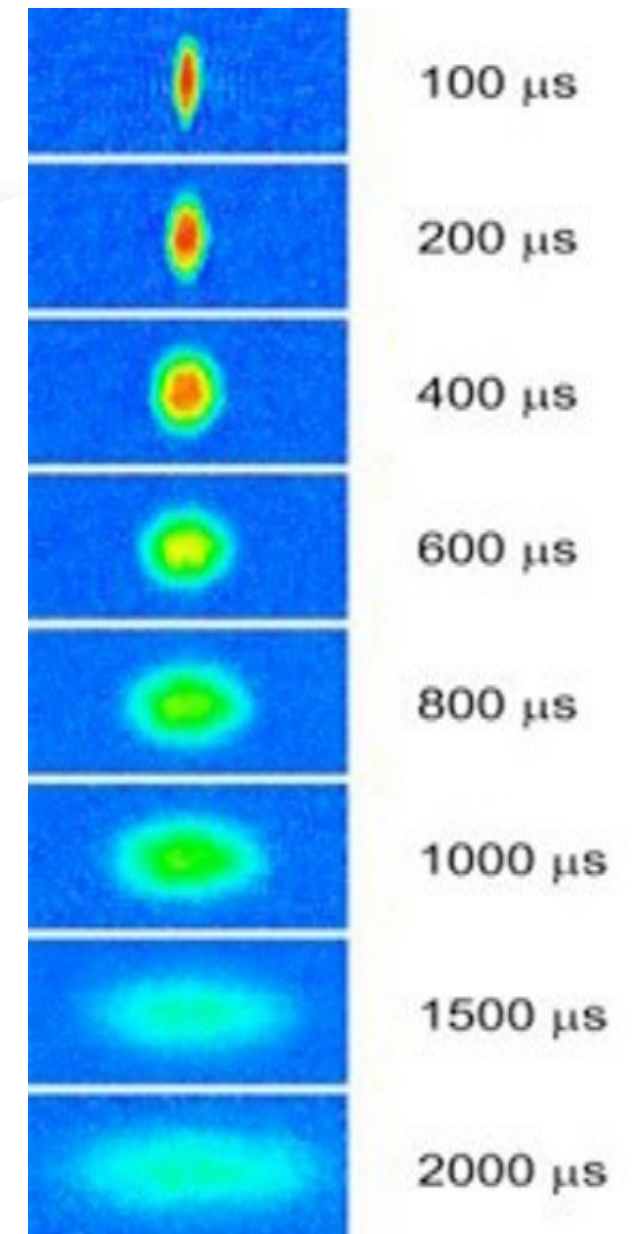
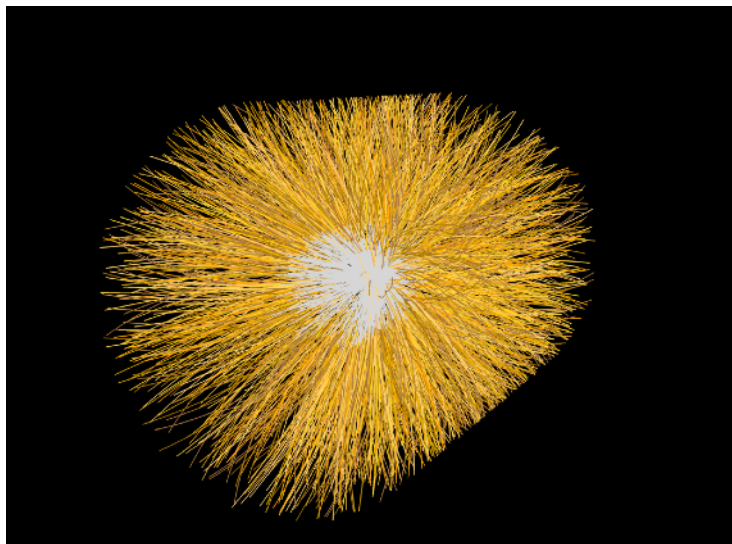
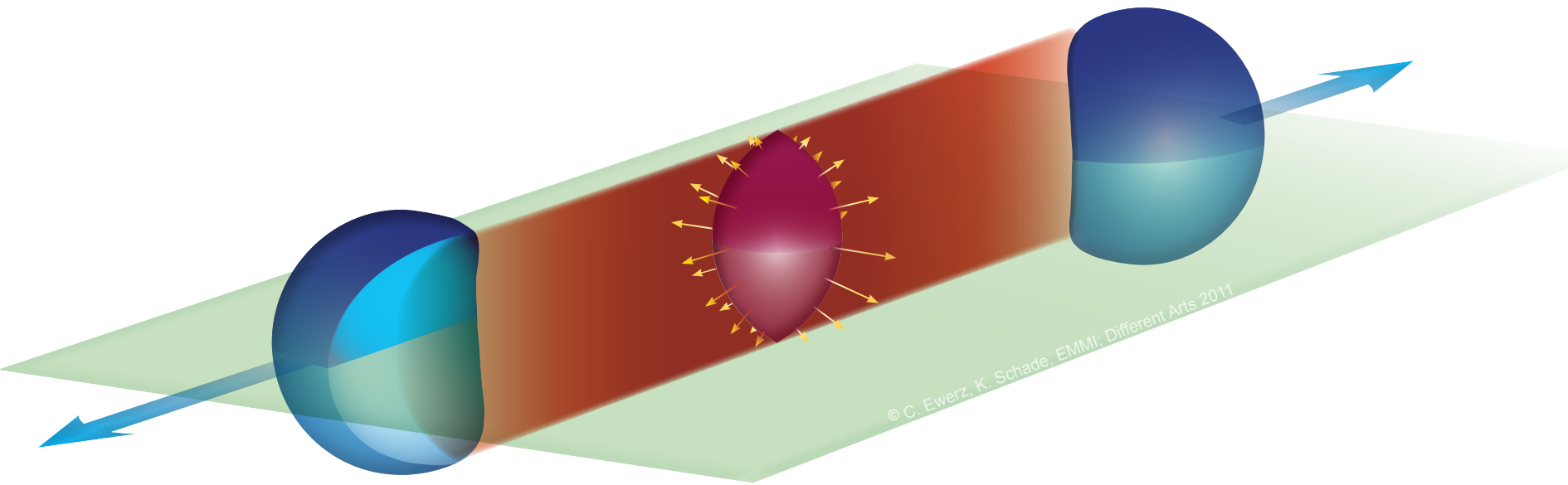
- applications in particle, nuclear, condensed matter physics
- model theories of strong coupling, search for new phenomena
- particularly successful for collective phenomena (microscopic theory often not essential)
- search for universal behavior in classes of holographic theories
- important observation:
universality of strongly coupled systems



Viscosity and Expansion

quark-gluon plasma

cold quantum gas



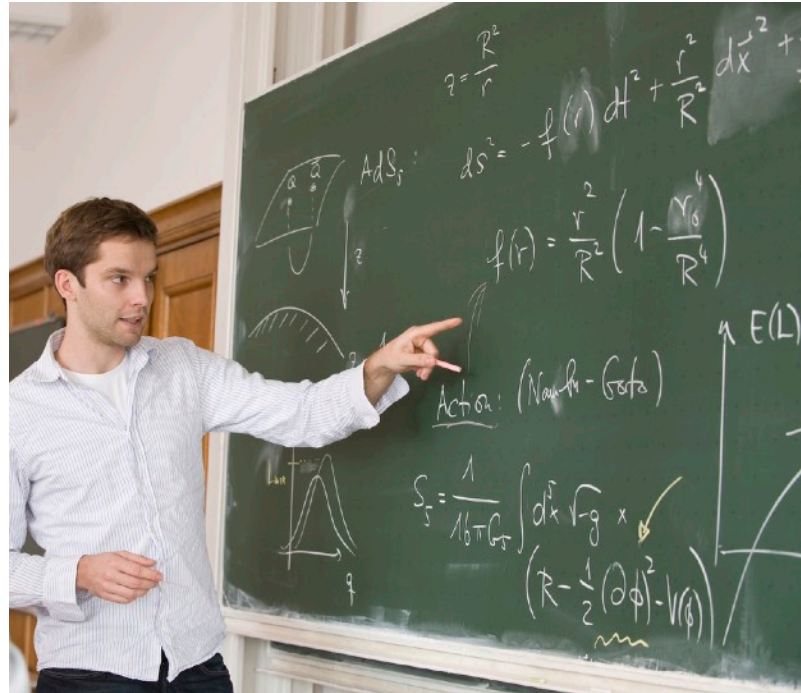
very small η/s , strongly coupled systems!

Key Problem for Applications of Holography

- Lagrangian, field content and parameters of dual field theory in general unknown (in particular for bottom-up models)
 - difficult to determine applicability to real-world systems or materials
 - parameters of dual field theory can sometimes be determined, first non-trivial example: holographic superfluid
- Wittmer, Schmied, Gasenzer, CE

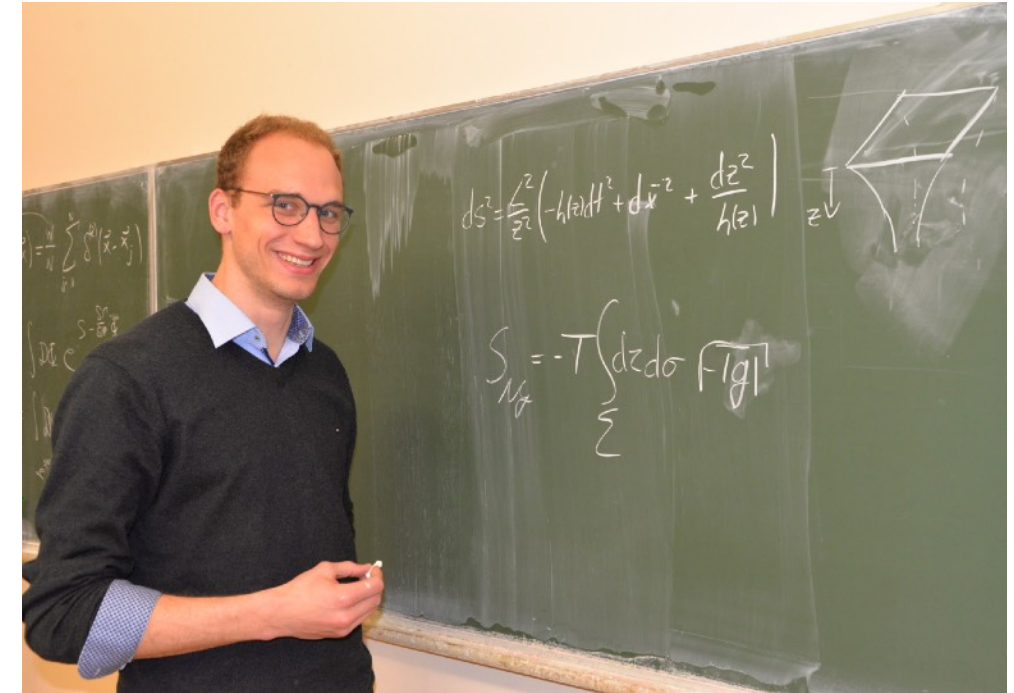
Holography for the Quark-Gluon Plasma: Heavy Quarks

some results from our group



Konrad Schade

The Team



Paul Wittmer



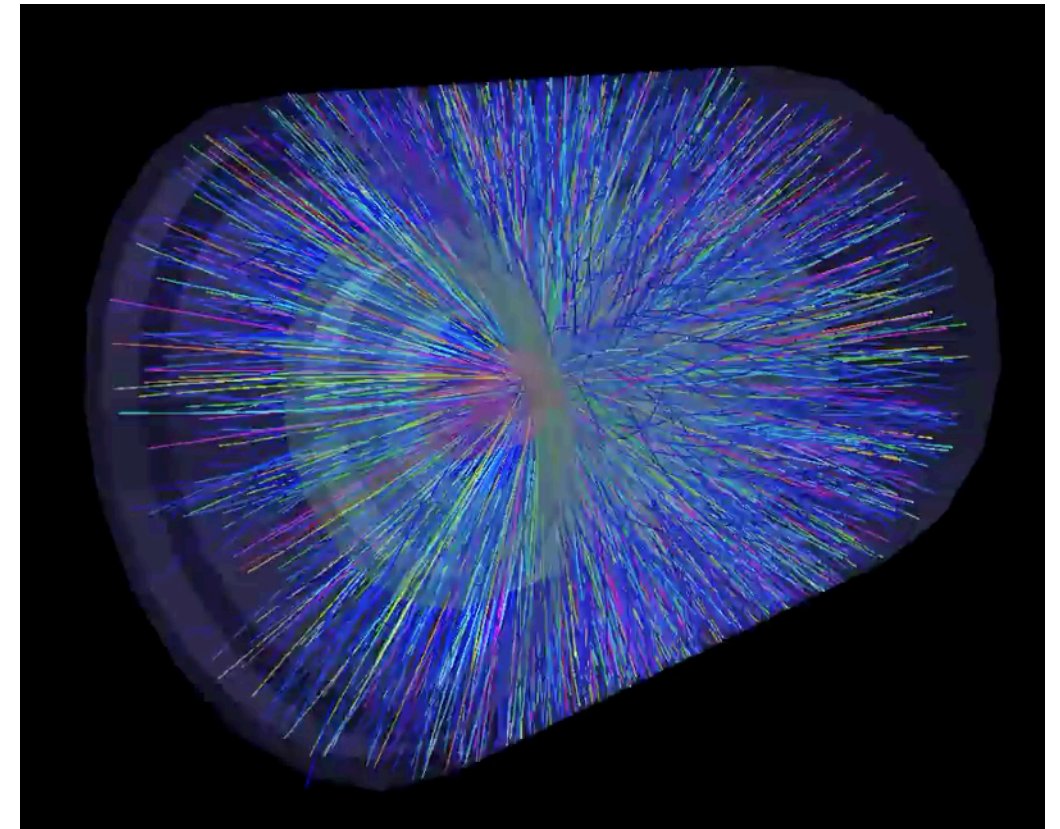
Andreas Samberg

Albert Bekov
Ling Lin
Enrico Brehm
Alexander Helmboldt

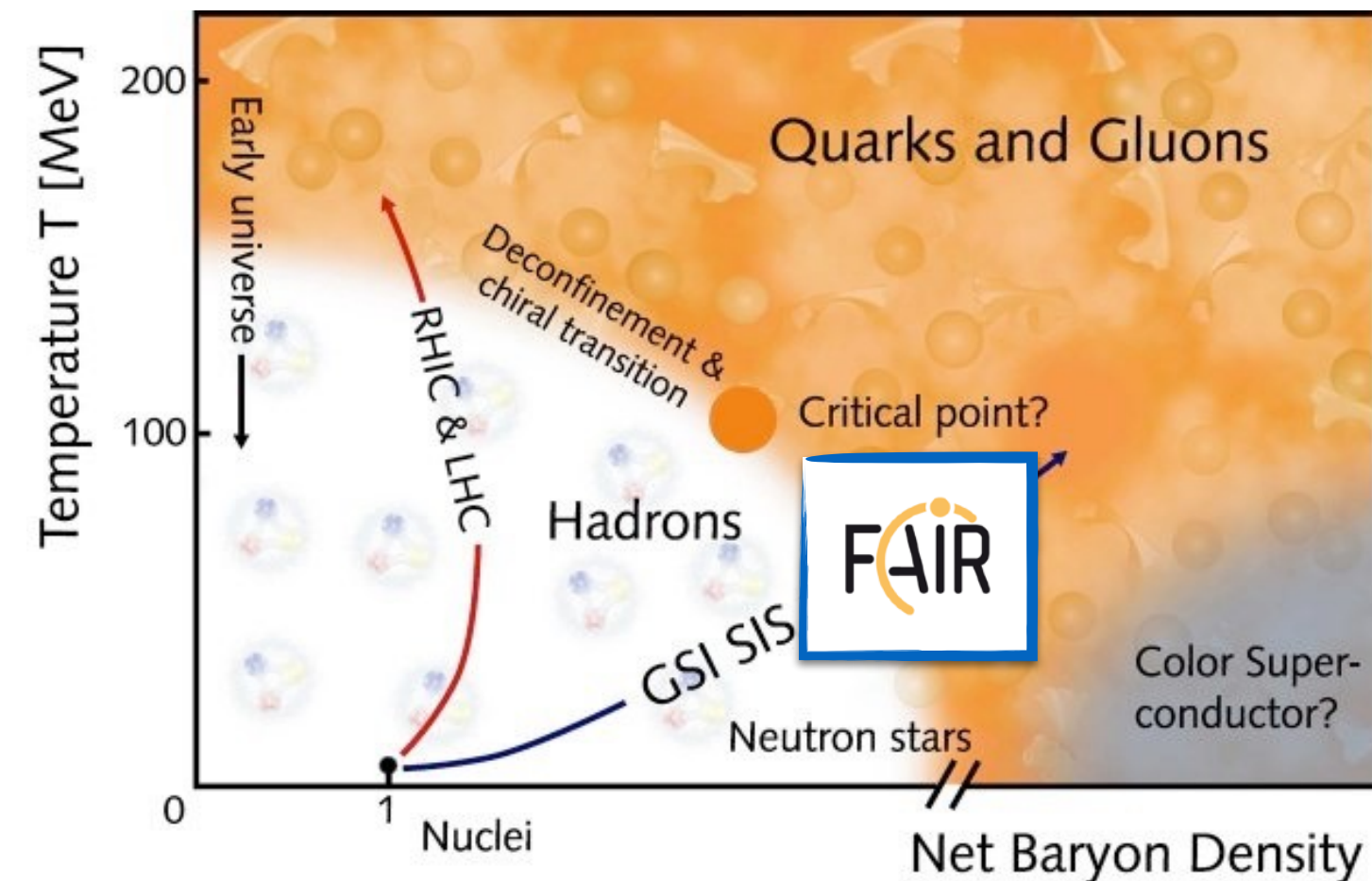
and with:
Olaf Kaczmarek

Motivation

quark-gluon plasma:
strongly coupled!
→ holography useful



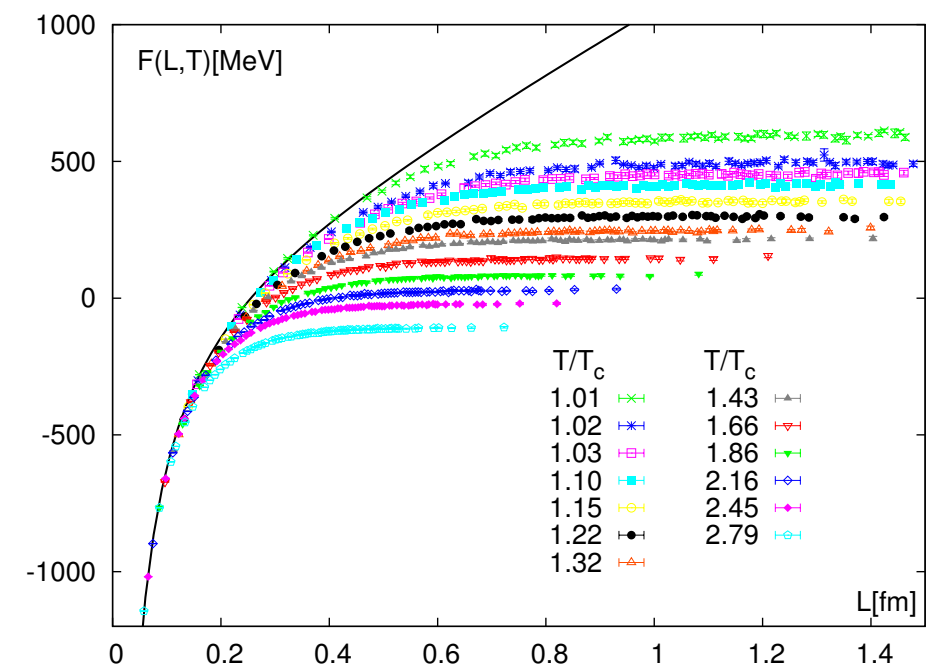
to approximate
real-world QCD
use non-conformal,
deformed AdS



Heavy Quarks in Quark-Gluon Plasma

Heavy quarks as probes of QGP:

- produced early in collision, witness full evolution of fireball to hadronization
- rich phenomenology in heavy-ion collisions
- heavy-quark bound states as sensitive probes of thermal medium
- well-studied in lattice QCD



Our aim

look for **universal** or **robust properties**
generically emerging in strongly coupled
theories

→ **classes** of holographic models

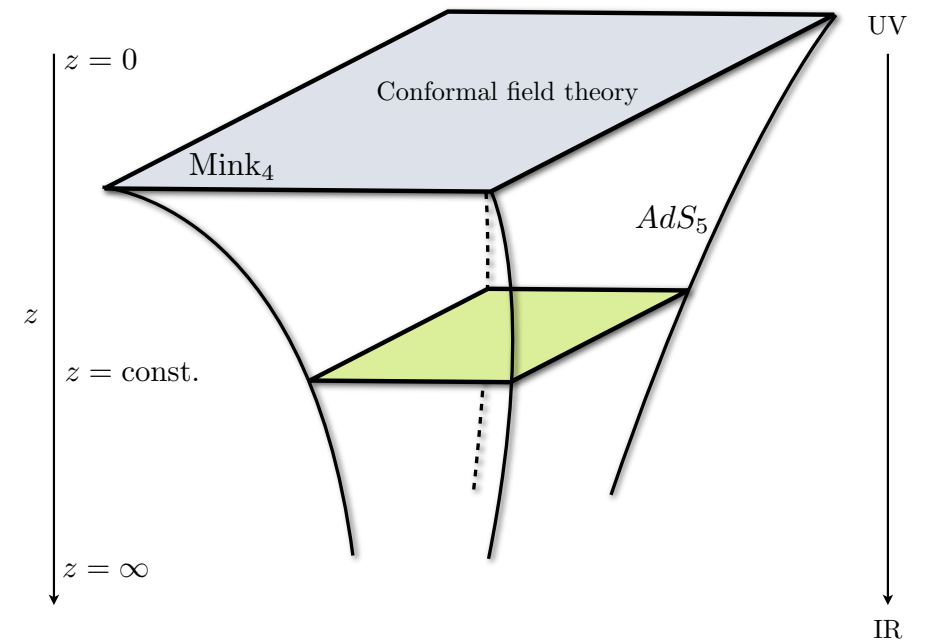
our aim is **not** to find a precise model for QCD

AdS Models for the Plasma

N=4 SYM at Finite Temperature

zero temperature: AdS_5

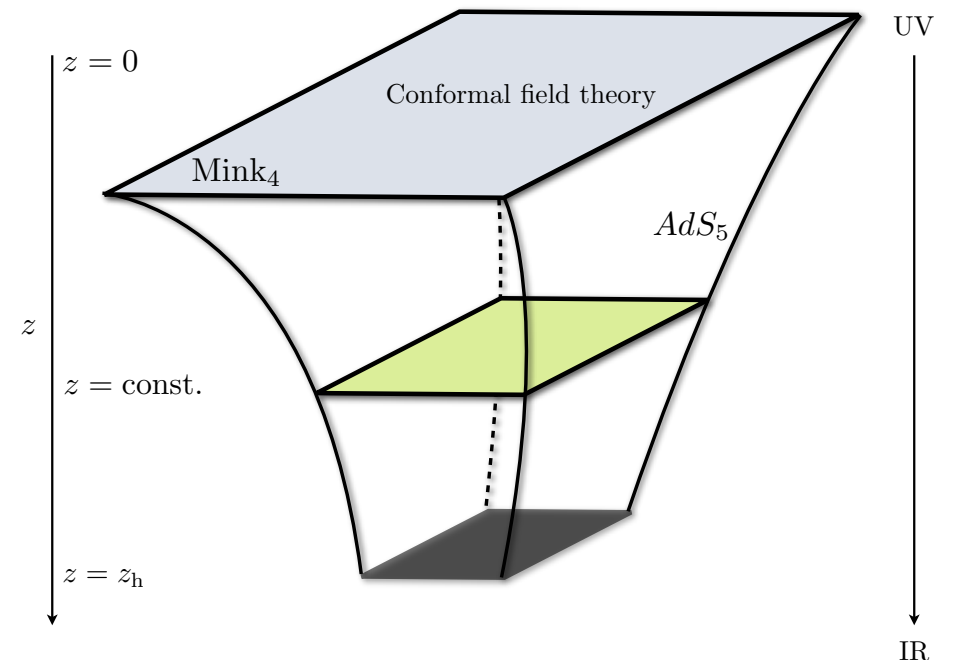
$$ds^2 = \frac{L^2}{z^2} \left(-dt^2 + d\vec{x}^2 + dz^2 \right)$$



finite temperature T:
 AdS_5 with black hole

$$ds^2 = \frac{L^2}{z^2} \left(-h dt^2 + d\vec{x}^2 + \frac{dz^2}{h} \right)$$

$$\text{with } h = 1 - \frac{z^4}{z_h^4} \text{ and } T = \frac{1}{\pi z_h}$$



Consistent Non-conformal Model

Start with five dimensional gravity action S_{EHs} :

$$S_{\text{EHs}} = \frac{1}{16\pi G_{\text{N}}^{(5)}} \int d^5x \sqrt{-G} \left(\mathcal{R} - \frac{1}{2} (\partial\Phi)^2 - V(\Phi) \right)$$

with general ansatz

$$ds^2 = e^{2A(z)} \left(-h dt^2 + d\vec{x}^2 \right) + e^{2B(z)} \frac{dz^2}{h}$$

$$T = e^{A(z_h) - B(z_h)} \frac{|h'(z_h)|}{4\pi}$$

leads to 3 independent equations of motion but 5 unknown functions V, Φ, A, B, h .

- 2-parameter model:
with parameters ϕ, c

$$e^{2A(z)} = e^{c z^2} \frac{L^2}{z^2} \quad \text{and} \quad \Phi(z) = \sqrt{\frac{3}{2}} \phi z^2$$

DeWolfe, Rosen; Gubser; 2009

$$\alpha = \frac{c}{\phi}$$

- 1-parameter model:
with parameter κ

$$e^{2A(z)} = \frac{L^2}{z^2} \quad \text{and} \quad \Phi(z) = \sqrt{\frac{3}{2}} \kappa z^2$$

Schade

Consistent Non-conformal Model

Start with five dimensional gravity action S_{EHs} :

$$S_{\text{EHs}} = \frac{1}{16\pi G_{\text{N}}^{(5)}} \int d^5x \sqrt{-G} \left(\mathcal{R} - \frac{1}{2} (\partial\Phi)^2 - V(\Phi) \right)$$

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$$T = e^{A(z_h) - B(z_h)} \frac{|h'(z_h)|}{4\pi}$$

scalar Φ :

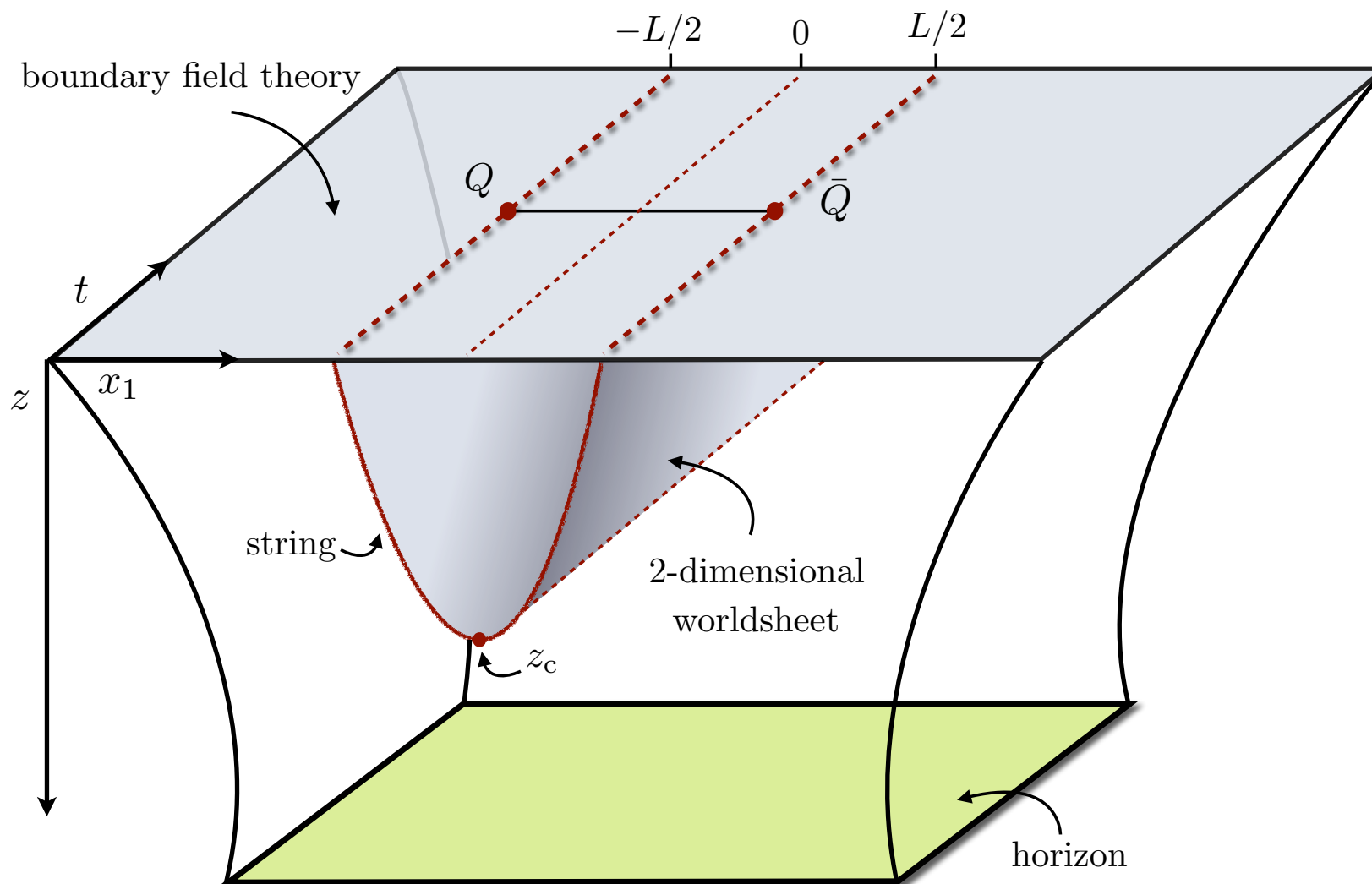
can be dilaton ('string frame model')
or not ('Einstein frame model')

We consider both possibilities as independent models.

Screening Distance

Konrad Schade, CE

Consider static (heavy) quark-antiquark pair



Expectation value of temporal Wegner-Wilson loop in boundary field theory dual to macroscopic string hanging into the bulk

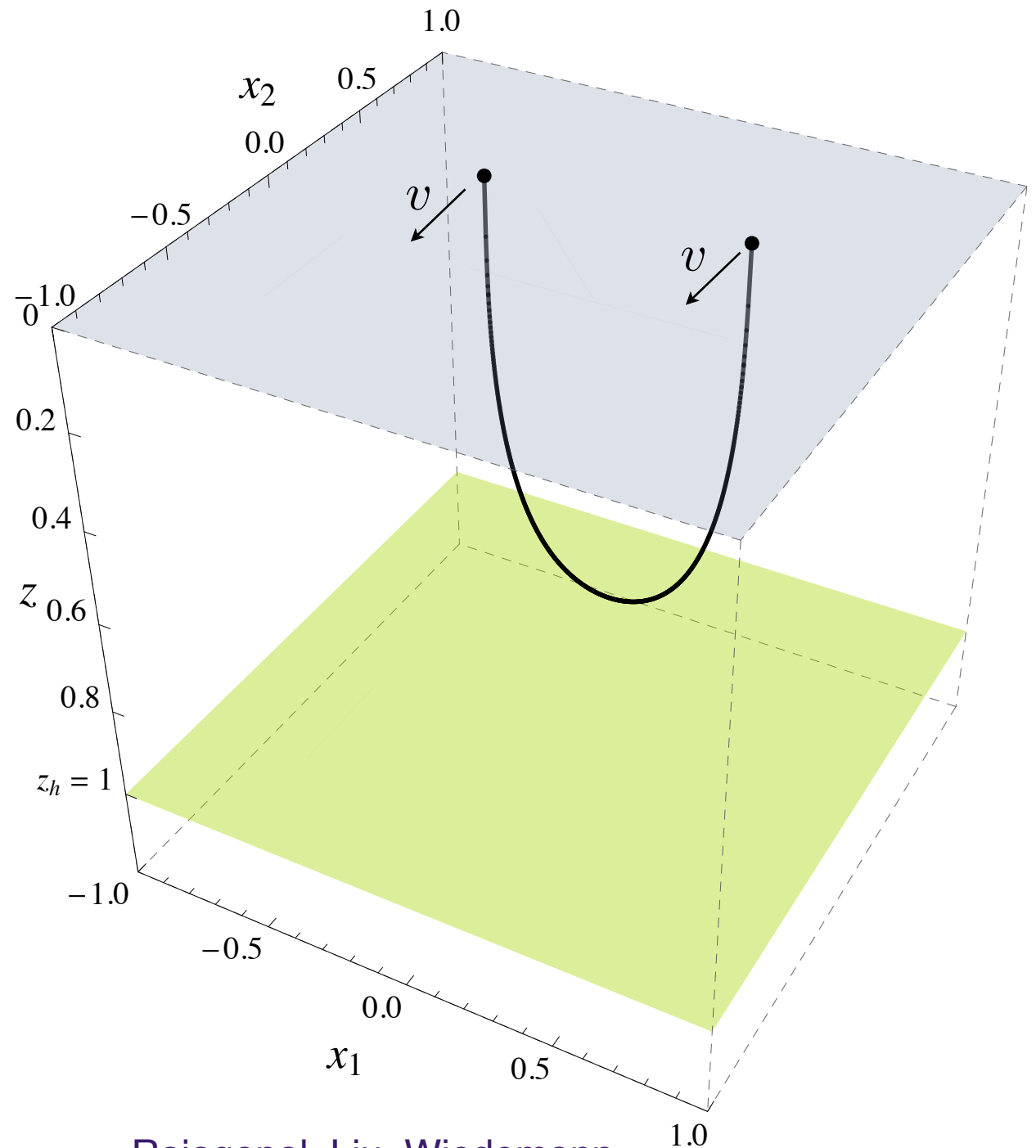
Screening Distance

- static $Q\bar{Q}$ -pair in a hot plasma wind blowing in x_2 -direction
- velocity is given by $v = \tanh \eta$
- orientation angle θ w.r.t. wind

- Nambu-Goto action:

$$S = \frac{1}{2\pi\alpha'} \int d\sigma d\tau \sqrt{-\det g_{\alpha\beta}}$$

with $g_{\alpha\beta} = G_{\mu\nu} \partial_\alpha x^\mu \partial_\beta x^\nu$

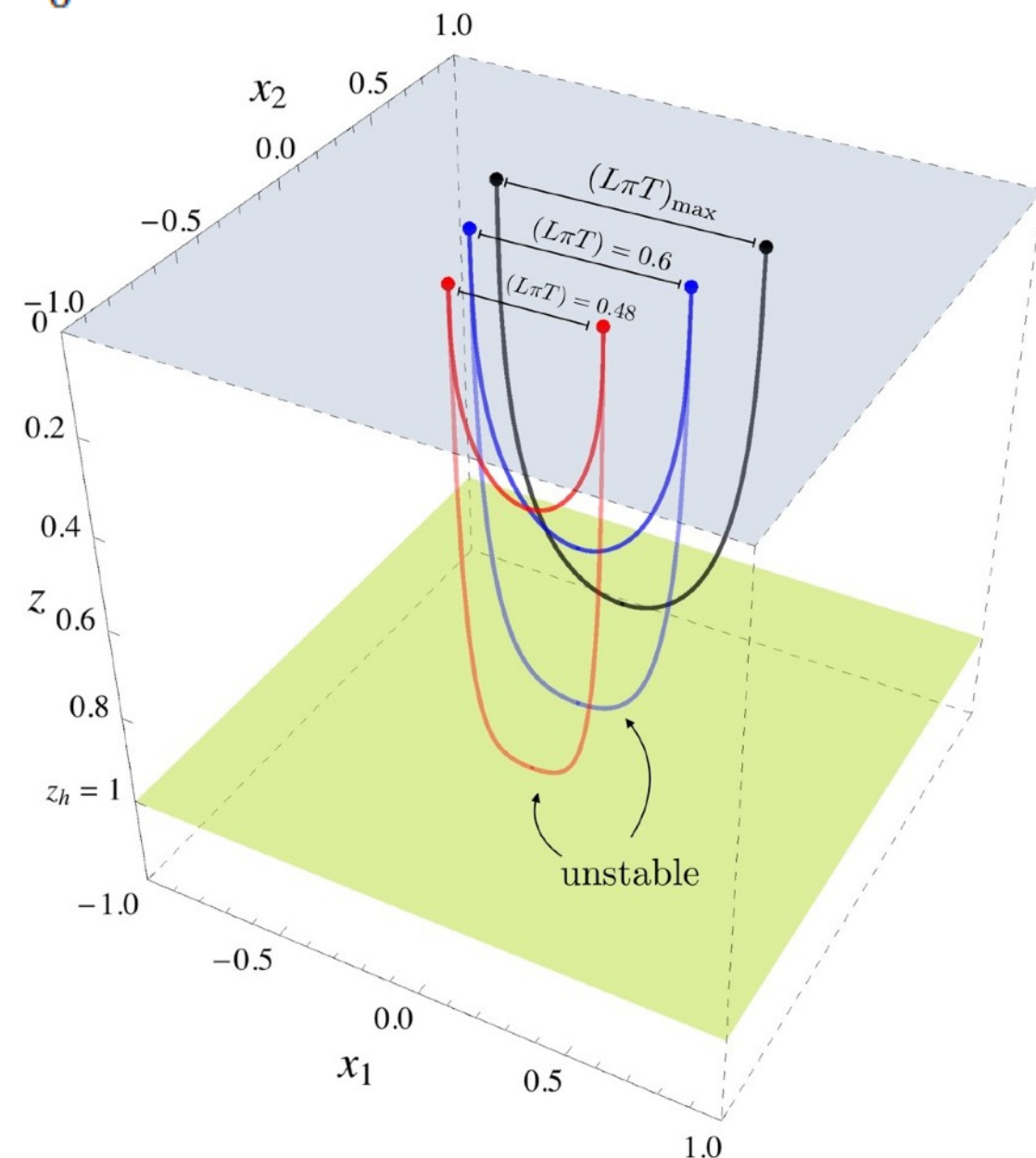


Screening Distance

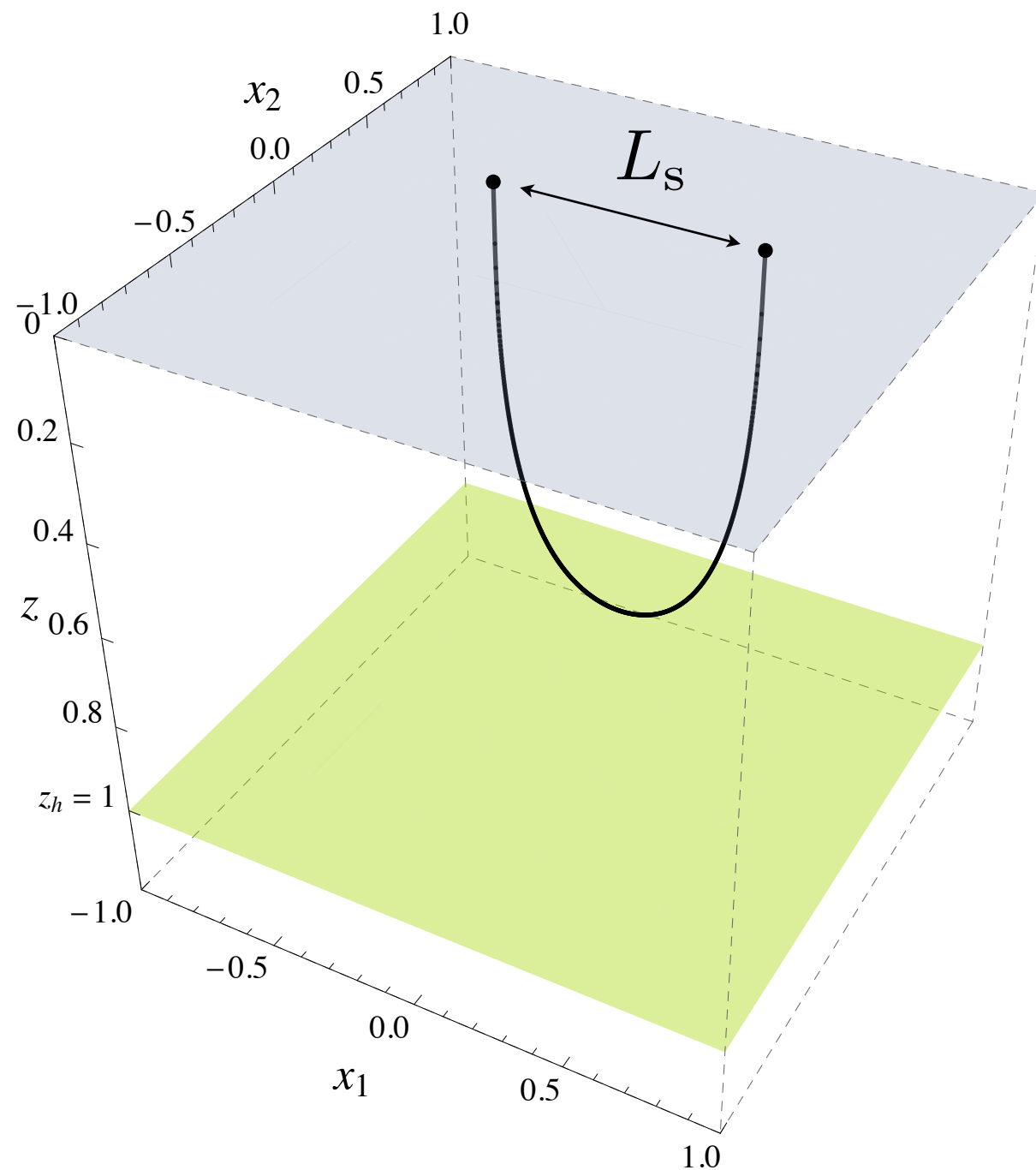
distance L:

$$L\pi T = 2\pi T \int_0^{L/2} d\sigma = 2\pi T \int_0^{L/2} \frac{dz}{z'} = 2\pi T \int_0^{z_c} dz \sqrt{\frac{h(z_c) e^{2B(z)+4A(z_c)} - 2A(z)}{h(z)^2 e^{4A(z)} - h(z) h(z_c) e^{4A(z_c)}}$$

lowest point z_c
parametrizes different
configurations



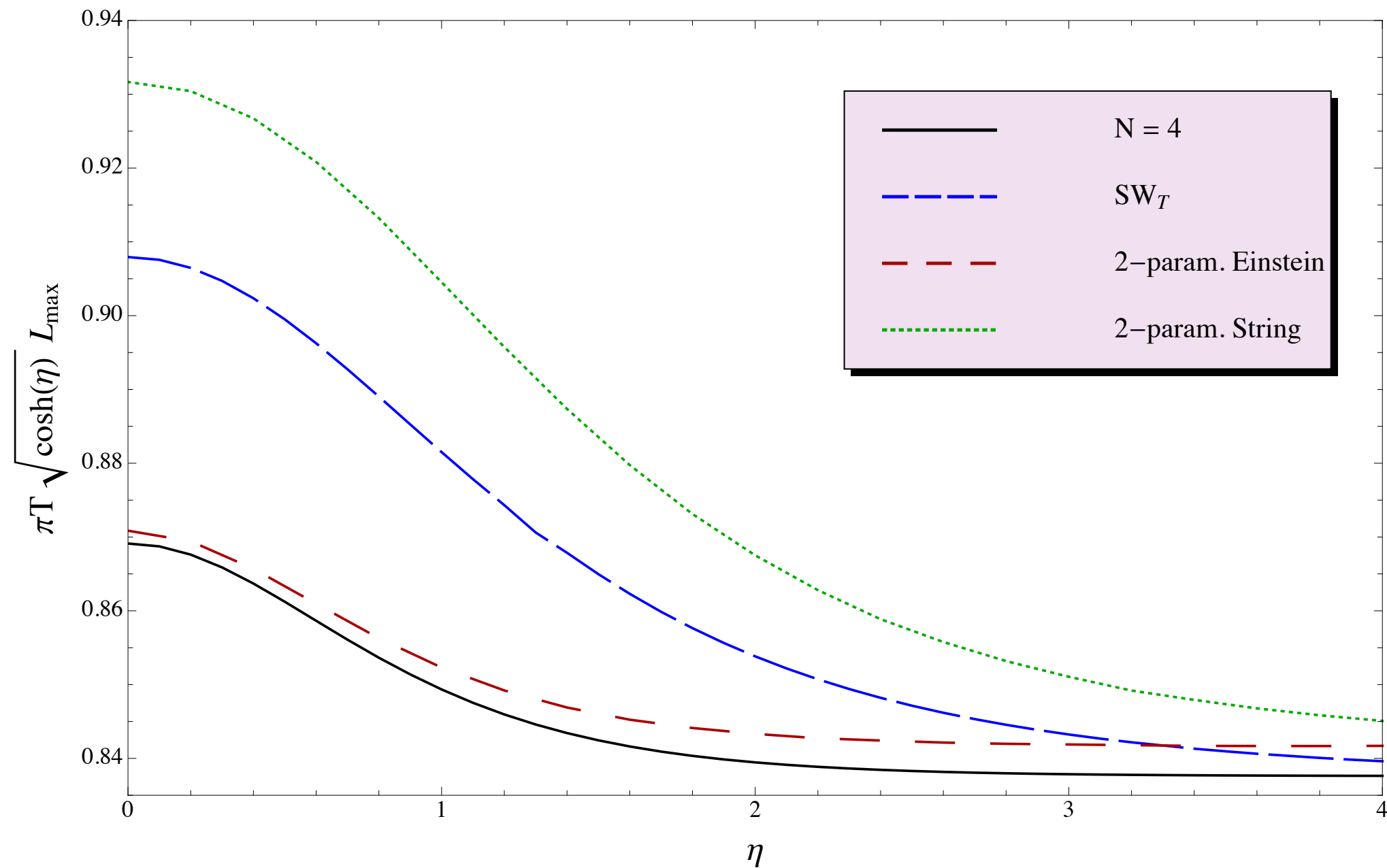
Screening Distance



maximal distance L_s
is **screening distance**

(different from Debye
screening length)

Screening Distance



- velocity lowers screening distance

$$\propto 1/\sqrt{\gamma} \propto (\text{boosted energy density})^{-1/4}$$

Energies

A. Samberg, O. Kaczmarek, CE

An old problem...

Free energy / potential of heavy quark-antiquark pair calculated in 1998:

Rey, Theisen, Yee;

Brandhuber, Itzhaki, Sonnenschein, Yankielowicz
and many times since then.

However, ... actually not.

Physical expectation:

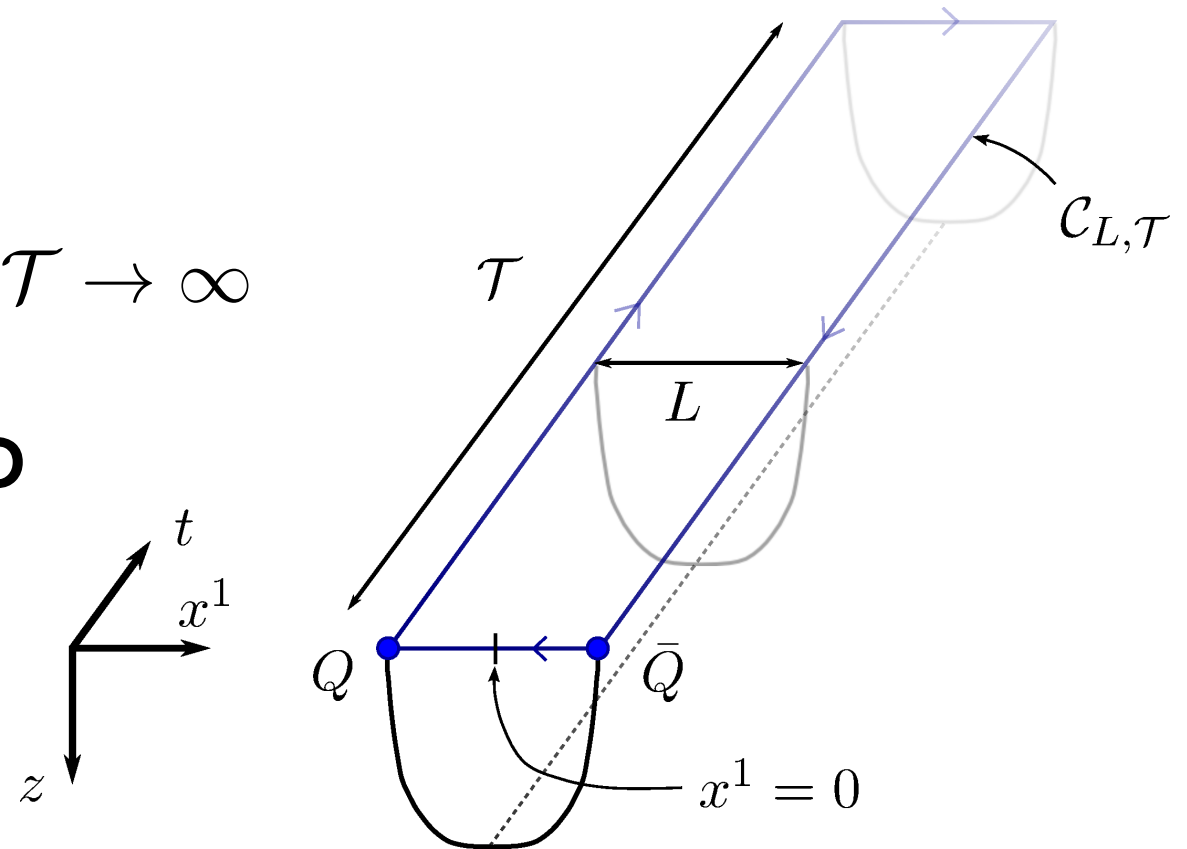
potential independent of T at small distances

Free Energy of Heavy QQbar pair

in field theory:

$$\langle W(\mathcal{C}_{L,\mathcal{T}}) \rangle \sim \exp(-iF_{Q\bar{Q}}(L)\mathcal{T}) , \quad \mathcal{T} \rightarrow \infty$$

for temporal Wilson loop



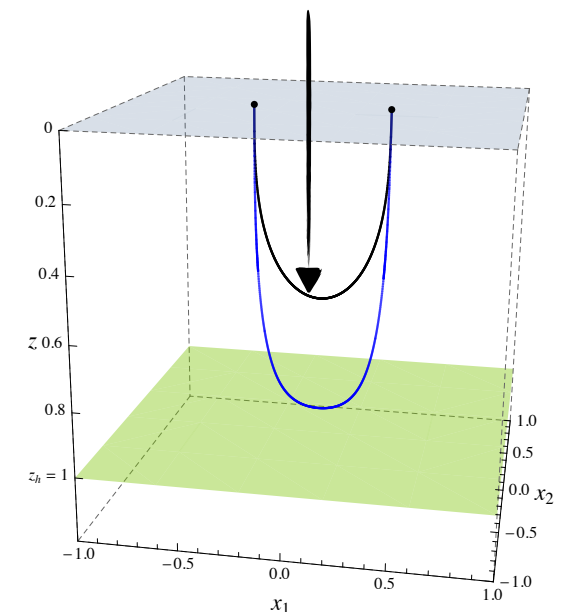
calculation in AdS/CFT:

$$\langle W(\mathcal{C}) \rangle \sim \exp(iS_{\text{NG}}[\mathcal{C}])$$

$$S_{\text{NG}} = -\frac{1}{2\pi\alpha'} \int d^2\sigma \sqrt{-\det g_{ab}} .$$

$$F_{Q\bar{Q}}(L) \sim -\frac{S_{\text{NG}}[\mathcal{C}_{L,\mathcal{T}}]}{\mathcal{T}} , \quad \mathcal{T} \rightarrow \infty$$

energetically favored string



Nambu-Goto action for hanging string

in general metric

$$ds^2 = e^{2A(z)} (-h(z) dt^2 + d\vec{x}^2) + \frac{e^{2B(z)}}{h(z)} dz^2$$

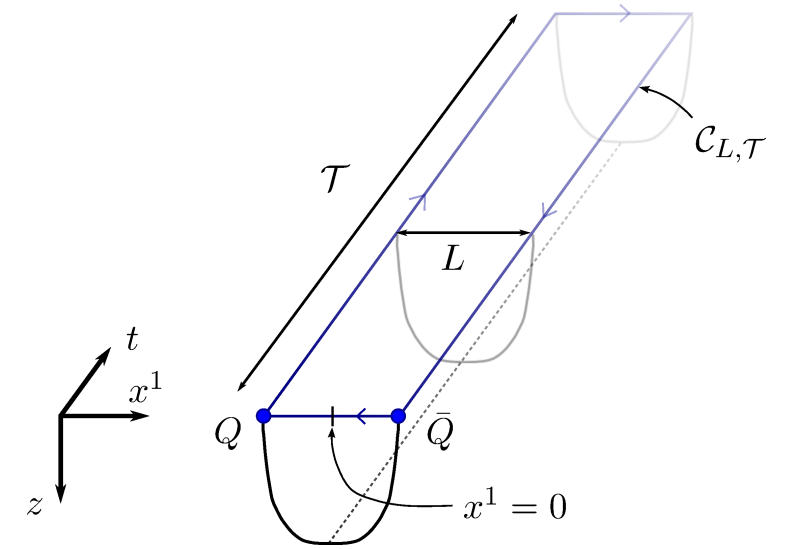
we have

$$S_{\text{NG}}[\mathcal{C}_{L,\mathcal{T}}] = -\frac{\mathcal{T}}{\pi\alpha'} \int_0^{z_t} dz e^{A+B} \sqrt{\frac{e^{4A}h}{e^{4A}h - e^{4A_t}h_t}}$$

z_t : turning point

UV divergent:

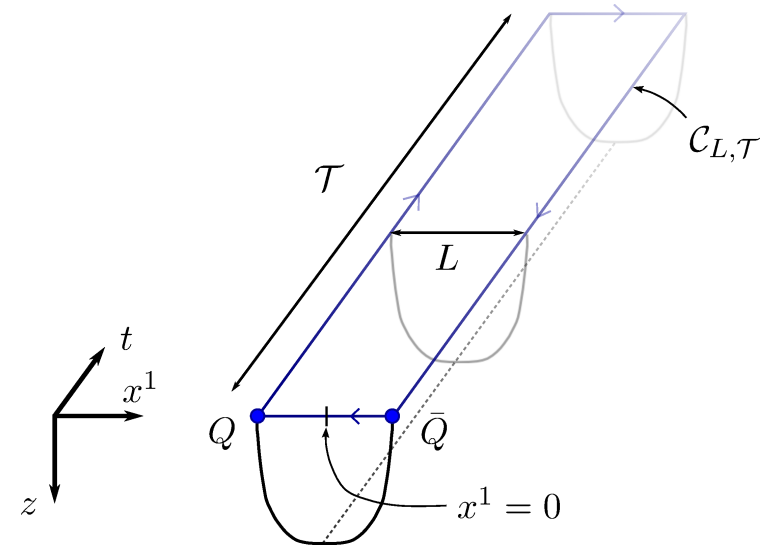
$$S_{\text{NG}}^{(\text{reg})}[\mathcal{C}_{L,\mathcal{T}}] = -\frac{\mathcal{T}}{\pi\alpha'} \int_{\varepsilon}^{z_t} dz e^{A+B} \sqrt{\frac{e^{4A}h}{e^{4A}h - e^{4A_t}h_t}} \sim -\frac{\mathcal{T}L_{\text{AdS}}^2}{\pi\alpha'} \left(\frac{1}{\varepsilon} + \dots \right)$$



Subtraction for Nambu-Goto action

subtraction required:

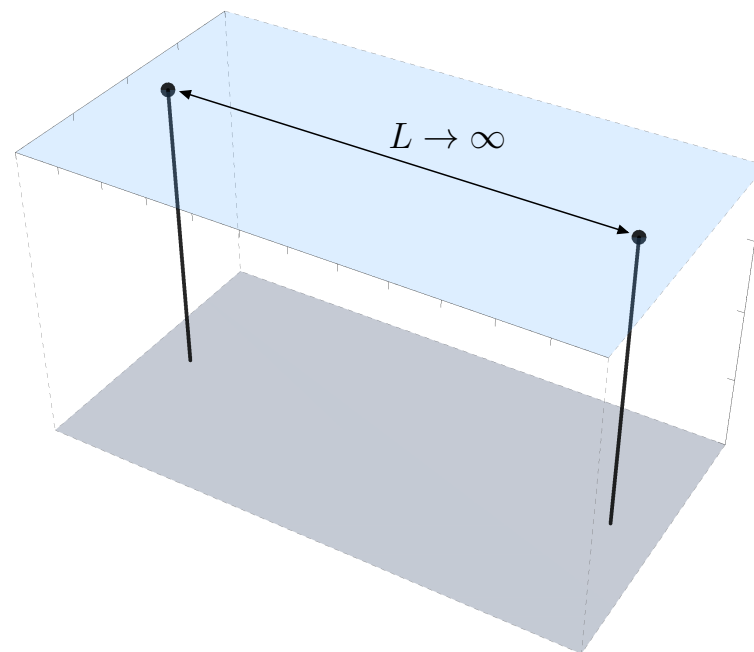
$$F_{Q\bar{Q}}^{(\text{ren})}(L) = \lim_{\mathcal{T} \rightarrow \infty} \left(-\frac{S_{\text{NG}}^{(\text{reg})}[\mathcal{C}_{L,\mathcal{T}}] - \Delta S}{\mathcal{T}} \right)$$



subtractions in the literature:

- non-interacting string hanging down into black hole (2x)

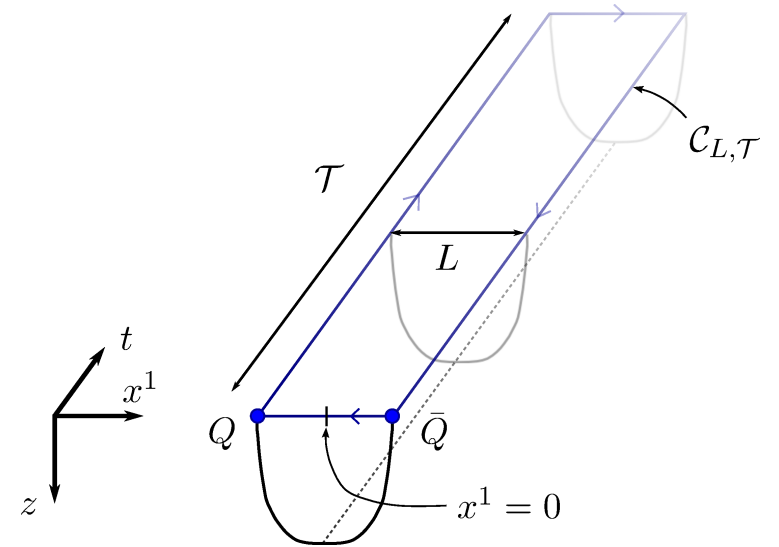
$$S_{\text{NG}}^{(\text{reg})}[\text{straight string}] = -\frac{\mathcal{T}}{2\pi\alpha'} \int_{\varepsilon}^{z_h} dz e^{A+B} \sim -\frac{\mathcal{T}L_{\text{AdS}}^2}{2\pi\alpha'} \left(\frac{1}{\varepsilon} + \dots \right)$$



Subtraction for Nambu-Goto action

subtraction required:

$$F_{Q\bar{Q}}^{(\text{ren})}(L) = \lim_{\mathcal{T} \rightarrow \infty} \left(-\frac{S_{\text{NG}}^{(\text{reg})}[\mathcal{C}_{L,\mathcal{T}}] - \Delta S}{\mathcal{T}} \right)$$



subtractions in the literature:

- non-interacting string hanging down into black hole (2x)

$$S_{\text{NG}}^{(\text{reg})}[\text{straight string}] = -\frac{\mathcal{T}}{2\pi\alpha'} \int_{\varepsilon}^{z_h} dz e^{A+B} \sim -\frac{\mathcal{T}L_{\text{AdS}}^2}{2\pi\alpha'} \left(\frac{1}{\varepsilon} + \dots \right)$$

- real part of action at $L=\infty$

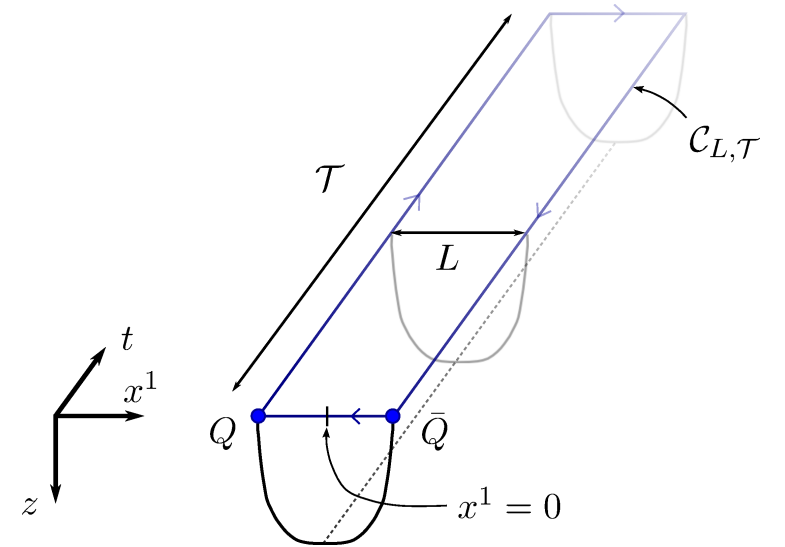
Albacete, Kovchegov, Taliotis

but: then F is T -dependent for small L - **unphysical!**

Subtraction for Nambu-Goto action

subtraction required:

$$F_{Q\bar{Q}}^{(\text{ren})}(L) = \lim_{\mathcal{T} \rightarrow \infty} \left(-\frac{S_{\text{NG}}^{(\text{reg})}[\mathcal{C}_{L,\mathcal{T}}] - \Delta S}{\mathcal{T}} \right)$$



correct subtraction: only UV singularity

$$\Delta S_{\text{min}} \equiv -\frac{\mathcal{T} L_{\text{AdS}}^2}{\pi \alpha'} \int_{\varepsilon}^{\infty} \frac{dz}{z^2} = -\frac{\mathcal{T} L_{\text{AdS}}^2}{\pi \alpha'} \frac{1}{\varepsilon}$$

then no unphysical T-dependence!

Similar problem with regularization occurs in calculation of entanglement entropy.

Binding Energy of Heavy QQbar pair

quantity with hanging-string subtraction is
binding energy

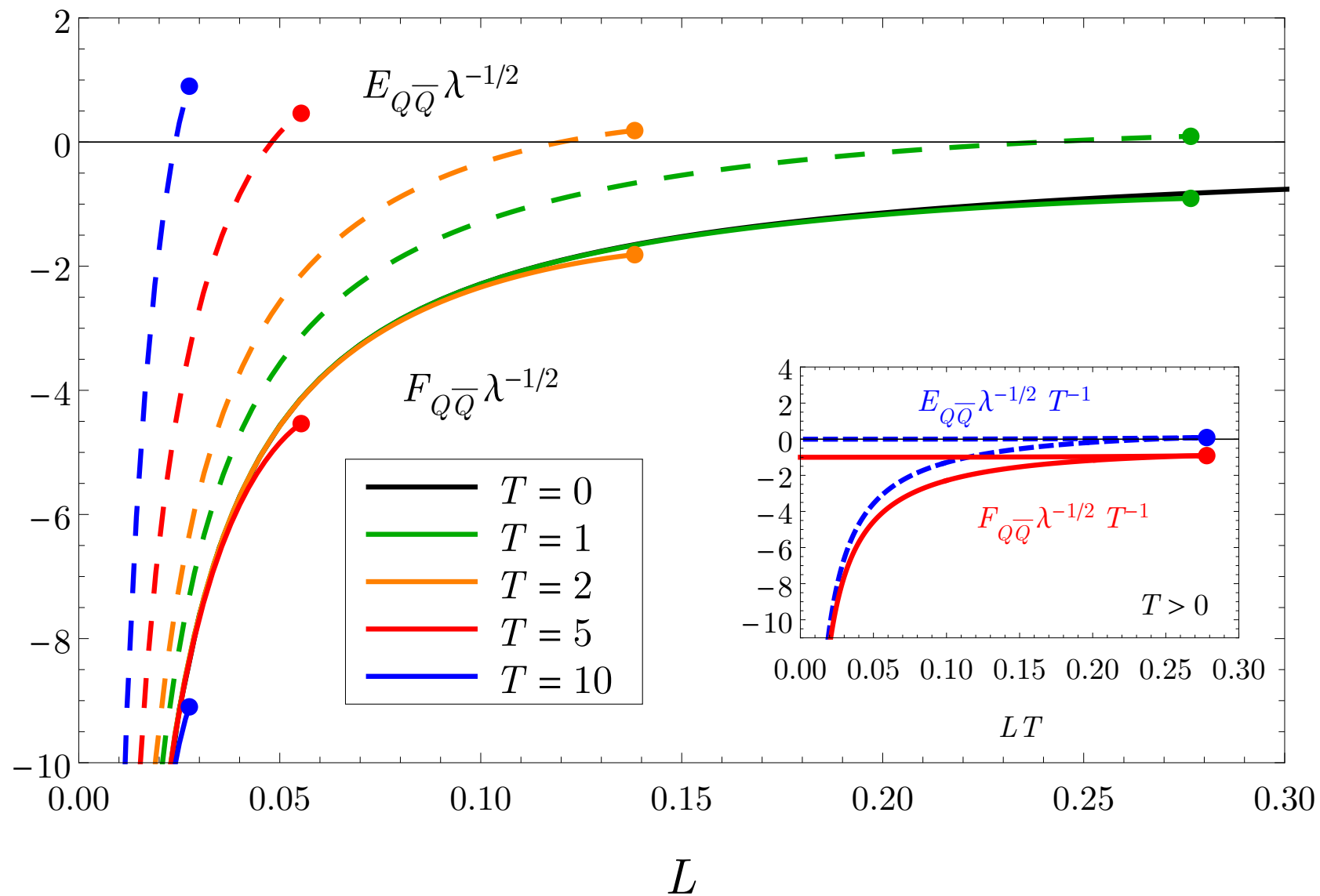
$$E_{Q\bar{Q}}(L) = \lim_{\mathcal{T} \rightarrow \infty} \left(-\frac{S_{\text{NG}}[\mathcal{C}_{L,\mathcal{T}}] - 2S_{\text{NG}}[\text{straight string}]}{\mathcal{T}} \right)$$

in fact difference of free energies:

$$\begin{aligned} E_{Q\bar{Q}}(L) &= \lim_{\mathcal{T} \rightarrow \infty} \left[-\frac{(S_{\text{NG}}[\mathcal{C}_{L,\mathcal{T}}] - \Delta S_{\text{min}}) - (2S_{\text{NG}}[\text{straight string}] - \Delta S_{\text{min}})}{\mathcal{T}} \right] \\ &= F_{Q\bar{Q}} - F_{Q;\bar{Q}}, \end{aligned}$$

(note: defines single-quark free energy)

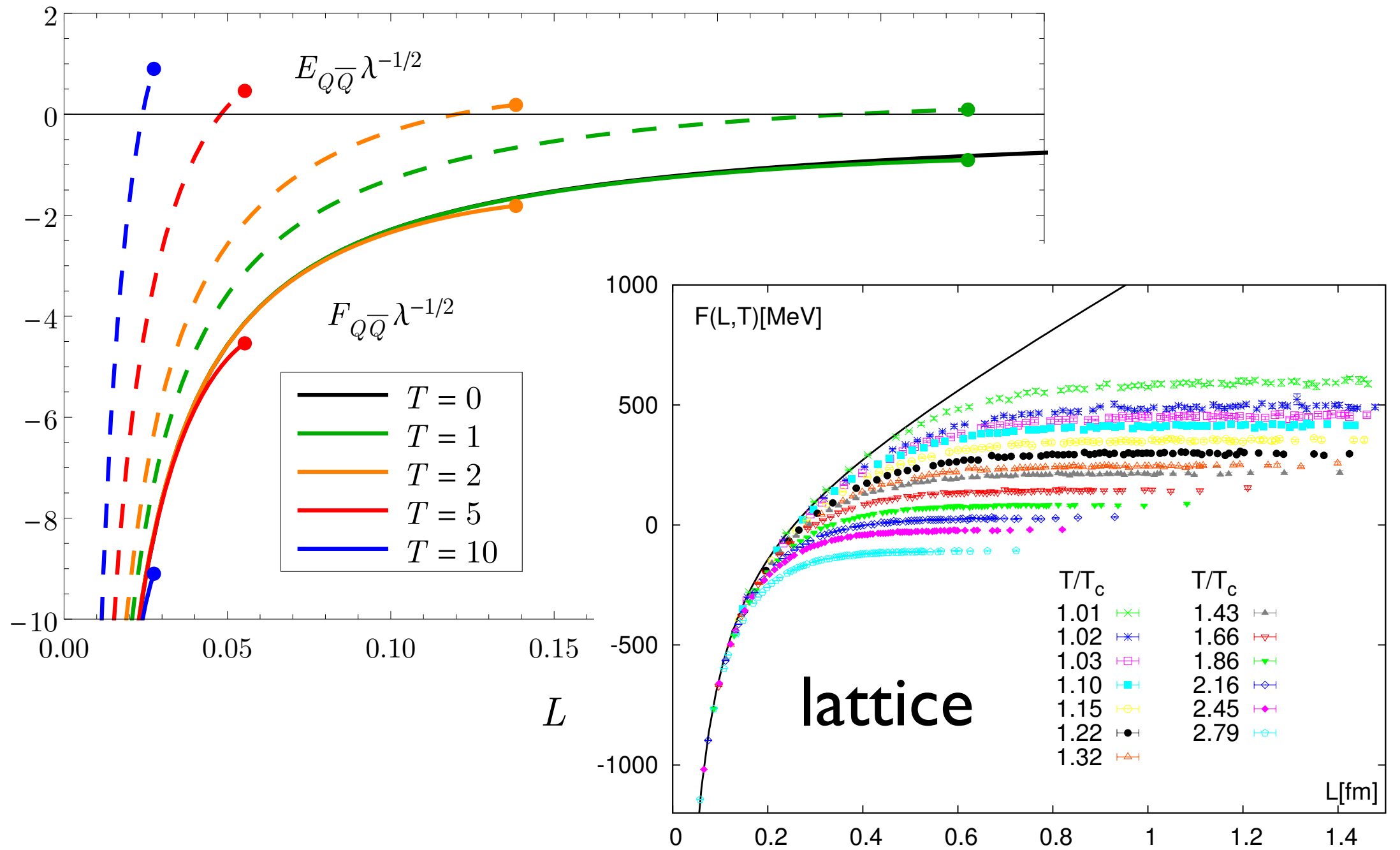
Free vs Binding Energy in N=4



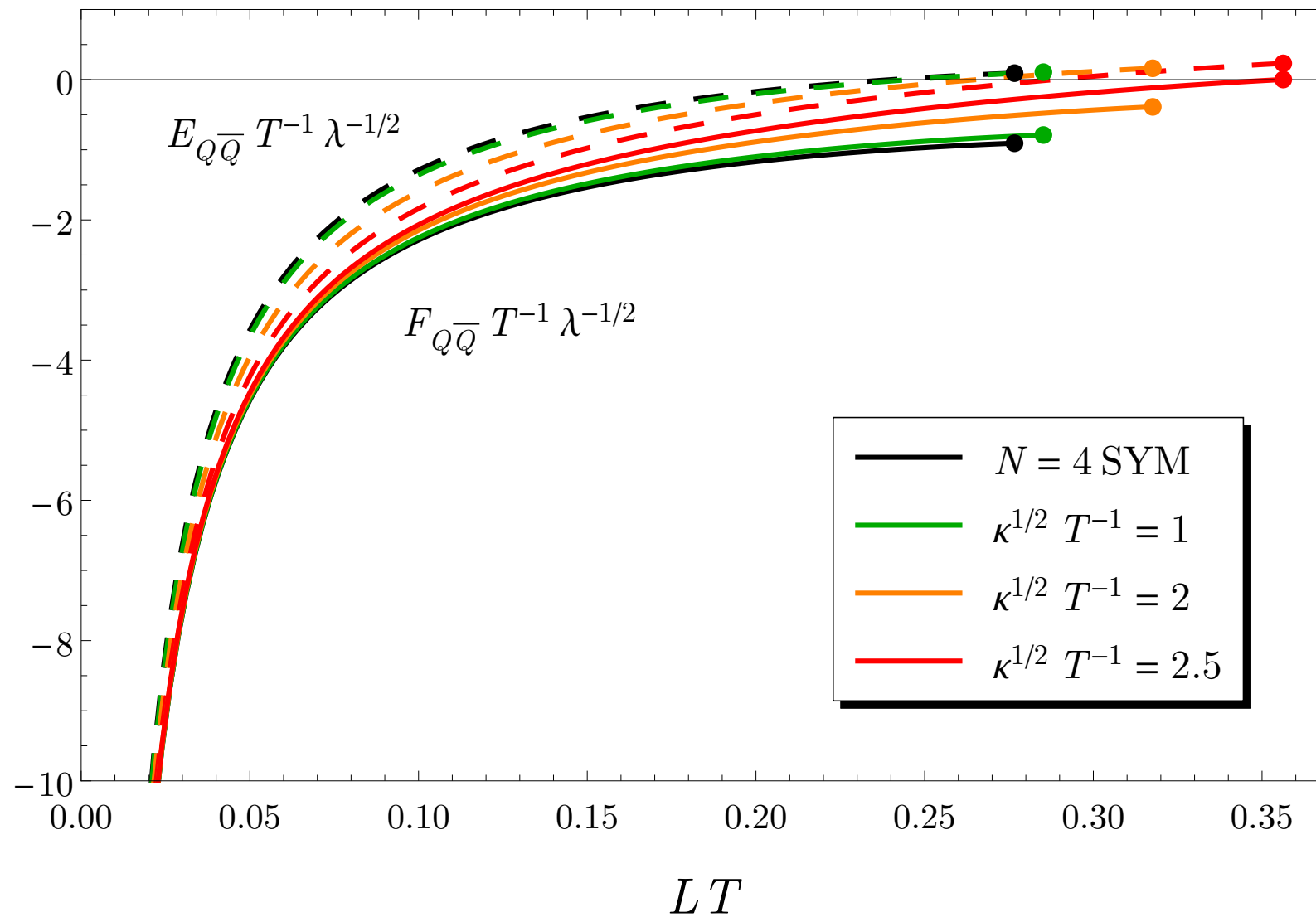
black: $T=0$ potential

$$V_{Q\bar{Q}}(L) = -\frac{4\pi^2\sqrt{\lambda}}{\Gamma^4\left(\frac{1}{4}\right)L}$$

Free vs Binding Energy in N=4



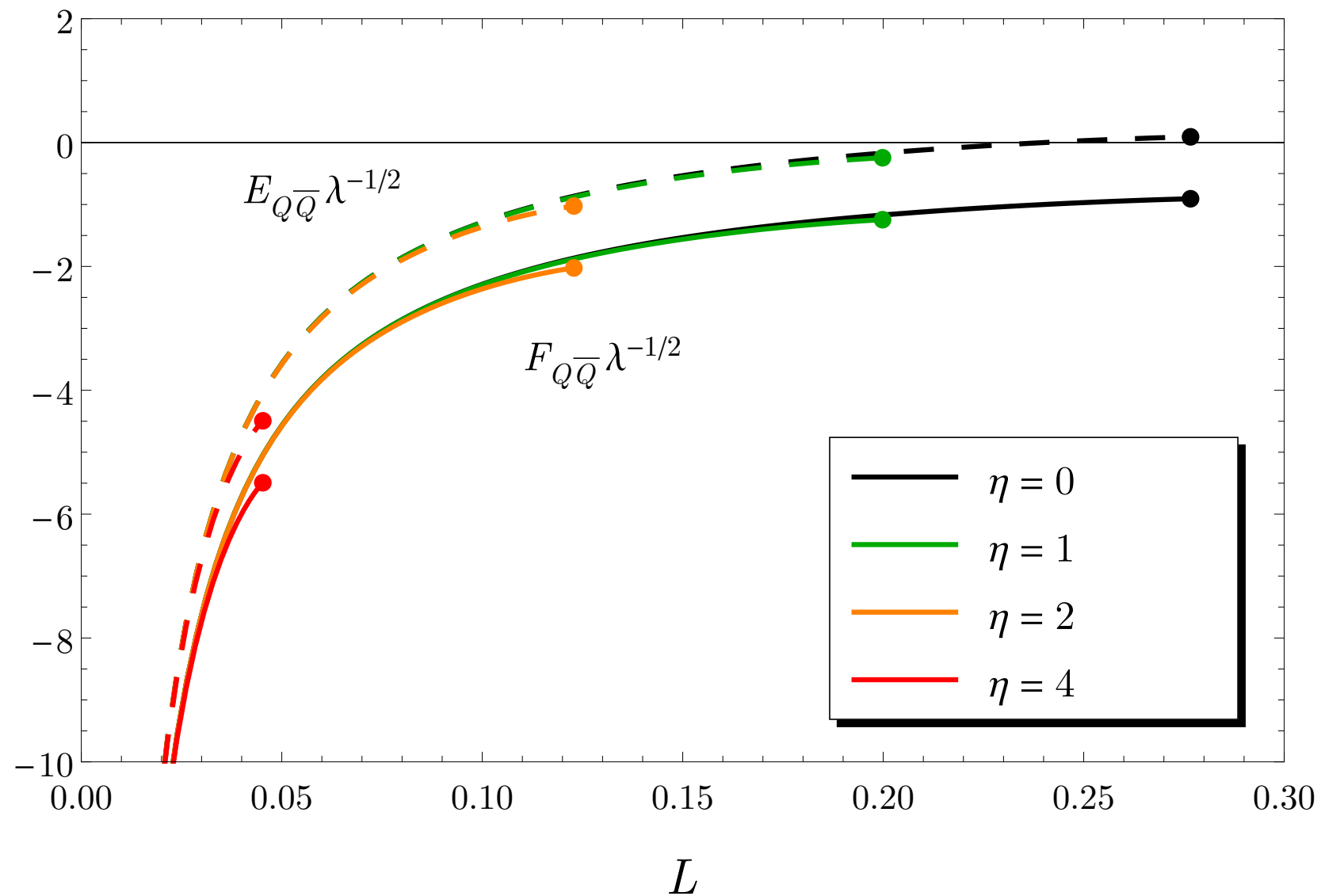
Free and Binding Energy - different non-conformalities



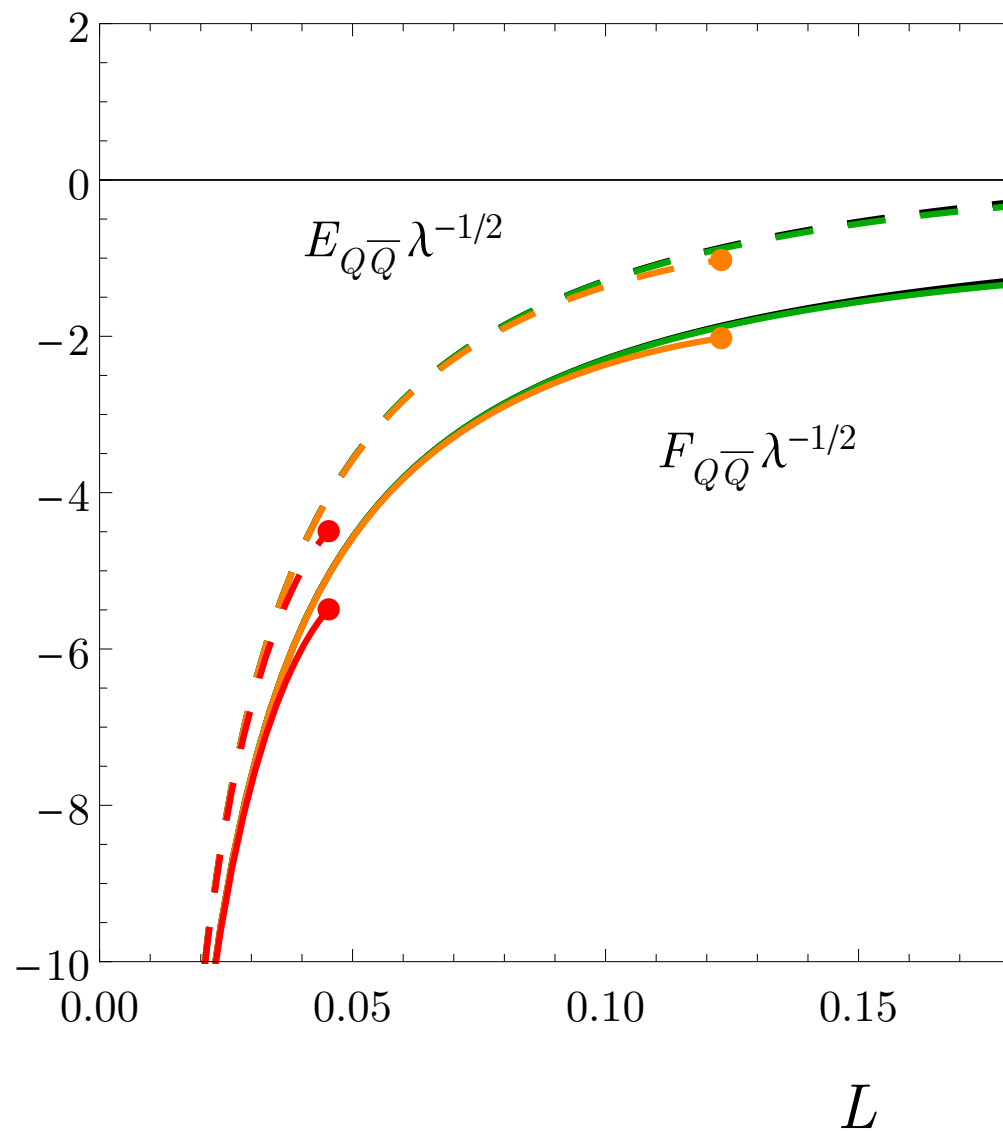
I-parameter string frame

Free and Binding Energy - quarks in motion

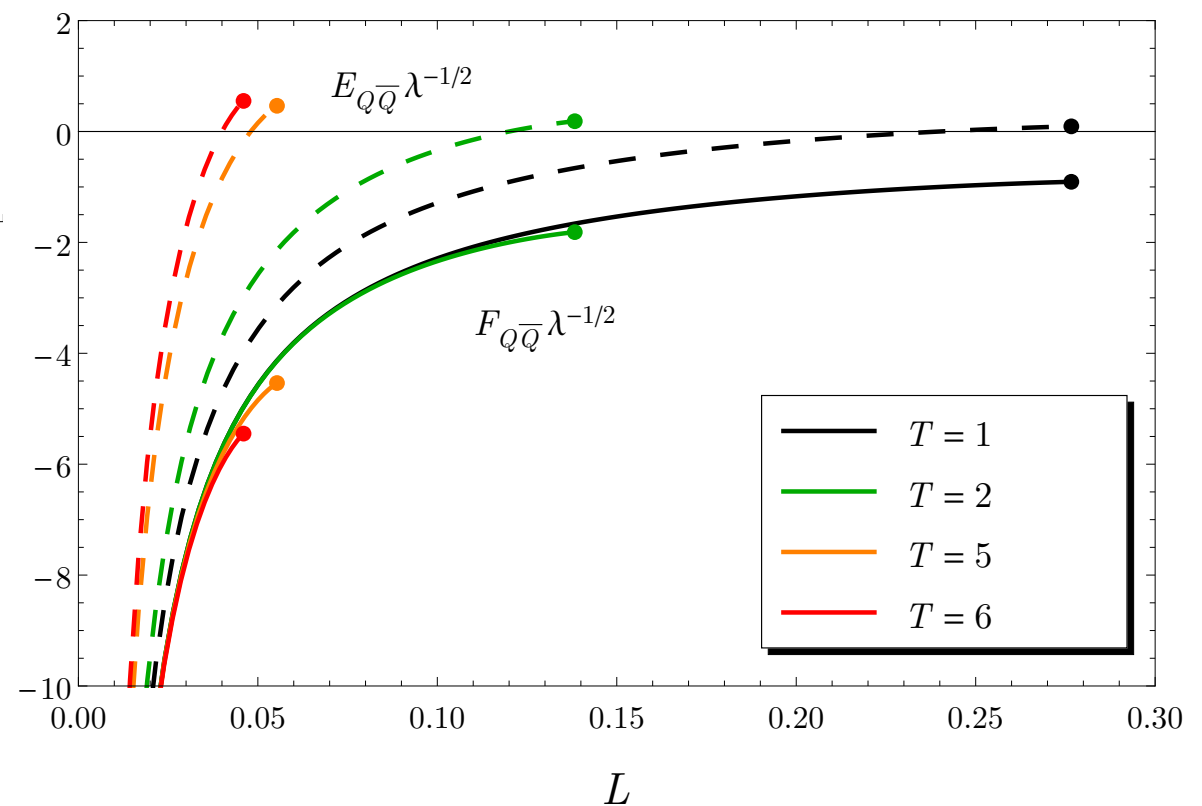
P. Wittmer, CE



Free and Binding Energy - quarks in motion



velocity not fully equivalent
to temperature change



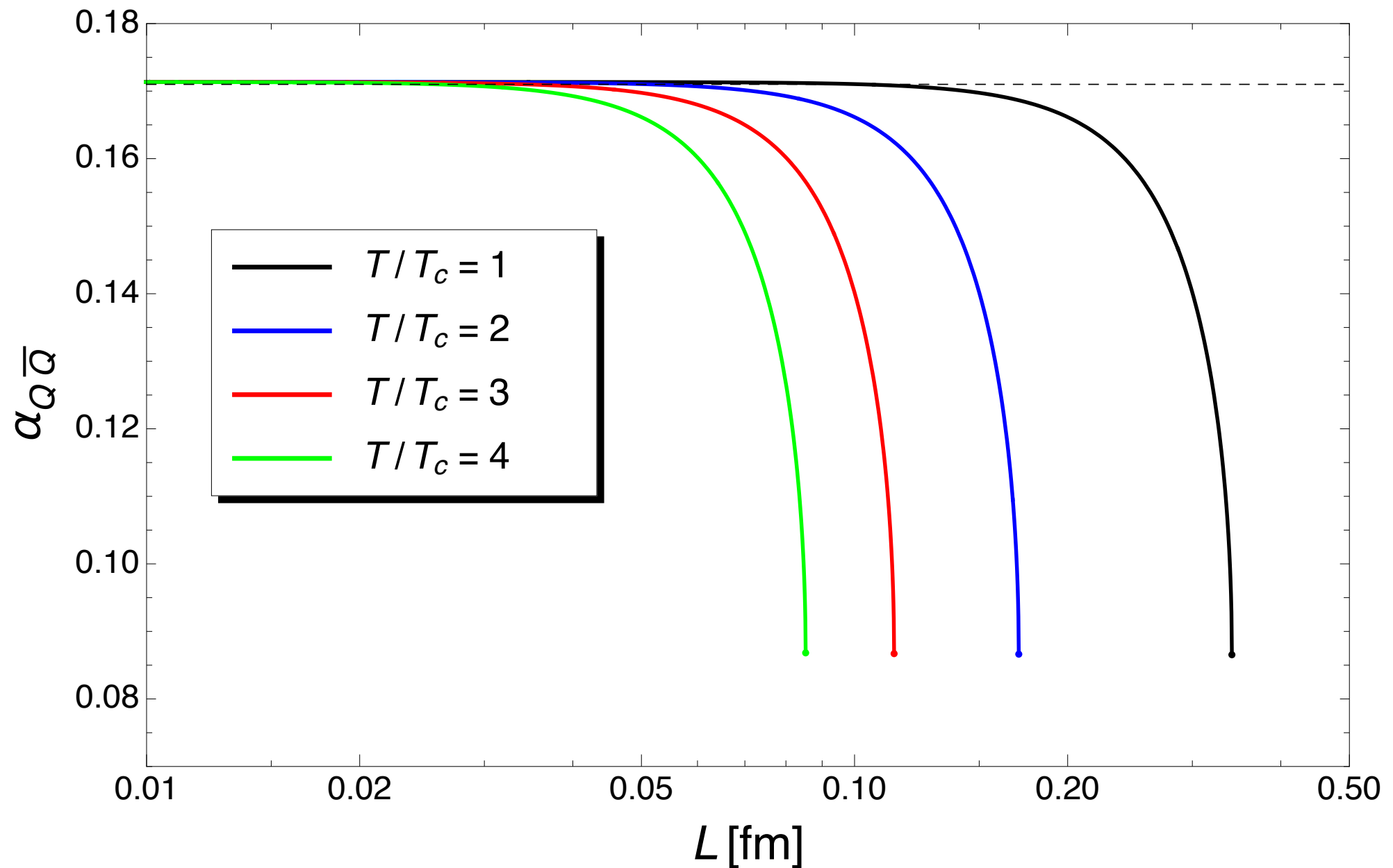
Running Coupling

Running Coupling $\alpha_{q\bar{q}}$

$$\alpha_{Q\bar{Q}} = \frac{3L^2}{4} \frac{dF(L, T)}{dL}$$

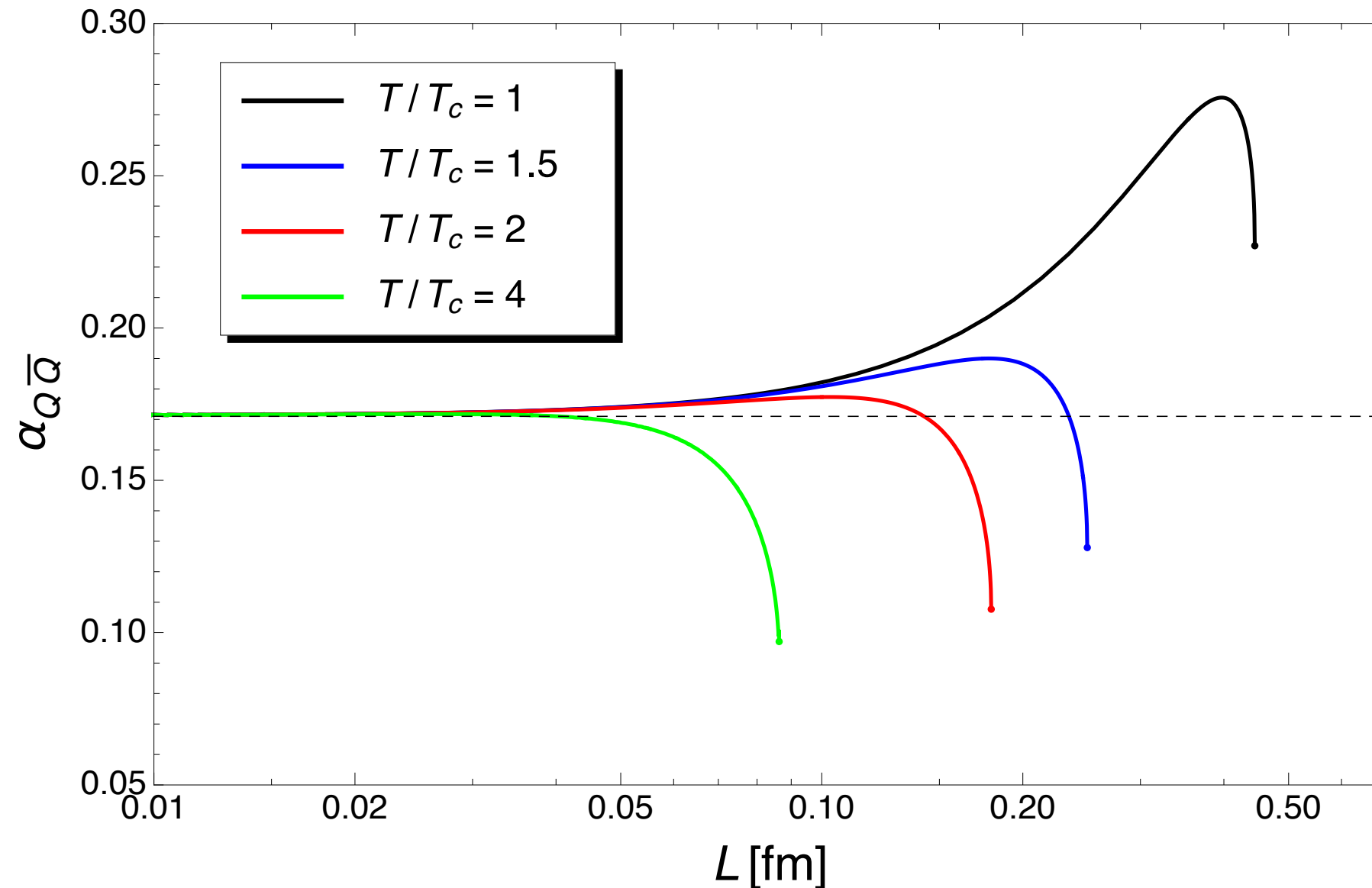
Konrad Schade, CE,
Paul Wittmer

N=4:



Running Coupling α_{qq}

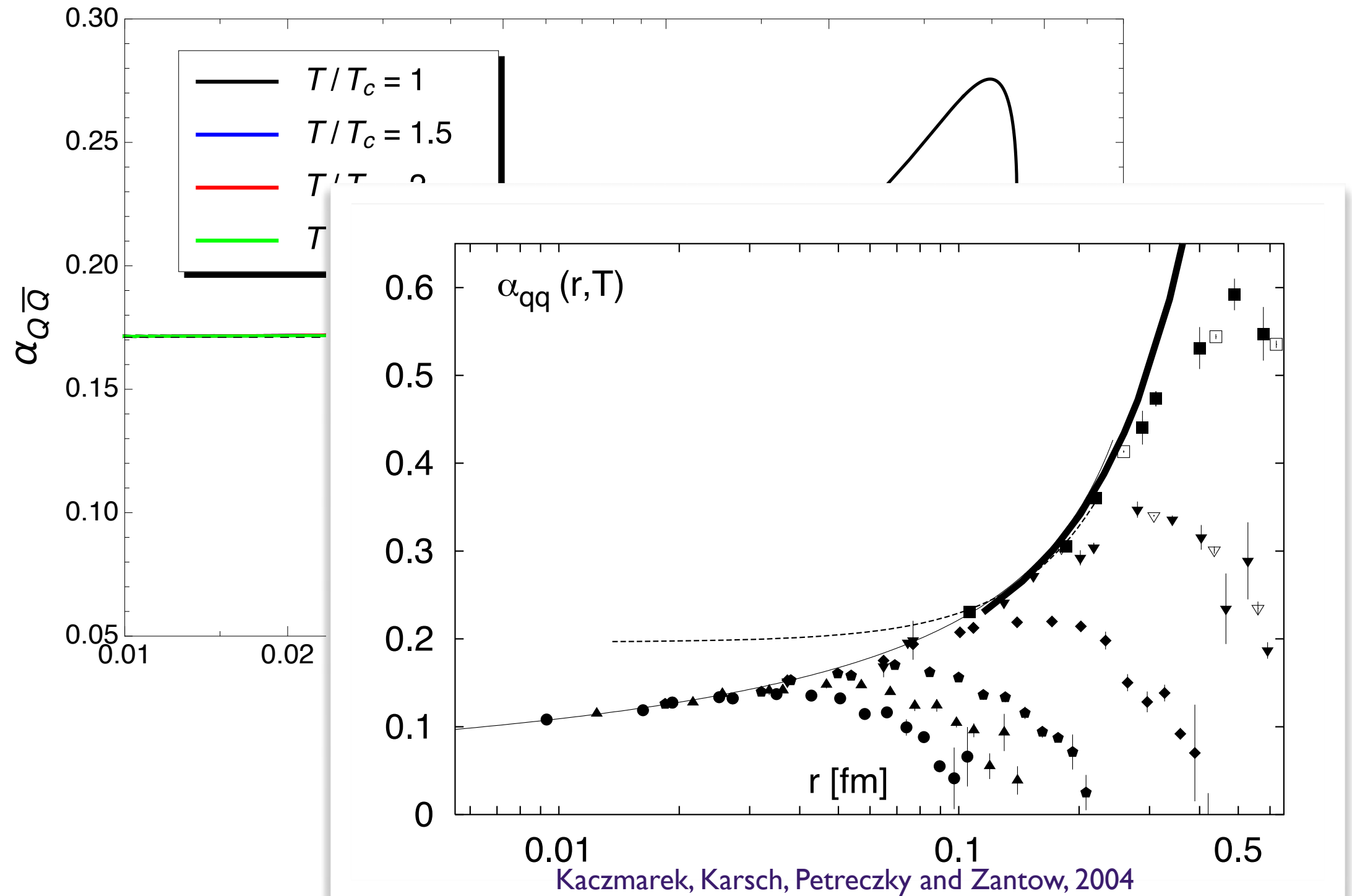
with non-conformal deformation:



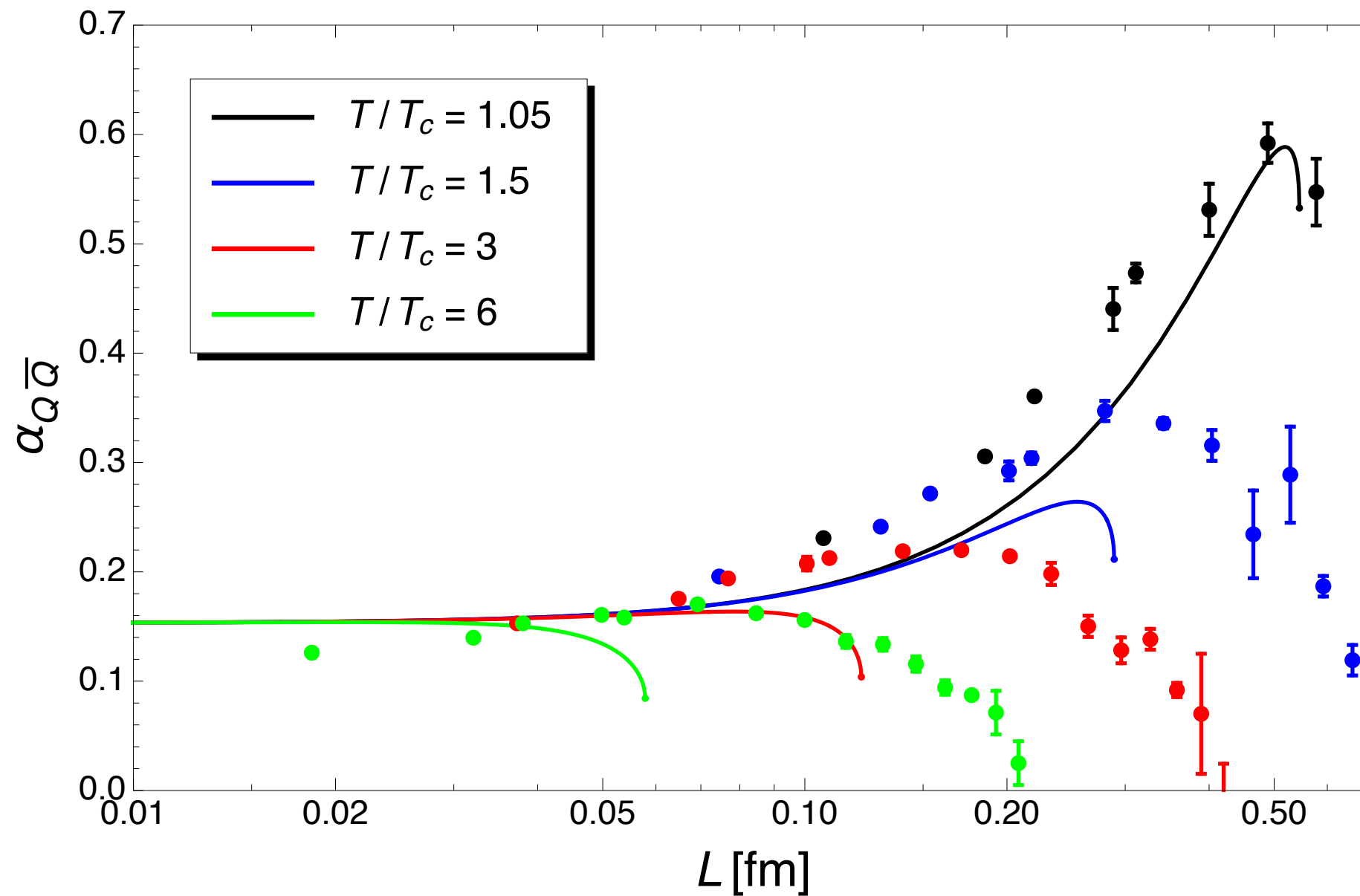
- universal rise above conformal value

Running Coupling α_{qq}

with non-conformal deformation:



Running Coupling α_{qq}

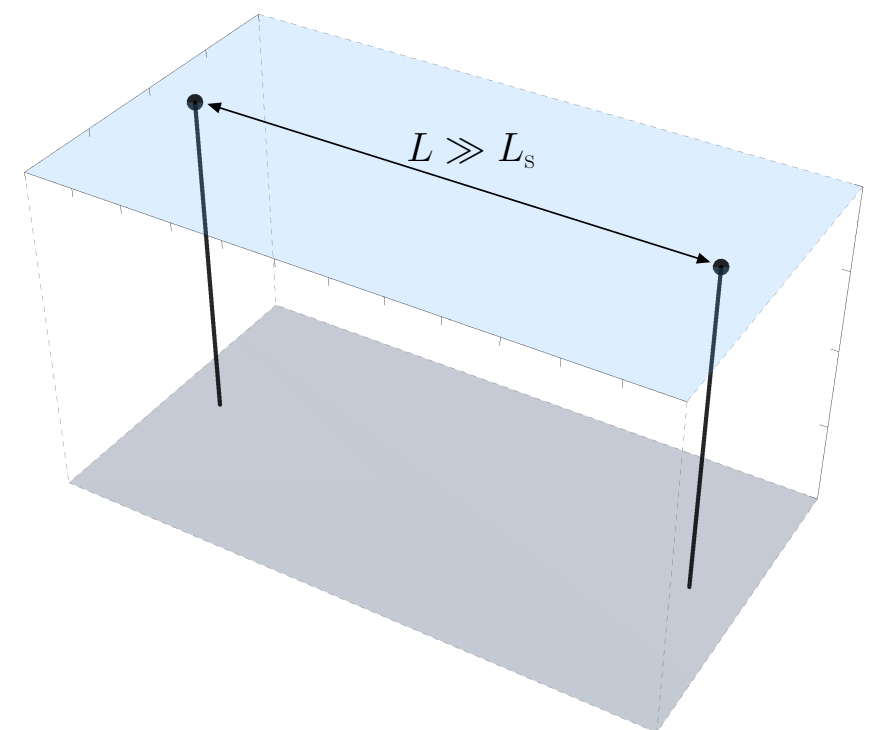
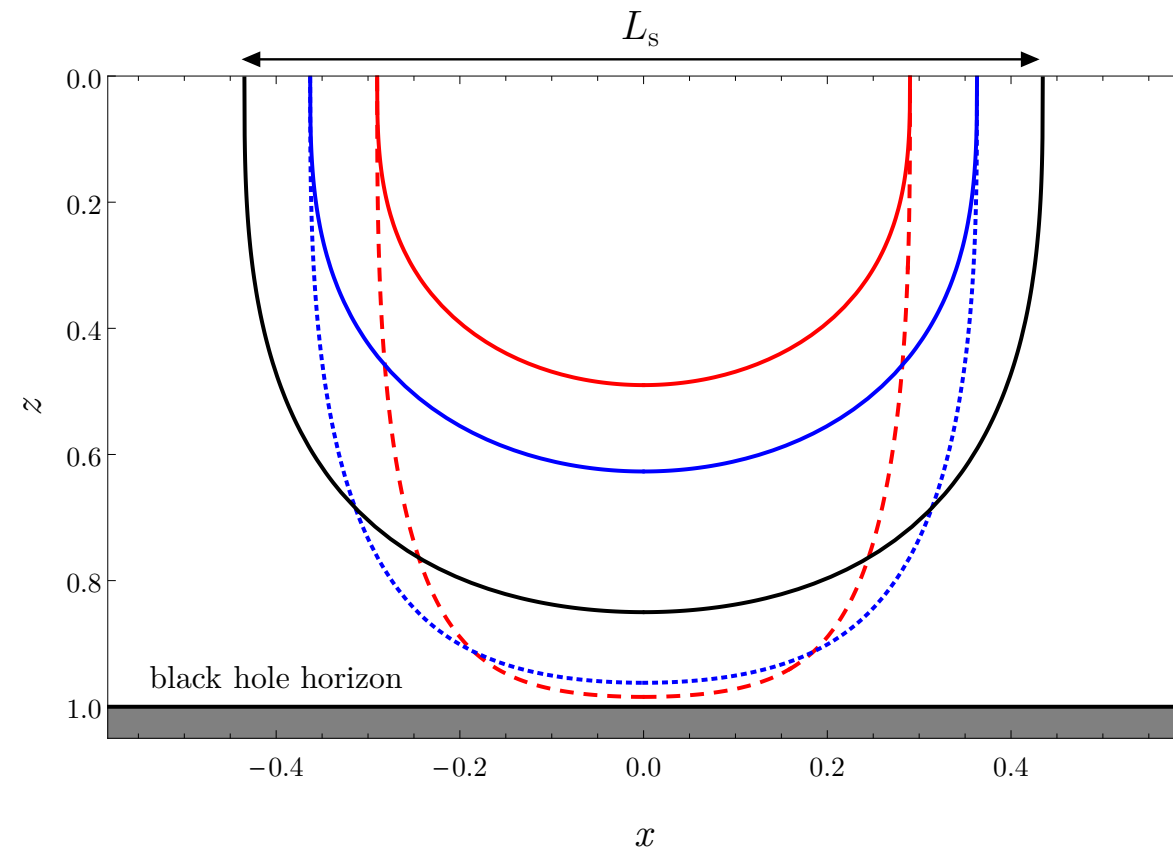


- Coming close to QCD data if free parameters properly adjusted, but not quite
- Parameters fixed from thermodynamics

Complex Static Potential

QQbar Potential at larger distances?

- small distances ($L < L_s$):
simple string configuration
- very large distances:
Debye screening,
due to supergravity mode
exchange between hanging
strings
(Bak, Karch, Yaffe)

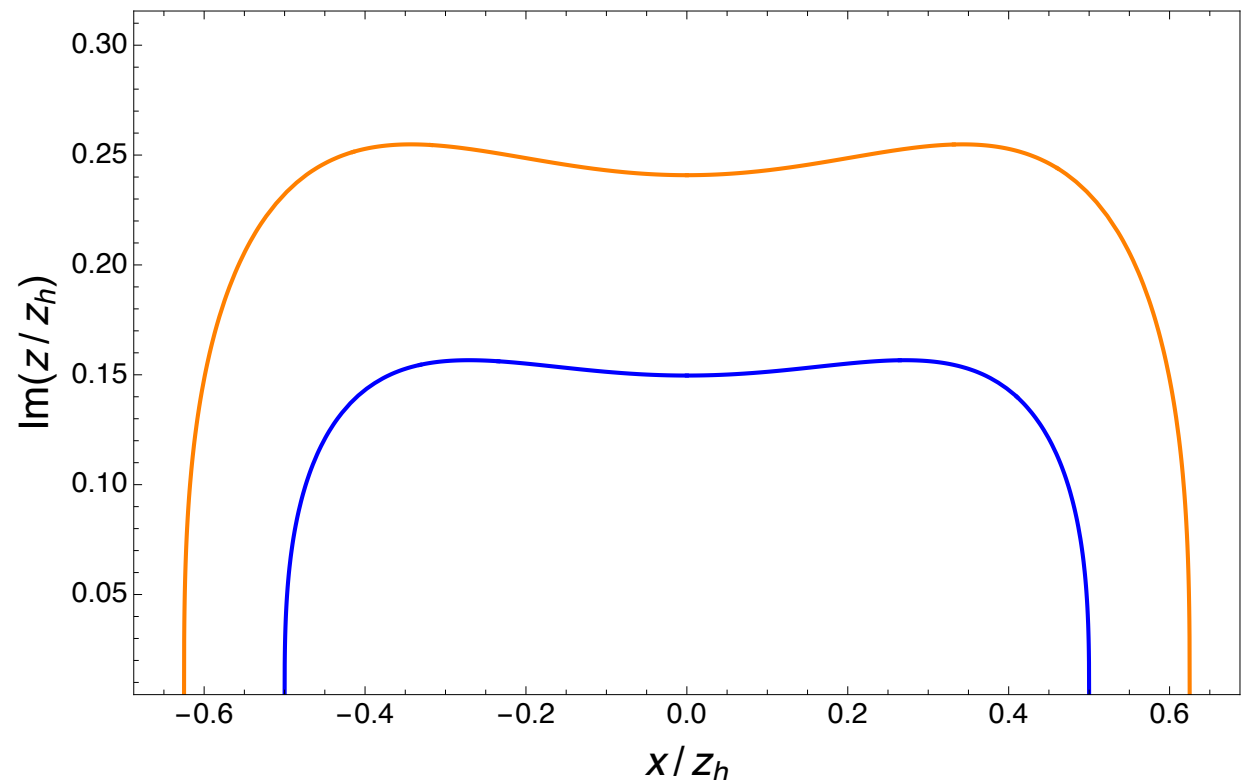
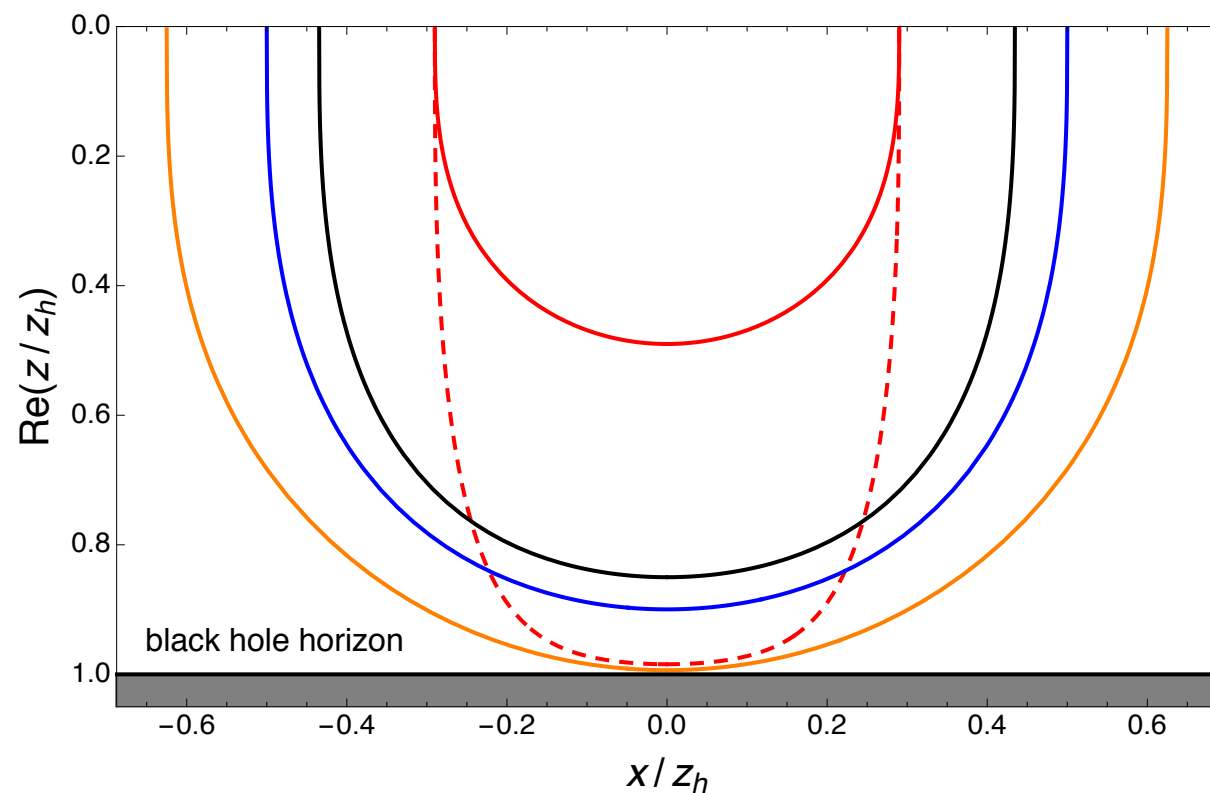


Complex QQbar Potential

Try to analytically continue simple string configuration beyond L_s

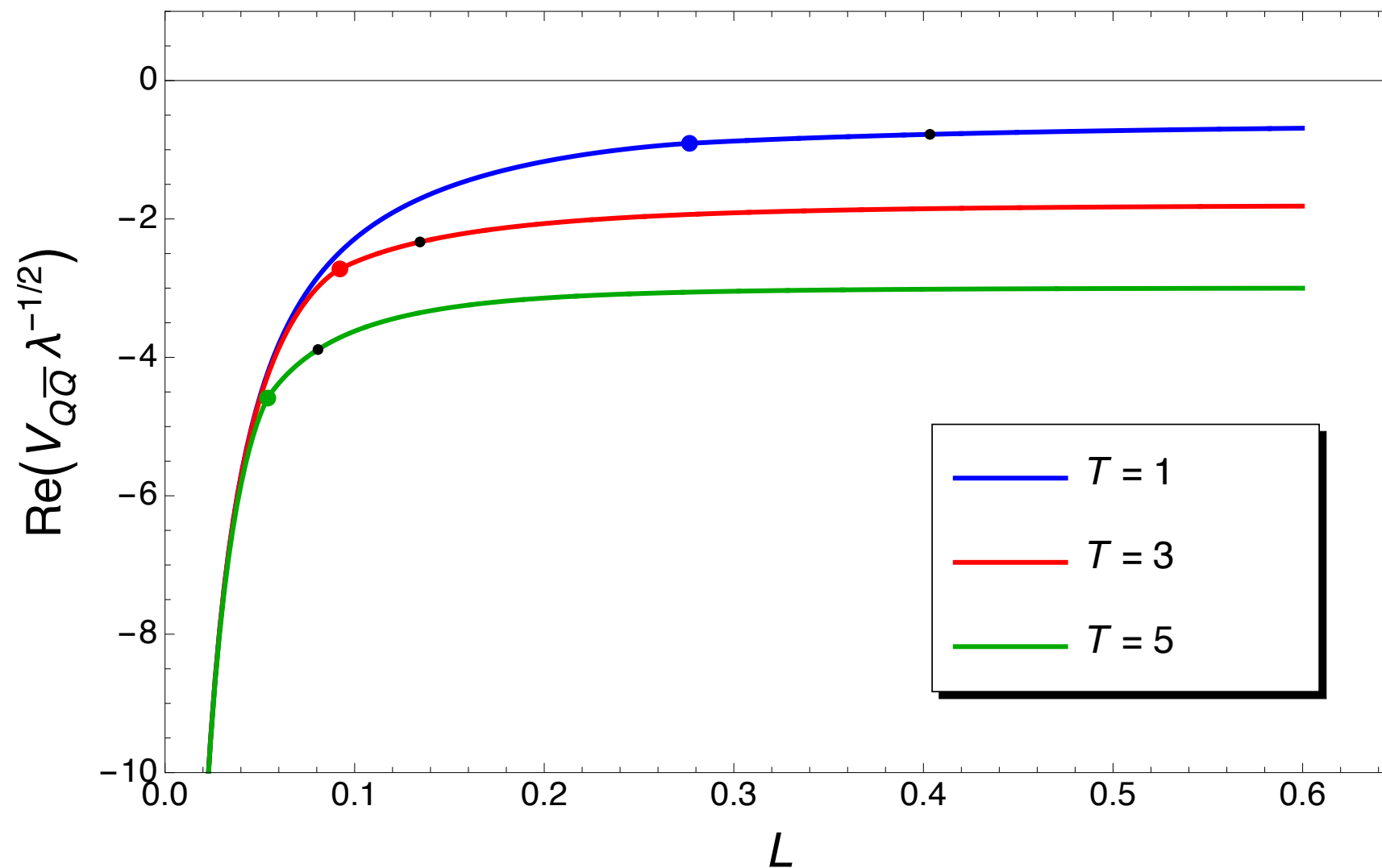
Kovchegov et al

String coordinates & potential become complex



Complex QQbar Potential

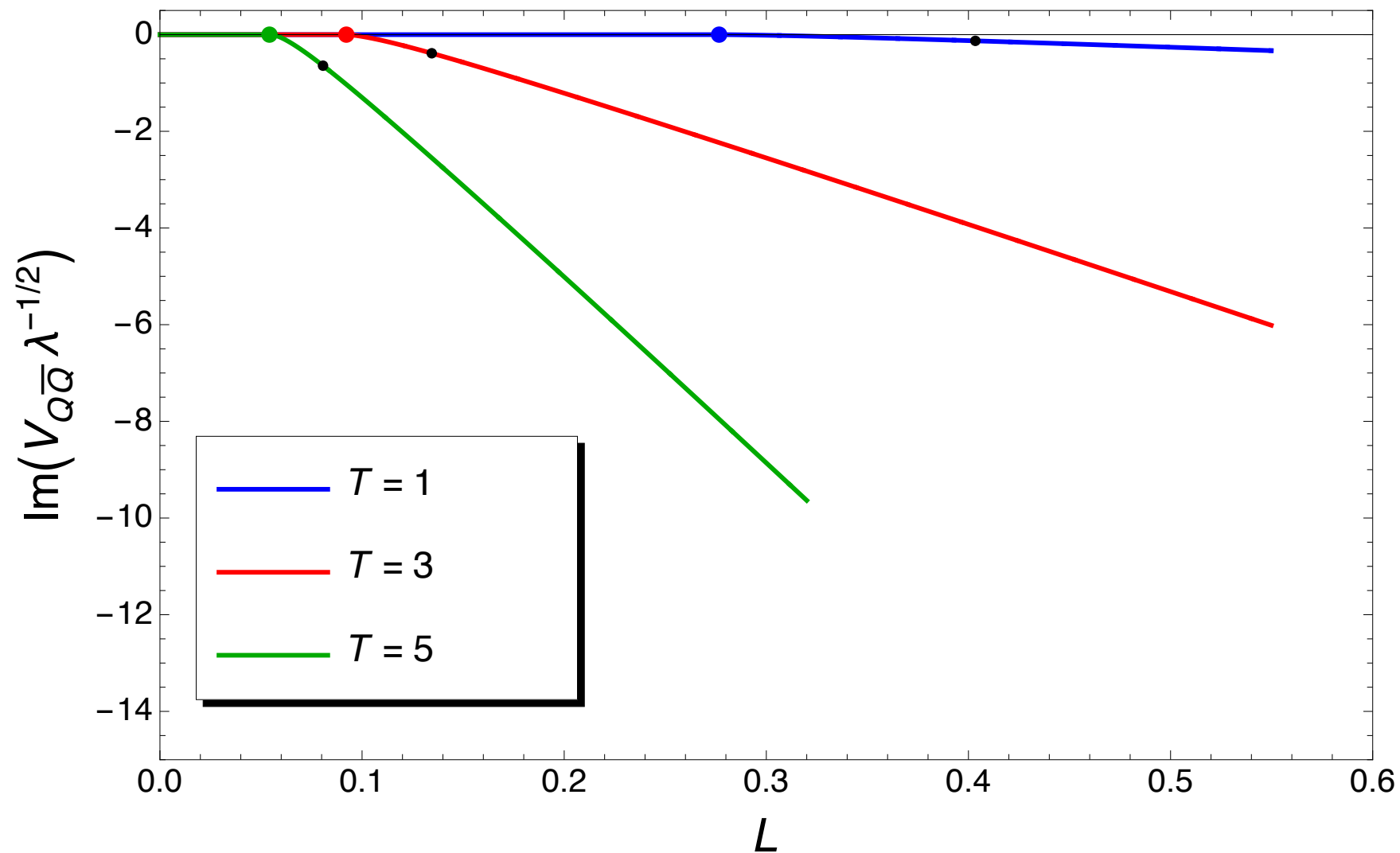
Real part of potential (with correct renormalization)



black points: L_h ; colored points: L_s

Complex QQbar Potential

Imaginary part of potential

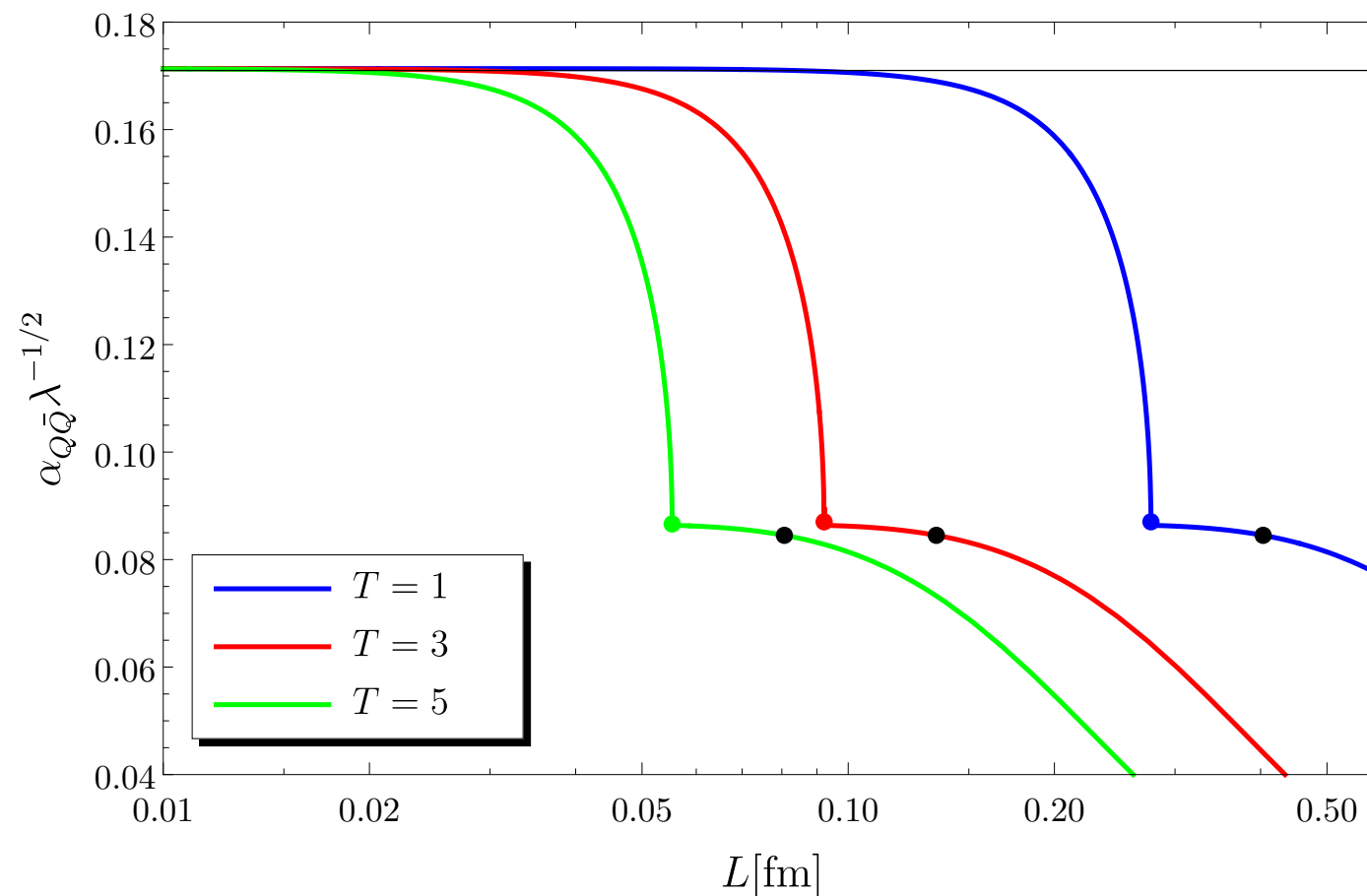


black points: L_h ; colored points: L_s

Complex QQbar Potential

However:

‘coupling strength’ shows unphysical behavior already above L_s



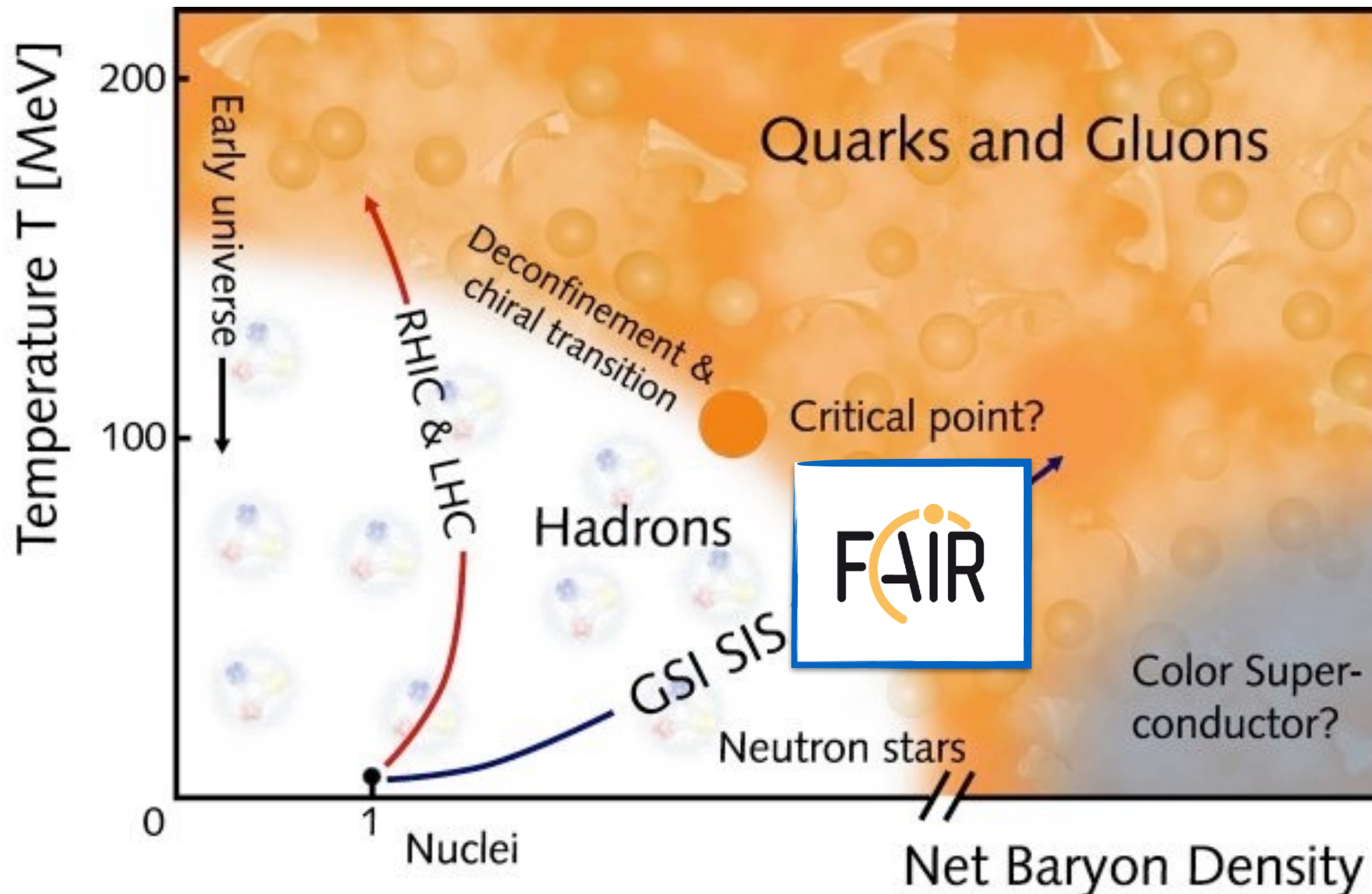
Better stop at L_s , dynamics above that not understood

Finite Chemical Potential

AdS Models for the Plasma at finite chemical potential

Finite Chemical Potential

Extending these calculations to finite μ

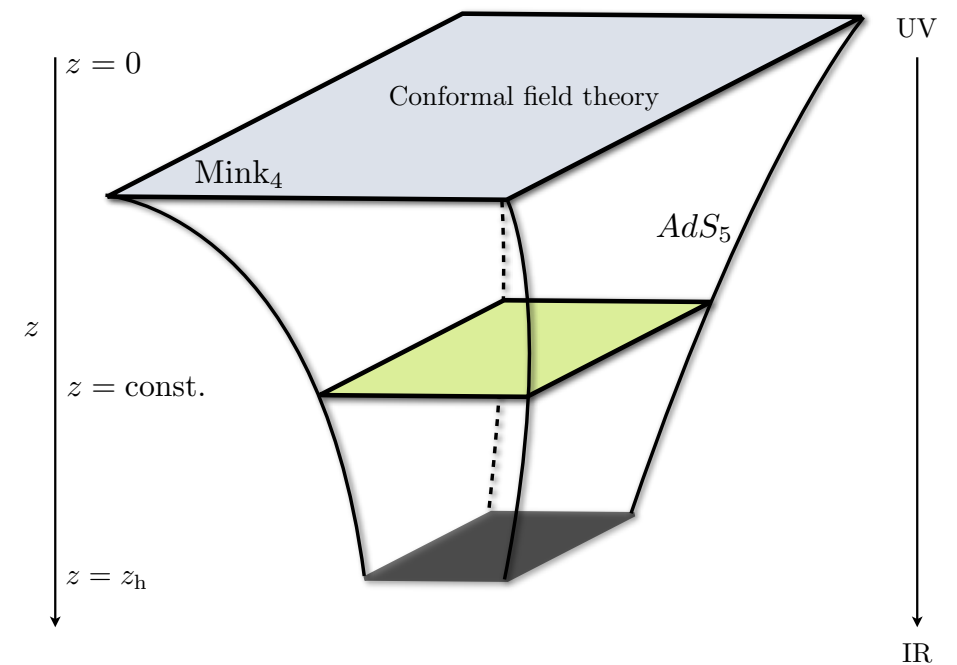


N=4 SYM at Finite T and μ

- finite T: AdS-BH

$$ds^2 = \frac{L^2}{z^2} \left(-h dt^2 + d\vec{x}^2 + \frac{dz^2}{h} \right)$$

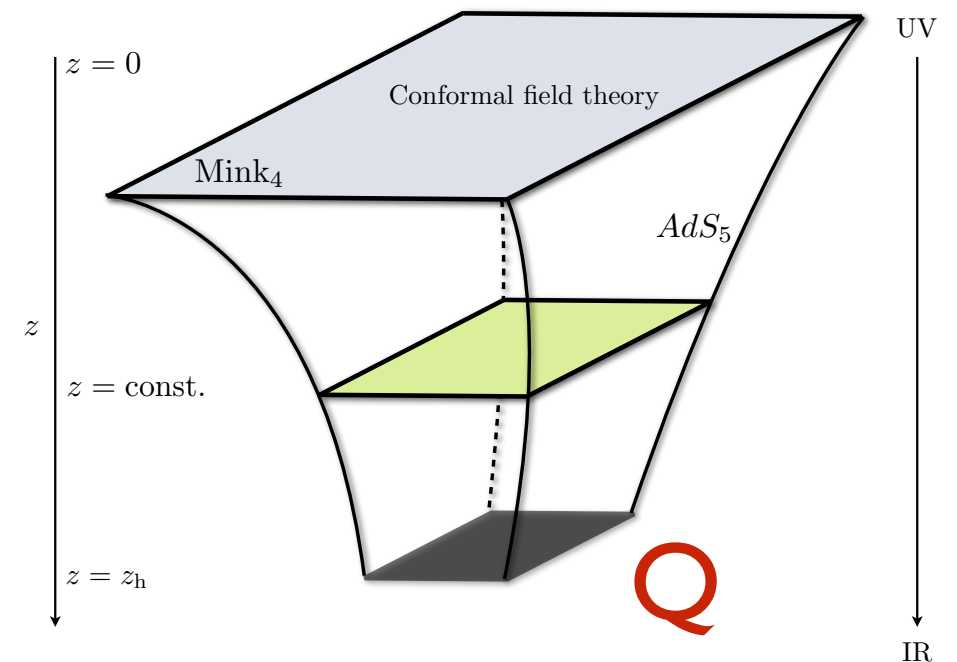
$$h = 1 - \frac{z^4}{z_h^4}$$



- finite T and μ : **charged** BH,
AdS Reissner-Nordström BH

$$ds^2 = \frac{L^2}{z^2} \left(-h dt^2 + d\vec{x}^2 + \frac{dz^2}{h} \right)$$

$$h(z) = 1 - \left(1 + \frac{\mu^2 z_h^2}{3} \right) \frac{z^4}{z_h^4} + \frac{\mu^2 z_h^2}{3} \frac{z^6}{z_h^6}$$



Finite Chemical Potential

consistent model:

Maxwell U(1) field

$$S = \frac{1}{2\kappa^2} \int d^5x \sqrt{-g} \left(\mathcal{R} - \frac{1}{2} \partial_\mu \phi \partial^\mu \phi - V(\phi) - \frac{f(\phi)}{4} F_{\mu\nu} F^{\mu\nu} \right)$$

(action as in DeWolfe, Gubser, Rosen)

solve with ansatz

$$ds^2 = e^{2A(z)} \left(-h dt^2 + d\vec{x}^2 \right) + e^{2B(z)} \frac{dz^2}{h}$$

$$A(z) = \log \left(\frac{L}{z} \right) \quad \phi(z) = \sqrt{\frac{3}{2}} \kappa z^2$$

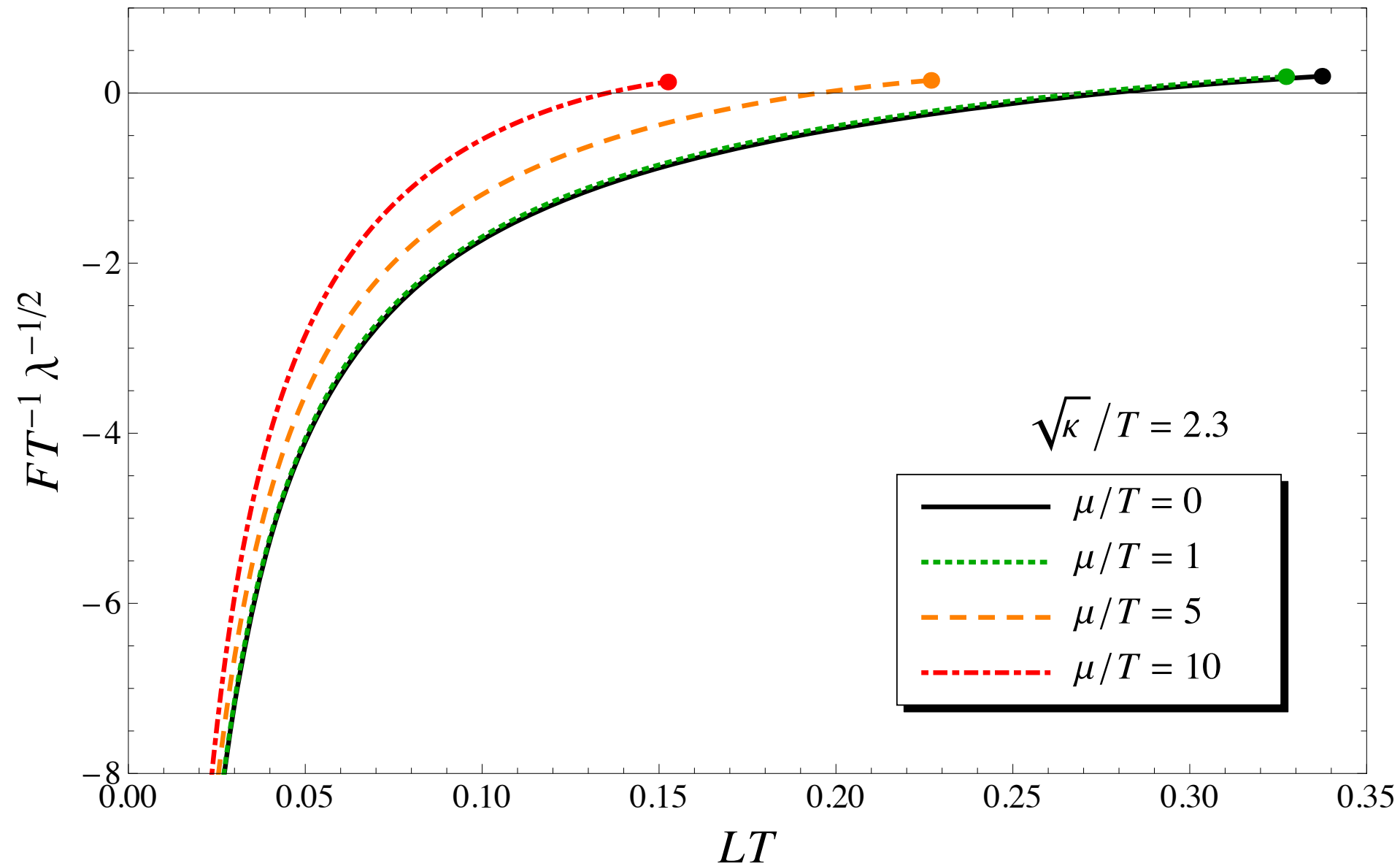
A. Samberg, CE

and choice $f(\phi) = \cosh(12/5) / \cosh(6(\phi - 2)/5)$

- solves 5d gravity action, consistent deformation with scale κ
- evades problems at small T, μ

Observables at Finite Chemical Potential

Binding Energy at Finite T, μ

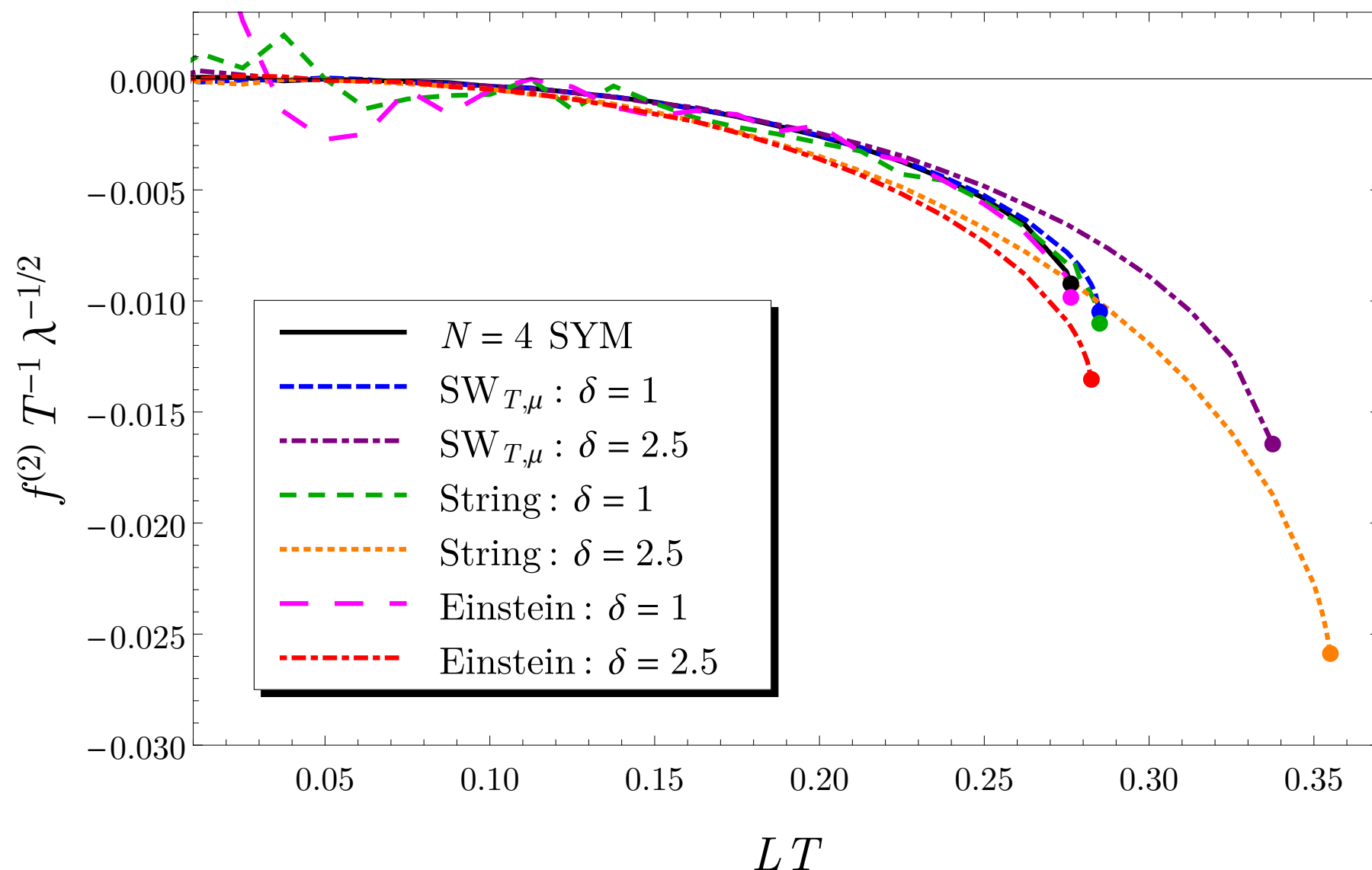


- increasing μ decreases binding strength

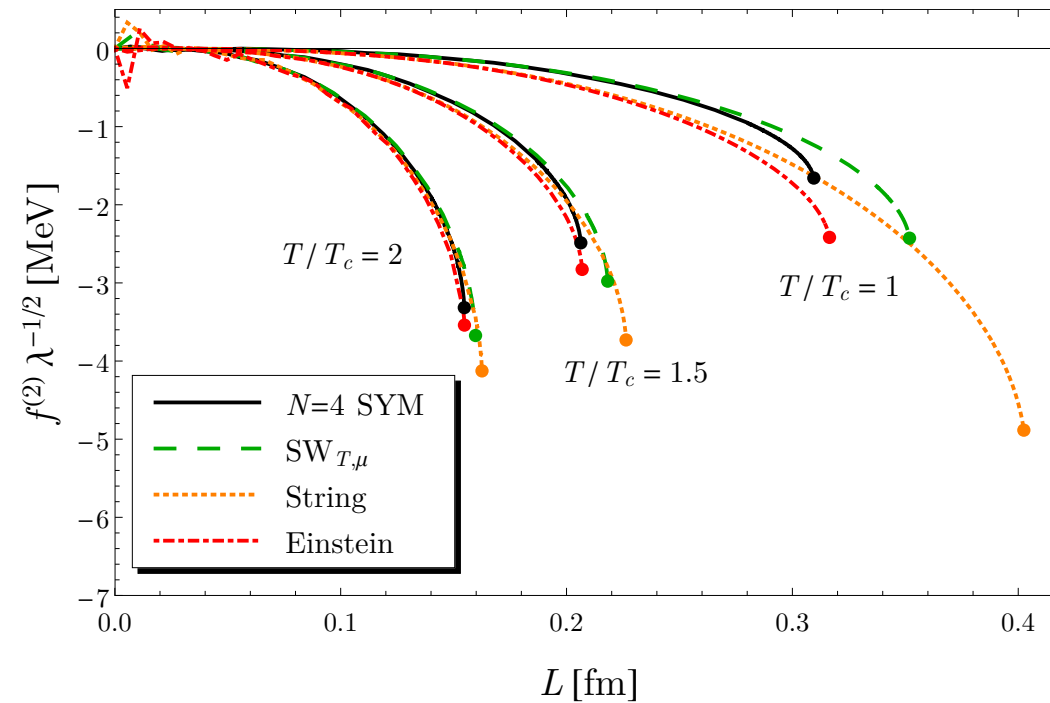
Taylor Coefficients in Expansion in μ

$$F_{Q\bar{Q}}(L; T, \mu) = \sum_{n=0}^{\infty} f^{(n)}(L; T) \left(\frac{\mu}{T} \right)^n$$

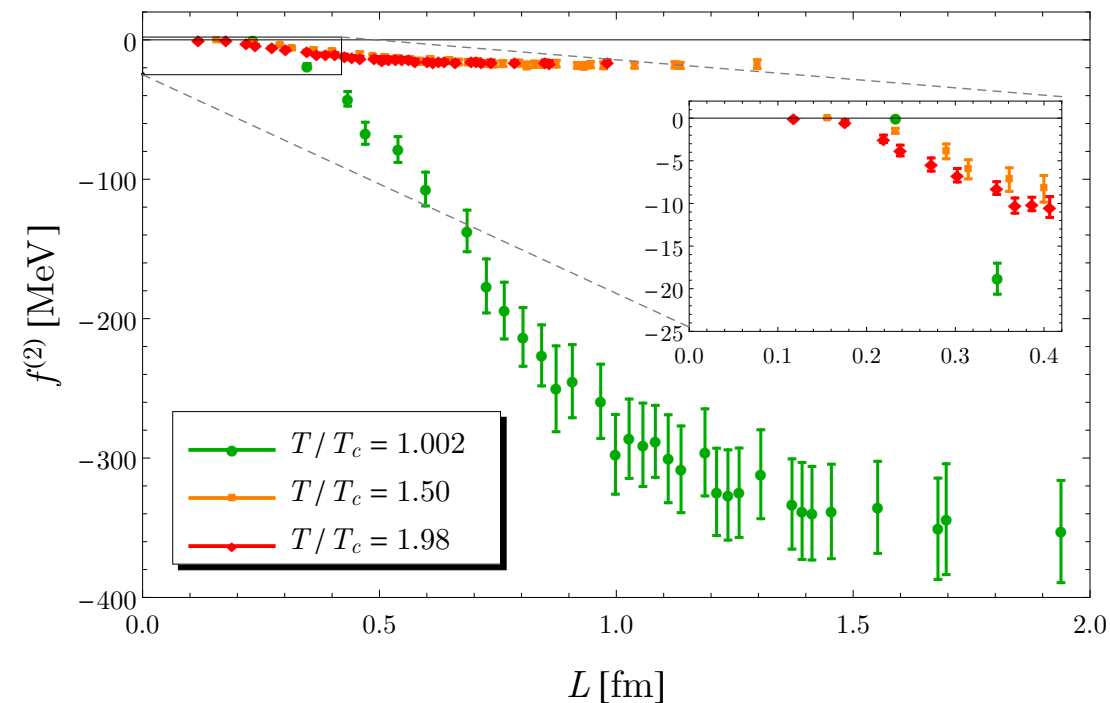
coefficient of order $(\mu/T)^2$:



Taylor Coefficients in Expansion in μ



(a) $f^{(2)}/\sqrt{\lambda}$ in $\mathcal{N} = 4$ SYM and our three non-conformal models. See the text for an explanation of the scale T_c used here. The noisy behavior for small L is a numerical artifact. Note the smaller range in L as compared to the lattice data (lower panel).

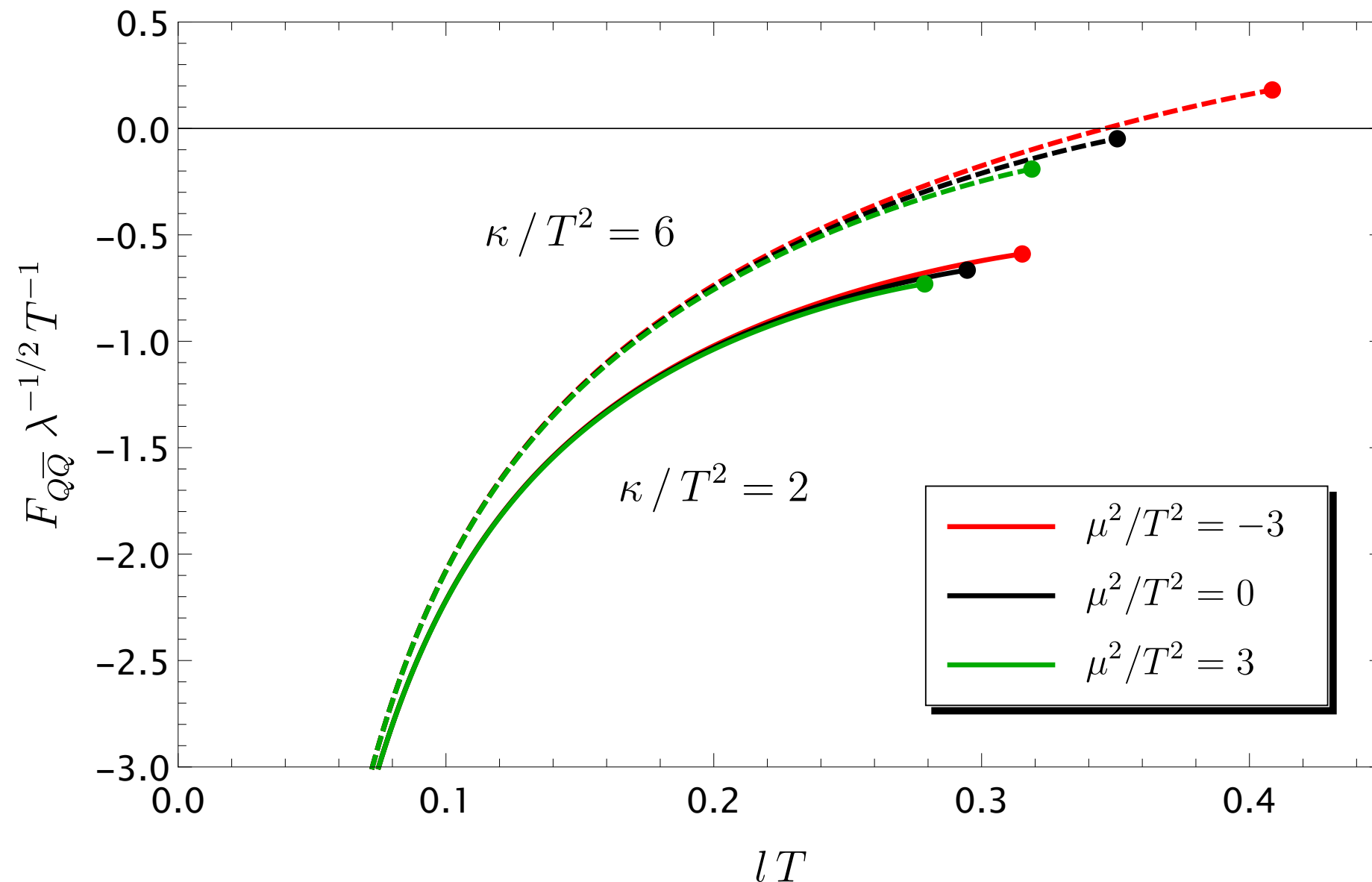


(b) $f^{(2)}$ in 2-flavor lattice QCD [226]. We have chosen from the data of [226] the three temperatures closest to the ones used in the holographic models (upper panel), and converted the data to physical units by using $\sqrt{\sigma} = 420$ MeV for the string tension.

Imaginary Chemical Potential

black hole configuration in AdS is stable for some range of **negative μ^2** (positive heat capacity of BH)

Albert Bekov, CE



Summary & Outlook

Summary & Outlook

- Holography applied to heavy quark observables, incl. static potential in strongly coupled plasma
- Screening distance conjecture:
 L_s is bounded from below by its value in $N=4$ SYM
- First systematic calculation of free (and internal) energy of $QQbar$ pair in holography (correct UV renormalization)
- Running coupling
- Universal strong-coupling behavior in these observables

Summary & Outlook

- Complex static potential: dynamics at distances above L_s not yet understood
- Finite chemical potential: same quantities calculated
- Taylor coefficients qualitatively agree with lattice results
- Imaginary chemical potential: comparison to lattice will be interesting!

Thanks for your interest!

One more thing ...

pbm 80

Best wishes for your birthday!

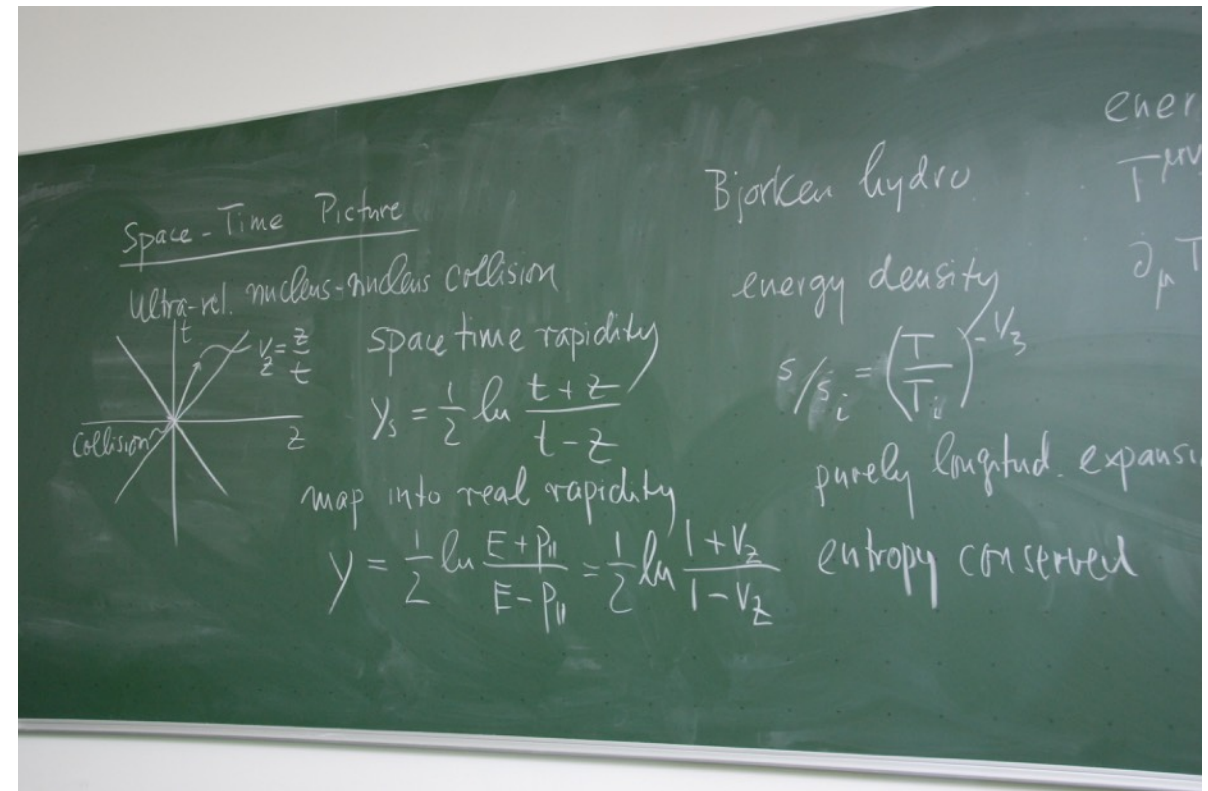


pbm 80



pbm 80

Achtzig Jahre ist er nun,
achtzig Jahre, achtzig!
Bringt den Tag zu mit Studieren,
wie die Ionen kollidieren,
denn vor allem Physizieren
macht ihn froh und lustig.



pbm 80

Mit Johanna tüftelt er
an so manchem Thema.
Forscht an dem was ihm gefällt,
reist auch häufig in die Welt,
wo er viele Vorträg' hält,
über Quarks im Plasma.



pbm 80

Geistesblitze und Ideen
mögen stets sich regen.
Forschung bleibe ihm Pläsier,
daß er nie die Lust verlier.
Zum Geburtstag wünschen wir
lauter Glück und Segen.

