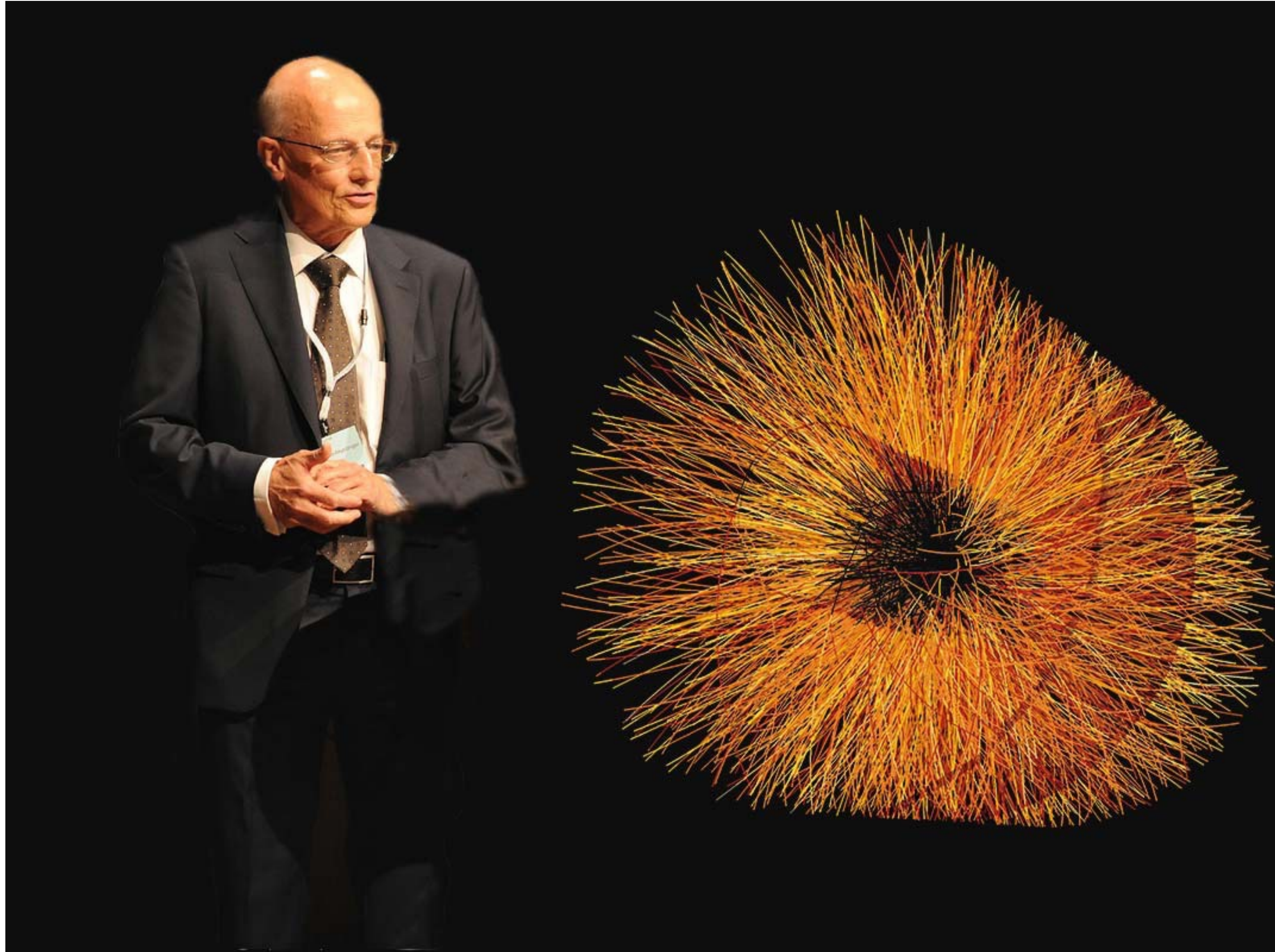


Properties of Baryonic Matter







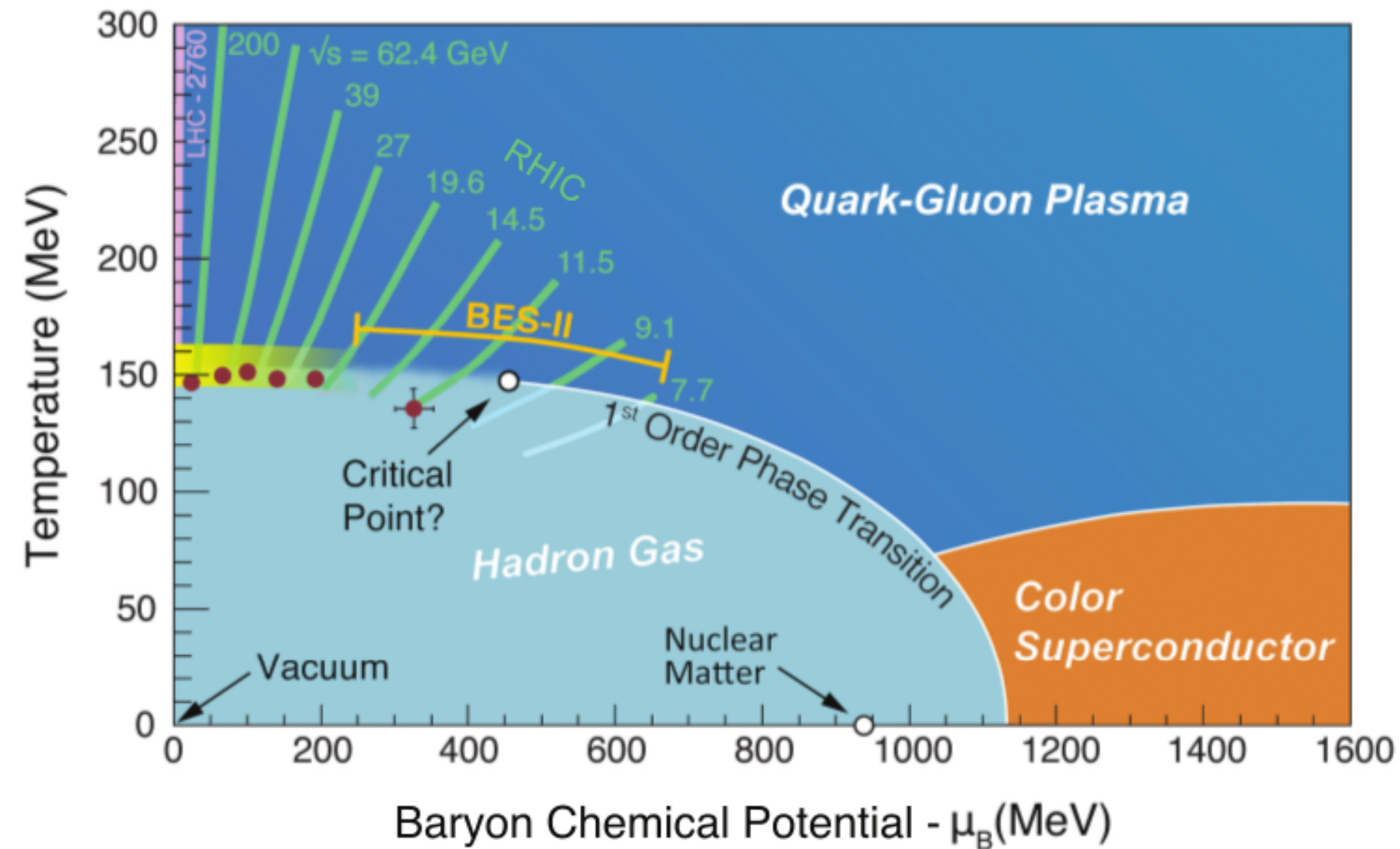


Properties of Baryonic Matter

- Introduction
- Revisit net charge fluctuations
- Cumulants and Correlations
- Non critical baseline
- Comparison with RHIC BES data
 - what have we learned so far
 - next steps
- Conclusions

Thanks to: Volodymyr Vovchenko, Jonathan Parra, Bengt Friman, Adam Bzdak

The phase diagram



Increase chemical potential by lowering the beam energy

In reality, we add baryons (nucleons) from target and projectile to mid-rapidity

What we know about the Phase Diagram

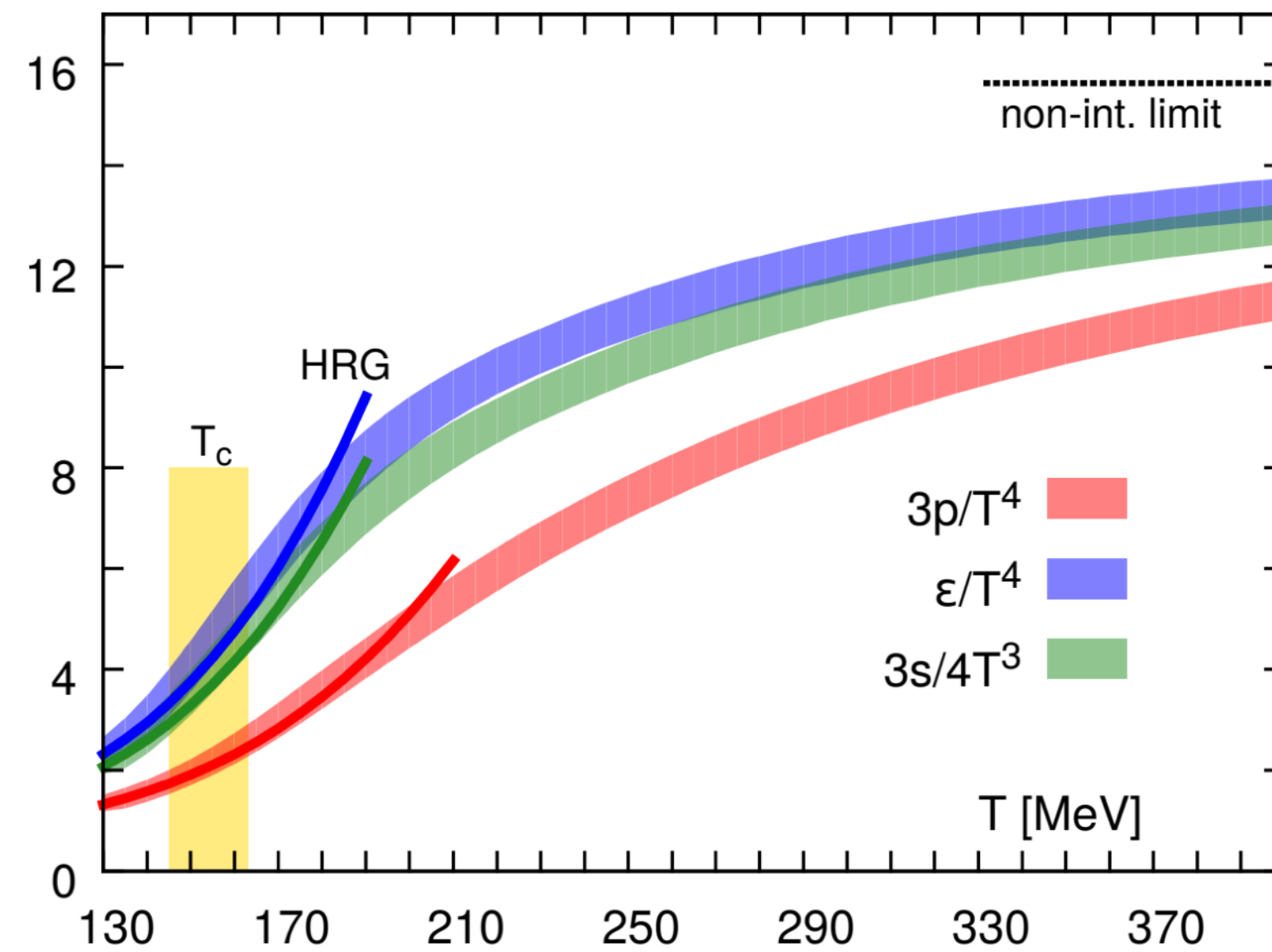
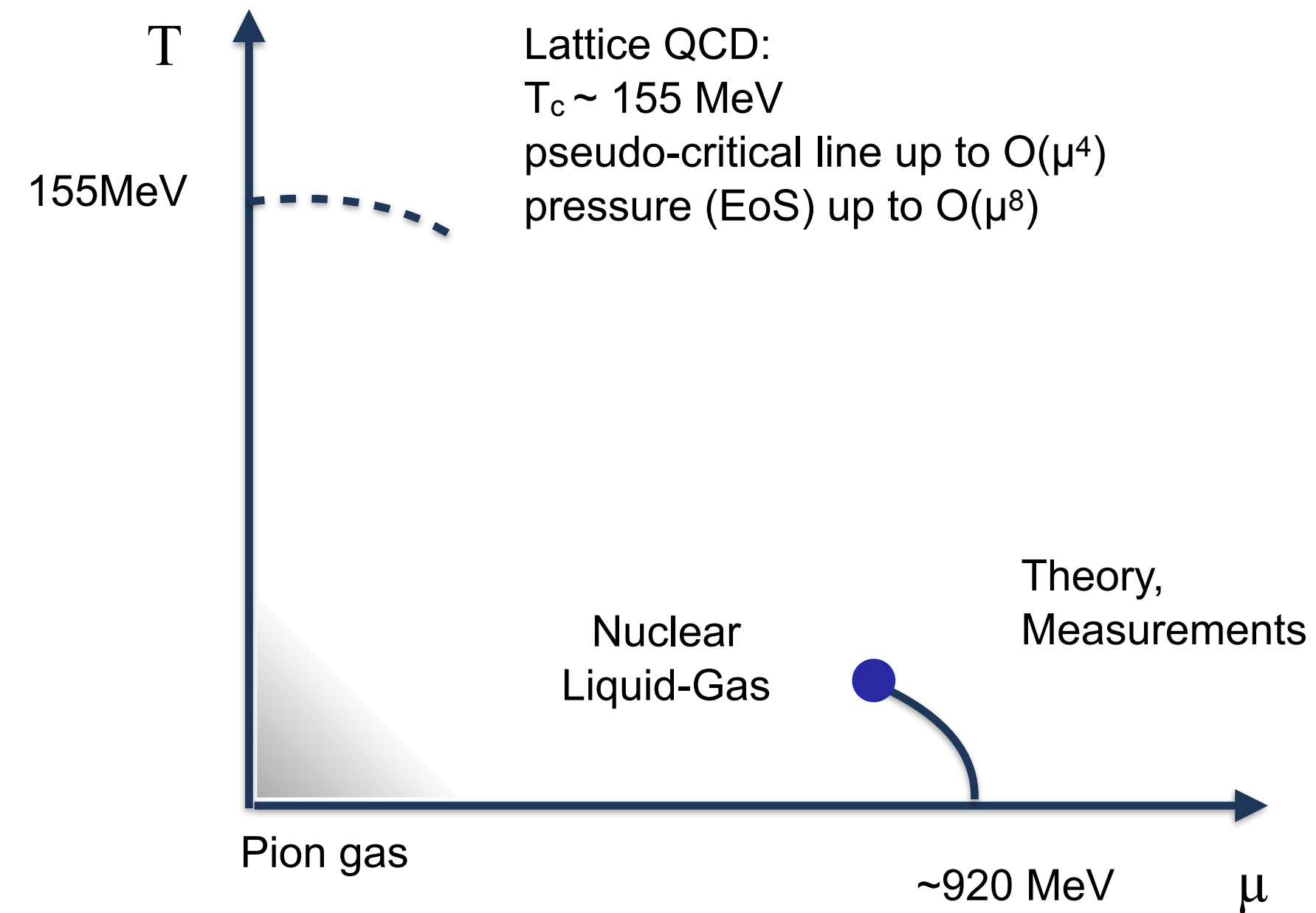
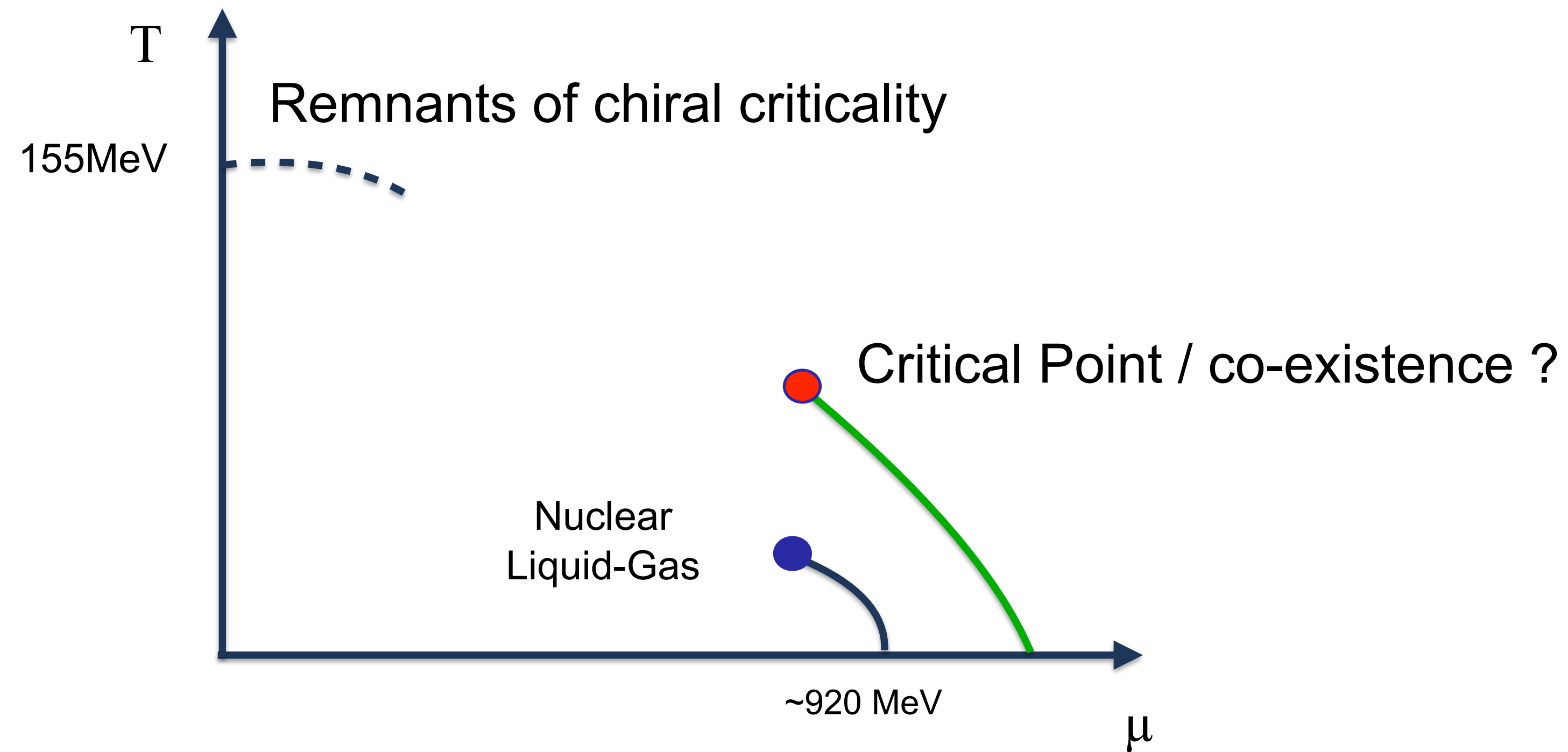


Figure from HotQCD coll., PRD '14



What we are looking for



We are dealing with small system of finite lifetime

NO real singularities!

Recent results on CP

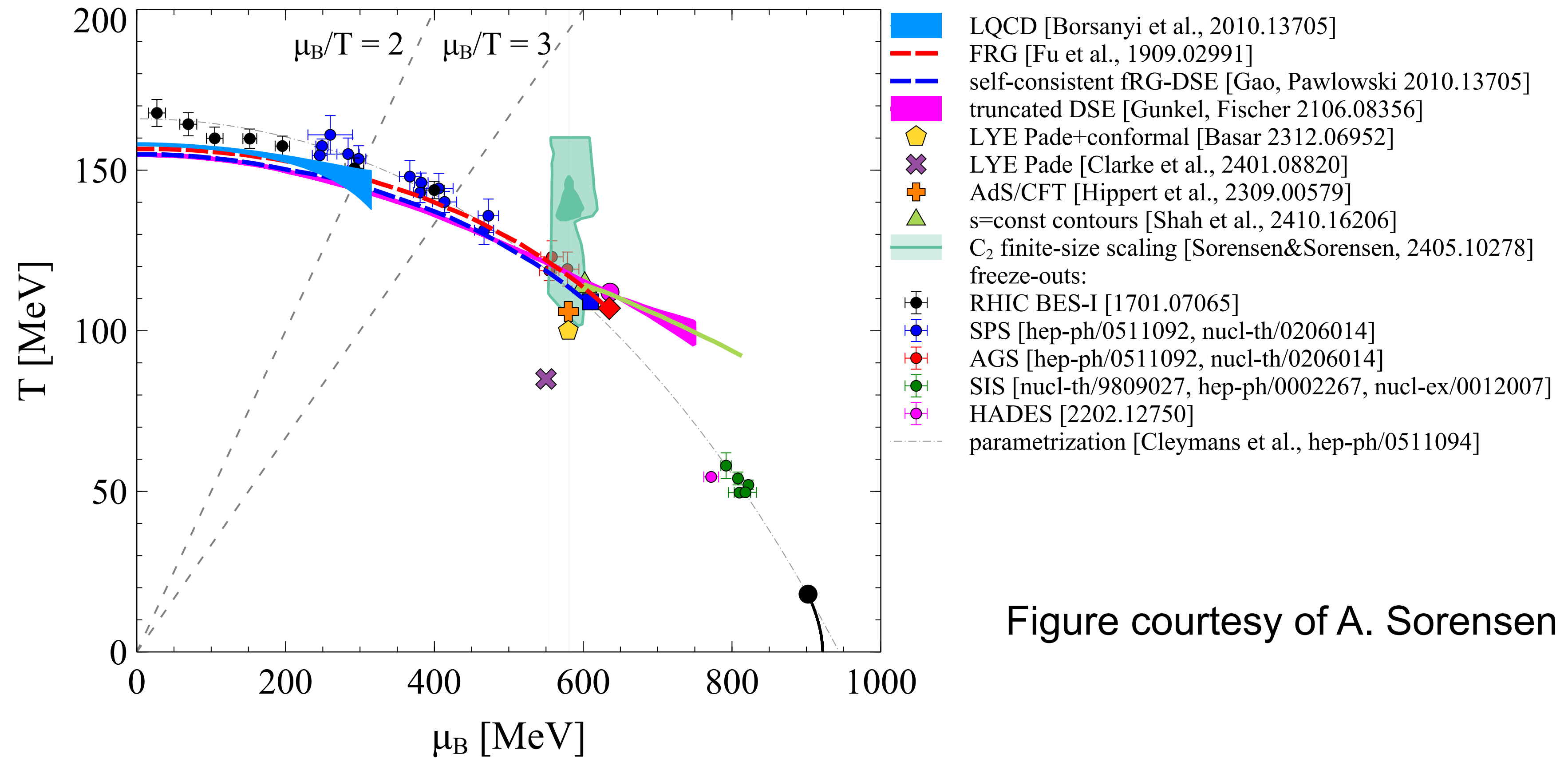


Figure courtesy of A. Sorensen

Fluctuations and the phase diagram

Fluctuations of conserved charges measure derivatives of the partition function (pressure)

$$\kappa_n(q) = \frac{\partial^n}{\partial(\mu_q/T)^n} \ln(Z); \quad q = B, Q, S$$

They are also sensitive to the degrees of freedom (fractional charges, correlations):

$$\kappa_2(B) \sim b^2 N_{\text{quarks}} = \frac{1}{9} N_{\text{quarks}}$$

Asakawa, Heinz and Müller, PRL 86, 2000

$$\kappa_2(Q) \sim q^2 N_{\text{quarks}} = \frac{1}{9} N_{up} + \frac{4}{9} N_{down}$$

Jeon, V.K. PRL 86, 2000

Update on Charge fluctuations

Proposed observable: (Jeon, V.K. PRL 86, 2000) $D = 4 \frac{\kappa_2(Q)}{\langle N_{ch} \rangle} = 4 \frac{\chi_2(Q)}{s/T^3} \frac{S}{\langle N_{ch} \rangle}$

$$\omega = \frac{\kappa_2[Q]}{\langle N_{ch}^{prim} \rangle}$$

variance at hadronization

charged multiplicity

Hadron gas: $\omega_{HG} \approx 1$ (Poisson statistics + Bose)

Free QGP*: $\omega_{QGP} \approx 0.36$ (Stefan-Boltzmann limit)

More generally:

$$\omega = \frac{\kappa_2[Q]}{\langle N_{ch}^{prim} \rangle} = \frac{VT^3 \chi_2^Q}{S} \frac{S}{\langle N_{ch}^{prim} \rangle}$$

$$= \frac{\chi_2^Q}{s/T^3} \frac{S}{\langle N_{ch} \rangle} \frac{\langle N_{ch} \rangle}{\langle N_{ch}^{prim} \rangle}$$

$\gamma_Q \approx 1.67$ (decays)

from thermal model



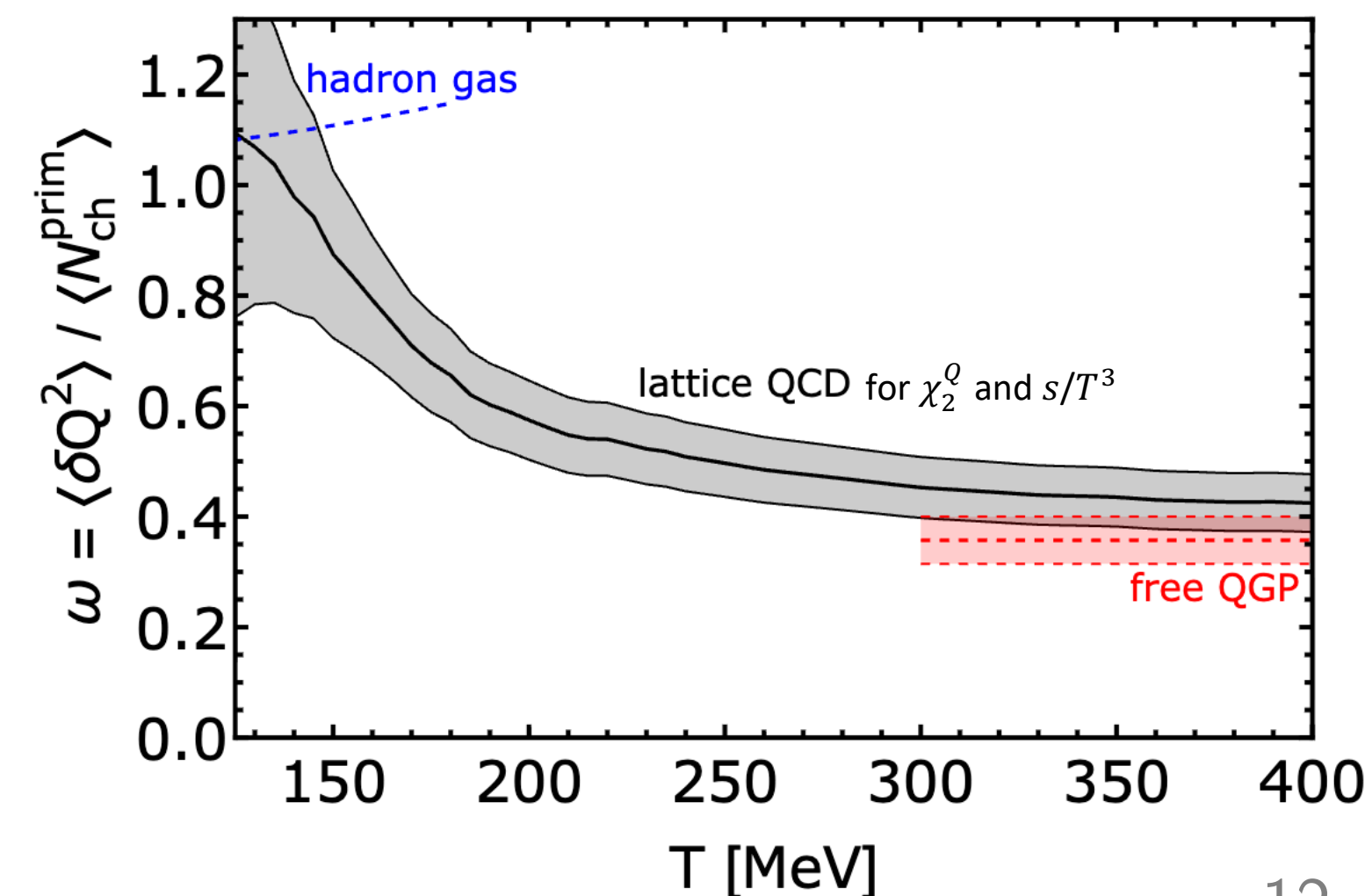
$$S/N_{ch} = 6.7 \pm 0.8 \text{ (LHC)}$$

Data-driven [P. Hanus, A. Mazeliauskas, K. Reygers, PRC (2019)]

The EoS

e.g. lattice QCD

ω from lattice QCD



$$D = 4 \frac{\omega}{\gamma_Q}$$

Courtesy V. Vovchenko

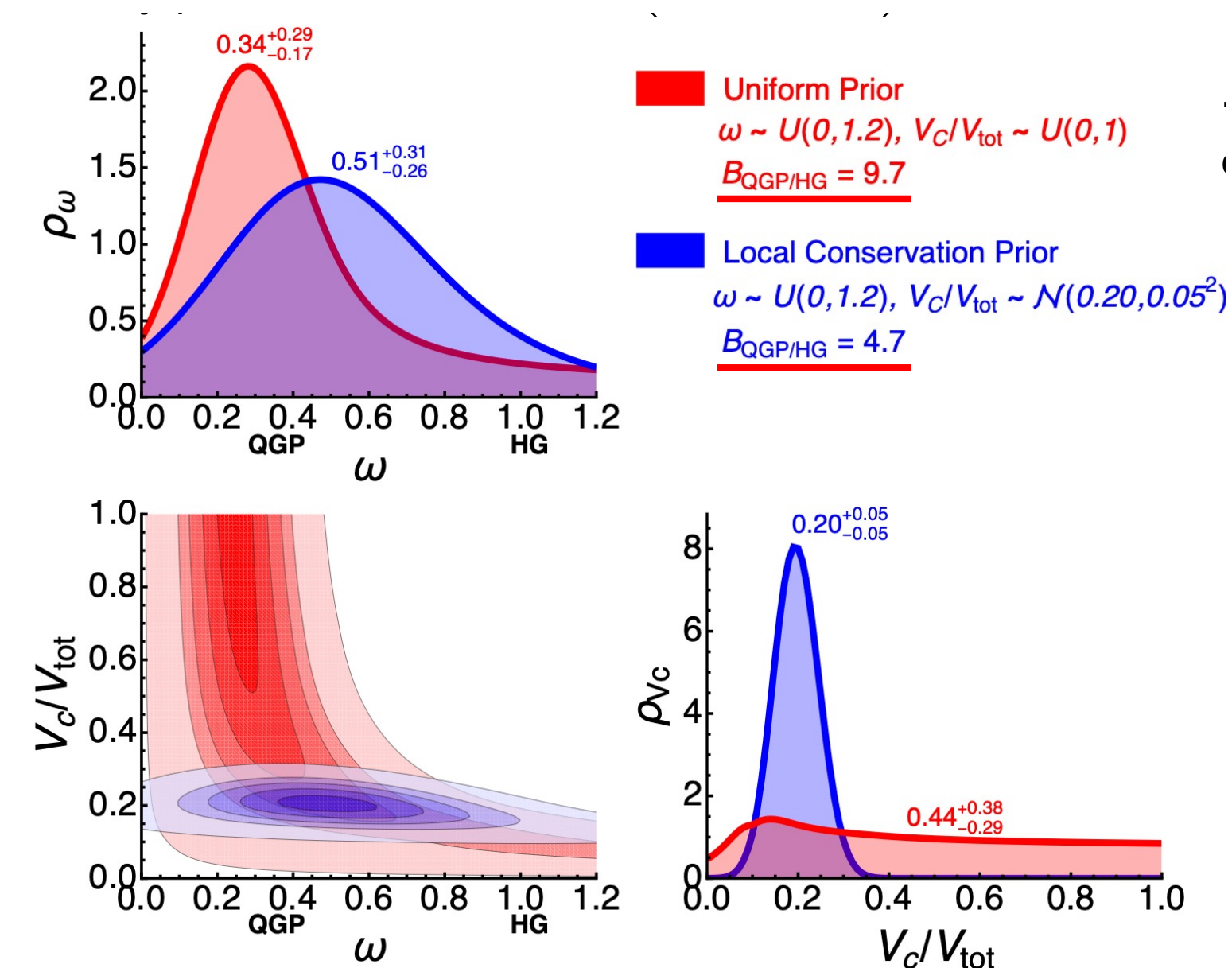
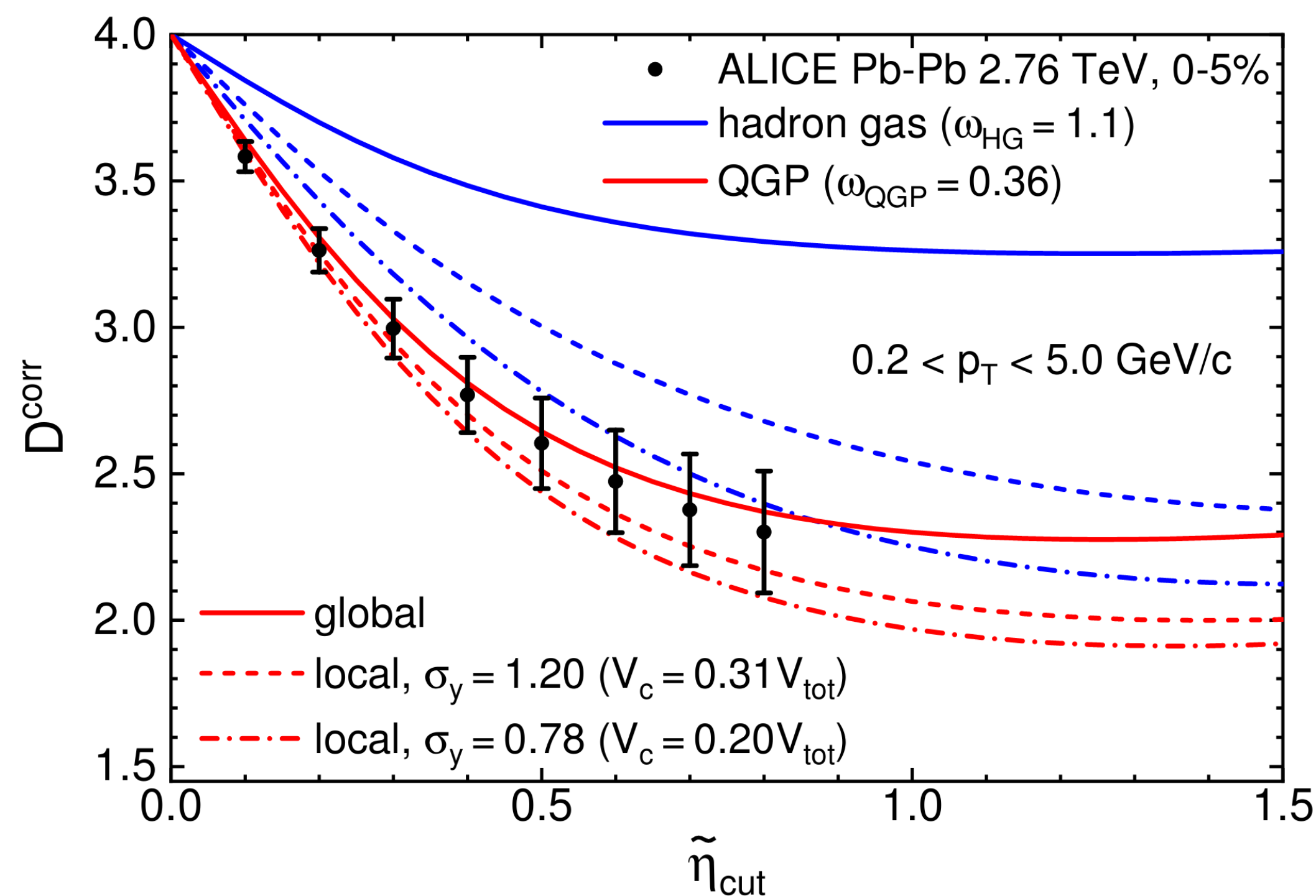
*Same/similar for SQGB scenario of Fujimoto et al., PRD 112, 074006 (2025)

Update on Charge fluctuations

J. Parra et al, PRL 135 (2025)

$$D = 4 \frac{\kappa_2(Q)}{\langle N_{ch} \rangle}$$

D_{corr} : simple corrections for charge conservation included



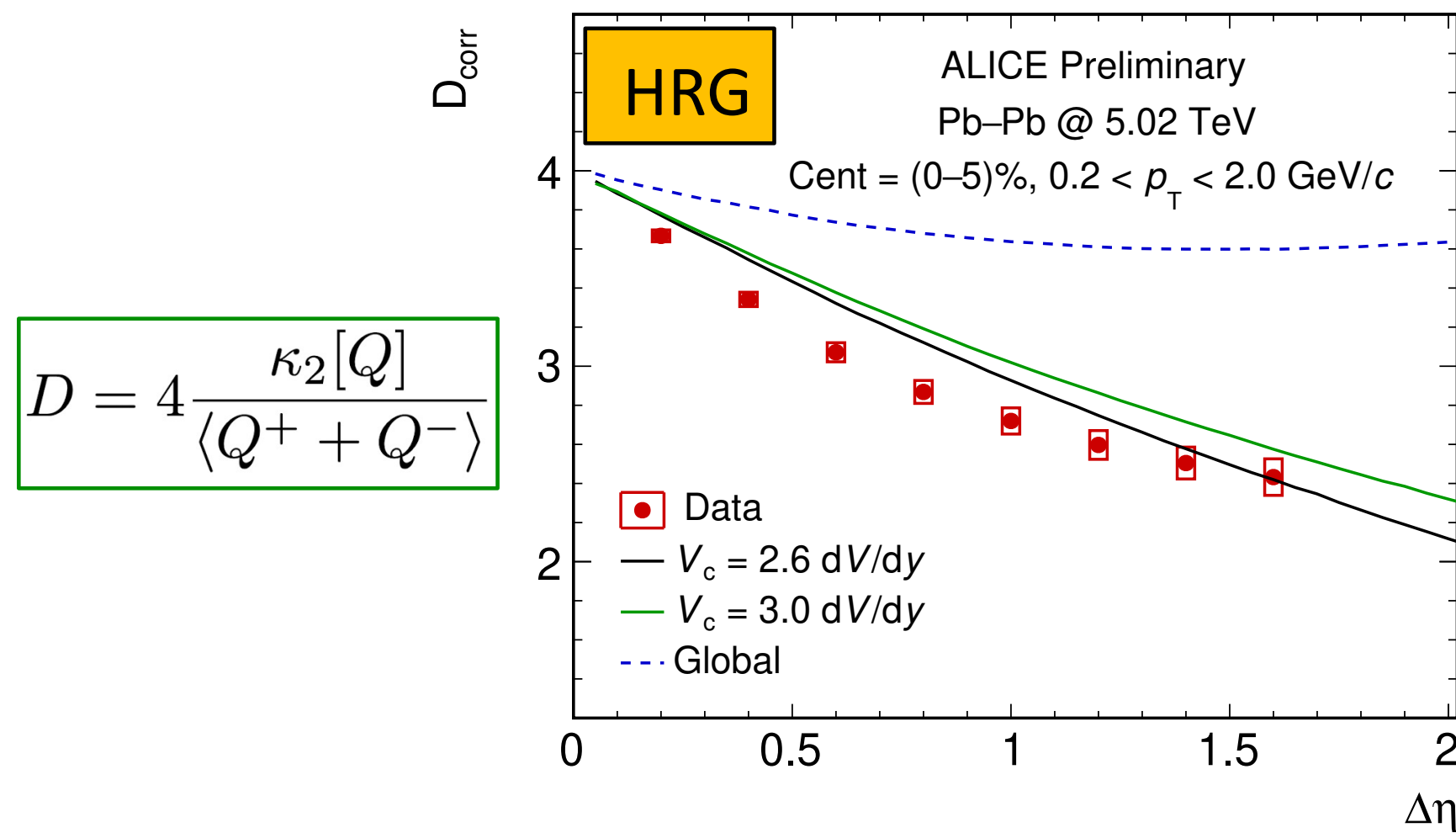
“Moderate” evidence for fractional charges

2nd order net-Q: Subvolume vs Correlation length

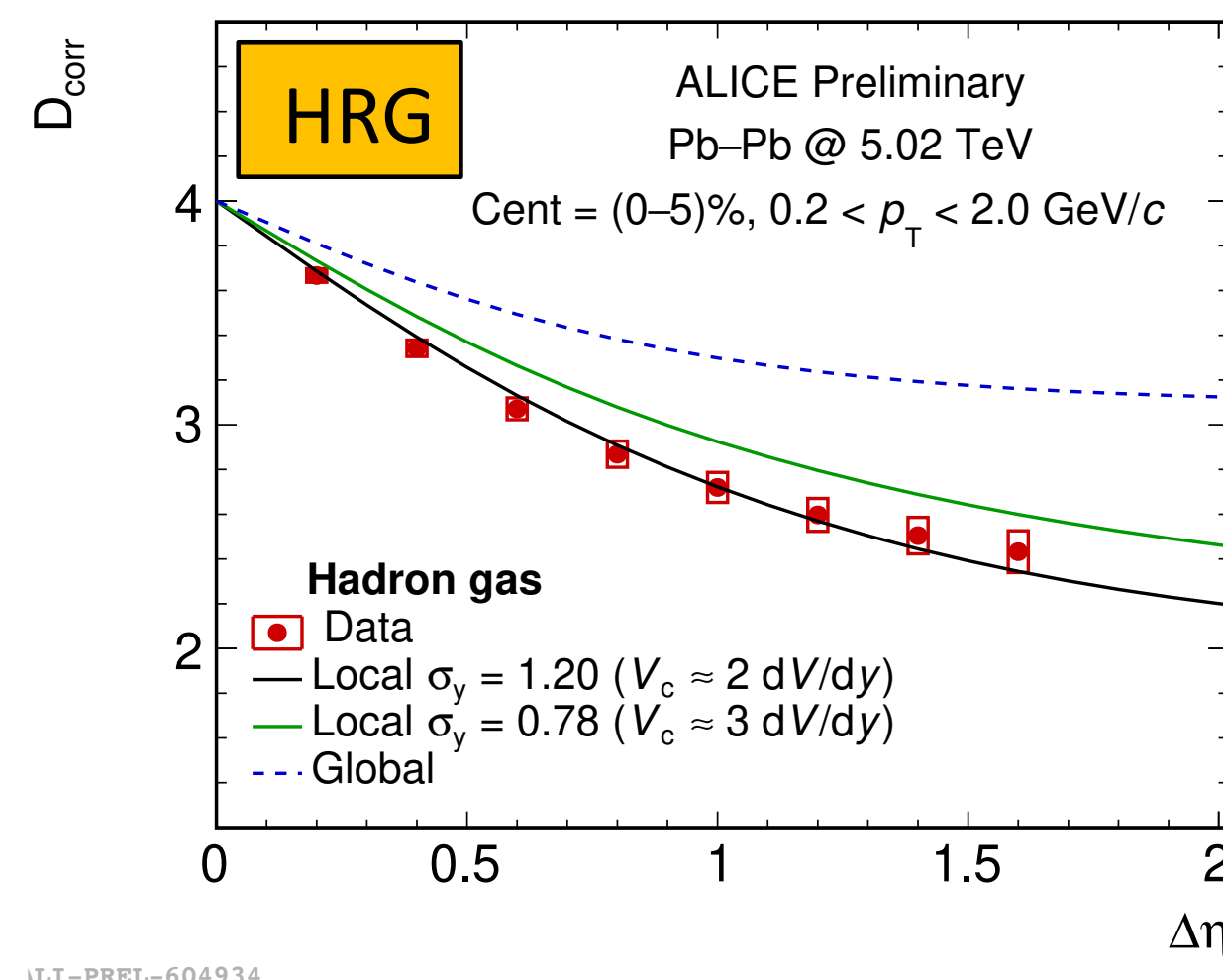
ALICE: RUN 2

➤ **Question:** What is the right modeling of charge conservation?

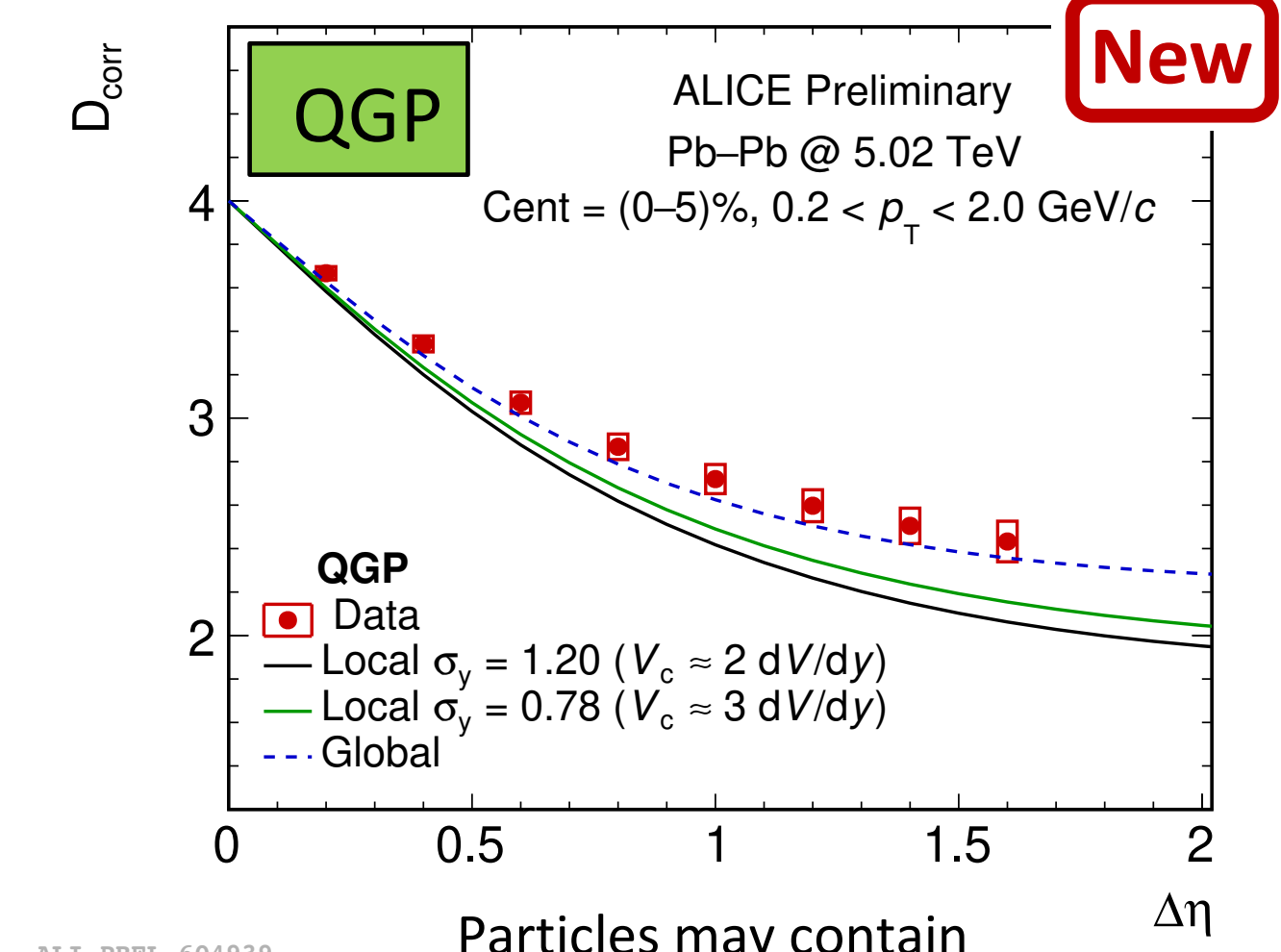
Subvolume approach



Gaussian correlation



Gaussian correlation



New

Particles may contain
the memory of where the
Q-fluctuations freeze out

$$D^{\text{corr}} = \frac{D' + D''}{2}$$

$$D' = D + 4\langle p(\eta) \rangle, \quad D'' = \frac{D}{1 - \langle p(\eta) \rangle}$$

direct comparison with UNCORRECTED D-measure preferred

Back to Critical Point

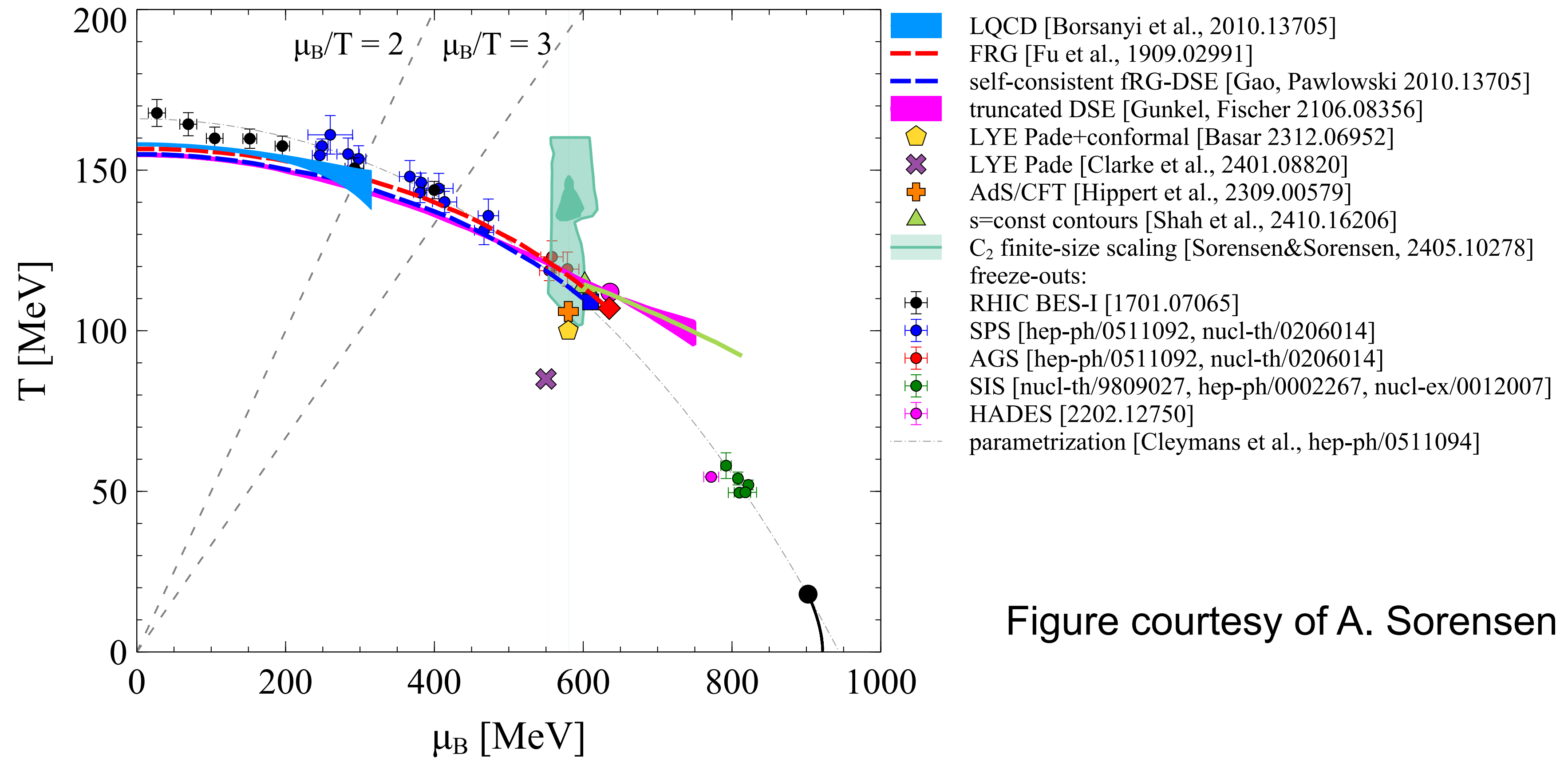


Figure courtesy of A. Sorensen

Conventions, conventions conventions

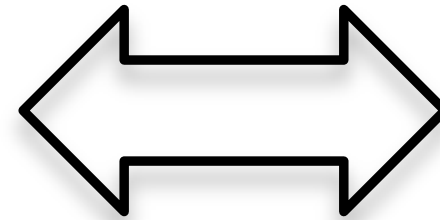
STAR

Cumulants (C)
Factorial cumulants (κ)



Others

Cumulants (κ), (K)
Factorial cumulants (C), (FC)



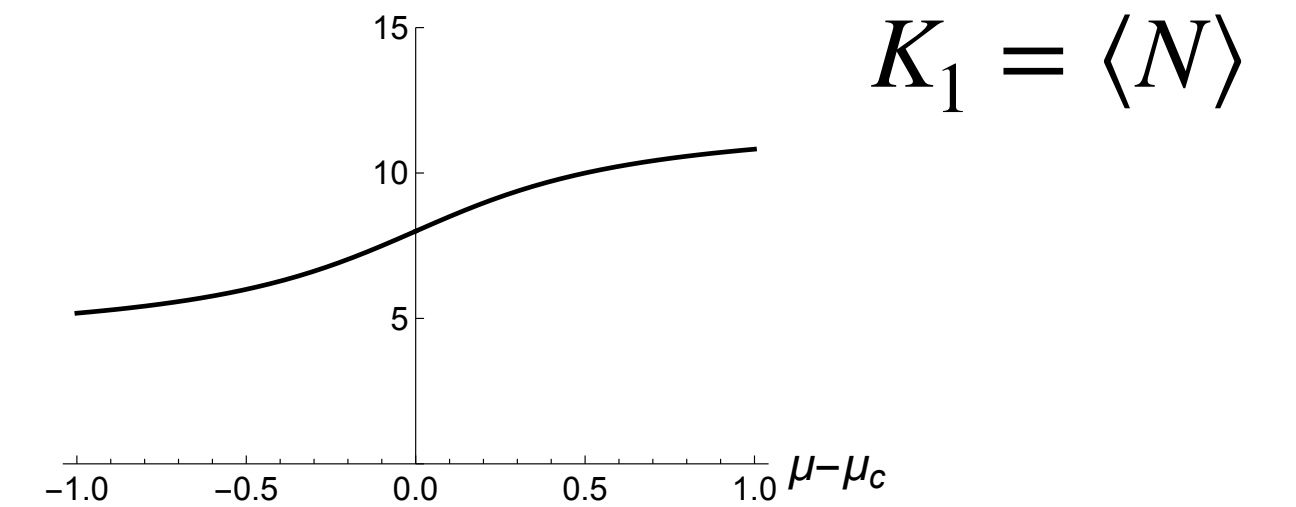
Arslandok, QM25

People love conventions...

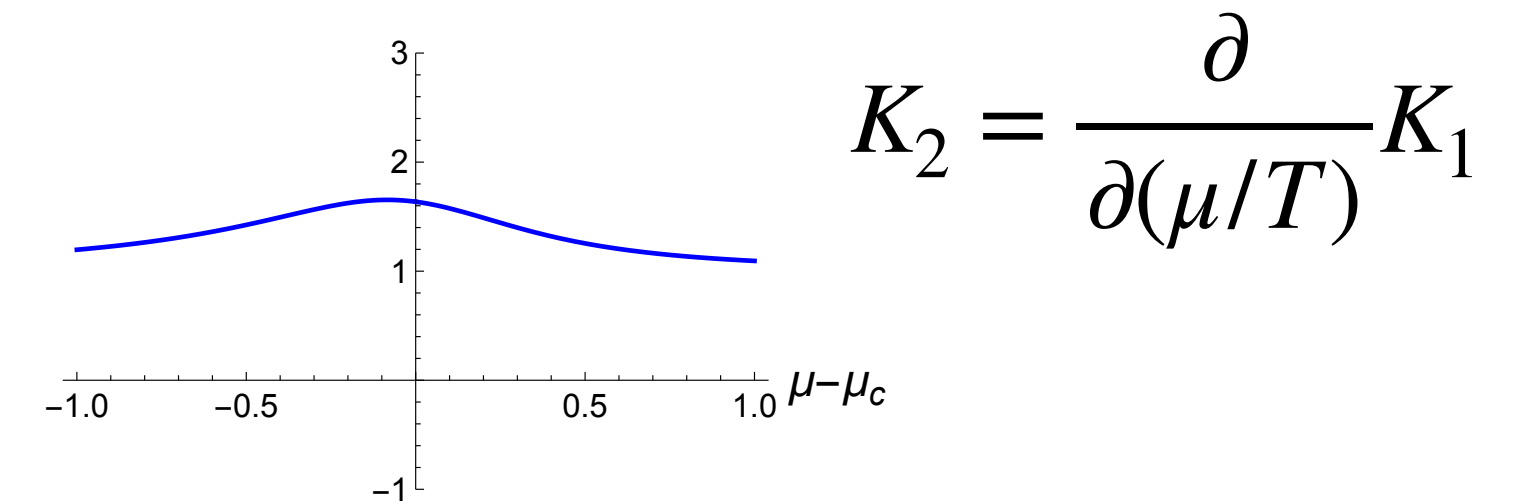
That's why we have so many of them!!

Cumulants of (Baryon) Number

$$K_n = \frac{\partial^n}{\partial(\mu/T)^n} \ln Z = \frac{\partial^{n-1}}{\partial(\mu/T)^{n-1}} \langle N \rangle$$

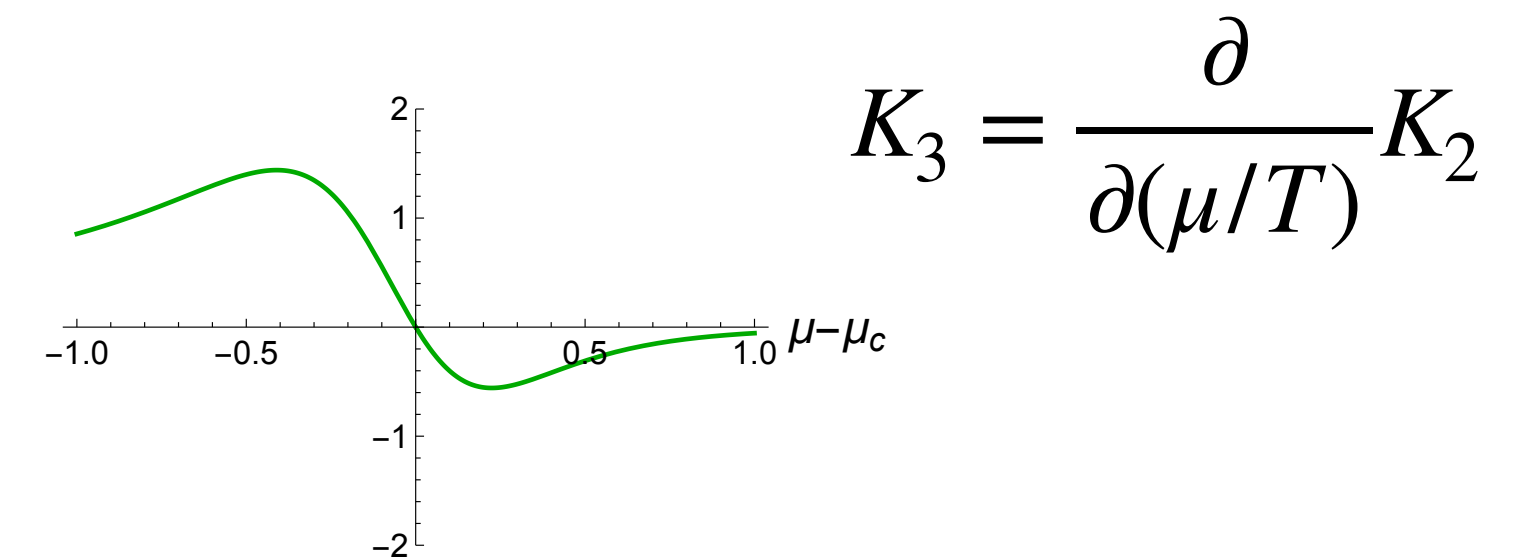


$$K_1 = \langle N \rangle, \quad K_2 = \langle N - \langle N \rangle \rangle^2, \quad K_3 = \langle N - \langle N \rangle \rangle^3$$

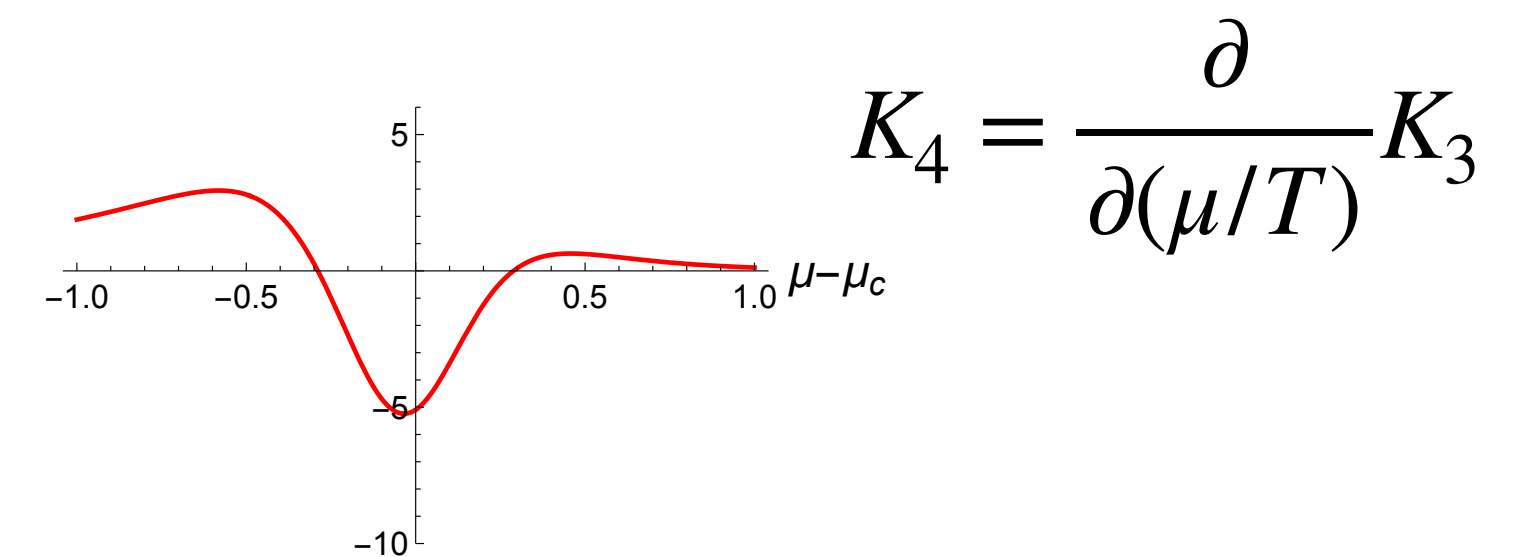


Cumulants scale with volume (extensive): $K_n \sim V$

Volume not well controlled in heavy ion collisions



Cumulant Ratios: $\frac{K_2}{\langle N \rangle}, \quad \frac{K_3}{K_2}, \quad \frac{K_4}{K_2}$



Cumulants and Factorial cumulants

Cumulants:

generating function: $g(t) = \ln \left[\sum_n P(n) e^{nt} \right]$

$$\kappa_n = \frac{\partial^n}{\partial t^n} g(t) \Big|_{t=0}$$

Gaussian:

$$\kappa_n = 0; \quad n \geq 2$$

Factorial Cumulants (no anti-particles):

generating function: $g_F(z) = \ln \left[\sum_n P(n) z^n \right]$

$$FC_n = \frac{\partial^n}{\partial z^n} g_F(z) \Big|_{z=1}$$

Poisson:

$$FC_n = 0; \quad n \geq 1$$

Relations : $g(t) = g_F(e^t)$

$$FC_n = \sum_{k=1}^n s(n, k) \kappa_k; \quad \kappa_n = \sum_{k=1}^n S(n, k) FC_k \quad s(n, k); \quad S(n, k) \text{ Sterling numbers 1st and 2nd kind}$$

For example:

$$FC_1 = \kappa_1 = \langle n \rangle; \quad FC_2 = \kappa_2 - \kappa_1; \quad FC_3 = \kappa_3 - 3\kappa_2 + 2\kappa_1;$$

Same singularity structure!

Factorial cumulants and correlation functions

$$\rho_1(p) = \frac{dN}{dp}; \quad \rho_2(p_1, p_2) = \frac{d^2N}{dp_1 dp_2}; \quad \dots$$

Two particle density: $\rho_2(p_1, p_2) = \rho_1(p_1)\rho_1(p_2) + C_2(p_1, p_2)$

Three particle density: $\rho_3(p_1, p_2, p_3) = \rho_1(p_1)\rho_1(p_2)\rho_1(p_3) + \rho_1(p_1)c_2(p_2, p_3) + \rho_1(p_2)c_2(p_1, p_3) + \rho_1(p_3)c_2(p_1, p_2) + c_3(p_1, p_2, p_3)$

$c_n(p_1, \dots, p_n)$ n-particle genuine correlations functions

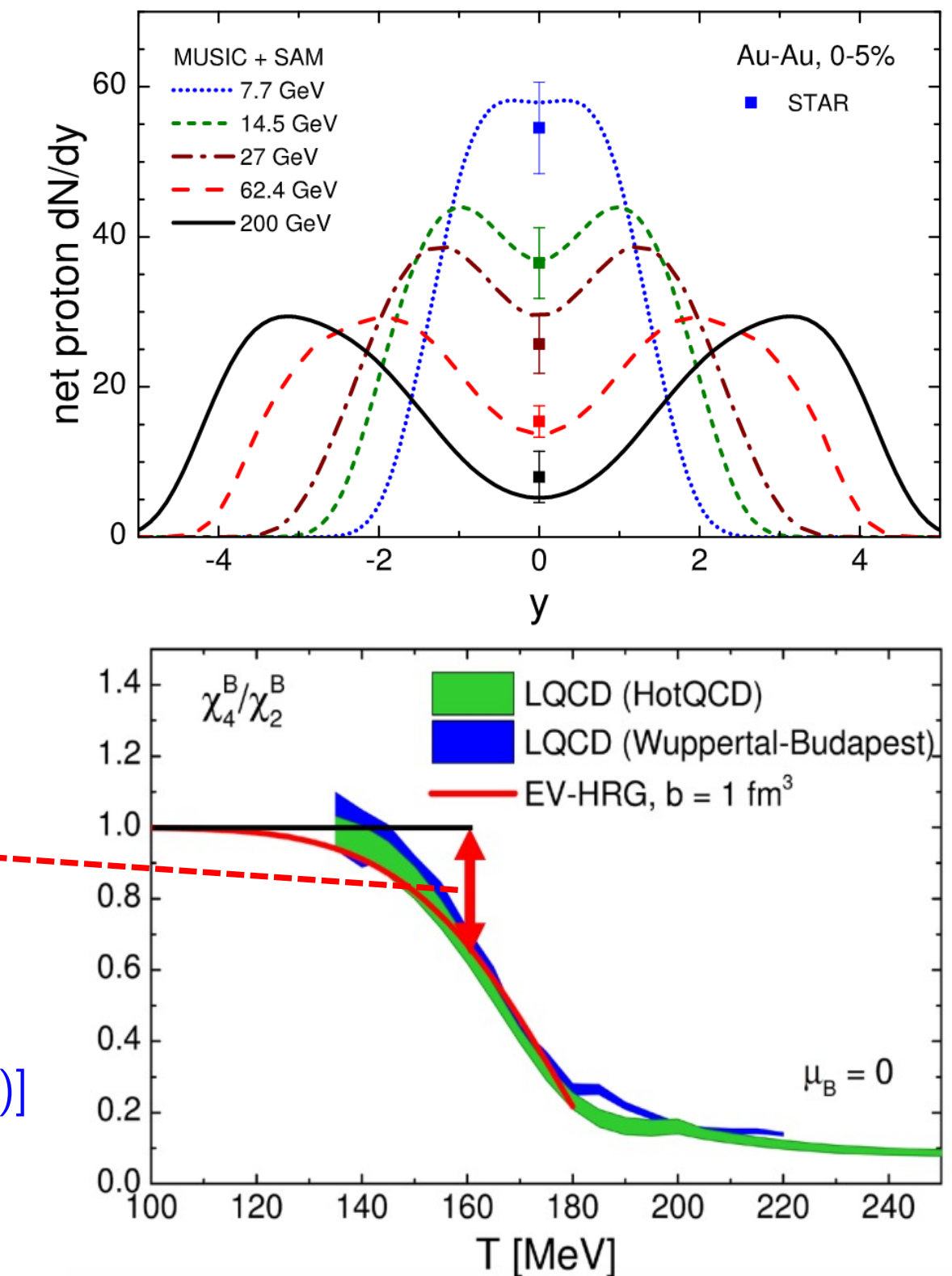
Factorial cumulants are integrals over correlation functions: $FC_n = \int_{\text{acceptance}} dp_1 \cdots dp_n c(p_1, \dots, p_n)$

Poisson: $FC_n = 0; \quad n \geq 1; \quad \text{in contrast with} \quad \kappa_n = \langle n \rangle$

Calculation of non-critical contributions at RHIC-BES

VV, V. Koch, C. Shen, Phys. Rev. C 105, 014904 (2022)

- (3+1)-D viscous hydrodynamics evolution (MUSIC-3.0)
 - Collision geometry-based 3D initial state [Shen, Alzhrani, PRC 102, 014909 (2020)]
 - Crossover equation of state based on lattice QCD [Monnai, Schenke, Shen, Phys. Rev. C 100, 024907 (2019)]
 - Cooper-Frye particlization at $\epsilon_{sw} = 0.26 \text{ GeV/fm}^3$
- Non-critical contributions are computed at particlization
 - QCD-like baryon number distribution (χ_n^B) via **excluded volume** $b = 1 \text{ fm}^3$ [VV, V. Koch, Phys. Rev. C 103, 044903 (2021)]
 - **Exact global baryon conservation*** (and other charges)
 - Subensemble acceptance method 2.0 (analytic) [VV, Phys. Rev. C 105, 014903 (2022)]
 - or FIST sampler (Monte Carlo) [VV, Phys. Rev. C 106, 064906 (2022)]
<https://github.com/vlvovch/fist-sampler>



- **Absent:** critical point, local conservation, initial-state/volume fluctuations, hadronic phase

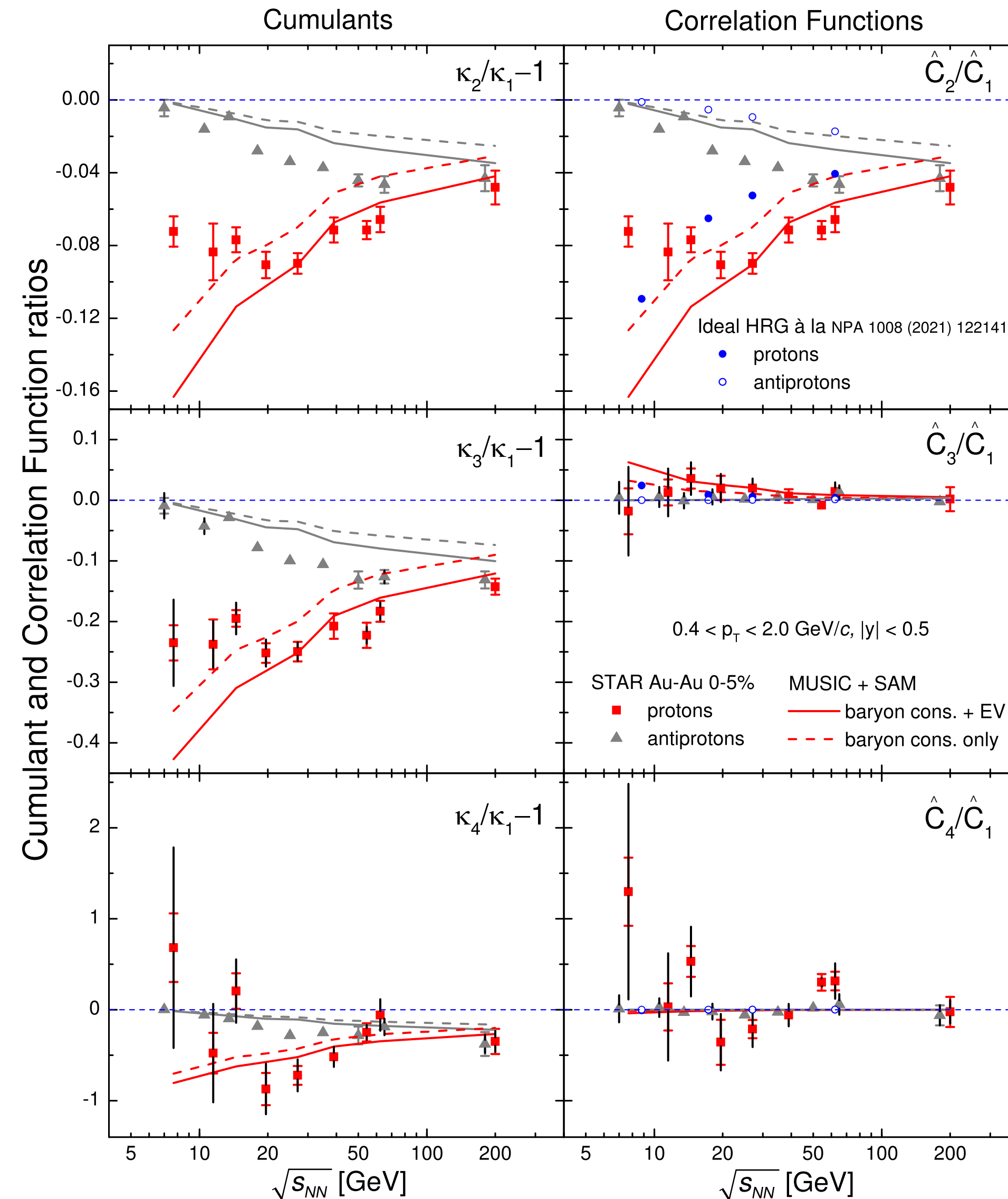
*If baryon conservation is the only effect (no other correlations), non-critical baseline can be computed without hydro
 Braun-Munzinger, Friman, Redlich, Rustamov, Stachel, NPA 1008, 122141 (2021)

Results (Prediction) for proton cumulants

Vovchenko, Shen, VK, 2107.00163

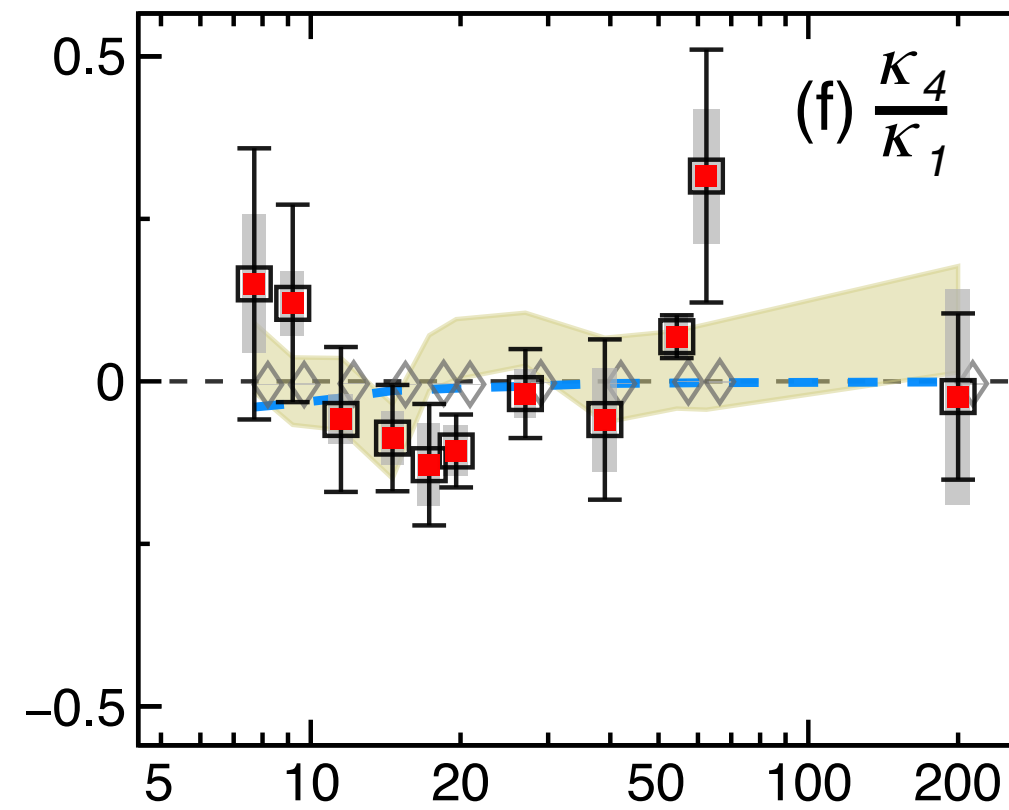
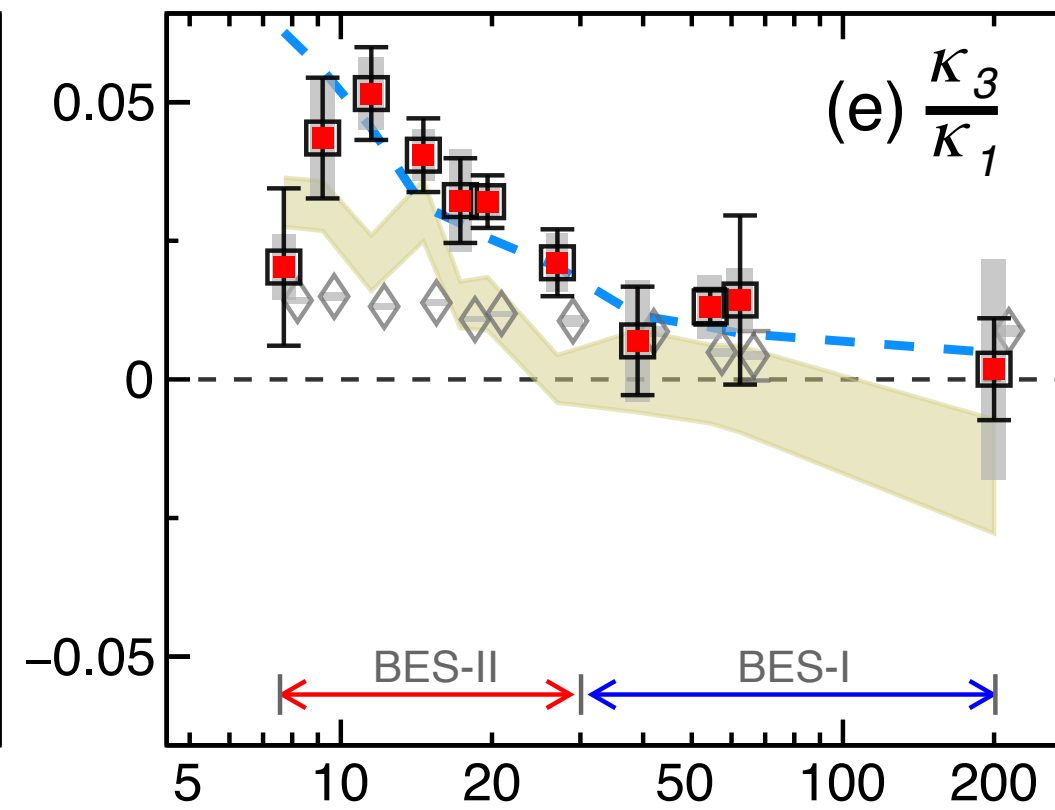
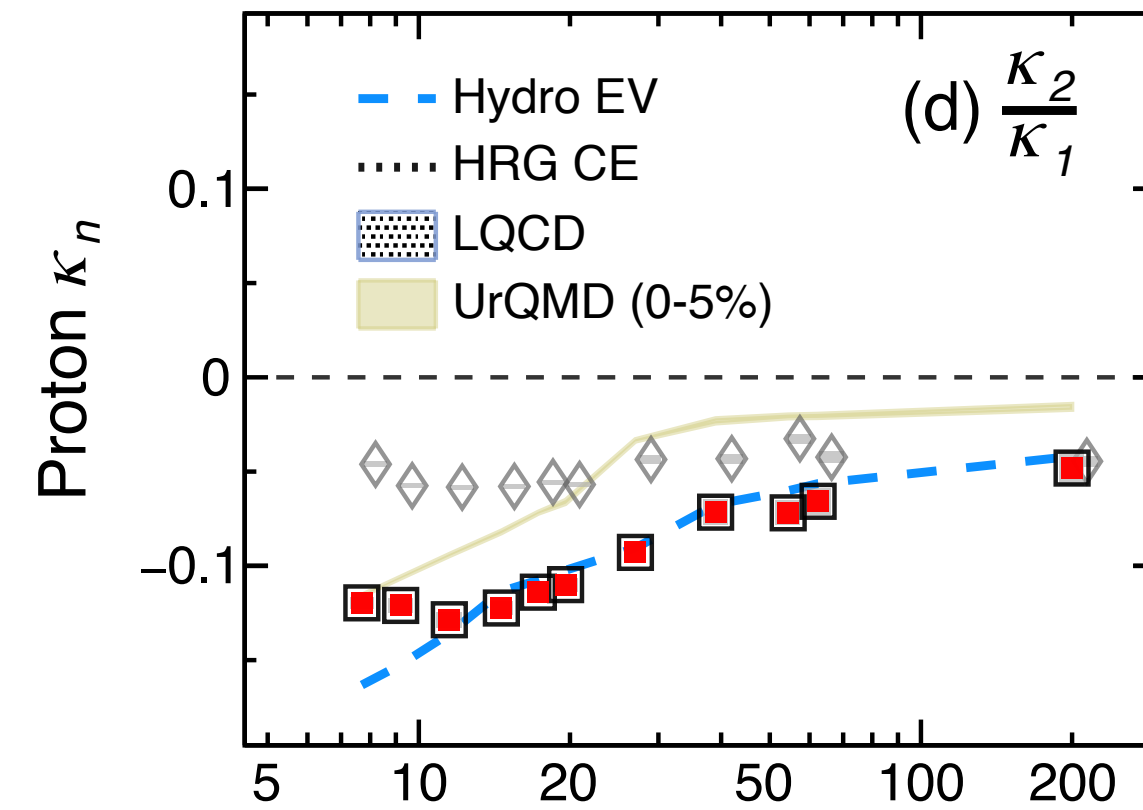
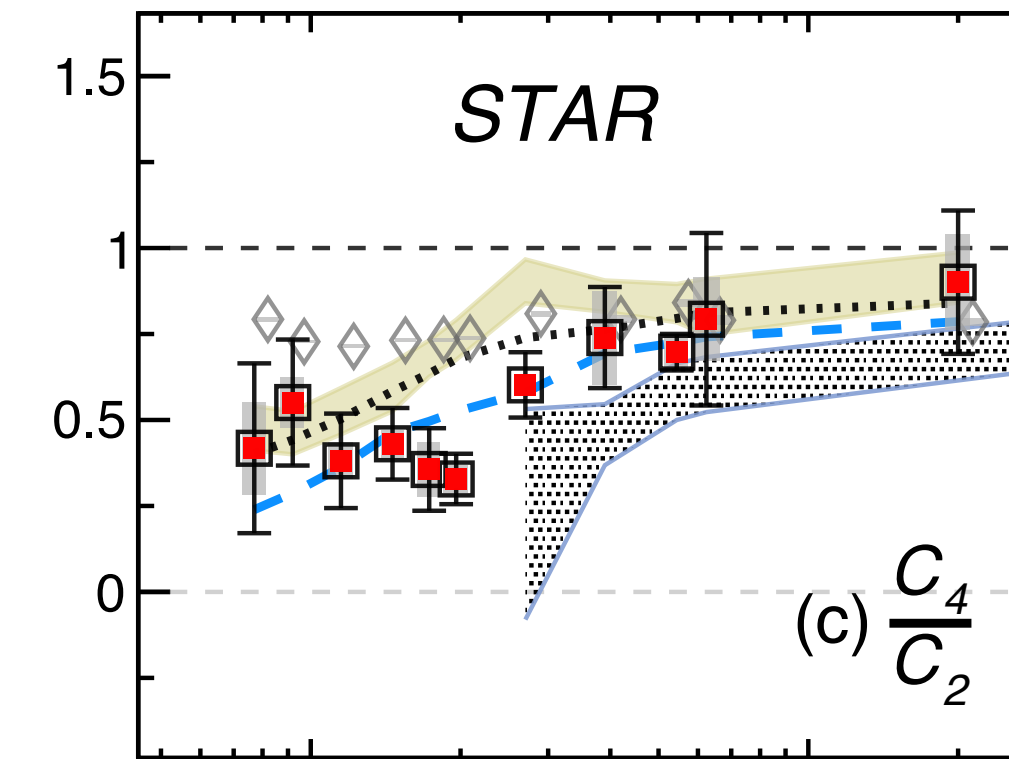
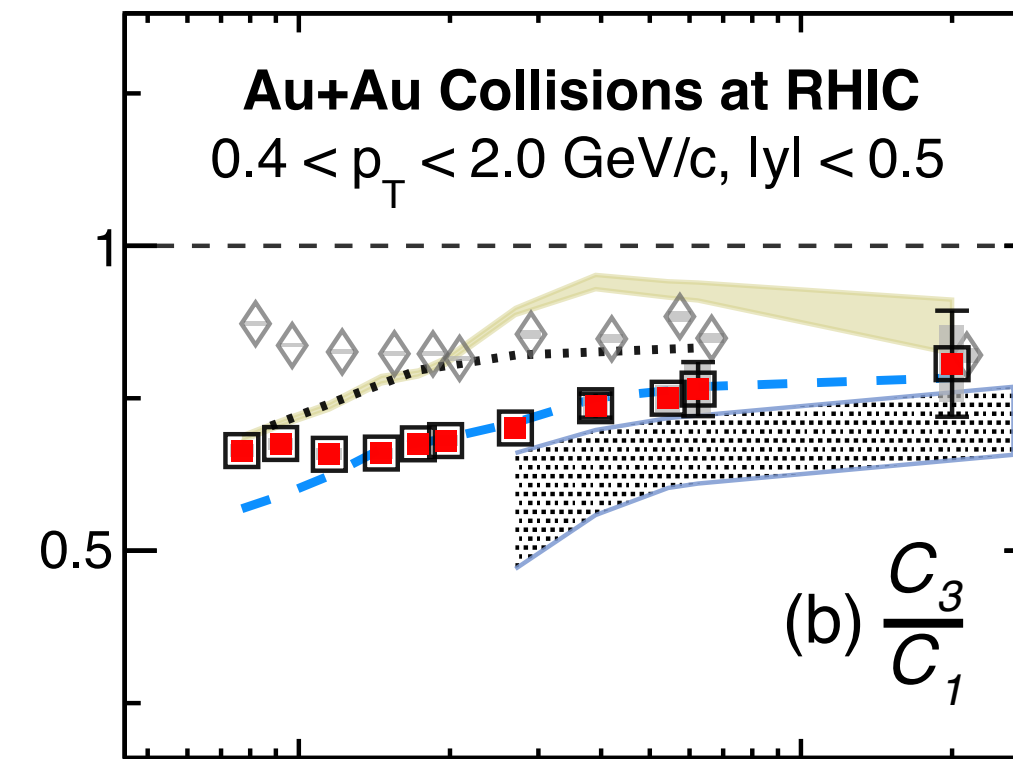
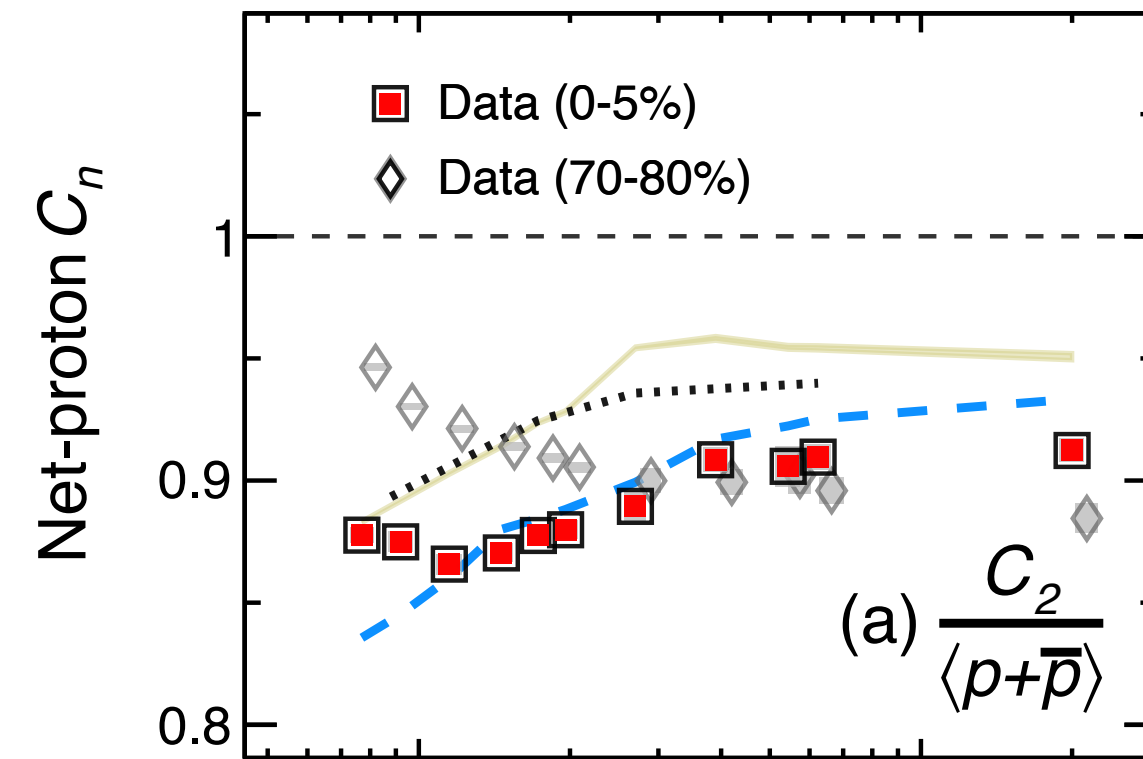
- Viscous hydro
- EOS tuned to LQCD
- Correct for global charge conservation
- Protons NOT baryons
- Baseline!
No critical point or phase transition
- No volume fluctuations

See also: Braun-Munzinger et al,
NPA 1008 (2021) 122141



STAR BES I data

Cumulants

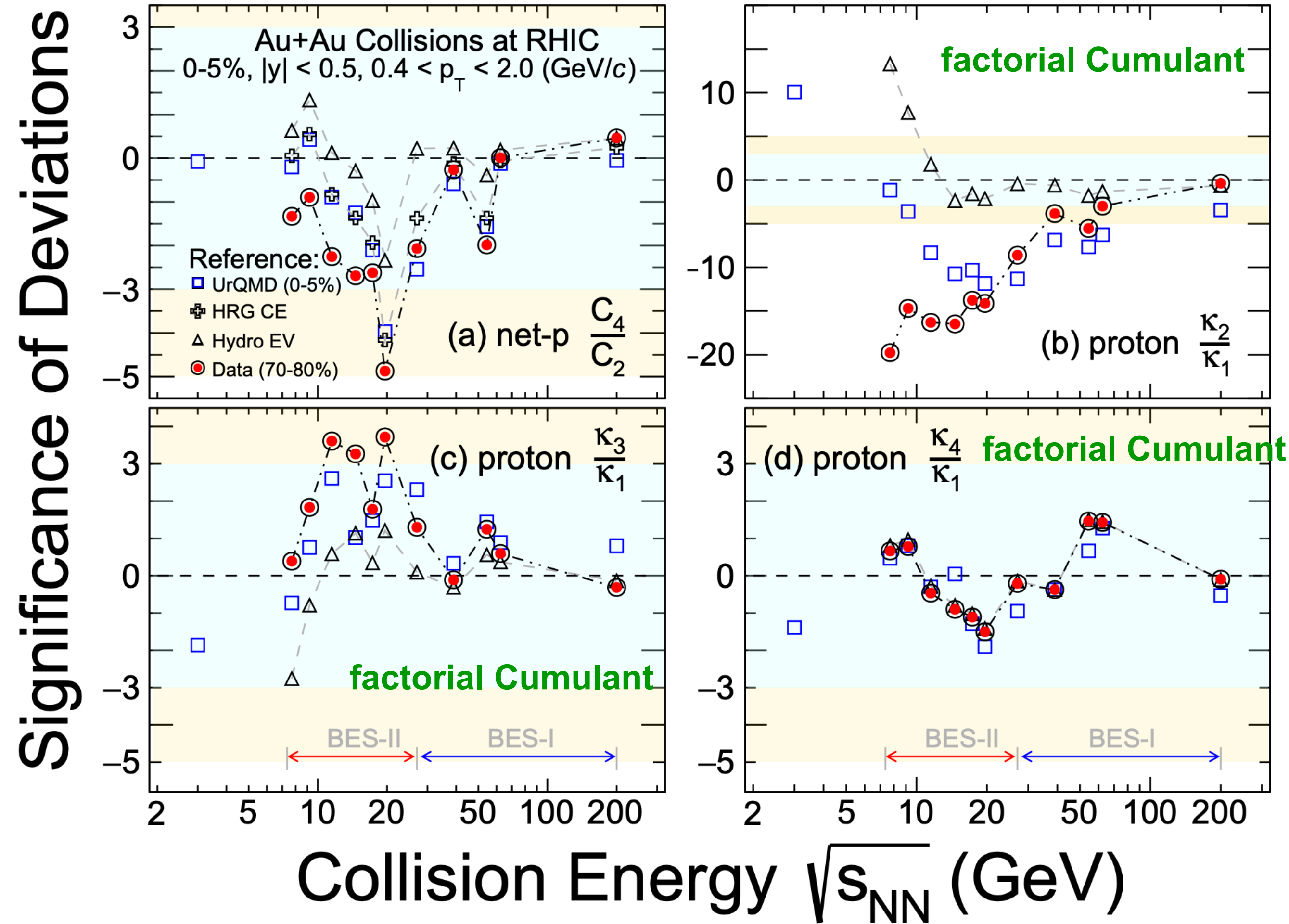


Factorial
Cumulants (FC)

Collision Energy $\sqrt{s_{NN}}$ (GeV)

Significance

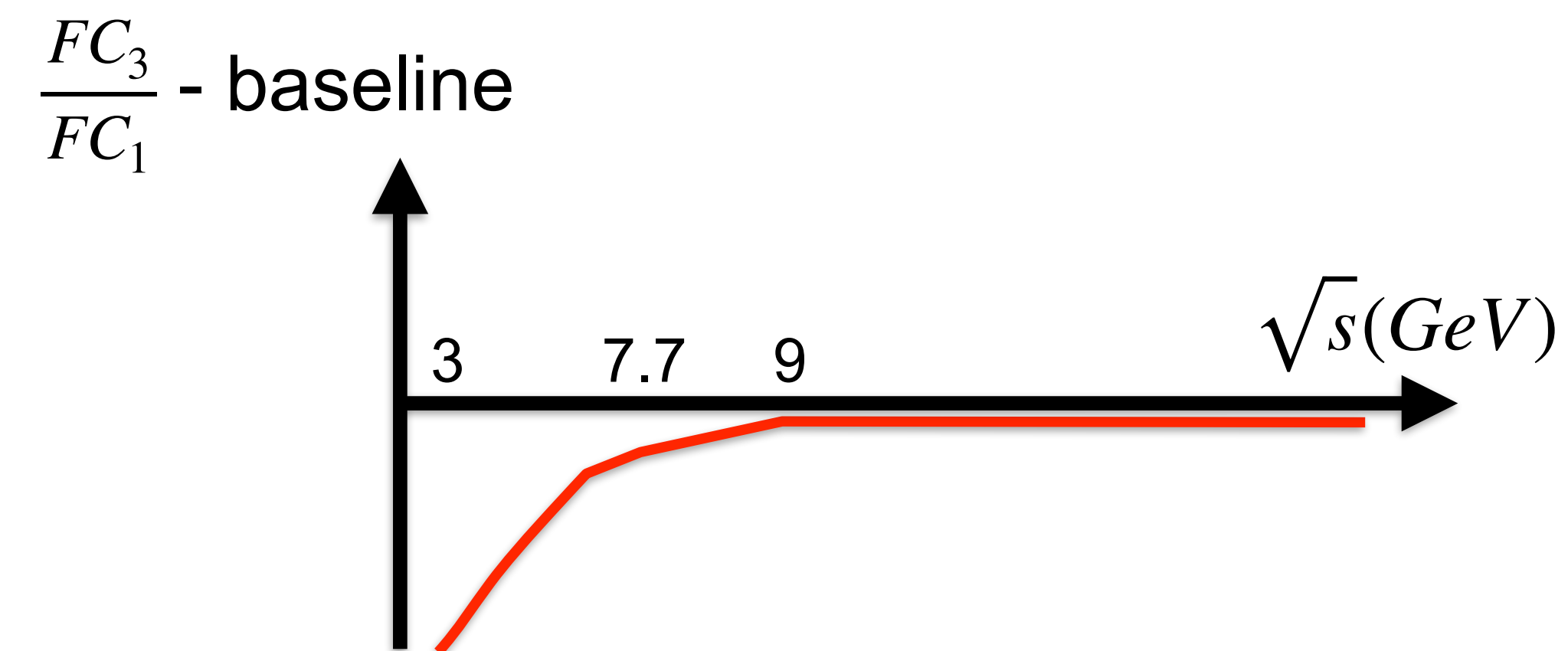
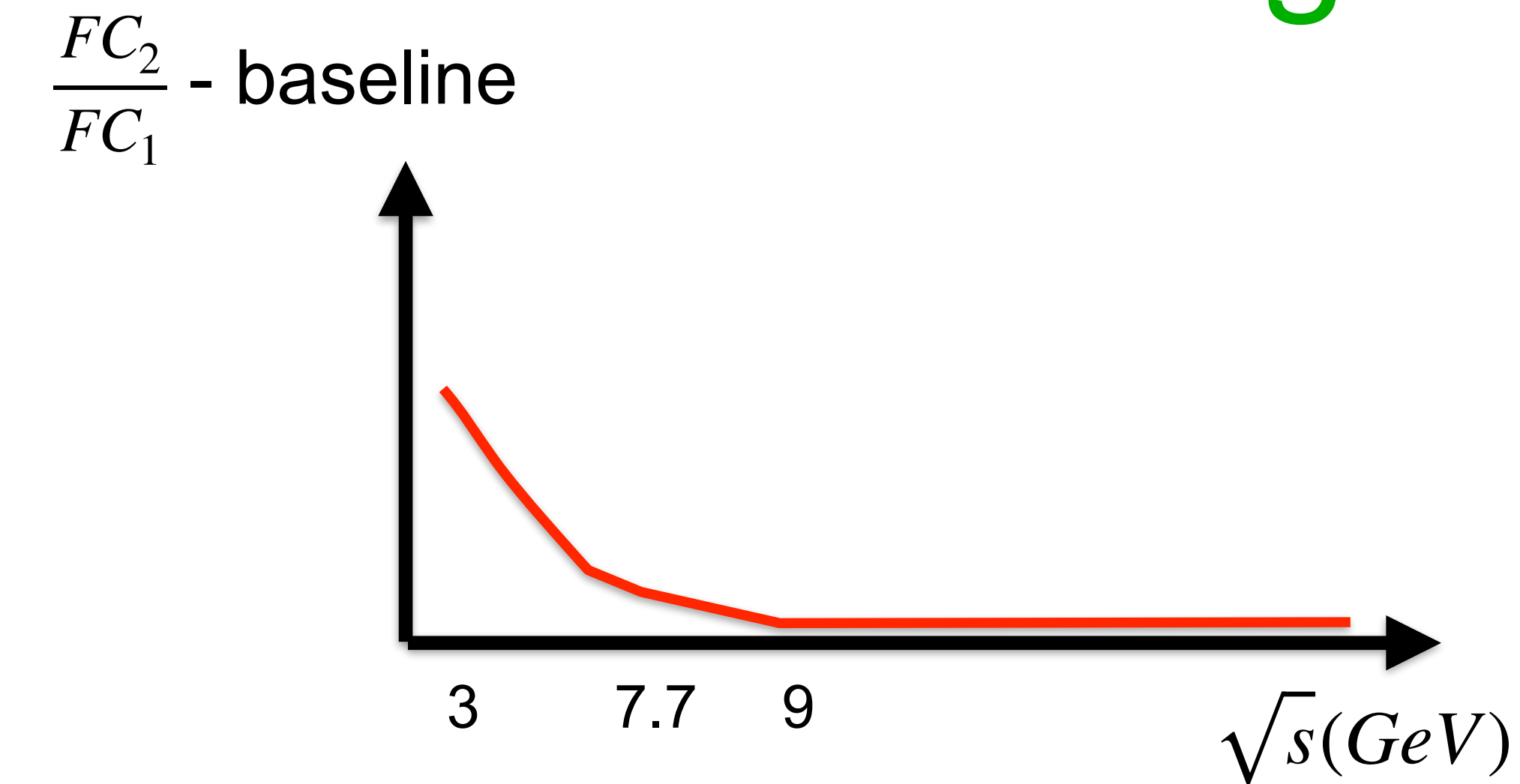
X. Dong, CPOD 2026



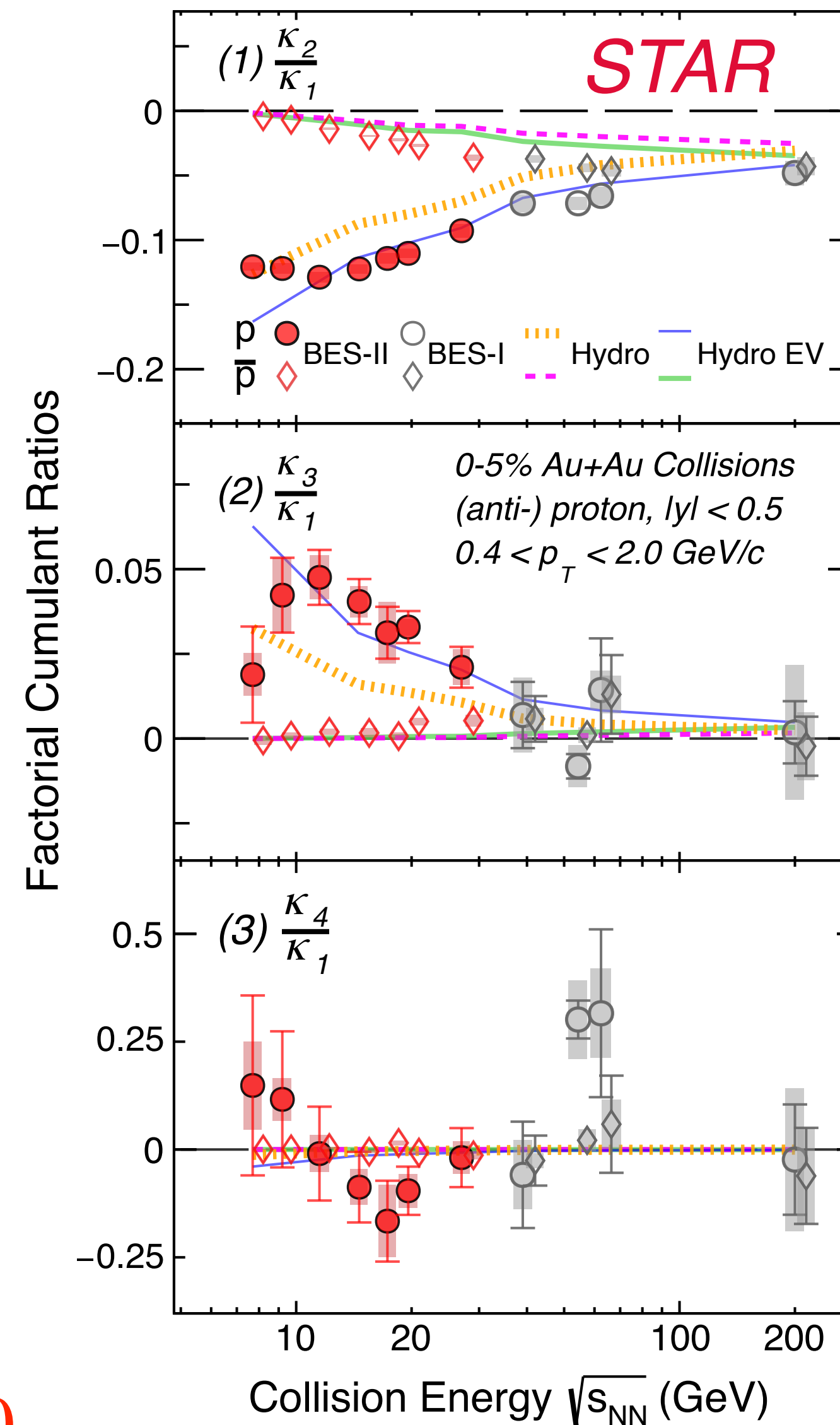
STAR, PRL 135 (2025) 142305

significant difference
between data and baselines
for $\frac{FC_2}{FC_1}$

The “signal” (relative to baseline)

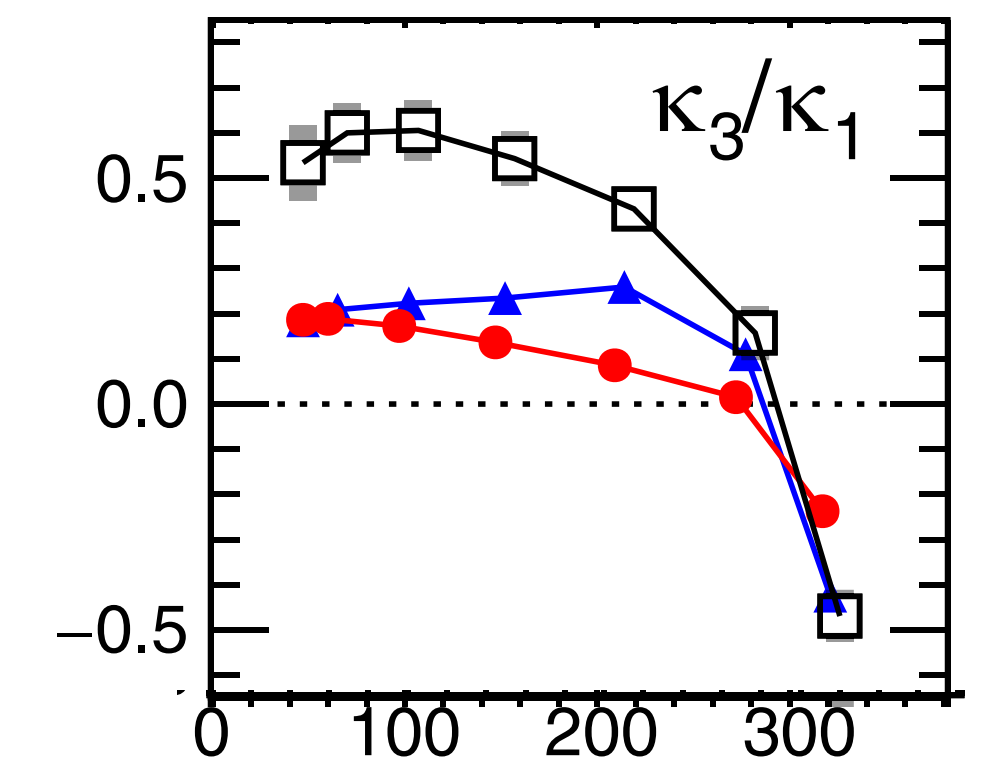
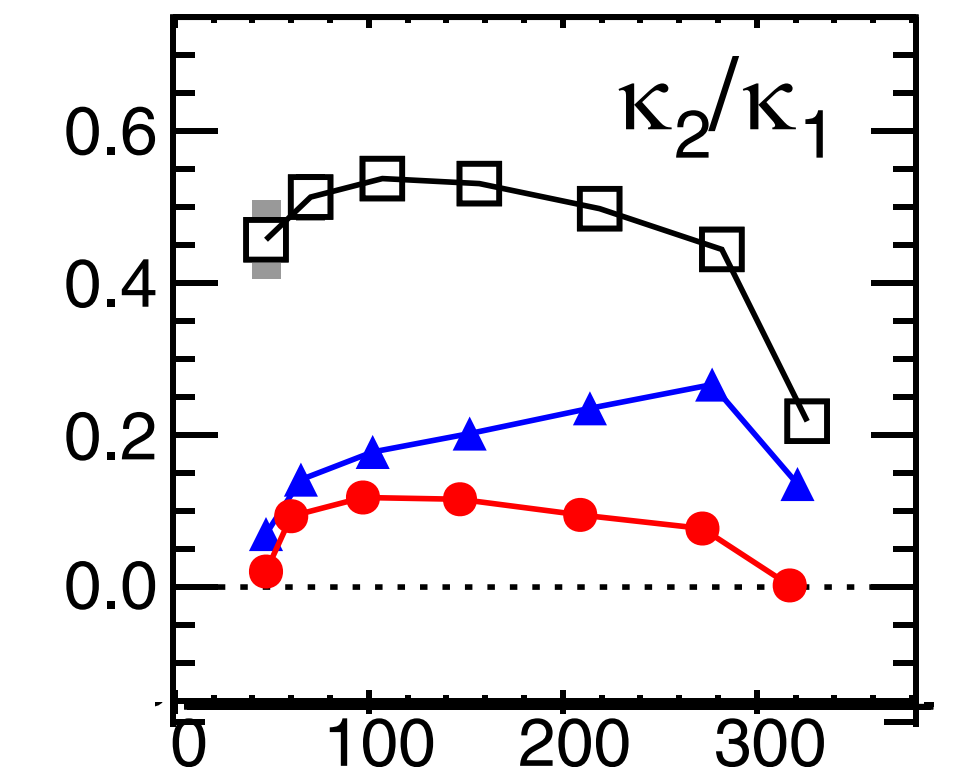


CAVEAT: 3 GeV has acceptance $-0.5 \leq Y \leq 0$



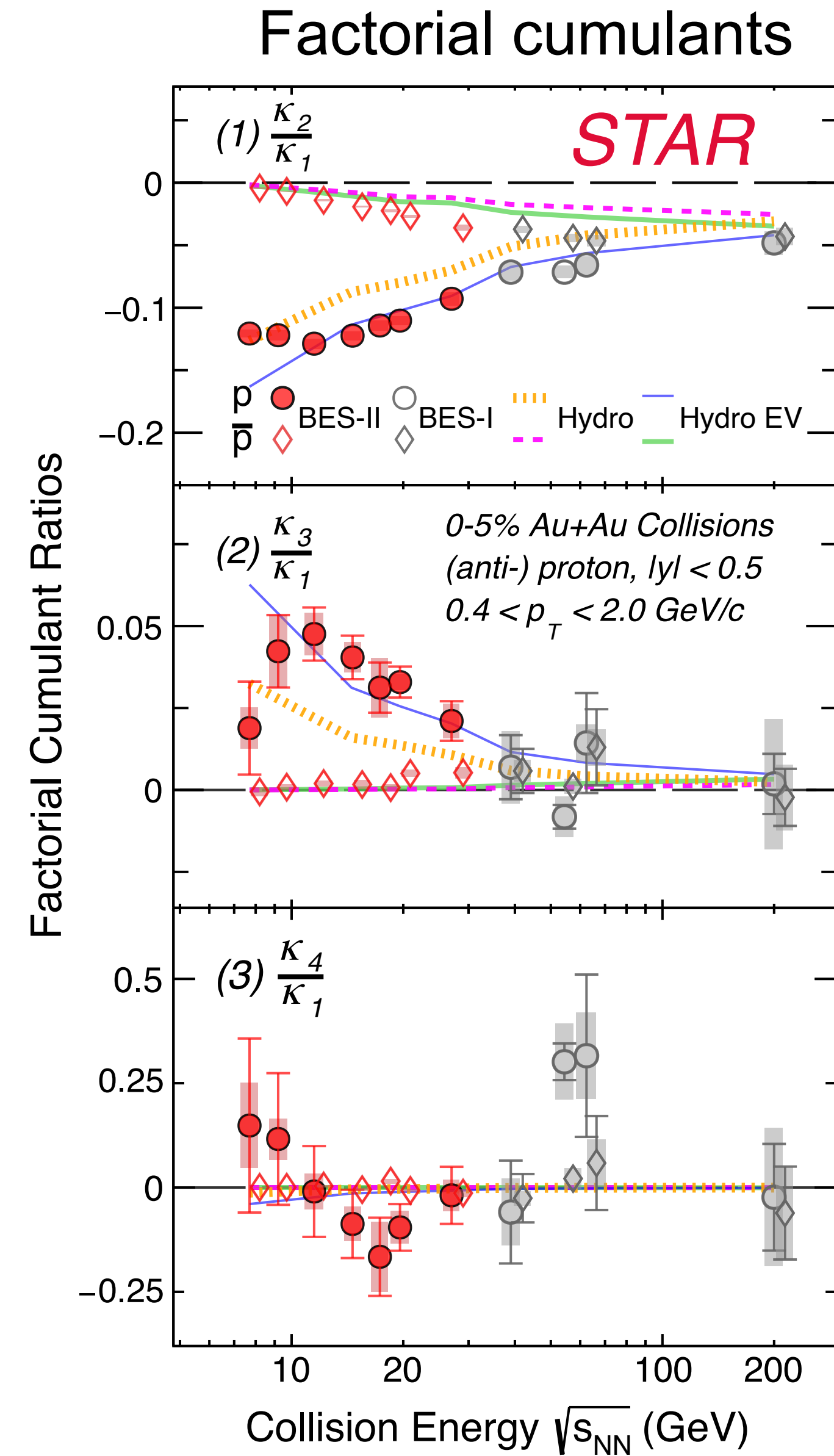
STAR 2209.11940

$\sqrt{s} = 3 \text{ GeV}$



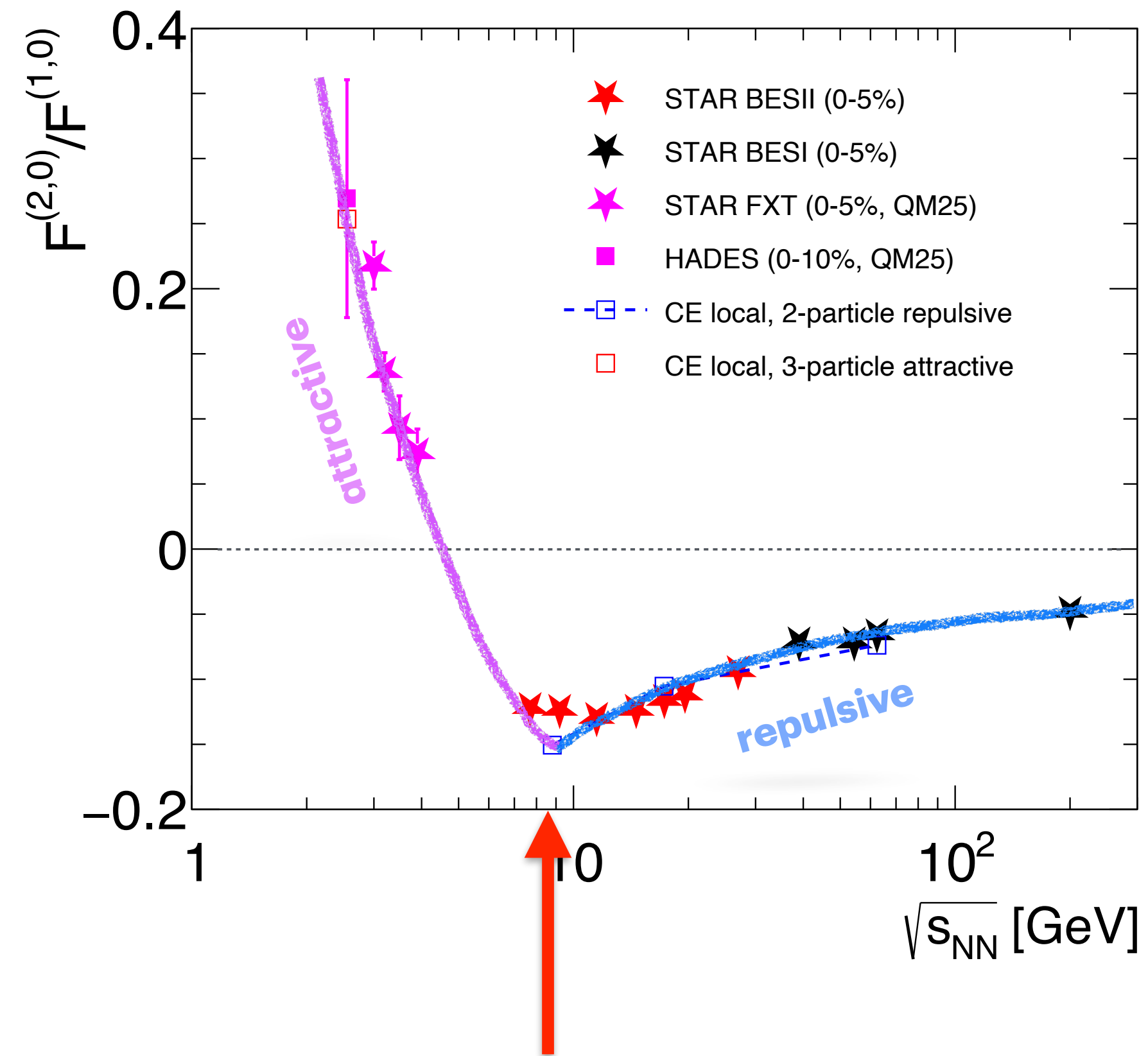
New STAR data (BESII)

Looks like we need some kind of attraction!



Attraction does the trick

Friman, Redlich, Rustamov, arXiv: 2508.18879



Interaction in momentum space
Strength of interaction is adjusted to fit the data

NOTE: change from attractive to repulsive at ~ 10 GeV

Beth-Uhlenbeck

B. Friman, V.K. in preparation

$$Z_{GC}(\mu_B, T) = \exp \left(e^{\mu_B/T} z_1 + e^{2\mu_B/T} z_2 \right)$$

$$z_1 = V \int \frac{d^3 p}{(2\pi)^3} e^{-\varepsilon_p/T}$$

$$z_2^0 = V \int \frac{d^3 p}{(2\pi)^3} \int_{2m_N}^{\infty} \frac{dM}{\pi} \sum_{j=0}^{\infty} \sum_{\text{(even)}} (2j+1) \frac{\partial \delta_{jj}^{10}}{\partial M} e^{-\sqrt{p^2 + M^2}/T}, \quad (S=0)$$

$\delta_{j,l}$ proton proton phase shift
 M = center of mass energy

$$z_2^1 = V \int \frac{d^3 p}{(2\pi)^3} \int_{2m_N}^{\infty} \frac{dM}{\pi} \sum_{\ell=1}^{\infty} \sum_{\text{(odd)}} \sum_{j=\ell-1}^{\ell+1} (2j+1) \frac{\partial \delta_{j\ell}^{11}}{\partial M} e^{-\sqrt{p^2 + M^2}/T}, \quad (S=1)$$

Phase shifts are extracted up to $M \simeq 2 \text{ GeV}$ (SAID data base, G.W. University)

Complication: for $T \simeq 150 \text{ MeV}$ inelastic channels are needed

Beth-Uhlenbeck grand canonical

$$Z_{GC}(\mu_B, T) = \exp \left(e^{\mu_B/T} z_1 + e^{2\mu_B/T} z_2 \right)$$

Cumulants:

$$\kappa_1 = \frac{\partial}{\partial(\mu/T)} \ln(Z) = e^{\mu_B/T} z_1 + 2e^{2\mu_B/T} z_2$$

$$\kappa_2 = e^{\mu_B/T} z_1 + 4e^{2\mu_B/T} z_2$$

$$\kappa_3 = e^{\mu_B/T} z_1 + 8e^{2\mu_B/T} z_2$$

$$FC_2 = \kappa_2 - \kappa_1 = 2e^{2\mu_B/T} z_2 \quad \text{2nd Order factorial cumulant “measures” the strength of the interaction}$$

$$FC_3 = \kappa_3 - 3\kappa_2 + 2\kappa_1 = 0 \quad \text{No 3-particle correlations}$$

Beth-Uhlenbeck canonical

$$Z_C(B, T; u, v) = \sum_{n=0}^{\infty} \sum_{m=0}^{\infty} \frac{z_1(u, v)^n}{n!} \frac{z_2(u, v)^m}{m!} \delta(B - n - 2m)$$

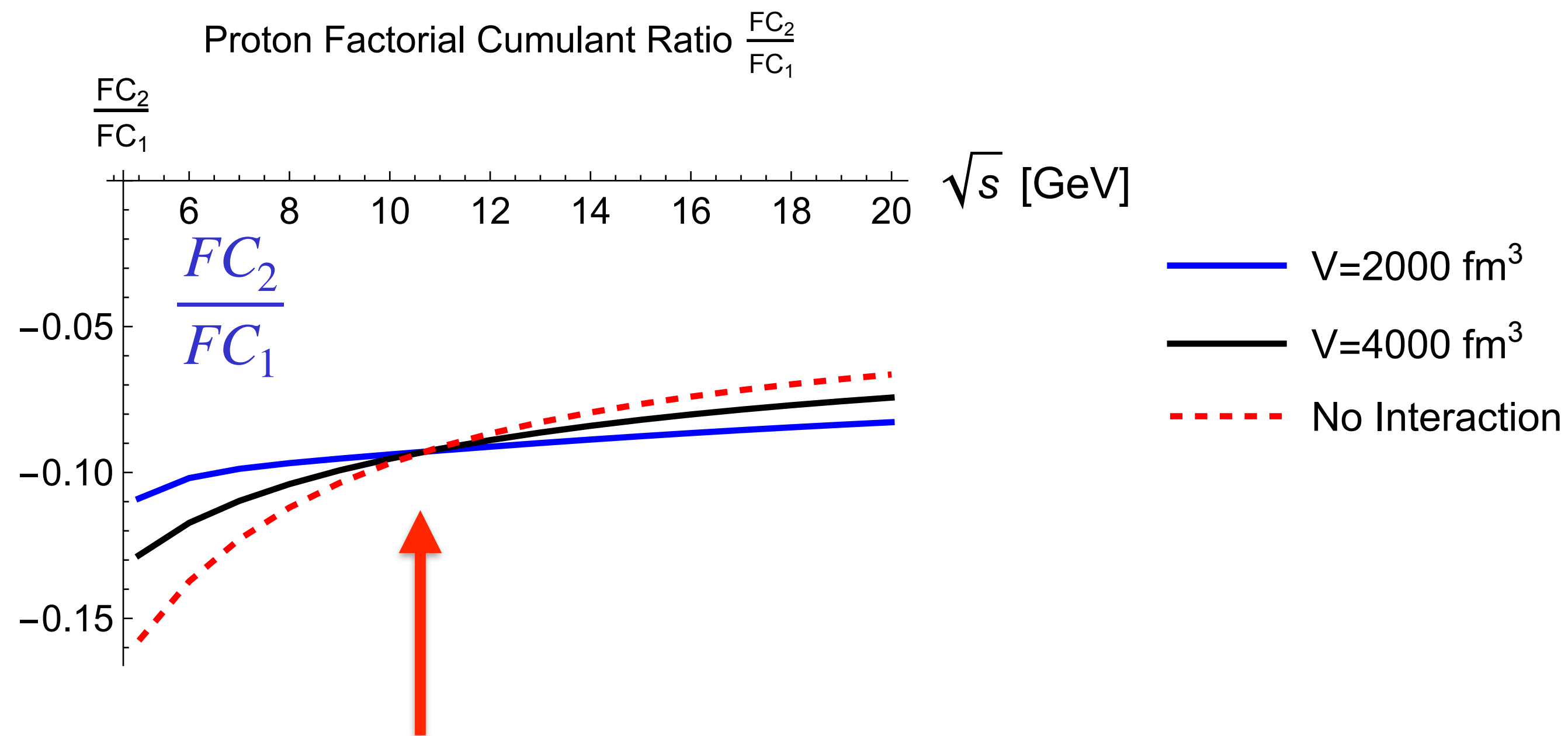
$$z_1(u, v) = u z_{1,p} + v z_{1,n}, \quad z_2(u, v) = u^2 z_{2,p} + v^2 z_{2,n} + v u z_{2,np}$$

For subsystem S:

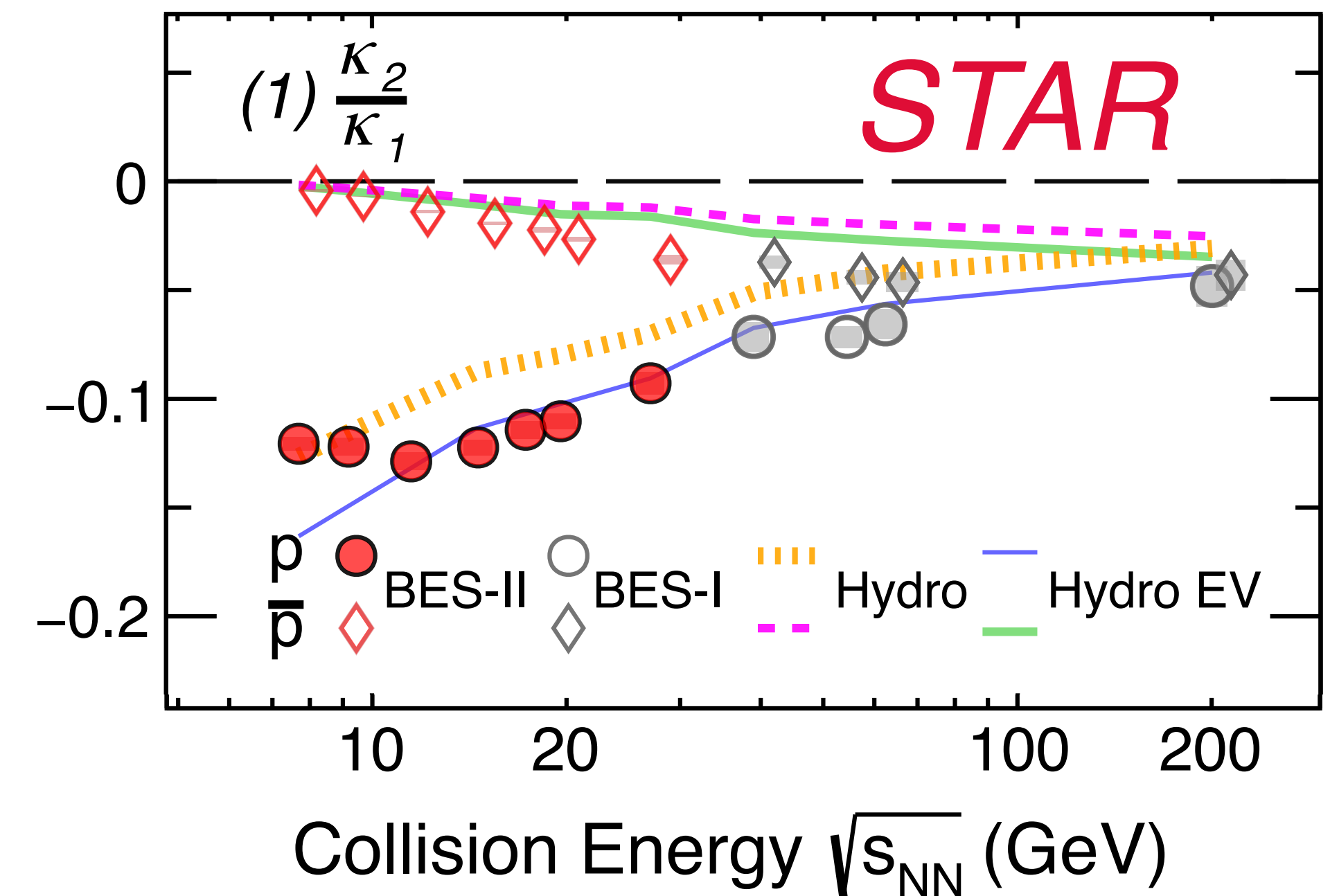
$$Z_{C,S}(B, T; u, v) = \sum_{B_S=0}^B Z_C(B_S, T; u, v) Z_C(B - B_S, T)$$

Factorial cumulants: $FC_{n,m} = \frac{\partial^n \partial^m}{\partial u^n \partial v^m} \log[Z_{C,S}(B, T; u, v)] \Big|_{u=0, v=0}$

Beth-Uhlenbeck FC_2/FC_1



attractive to repulsive at $\sim 10\text{GeV}$!



Use freeze out parametrization Andronic et al, Nature 561 (2018)

Use $\alpha(\sqrt{s}) = \frac{N_p}{N_B}$ from PBM et al, NPA1008 (2021); Fit provided by A. Rustamov

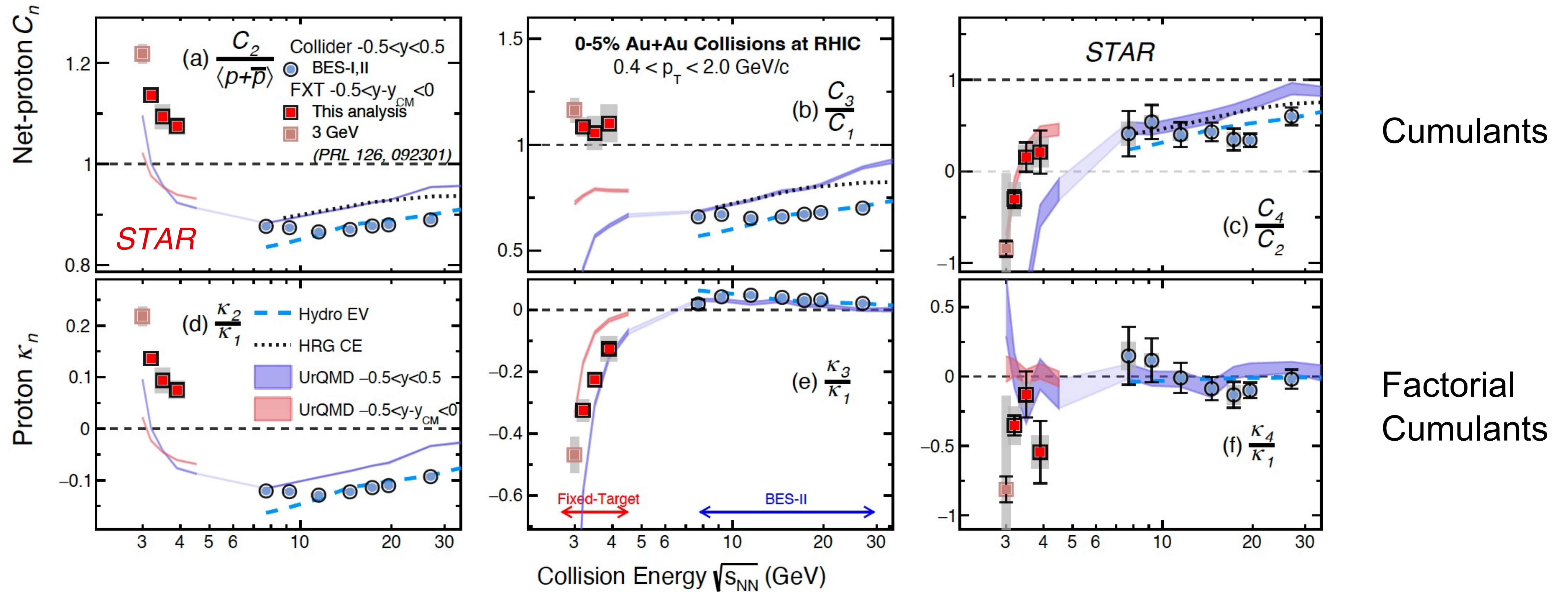
Limitations and open questions

- No inelastic channels (yet)
- Only protons and neutrons; no resonances
- NOT (yet) ready for comparison with data!!!
- Elastic interactions effective even after chemical freeze out!!

However....

Possible Spoiler

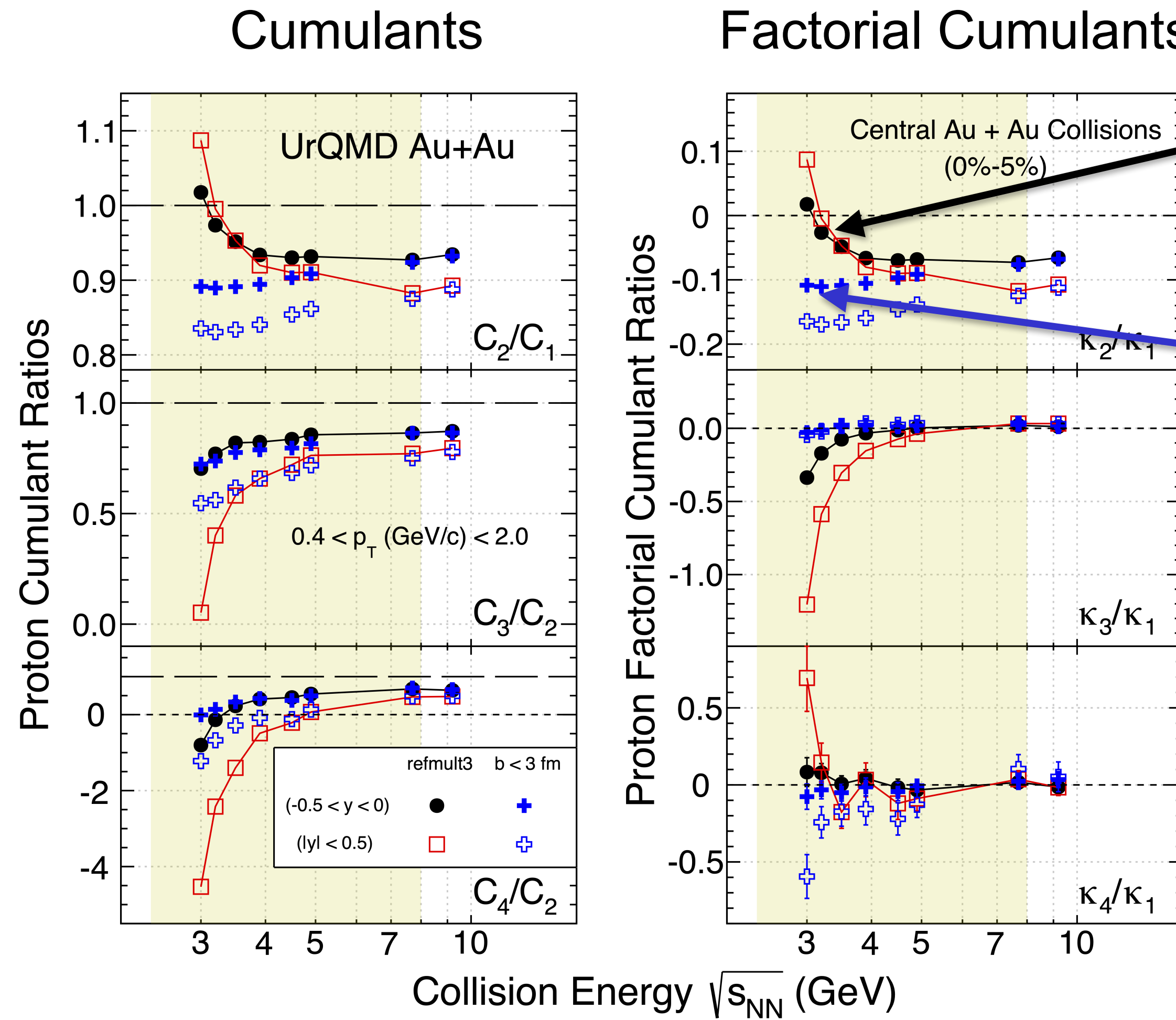
Zachary Sweger (for STAR) QM 2025



UrQMD (WITHOUT mean field) get energy dependence qualitatively right!!!

The (possible) culprit

X. Zhang, Y. Zhang, X. Luo, N. Xu, arXiv: 2506.18832



Fluctuating impact parameter
STAR centrality selection

Fixed impact parameter (3=3 fm)
minimal volume fluctuations.

N.B.: Centrality Bin Width Corrections
(CBWC) applied to both

Possible culprit:

- volume fluctuations
 - CBWC does not work well at low energies
(Friman, VK 2511.11869, NPA in print)
 - New methods (Rustamov et al, 2211.14849;
Wang et al, 2505.03666)

Test for Baseline

A. Bzdak, V. Vovchenko, VK, arXiv:2503.16405

Both global charge conservation and volume fluctuations are **long range** correlations

Factorial cumulant:

$$FC_n = \int_{\Delta_Y} dy_1 \cdots \int_{\Delta_Y} dy_n c(y_1, \cdots, y_n)$$

$c(y_1, \cdots, y_n)$: n-particle correlations function

Long range correlations: $c(y_1, \cdots, y_n) = \text{const}$ within ΔY

$$\Rightarrow C_n \sim (\Delta Y)^n$$

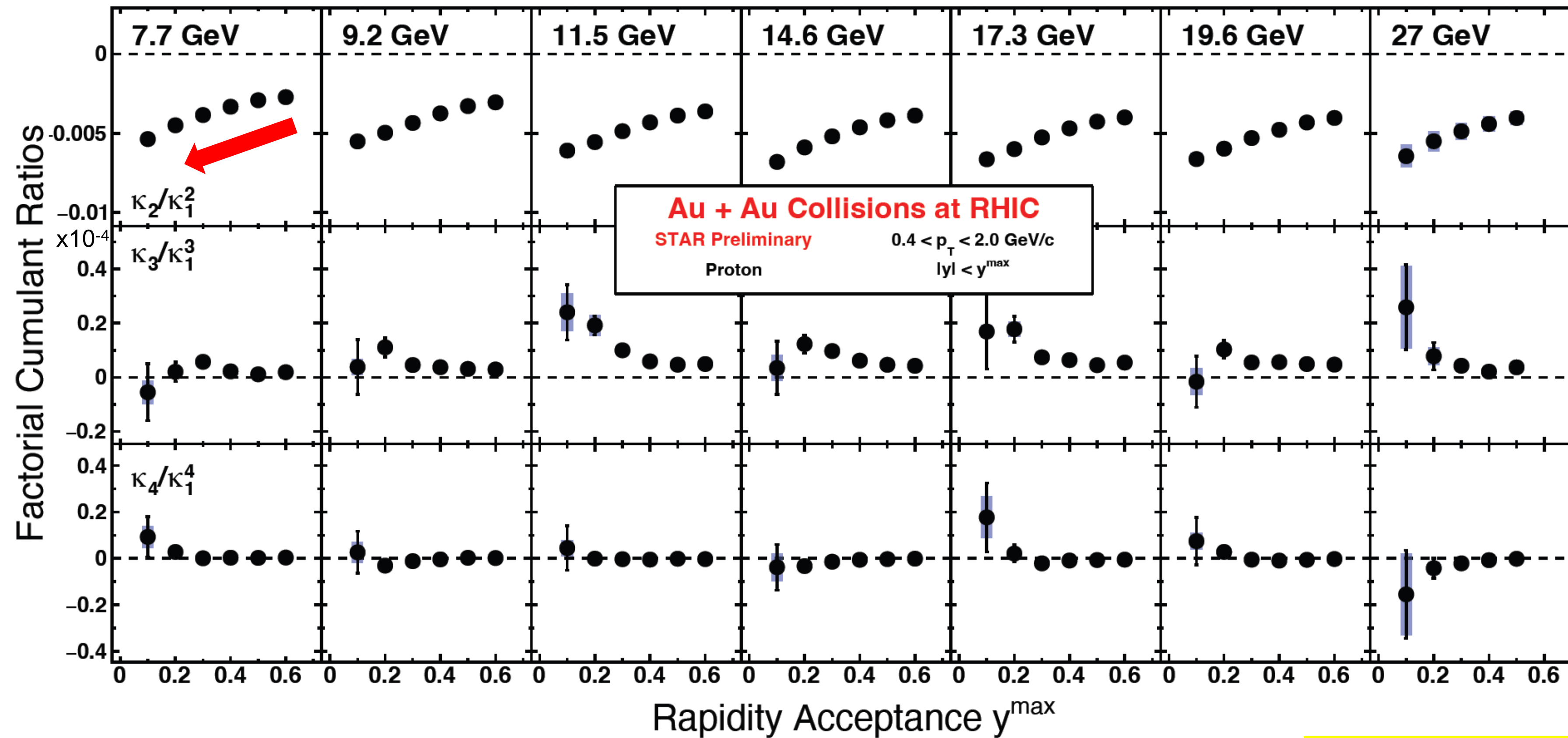
$$\Rightarrow \frac{FC_n}{FC_1^n} = \text{const} \text{ as function of } \Delta Y$$

Test of Baseline

Both GLOBAL charge conservation and volume fluctuations introduce only

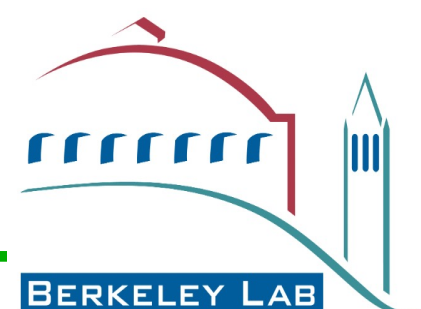
LONG Range correlations in rapidity (larger than acceptance)

Acceptance Dependence



Yige Huang, PHD 25

Looks like REPULSION even at $\sqrt{s} = 7.7 \text{ GeV}$



Summary

- Net charge fluctuations may finally say something about fractional charges
- STAR has delivered on the BESII data!
- Baseline with baryon number conservation and repulsive interaction tuned to LQCD agrees with data down to $\sqrt{s} \sim 10 \text{ GeV}$
- Known and measured nucleon-nucleon interaction gets this trend qualitatively right
 - Work is still needed to get a quantitative result
- HOWEVER, UrQMD get the trend in the energy dependence right. Volume fluctuations at low energy?
- Possible test of a baseline involving baryon number conservation and volume fluctuations via ratio of factorial cumulants $\frac{FC_n}{FC_1^N}$
 - Does not seem to scale in BES II data. Looks like repulsion down to $\sqrt{s} = 7.7 \text{ GeV}$

Happy Birthday, Peter!

