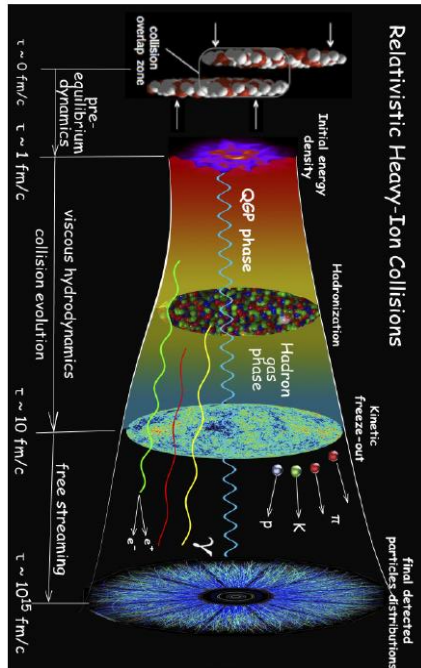


Thermal origin of light and heavy flavor hadrons produced in HIC

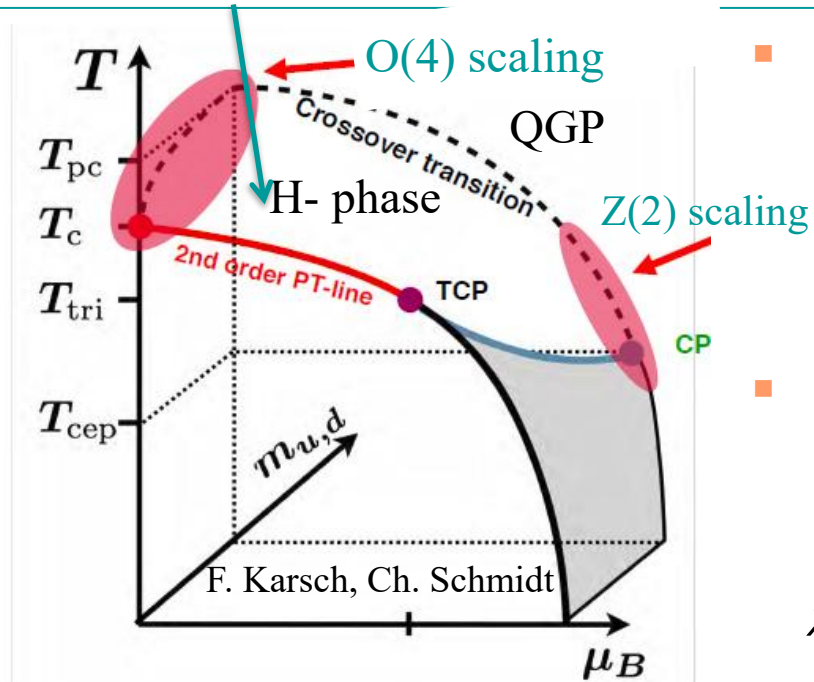
K. Redlich Uni. Wroclaw, Poland



- Equation of state of hadronic phase
From S-matrix Hadron Resonance Gas, (SHM)
 $\Leftrightarrow$ LQCD EoS
- Linking SHM equation of state with hadron multiplicity data in HIC
 $\Rightarrow$ energy and system size dependence from SIS to LHC
- Thermal origin of quarkonia and open charm hadrons
 $\Rightarrow$ Peter Braun-Munzinger and Johanna Stachel
hadronization model of charm and its link to data

Based on the collaboration with: **P. Braun-Munzinger:** A. Andronic & Johanna Stachel
and with: J. Cleymans, B. Friman, F. Karsch, P.M. Lo, M. Marczenko,
L. McLerran, H. Oeschler, A. Rustamov, C. Sasaki, N. Sharma, Nu Xu.

LQCD phase diagram and hadronic equation of state



- Due to expected $O(4)/Z(2)$ scaling of QCD, the free energy can be decomposed as:

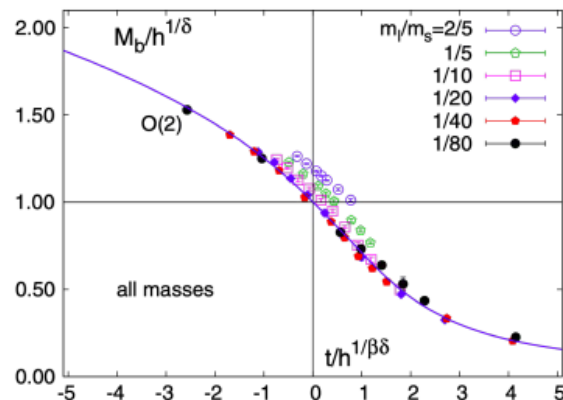
$$F = F_R(T, \mu_q, \mu_l) + b^{-1} F_S(b^{(2-\alpha)^{-1}} t(\mu), b^{\beta\delta/\nu} h)$$

- Direct delineation of chiral symmetry restoration via generalized susceptibilities linked to cumulants

LQCD		HIC
$\chi_B^{(n)} = \frac{\partial^n (P/T^4)}{\partial (\mu_B/T)^n}$	➡	$VT^3 \chi_n^B = \kappa_n^B (N_B - N_{\bar{B}})$

S. Ejiri, F. Karsch, K.R., Phys. Lett. B633 (2006)

- Pioneering LQCD studies of different charge susceptibilities by Bielefeld-Swansea collaboration
C. R. Allton, ET AL., Phys. Rev. D71 (2005)
- Pioneering HIC experimental studies of net-proton cumulants in AA collisions done by BES-STAR, ALICE and HADES experiments



S. Ejiri, F. Karsch et al., Phys. Rev. D80 (2009)

Modelling QCD thermodynamic potential in hadronic phase

Pressure of an interacting, $a+b \Leftrightarrow a+b$, hadron gas in equilibrium

$$P(T) \approx P_a^{id} + P_b^{id} + P_{ab}^{int}$$

The leading order interactions, determined by the two-body scattering phase shift, which is equivalent to the second virial coefficient

$$P^{int} = \sum_{I,j} \int_{m_{th}}^{\infty} dM B_j^I(M) P^{id}(T, M)$$

$$B_j^I(M) = \frac{1}{\pi} \frac{d}{dM} \delta_j^I(M)$$

Effective weight function *Scattering phase shift*

- Interactions driven by narrow resonance of mass M_R

$$B(M) = \delta(M - M_R) \Rightarrow P^{int} = P^{id}(T, M_R) \Rightarrow HRG$$

For finite and small width of resonance, $B(M) \Rightarrow$ Breit-Wigner form

- For non-resonance interactions or for broad resonances $P_{ab}^{int}(T)$ should be linked to the phase shifts

R. Dashen, S. K. Ma and H. J. Bernstein,
Phys. Rev. 187, 345 (1969)

R. Venugopalan, and M. Prakash,
Nucl. Phys. A 546 (1992) 718.

W. Weinhold, and B. Friman,
Phys. Lett. B 433, 236 (1998).

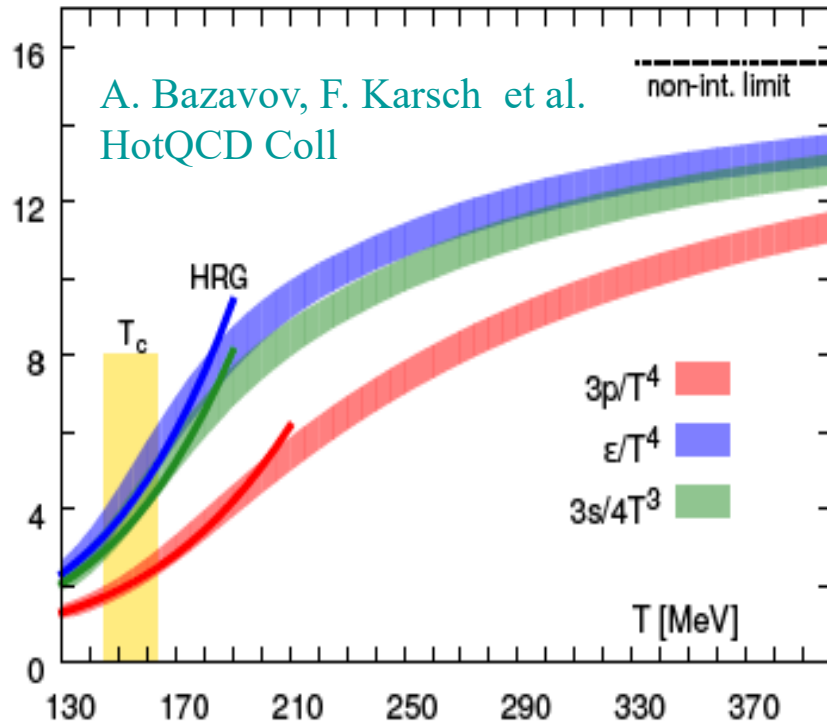
Pok Man Lo, Eur. Phys.J. C77 (2017) no.8, 533

A. Andronic, P. Braun-Munzinger,
P. M. Lo, B. Friman, J. Stachel & K.R.
Phys. Lett. B 792, 304 (2019)

Quark-Hadron duality near the QCD phase boundary

$$P(T, \vec{\mu}) \approx \sum_H P_H^{id} + \sum_R P_R^i$$

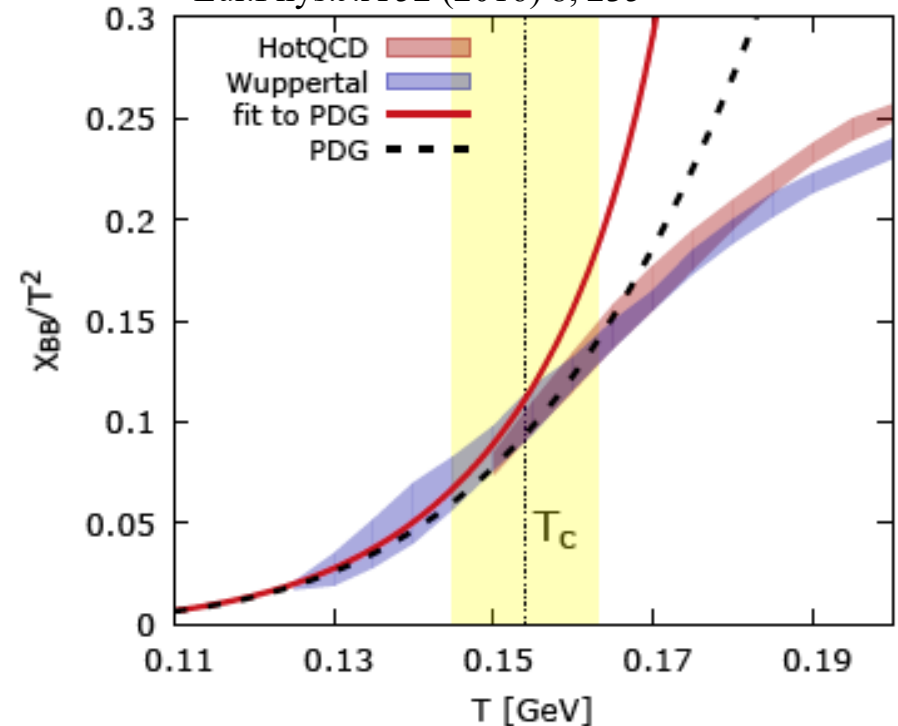
$$P_R^i = \pm \frac{T g_i}{2\pi^2} \int p^2 dp \int dM \ln(1 \pm e^{-\beta(E_i - \vec{q}_i \vec{\mu}_i)}) F_R^{BW}(M)$$



see e.g.: J. Goswami, et al., 2011.02812 [hep-lat]
HotQCD Coll., (A. Bazavov et al.),
Phys. Rev. D86, (2012)

S. Borsányi, et. al 2102.06660 [hep-lat]

Pok Man Lo, M. Marczenko et. al.
Eur.Phys.J.A 52 (2016) 8, 235

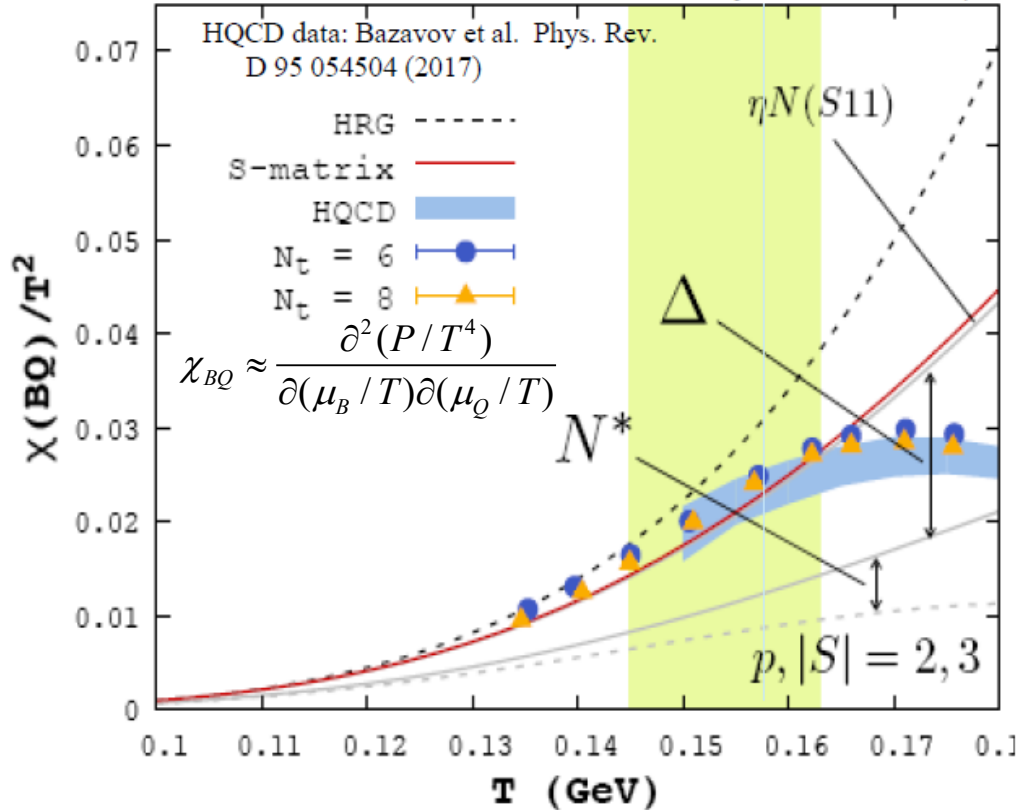


- SM Hadron Resonance Gas thermodynamic potential provides good approximation of the QCD equation of states in confined phase

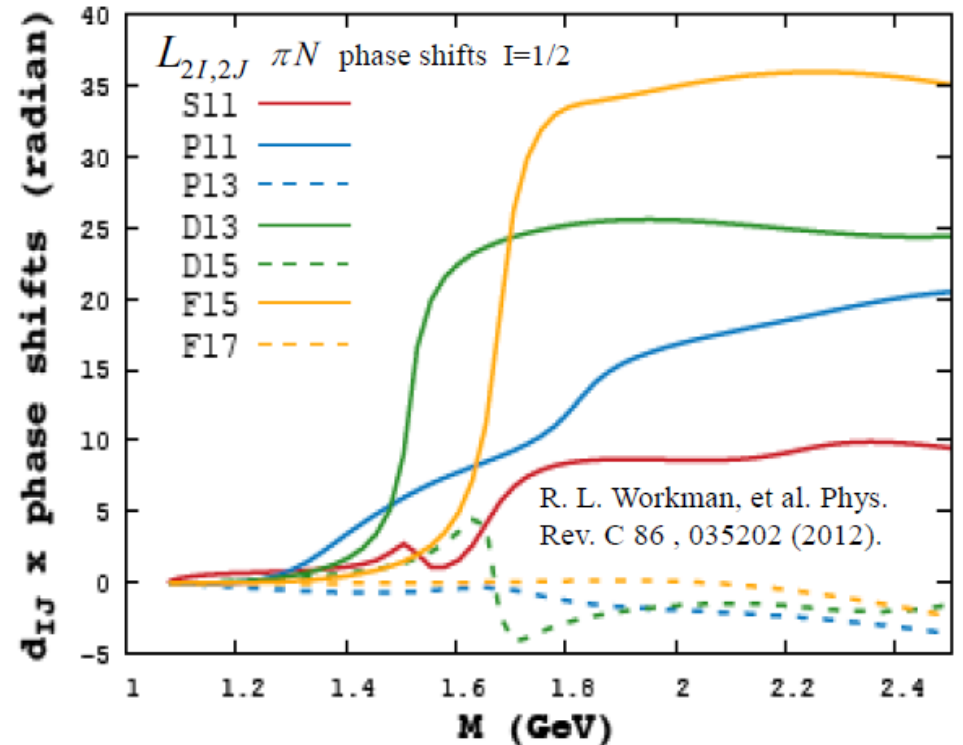
Consistent description of the net-baryon number fluctuations and in further sectors of hadronic quantum numbers

Probing non-strange baryon sector in πN - system

Pok Man Lo, B. Friman, C. Sasaki & K.R., Phys.Lett. B778 (2018)



$$\chi_{BQ} = (\chi_{BB} - |\chi_{BS}|) / 2$$

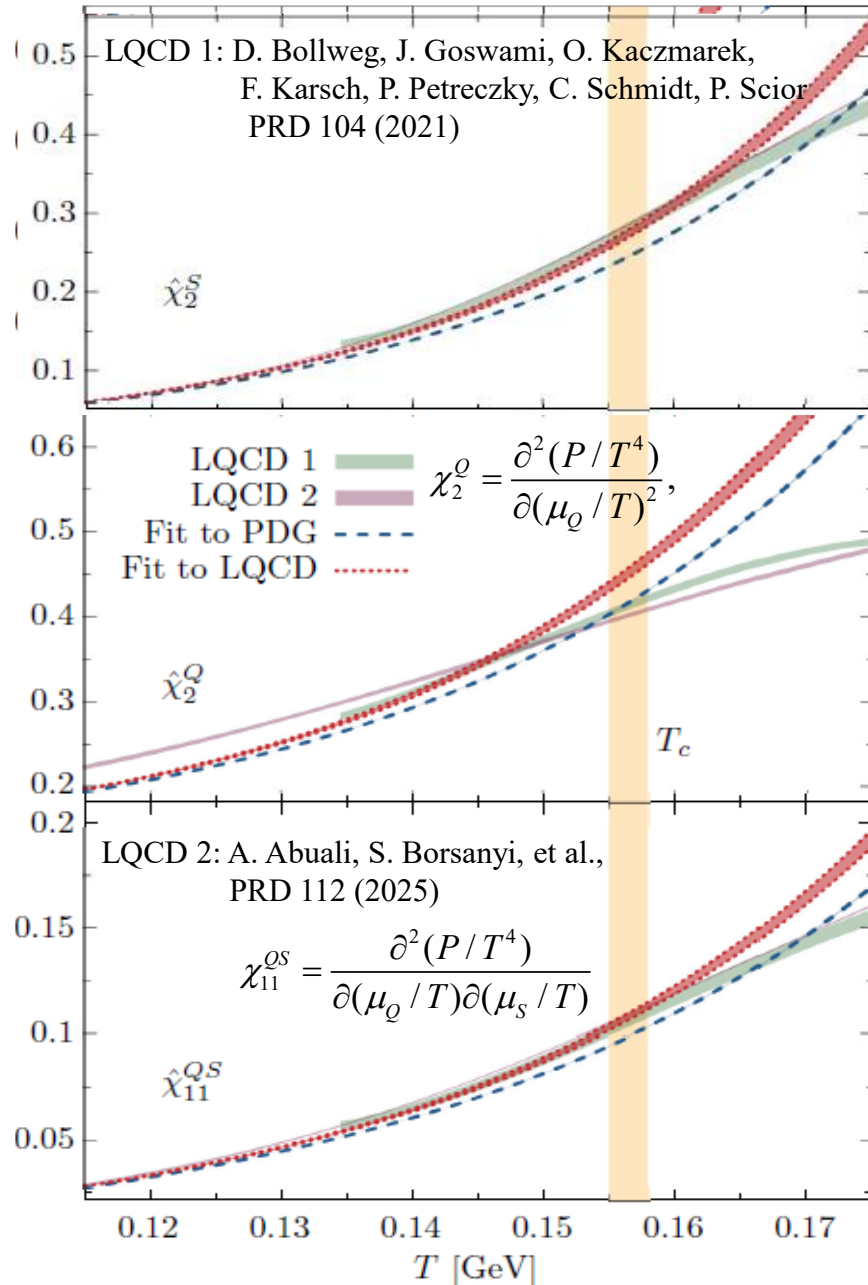


- Considering contributions of all πN $\delta_j^{I=(1/2), (3/2)}$ (N^*, Δ^* resonances) to χ_{BQ} within S-matrix approach, reduces the HRG predictions towards the LQCD in the chiral crossover $0.15 < T < 0.16$ GeV

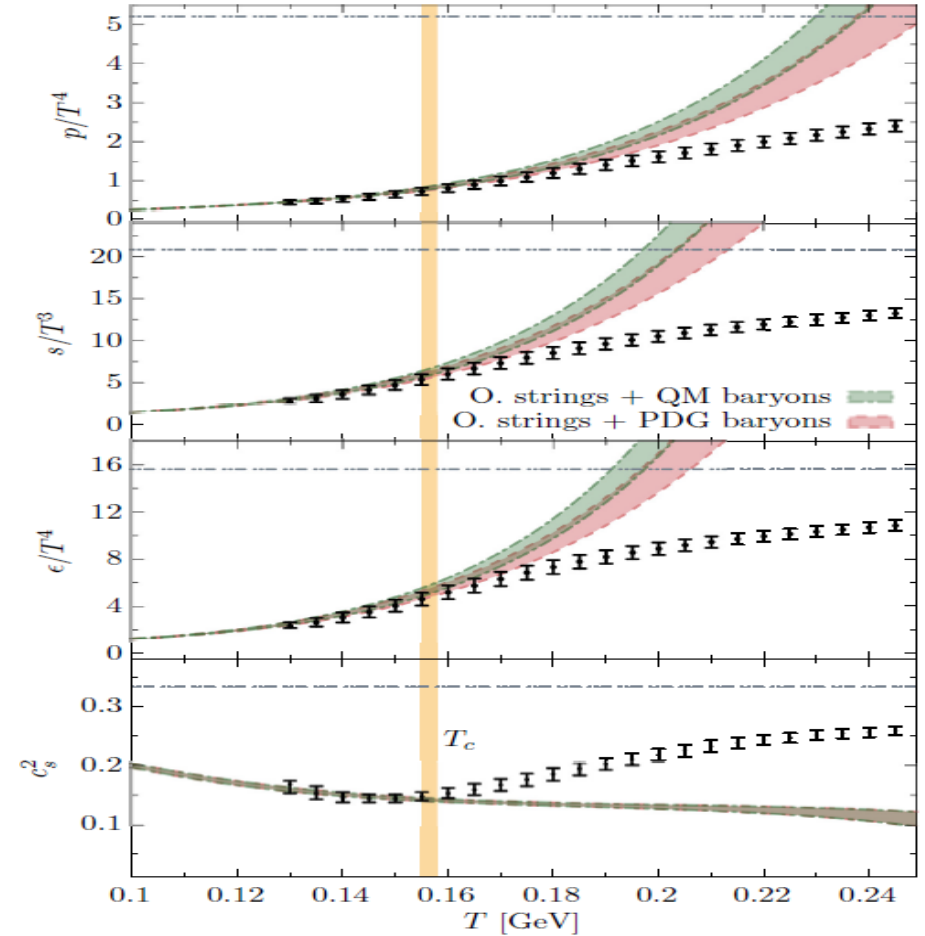
$$\Delta\chi_{BQ} \approx \sum_{I_z, j, B} d_j BQ \int dM \int d^3p \frac{1}{T} \frac{d\delta_j^I}{dM} \times e^{-\beta\sqrt{p^2+M^2}} (1 + e^{-\beta\sqrt{p^2+M^2}})^{-2}$$

A. Andronic, P. Braun-Munzinger, Pok Man Lo, B. Friman, J. Stachel & K.R. Phys. Lett. B 792, 304 (2019)

Cumulants in LQCD and S-matrix HRG model



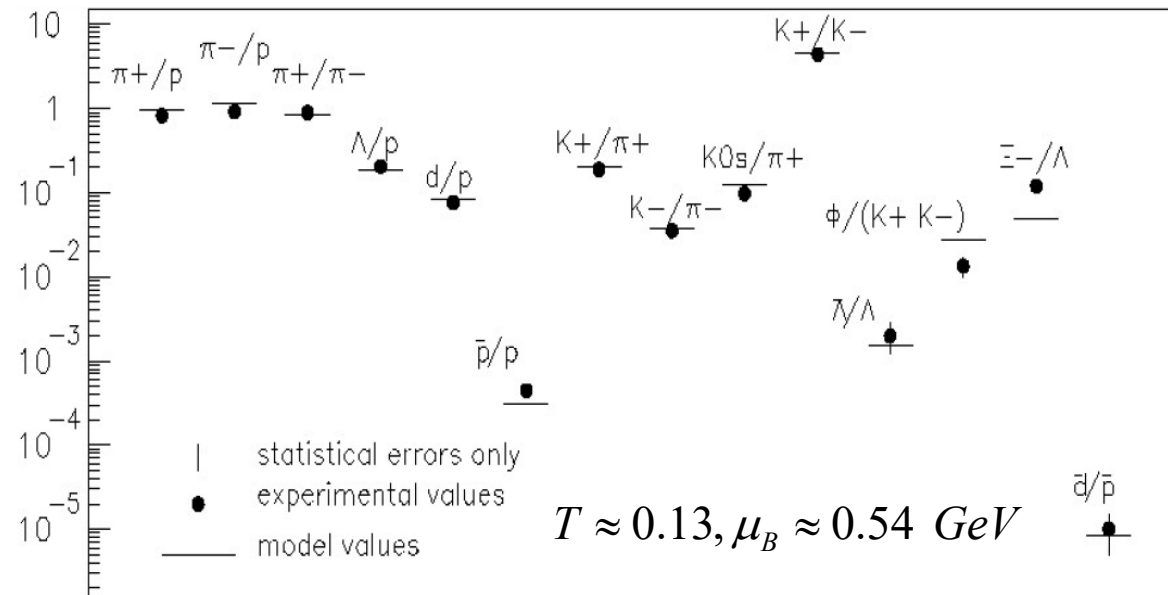
M. Marczenko, L. McLerran, K.R., Phys.Rev.C 114 (2026)
 ____Phys.Rev.D 112 (2025), [2603.28668](#) [hep-ph]



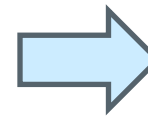
- Good quantitative agreement of HRG with PDG mass spectrum and LQCD in different sectors of quantum numbers. Further improvement with string mass spectrum

Thermalization and freezeout in HIC at the AGS,SPS

With a comprehensive set of 14.6 A GeV/c Si-Au data at BNL AGS in 1993 from E814,E810, E802



P. Braun-Munzinger, J. Stachel, J. P. Wessels, N. Xu,
 PLB 344 (1995) 43 for AGS: PLB 365 (1996) 1 for SPS data



Thermal freezeout at AGS and SPS established with HRG model where:

- The multiplicity of particle type p , in a fireball of volume V at the temperature T and ch. potentials $\vec{\mu}$ is calculated from:

$$\langle N_p \rangle = V \cdot n^{th}(m_p, T, \vec{\mu}) + V \sum_R \Gamma(R \rightarrow p) n^{th}(m_R, T, \vec{\mu})$$

First successful application of thermal hadronisation model to quantify production yields of hadrons with (T, μ_B) -parameters over dynamical range of 6-orders

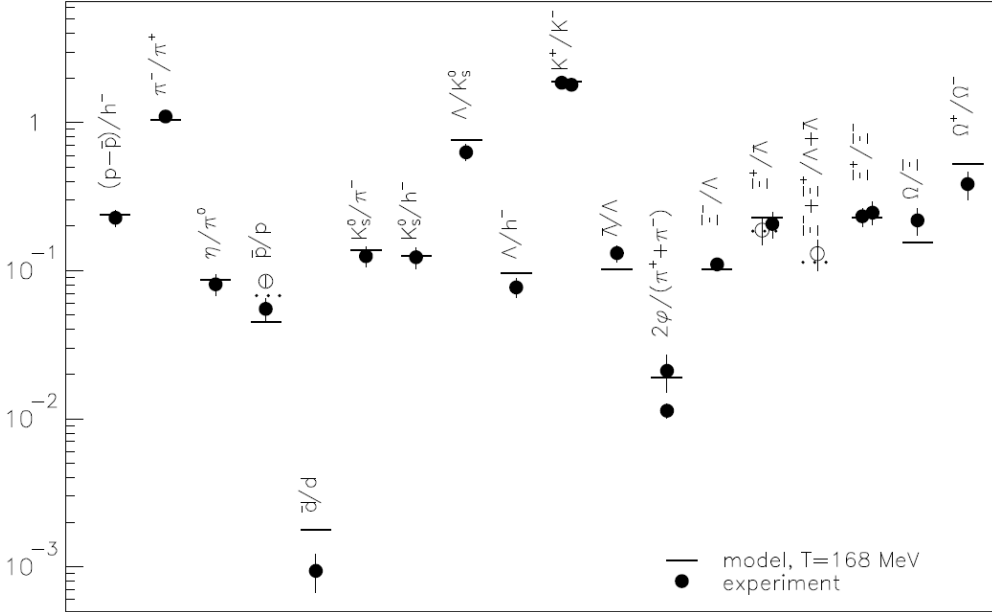
P. Braun-Munzinger, D. Magestro, K. Redlich and J. Stachel,
 Phys. Lett. B518 (2001), 41 (cited 730) RHIC-energy

The same approach showing thermal origin of particle production at SPS energies

P. Braun-Munzinger, I. Heppe, J. Stachel
 PLB 465 (1999), 15 (673 citations)

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With a comprehensive set of 14.6 A GeV/c Si-Au data at BNL AGS in 1993 from E814, E810, E802

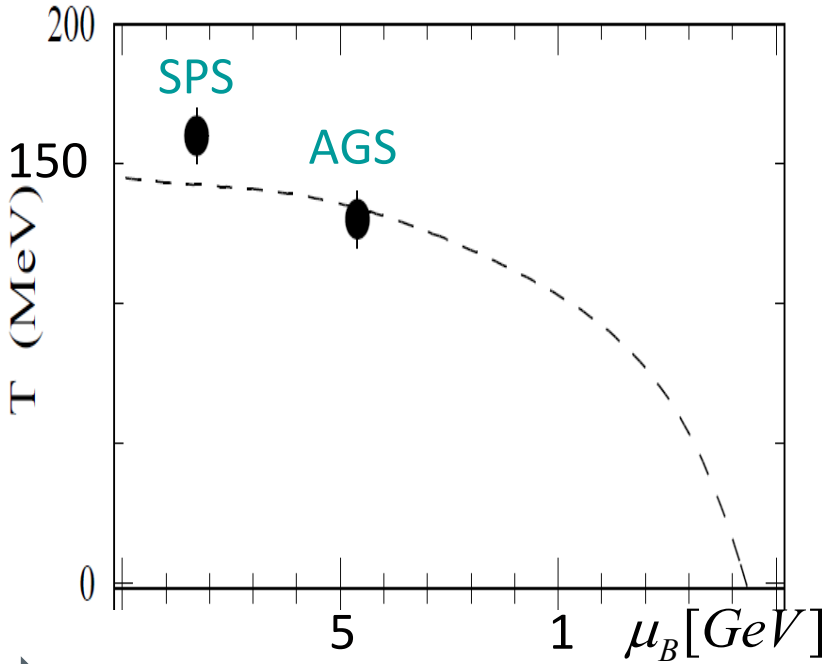


**P. Braun-Munzinger, J. Stachel, J. P. Wessels, N. Xu,
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P. Braun-Munzinger, D. Magestro, K. Redlich and J. Stachel,
Phys. Lett. B518 (2001), 41 (cited 730) RHIC-energy

Freezeout points at SPS & AGS and model phase boundary



Summarized citations 1120

The same approach showing thermal origin of particle production at SPS energies

P. Braun-Munzinger, I. Heppe, J. Stachel
PLB 465 (1999), 15 (673 citations)

Strangeness canonical suppression

- If the number of s-particles is small, then strangeness conservation must be exact

$$Z^{GC}(\mu) = \text{Tr}[e^{-\beta(H-\mu S)}] \Rightarrow Z_S^C = \text{Tr}[e^{-\beta H} \delta_S]$$

$$Z^{GC}(\lambda) = \sum_{S=-\infty}^{\infty} \lambda^S Z_S^C \Rightarrow Z_S^C \simeq \int_{-\pi}^{\pi} d\phi e^{i\phi S} e^{\text{Ln}(Z^{GC}(\mu_s \rightarrow i\phi))}$$

$$\ln Z^{GC} = \sum_{s=-3}^{s=+3} z_s e^{s\mu_s/T}$$

- This implies strangeness suppression effect

$$\langle N_s \rangle_A^C \approx V_A n^{GC} \cdot \frac{I_s(x)}{I_0(x)} \quad \text{with} \quad x = 2V_C \sqrt{z_{s=1} z_{s=-1}}$$

Volume parameters $V_{A(C)} \sim A_{part} \sim dN_{ch} / d\eta$

V_C - volume where S is exactly conserved

V_A - effective fireball volume in the acceptance

The suppression factor $I_s(x) / I_0(x) \leq 1$,
decreases with decreasing x, and increasing
strange s-quantum number of hadron.

R. Hagedor & K.R. *Z.Phys.C* 27 (1985) 541

J. Cleymans, E. Suhonen & K.R.
Z.Phys.C 51 (1991) 137; C76 (1997)

J. Cleymans, H. Oeschler and K.R.
Phys. Rev. C 59 (1999) 1663

Exact result for $\langle N_s \rangle_A^C$
see:

P. Braun-Munzinger, J. Cleymans,
H. Oeschler, K. R. *Nucl.Phys.A* 697
(2002), 902

P. Braun-Munzinger, K.R., J. Stachel
in R. Hwa: "QGP 3",
e-Print: [nucl-th/0304013](https://arxiv.org/abs/nucl-th/0304013) [nucl-th]

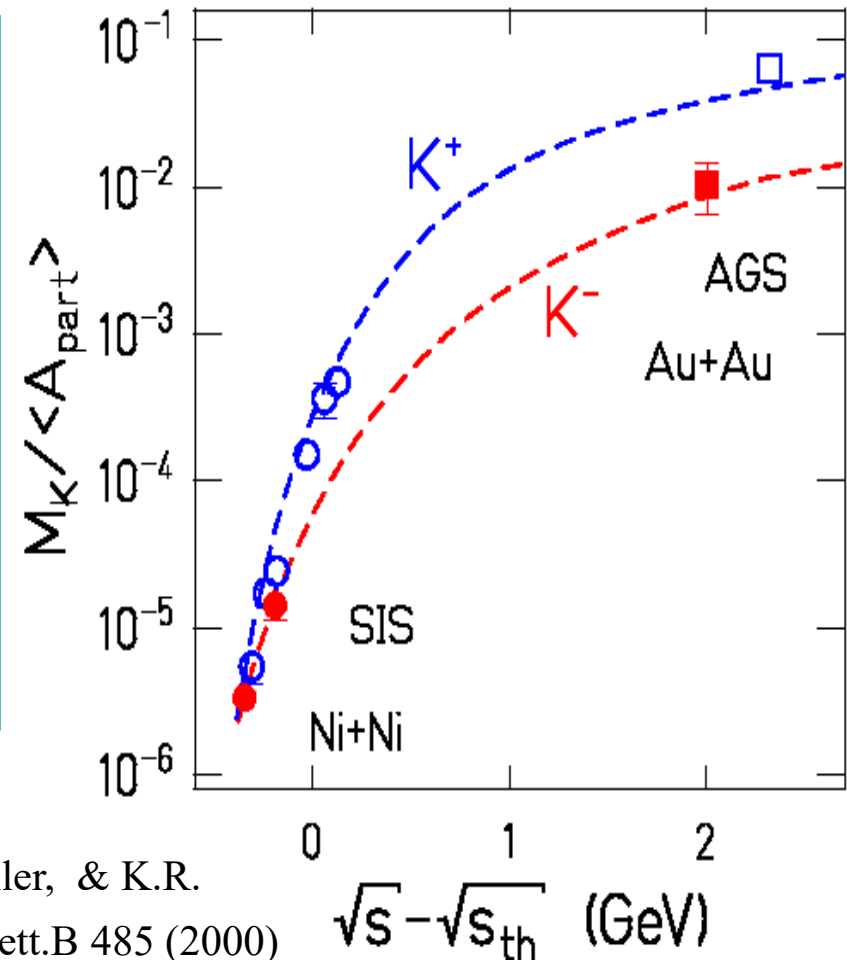
Correlated strangeness production



$$\langle K^+ \rangle^C \sim V_A e^{-m_k/T} V_C e^{-(m_Y - \mu_B)/T}$$

Suppression Factor

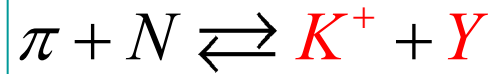
- If the number of produced particles carrying conserved charges is small, then it is essential to describe their thermodynamics in the canonical ensemble with respect to conservation laws



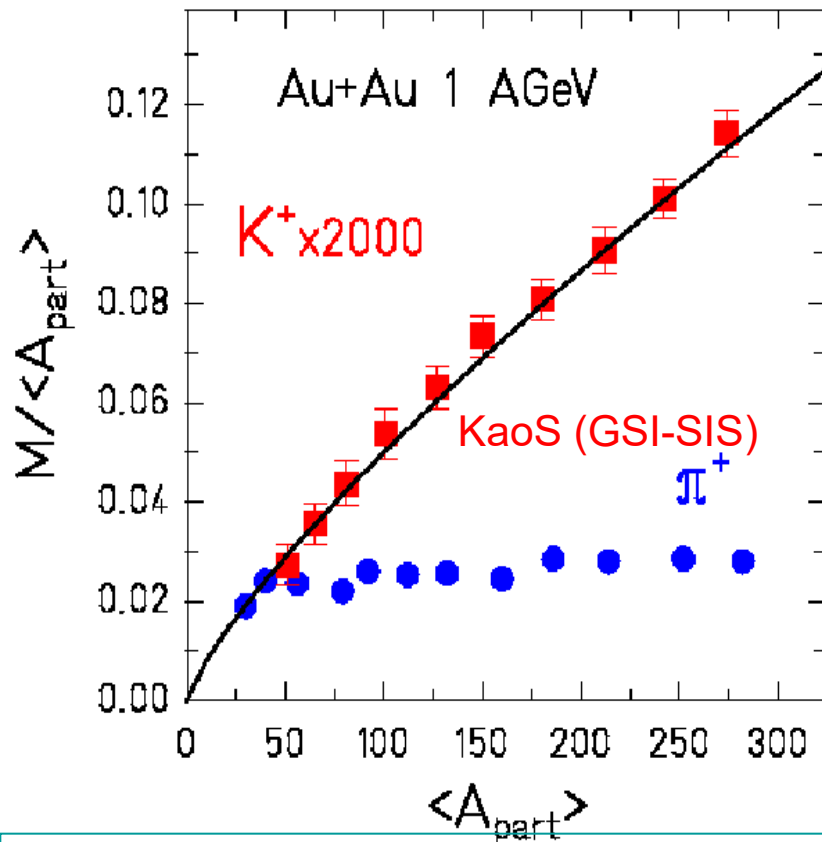
J. Cleymans, H. Oeschler, & K.R.

Phys.Rev.C 59 (1999), Phys.Lett.B 485 (2000)

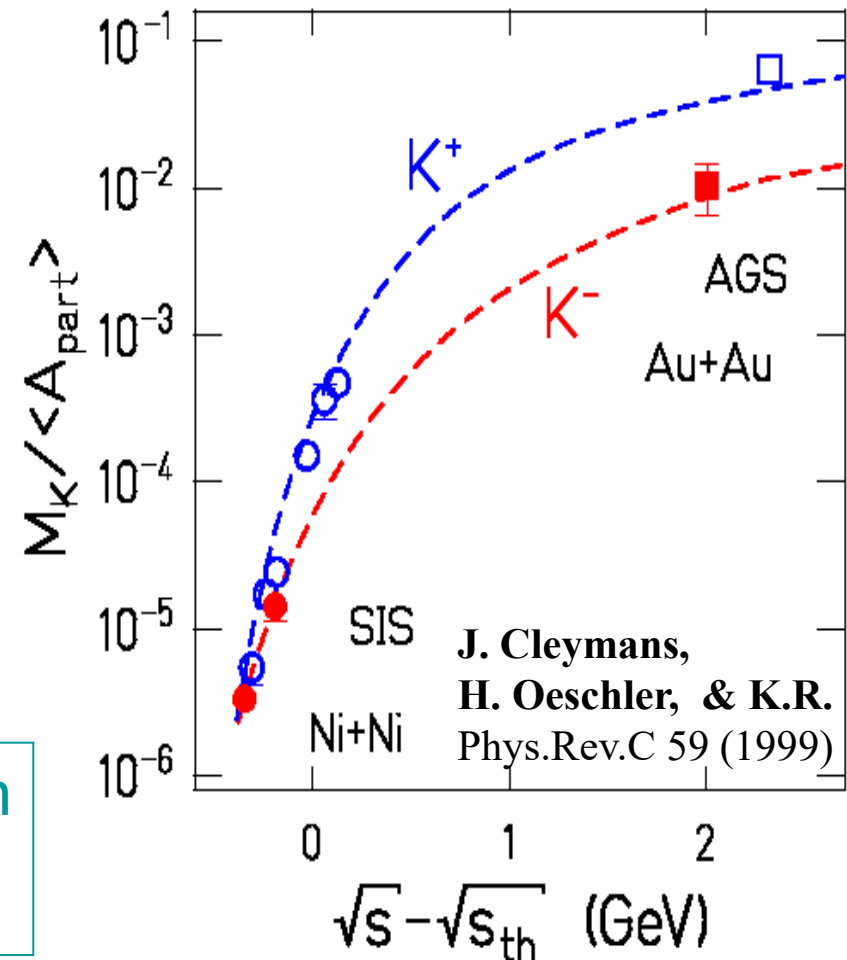
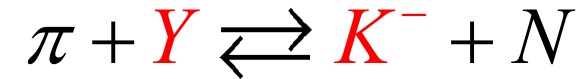
Correlated strangeness production



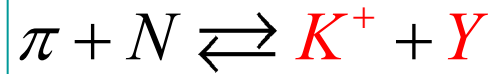
$$\langle K^+ \rangle^C \sim A_{part}^2 e^{-m_k/T} e^{-(m_Y - \mu_B)/T}$$



- Nearly quadratic increase of Kaon yield with a number of participant

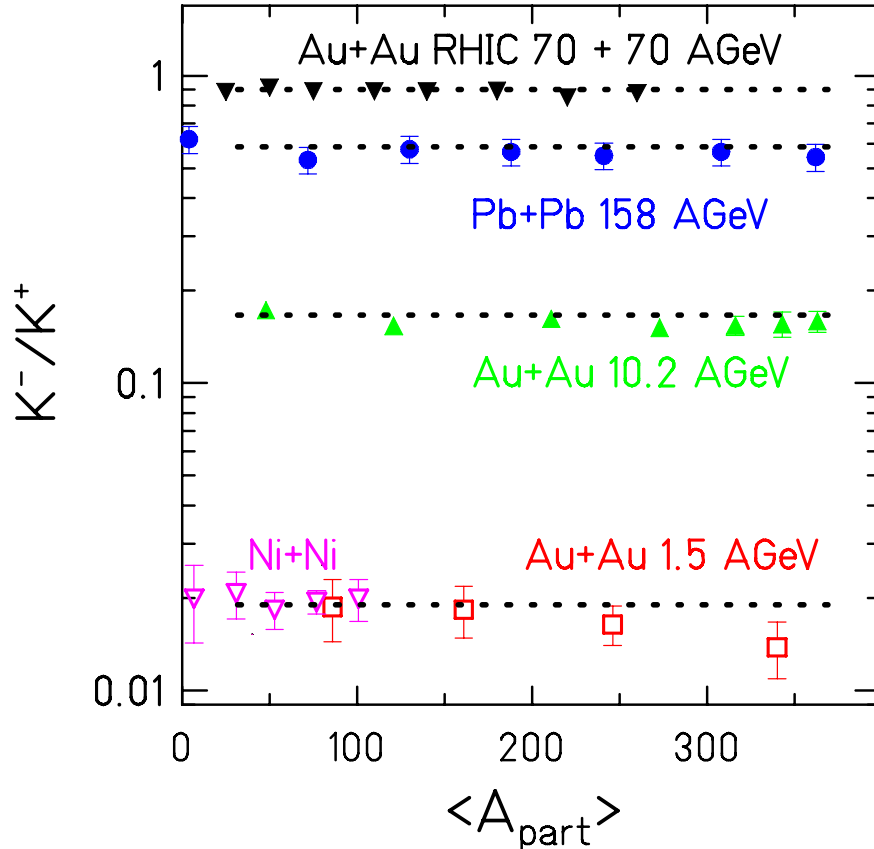


Correlated strangeness production

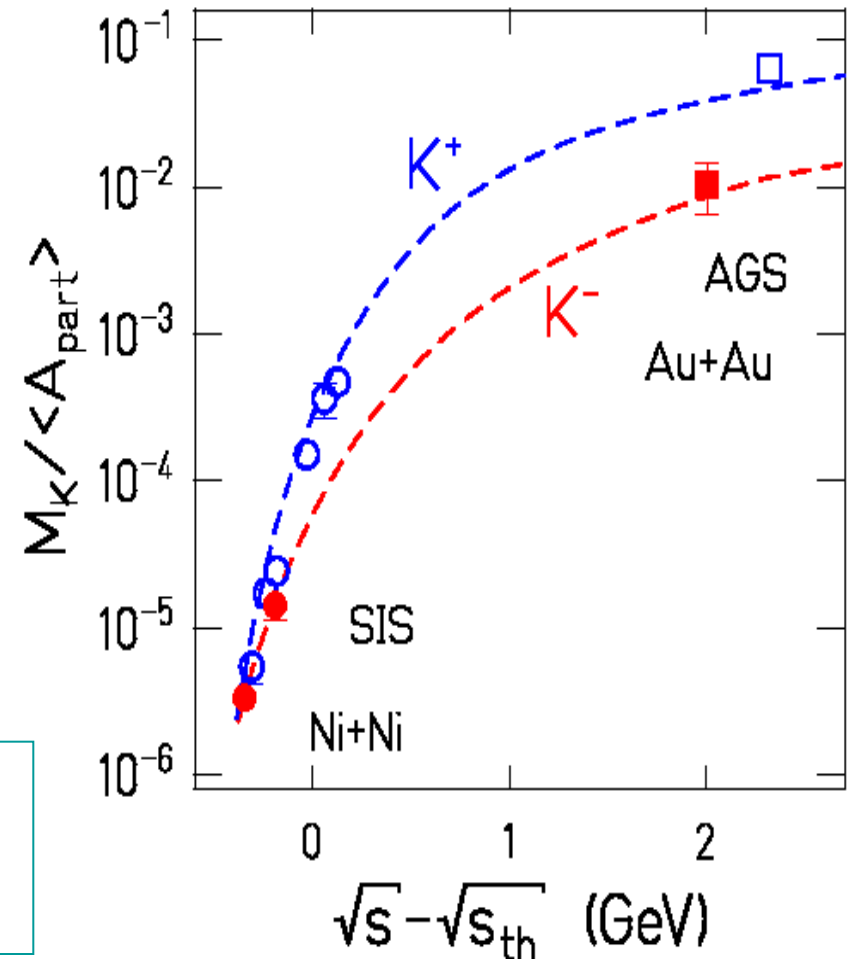


$$\langle K^+ \rangle^C \sim A_{part}^2 e^{-m_k/T} e^{-(m_Y - \mu_B)/T}$$

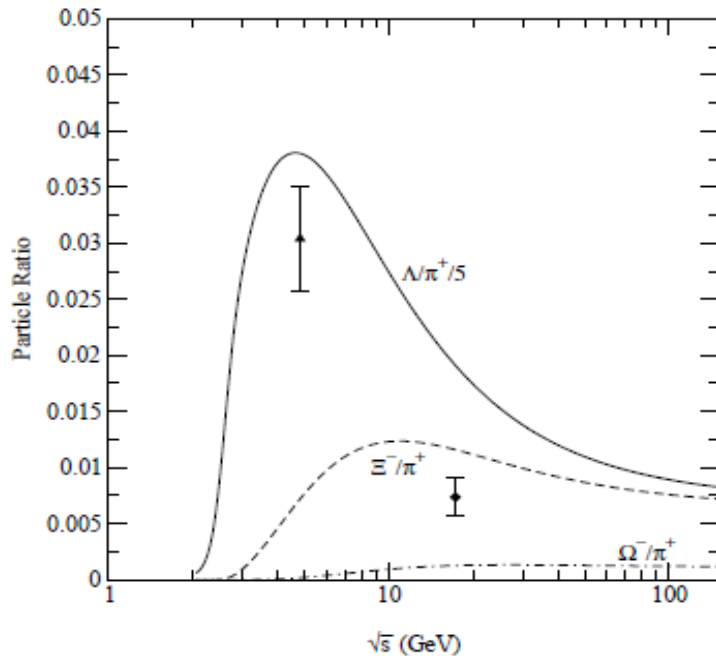
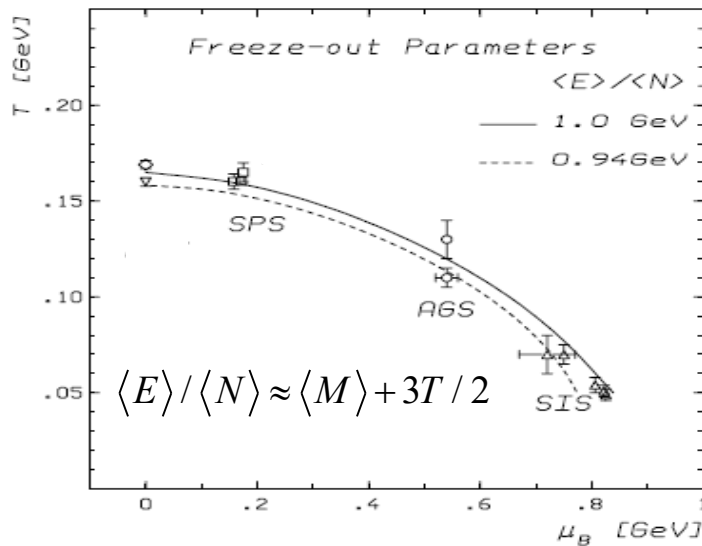
J. Cleymans, H. Oeschler, K.R.
Phys.Lett.B 485 (2000) 27



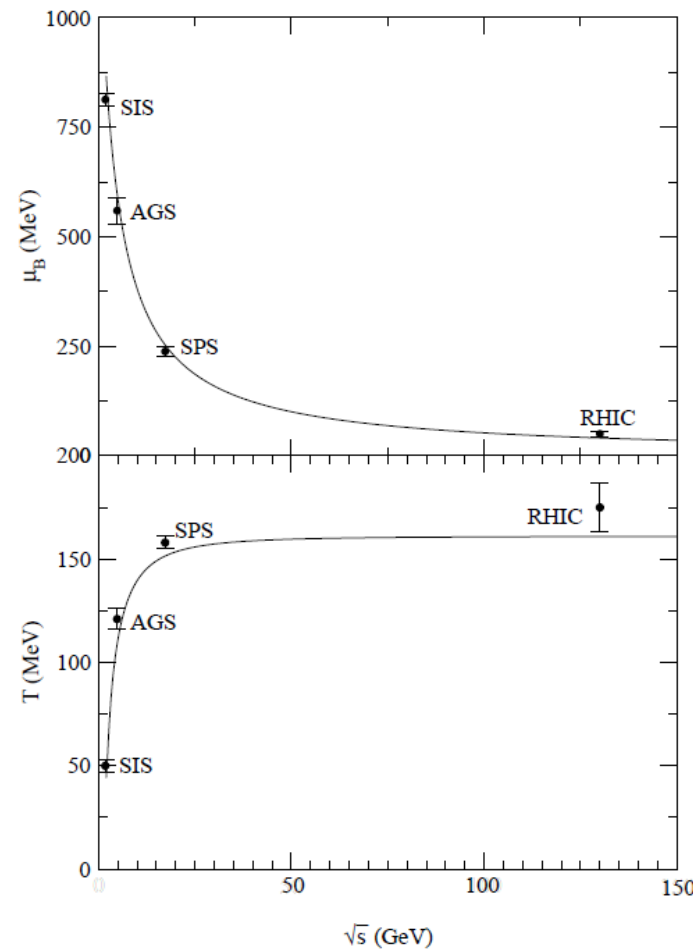
- Ratio independent of the number of participant



Particle production and energy dependence



Freezeout parameters



J. Cleymans, & K.R.
Phys Rev. Lett. 81 (1998)

P. Braun-Munzinger,
J. Cleymans, H. Oeschler,
K. Redlich *Nucl.Phys.A*
697 (2002), 902

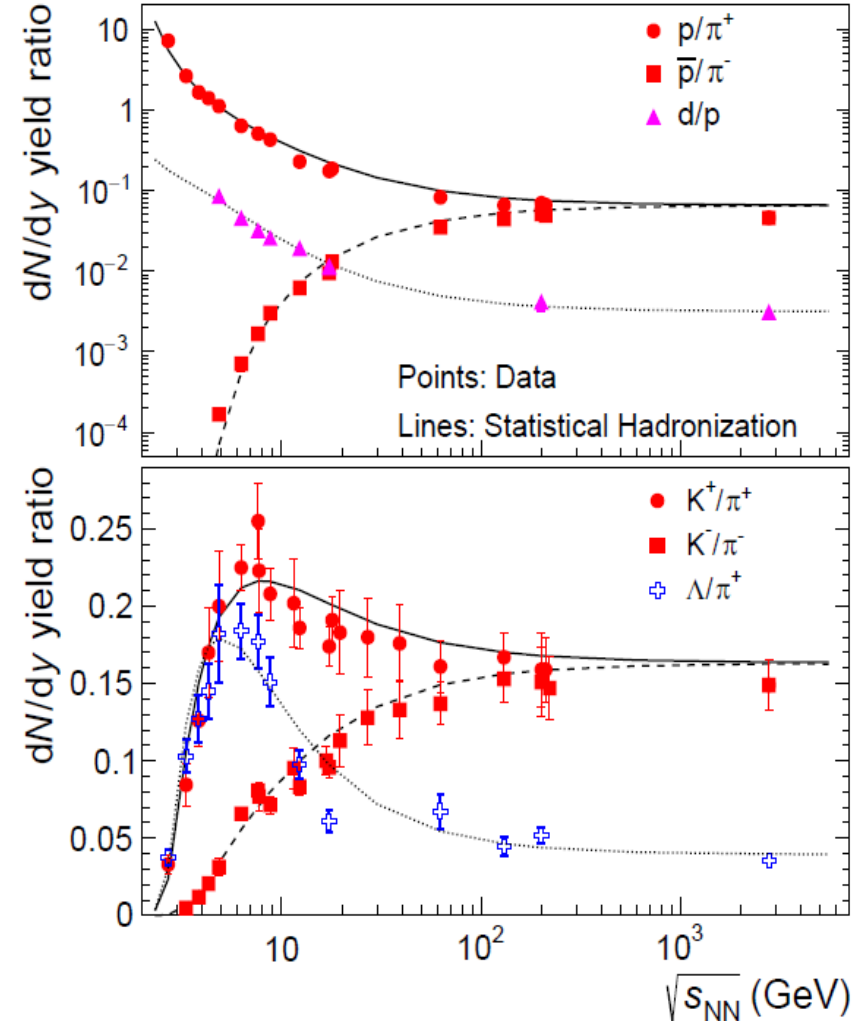
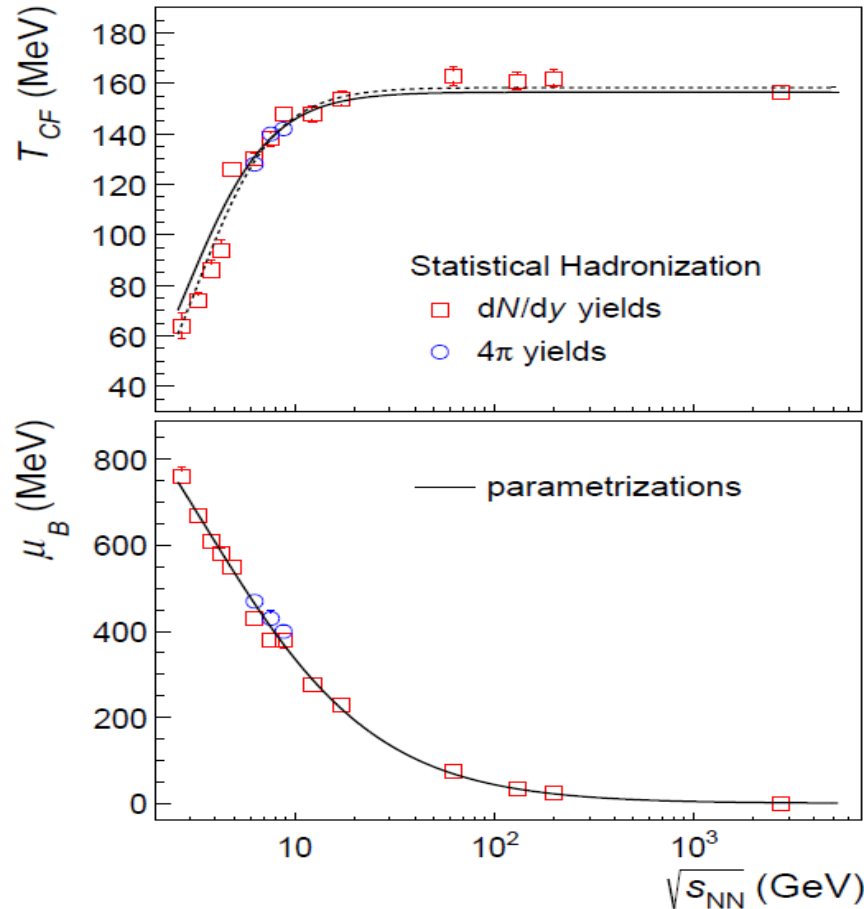
A. Andronic, D. Blaschke,
P. Braun-Munzinger,
J. Cleymans, et al.,
Nucl.Phys.A 837 (2010) 65

J. Cleymans, H. Oeschler, K.R.
Phys.Lett.B 485 (2000) 27

Thermal origin of light flavors in HIC: Beam Energy dependence

A. Andronic, P. Braun-Munzinger, J. Stachel, K.R.

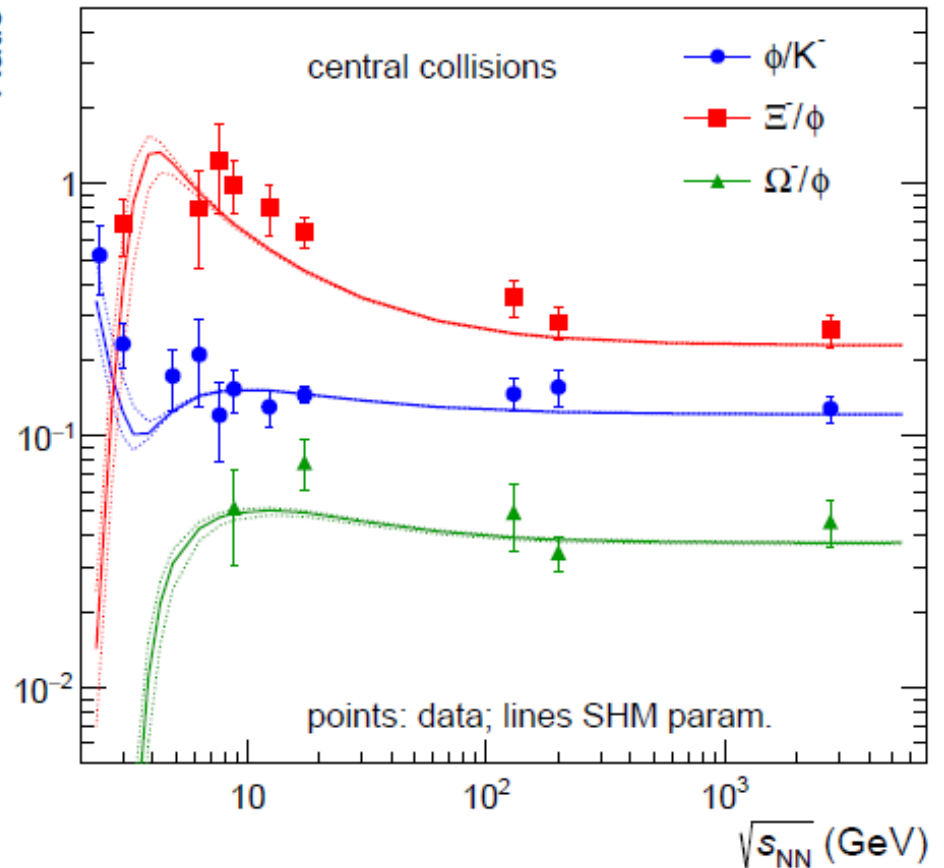
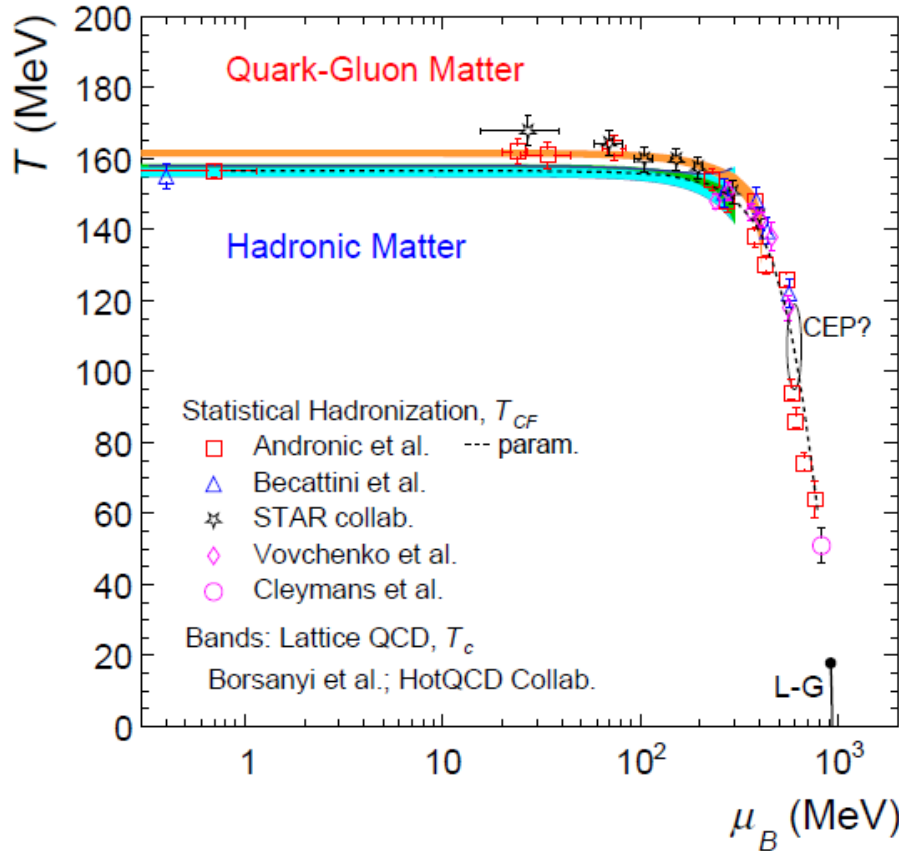
Nature 561 (2018), and [2606.14799 \[nucl-ex\]](#) (2026)



- Hadron yields in heavy-ion collisions from SIS to LHC are consistent with the SHM –
- A. Andronic, P. Braun-Munzinger and J. Stachel Nucl. Phys. A772 (2006) 167 (920 citations)
- A. Andronic, P. Braun-Munzinger and J. Stachel Phys. Lett. B673 (2009) 142 (539 citations)

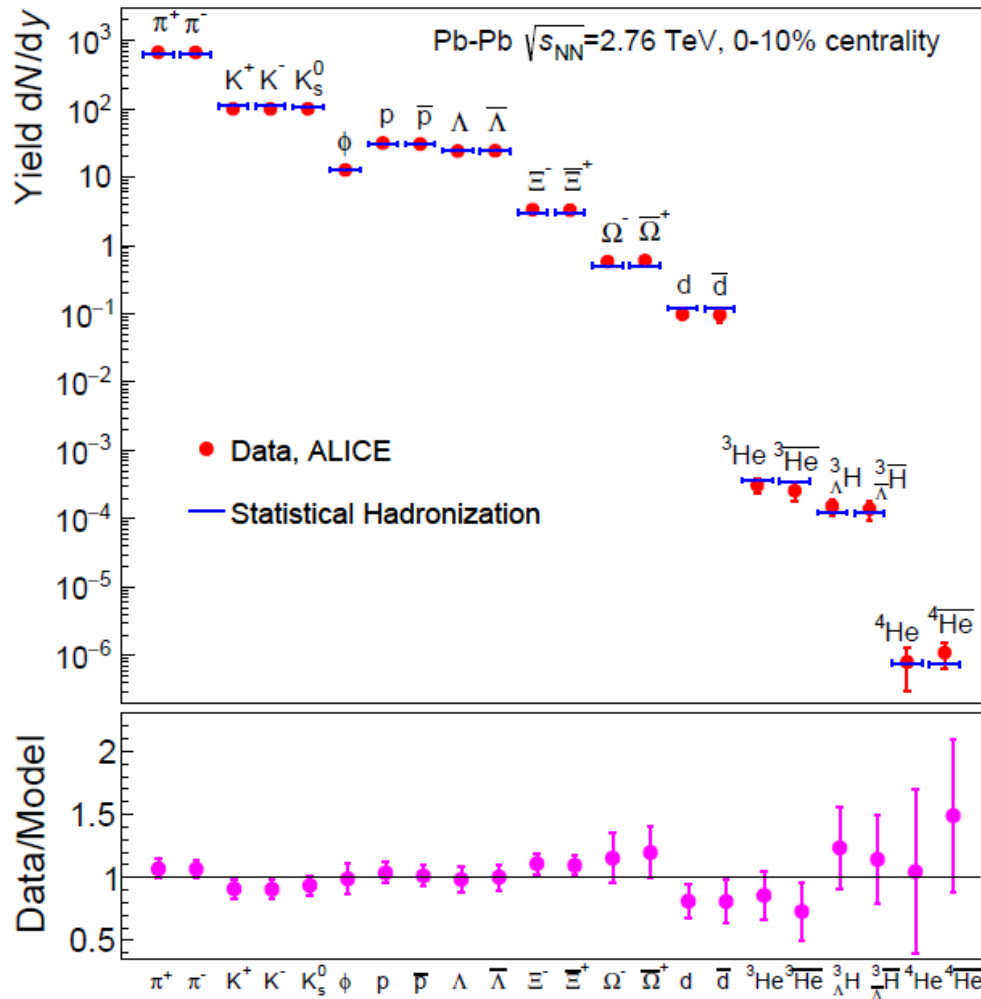
Thermal origin of light flavors in HIC: Beam Energy dependence

A. Andronic, P. Braun-Munzinger, J. Stachel, K.R. e-Print: [2606.14799](#) [nucl-ex] (2026)



- Hadron yields in heavy-ion collisions from SIS to LHC are consistent with the SHM. At and above SPS, particle produced at QCD phase-boundary from hadronizing QGP!
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- A. Andronic, P. Braun-Munzinger and J. Stachel Phys. Lett. B673 (2009) 142 (539 citations)

S-matrix HRG and proton yield puzzle in Pb-Pb collisions at the LHC



The S-matrix HRG model that includes empirical information on pion-nucleon interactions provides a very good description of LQCD results on charge-baryon correlations χ_{QB} and implies a reduction of proton yield towards LHC data $\Rightarrow$ proton puzzle

Excellent synergy:

LQCD $\Leftrightarrow$ HIC yield data

■ Yields reproduced at

$$T = 156.6 \pm 1.7 \text{ MeV}$$

$$\mu = 0.7 \pm 3.8 \text{ MeV}$$

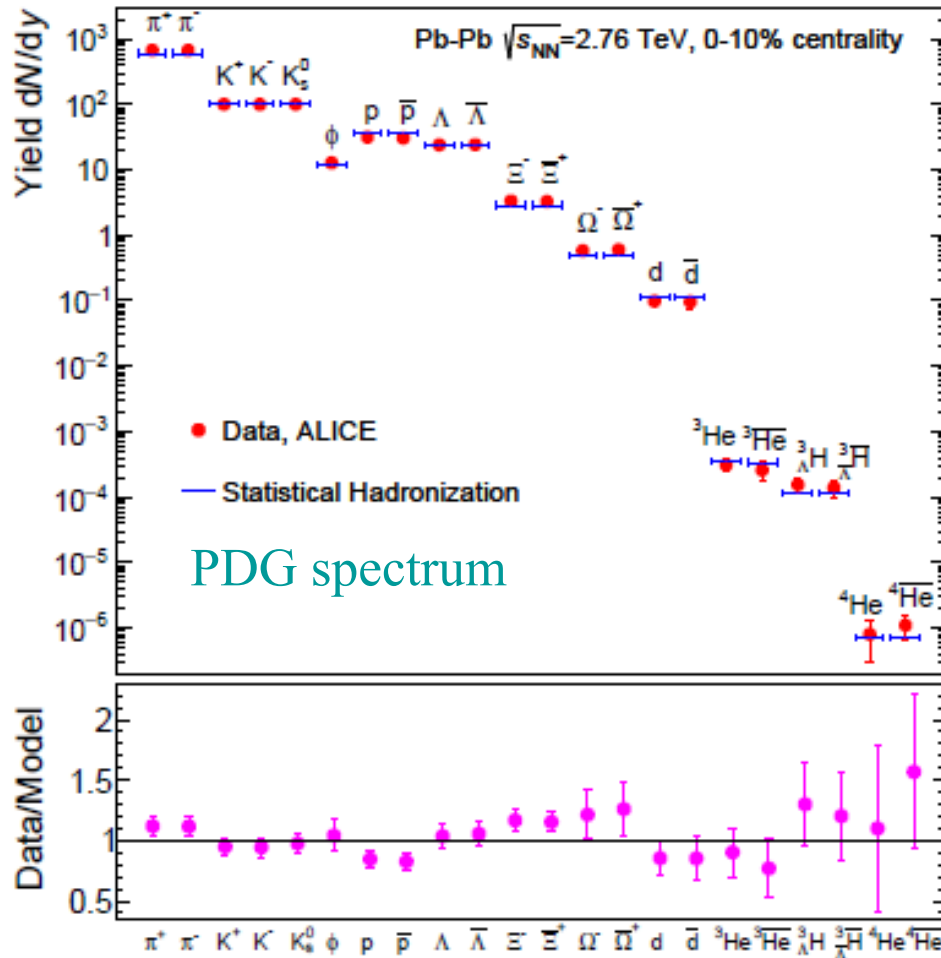
$$V_{\Delta y=1} = 4175 \pm 380 \text{ fm}^3$$

$$\chi^2 / \text{dof} = 16.7 / 19$$

■ A fireball in central Pb-Pb collisions is matter at the QCD phase boundary

A. Andronic, P. Braun-Munzinger, Pok Man Lo, B. Friman, J. Stachel & K.R. Phys. Lett. B 792, 304 (2019)
A. Andronic, P. Braun-Munzinger, J. Stachel & K.R., Nature 561, 302 (2018)

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A. Andronic, P. Braun-Munzinger, J. Stachel & K.R., Nature 561, 302 (2018)

Strangeness canonical suppression with yields of charged particles

- If the number of s-particles is small, then the strangeness conservation must be exact

$$Z^{GC}(\mu) = \text{Tr}[e^{-\beta(H-\mu S)}] \Rightarrow Z_S^C = \text{Tr}[e^{-\beta H} \delta_S]$$

$$Z^{GC}(\lambda) = \sum_{S=-\infty}^{\infty} \lambda^S Z_S^C \Rightarrow Z_S^C \simeq \int_{-\pi}^{\pi} d\phi e^{i\phi S} e^{\ln(Z^{GC}(\mu \rightarrow i\phi))}$$

This implies the strangeness suppression effect

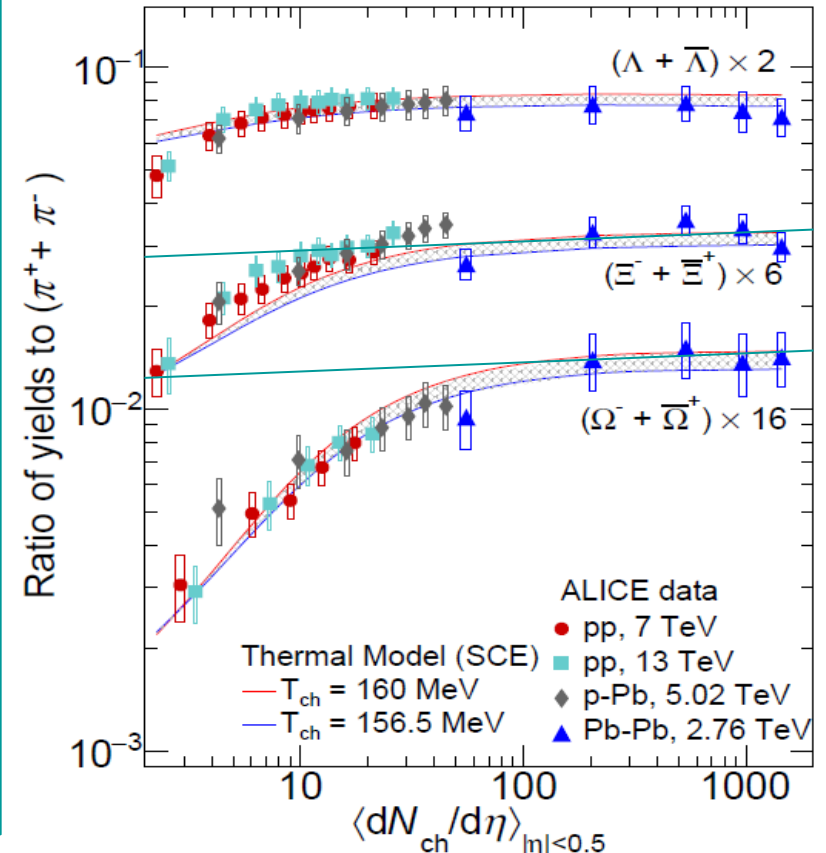
$$\langle N_s \rangle_A^C \approx V_A n^{GC} \cdot \frac{I_s(2V_C n_{s=1}^{th}(T))}{I_0(2V_C n_{s=1}^{th}(T))}$$

volume parameters $V_{A(C)} \sim dN_{ch} / d\eta$

V_C - volume where S is exactly conserved

V_A - effective fireball volume

J. Cleymans, Pok Man Lo, N. Sharma & K.R. Phys. Rev. C103 014904 (2021)



The suppression factor $I_s(x) / I_0(x) \leq 1$ decreases with decreasing x , and increasing strange s-quantum number

J. Cleymans, K.R., E. Suhonen Z. Phys. C51 (1999)

S. Hamieh, K.R., A. Tounsi, Phys.Lett.B486 (2000)

P. Braun-Munzinger, J.Cleymans, H.Oeschler, K.R. Nucl.Phys.A697 (2002)

The thermal origin of charmed-hadrons in HIC

A. Andronic, P. Braun-Munzinger,
J. Stachel & K.R, Nature (2018)

In HRG, yield of J/Psi differs by a huge factor 900 from model expectation:

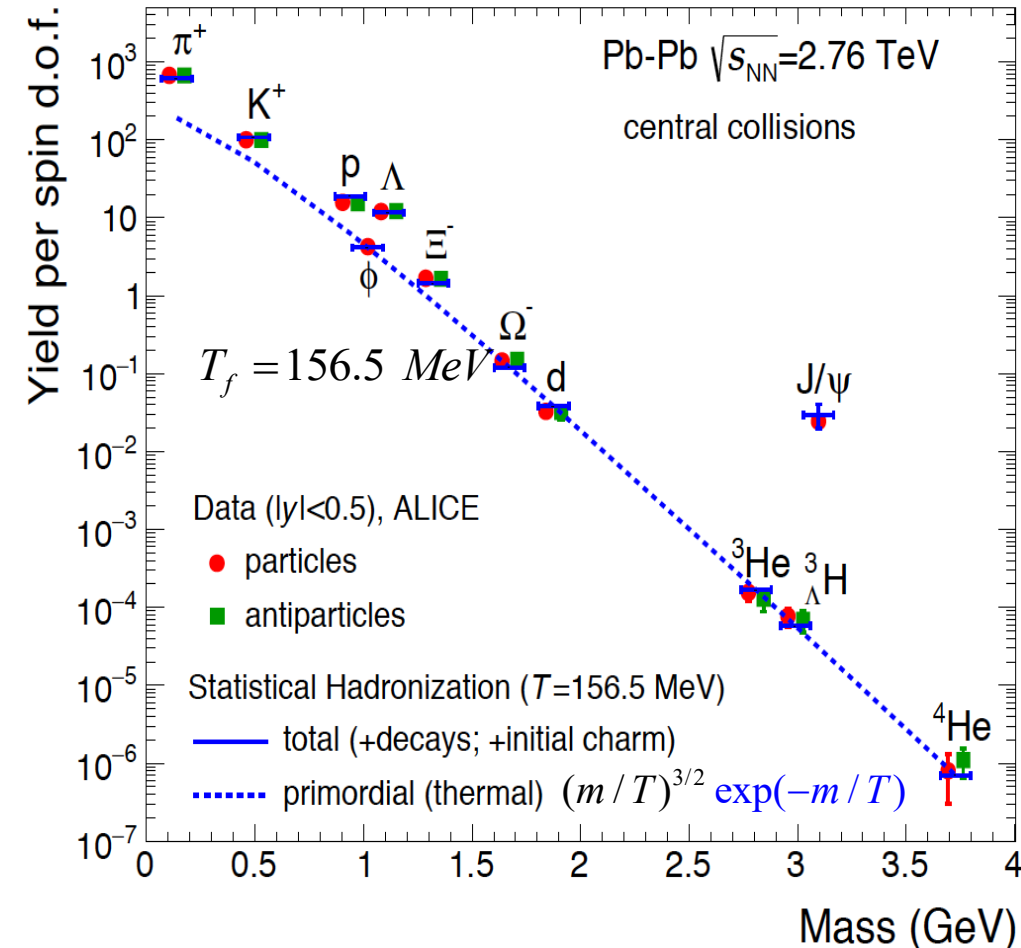
There are too few charm quark pairs in equilibrium QGP at T_c !

The way out: Statistical Hadronization Model of Charm (SHMc) introduced by:

**Peter Braun-Munzinger & Johanna Stachel
Phys. Lett. B490 (2000) 196.**

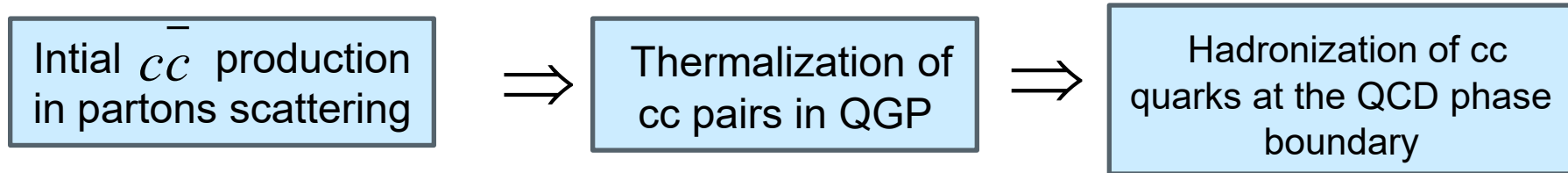
See also:

A. Andronic, P. Braun-Munzinger, J. Stachel & K.R,
NPA 789 (2007) 334, *Phys.Lett.B* 652 (2007) 259



SHMc charm statistical hadronization: Yields and Spectra of Open Charmed Hadrons

Peter Braun-Munzinger & Johanna Stachel Phys. Lett. B490 (2000) 196.



Formation of $c\bar{c}$ pairs in hard initial scattering comparable to QGP formation

Charm quarks as external source in QGP: annihilation and production of charm quarks in QGP negligible

Kinetic calculations: P. Braun-Munzinger and K.R.,
Eur. Phys.J.C 16 (2000) 519

Charm quarks thermalize inside QGP: strong evidence through observed elliptic flow, and energy loss (BDMPS)

⇒ Justifying application of charm statistical hadronization

- The final number of charm-anticharm quark pairs bound in the produced hadrons is the same as in the initial state

SHMc: linking initial with final state

- The charm balance equation to preserve N_{cc} quark pairs

$$2N_{cc} = g_c V \sum_{h_{oc,1}^i} n_i^{th} + g_c^2 V \sum_{h_{oc,2}^i} n_i^{th} + g_c^2 V \sum_{h_{hc}^i} n_i^{th} \quad \text{with } n_i^{th} \simeq d_{i,J} m_i^2 TK_2(m_i / T)$$

obtained from measured
charm cross section
or from QCD+Glauber

thermal charm hadrons

- In general for small N one needs to include canonical suppression

$$2N_{cc} \simeq \sum_{\alpha=1,2} N_{oc,\alpha} \frac{I_\alpha(N_{oc,1})}{I_0(N_{oc,1})} + N_{hc} \quad \text{defining:} \quad \begin{aligned} N_{oc,1} &= V g_c \sum_{h_{oc,1}^i} n_i^{th} \\ N_{oc,2} &= V g_c^2 \sum_{h_{oc,2}^i} n_i^{th} \\ N_{hc} &= V g_c^2 \sum_{h_{hc}^i} n_i^{th} \end{aligned}$$

- For a given N_{cc} and knowing T, V from thermal analysis of light hadron yields:

➡ solve the above balance equation to get g_c , consequently

Heavy flavor particle yields in SHMc

- The rapidity density of open and hidden charm hadrons in SHMc:

$$\frac{dN_{i,\alpha=1,2}}{dy} = g_c^\alpha V n_i^{th} \cdot \frac{I_\alpha(2V g_c \sqrt{n_{c=1}^{tot} n_{c=-1}^{tot}})}{I_0(2V g_c \sqrt{n_{c=1}^{tot} n_{c=-1}^{tot}})}$$

Total thermal density of $c = \pm 1$

$$n_{c=\pm 1}^{tot} = \sum_k n_{k,c=\pm 1}^{th}$$

contribution of resonances:

$$n_i^{th} = n_i^{prompt} + \sum_j Br(j \rightarrow i) n_j^r$$

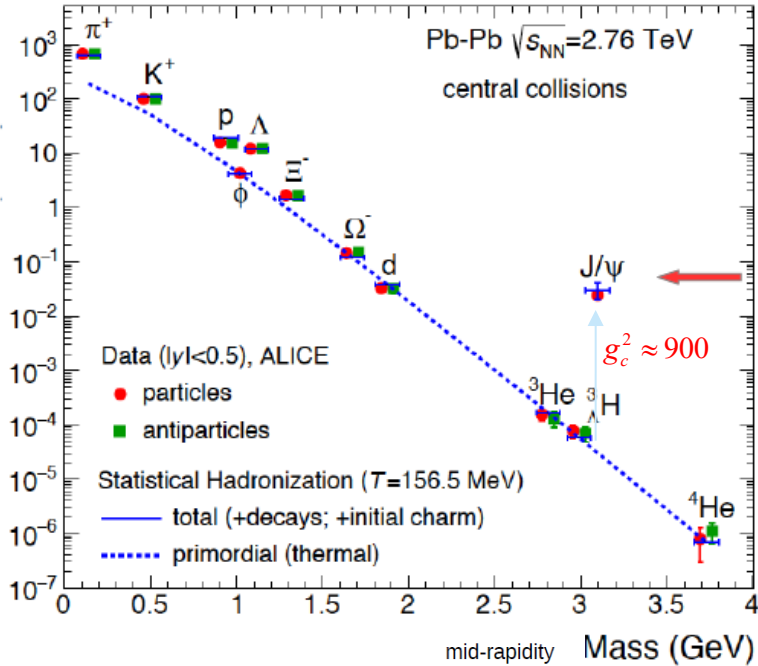
$$\frac{dN_{i,hc}}{dy} = g_c^2 V n_i^{th}$$

- The essential difference with light particle thermal yields is due to the fugacity factor $g_c = g_c(N_{cc}, V, T, \vec{\mu})$ which guarantees conservation of N_{cc} pairs from the initial partonic to the final hadronic state.

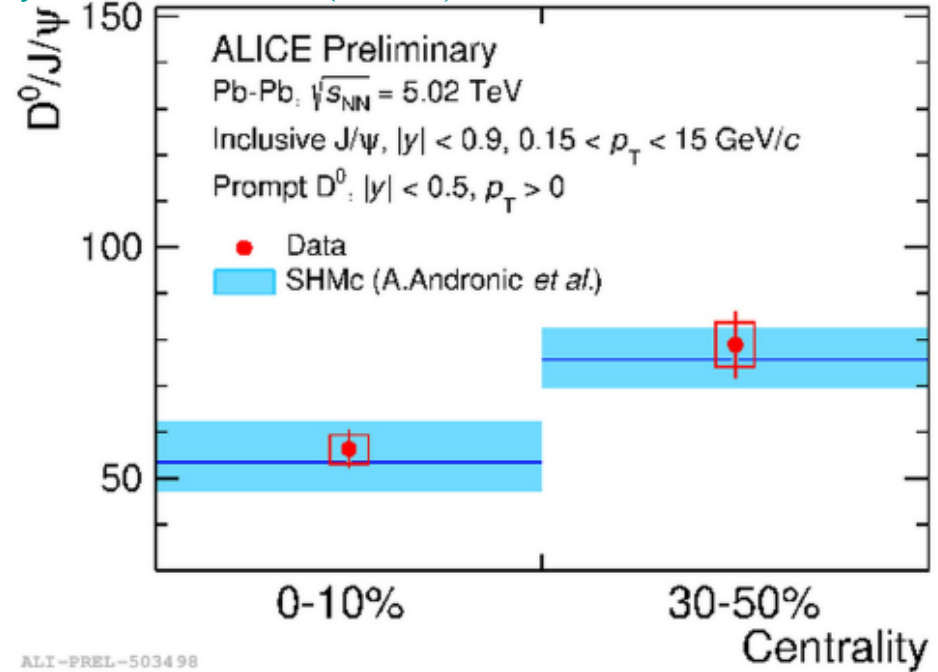
Model comparison with ALICE data

Phys.Lett.B 849 (2024) 138451 arXiv:2303.13361

A.Andronic et al., PLB 797 (2019) 134836

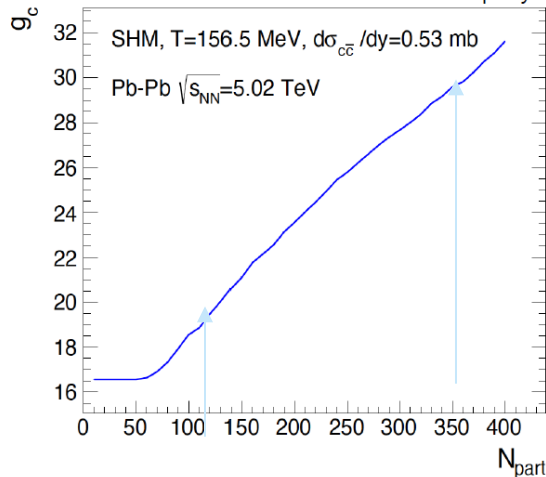


ALI-PREL-503498



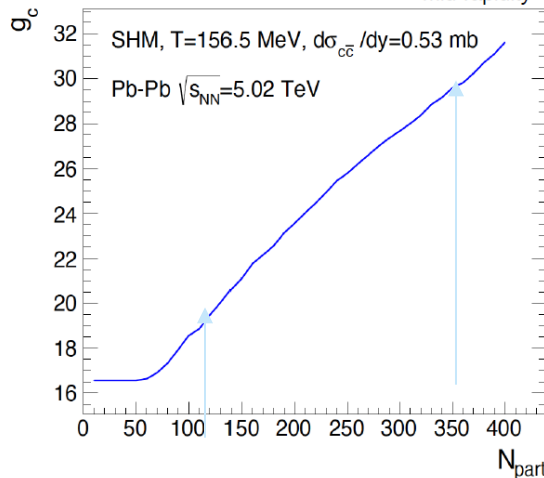
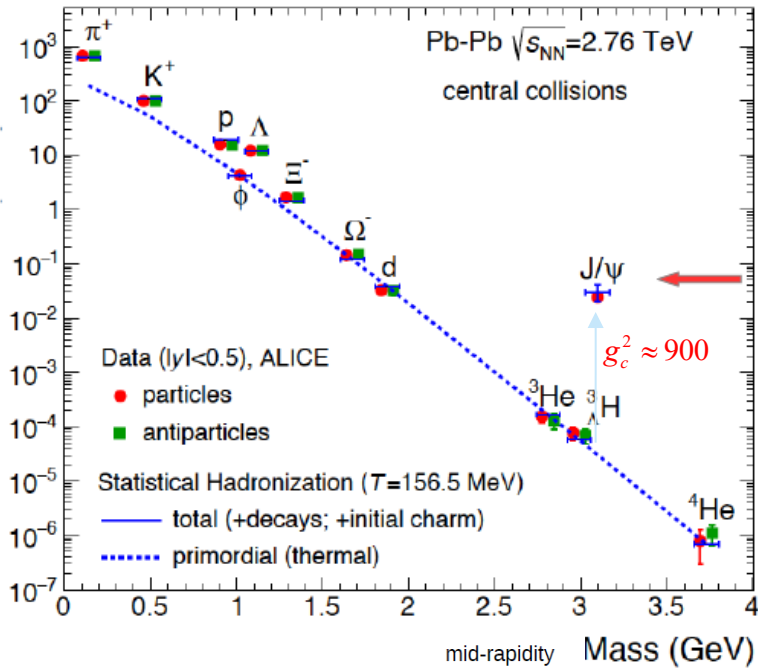
- Strong increase of J/ψ yields due to off chemical equilibrium factor $g_c^2 \approx 900$ in central collisions
- First data on $D^0 / (J/\psi)$ and centrality dependence well described by SHMc. Increasing ratio towards non central collisions due to decreasing g_c since:

$$D^0 / (J/\psi) \sim f(T)(I_1 / I_0) g_c^{-1}$$

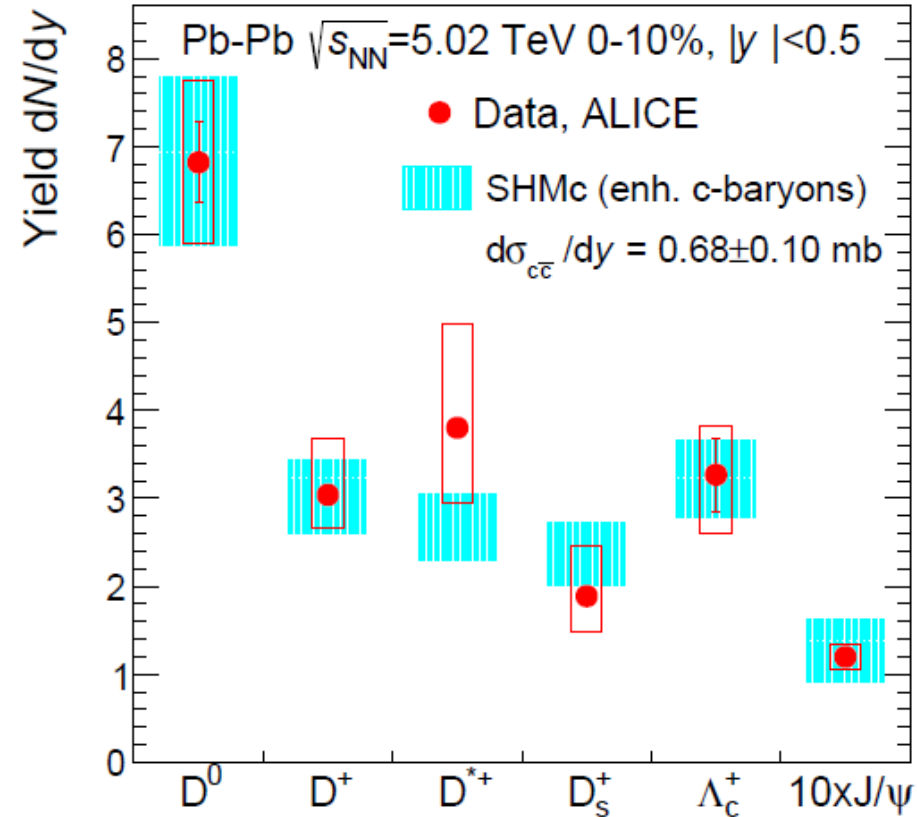


Model comparison with ALICE data

A.Andronic et al., PLB 797 (2019) 134836

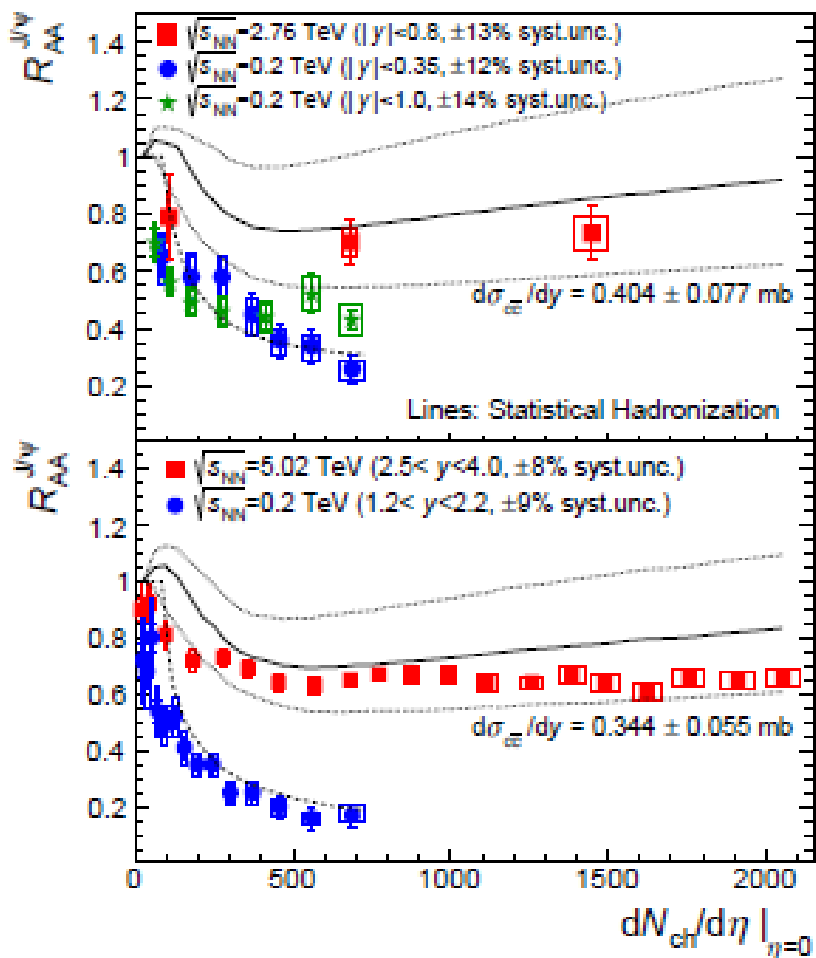


A. Andronic, P. Braun-Munzinger, J. Stachel, K.R.
e-Print: [2606.14799](https://arxiv.org/abs/2606.14799) [nucl-ex] (2026)

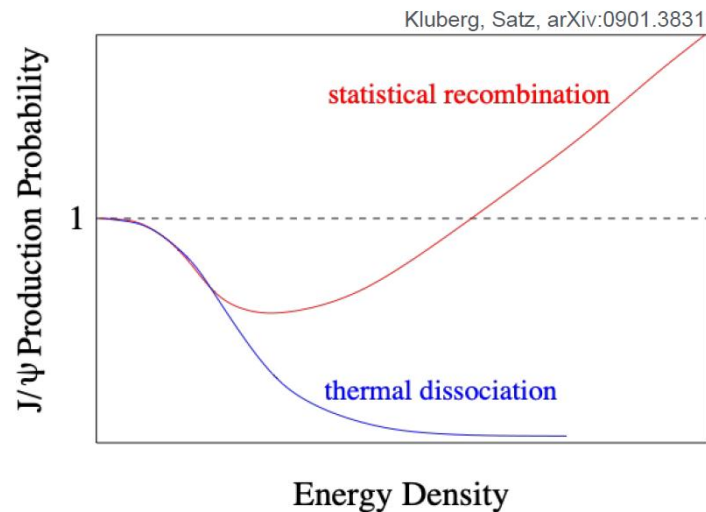


- Very good agreement of SHMc model results and ALICE data for open and hidden charm hadrons
- Charm hadrons produced at the QCD phase boundary as all other particles

Essential predictions of SHMc: J/Psi suppression



Predictions successfully applied to quantify SPS, RHIC and LHC data: **Regeneration of J/Psi at the QCD phase boundary and increasing number of Ncc with energy imply that J/Psi suppression observed at SPS decreases with increasing energy towards LHC**



Melting scenario at the LHC not observed

Emergence of Systematics of Open Charm production at High Energy Collisions

- Consider SHMc balance equation and its approximation:

$$2N_{cc} \simeq Vg_c \frac{I_1(N_{oc,1}(g_c, V))}{I_0(N_{oc,1})} n_{oc,1}^{tot} + Vg_c^2 \frac{I_2(N_{oc,1}(g_c, V))}{I_0(N_{oc,1})} n_{oc,2}^{tot} + Vg_c^2 n_{hc}$$

However, contributions of $c = \pm 2$ and hidden charm hadrons less than 3%, thus can be neglected to extract g_c

- Consequently:

$$Vg_c \frac{I_1(N_{oc,1})}{I_0(N_{oc,1})} \approx \frac{2N_{cc}}{n_{oc,1}^{tot}}$$

and since

$$\frac{dN_{i,c=\pm 1}}{dy} = Vg_c \frac{I_1(N_{oc,1})}{I_0(N_{oc,1})} n_i^{th}$$

- the open charm hadron rapidity density with $c = \pm 1$,

$$\frac{dN_{i,c=\pm 1}}{dy} \simeq 2 \frac{n_i^{th}(T)}{n_{oc,1}^{tot}(T)} N_{cc}$$

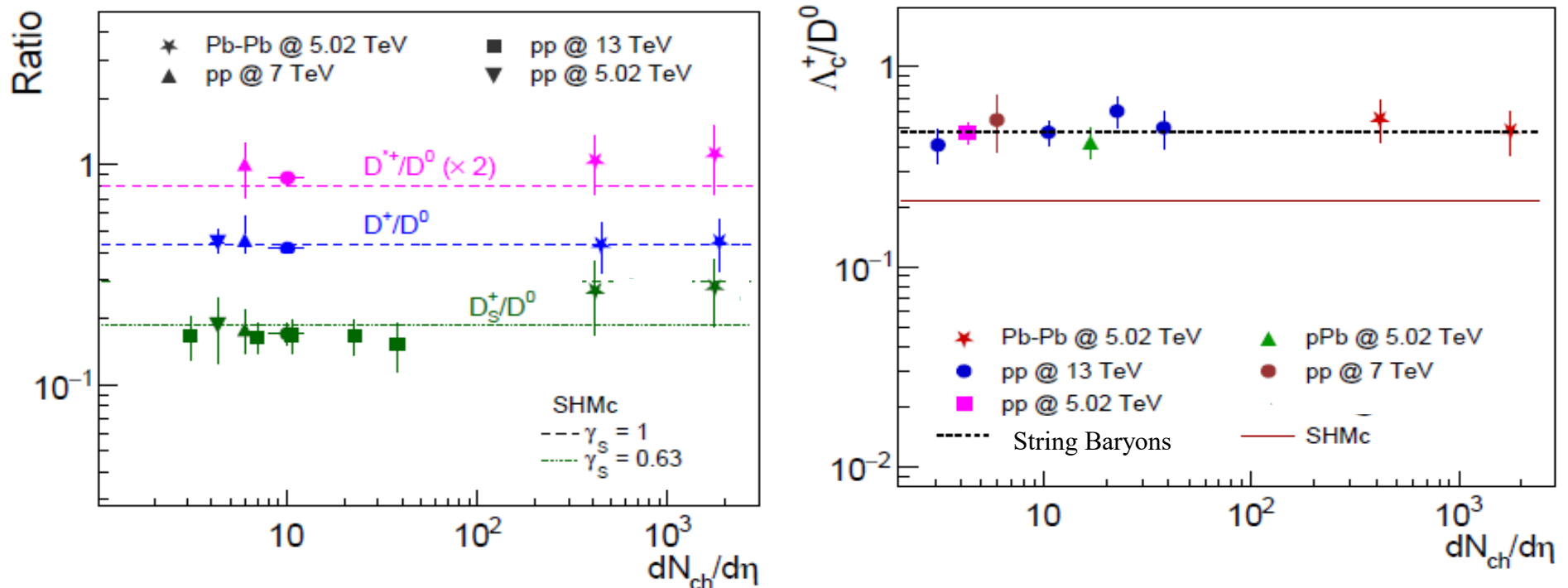
$$n_i^{th} = n_i^{prompt} + \sum_j Br(j \rightarrow i) n_j^r$$

$$n_{c=\pm 1}^{tot} = \sum_k n_{k,c=\pm 1}^{th}$$

is fully determined by N_{cc} (from experiment or model) and the temperature, which at high energy is just chiral crossover $T = T_c = 156.5$ MeV.

Basic properties for different open charm yield ratios

$$N_{i,c=\pm 1} / N_{k,c=\pm 1} = n_i^{th}(T) / n_k^{th}(T) \quad \text{P. Braun-Munzinger, K.R. , N. Sharma, J. Stachel, JHEP 04 (2025)}$$



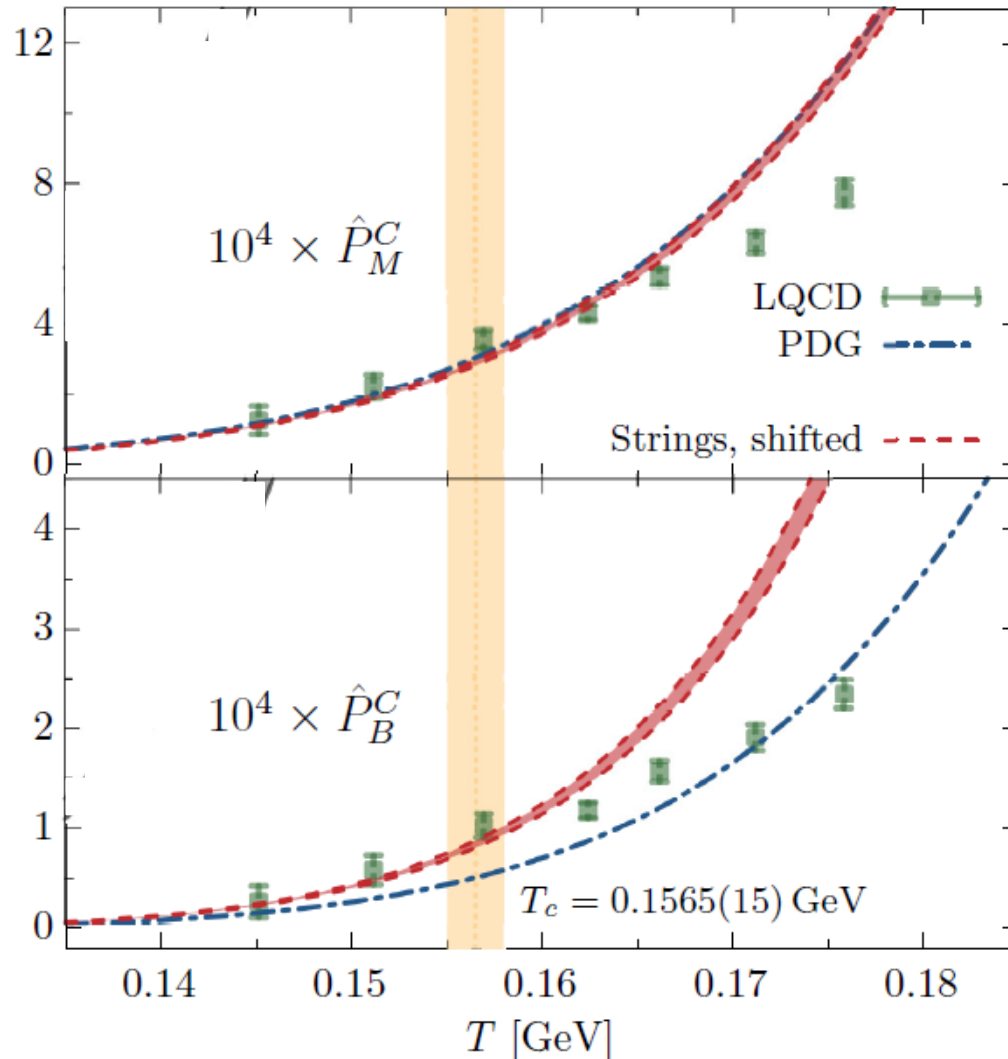
Ratios in pp, pA and AA collisions are within uncertainties the same and independent of associated charged particle pseudo-rapidity density, as expected in SHM.

An increase of D_s^+ / D^0 from pp to AA is possible and needs more data. Quantitative agreement of D^{*+} and D^+ to D^0 ratios with SHMc, however suppression of D_s^+ / D^0 from AA to pp. Λ_c^+ / D^0 Larger by a factor 2.2 ± 0.15 than SHMc =>

Missing Baryon-Resonances in charm-baryon spectrum

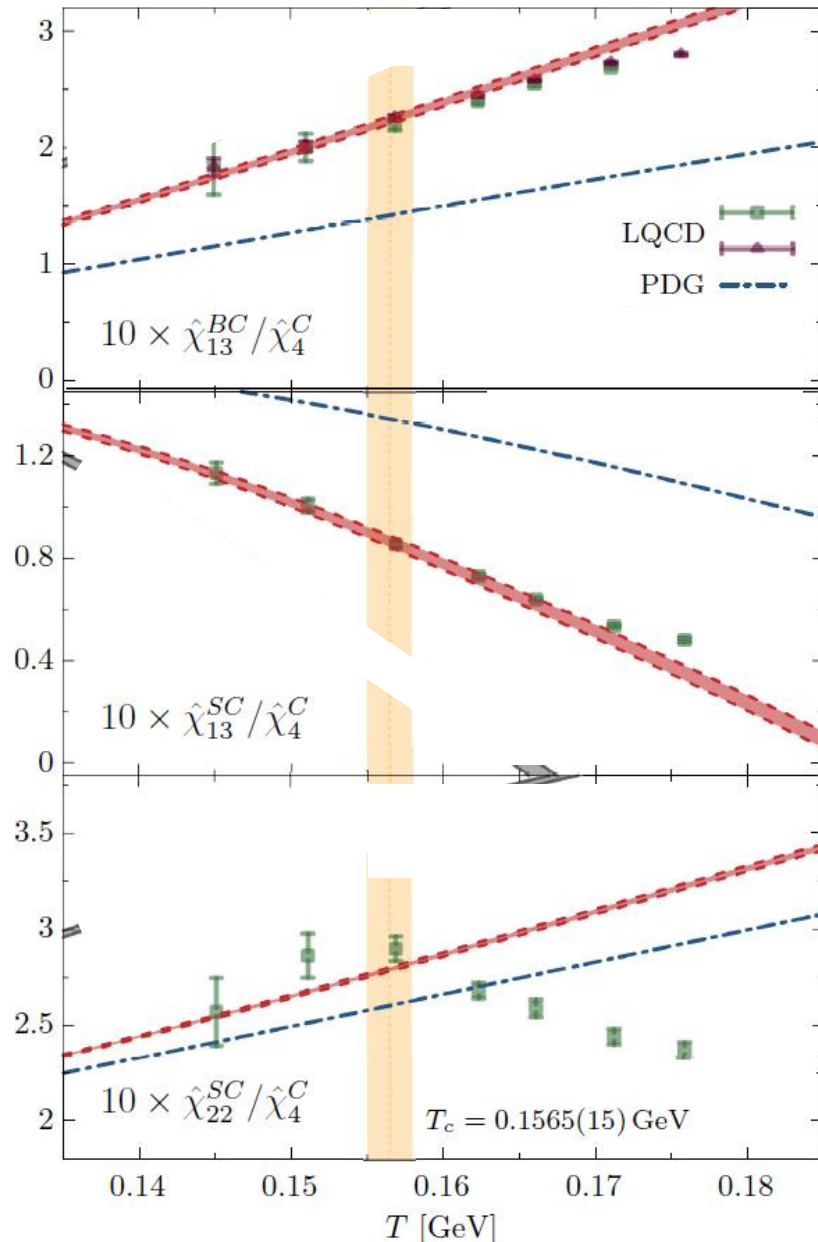
LQCD: Olaf Kaczmarek, Frithjof Karsch, Peter Petreczky, Christian Schmidt, Sipaz Sharma, PRD (2025)

M. Marczenko, L. McLerran and K.R.:
(2026) hep-ph/ 2603.28668



- The PDG charm-meson spectrum is consistent with LQCD
- In the charm-baryon sector, there are missing resonance states.
- The QM or the string mass spectrum increases the pressure by a factor of 1.9 at T_c towards LQCD

M. Marczenko, L. McLerran and K.R.
(2026) hep-ph/ 2603.28668



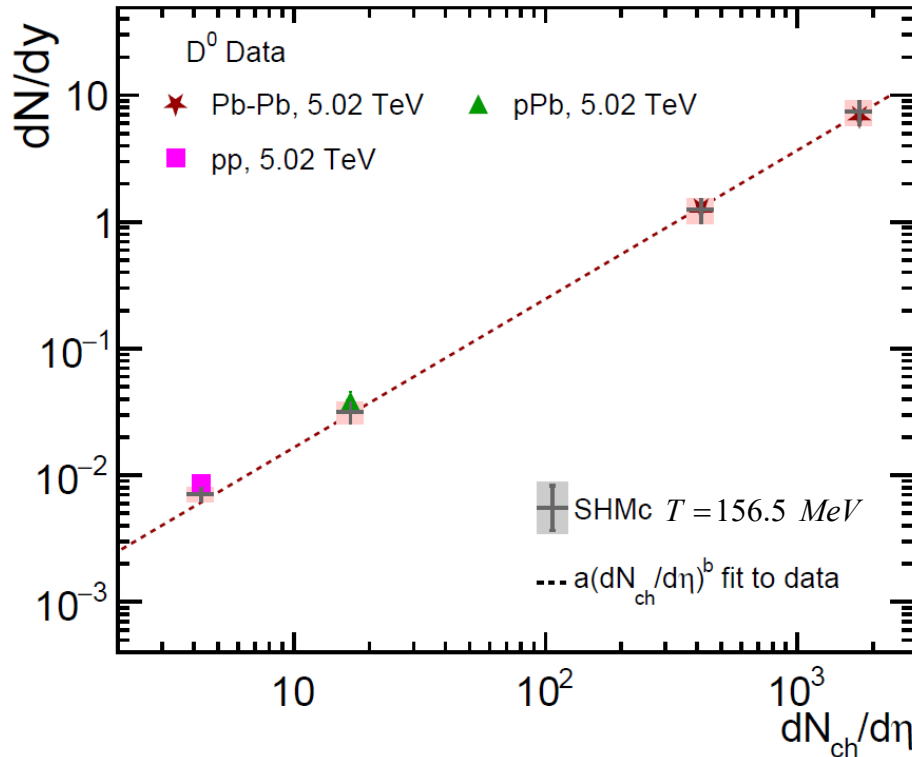
- The quantitative coincidence of thermodynamics with open string mass spectrum and LQCD data is extended to ratios of the 4th order susceptibilities.

$$\chi_{jklm}^{BQSC} = \frac{\partial^n P}{\partial \mu_B^j \partial \mu_Q^k \partial \mu_S^l \partial \mu_C^m} \quad j+k+l+m=4$$

- The ratios indicate relative contributions of strange-charmed mesons and baryons, as well as increasing contribution of |S=2| baryons towards chiral crossover.

Quantifying rapidity densities of open charm hadrons

P. Braun-Munzinger, K.R. , N. Sharma, J. Stachel, *JHEP* 04 (2025)



$$\frac{dN_{i,c=\pm 1}}{dy} \simeq 2 \frac{n_i^{th}(T)}{n_{oc,1}^{tot}(T)} N_{cc}$$

$$N_{cc} = \begin{cases} \sigma_{cc}^{pp} / \sigma_{inel}^{pp} & \text{in pp} \\ \sigma_{cc}^{pA} / \sigma_{inel}^{pA} & \text{in pA} \\ \alpha_A \sigma_{cc}^{pp} T_{AA} & \text{in AA} \end{cases}$$

Cross sections from data: $T=156.5$ MeV

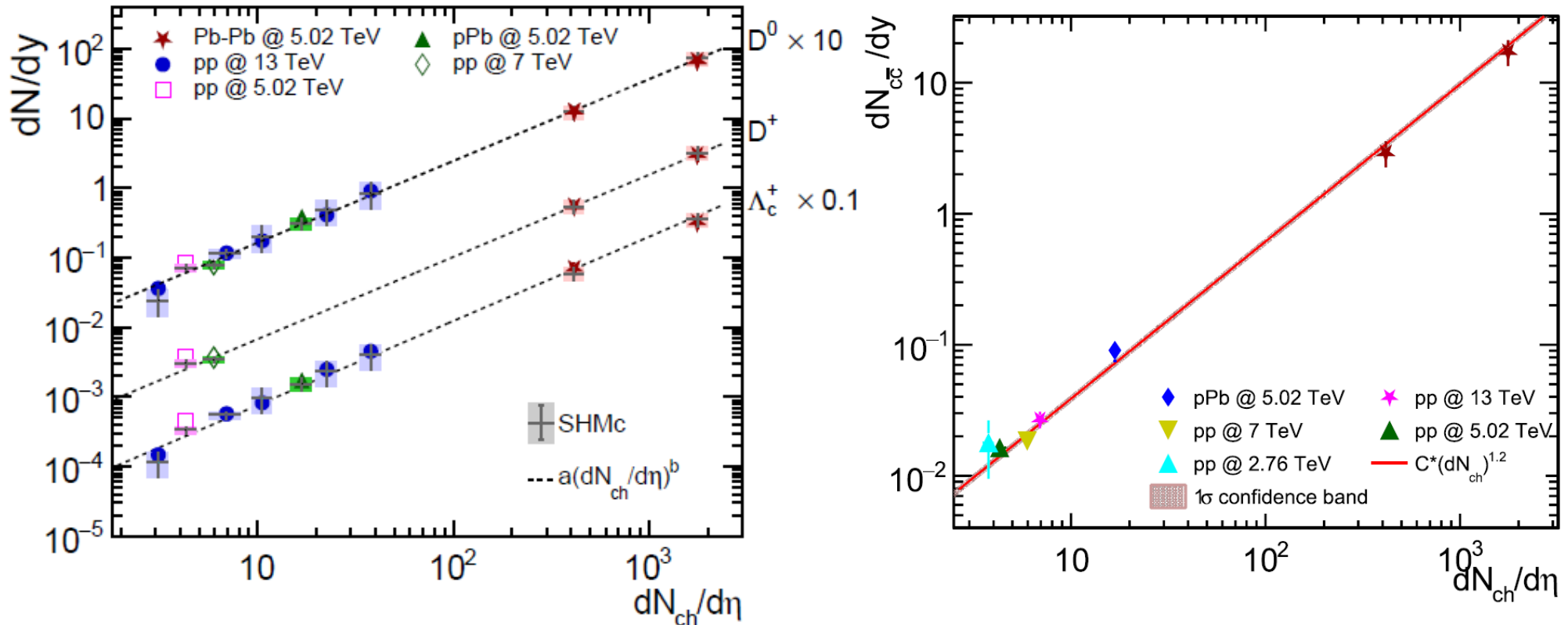
Thickness function from Glauber model.

Factor α_A accounts for nuclear modification effects such as shadowing, energy loss or saturation.

- SHMc provides consistent description of data from pp, pA and AA
- Data at LHC exhibit power law scaling: $dN / dy = a(dN_{ch} / d\eta)^b$ with $b = 1.2 \pm 0.02$ and $a = (1.1 \pm 0.1) \times 10^{-3}$.

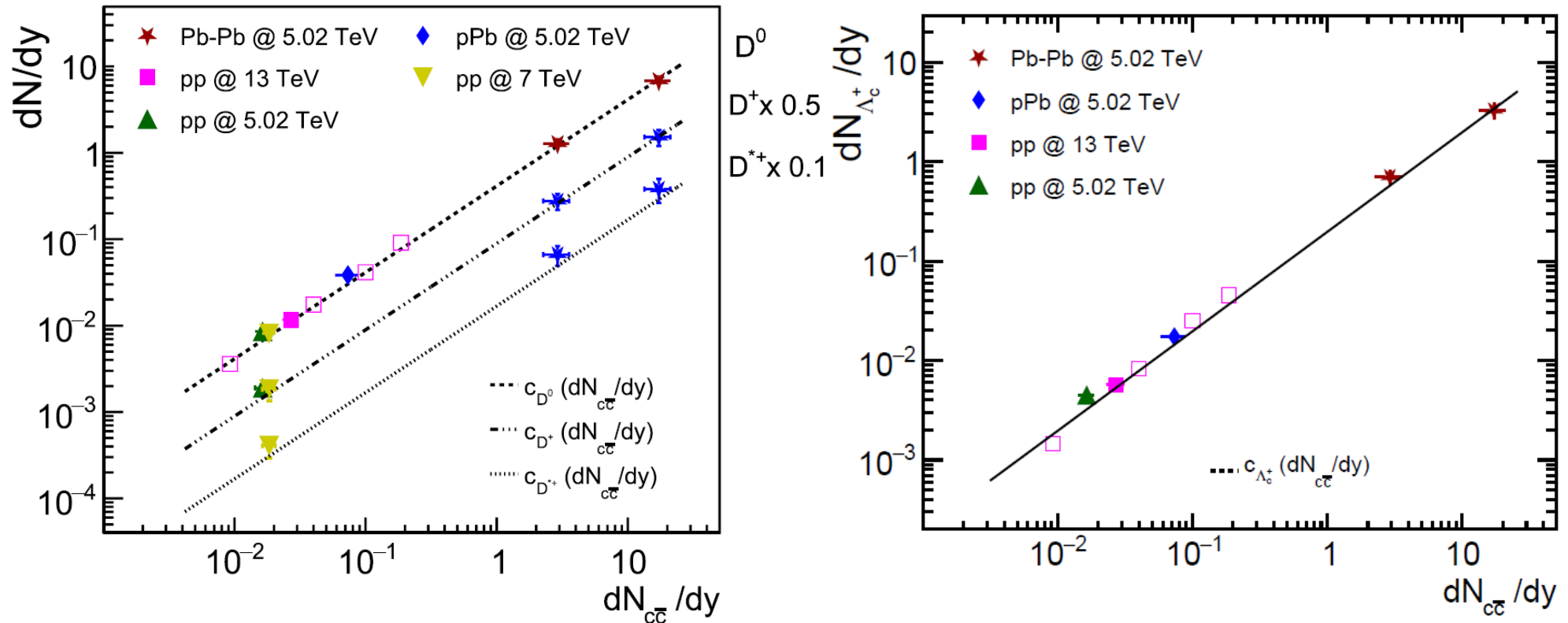
Including predictions for pp at different N_{ch}

P. Braun-Munzinger, K.R. , N. Sharma, J. Stachel, *JHEP* 04 (2025)



- In a narrow rapidity window N_{cc} fitted with power-law in N_{ch} . This allows to extract experimentally unknown values of N_{cc} at $N_{ch} = 3.1, 10.5, 22.6, 37.8$ where D^0 and Λ_c were measured in pp collisions at $\sqrt{s_{NN}} = 13$ TeV.
- All data follow the observed power law scaling with N_{ch} . The yields are also well quantified by the SHMc.

Charm quarks fragmentation/hadronization in the SHMc



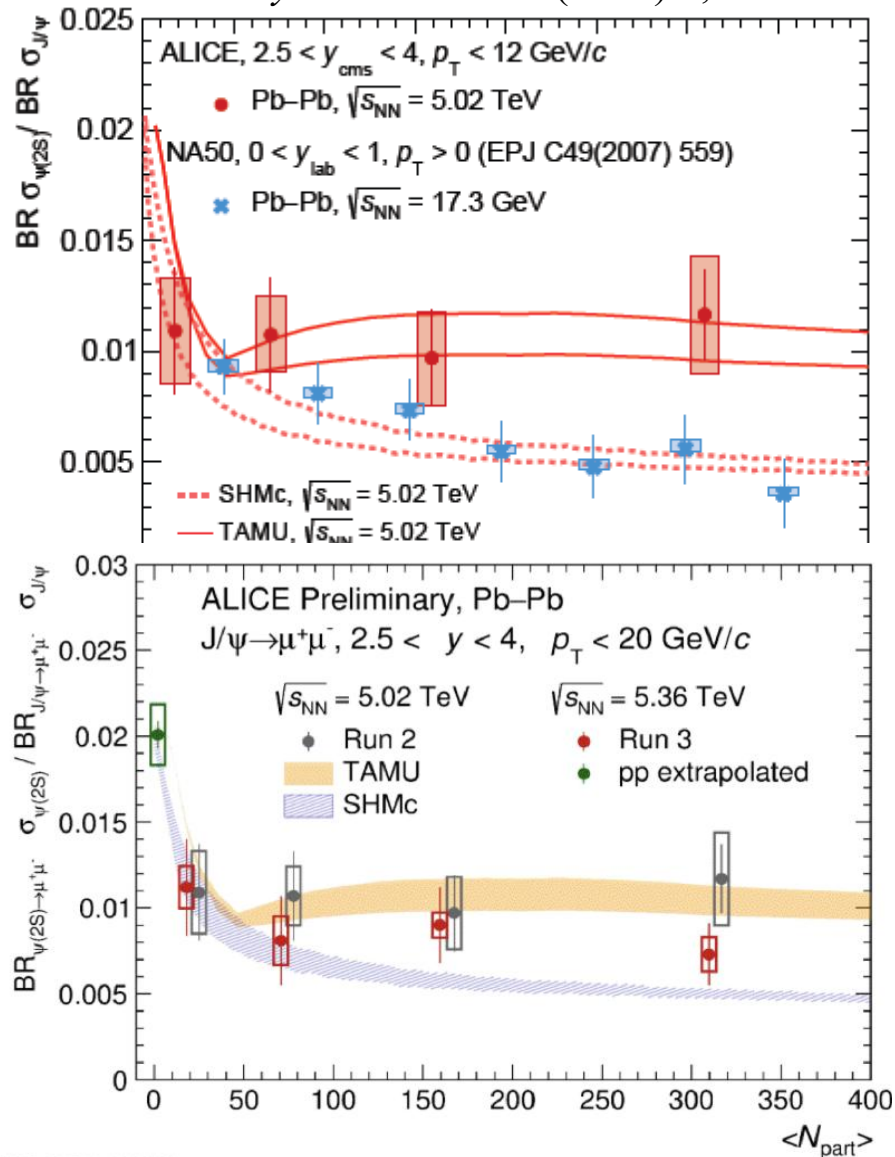
- In SHMc the rapidity density of open charm hadrons in high energy pp, pA and AA collisions should closely follow the proportional scaling with rapidity density of the number of $c\bar{c}$ pairs:

$$\frac{dN_{i,c=\pm 1}}{dy} \simeq 2 \frac{n_i^{th}(T)}{n_{oc,1}^{tot}(T)} N_{cc} \rightarrow T = 156.5 \text{ MeV} \rightarrow \frac{dN_{i,c=\pm 1}}{dy} = \begin{cases} 0.43 \times N_{cc} & \text{for } D^0 \\ 0.21 \times N_{cc} & \text{for } \Lambda_c^+ \end{cases}$$

- Data follow SHMc model expectations indicating that it provides a good description of charm fragmentation/hadronization in high energy collisions.

Quarkonia ratios measure deconfinement T in QCD

ALICE: *Phys.Rev.Lett.* 132 (2024) 4, 042301



- Within SHMc and in central collisions the ratio depends only on temperature,

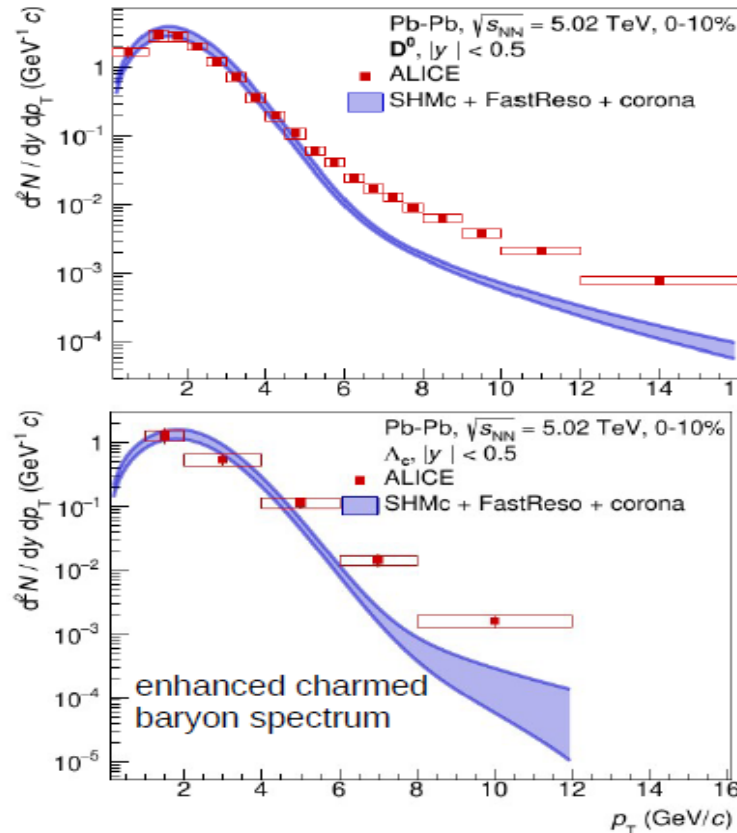
$$\frac{\psi(2s)}{J/\psi} = \frac{n_{\psi(2s)}(T)}{n_{J/\psi}(T)}$$

consequently since freezeout temperature at SPS is similar as in LHC, the ratio should coincide at these energies

- Quarkonia ratios excellent measure of QGP hadronization and deconfinement T_c
- SHMc Ratios $\psi(2s)/\psi(1s)$ consistent with data for $T=156.5$ MeV.

SHMc: A. Andronic, P.Braun-Munzinger, J. Stachel

Spectra of charm hadrons established at QCD phase boundary



Charm quarks in QGP follow collective flow and hadronise at $T_c=156.5$ MeV :

Use blast-wave parametrisation of particle spectra

$$\frac{d^2N}{2\pi p_T dp_T dy} = \frac{2J+1}{(2\pi)^3} \int d\sigma_\mu p^\mu f(p)$$

with the parametrisation of a freezeout hypersurface from 3+1 dim hydrodynamics to verify that transverse momentum spectra of quarkonia and open charm hadrons are established at the QCD phase boundary

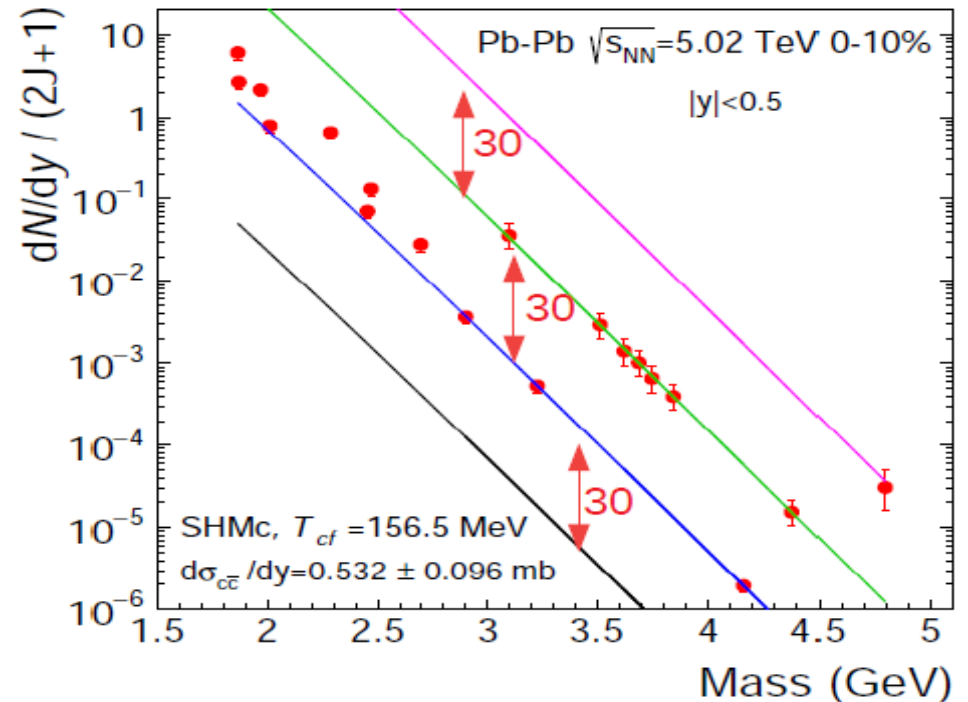
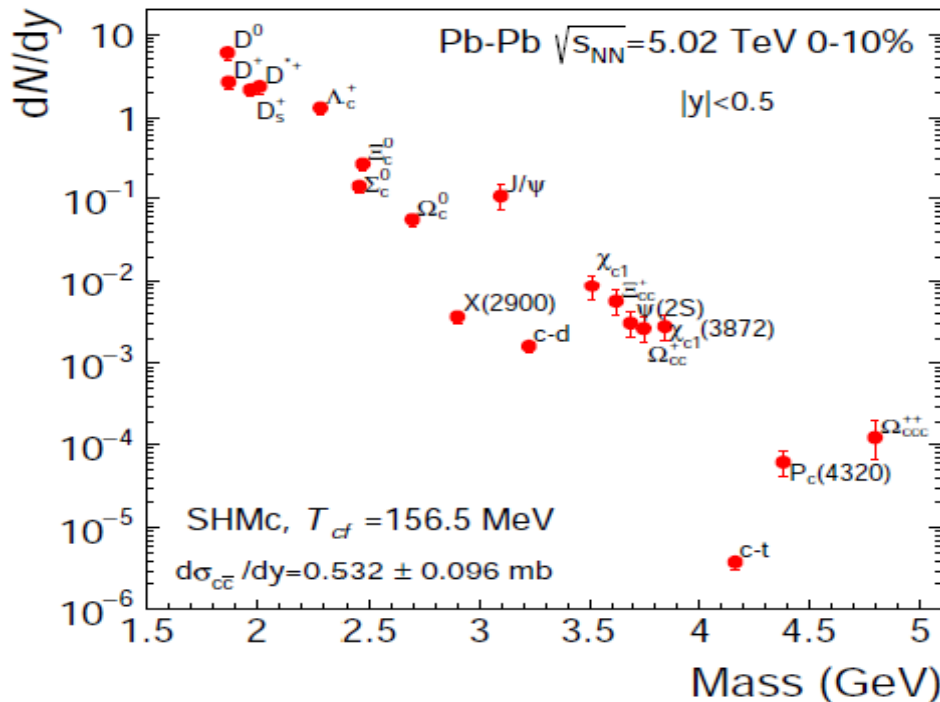
A. Andronic, P. Braun-Munzinger, J. Brunßen, J. Crkovska, J. Stachel, V. Vislavicius, M. Völkl, arXiv: 2308.14821 JHEP 10 (2024) 229

A. Andronic, P. Braun-Munzinger, M. Koehler, A. Mazeliauskas, K. R., J. Stachel, V. Vislavicius, JHEP 07 (2021) 035

A. Andronic, P. Braun-Munzinger, M. Koehler, K. R., J. Stachel, PLB 797 (2019) 134836

SHMc predictions for different charm states

A. Andronic, P. Braun-Munzinger, M. Koehler, A. Mazeliauskas, K. Redlich J. Stachel, V. Vislavicius, JHEP 07 (2021) 035



- Within SHMc model we can also make predictions for yet unmeasured charm and multi-charm hadron species and exotic states like e.g. X-state.
- There are also interesting systematics of particle yields and hierarchy, not only with a mass but also with the charm quark content of hadron due to the $g_c = 30$ fugacity factor.

CONCLUSIONS 1:

- The S-matrix (Hadron Resonance Gas) thermodynamic potential provides a good approximation of the LQCD equation of state and 2nd-order fluctuations and correlations in confined phase up to the chiral crossover.
- The QCD thermodynamic potential is encoded in nuclear collisions
- Particle yields in the light-flavor sector from SIS up to LHC follow the expectations and systematics of the thermal model.
 - From SPC up to LHC, chemical freezeout “coincides” with the LQCD phase boundary
- Clear experimental evidence for charm thermalisation and regeneration in AA collisions, and a parameter-free quantitative description of charmonium and open-charm multiplicity within SHMc, introduced in year 2000 by Peter Braun-Munzinger and Johanna Stachel.
 - Interesting scaling of open charm production yields with charge particle multiplicity and number of cc pairs that link AA, pA and pp data

2019 Stern-Gerlach Medal of the DPG



For the outstanding contributions to the interpretation of heavy ion collisions and the development and establishment of new experimental techniques.”

- 2023 (Peter) and (2026) (Johanna) selected as a member of the Polish Academy of Arts and Sciences, PAU in Krakow

2014 Lise Meitner Prize of the EPS



*For the outstanding contributions to the experimental exploration of the QGP and the design and construction of **ALICE experiment** at CERN*

Dear Pater

**Many Happy Returns,
and further success in
your excellent research**

Strings representation of confined phase constituents

Y. Fujimoto, K. Fukushima, Y. Hidaka, and L. McLerran: PRD 112 (2025) hep-ph/ 2506.0023

String Mass Spectrum in D=4

M.B. Green, J.H. Schwarz, E. Witten (1988)

H.B. Meyer Phys. Rev. D80, (2009)

Open Strings (Mesons)

$$\rho_{Mes}(m) = \frac{\sqrt{2\pi}}{6} \frac{1}{T_H} \left(\frac{T_H}{m} \right)^{3/2} \exp(m / T_H)$$

$$T_H = \sqrt{\frac{3}{2\pi}} \sqrt{\sigma}$$

Closed Strings (Glueballs)

$$\rho_{gb}(m) = \left(\frac{2\pi}{3} \right)^3 \frac{1}{T_H} \left(\frac{T_H}{m} \right)^4 \exp(m / T_H)$$

- For $\sqrt{\sigma} = 0.44 - 0.48 \text{ GeV}$, $T_H \approx 0.3 - 0.34$ common for glue. and mesons

- Diquark-quark structure for **baryons** is energetically favored: $3_c \otimes 3_c = \bar{3}_c \oplus 6_c$
The baryon mass spectrum as for open strings with different degeneracy and mass threshold

Y. Fujimoto: PRD 113 (2026) hep-ph/ 2512.2882

M. Marczenko, G. Kovacs, L. McLerran and K.R.: PRD 112 (2025) hep-ph/ 2508.11626

M. Marczenko, and K.R.: Phys. Rev. C (2026) hep-ph/ 2601.22902

M. Marczenko, L. McLerran and K.R.: (2026) hep-ph/ 2603.28668

Thermodynamics with continue mass spectrum

- The normalized pressure $P = P / T^4$:

$$P(T, \mu) = \int dm \rho_{str}(m) p(T, \mu, m) \quad \text{with} \quad \mu = B\mu_B + S\mu_S + Q\mu_Q$$
$$p(T, \mu, m) = \frac{1}{2\pi^2} \left(\frac{m}{T} \right)^2 K_2(m/T) \exp(\mu/T)$$

- The mass density of mesons:

$$\rho_M(m) = \sum_{i=\pi, K, \eta} d_i \delta(m - m_i) + \sum_{k=\rho, \phi, K^*} d_k \Theta(m - m_k^{thr}) \rho_{str}(m)$$

- The mass density of baryons:

$$\rho_B(m) = \sum_{k=N, \Lambda, \Sigma, \Delta, \Xi, \Omega} d_k \Theta(m - m_k^{thr}) \rho_{str}(m)$$

From discrete to continuum Hagedorn's mass spectrum

- Consider the cumulative mass spectrum QM

$$N(m) = \int_0^m dm' \rho(m')$$

- The density of states

$$\rho(m) = \left. \frac{dN(m')}{dm'} \right|_{m'=m}$$

- $N(m)$ Counts the number of degrees of freedom with mass below m

- For discrete mass spectrum of particle m_i and spin degeneracy d_i

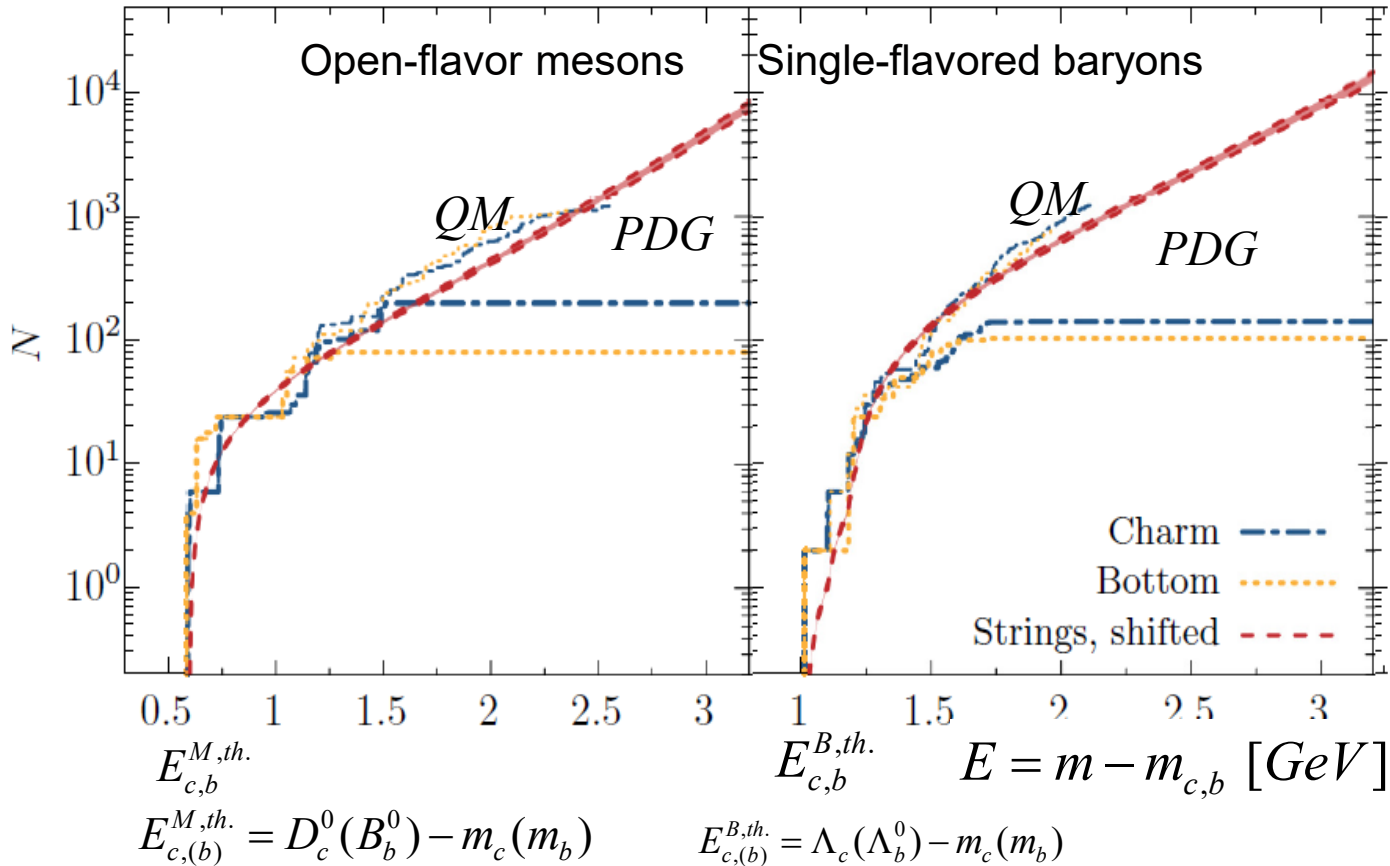
$$N(m) = \sum_i d_i \theta(m - m_i)$$

$$\rho(m) = \sum_i d_i \delta(m - m_i)$$

- Consider the PDG hadron mass spectrum, plot $N(m)$ and compare to string mass spectrum

Comparing charm and bottom spectra

M. Marczenko, L. McLerran and K.R. (2026) hep-ph/ 2603.28668



- A common string excitation energy spectrum for all flavors

$$\rho(E) = \frac{\sqrt{2\pi}}{6T_H} \left(\frac{T_H}{E} \right)^{3/2} \exp(E / T_H)$$

- For the PDG s, c, b quark current masses:

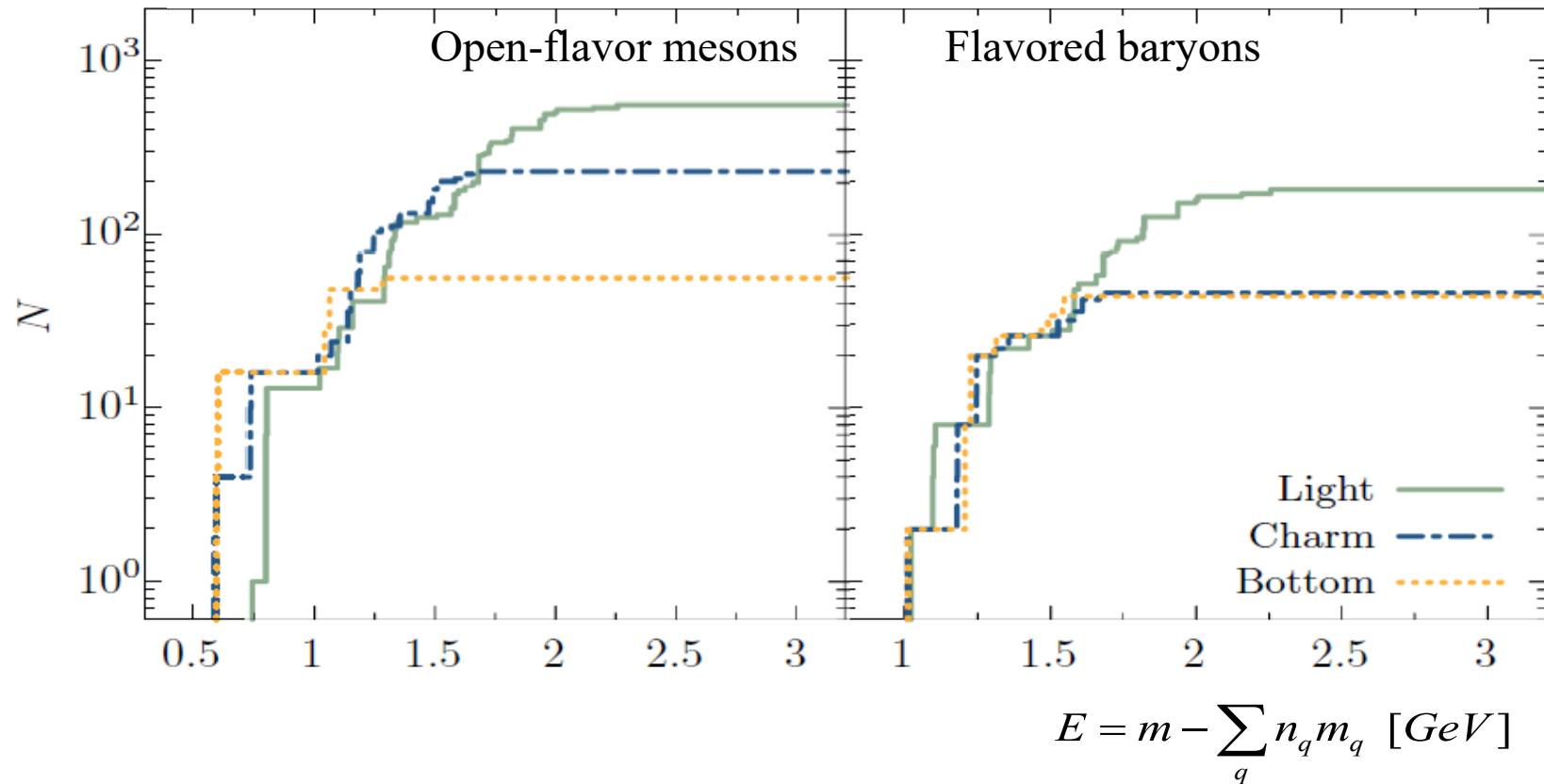
$$m_{s,c,b} = 93.5 [MeV], 1.273, 4.6 [GeV]$$

The threshold energy $E_{c,b}^{th.}$ for c and b meson and baryon coincides

- After subtracting the charm and bottom quark masses, the cumulative spectra overlap in the meson and baryon sectors
- String excitation spectrum beyond quark masses is identical for charm and bottom hadrons

Universal nature of hadron spectra in QCD for all quark flavors

M. Marczenko, L. McLerran and K.R. (2026) hep-ph/ 2603.28668



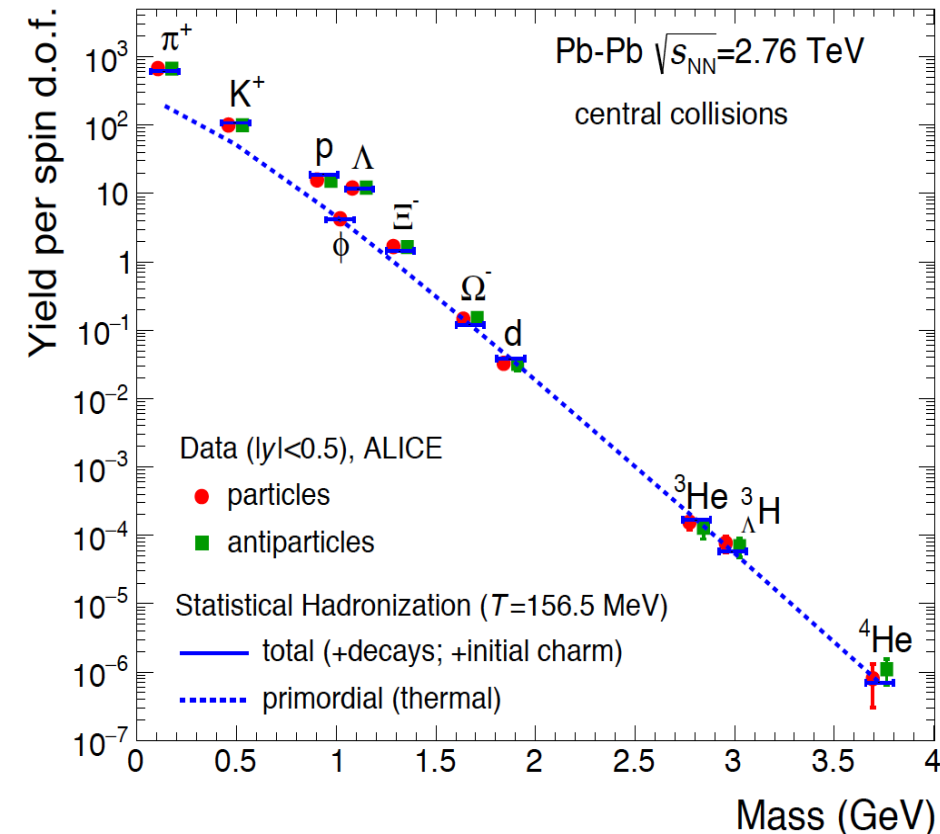
- The exponential growth of density of states above the **energy threshold** is **universal across all flavors** and is governed by string dynamics characterized by the same Hagedorn's temperature linked to the string tension as: $T_H = \sqrt{3/2\pi} \cdot \sqrt{\sigma} = 332 \pm 6 \text{ MeV}$

String theory and confined phase in QCD

- Using first-principles physics theory, it is possible to successfully describe the spectra and EoS of a confined phase in (2+1+1)-flavour QCD and SU(3) pure gauge theory. This is done with no free parameters in string theory, where hadrons are open- and glueballs are closed-strings with the exponentially growing mass spectrum quantified by the universal Hagedorn temperature fixed by the string tension,
$$T_H = (3 / 2\pi)^{1/2} \sqrt{\sigma} \approx 320 MeV$$
- This points towards a fundamental feature of the QCD confined phase.

Charm particle production in HIC

A. Andronic, P. Braun-Munzinger,
J. Stachel & K.R, Nature (2018)



- Light quark hadrons produced at the QCD phase boundary during hadronization of QGP and their yields are consistent with QCD EQS

- The seminal paper by T. Matsui & H. Satz *J/ψ Suppression by Quark-Gluon Plasma Formation*, Phys. Lett. B178 (1986), cited 3800 times, pointing out that:
- Colour screening can prevent quarkonium binding in QGP, the physics confirmed by LQCD calculations, has been driving theoretical and experimental studies of heavy flavour production for the last four decades
- Once dissociated in QGP, can quarkonium be regenerated during hadronisation of QGP?

Fluctuation of the net-baryon number in the acceptance window

- Calculate probability distribution in the acceptance $P_A(B_A)$ from B-can. Ensemble

$$Z_B^C = \text{Tr}[e^{-\beta H} \delta_B] = \sum_{N_B, N_{\bar{B}}} \frac{z_B^{N_B}}{N_B!} \frac{z_{\bar{B}}^{N_{\bar{B}}}}{N_{\bar{B}}!} \delta(N_B - N_{\bar{B}} - B)$$

- Resulting in: mean number of $\pm B$ baryons

$$N_{\pm B} = z \frac{I_{B \mp 1}(2z)}{I_B(2z)} \quad \text{and} \quad N_{\pm B}^A = \alpha_{\pm B} z \frac{I_{B \mp 1}(2z)}{I_B(2z)}$$

- probability to find net baryon B_A in acceptance

$$P_A(B_A) = I_B(2z) (\alpha_B / \alpha_{\bar{B}})^{B_A/2} ((1 - \alpha_B) / (1 - \alpha_{\bar{B}}))^{(B - B_A)/2} \\ \times I_{B_A}(2z \sqrt{\alpha_B \alpha_{\bar{B}}}) I_{B - B_A}(2z \sqrt{(1 - \alpha_B)(1 - \alpha_{\bar{B}})})$$

- Moments of the net baryon number in acceptance

$$\mu_n = \langle (N_B^A - N_{\bar{B}}^A)^n \rangle = \sum_{B_A} (B_A)^n P_A(B_A)$$

Full phase space

$$N_B, N_{\bar{B}} \Rightarrow B = N_B - N_{\bar{B}} \\ z_B, z_{\bar{B}} \Rightarrow z = \sqrt{z_B z_{\bar{B}}} \\ z_{\pm B} = \int d^3x \int \frac{d^3p}{(2\pi)^3} e^{-\beta(E \mp q \mu_q)} \\ \chi_B^{(n)} = 0 \text{ for } n > 1 \\ \text{due to exact baryon number conservation}$$

Acceptance subspace

$$N_B^A, N_{\bar{B}}^A \Rightarrow B_A = N_B^A - N_{\bar{B}}^A \\ z_B^A, z_{\bar{B}}^A \quad \alpha_{\bar{B}} = \frac{N_{\bar{B}}^A}{N_{\bar{B}}} \quad \alpha_B = \frac{N_B^A}{N_B} \\ P_A(B_A) = f(B_A, \alpha_B, \alpha_{\bar{B}}, B, z) \\ \chi_{B_A}^{(n)} \neq 0$$

A.Bzdak, V. Koch and V. Skokov,
Phys. Rev. C87, 014901 (2013)

P. Braun-Munzinger, B. Friman, A. Rustamov,
J. Stachel & K.R. Nucl. Phys. A (2021)

2007.02463 [nucl-th]

- The corresponding cumulants are Bell-polynomials in the moments

$$\kappa_{B_A}^{(n)} = \sum_{k=1}^n (-1)^{k-1} (k-1)! B_{n,k}(\mu_1, \dots, \mu_{n-k+1})$$

Degeneracy factors for strings

Channel	mass [GeV]	I	Q	S	deg
$\rho (l\bar{l})$	0.770	1	$0, \pm 1$	0	16
$K^* (l\bar{s}, s\bar{l})$	0.896	1/2	$0, \pm 1$	± 1	16
$\phi (s\bar{s})$	1.019	0	0	0	4

Channel	mass [GeV]	I	Q	S	deg
$N (l[l])$	0.938	1/2	(0, 1)	0	4
$\Lambda (s[l])$	1.116	0	(0)	-1	2
$\Sigma (l[ls])$	1.189	1	(0, ± 1)	-1	6
$\Delta (\{ll\})$	1.232	3/2	(0, ± 1 , 2)	0	16
$\Xi (s[ls])$	1.315	1/2	(0, -1)	-2	4
$\Omega (s\{ss\})$	1.672	0	(-1)	-3	4

$$\rho_{\text{mes}}(m) = \sum_{i=\pi, K, \eta} d_i \delta(m - m_i) + d_{\text{mes}}(m) \rho_{\text{str}}(m),$$

$$d_{\text{mes}}(m) = \sum_i d_i \theta(m - m_i),$$

$$d_{\text{bar}}(m) = \sum_i d_i \theta(m - m_i).$$

It was recently demonstrated that the baryon spectrum can be described by modeling baryons as open relativistic strings with a quark attached to one endpoint and a diquark attached to the other [30]. In this description, the string dynamics and the resulting excitation spectrum are identical to those governing mesons, such that the asymptotic density of states is given by the same open-string expression. Baryon-specific properties enter only through the physical threshold masses and the degeneracy factors associated with the quantum numbers of the string endpoints.