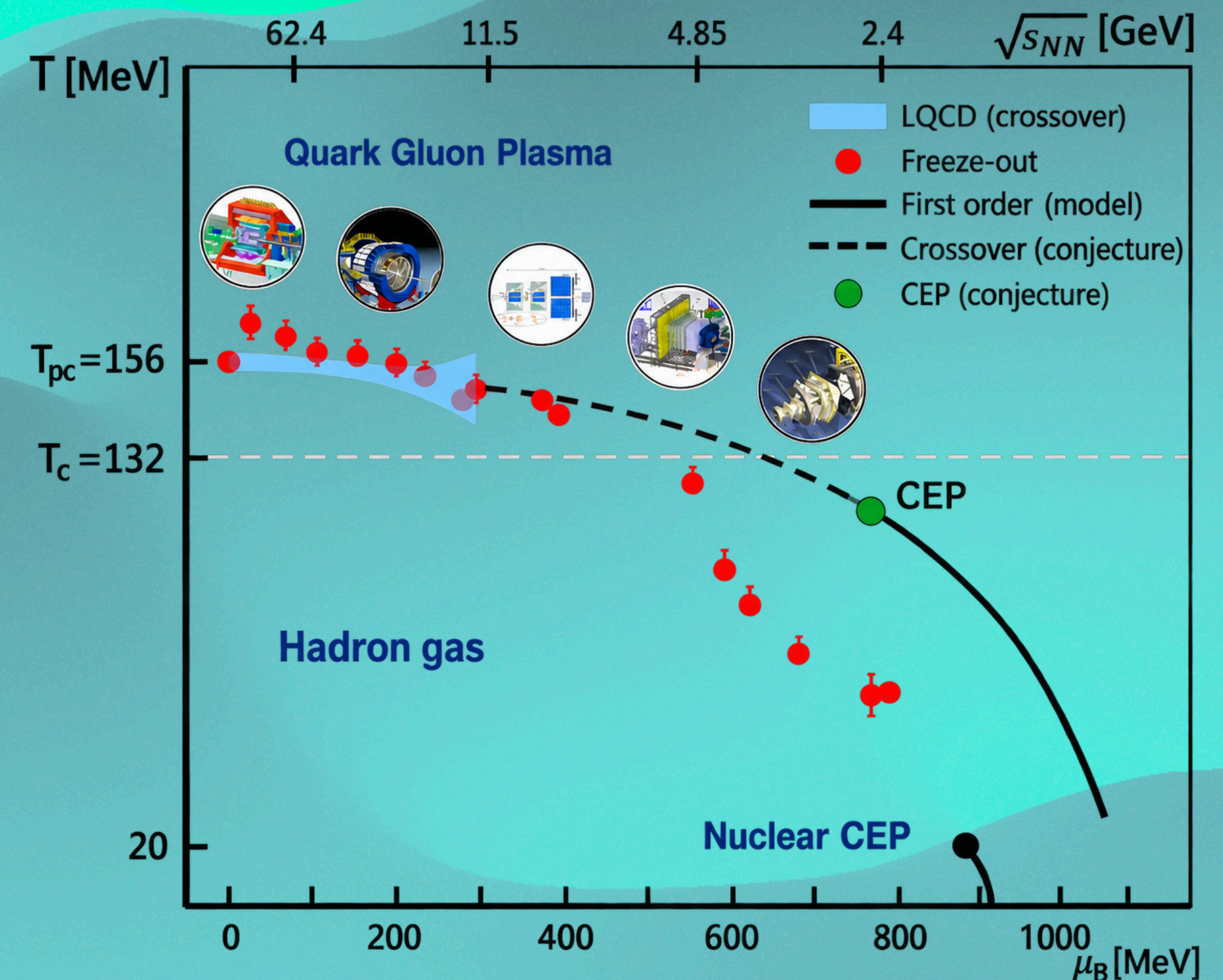


Precision QCD Thermodynamics

from fluctuation measurements to precision thermal baselines

Anar Rustamov



Looking for Trends



At my PhD defense (2006)

- Our only photo together from my entire PhD!
- Those were different times...

Looking for Trends



At my PhD defense (2006)

- Our only photo together from my entire PhD!
- Those were different times...



20 years later...

- **Trend:** 2006 → 2026 →
- At this rate, the next photo is already scheduled

Looking for Trends



First question in 2002:
why does the pion have negative
parity?

$$P_{q\bar{q}} = (-1)^{L+1} \xrightarrow{L=0} P = -1$$

At my PhD defense (2006)

- Our only photo together from my entire PhD!
- Those were different times...

20 years later...

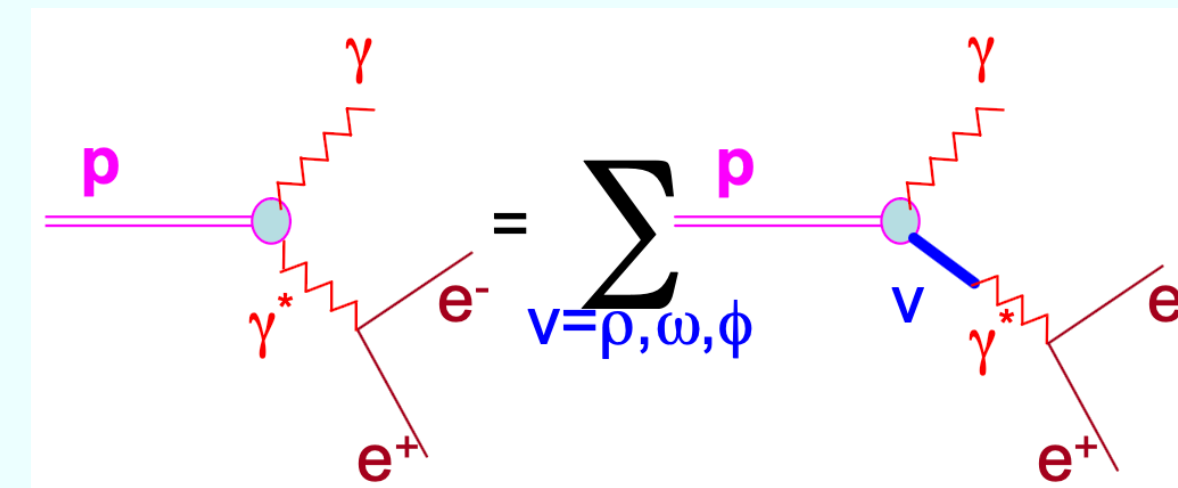
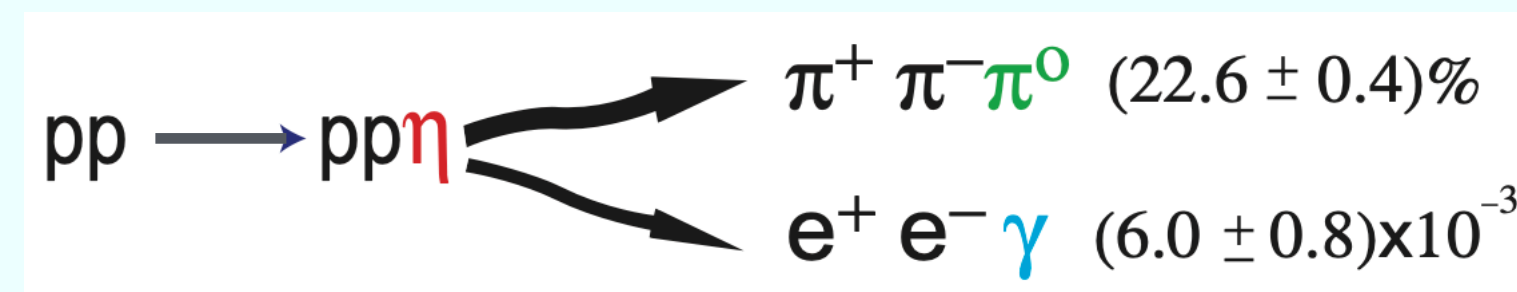
- **Trend:** 2006 → 2026 →
- At this rate, the next photo is already scheduled



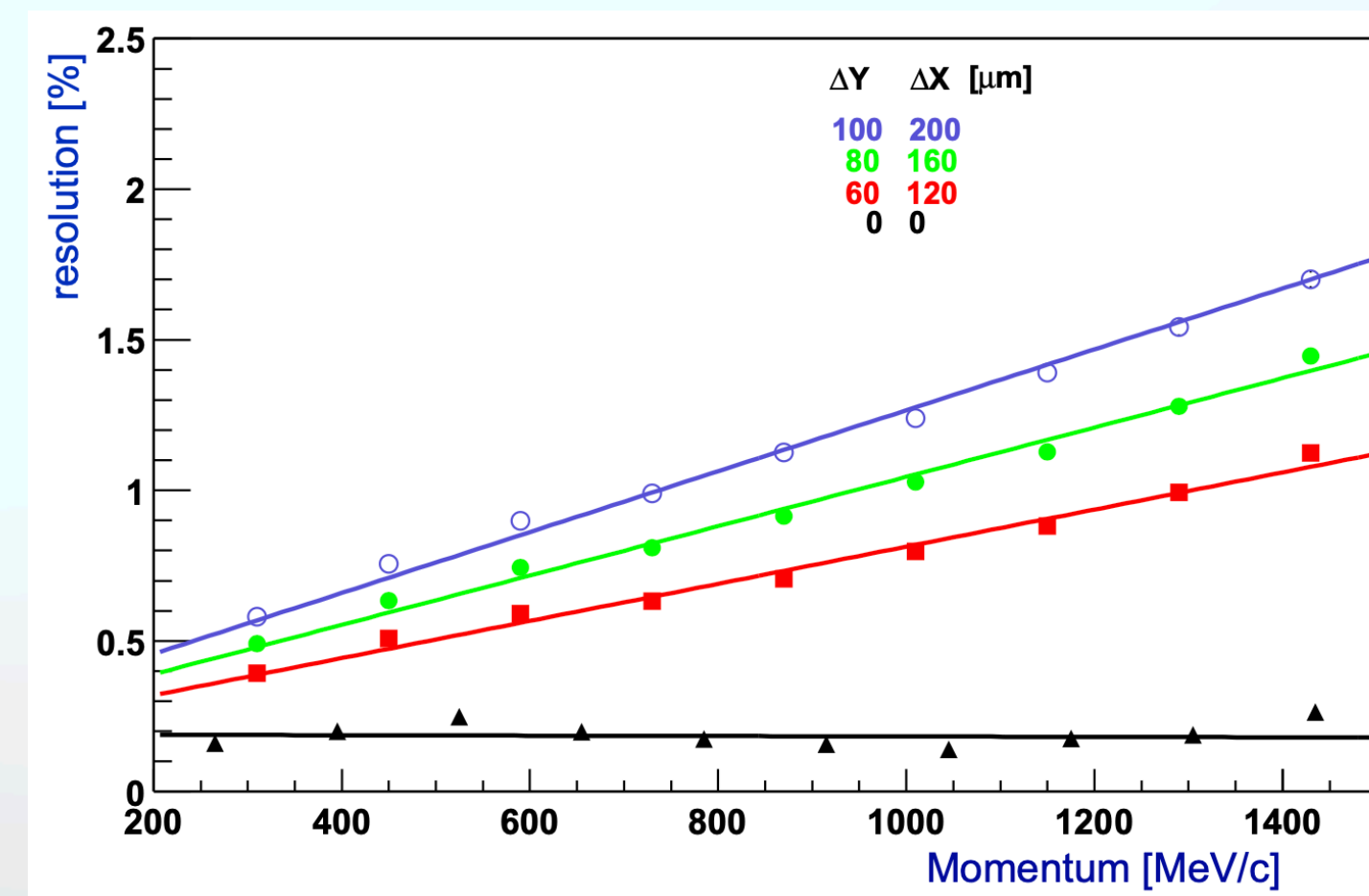
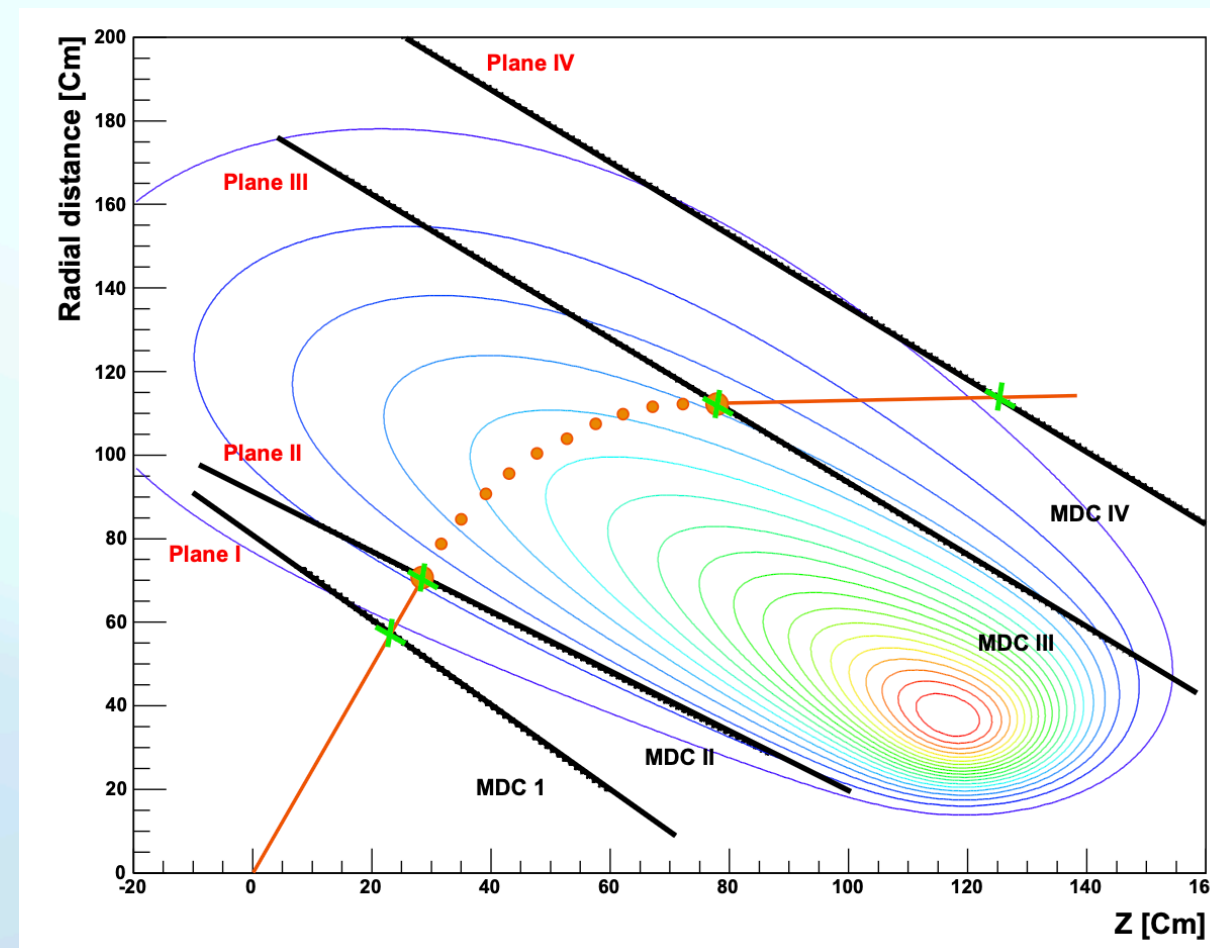
Elementary p+p reactions in HADES

One innocent question → a PhD in HADES

Electromagnetic transition form factor of the η meson



Charged-particle momentum reconstruction in a toroidal magnetic field

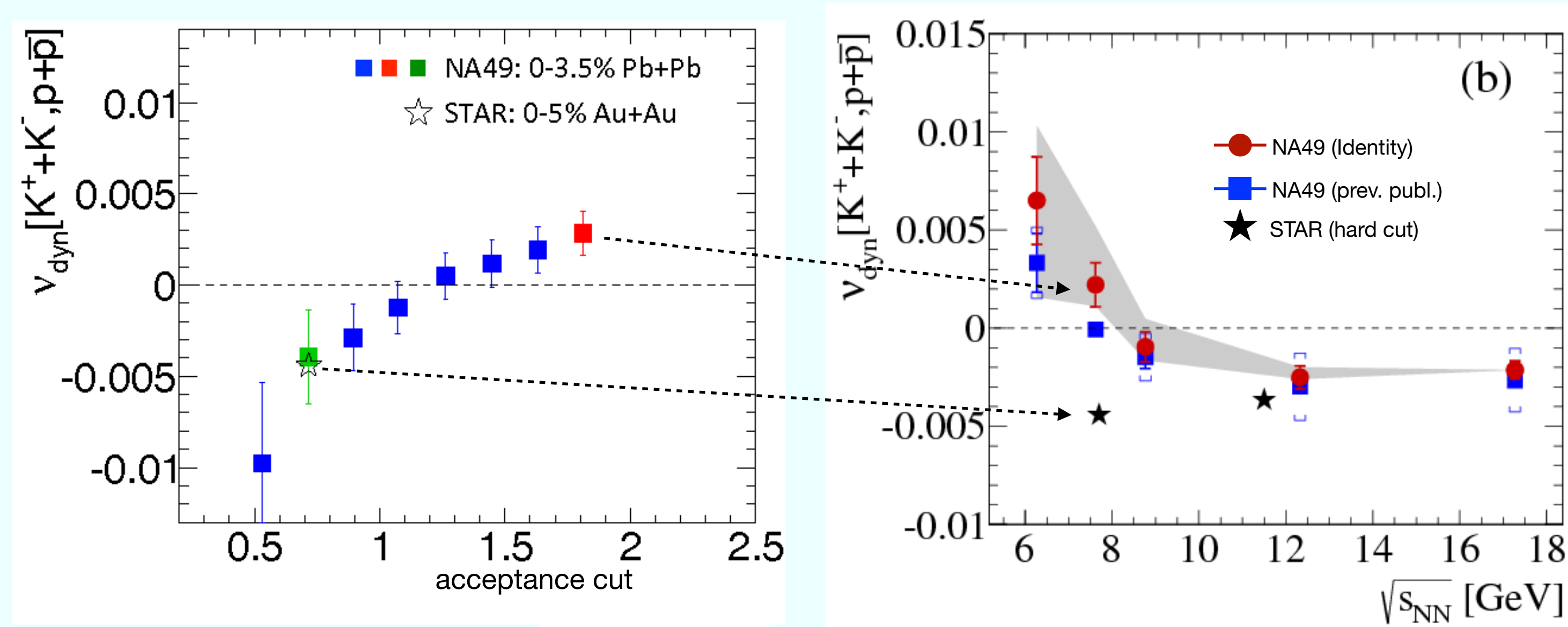


► A.R., PhD Thesis, 2006

Fortunately, today there will be no questions from the committee

Big transition in 2011

from p+p to A+A collisions: extending from HADES to NA49, NA61/SHINE



NA49: Phys.Rev.C 89 (2014) 5, 054902

An innovative method for fluctuation analysis, **Identity Method**

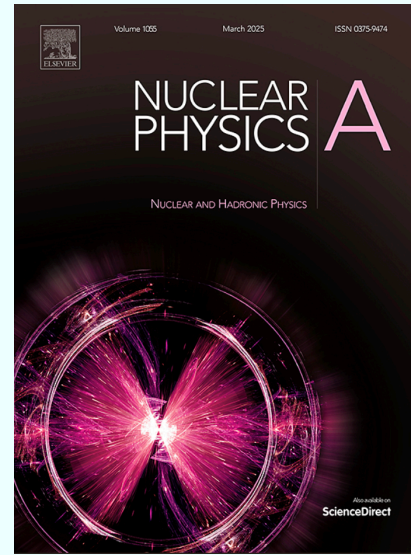
AR, M. I. Gorenstein, PRC 86, 044906 (2012)

M. Arslanok, AR, NIM A946, 162622 (2019)

A. Rustamov, Phys.Rev.C 110 (2024) 6, 064910

In 2016, the circle closed: back to Peter, this time in ALICE

Our first paper



Bridging the gap between event-by-event fluctuation measurements and theory predictions in relativistic nuclear collisions, [P. Braun-Munzinger, A. Rustamov, J. Stachel, Nucl. Phys. A 960 \(2017\) 114-130.](#)

fixed volume/sources

$$\langle N \rangle \sim V, \quad \langle N \rangle \sim N_W$$

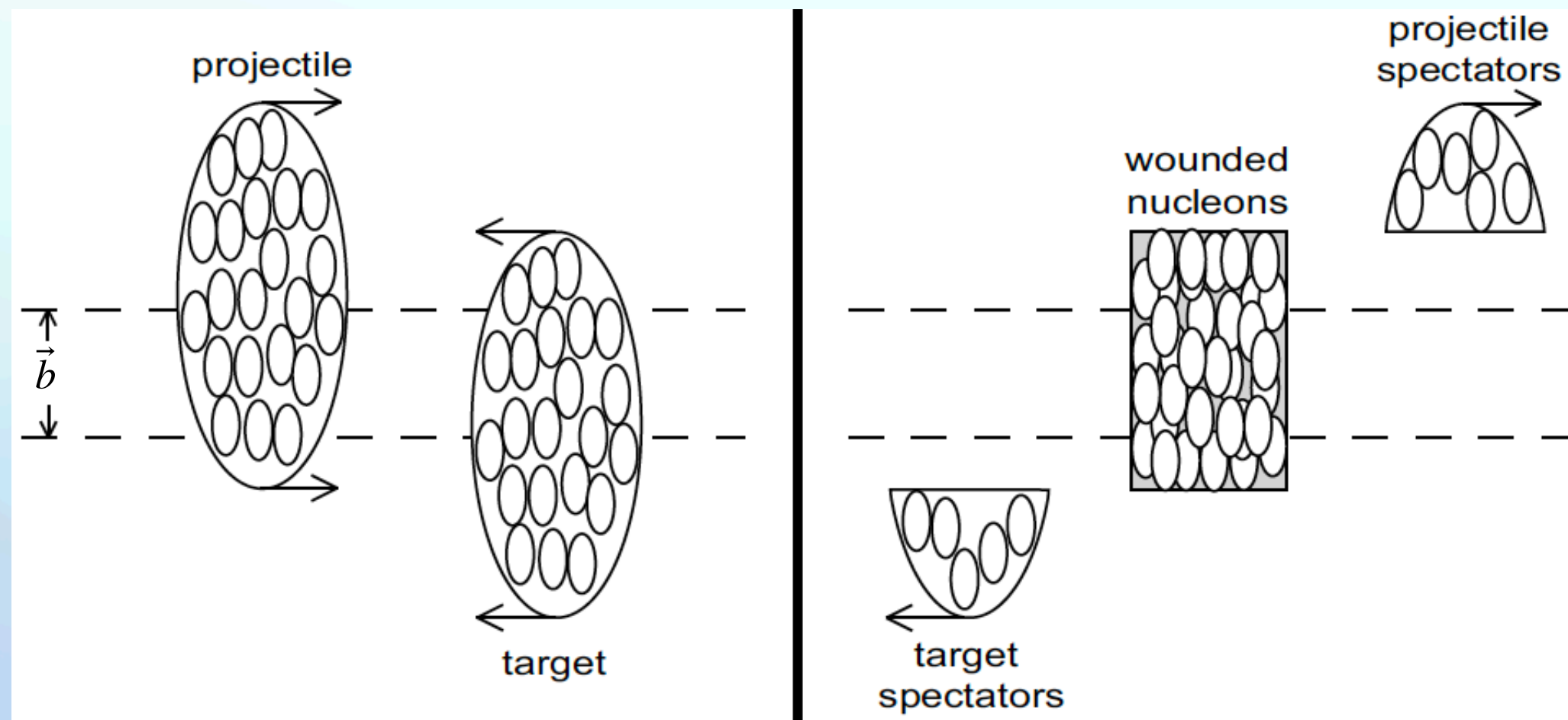
$$\kappa_2(N) \sim V, \quad \kappa_2(N) \sim N_W$$

[V. Skokov, B. Friman, and K. Redlich, Phys.Rev. C88 \(2013\) 034911](#)

fluctuating sources

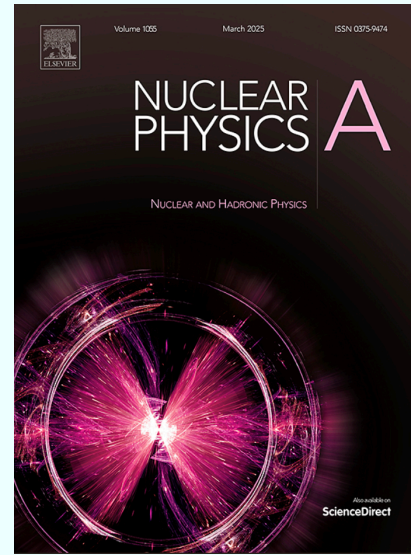
$$\langle N \rangle = \langle n \rangle \langle N_W \rangle$$

$$\kappa_2(N) = \kappa_2(n) \langle N_W \rangle + \langle N \rangle^2 \frac{\kappa_2(N_W)}{\langle N_W \rangle^2}$$



Wounded nucleons, N_W : Nucleons which collided at least once inelastically

Our first paper



Bridging the gap between event-by-event fluctuation measurements and theory predictions in relativistic nuclear collisions, [P. Braun-Munzinger, A. Rustamov, J. Stachel, Nucl. Phys. A 960 \(2017\) 114-130.](#)

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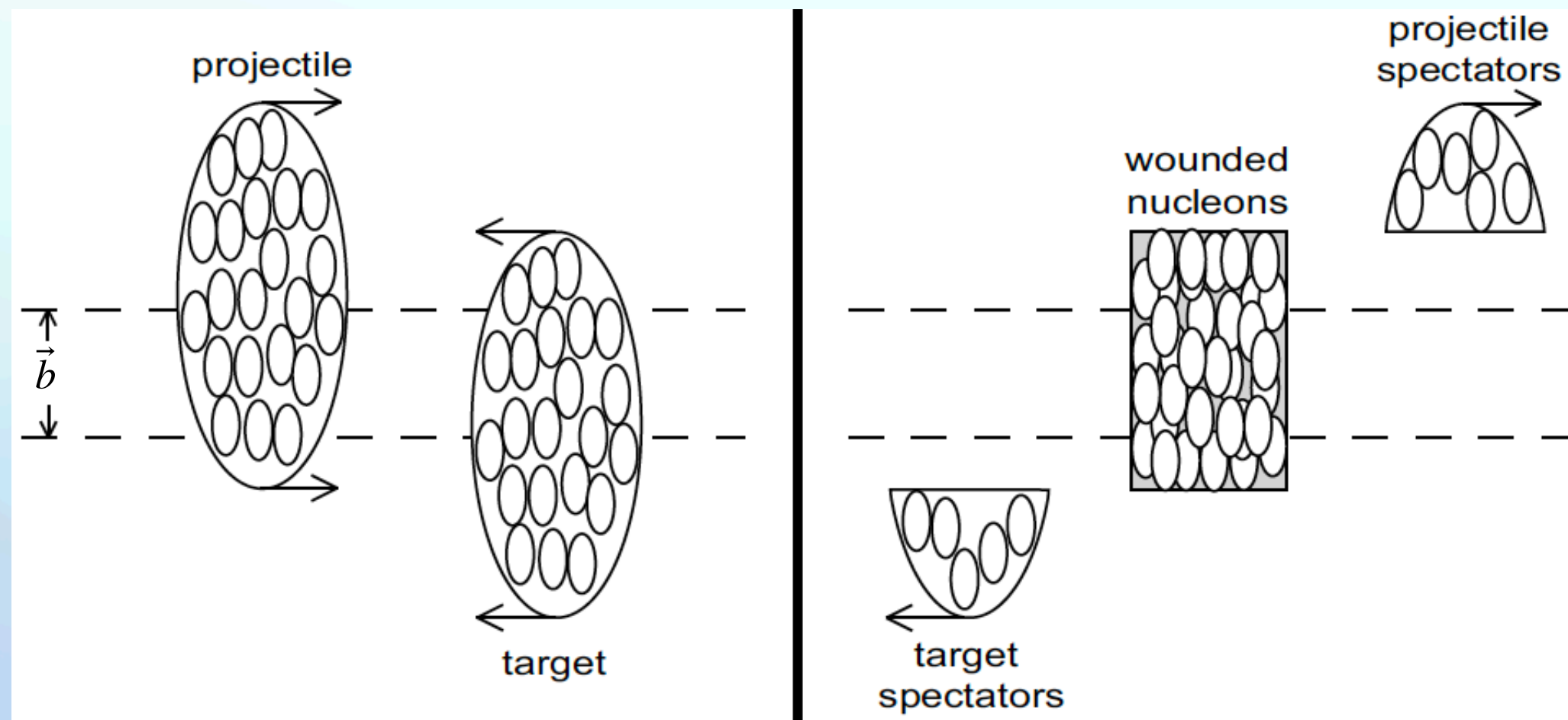
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[V. Skokov, B. Friman, and K. Redlich, Phys.Rev. C88 \(2013\) 034911](#)

fluctuating sources

$$\langle N \rangle = \langle n \rangle \langle N_W \rangle$$

$$\kappa_2(N) = \kappa_2(n) \langle N_W \rangle + \langle N \rangle^2 \frac{\kappa_2(N_W)}{\langle N_W \rangle^2}$$



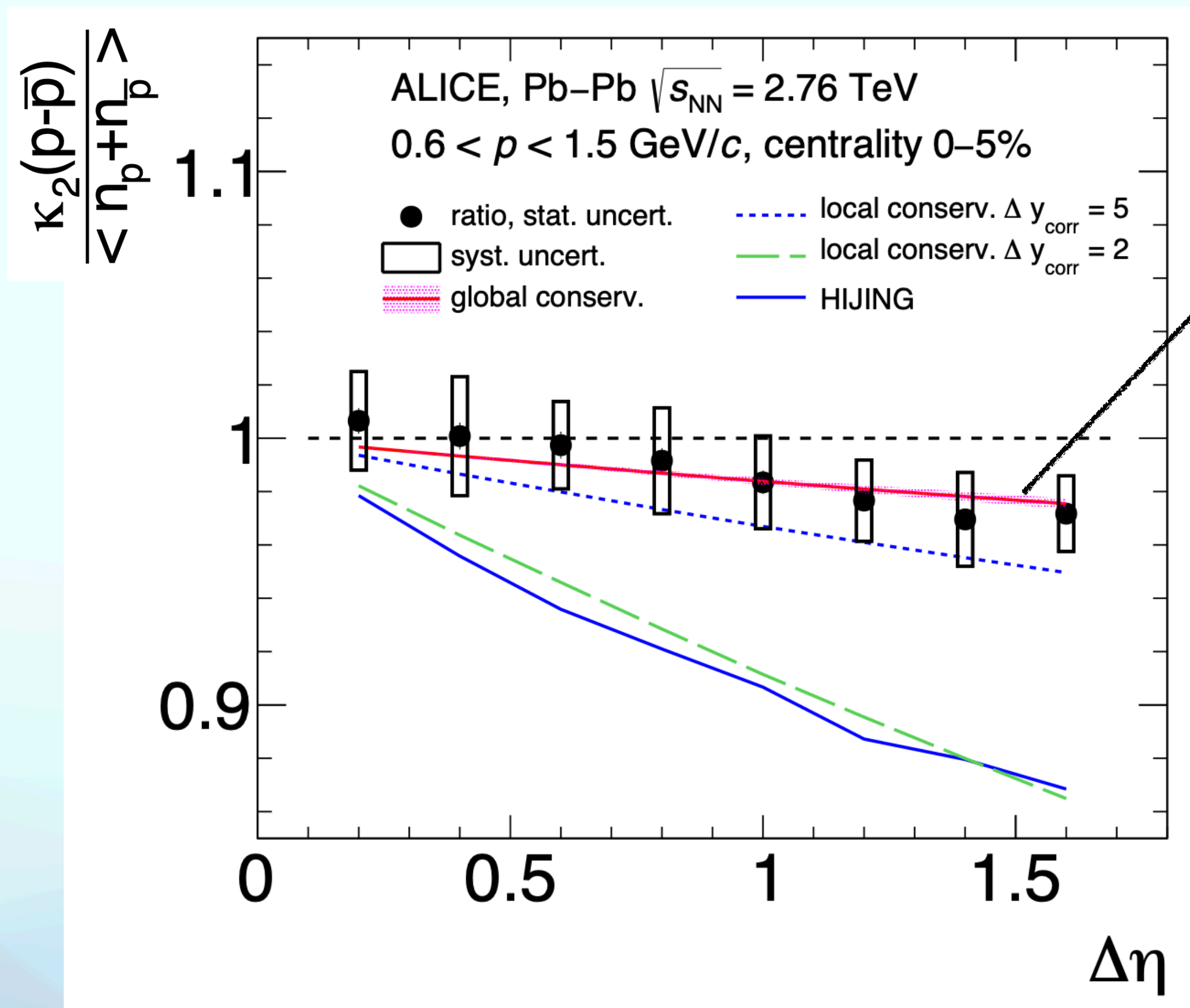
Artifact of baryon number conservation

$$\frac{\kappa_2(p - \bar{p})}{\langle n_p + n_{\bar{p}} \rangle} = 1 - \alpha$$

α - fraction of accepted protons

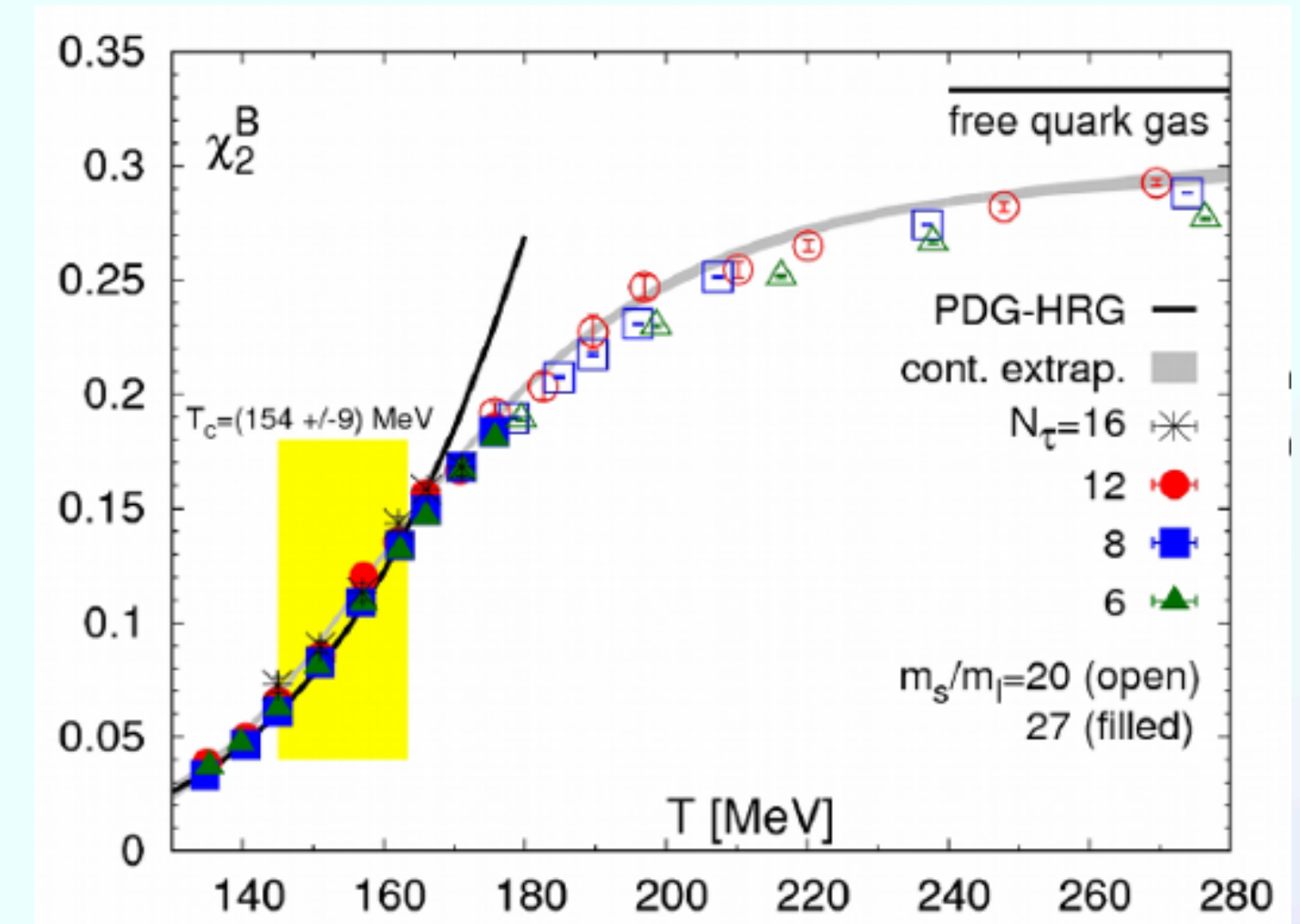
Wounded nucleons, N_W : Nucleons which collided at least once inelastically

First Alice results with the Identity Method



$$1 - \alpha(\Delta\eta)$$

$$\alpha_{\Delta\eta} = \frac{\langle n_p \rangle^{acc}(\Delta\eta)}{\langle N_B \rangle^{4\pi}}$$



A. Bazavov et al [HotQCD], PRD 101 (2020) 074502
 A. Bazavov et al., Phys.Rev. D85 (2012) 054503

$$\frac{\kappa_n(N_B - N_{\bar{B}})}{VT^3} = \frac{1}{VT^3} \frac{\partial^n \ln Z(V, T, \mu_B)}{\partial (\mu_B/T)^n} \equiv \hat{\chi}_n^B$$

first experimental verification of this LQCD prediction

Consequences:

- Support for the validity of the HRG model
- Further support for freeze-out at the phase boundary

A. R., Nucl.Phys.A 967 (2017) 453-456 (QM 17)

ALICE: Phys. Lett. B 807 (2020) 135564,
 Phys. Lett. B (2022) 137545

Paper committee: M. Arslanok, P. Braun-Munzinger, A. R., J. Stachel

Exact conservation suppresses fluctuations

$$Z_B(V, T) = \int_0^{2\pi} \frac{d\phi}{2\pi} e^{-iB\phi} \exp(\lambda_B z_B e^{i\phi} + \lambda_{\bar{B}} z_{\bar{B}} e^{-i\phi}) = \left(\frac{\lambda_B z_B}{\lambda_{\bar{B}} z_{\bar{B}}} \right)^{\frac{B}{2}} I_B(2 z \sqrt{\lambda_B \lambda_{\bar{B}}}) \quad \mu_B \rightarrow iT\phi$$

B net baryon number, conserved in each event

I_B modified Bessel function of the first kind

$z_B, z_{\bar{B}}$ single particle partition functions for baryons, anti baryons

$\lambda_B, \lambda_{\bar{B}}$ auxiliary parameters for calculating cumulants of baryons, anti baryons

► P. Braun-Munzinger, B. Friman, K. Redlich, AR., J. Stachel, NPA 1008 (2021) 122141

► A. Bzdak, V. Koch, V. Skokov, Phys.Rev.C 87 (2013) 1, 014901

$$\frac{\kappa_2(B - \bar{B})}{\langle n_B + n_{\bar{B}} \rangle} = 1 - \frac{\alpha_B \langle n_B \rangle + \alpha_{\bar{B}} \langle n_{\bar{B}} \rangle}{\langle n_B + n_{\bar{B}} \rangle} + (z^2 - \langle N_B \rangle \langle N_{\bar{B}} \rangle) \frac{(\alpha_B - \alpha_{\bar{B}})^2}{\langle n_B + n_{\bar{B}} \rangle}$$

$\langle N_B \rangle, \langle N_{\bar{B}} \rangle$ - in 4π

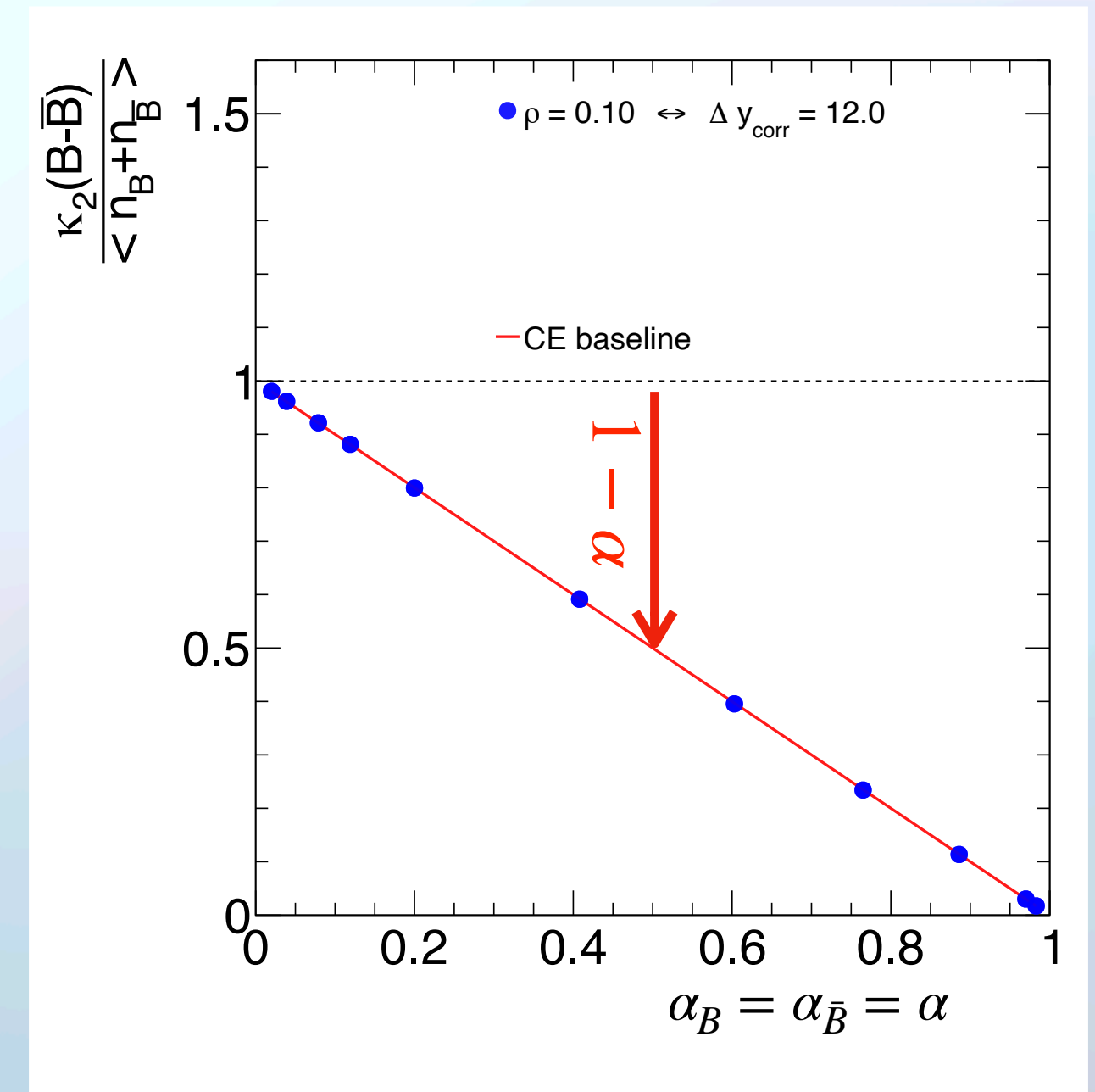
canonical suppression

$\langle n_B \rangle, \langle n_{\bar{B}} \rangle$ - inside acceptance

$\alpha_B = \langle n_B \rangle / \langle N_B \rangle$ - acceptance for B

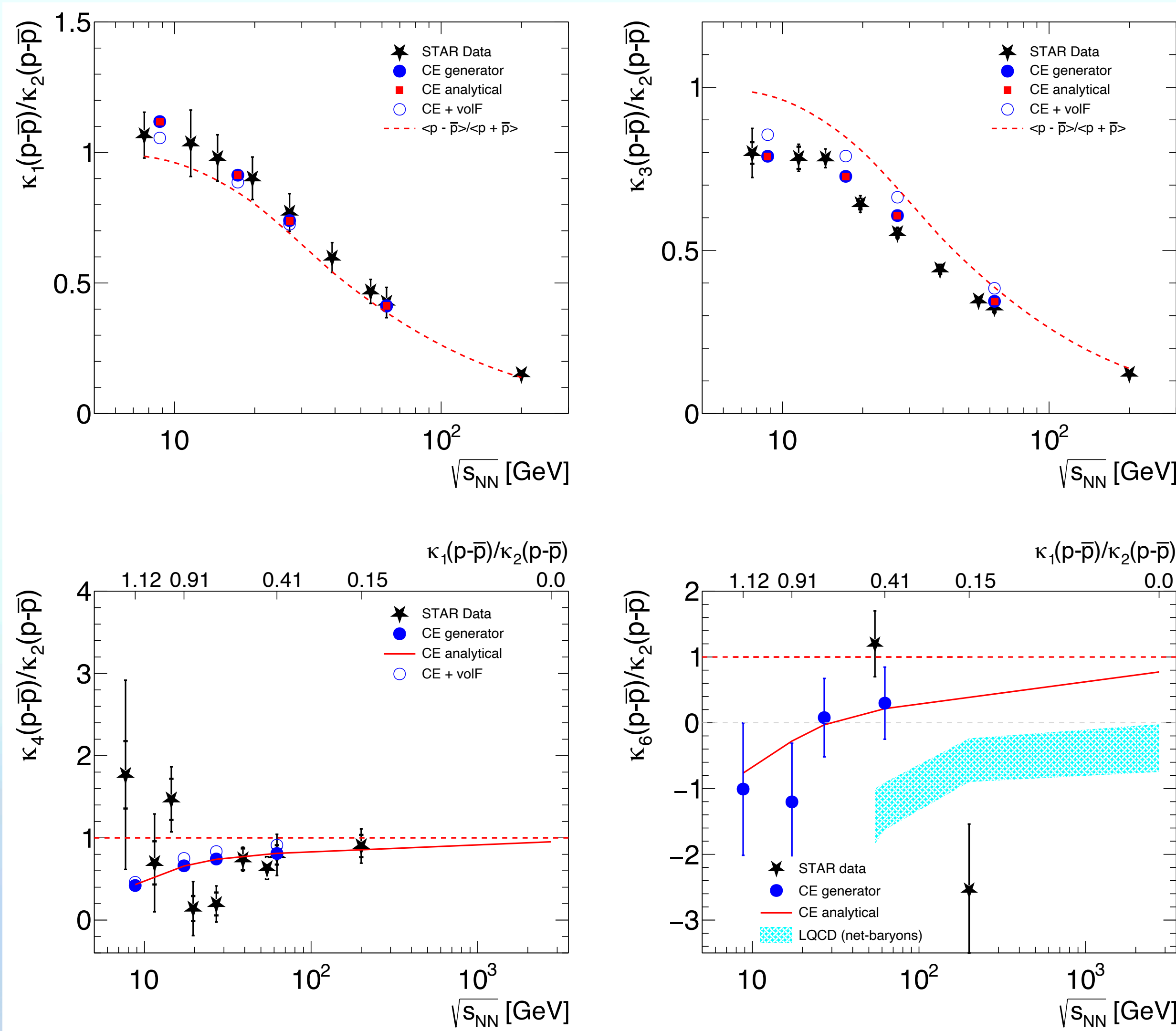
$\alpha_{\bar{B}} = \langle n_{\bar{B}} \rangle / \langle N_{\bar{B}} \rangle$ - acceptance for $\bar{B}$

z - single baryon partition function



Canonical baselines explain much of STAR BES-I

the first quantitative canonical baselines



remarkable agreement between
canonical baseline and STAR BES-I data

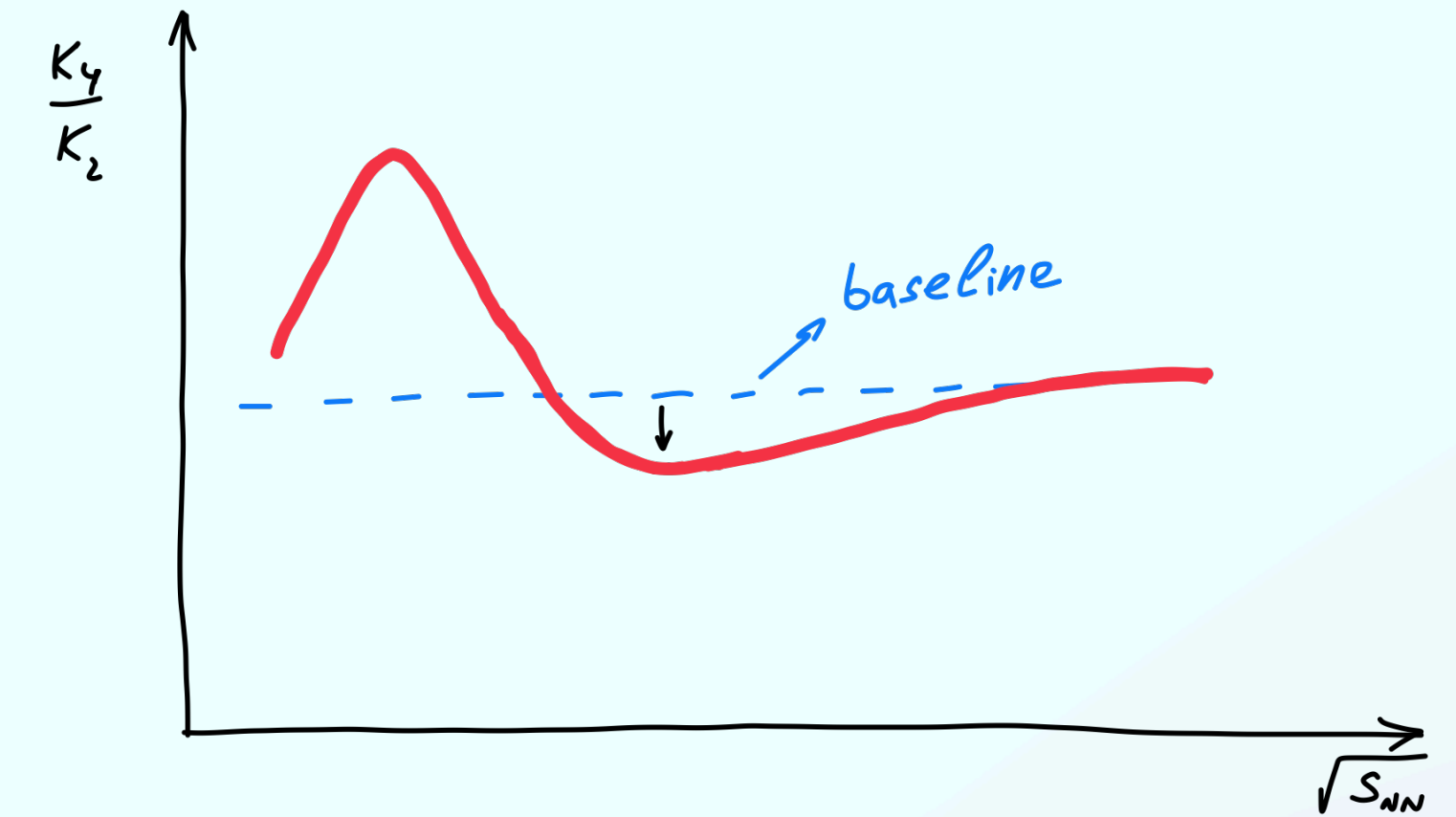
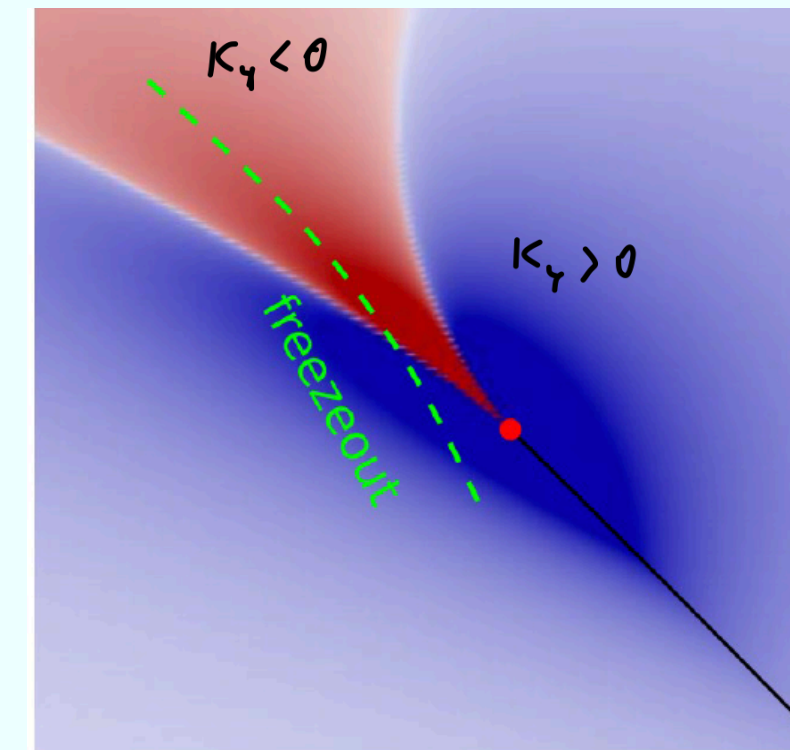
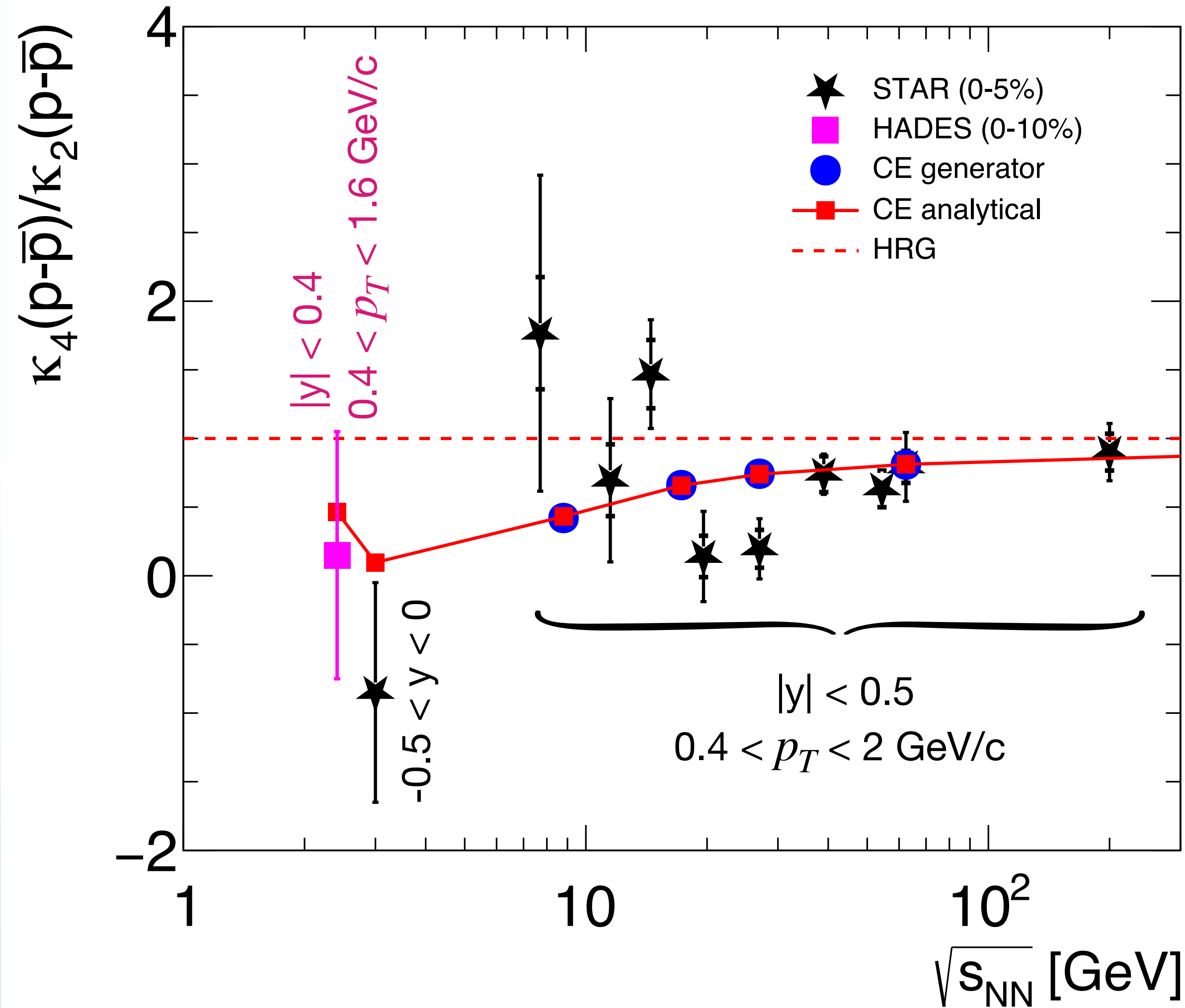
- significant reduction of canonical baseline for κ_6/κ_2 going from positive values at LHC to negative values at lower energies
- STAR DATA for κ_6/κ_2 show tension with the LQCD prediction

STAR: PRL 126 (2021) 9, 092301, PRL 130 (2023) 8, 082301
P. Braun-Munzinger, B. Friman, K. Redlich, A.R., J. Stachel, NPA 1008 (2021) 122141

The significance depends on the baseline

HADES: Phys.Rev.C 102 (2020) 2, 024914

STAR: Phys.Rev.Lett. 126 (2021) 9, 092301



a dip in the excitation function is generic

M. Stephanov, PRL102.032301(2009), PRL107.052301(2011)

M.Cheng et al, PRD79.074505(2009)

STAR: Phys.Rev.Lett. 126 (2021) 9, 092301

“non-monotonic behavior with a significance of 3.1σ relative to GCE expectation”

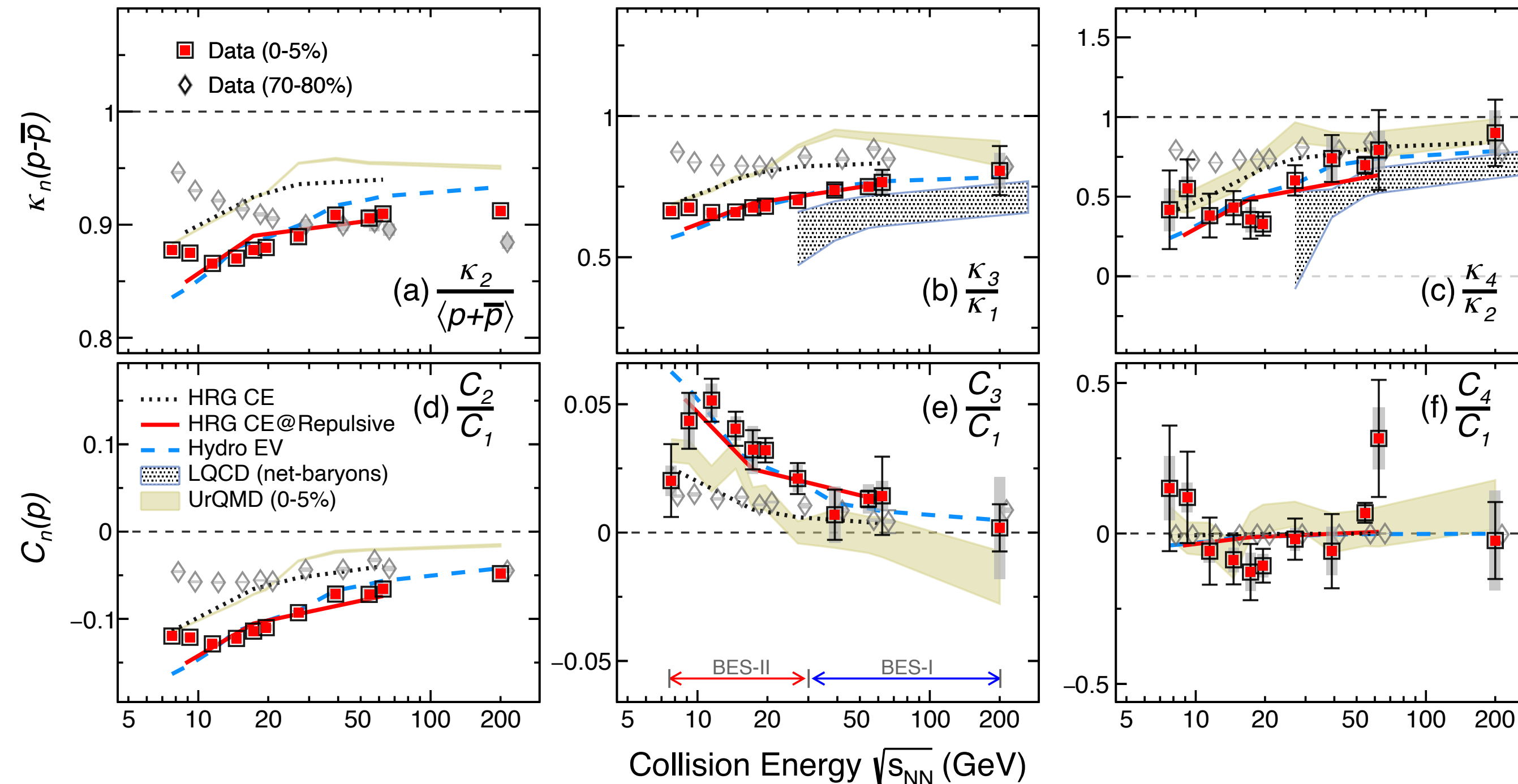
CE Baseline: P. Braun-Munzinger, B. Friman, K. Redlich, A.R., J. Stachel, NPA 1008 (2021) 122141

no statistically significant difference between the data and the canonical baseline (KS test: 1.2σ , χ^2 test: 1.5σ)

Inter-baryon interactions are decisive

PRL 135 (2025) 14, 142301

0-5% Au+Au Collisions at RHIC, $0.4 < p_T(\text{GeV}/c) < 2.0$, $|y| < 0.5$



Clear deviations from canonical baselines
Better description with inter-baryon interactions

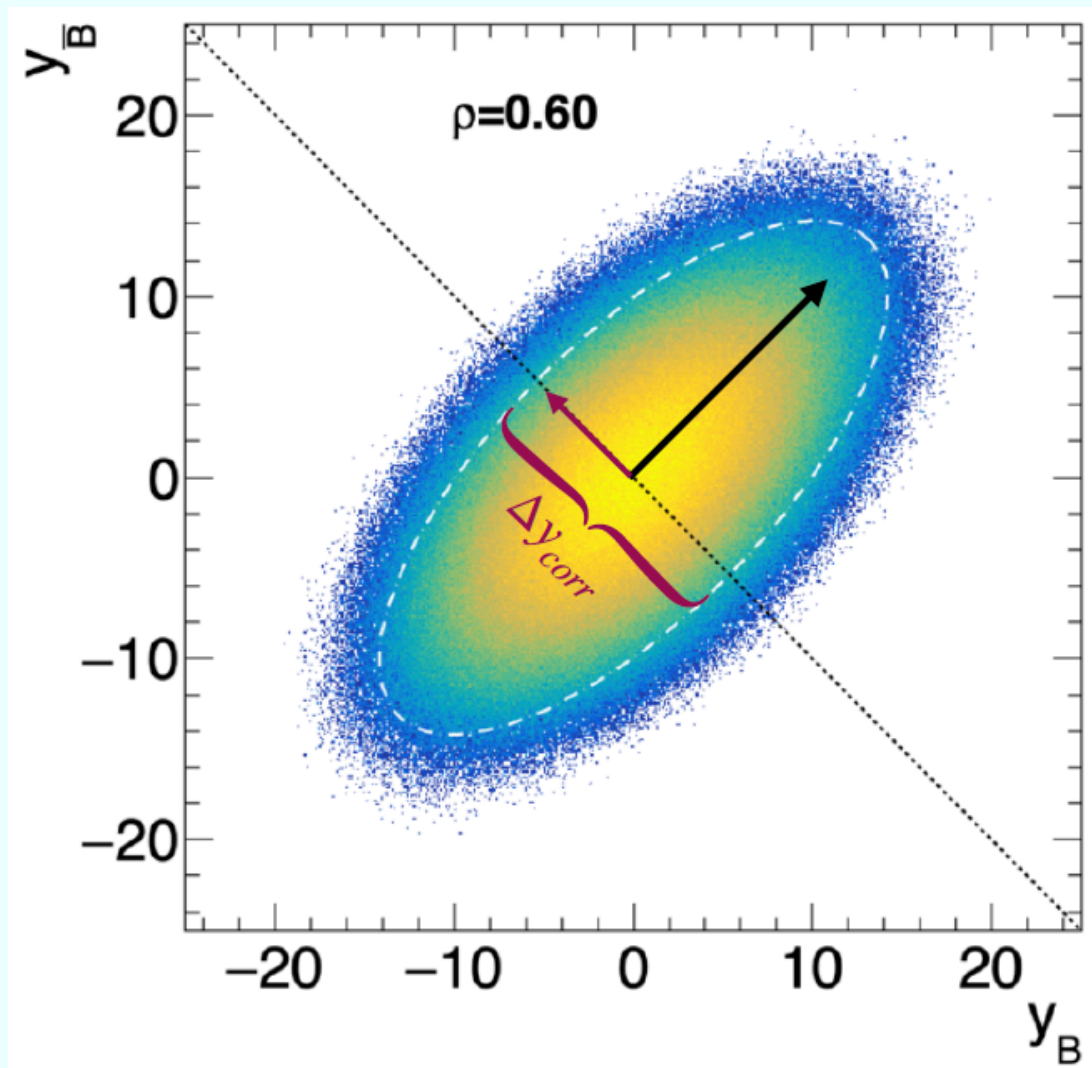
- **At energies > 10 GeV:**
 - Dominance of **repulsive interactions**
 - B. Friman, A. R., K. Redlich, e-Print: 2508.18879 [nucl-th]
 - V. Vovchenko, V. Koch, Ch. Shen, PRC 105, 014904 (2022)
- **At lower energies**
 - Repulsive part weakens
 - Do attractive interactions take over?
 - B. Friman, A. R., K. Redlich, JHEP 07 (2026) 20

κ_n - ordinary cumulants

C_n - factorial cumulants

P. Braun-Munzinger, A. R. and N. Xu,
arXiv:2601.18666, to appear in Ann. Rev. Nucl. Part. Sci.

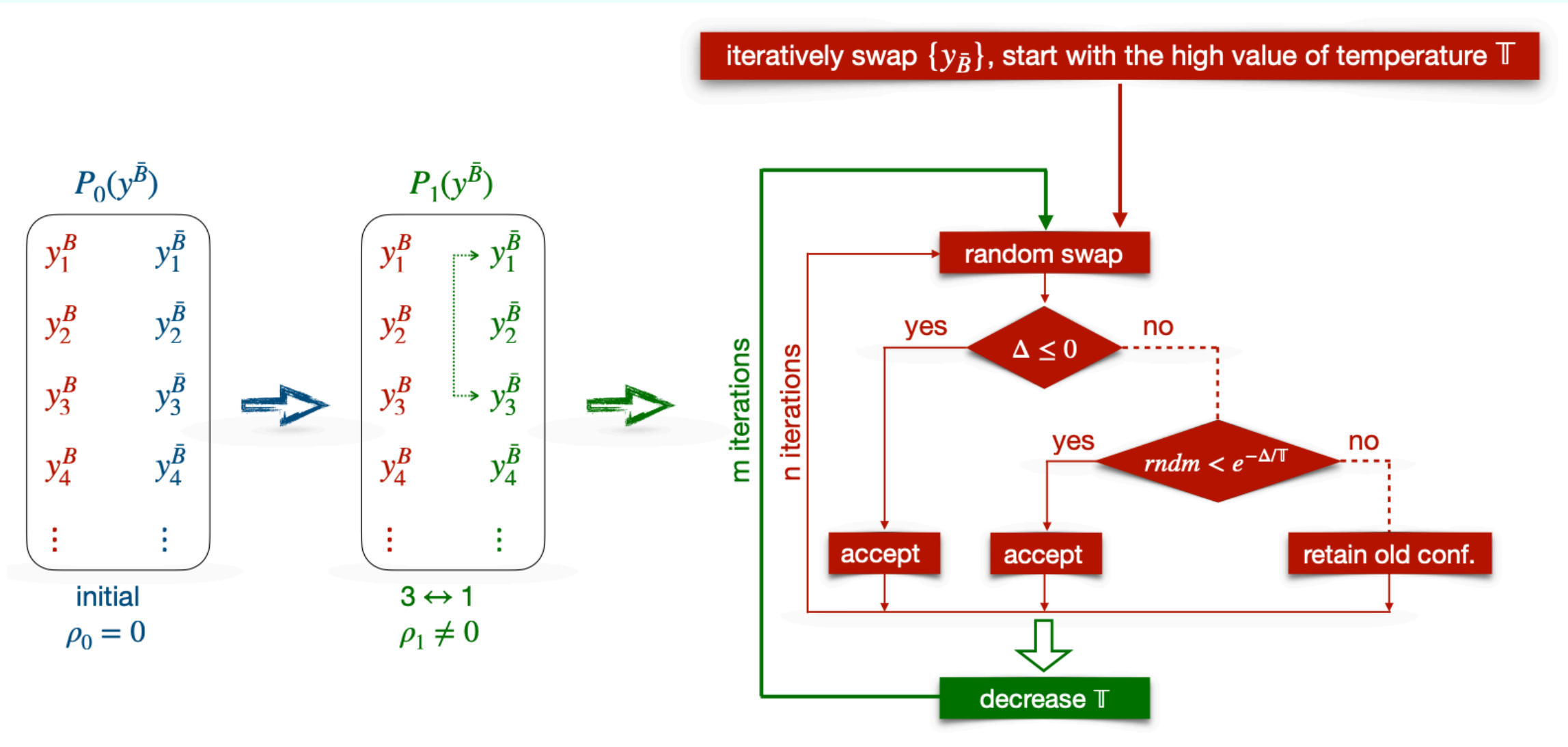
Conservation can be local



- Introducing correlations between baryon-antibaryon rapidities
- Minimizing cost function using Metropolis and Simulated Annealing
- Integrating with the Canonical Ensemble partition function

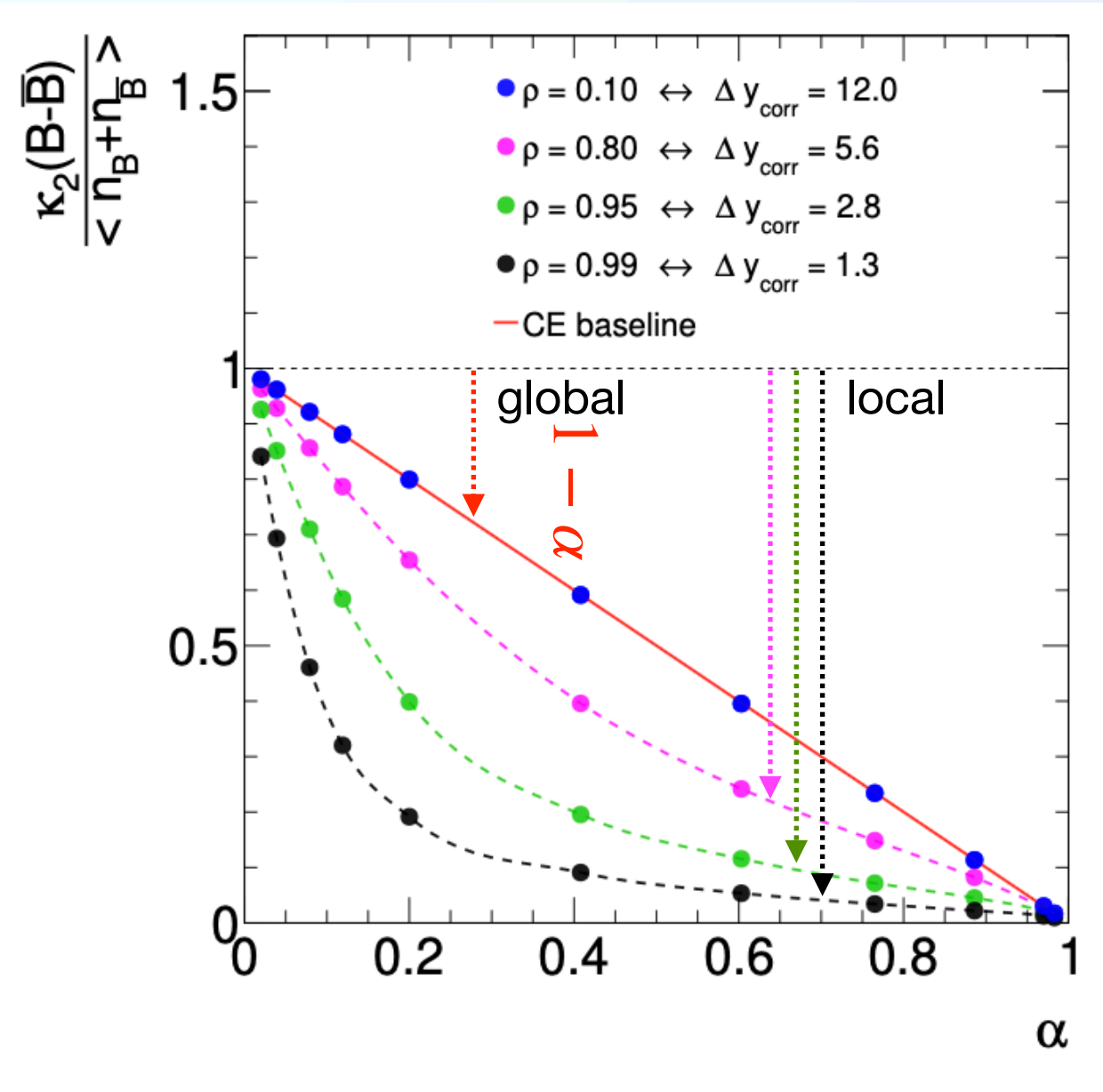
$$\rho = \frac{\langle y_B y_{\bar{B}} \rangle - \langle y_B \rangle \langle y_{\bar{B}} \rangle}{\sigma_{y_B} \sigma_{y_{\bar{B}}}}$$

► P. Braun-Munzinger, K. Redlich, A.R., J. Stachel, JHEP 08 (2024) 113



Metropolis algorithm

+ Canonical Ensemble

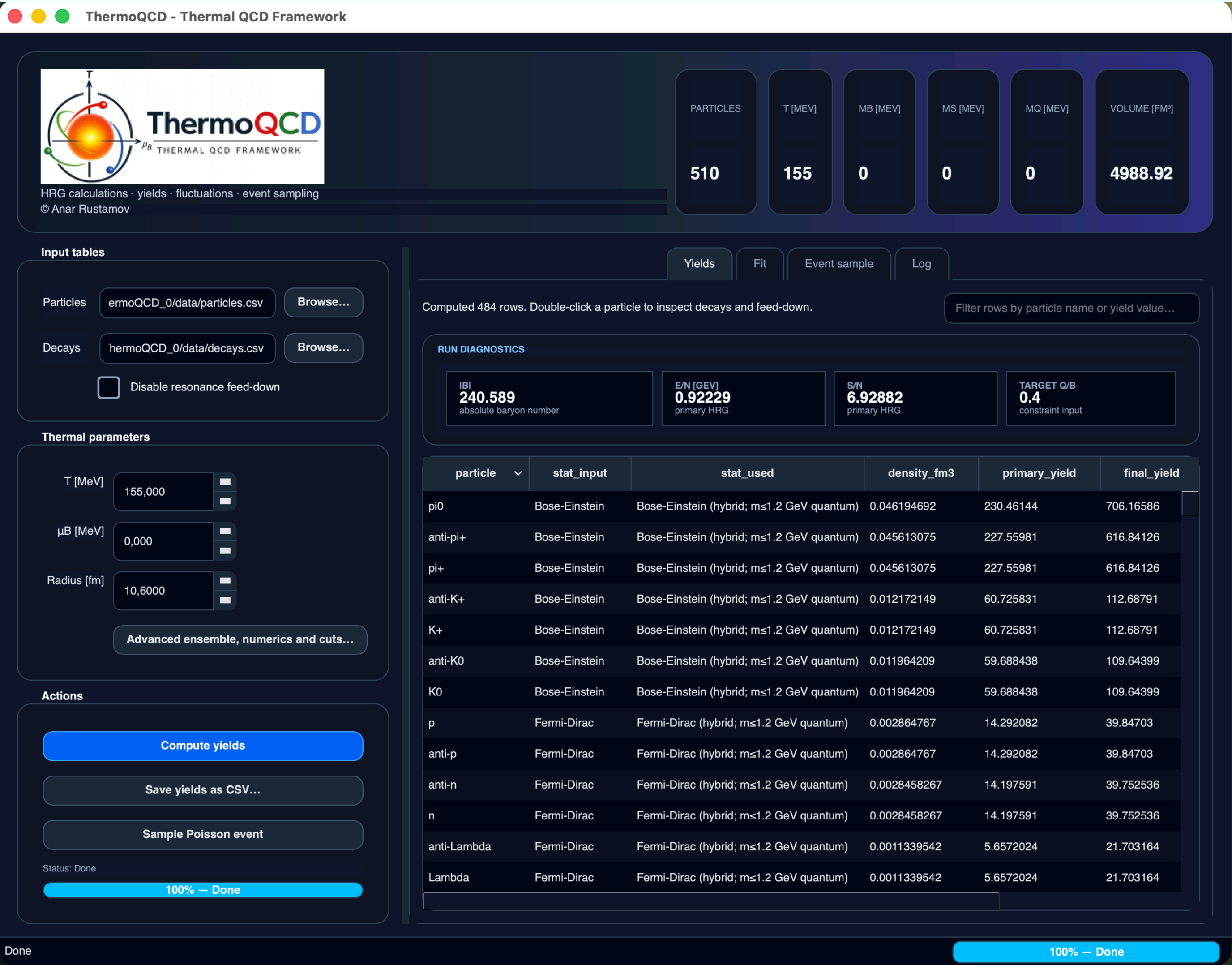




Putting precision thermal baselines into one framework

For Peter, on his 80th birthday

ThermoQCD - Thermal QCD Framework



Physics

HRG, GCE/CE, B/S/Q/C

Analysis

yields, fitting, event generation

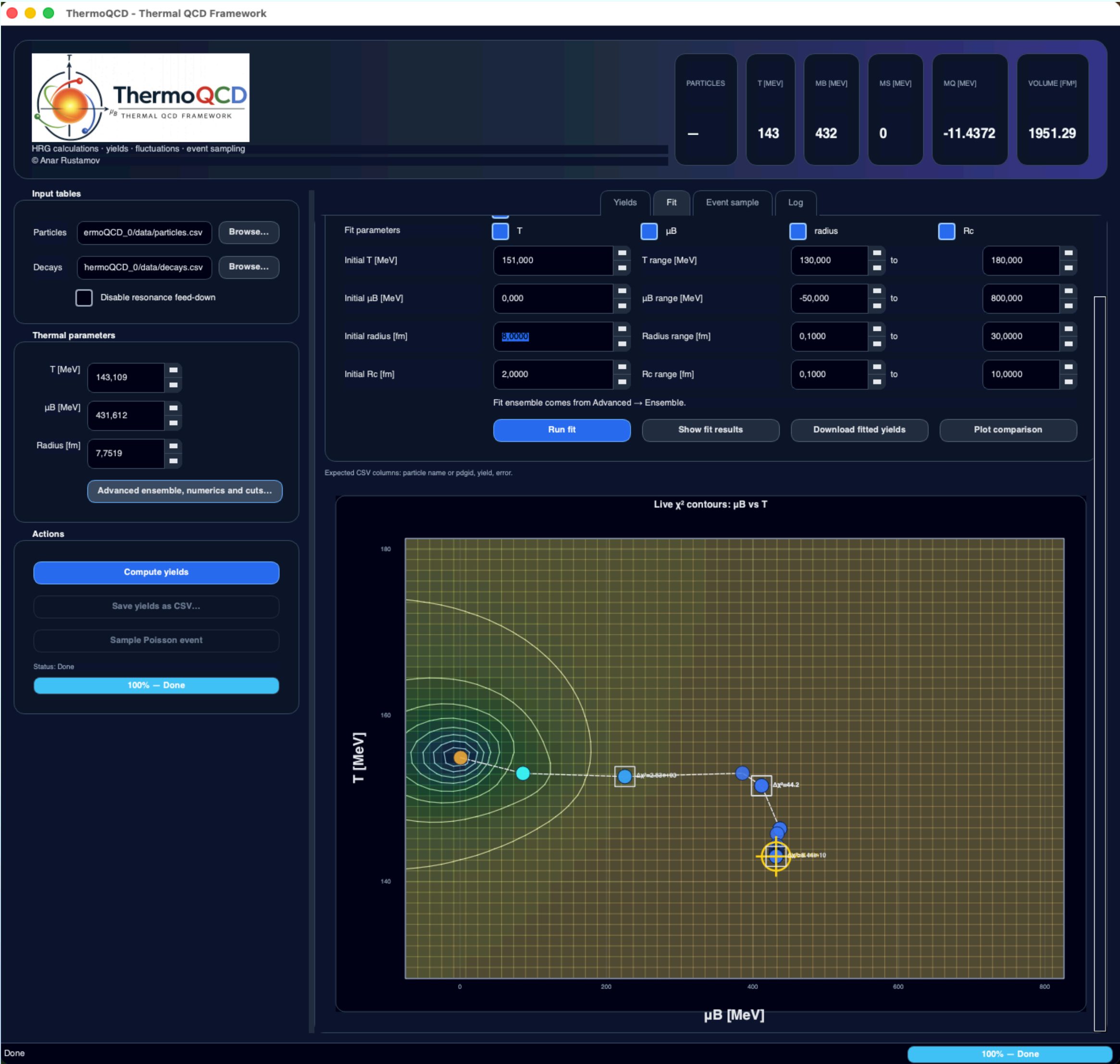
Interface

Python, with CERN/ROOT integration

External particle lists

Compatible with particle-lists/decays from Thermal-FIST and GSI/Heidelberg

ThermoQCD - GUI for Fitting



ThermoQCD reproduces established HRG thermodynamics

HRG Partition function

$$\ln Z_{HRG} = \sum_{i \in \text{hadrons}} \ln Z_i$$

Pressure

$$P = T \frac{\partial \ln Z_{HRG}}{\partial V}$$

Particle density

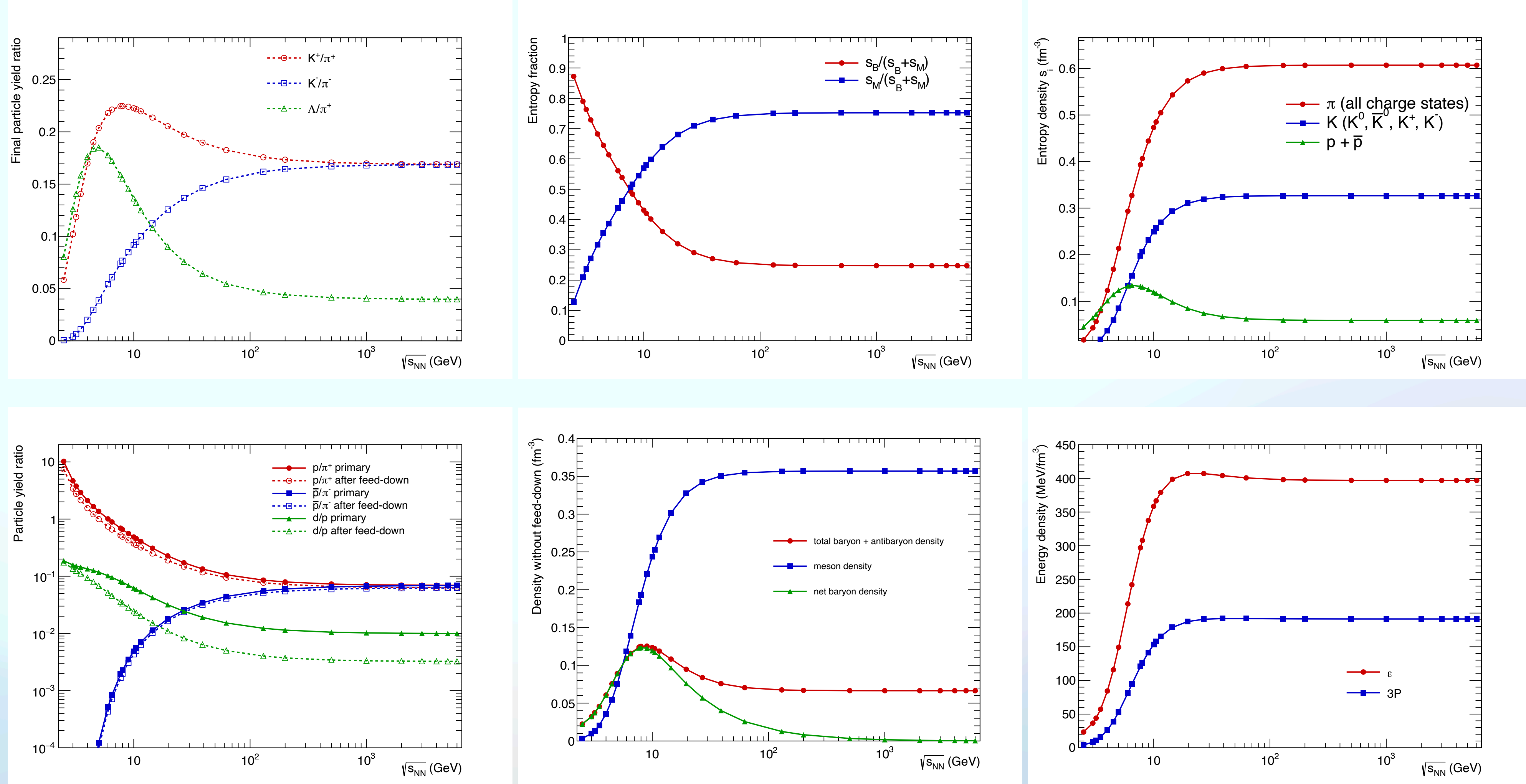
$$n_i = \frac{\partial P}{\partial \mu_i}$$

Entropy density

$$s = \frac{\partial P}{\partial T}$$

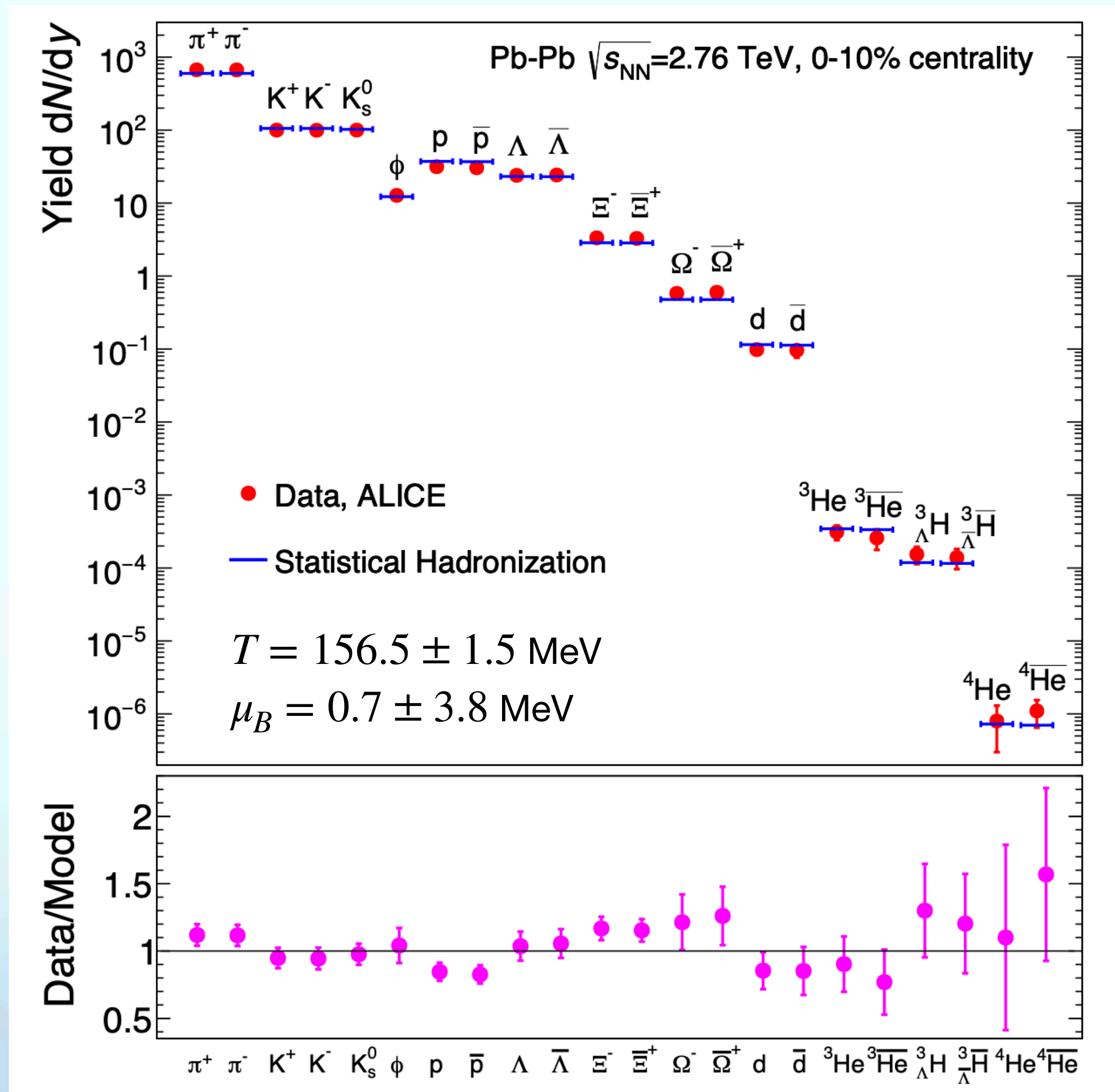
Energy density

$$\varepsilon = \frac{T^2}{V} \frac{\partial \ln Z_{HRG}}{\partial T} + \sum_i \mu_i n_i$$

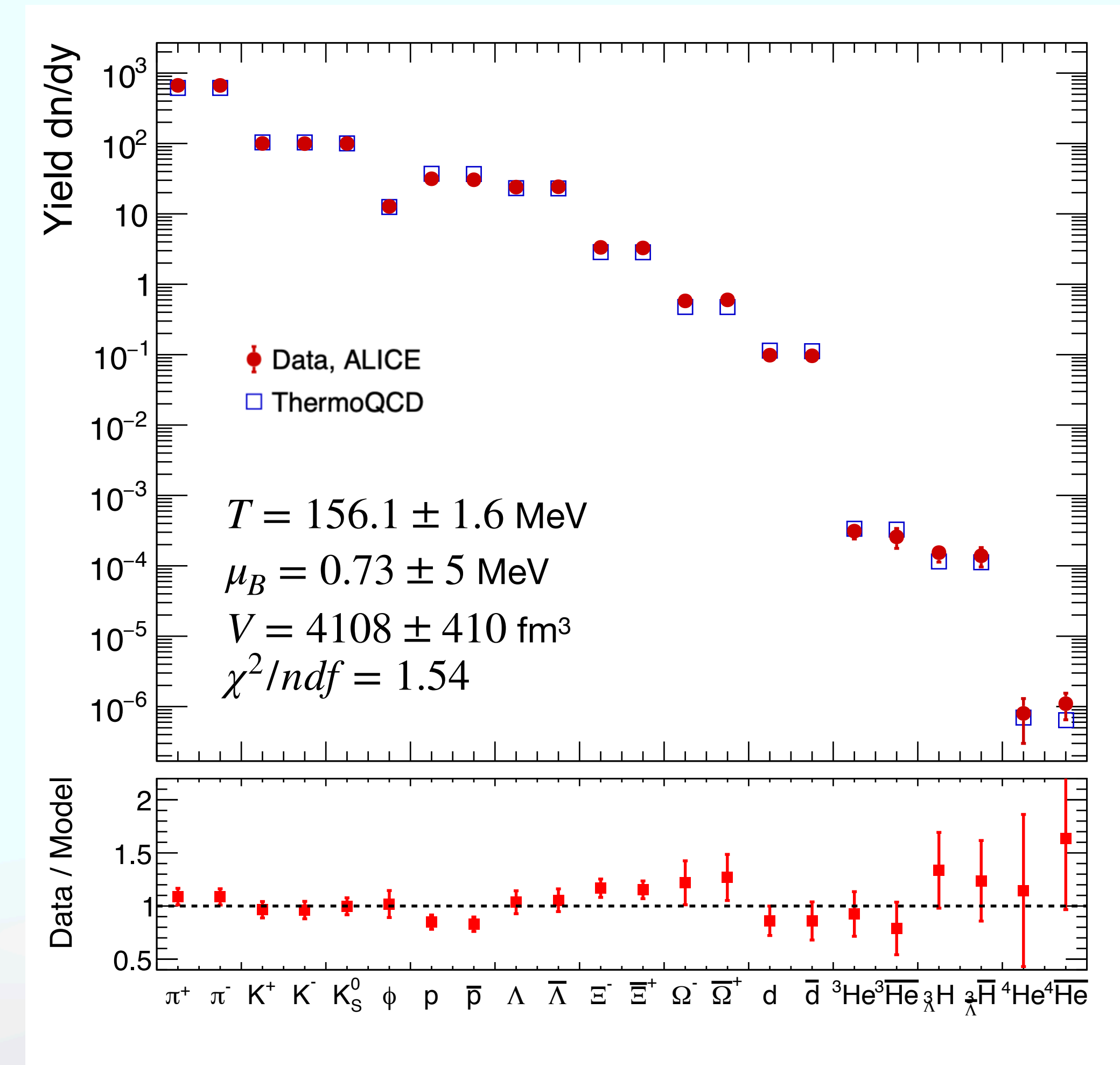


Good agreement with published HRG calculations: ► [A. Andronic, P. Braun Munzinger, K. Redlich, J. Stachel, e-Print: 2606.14799 \[nucl-ex\]](#)

ThermoQCD reproduces established HRG thermodynamics



A. Andronic, P. Braun-Munzinger, K. Redlich and J. Stachel,
Nature 561, 321–330 (2018)

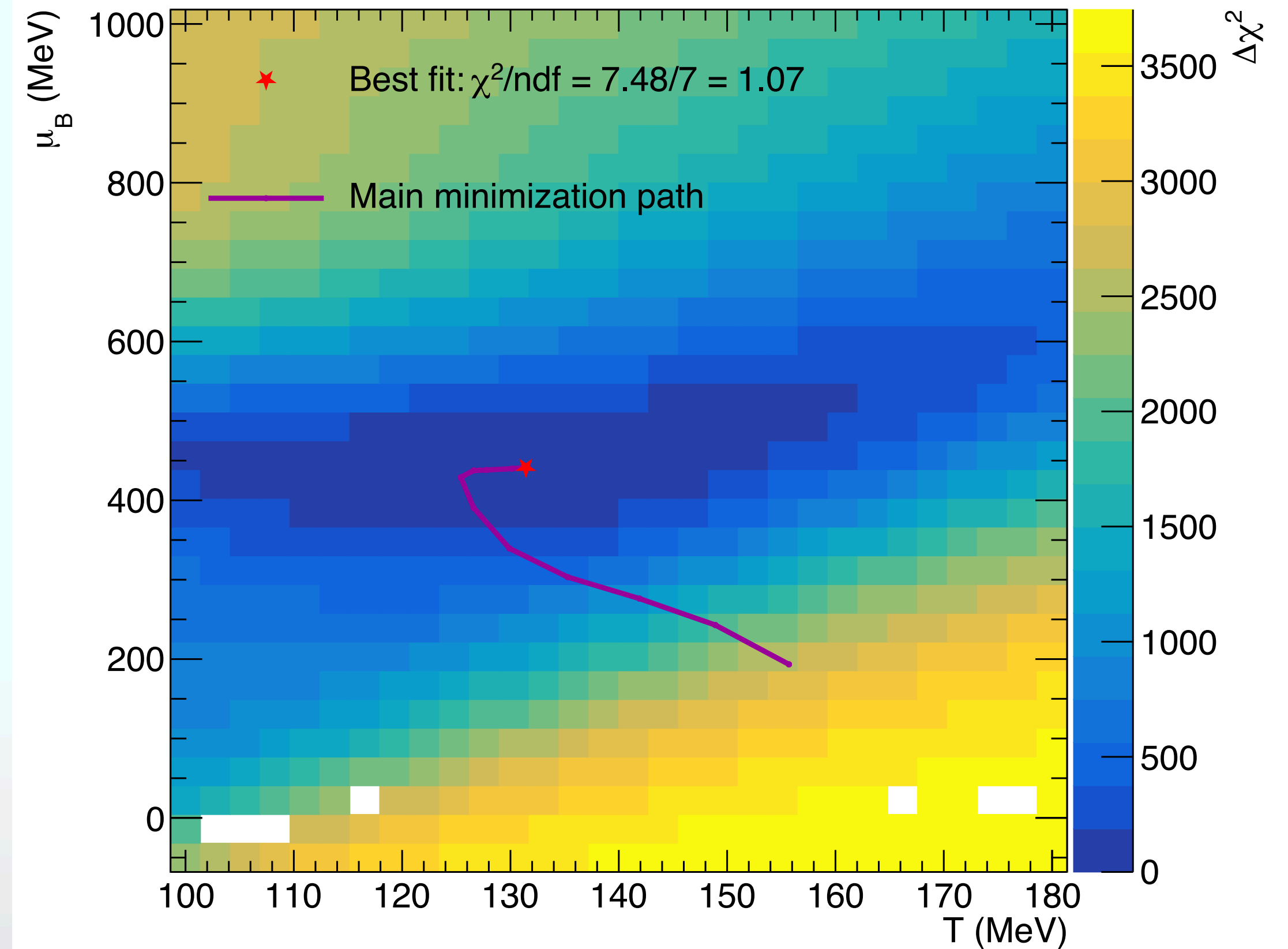
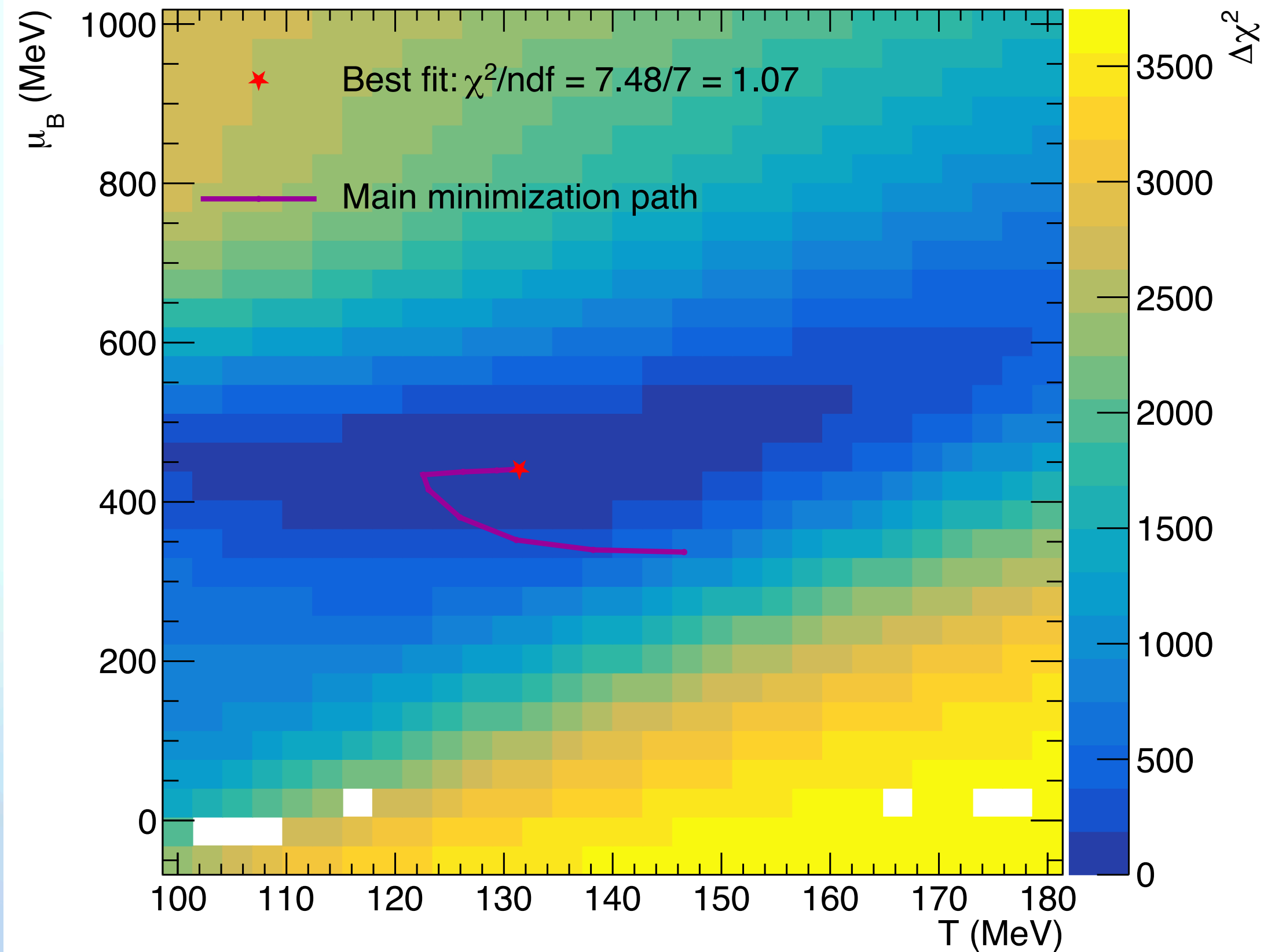


ThermoQCD

Fit convergence and χ^2 profiling

Fitted: $T = 131.4 \pm 2$ MeV, $\mu_B = 441.3 \pm 6$ MeV, $V = 1155 \pm 152$ fm³

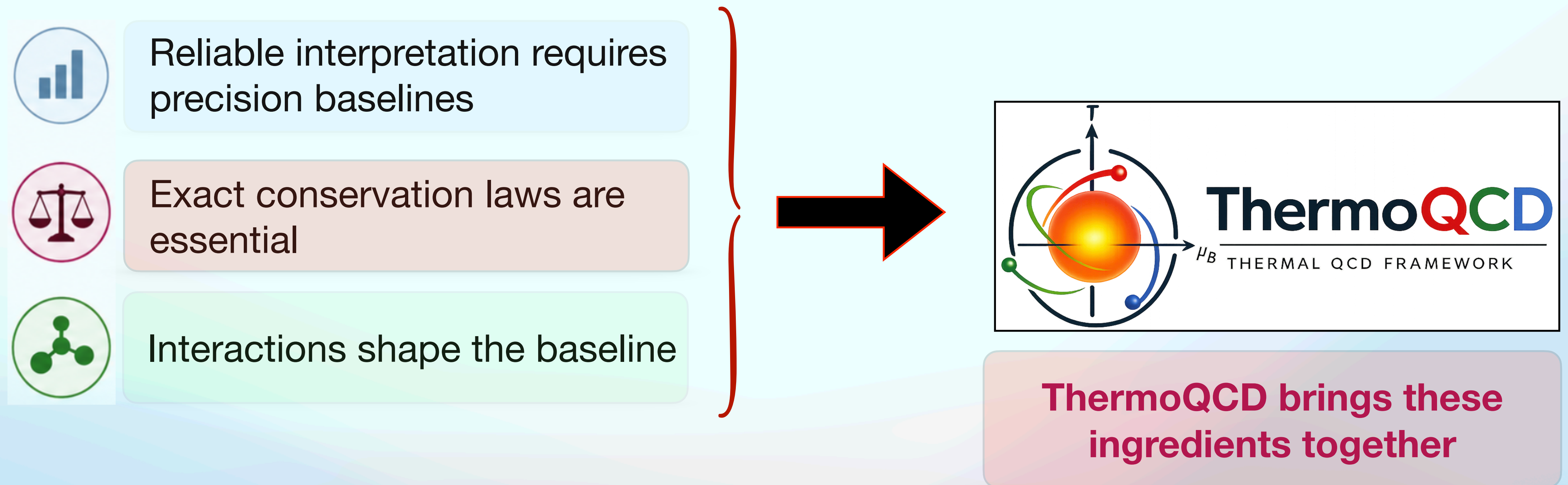
Constraint-solved: $\mu_S = 99.8$ MeV, $\mu_Q = -11.4$ MeV; Constraints: $\langle S \rangle = 0$, $\langle Q \rangle / \langle B \rangle = 0.4$



different initial conditions $\rightarrow$ different optimization trajectories $\rightarrow$ **identical best-fit parameters**

What did we learn?

Particle-Number Fluctuations: A Window into the QCD Phase Structure





80 YEARS OF LEGENDARY!



EIGHT DECADES
OF



CURIOSITY



DISCOVERY



RICH EXPERIENCES



80TH HAPPY
BIRTHDAY,
Peter!



2006



Serious Meetings

2026



Relaxing

80 YEARS OF
LEGENDARY!



EIGHT DECADES



OF

CURIOSITY



DISCOVERY



RICH EXPERIENCES

please don't forget: our next photo is already scheduled!

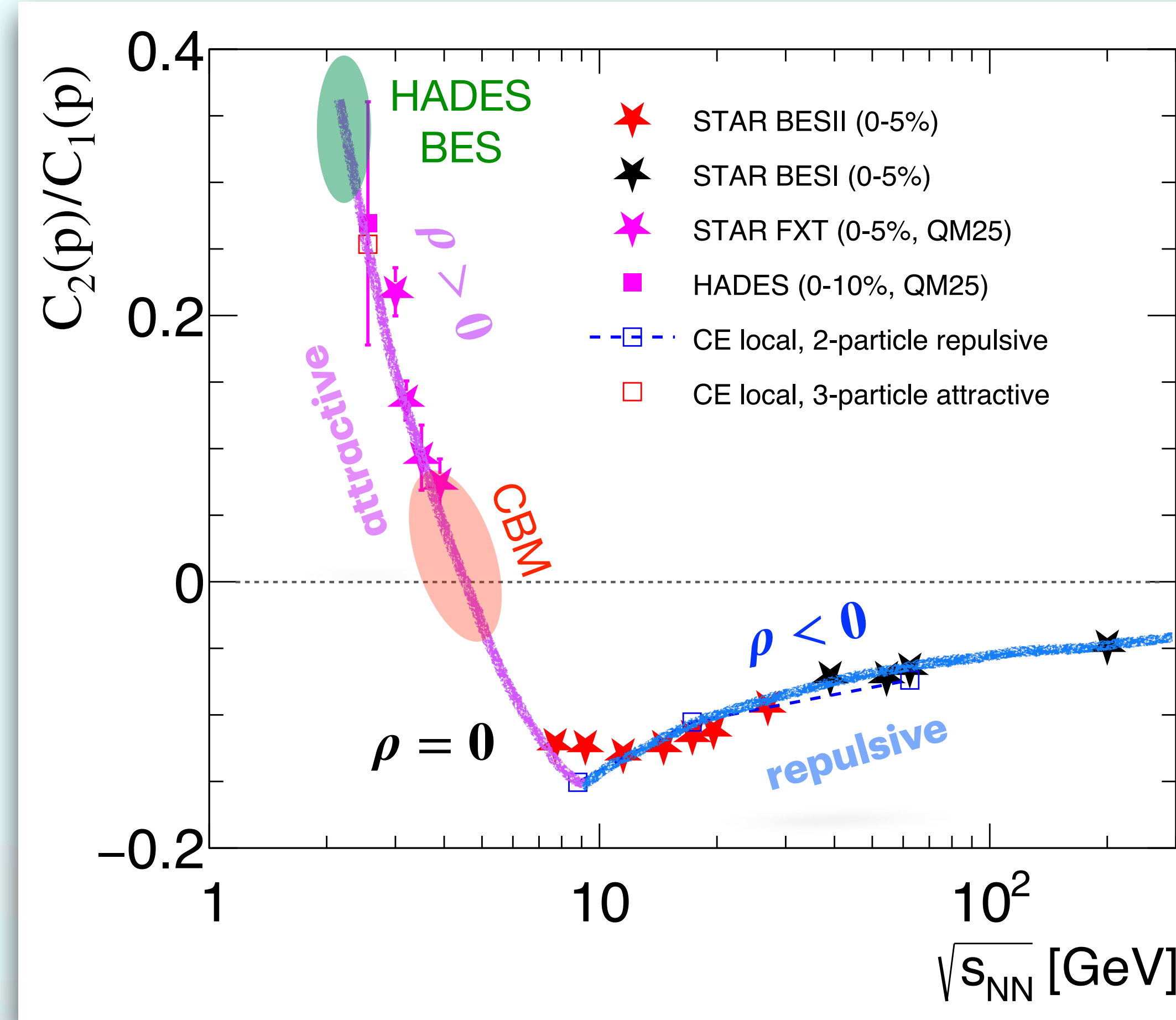
80TH  2046,

See you at 100! 





Putting It All Together



$$\rho = \frac{\langle y_1 y_2 \rangle - \langle y_1 \rangle \langle y_2 \rangle}{\sigma_{y_1} \sigma_{y_2}}$$

B. Friman, A. R., K. Redlich, JHEP 07 (2026) 20

A Striking non-monotonic Pattern: Repulsive at High Energies → Attractive at Low Energies