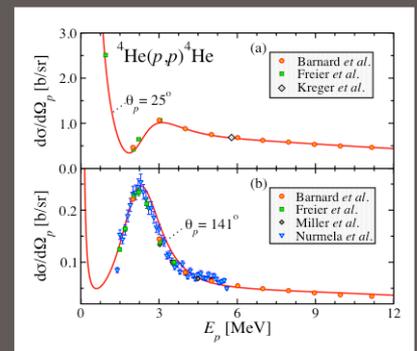
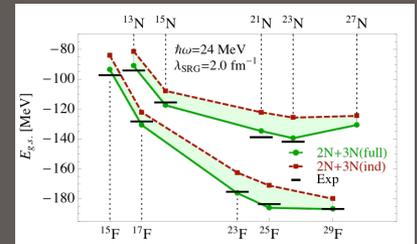


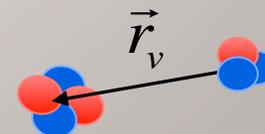
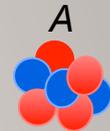
# Ab initio calculations in nuclear physics

International Conference on Science and Technology for FAIR in Europe  
 Worms, Germany  
 October 13-17, 2014

Petr Navratil | TRIUMF



- What is meant by *ab initio* in nuclear physics
- *Ab initio* nuclear structure and reaction approaches
  - Exact few-body calculations ( $A=3,4$ )
  - Quantum Monte Carlo ( $A \leq 12$ )
  - Nuclear Lattice EFT ( $A=4,8,12,16, 20, 24, 28$ )
  - Coupled Cluster Method ( $A \leq 132$ , magic, semi-magic)
  - In-medium Similarity Renormalization Group ( $A \leq 90$ , open shells)
  - Self-Consistent Green's Function Method ( $A \leq 78$ , open shells)
- No-core shell model ( $A \leq 26$ , hypernuclei)
- Including the continuum with the resonating group method
  - NCSM/RGM
  - NCSM with continuum
- Outlook



# What is meant by *ab initio* in nuclear physics?

- **First principles for Nuclear Physics:**

- QCD**

- Non-perturbative at low energies
    - Lattice QCD in the future

- **Degrees of freedom: NUCLEONS**

- Nuclei made of nucleons
  - Interacting by nucleon-nucleon and three-nucleon potentials

- *Ab initio*
  - ✧ All nucleons are active
  - ✧ Exact Pauli principle
  - ✧ Realistic inter-nucleon interactions
    - ✧ Accurate description of NN (and 3N) data
  - ✧ Controllable approximations

# Chiral Effective Field Theory

- **First principles for Nuclear Physics:**

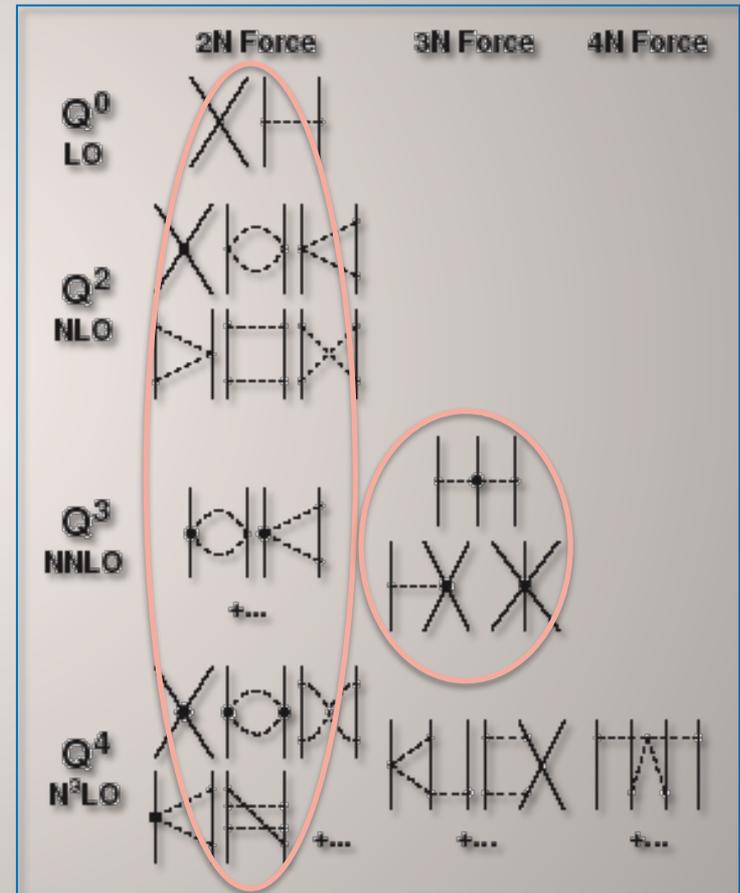
- **QCD**

- Non-perturbative at low energies
    - Lattice QCD in the future

- ***For now a good place to start:***

- **Inter-nucleon forces from chiral effective field theory**

- Based on the symmetries of QCD
    - Chiral symmetry of QCD ( $m_u \approx m_d \approx 0$ ), spontaneously broken with pion as the Goldstone boson
    - Degrees of freedom: nucleons + pions
  - Systematic low-momentum expansion to a given order ( $Q/\Lambda_\chi$ )
  - Hierarchy
  - Consistency
  - Low energy constants (LEC)
    - Fitted to data
    - Can be calculated by lattice QCD



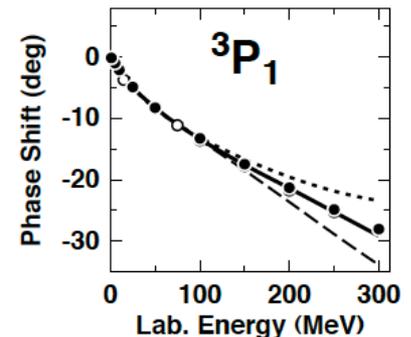
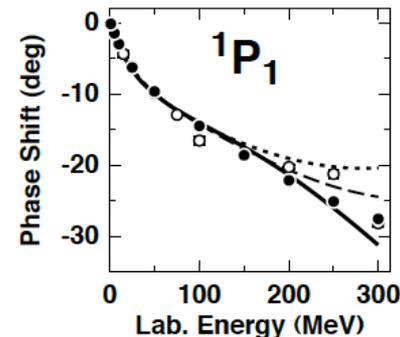
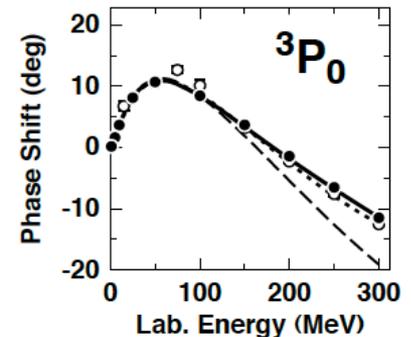
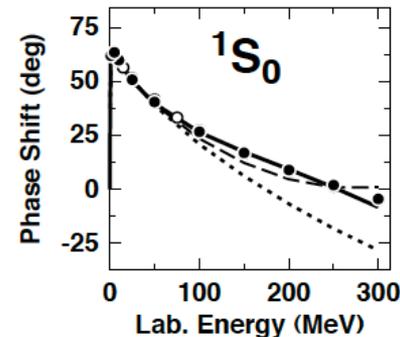
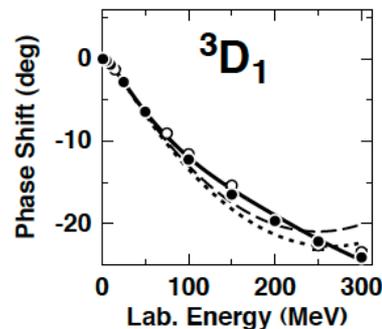
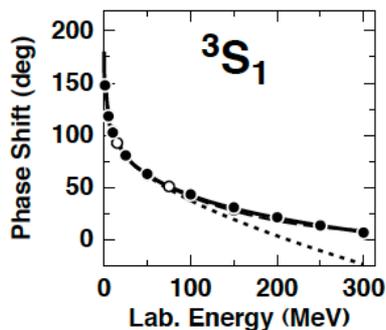
$\Lambda_\chi \sim 1 \text{ GeV}$  :  
Chiral symmetry breaking scale

# The NN interaction from chiral EFT

PHYSICAL REVIEW C **68**, 041001(R) (2003)

## Accurate charge-dependent nucleon-nucleon potential at fourth order of chiral perturbation theory

D. R. Entem<sup>1,2,\*</sup> and R. Machleidt<sup>1,†</sup>



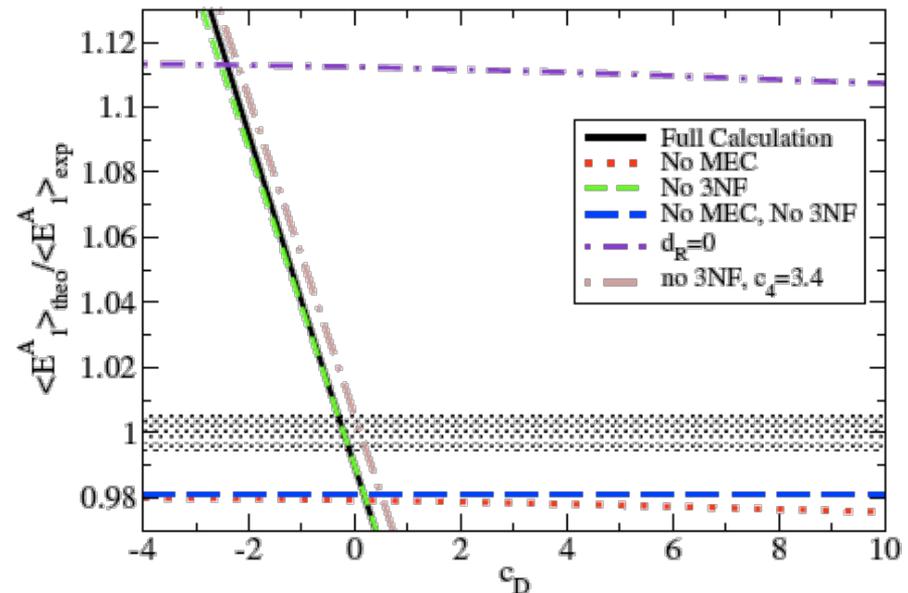
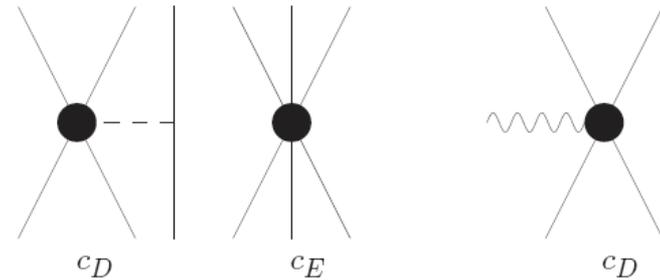
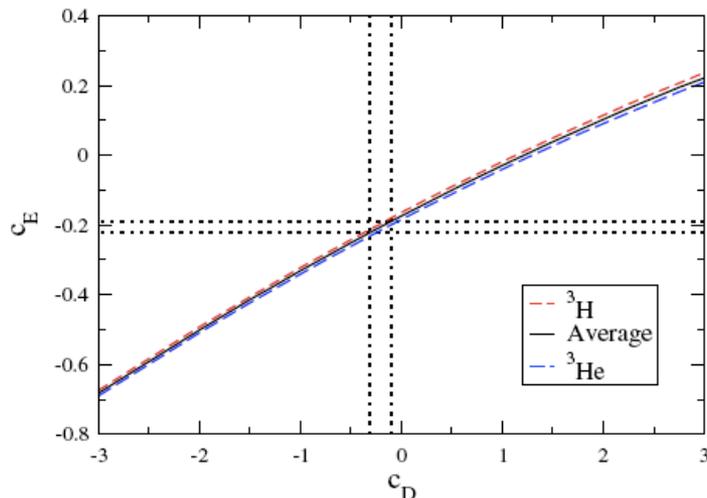
- 24 LECs fitted to the  $np$  scattering data and the deuteron properties
  - Including  $c_i$  LECs ( $i=1-4$ ) from pion-nucleon Lagrangian

# Determination of NNN LECs $c_D$ and $c_E$ from the triton binding energy and the half life

- **Chiral EFT:**  $c_D$  also in the two-nucleon contact vertex with an external probe
- Calculate  $\langle E_1^A \rangle = |\langle {}^3\text{He} || E_1^A || {}^3\text{H} \rangle|$ 
  - Leading order GT
  - N<sup>2</sup>LO: one-pion exchange plus contact

- **A=3 binding energy constraint:**

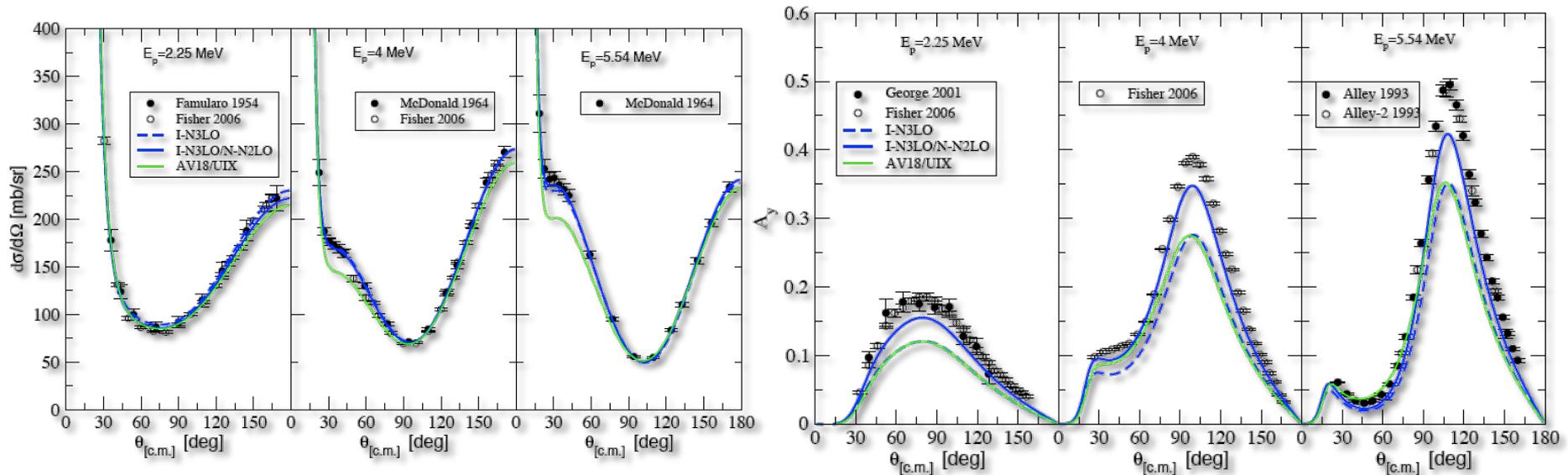
$$c_D = -0.2 \pm 0.1 \quad c_E = -0.205 \pm 0.015$$



# Exact few-body calculations ( $A=3,4$ )

## Proton- $^3\text{He}$ elastic scattering with $\chi\text{EFT NN+NNN}$

- **Hyperspherical-harmonics variational calculations**
  - M. Viviani, L. Girlanda, A. Kievski, L. E. Marcucci, and S. Rosati, EPJ Web Conf. **3** (2010) 05011; Few Body Syst. **54** (2013) 885
- **$A_y$  puzzle (almost) resolved with the chiral  $\text{N}^3\text{LO}$  NN plus local chiral  $\text{N}^2\text{LO}$  NNN**
  - *used with the NCSM and other methods*



# Quantum Monte Carlo

Variational Monte Carlo (VMC): construct  $\Psi_V$  that

- Are fully **antisymmetric** and **translationally invariant**
- Have **cluster structure** and correct asymptotic form
- Contain non-commuting 2- & 3-body **operator correlations** from  $v_{ij}$  &  $V_{ijk}$
- Are orthogonal for multiple  $J^\pi$  states
- Minimize  $E_V = \langle \Psi_V | H | \Psi_V \rangle \geq E$  integrating by Metropolis Monte Carlo

These are  $\sim 2^A$  ( $\frac{A}{Z}$ ) component (270,336 for  $^{12}\text{C}$ ) spin-isospin vectors in  $3A$  dimensions

Green's function Monte Carlo (GFMC): project out the exact eigenfunction

- $\Psi(\tau) = \exp[-(H - E_0)\tau]\Psi_V = \sum_n \exp[-(E_n - E_0)\tau]a_n\Psi_n \Rightarrow \Psi_0$  at large  $\tau$
- Propagation done stochastically in small time slices  $\Delta\tau$
- Exact  $\langle H \rangle$  for local potentials; mixed estimates for other  $\langle O \rangle$
- **Constrained-path propagation** controls fermion sign problem for  $A \geq 8$
- Multiple excited states for same  $J^\pi$  stay orthogonal

Many tests demonstrate 1–2% accuracy for realistic  $\langle H \rangle$

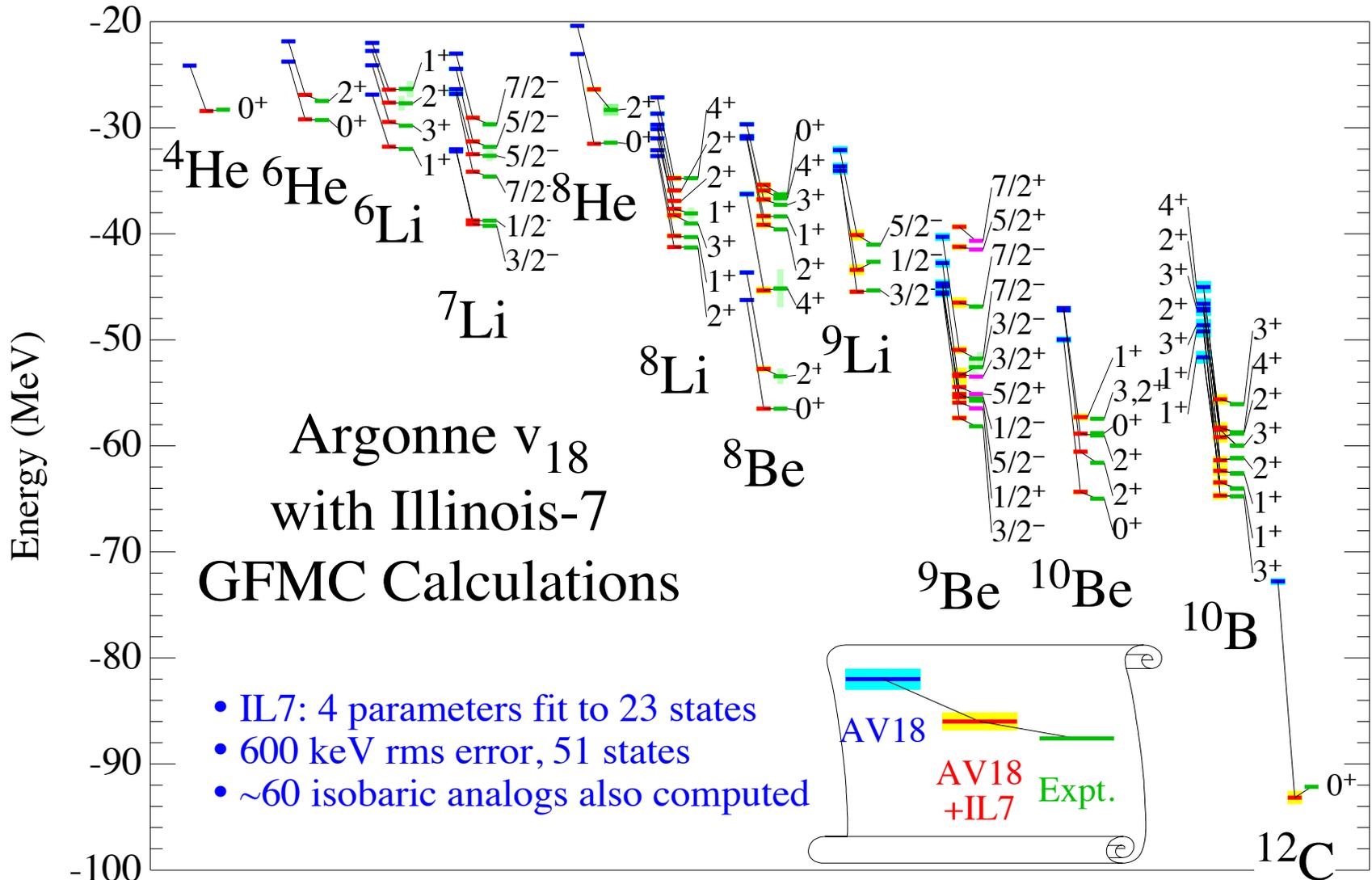
Wiringa, Pieper, Carlson, & Pandharipande, PRC **62**, 014001 (2000)

Pieper, Varga, & Wiringa, PRC **66**, 044310 (2002)

Pieper, Wiringa, & Carlson, PRC **70**, 054325 (2004)

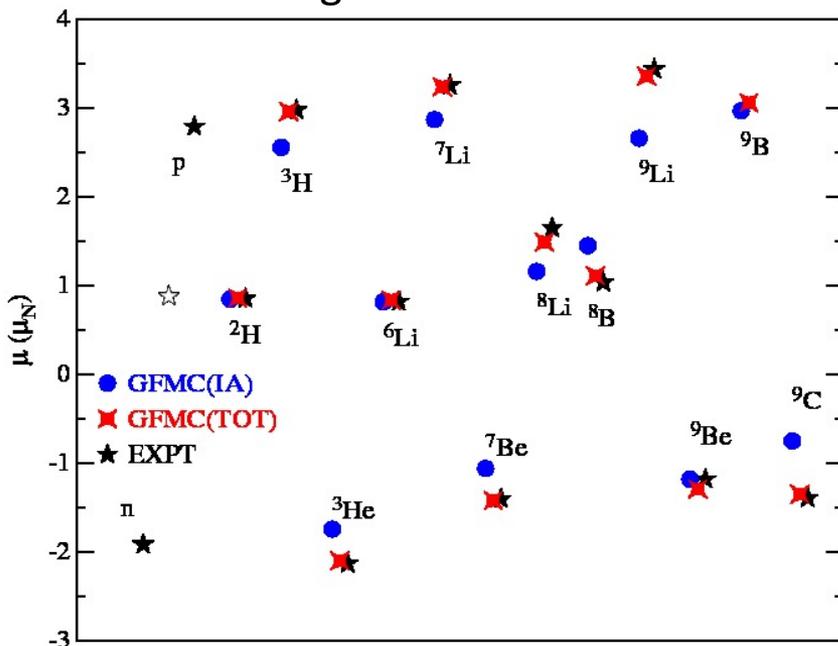
Pieper, NPA **751**, 516c (2005)

# Quantum Monte Carlo: Eigenenergies of light nuclei



# Quantum Monte Carlo: Magnetic moments and transitions light nuclei

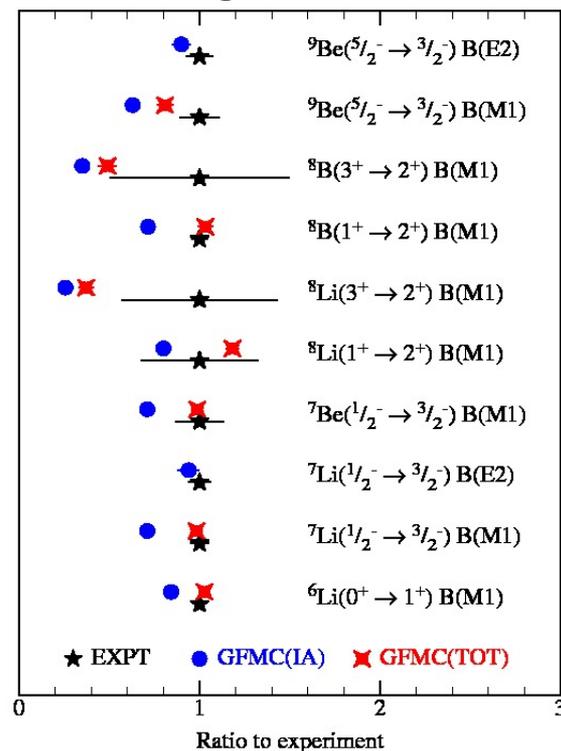
## Magnetic Moments



Green's Function Monte Carlo (GFMC) calculations of light nuclei give accurate energies but a **lowest-order theory of one-body currents (blue)** disagrees with experiment (black).

Including **two-nucleon currents based on effective field theory (red)** improves all predictions!

## Electromagnetic Transitions



PHYSICAL REVIEW C **87**, 035503 (2013)



Quantum Monte Carlo calculations of electromagnetic moments and transitions in  $A \leq 9$  nuclei with meson-exchange currents derived from chiral effective field theory

S. Pastore,<sup>1,\*</sup> Steven C. Pieper,<sup>1,†</sup> R. Schiavilla,<sup>2,3,‡</sup> and R. B. Wiringa<sup>1,§</sup>

# Nuclear Lattice Effective Field Theory Calculations

E. Epelbaum, H. Krebs, T. Lahde, D. Lee, U.-G. Meissner

Discretized version of  
chiral EFT for nuclear  
dynamics

$$\left[ \left( \sum_{i=1}^A \frac{-\vec{\nabla}_i^2}{2m_N} + \mathcal{O}(m_N^{-3}) \right) + \underbrace{V_{2N} + V_{3N} + V_{4N} + \dots}_{\text{derivable within ChPT}} \right] |\Psi\rangle = E|\Psi\rangle$$

Borasoy, E.E., Krebs, Lee, Meißner, Eur. Phys. J. A31 (07) 105,

Eur. Phys. J. A34 (07) 185,

Eur. Phys. J. A35 (08) 343,

Eur. Phys. J. A35 (08) 357,

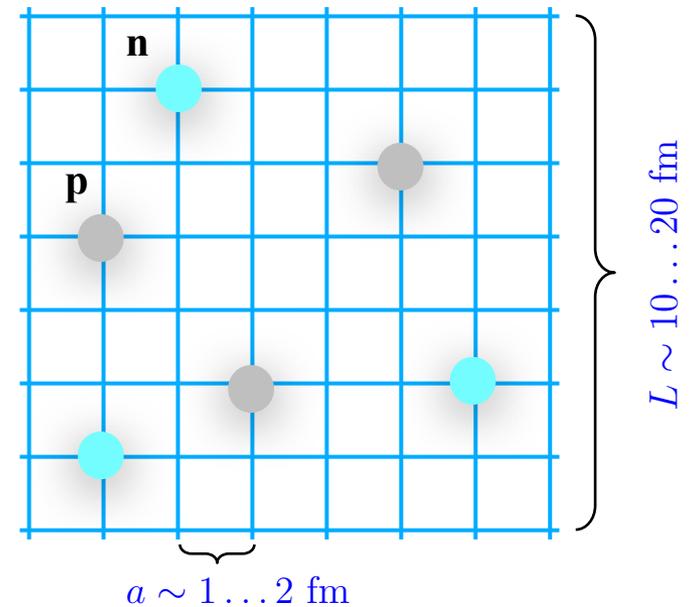
E.E., Krebs, Lee, Meißner, Eur. Phys. J A40 (09) 199,

Eur. Phys. J A41 (09) 125,

Phys. Rev. Lett 104 (10) 142501,

Eur. Phys. J. 45 (10) 335,

Phys. Rev. Lett. 106 (11) 192501



Physics Letters B 732 (2014) 110–115

Contents lists available at ScienceDirect

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www.elsevier.com/locate/physletb




Lattice effective field theory for medium-mass nuclei

Timo A. Lähde<sup>a,\*</sup>, Evgeny Epelbaum<sup>b</sup>, Hermann Krebs<sup>b</sup>, Dean Lee<sup>c</sup>, Ulf-G. Meißner<sup>a,d,e</sup>, Gautam Rupak<sup>f</sup>

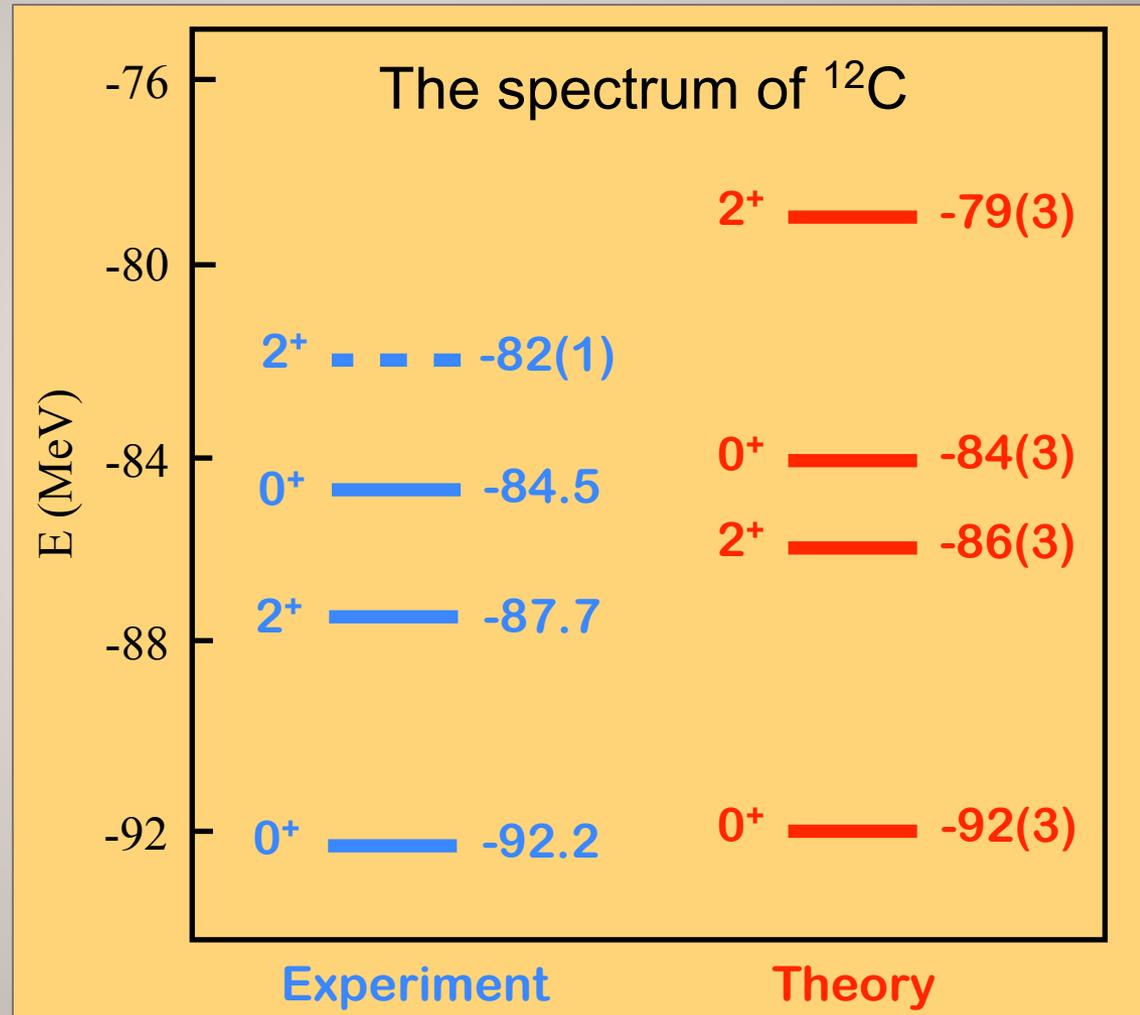
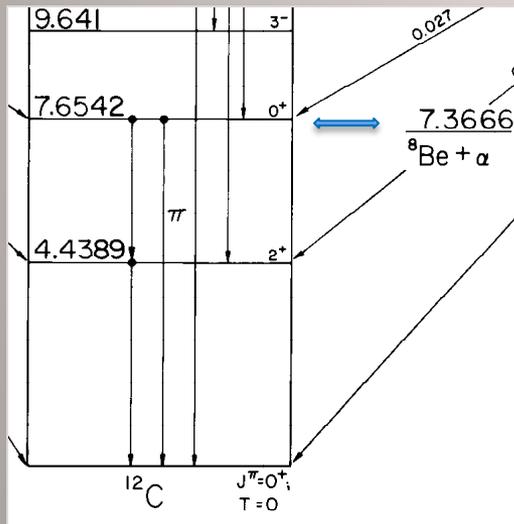
Ground states of alpha nuclei from  ${}^4\text{He}$  to  ${}^{28}\text{Si}$

# Nuclear Lattice Effective Field Theory Calculations

E. Epelbaum, H. Krebs, T. Lahde, D. Lee, U.-G. Meissner

## The Hoyle state

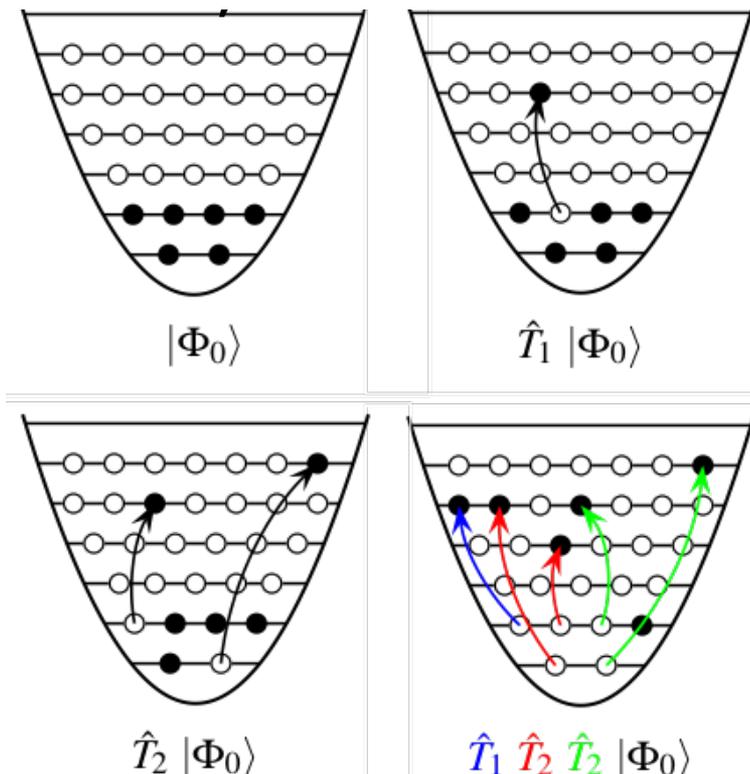
$^{12}\text{C}$  production  
in the Universe



# Coupled-Cluster Method

- **exponential Ansatz** for wave operator  $|\Psi\rangle = e^{\hat{T}} |\Phi_0\rangle$

- **CCSD**: truncate  $\hat{T}$  at the **2p2h excitation** level,  $\hat{T} = \hat{T}_1 + \hat{T}_2$

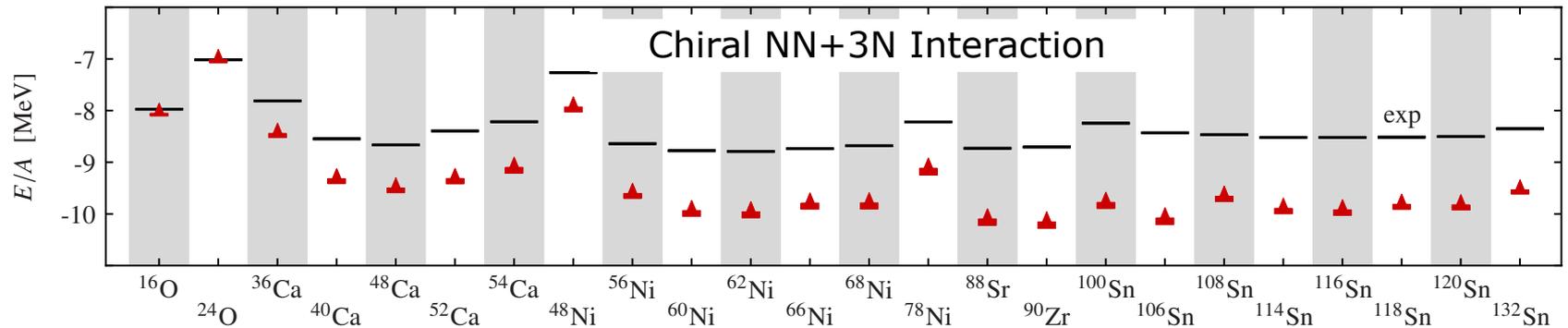


$$\hat{T}_n = \frac{1}{(n!)^2} \sum_{\substack{ijk\dots \\ abc\dots}} t_{ijk\dots}^{abc\dots} \{ \hat{a}_a^\dagger \hat{a}_b^\dagger \hat{a}_c^\dagger \dots \hat{a}_k \hat{a}_j \hat{a}_i \}$$

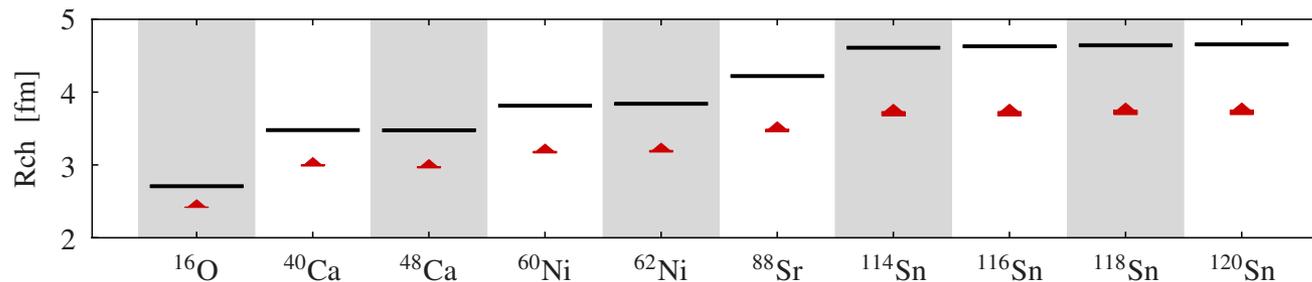
- effects of  $T_3$  clusters **included approximately** in ground-state calculations via  $\Lambda$ CCSD(T) or CR-CC(2,3) method

State-of-the-art:  $\Lambda$ -CCSD(T) with 3N interaction

# Coupled-Cluster calculations for heavy nuclei with chiral interactions



- **current** chiral Hamiltonians capable of describing the **experimental trend** of binding energies
- systematic overbinding indicates that there are still **deficiencies**
  - ➔ **consistent 3N** interaction at  $N^3LO$ , and **4N** interactions
- **charge radii** are considerably **too small**

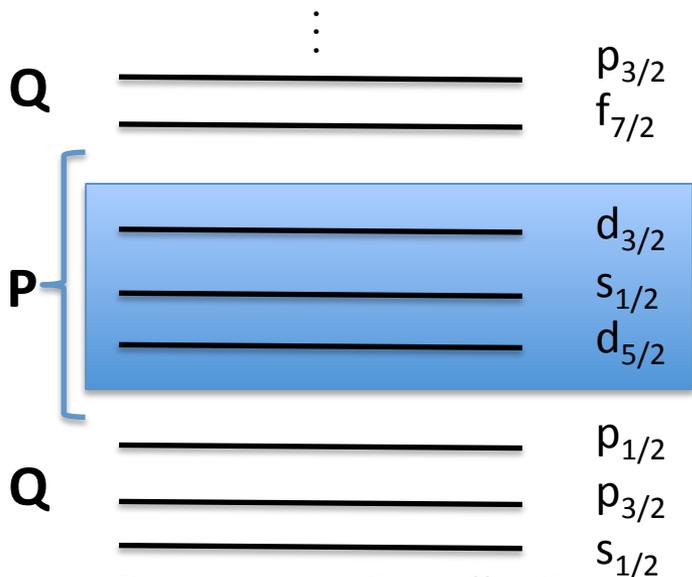


# Coupled-cluster effective interactions (CCEI) for the shell model

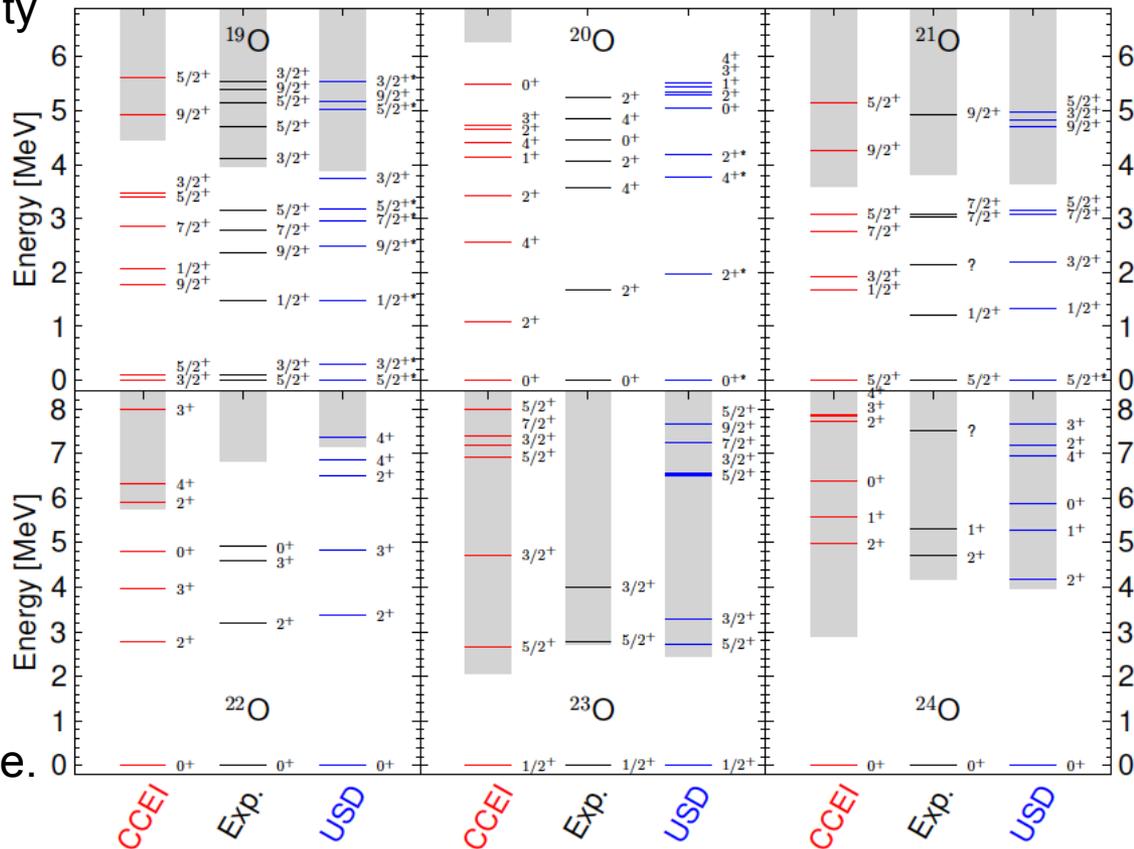
G. R. Jansen, J. Engel, G. Hagen, P. Navratil, A. Signoracci, Phys. Rev. Lett. **113**, 142502 (2014).

- Start from chiral NN(N3LO<sub>EM</sub>) + 3NF(N2LO) interactions
- Solve for A+1 and A+2 using CC. Project A+1 and A+2 CC wave functions onto the s-d model space using Lee-Suzuki similarity transformation.

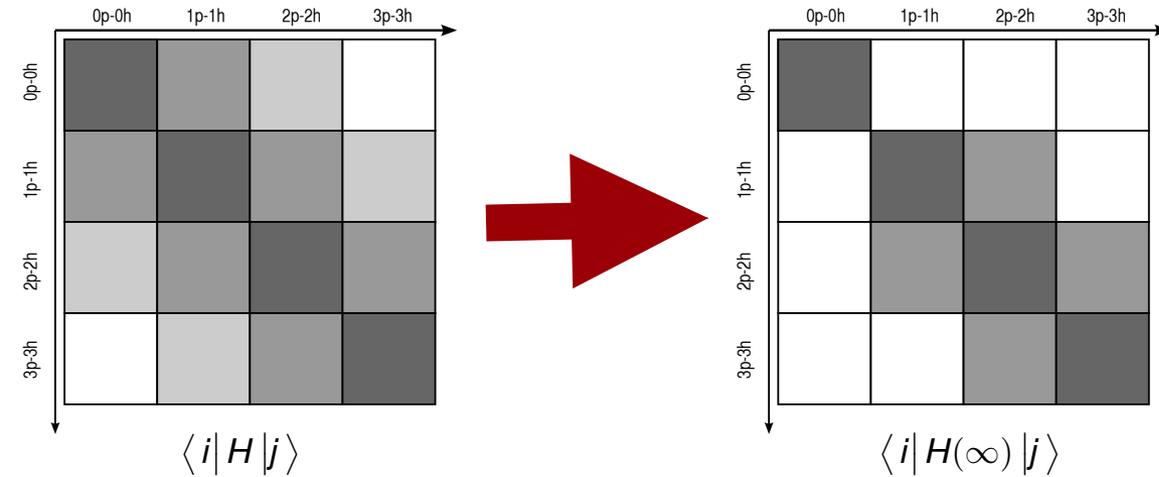
Spectra of oxygen isotopes computed with coupled-cluster effective interaction (CCEI), and compared to experimental data and the phenomenological USD shell model interaction.



- Diagonalize the effective hamiltonian in the valence space.



# In-medium SRG approach: Application to Oxygen isotopes



**aim:** decouple reference state  
(0p-0h) from excitations

$$\frac{d}{ds} H(s) = [\eta(s), H(s)], \quad \eta(s) = \frac{dU(s)}{ds} U^\dagger(s) = -\eta^\dagger(s)$$

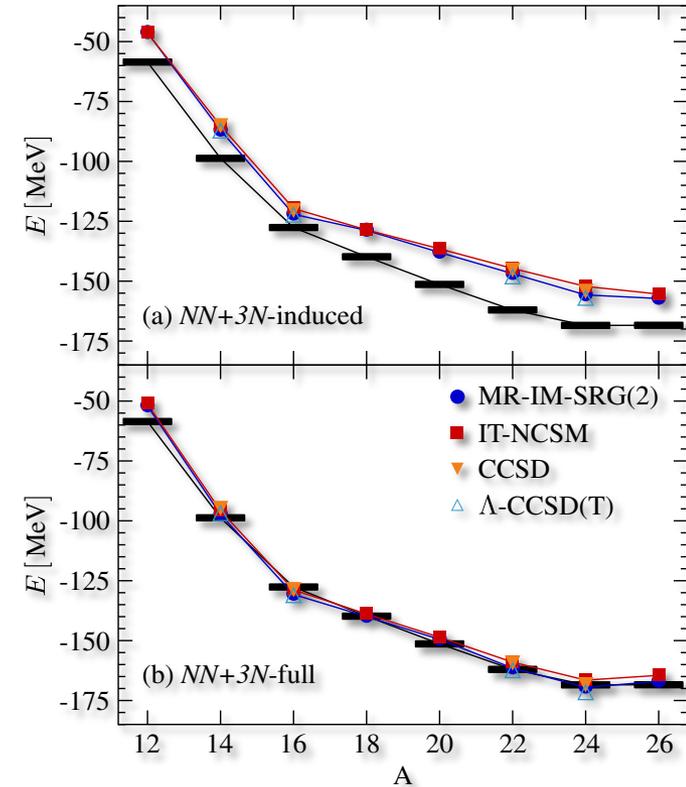
- Wegner

$$\eta^l = [H^d, H^{od}]$$

- White (J. Chem. Phys. 117, 7472)

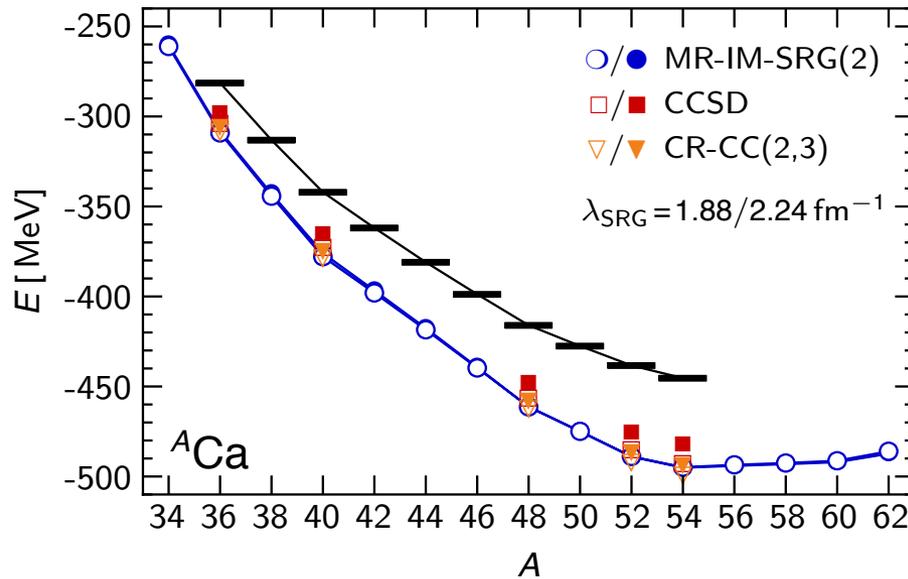
$$\eta^{ll} = \sum_{ph} \frac{f_h^p}{E_p - E_h} : A_h^p : + \sum_{pp'hh'} \frac{\Gamma_{hh'}^{pp'}}{E_{pp'} - E_{hh'}} : A_{hh'}^{pp'} : + \text{H.c.}$$

$E_p - E_h, E_{pp'} - E_{hh'}$  : approx. 1p1h, 2p2h excitation energies

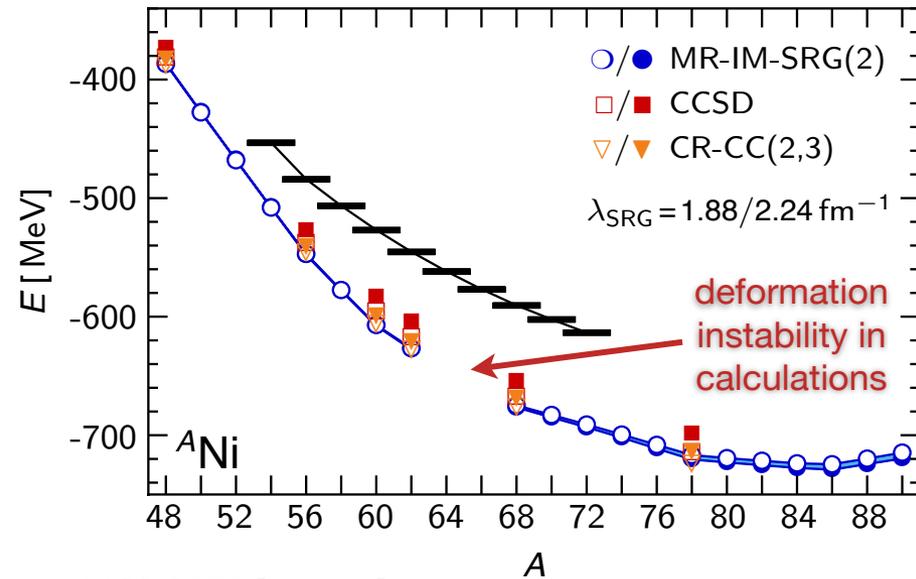


# In-medium SRG approach: Application to Ca and Ni isotopes

NN + 3N-full(400)



NN + 3N-full(400)



H. Hergert et al., arXiv:1408.6555 [nucl-th]

- IM-SRG calculations for  $A \sim 100$  are routine, tin isotopes in progress
- controlled uncertainties & consistent results for different ab-initio methods
- systematic overbinding due to current chiral Hamiltonians - results for new generation of chiral Hamiltonians soon

# Self-Consistent Green's Function Method: Oxygen, Fluorine, Nitrogen isotopes

Magic and semi-magic nuclei

$$g_{\alpha\beta}(\omega) = \sum_n \frac{\langle \Psi_0^A | c_\alpha | \Psi_n^{A+1} \rangle \langle \Psi_n^{A+1} | c_\beta^\dagger | \Psi_0^A \rangle}{\omega - \varepsilon_n^{A+1} + i\eta} + \sum_k \frac{\langle \Psi_0^A | c_\beta^\dagger | \Psi_k^{A-1} \rangle \langle \Psi_k^{A-1} | c_\alpha | \Psi_0^A \rangle}{\omega - \varepsilon_k^{A-1} - i\eta},$$

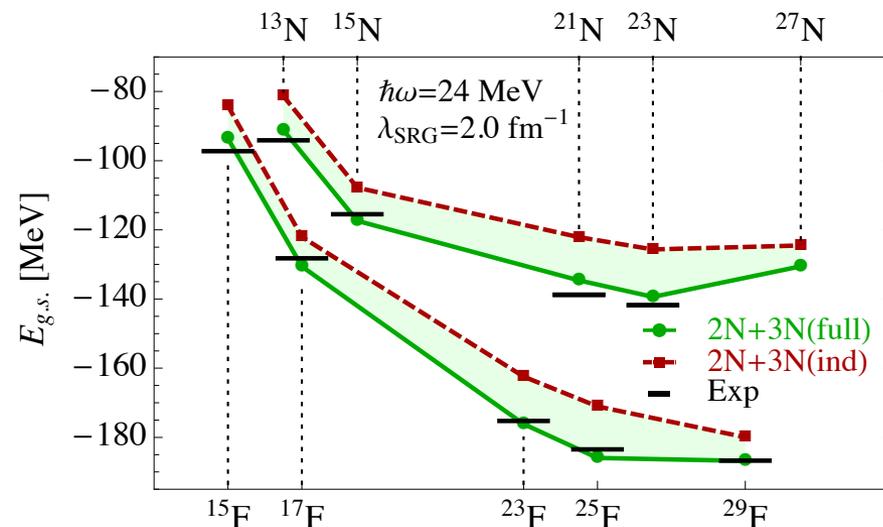
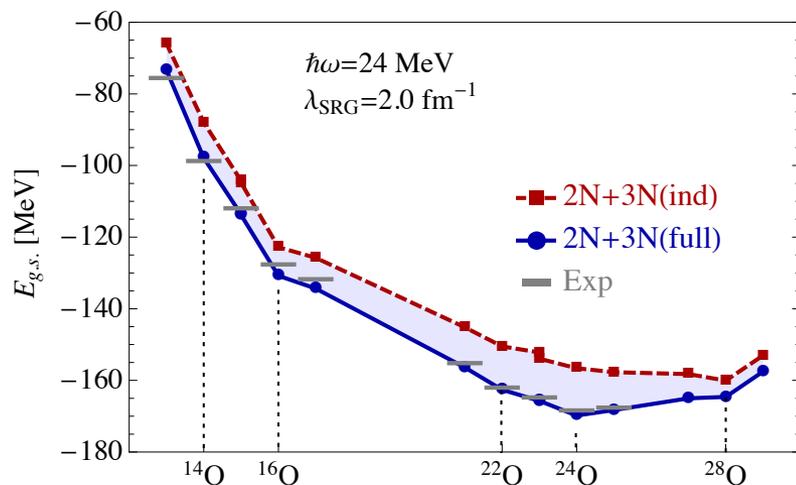
PRL 111, 062501 (2013)

PHYSICAL REVIEW LETTERS

week ending  
9 AUGUST 2013

Isotopic Chains Around Oxygen from Evolved Chiral Two- and Three-Nucleon Interactions

A. Cipollone,<sup>1</sup> C. Barbieri,<sup>1,\*</sup> and P. Navrátil<sup>2</sup>

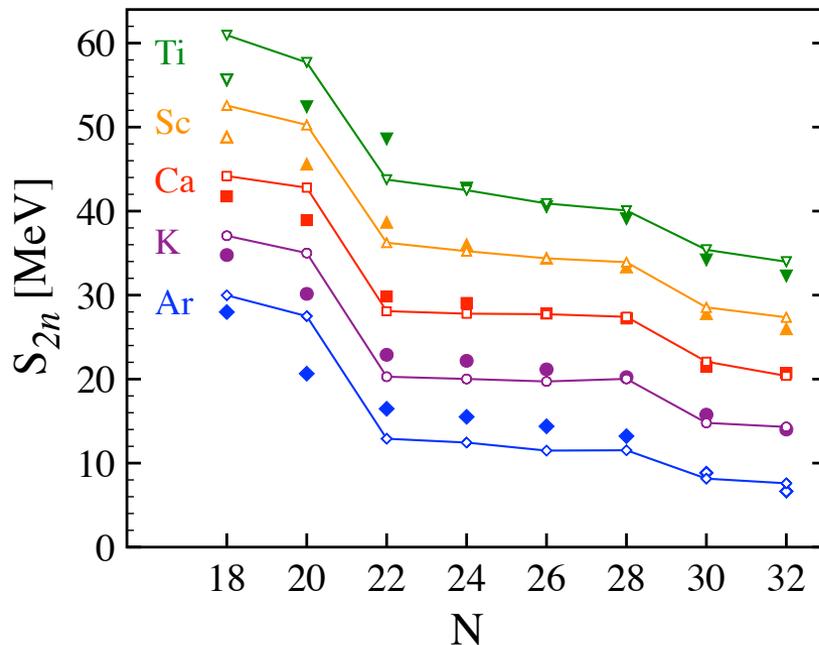


- 3NF crucial for reproducing binding energies and driplines around oxygen
- $d_{3/2}$  raised by genuine 3NF

# Green's functions in medium-mass nuclei

Gorkov GF go **beyond standard expansion schemes** and are not limited to doubly closed-shells

- Expansion around a Bogoliubov vacuum
- **From few tens to hundreds** of medium-mass open-shell systems (→ complete chains)



Open shell nuclei

PHYSICAL REVIEW C **89**, 061301(R) (2014)

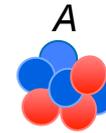
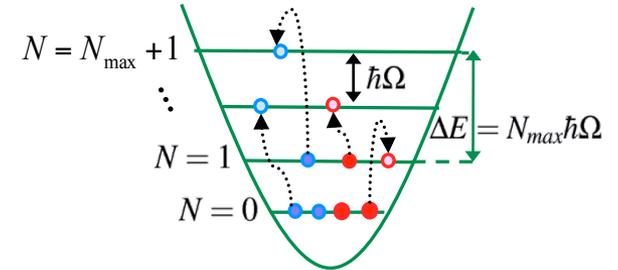
**Chiral two- and three-nucleon forces along medium-mass isotope chains**

V. Somà,<sup>1,2,3,\*</sup> A. Cipollone,<sup>4</sup> C. Barbieri,<sup>4,†</sup> P. Navrátil,<sup>5</sup> and T. Duguet<sup>3,6,‡</sup>

- Systematic overbinding of medium-mass nuclei (in **agreement with other ab initio methods**)
- initial (full) 3NF are **necessary** to reproduce relative trends
- Relative energies ( $S_{2n}$ ) well reproduced

# No-core shell model

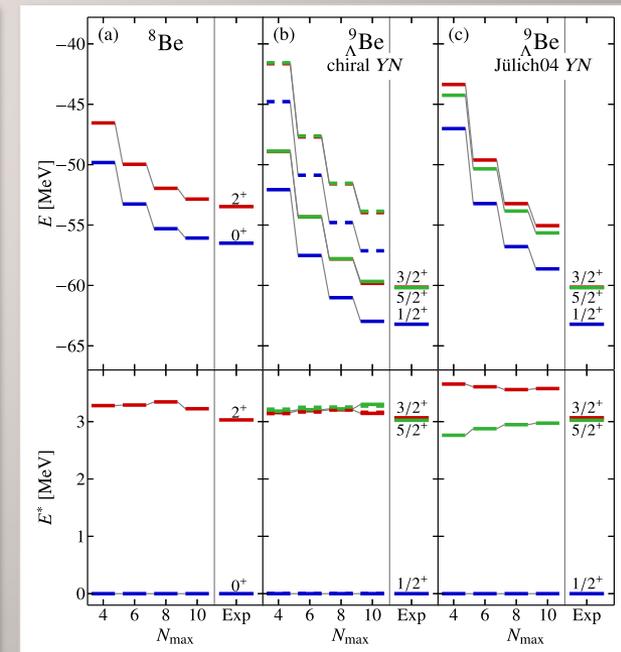
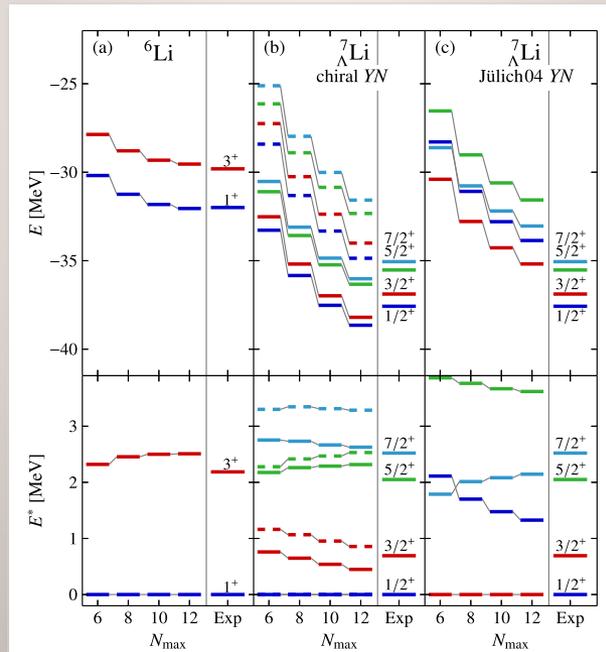
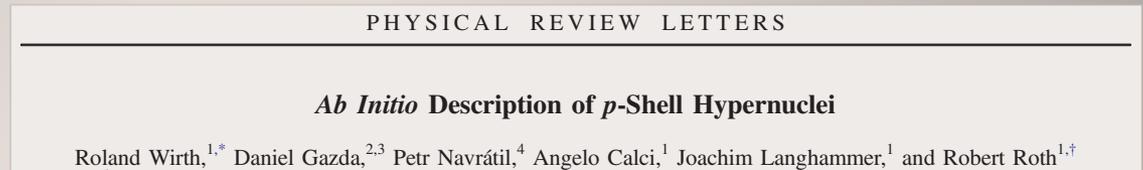
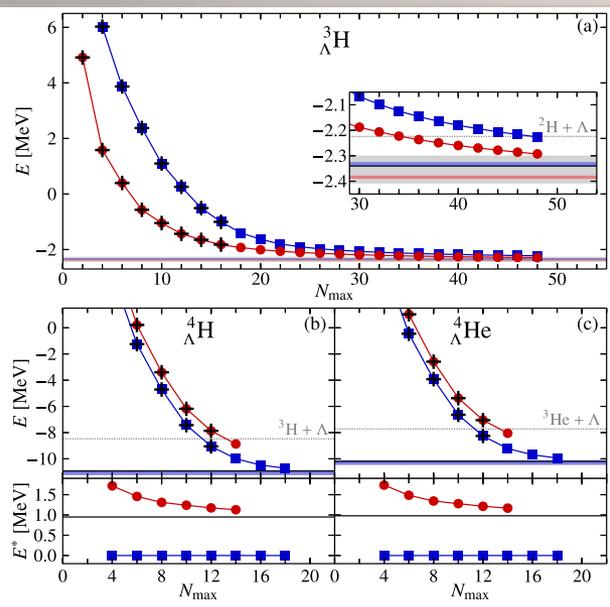
- No-core shell model (NCSM)
  - $A$ -nucleon wave function expansion in the harmonic-oscillator (HO) basis
  - short- and medium range correlations
  - Bound-states, narrow resonances



$$\Psi^A = \sum_{N=0}^{N_{\max}} \sum_i c_{Ni} \Phi_{Ni}^A$$

# NCSM calculations for light nuclei and *hypernuclei*

- Flexible approach capable performing exact calculations for few-nucleon systems and accurate calculations for nuclei with  $A \leq 24$  & **hypernuclei**



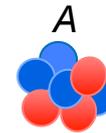
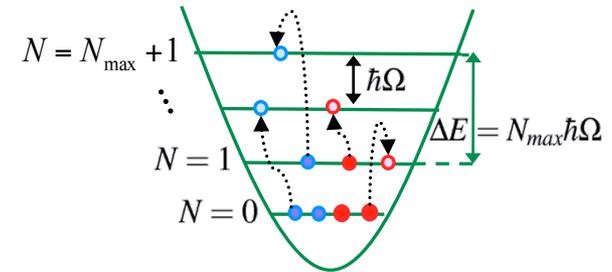
Testing chiral LO NY potentials with  $\Lambda$ - $\Sigma$  mixing included

...outperform the Julich '04 YN potential

# No-core shell model with continuum

- **No-core shell model (NCSM)**

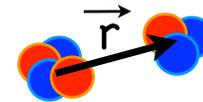
- $A$ -nucleon wave function expansion in the harmonic-oscillator (HO) basis
- short- and medium range correlations
- Bound-states, narrow resonances



$$\Psi^A = \sum_{N=0}^{N_{\max}} \sum_i c_{Ni} \Phi_{Ni}^A$$

- **NCSM with Resonating Group Method (NCSM/RGM)**

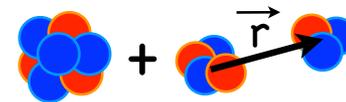
- cluster expansion
- proper asymptotic behavior
- long-range correlations



$$\Psi^A = \sum_{\nu} \int d\vec{r} \varphi_{\nu}(\vec{r}) \mathcal{A}_{\nu} \Phi_{1\nu}^{(A-a)} \Phi_{2\nu}^{(a)} \delta(\vec{r} - \vec{r}_{A-a,a})$$

- **NCSM with continuum (NCSMC)**

- unified description of bound and unbound states



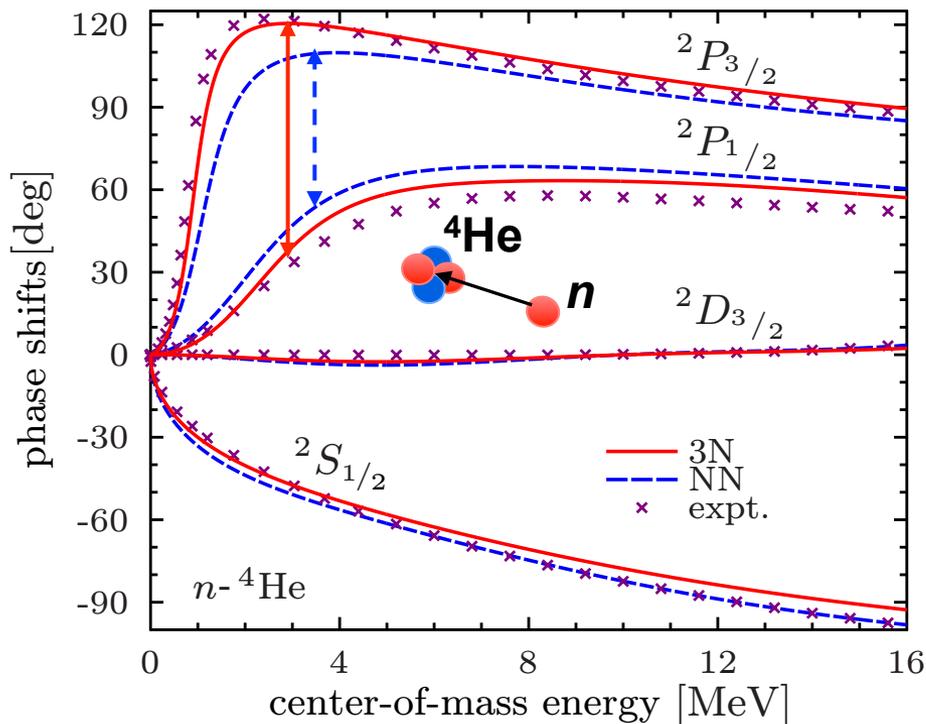
# Coupled NCSMC equations

$$\begin{array}{c}
 \begin{array}{c}
 \boxed{E_{\lambda}^{NCSM} \delta_{\lambda\lambda'}} \\
 \downarrow \text{blue} \\
 \left( \begin{array}{cc}
 H_{NCSM} & h \\
 h & H_{RGM}
 \end{array} \right) \begin{pmatrix} \textcircled{C} \\ \textcircled{\gamma} \end{pmatrix}
 \end{array}
 \quad = \quad E \quad
 \begin{array}{c}
 \boxed{\delta_{\lambda\lambda'}} \\
 \downarrow \text{blue} \\
 \left( \begin{array}{cc}
 1_{NCSM} & g \\
 g & N_{RGM}
 \end{array} \right) \begin{pmatrix} \textcircled{C} \\ \textcircled{\gamma} \end{pmatrix}
 \end{array}
 \\
 \\
 \begin{array}{c}
 \boxed{\langle (A) \left| H \hat{A}_v \right| (a) (A-a) \rangle} \\
 \downarrow \text{green} \\
 h \\
 \uparrow \text{red} \\
 \boxed{\langle (A-a) (a) \left| \hat{A}_v H \hat{A}_v \right| (a) (A-a) \rangle}
 \end{array}
 \quad
 \begin{array}{c}
 \boxed{\langle (A) \left| \hat{A}_v \right| (a) (A-a) \rangle} \\
 \downarrow \text{green} \\
 g \\
 \uparrow \text{red} \\
 \boxed{\langle (A-a) (a) \left| \hat{A}_v \hat{A}_v \right| (a) (A-a) \rangle}
 \end{array}
 \end{array}$$

Scattering matrix (and observables) from matching solutions to known asymptotic with microscopic  $R$ -matrix on Lagrange mesh

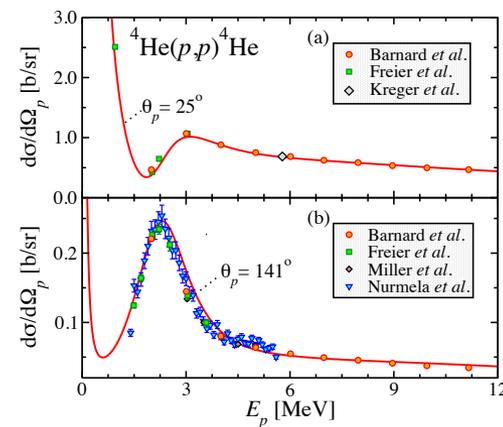
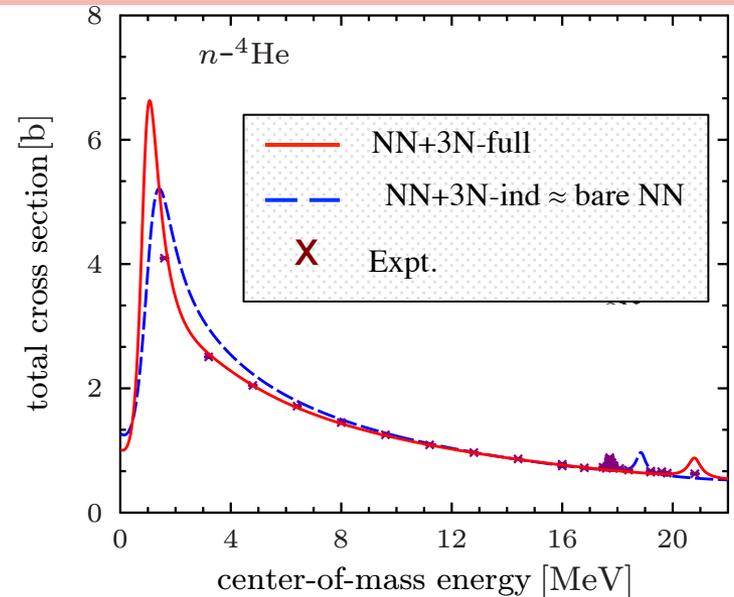
# $n$ - $^4\text{He}$ & $p$ - $^4\text{He}$ scattering within NCSMC

$n$ - $^4\text{He}$  scattering phase-shifts for chiral NN and NN+3N potential



3N force enhances  $1/2^- \leftrightarrow 3/2^-$  splitting; essential at low energies!

Total  $n$ - $^4\text{He}$  cross section with NN and NN+3N potentials



Differential  $p$ - $^4\text{He}$  cross section with NN+3N potentials

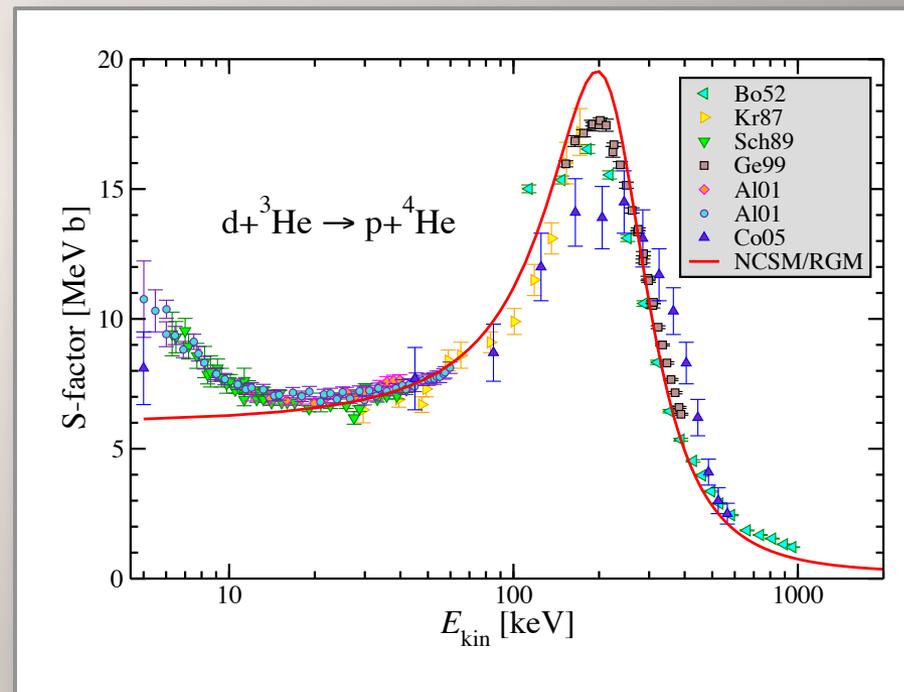
# NCSM/RGM calculations of transfer reactions

$$\int dr r^2 \left( \begin{array}{c} \left\langle \begin{array}{c} \mathbf{r}' \\ \alpha \\ n \end{array} \left| \hat{A}_1 (H - E) \hat{A}_1 \right| \begin{array}{c} \mathbf{r} \\ \alpha \\ n \end{array} \right\rangle \\ \left\langle \begin{array}{c} \mathbf{r}' \\ d \text{ } ^3\text{H} \end{array} \left| \hat{A}_2 (H - E) \hat{A}_1 \right| \begin{array}{c} \mathbf{r} \\ \alpha \\ n \end{array} \right\rangle \\ \left\langle \begin{array}{c} \mathbf{r}' \\ \alpha \\ n \end{array} \left| \hat{A}_1 (H - E) \hat{A}_2 \right| \begin{array}{c} \mathbf{r} \\ ^3\text{H} \\ d \end{array} \right\rangle \\ \left\langle \begin{array}{c} \mathbf{r}' \\ d \text{ } ^3\text{H} \end{array} \left| \hat{A}_2 (H - E) \hat{A}_2 \right| \begin{array}{c} \mathbf{r} \\ ^3\text{H} \\ d \end{array} \right\rangle \end{array} \right) \begin{pmatrix} \frac{g_1(r)}{r} \\ \frac{g_2(r)}{r} \end{pmatrix} = 0$$

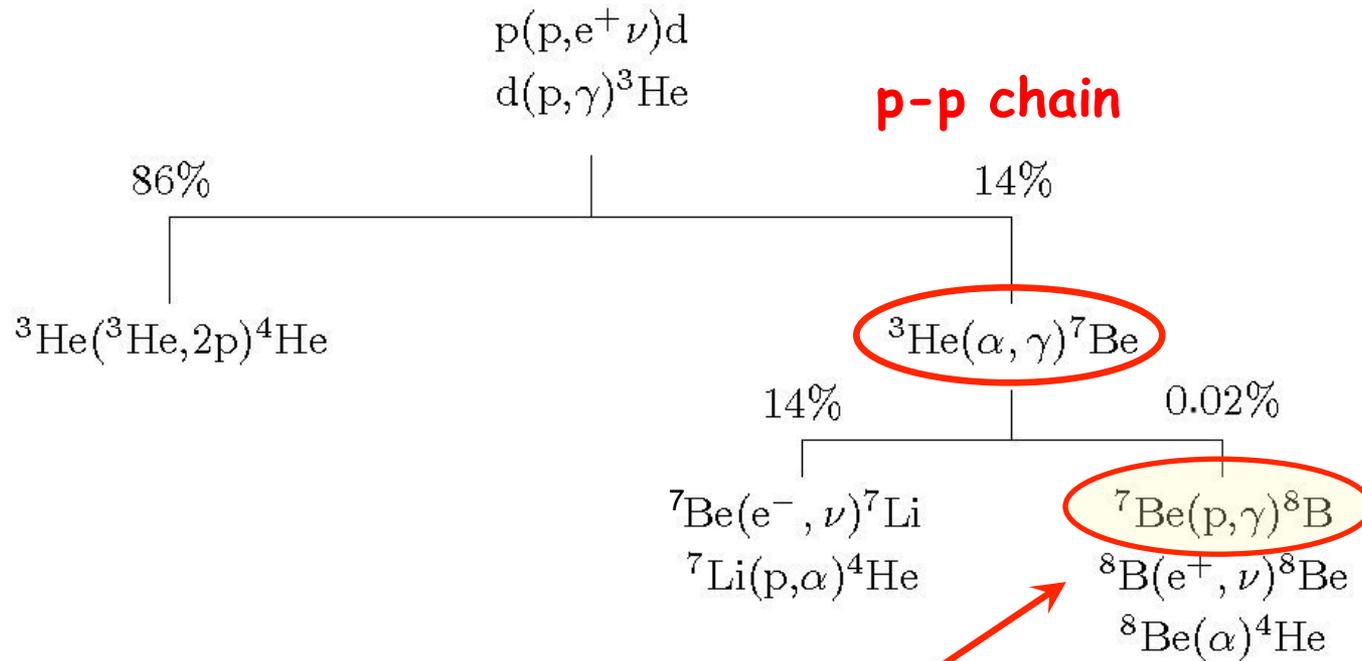
Straightforward to couple different mass partitions in the NCSM/RGM formalism

Applications to (d,p) and (d,n) reactions  
Example:  $^3\text{He}(d,p)^4\text{He}$

Work in progress:  
 $^7\text{Li}(d,p)^8\text{Li}$  &  $^8\text{Li}(d,p)^9\text{Li}$



# Solar $p$ - $p$ chain



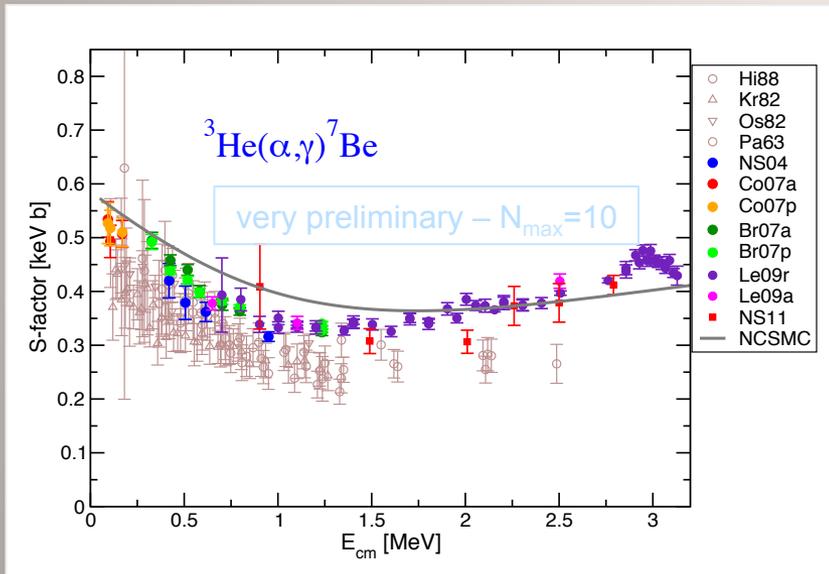
**Solar neutrinos**  
 $E_\nu < 15 \text{ MeV}$

# $^3\text{He}(^4\text{He}, \gamma)^7\text{Be}$ & $^7\text{Be}(p, \gamma)^8\text{B}$ radiative capture

- NCSMC & NCSM/RGM calculations
  - Soft NN potential (chiral SRG- $N^3\text{LO}$  with  $\Lambda = 2.1 \text{ fm}^{-1}$  &  $\Lambda = 1.86 \text{ fm}^{-1}$ )

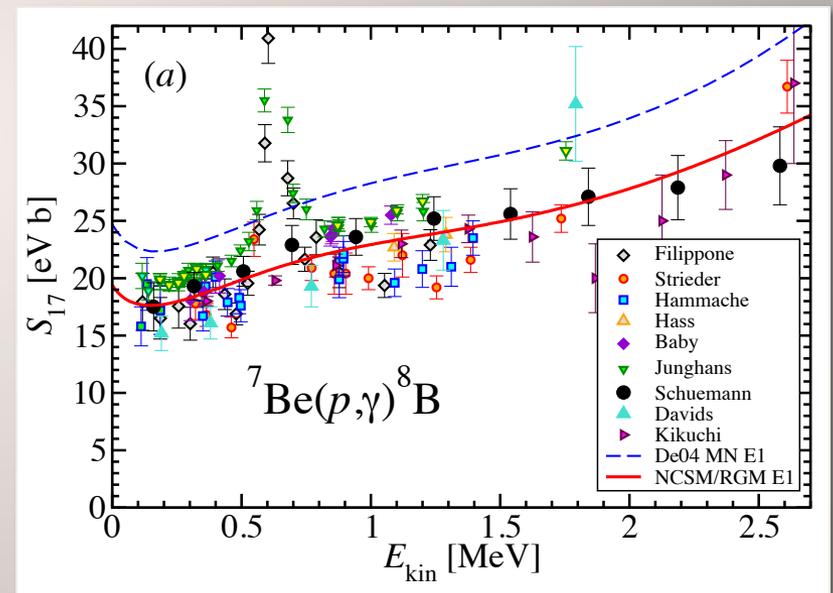
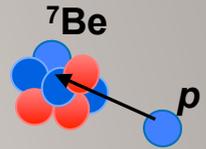
In progress

J. Dohet-Eraly, P.N., S. Quaglioni, W. Horiuchi, G. Hupin



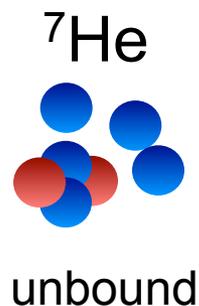
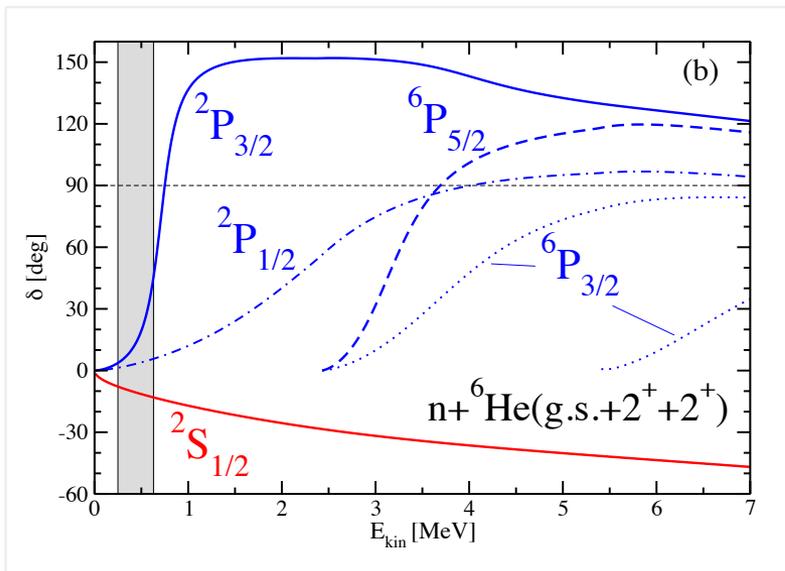
Preliminary:  $N_{\text{max}}=10$   
 $E_{\text{th}}(^7\text{Be}) = -1.32 \text{ MeV}$  (Expt.  $-1.59 \text{ MeV}$ )

P.N., R. Roth, S. Quaglioni,  
 Physics Letters B 704 (2011) 379



$^8\text{B}$   $2^+$  g.s. **bound** by 136 keV (expt. 137 keV)  
 $S(0) \sim 19.4(0.7) \text{ eV b}$   
 Current data evaluation:  $S(0) = 20.8(2.1) \text{ eV b}$

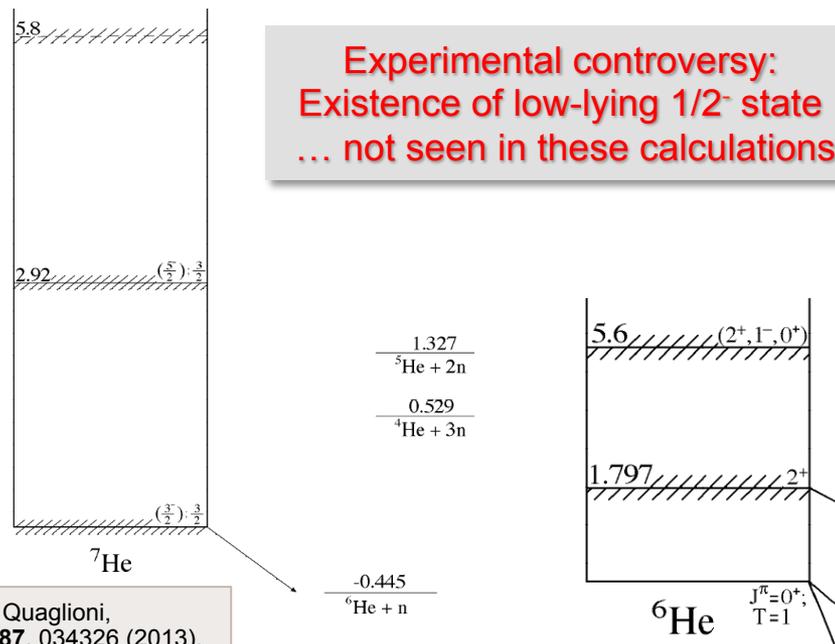
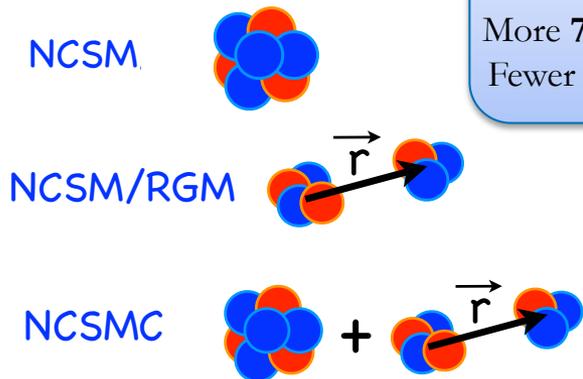
# NCSM with continuum: ${}^7\text{He} \leftrightarrow {}^6\text{He}+n$



$J^\pi$	experiment			NCSMC	
	$E_R$	$\Gamma$	Ref.	$E_R$	$\Gamma$
$3/2^-$	0.430(3)	0.182(5)	[2]	0.71	0.30
$5/2^-$	3.35(10)	1.99(17)	[40]	3.13	1.07
$1/2^-$	3.03(10)	2	[11]	2.39	2.89
	3.53	10	[15]		
	1.0(1)	0.75(8)	[5]		

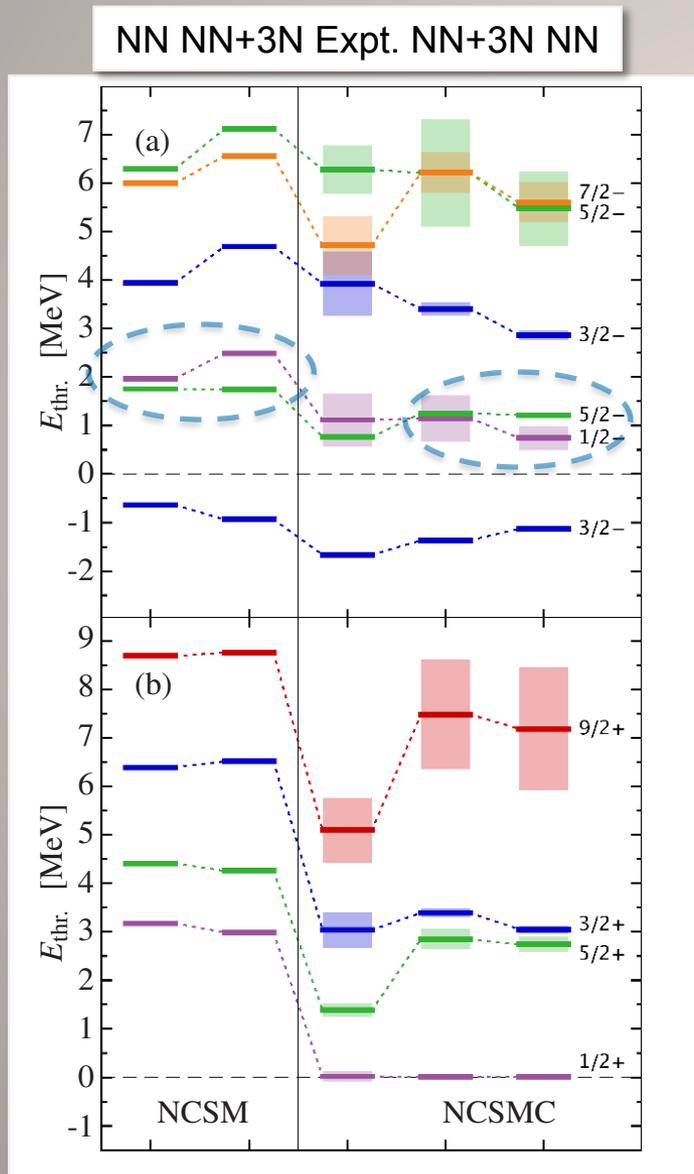
[11] A. H. Wuosmaa *et al.*, Phys. Rev. C **72**, 061301 (2005).

NCSMC  
with three  ${}^6\text{He}$  states  
and ten  ${}^7\text{He}$  eigenstates  
More 7-nucleon correlations  
Fewer  ${}^6\text{He}$ -core states needed



S. Baroni, P. N., and S. Quaglioni,  
PRL **110**, 022505 (2013); PRC **87**, 034326 (2013).

# Structure of ${}^9\text{Be}$ : bound states and resonances



${}^9\text{Be}$  is a stable nucleus  
 ... but all its excited states unbound  
 A proper description requires to include  
 effects of continuum

Three-nucleon interaction *and* continuum  
 improve agreement with experiment for  
 negative parity states

Continuum crucial for the description of  
 positive-parity states

J. Langhammer, P. N., G. Hupin, S. Quaglioni, A. Calci, R. Roth,  
 in preparation

# p+<sup>10</sup>C scattering: structure of <sup>11</sup>N resonances

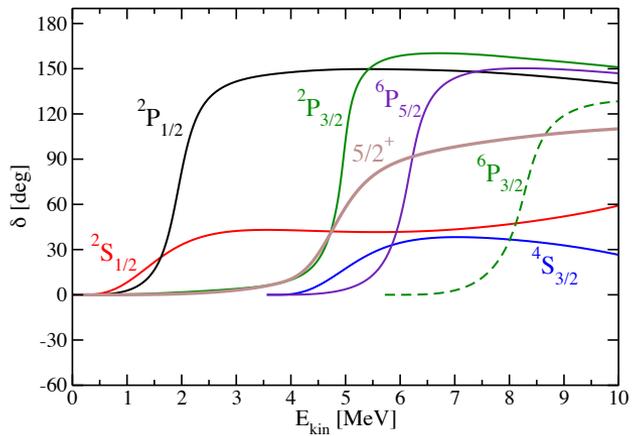
<sup>11</sup>N from chiral NN+3N within NCSMC

<sup>11</sup>N Expt. (TUNL evaluation)

– Preliminary

$J^\pi$	T	$E_{\text{res}}$ [MeV]	$E_x$ [MeV]	$\Gamma$ [keV]
				“4100”
✓ 1/2 <sup>+</sup>	3/2	1.35	0	580
✓ 1/2 <sup>-</sup>	3/2	1.94	0.59	280
✓ 3/2 <sup>-</sup>	3/2	4.69	3.34	1790
5/2 <sup>+</sup>	3/2	4.75	3.40	“4760”
3/2 <sup>+</sup>	3/2	4.95	3.60	470
5/2 <sup>-</sup>	3/2	5.95	4.60	620
3/2 <sup>-</sup>	3/2	7.68	6.33	

$E_{\text{res}}$ (MeV ± keV)	$E_x$ (MeV ± keV)	$J^\pi; T$	$\Gamma$ (keV)
1.49 ± 60	0	$\frac{1}{2}^+; \frac{3}{2}$	830 ± 30
2.22 ± 30	0.73 ± 70	$\frac{1}{2}^-$	600 ± 100
3.06 ± 80	(1.57 ± 80)		< 100
3.69 ± 30	2.20 ± 70	$\frac{5}{2}^+$	540 ± 40
4.35 ± 30	2.86 ± 70	$\frac{3}{2}^-$	340 ± 40
5.12 ± 80	(3.63 ± 100)	$(\frac{5}{2}^-)$	< 220
5.91 ± 30	4.42 ± 70	$(\frac{5}{2}^-)$	
6.57 ± 100	5.08 ± 120	$(\frac{3}{2}^-)$	100 ± 60



$$\Gamma = \frac{2}{\left. \frac{\partial \delta(E_{\text{kin}})}{\partial E_{\text{kin}}} \right|_{E_{\text{kin}}=E_R}}$$

Negative parity 1/2<sup>-</sup> and 3/2<sup>-</sup> resonances in a good agreement with the current evaluation

Positive parity resonances too broad  
– N<sub>max</sub> convergence

# p+<sup>10</sup>C scattering: structure of <sup>11</sup>N resonances

<sup>11</sup>N from chiral NN+3N within NCSMC

<sup>11</sup>N Expt. (TUNL evaluation)

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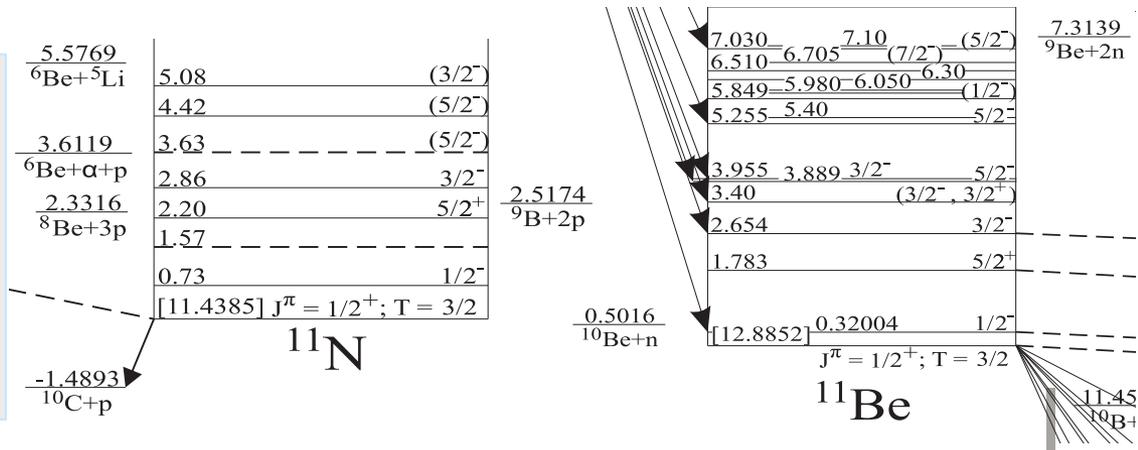
$E_{\text{res}}$ (MeV ± keV)	$E_x$ (MeV ± keV)	$J^\pi; T$	$\Gamma$ (keV)
1.49 ± 60	0	$\frac{1}{2}^+; \frac{3}{2}$	830 ± 30
2.22 ± 30	0.73 ± 70	$\frac{1}{2}^-$	600 ± 100
→ 3.06 ± 80	(1.57 ± 80)		< 100
3.69 ± 30	2.20 ± 70	$\frac{5}{2}^+$	540 ± 40
4.35 ± 30	2.86 ± 70	$\frac{3}{2}^-$	340 ± 40
→ 5.12 ± 80	(3.63 ± 100)	$(\frac{5}{2}^-)$	< 220
→ 5.91 ± 30	4.42 ± 70	$(\frac{5}{2}^-)$	
6.57 ± 100	5.08 ± 120	$(\frac{3}{2}^-)$	100 ± 60

No candidate for 3.06 MeV resonance

We predict only one 5/2<sup>-</sup> resonance below the 3/2<sup>-</sup><sub>2</sub>

Calculations suggest that either 5.12 MeV or 5.91 MeV resonance might be 3/2<sup>+</sup> instead

NCSMC resonance predictions more in line with assignments in <sup>11</sup>Be



# NCSM/RGM for three-body clusters: Structure of ${}^6\text{He}$

${}^4\text{He} + n + n$

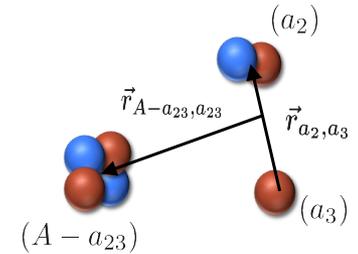
PRL 113, 032503 (2014) PHYSICAL REVIEW LETTERS week ending 18 JULY 2014

**${}^4\text{He} + n + n$  Continuum within an *Ab initio* Framework**

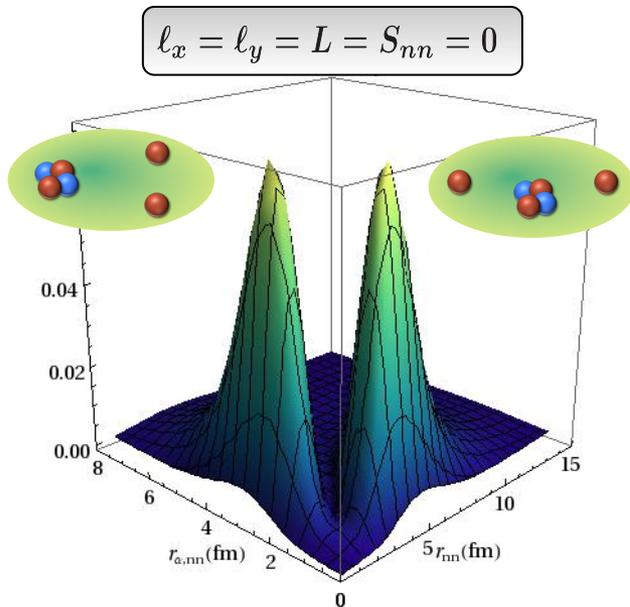
Carolina Romero-Redondo,<sup>1,\*</sup> Sofia Quaglioni,<sup>2,†</sup> Petr Navrátil,<sup>1,‡</sup> and Guillaume Hupin<sup>2,§</sup>

<sup>1</sup>TRIUMF, 4004 Wesbrook Mall, Vancouver, British Columbia V6T 2A3, Canada

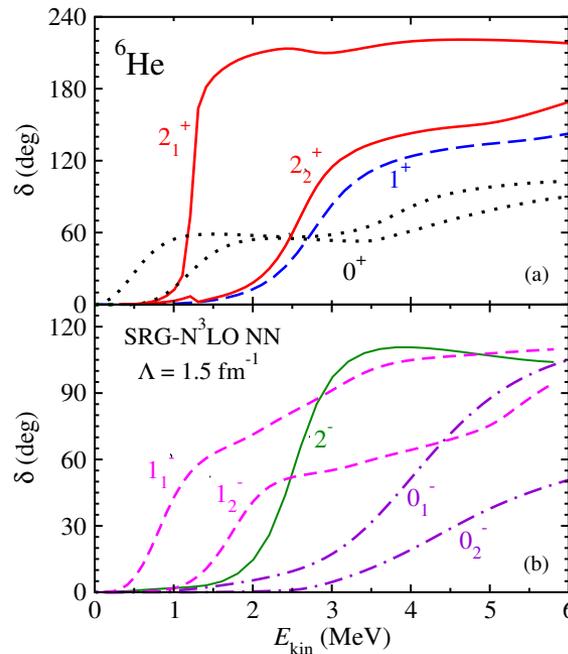
<sup>2</sup>Lawrence Livermore National Laboratory, P.O. Box 808, L-414, Livermore, California 94551, USA



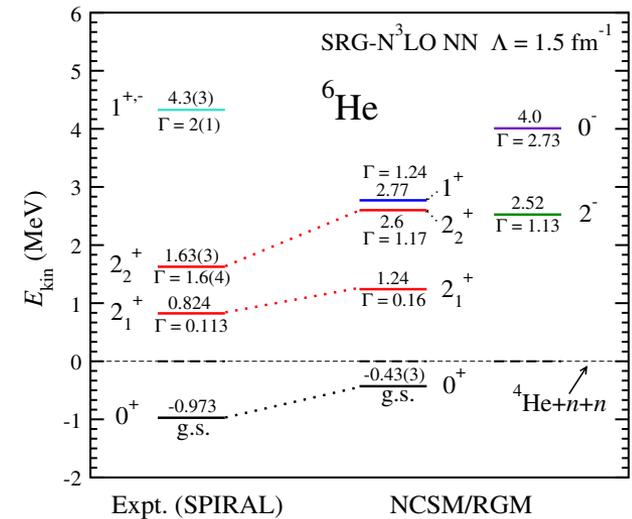
${}^6\text{He}$  bound  $0^+$  ground state



${}^6\text{He}$  resonances and continuum



Comparison to recent experiment



${}^5\text{H} \approx {}^4\text{He} + n + n$  in progress

# Conclusions and Outlook

- *Ab initio* calculations of nuclear structure & reactions is a dynamic field with rapid advances
- Several exact methods applicable to few-nucleon systems ( $A=3,4$ )
- Significant progress in *ab initio* approaches for  $p$ -shell nuclei
- New very successful approaches to medium mass nuclei
- We developed a new unified approach to nuclear bound and unbound states
  - Merging of the NCSM and the NCSM/RGM = **NCSMC**
- **Outlook:**
  - Applications to astrophysics
    - nuclear reactions important for astrophysics (and fusion energy generation)
    - equation of state, symmetry energy
  - Neutrino physics
    - neutrino-nucleus cross sections
    - double beta decay nuclear matrix elements
  - Fundamental symmetries
    - nuclear corrections (CKM unitarity...)
  - Strangeness
    - hypernuclei