

Dilepton production in pion-nucleon and pion-nucleus reactions

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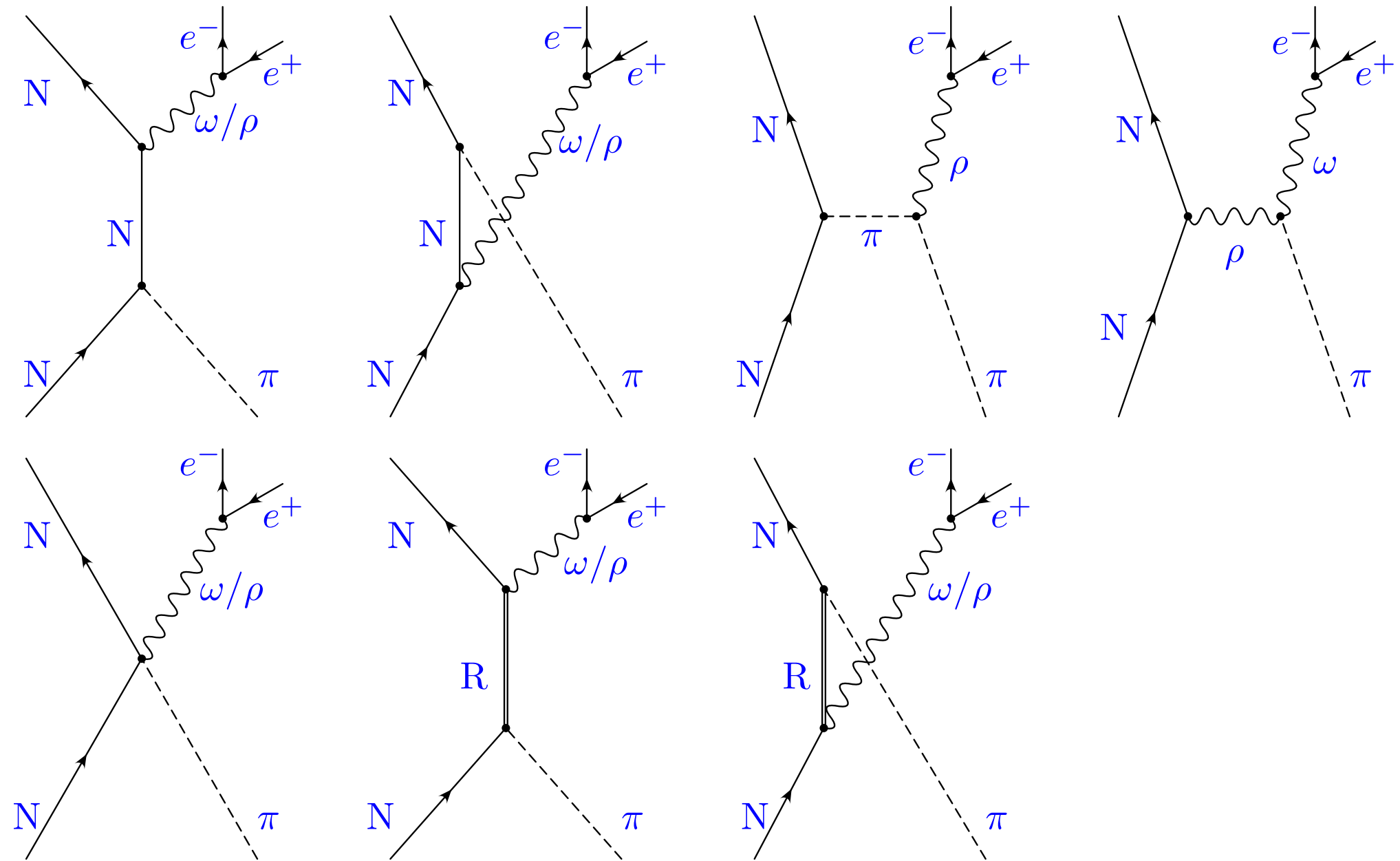
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Wigner RCP, Budapest

- πN reaction
- Transport equations for spectral functions
- Quantum interference in nuclear matter
- Summary

Feynman diagrams for $\pi + N \rightarrow N + e^+e^-$



Effective Field Theory for $\pi + N \rightarrow N + e^+e^-$

$$\mathcal{L}_{NN\pi} = -\frac{f_{NN\pi}}{m_\pi} \bar{\psi}_N \gamma_5 \gamma^\mu \vec{\tau} \psi_N \cdot \partial_\mu \vec{\pi}.$$

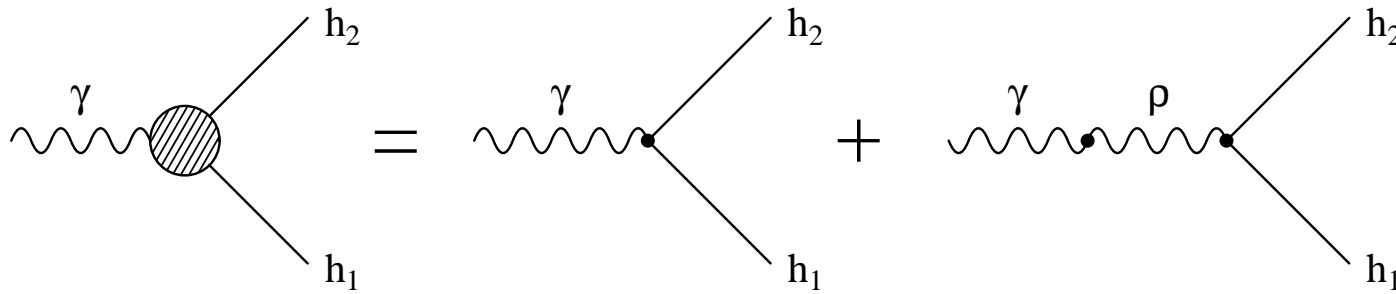
$$\mathcal{L}_{\omega\rho\pi} = \frac{g_{\omega\rho\pi}}{2} \epsilon^{\mu\nu\alpha\beta} \partial_\mu \omega_\nu \text{Tr}((\partial_\alpha \vec{\rho}_\beta \cdot \vec{\tau})(\vec{\pi} \cdot \vec{\tau}))$$

$$\mathcal{L}_{NN\rho} = g_\rho \bar{\psi}_N \left(\vec{\rho} - \kappa_\rho \frac{\sigma_{\mu\nu}}{4m_N} \vec{\rho}^{\mu\nu} \right) \cdot \vec{\tau} \psi_N, \quad \mathcal{L}_{NN\omega} = g_\omega \bar{\psi}_N \left(\psi - \kappa_\omega \frac{\sigma_{\mu\nu}}{4m_N} \omega^{\mu\nu} \right) \psi_N.$$

ρ_0 couples to $\bar{\psi}_N \tau_0 \psi_N$ so to p and to n with different signs, while ω with the same sign

Considering $\pi^- p \rightarrow n e^+ e^-$ and $\pi^+ n \rightarrow p e^+ e^-$ in one channel constructive and in the other channel destructive interference

Vector meson dominance



- $\mathcal{L}_{VDM1} = -\frac{em_\rho^2}{g_\rho} \rho_\mu^0 A^\mu$

The width of $R \rightarrow N\gamma$ and $R \rightarrow N\rho$ are not independent photons from ρ (ρ -width taken from PDG) already overestimate the γ -width

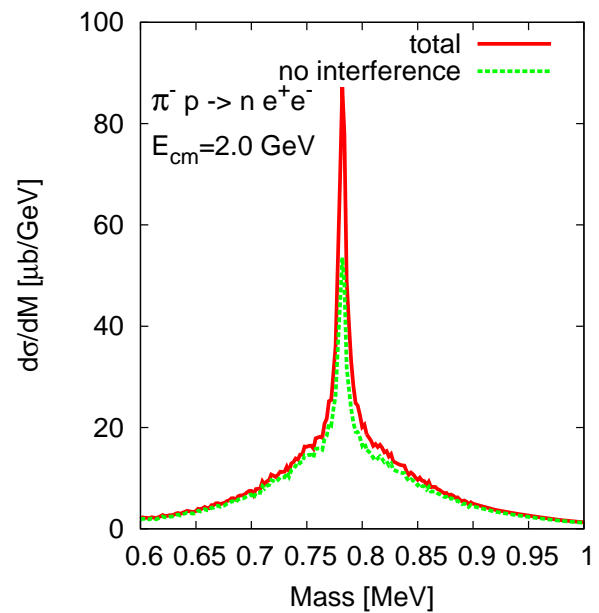
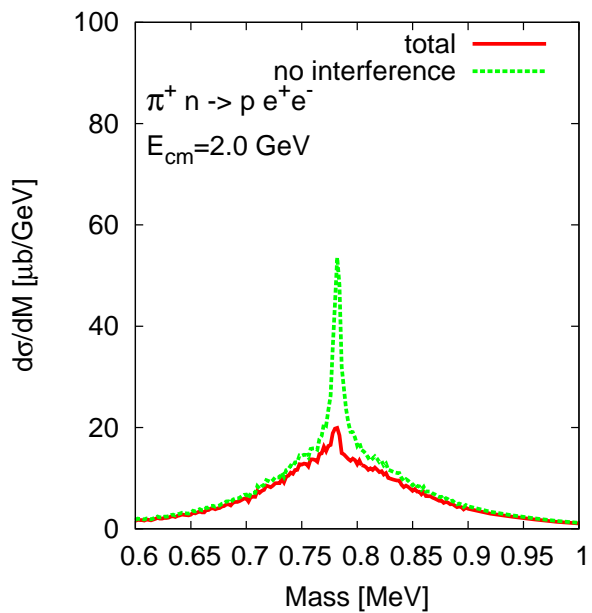
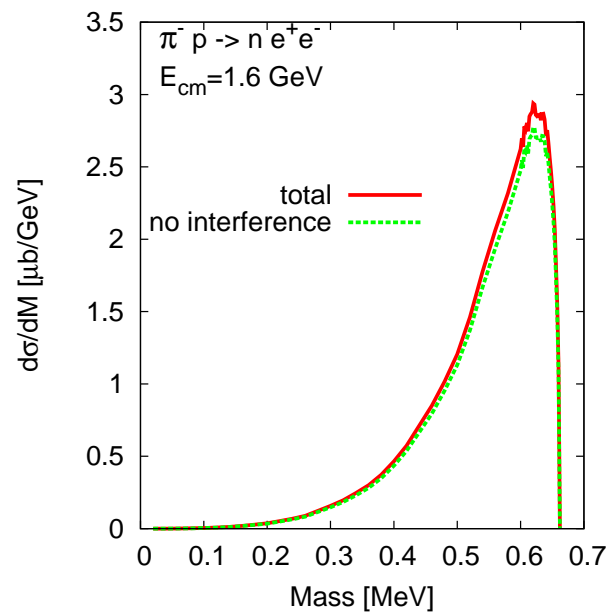
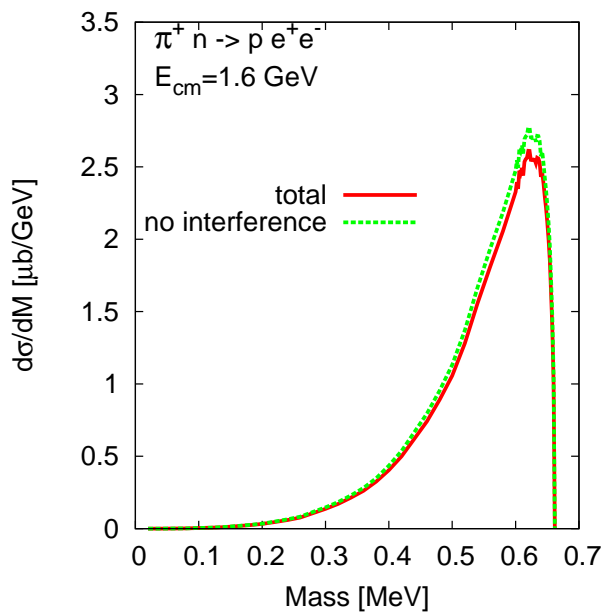
- $\mathcal{L}_{VMD2} = -\frac{e}{2g_\rho} F^{\mu\nu} \rho_{\mu\nu}^0$

From ρ -width the contribution to the photonic decay can be obtained by multiplying it with $\frac{e}{g_\rho} \frac{k^2}{m_\rho^2 - k^2 - iq\Gamma_\rho(k^2)}$

Decay through ρ does not contribute to the real photonic width.

We use VMD2. The final result depend on the choice, the ratio:

$$M_{dil}^2 / m_\rho^2$$



Dileptons in pion-nucleus collisions

- $\frac{d\sigma}{dM} \pi^- p \rightarrow n e^+ e^- (m_\omega) \approx 4$ because of the interference
- $\frac{d\sigma}{dM} \pi^+ n \rightarrow p e^+ e^- (m_\omega)$
- The effect is strong if ρ and ω production cross section is similar
- resonance contribution to ω production is neglected, taking them into account may change the result
- coupling constants were taken from the literature,
- M.F.M. Lutz, B. Friman, M. Soyeur, Nucl. Phys. A713 (2003) 97: the same problem was studied. They had much smaller ρ cross section, so the effect was strong at lower \sqrt{s}
- How much of this coherence survive in a nucleus?
- In a nucleus coherence is lost if one of the vector meson collides
- In a potential coherence decays by $\cos(E_\omega - E_\rho)dt$

- Boltzmann-Ühling-Uhlenbeck equation

$$\frac{\partial F}{\partial t} + \frac{\partial H}{\partial \mathbf{p}} \frac{\partial F}{\partial \mathbf{x}} - \frac{\partial H}{\partial \mathbf{x}} \frac{\partial F}{\partial \mathbf{p}} = \mathcal{C}, \quad H = \sqrt{(m_0 + U(\mathbf{p}, \mathbf{x}))^2 + \mathbf{p}^2}$$

- potential: momentum dependent, soft: $K=215$ MeV

$$U^{nr} = A \frac{n}{n_0} + B \left(\frac{n}{n_0} \right)^\tau + C \frac{2}{n_0} \int \frac{d^3 p'}{(2\pi)^3} \frac{f_N(x, p')}{1 + \left(\frac{\mathbf{p} - \mathbf{p}'}{\Lambda} \right)^2},$$

S. Teis, W. Cassing, M. Effenberger, A. Hombach, U. Mosel, Gy.

Wolf, Z. Phys. A359 (1997) 297-304,

Gy. Wolf et al., Phys.Atom.Nucl. 75 (2012) 718-720

- testparticle method

$$F = \sum_{i=1}^{N_{test}} \delta^{(3)}(\mathbf{x} - \mathbf{x}_i(t)) \delta^{(4)}(p - p_i(t)).$$

Collision term

- $NN \leftrightarrow NR, NN \leftrightarrow \Delta\Delta$
- baryon resonance can decay via 9 channels
 $R \leftrightarrow N\pi, N\eta, N\sigma, N\rho, N\omega, \Delta\pi, N(1440)\pi, K\Lambda, K\Sigma$
- 24 baryon resonances + Λ and Σ baryons
 $\pi, \eta, \sigma, \rho, \omega$ and kaons
- $\pi\pi \leftrightarrow \rho, \pi\pi \leftrightarrow \sigma, \pi\rho \leftrightarrow \omega$
- for resonances: energy dependent with
- $\frac{d\sigma^{X \rightarrow NR}}{dM_R} \sim A(M_R) \lambda^{0.5}(s, M_R^2, M_N^2)$

Spectral equilibration

- medium effects on the spectrum of hadrons (vector mesons)
- how they get on-shell (energy-momentum conservation)
- Field theoretical method (Kadanoff-Baym equation)
B. Schenke, C. Greiner, Phys.Rev.C73:034909,2006
- Off-shell transport
W. Cassing, S. Juchem, Nucl.Phys. A672 (2000) 417
S. Leupold, Nucl.Phys. A672 (2000) 475
- Spectral equilibration: Markov or memory effect

Off-shell transport

- Kadanoff-Baym equation for retarded Green-function
Wigner-transformation, gradient expansion

- transport equation for $F_\alpha = f_\alpha(x, p, t)A_\alpha$

$$A(p) = -2ImG^{ret} = \frac{\hat{\Gamma}}{(E^2 - \mathbf{p}^2 - m_0^2 - \text{Re}\Sigma^{ret})^2 + \frac{1}{4}\hat{\Gamma}^2},$$

W. Cassing, S. Juchem, Nucl.Phys. A672 (2000) 417

S. Leupold, Nucl.Phys. A672 (2000) 475

- testparticle approximation

Transport equations

- $$\frac{d\vec{X}_i}{dt} = \frac{1}{1-C_{(i)}} \frac{1}{2\epsilon_i} \left[2\vec{P}_i + \vec{\nabla}_{P_i} \text{Re}\Sigma_{(i)}^{\text{ret}} + \frac{\epsilon_i^2 - \vec{P}_i^2 - M_0^2 - \text{Re}\Sigma_{(i)}^{\text{ret}}}{\text{Im}\Sigma_{(i)}^{\text{ret}}} \vec{\nabla}_{P_i} \text{Im}\Sigma_{(i)}^{\text{ret}} \right]$$

$$\frac{d\vec{P}_i}{dt} = -\frac{1}{1-C_{(i)}} \frac{1}{2\epsilon_i} \left[\vec{\nabla}_{X_i} \text{Re}\Sigma_{(i)}^{\text{ret}} + \frac{\epsilon_i^2 - \vec{P}_i^2 - M_0^2 - \text{Re}\Sigma_{(i)}^{\text{ret}}}{\text{Im}\Sigma_{(i)}^{\text{ret}}} \vec{\nabla}_{X_i} \text{Im}\Sigma_{(i)}^{\text{ret}} \right]$$

$$\frac{d\epsilon_i}{dt} = \frac{1}{1-C_{(i)}} \frac{1}{2\epsilon_i} \left[\frac{\partial \text{Re}\Sigma_{(i)}^{\text{ret}}}{\partial t} + \frac{\epsilon_i^2 - \vec{P}_i^2 - M_0^2 - \text{Re}\Sigma_{(i)}^{\text{ret}}}{\text{Im}\Sigma_{(i)}^{\text{ret}}} \frac{\partial \text{Im}\Sigma_{(i)}^{\text{ret}}}{\partial t} \right]$$

- where $C_{(i)}$ renormalization factor

$$C_{(i)} = \frac{1}{2\epsilon_i} \left[\frac{\partial}{\partial \epsilon_i} \text{Re}\Sigma_{(i)}^{\text{ret}} + \frac{\epsilon_i^2 - \vec{P}_i^2 - M_0^2 - \text{Re}\Sigma_{(i)}^{\text{ret}}}{\text{Im}\Sigma_{(i)}^{\text{ret}}} \frac{\partial}{\partial \epsilon_i} \text{Im}\Sigma_{(i)}^{\text{ret}} \right]$$

- the last equation for homogenous system can be rewritten as

$$\frac{dM_i^2}{dt} = \frac{d(\epsilon_i^2 - P_i^2)}{dt} = \frac{d\text{Re}\Sigma_{(i)}^{\text{ret}}}{dt} + \frac{M_i^2 - M_0^2 - \text{Re}\Sigma_{(i)}^{\text{ret}}}{\text{Im}\Sigma_{(i)}^{\text{ret}}} \frac{d\text{Im}\Sigma_{(i)}^{\text{ret}}}{dt}$$

Analytical solution for homogenous system

$$\frac{d}{dt}(M_i(t)^2 - M_0^2 - \text{Re}\Sigma_i) = \frac{M_i^2(t) - M_0^2 - \text{Re}\Sigma_i}{\text{Im}\Sigma_i} \frac{d\text{Im}\Sigma_i}{dt}$$

If the mass of the testparticle as just at the peak ($M_0^2 + \text{Re}\Sigma_i$) then it remains there

$$\frac{M_i^2(t) - M_0^2 - \text{Re}\Sigma_i(t)}{M_i^2(0) - M_0^2 - \text{Re}\Sigma_i(0)} = \frac{\text{Im}\Sigma_i(t)}{\text{Im}\Sigma_i(0)}$$

If $A(M^2, t)$ is the spectral function of a Breit-Wigner form at t then

$$f(M'^2, t) = f(M^2, 0) \frac{dM^2}{dM'^2} = A(M^2, 0) \frac{dM^2}{dM'^2} = A(M^2, 0) \frac{\text{Im}\Sigma(0)}{\text{Im}\Sigma(t)} = A(M'^2, t)$$

The mass distribution agrees always with the spectral function at that point.

Medium effects

- imaginary part (collisional broadening):

$$\Gamma = \Gamma_{vac} + nv\sigma\gamma$$

- real part (mass shift)

$$M = M_{vac} + n/n_o\Delta M$$

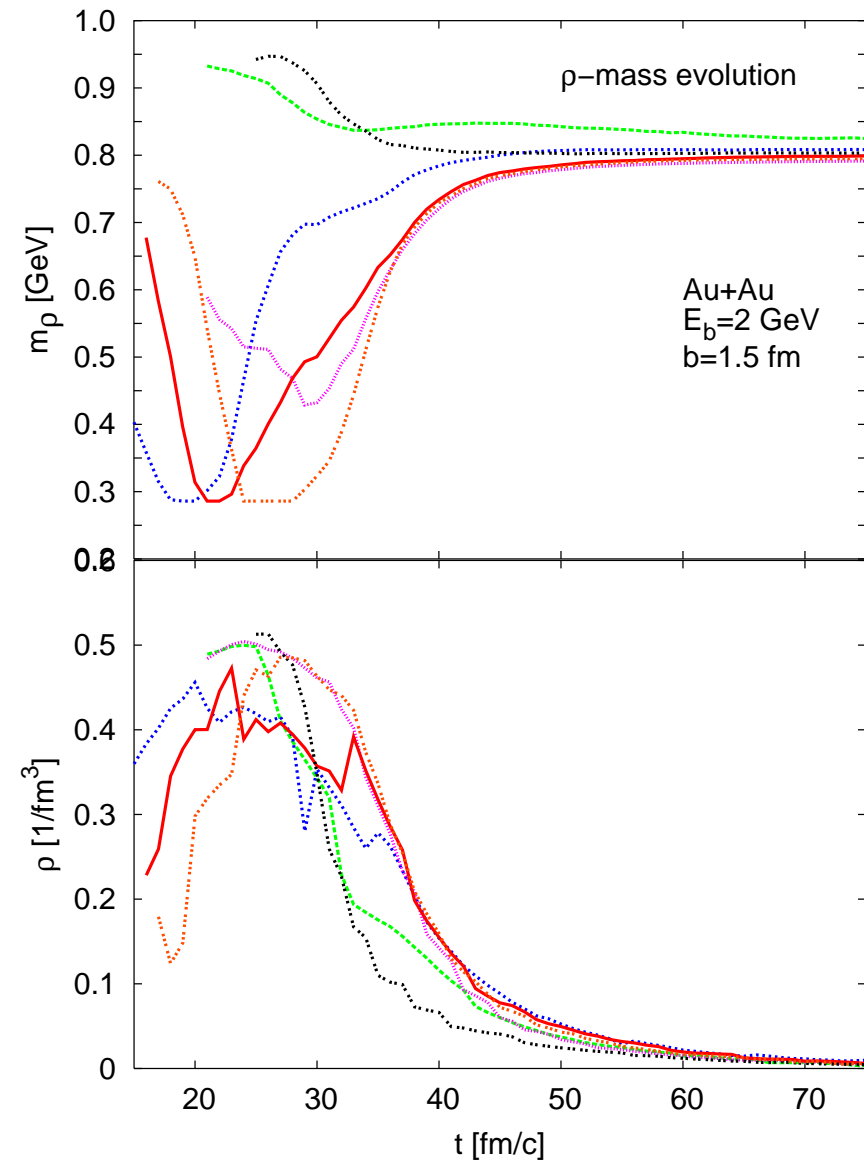
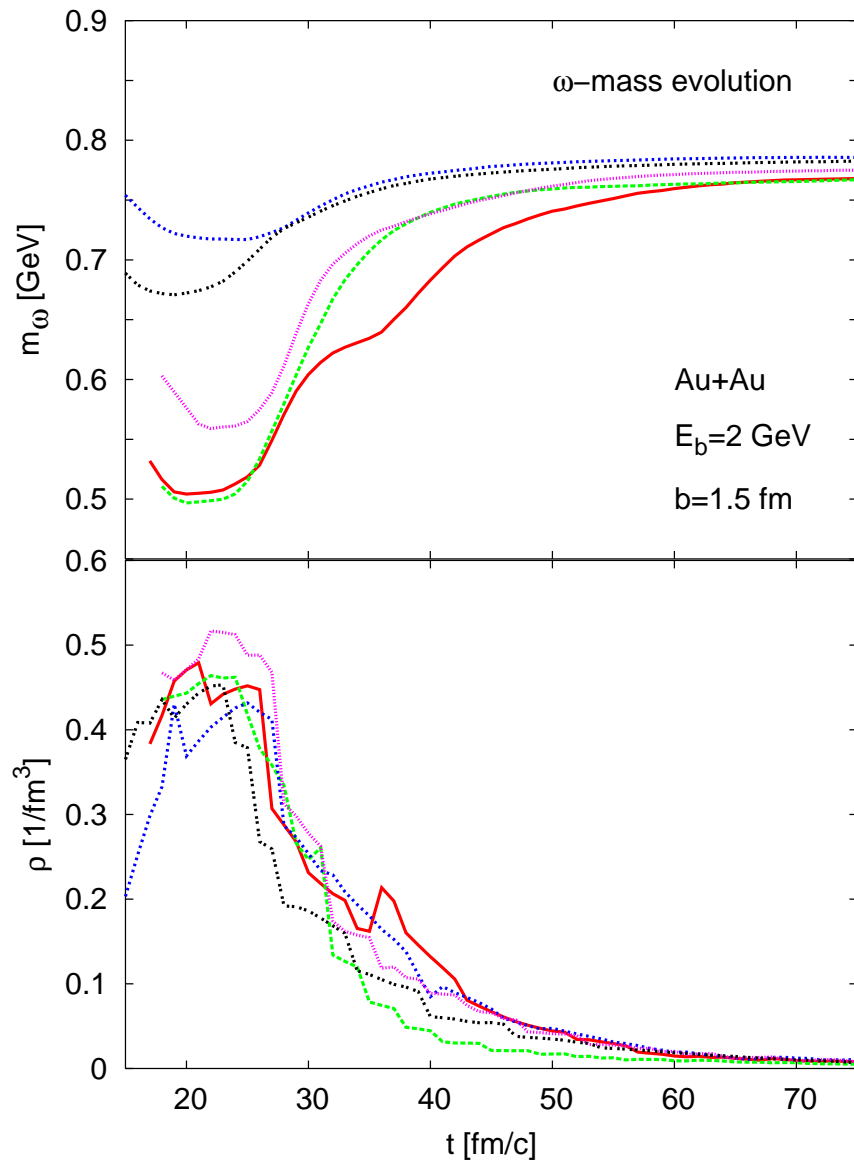
$$\Delta M_\omega = -50 \text{ MeV}, \Delta M_\rho = -120 \text{ MeV}$$

- danger of double counting

collision term already contains partly the mixing of mesons with resonance-hole excitations

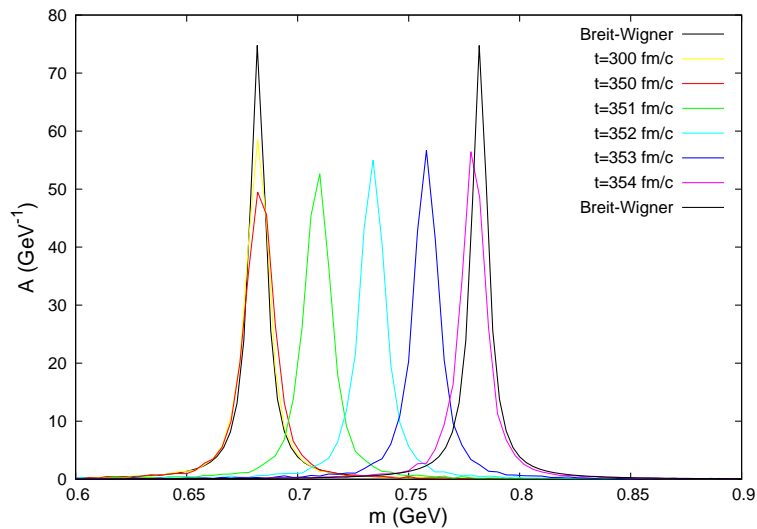
but sum up only to finite order

Evolution of masses in heavy ion collisions

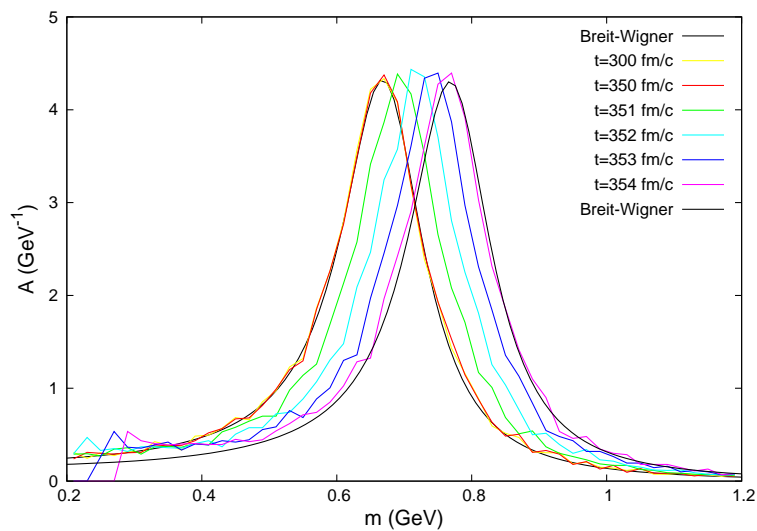


Evolution of mass distribution in a box

the vector meson masses are shifted linearly with density, and change the density linearly from ρ_0 to 0 in 4 fm/c:



ω



ρ

Simulation of π A collisions

- Same as usually except for $\pi N \rightarrow Ne^+e^-$
- in case of a πN collision several “doublets” are created.
(The original π and N do not change their state.)
A doublet consists of 2 perturbative particles ρ and ω with their cross sections and the “cross section” of the interference term. ρ and ω are created with the same position, momentum and mass.
- They propagate, decay and can be absorbed. The interference term contribute to the “decays”.
- Propagation: perturbative ρ 's and ω 's propagate in the surrounding medium
- Absorption: ρ 's and ω 's can be absorbed by a nucleon

Decays

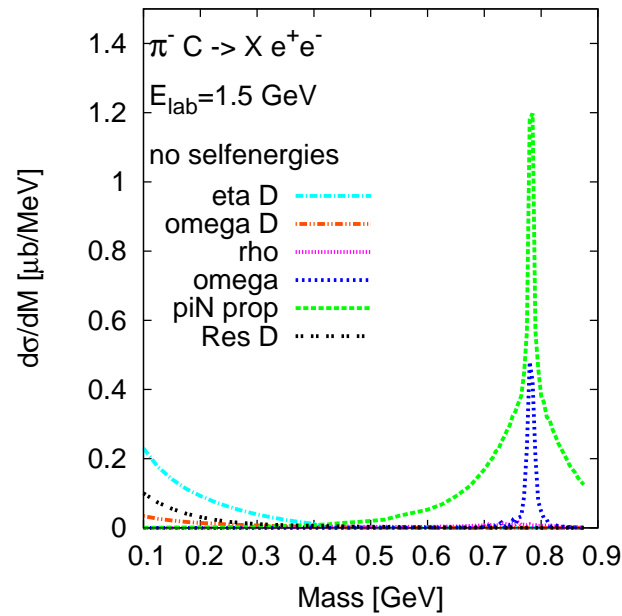
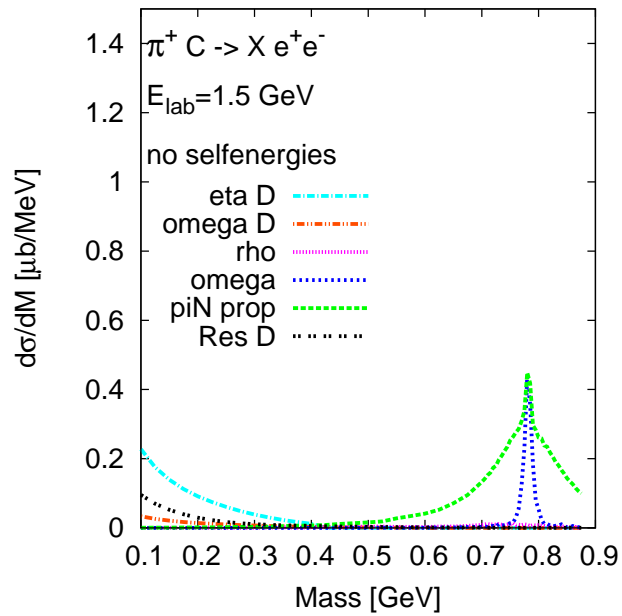
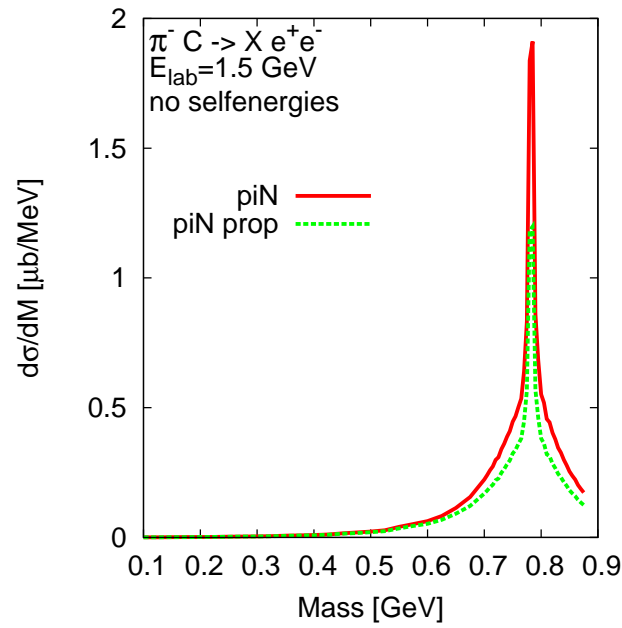
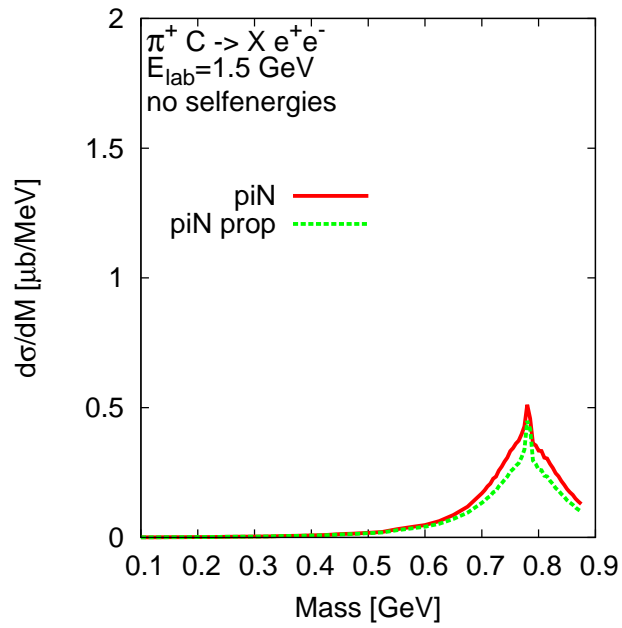
- Denote the probability that the ρ and ω decay until the i th timestep as α_i and β_i , respectively (they decay according to their total width).
- $\alpha_i \leq 1$. At the end it is 1 if not absorbed. The same is for ω .
- In the n th timestep the ρ contribution to the dilepton yield:
 $(\alpha_n - \alpha_{n-1}) \sigma^{\pi N \rightarrow N \rho \rightarrow N e^+ e^-}$. Similarly for ω .

The contribution of the interference term is:

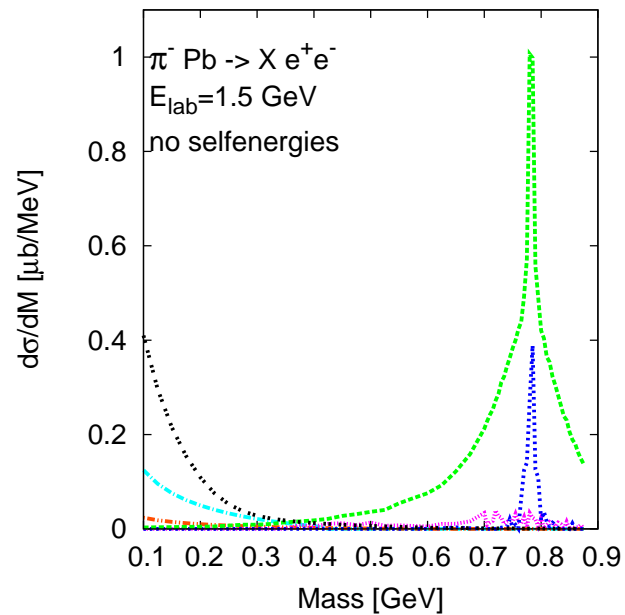
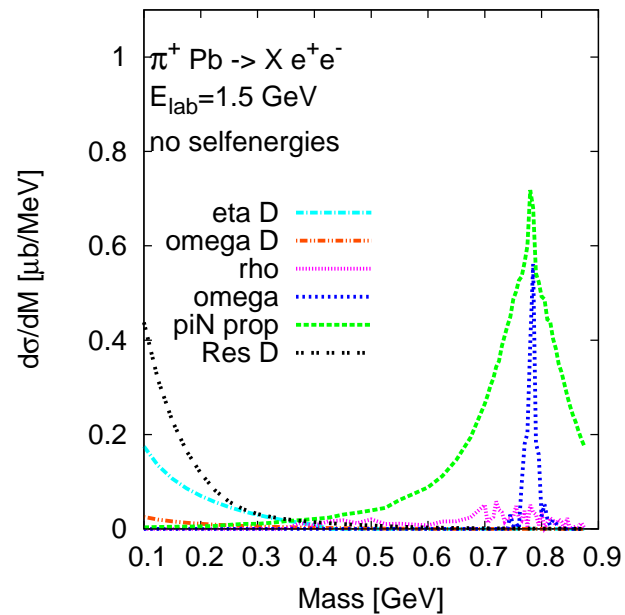
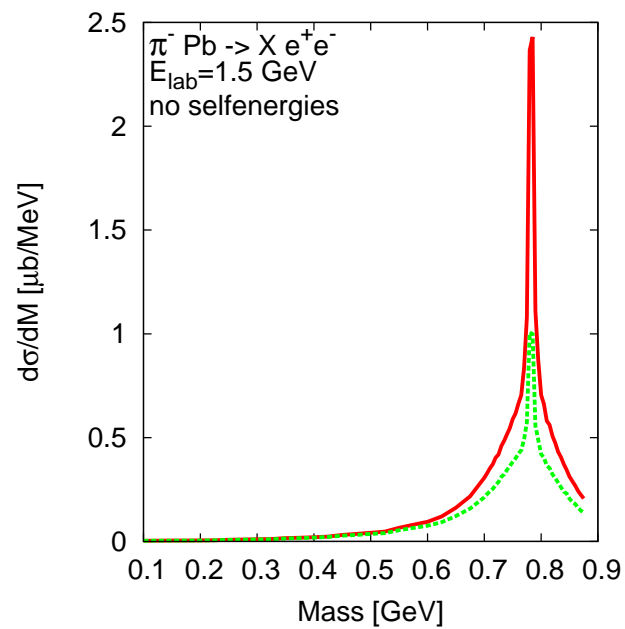
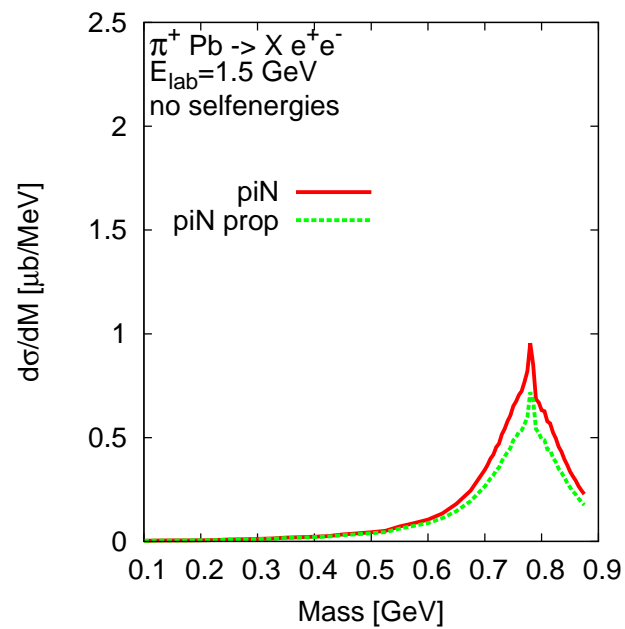
$$(\alpha_n \beta_n - \alpha_{n-1} \beta_{n-1}) \cos((E_\rho - E_\omega) \Delta t) \sigma^{\pi N \rightarrow N \rho - \omega \rightarrow N e^+ e^-}.$$

In vacuum it reproduces the original cross section.

π C, 1.5 GeV, no selfenergies, Preliminary results



π Pb, 1.5 GeV, no selfenergies, Preliminary results



Dileptons in pion-nucleus collisions

- $\frac{\frac{d\sigma}{dM} \pi^- p \rightarrow n e^+ e^- (m_\omega)}{\frac{d\sigma}{dM} \pi^+ n \rightarrow p e^+ e^- (m_\omega)} \approx 4$
- $\frac{\frac{d\sigma}{dM} \pi^- C^{12} \rightarrow X e^+ e^- (m_\omega)}{\frac{d\sigma}{dM} \pi^+ C^{12} \rightarrow X e^+ e^- (m_\omega)} \approx 2.9$
- $\frac{\frac{d\sigma}{dM} \pi^- Pb^{207} \rightarrow X e^+ e^- (m_\omega) / N_p}{\frac{d\sigma}{dM} \pi^+ Pb^{207} \rightarrow X e^+ e^- (m_\omega) / N_n} \approx 2.0$

In case of complete decoherence these ratios should be 1.

- Experimentally the decoherence can be observed in strongly interacting matter.

Summary

- Dilepton production in πN and πA an unique way to study quantum interference inside strongly interacting matter by measuring on nucleon, on light and on heavy nuclei.
- Dilepton production in πN and πA provides us the possibility to study the vector meson spectral function in matter.
- Make own fit to vector meson production (including the resonances and their interference)
- Take into account the selfenergies for the vector mesons

Cross sections

Elastic baryon-baryon cross section is fitted to the elastic pp data

Meson absorption cross sections are given by

$$\sigma_{\pi N \rightarrow R} = \frac{4\pi}{p^2} (\text{spin factors}) \frac{\Gamma_{in} \Gamma_{tot}}{(s - m_R^2) + s \Gamma_{tot}^2}$$

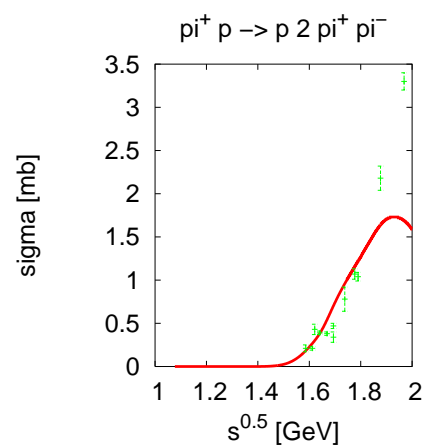
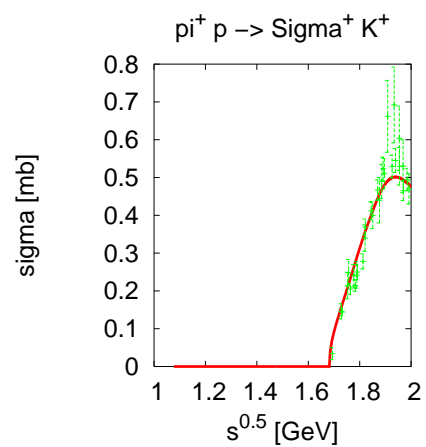
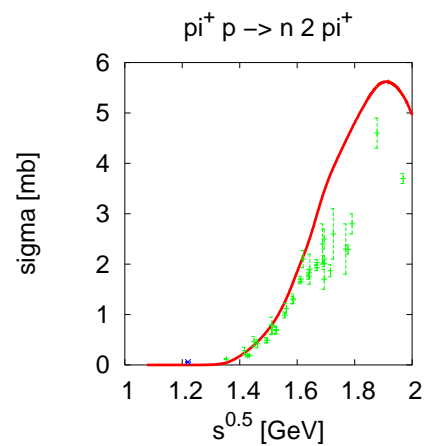
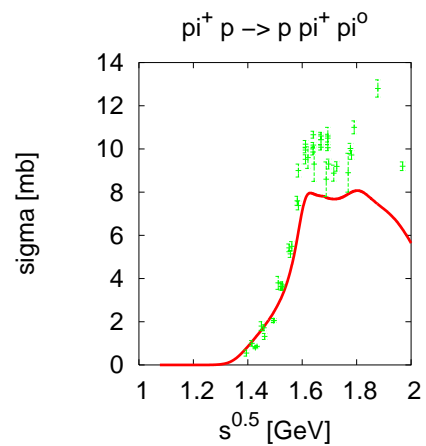
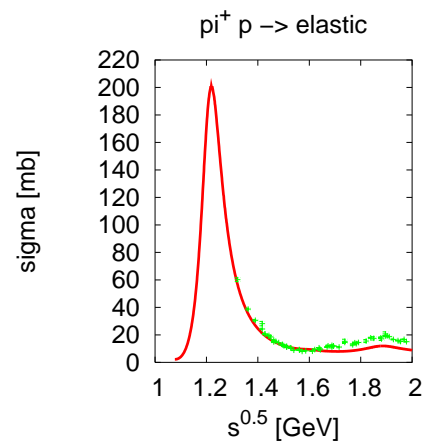
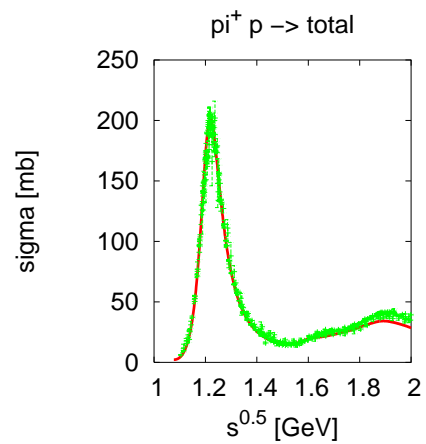
Baryon resonance parameters: mass, width, branching ratios are fitted by describing the meson production channels in πN collisions:

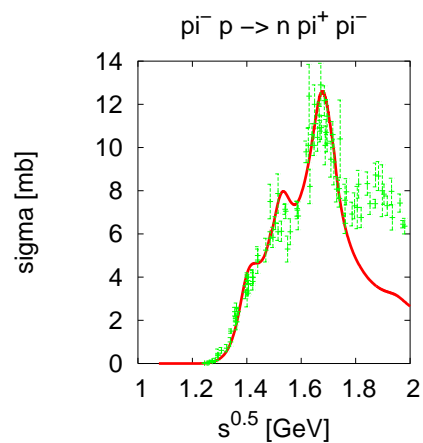
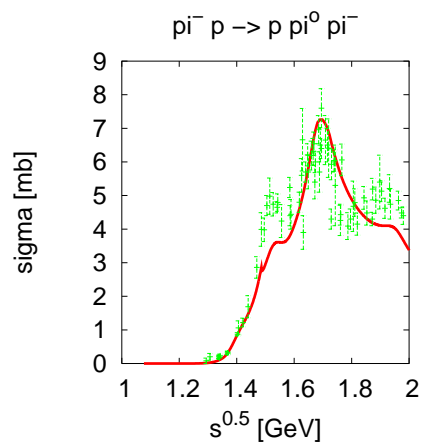
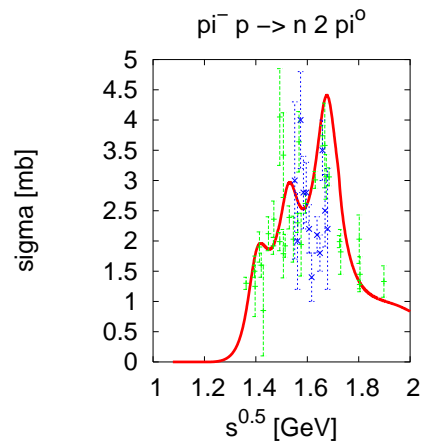
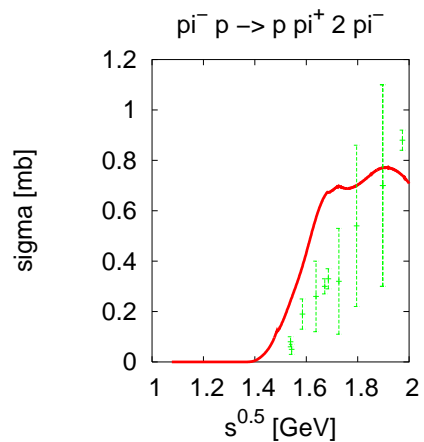
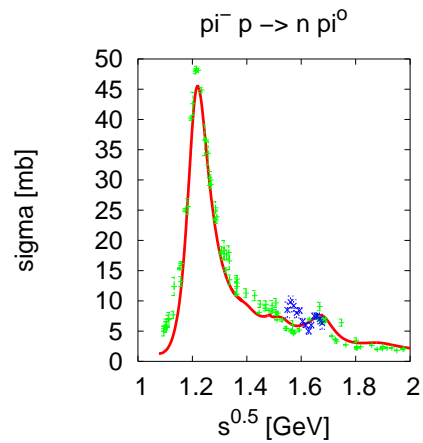
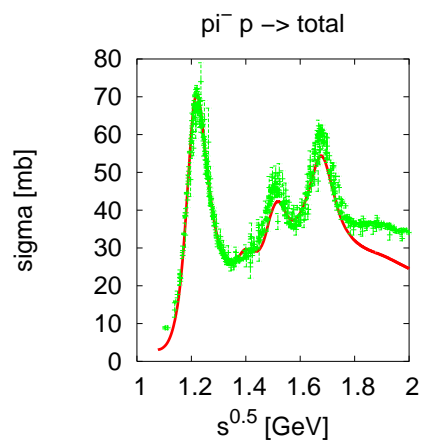
$$\sigma_{\pi N \rightarrow NM} = \sum_R \sigma_{\pi N \rightarrow R} \frac{\Gamma_{R \rightarrow NM}}{\Gamma_{tot}}$$

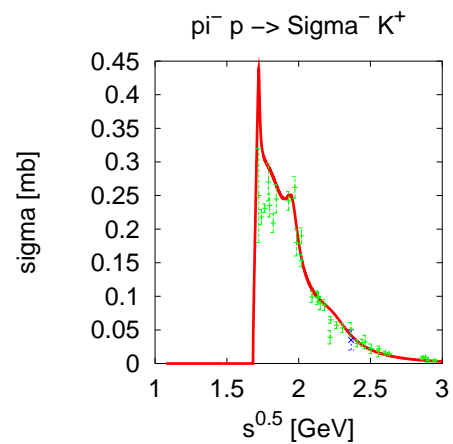
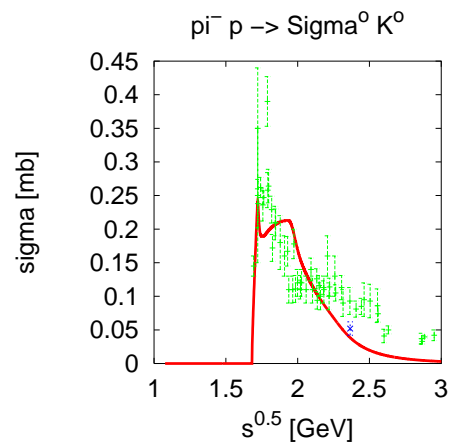
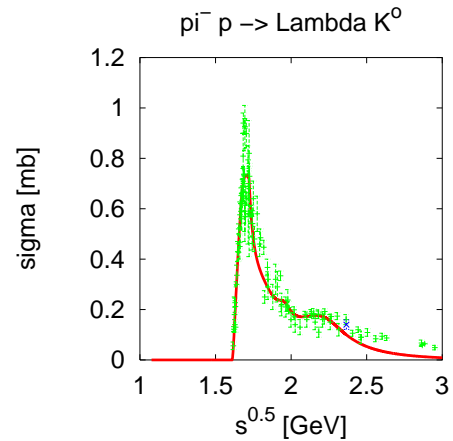
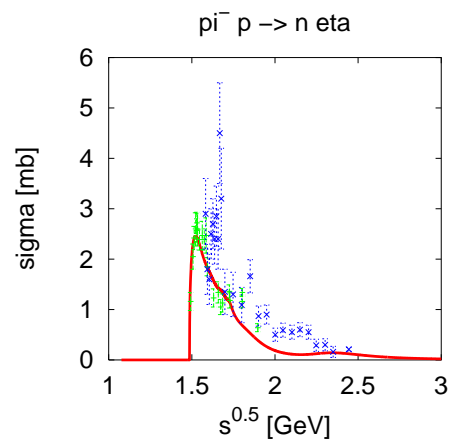
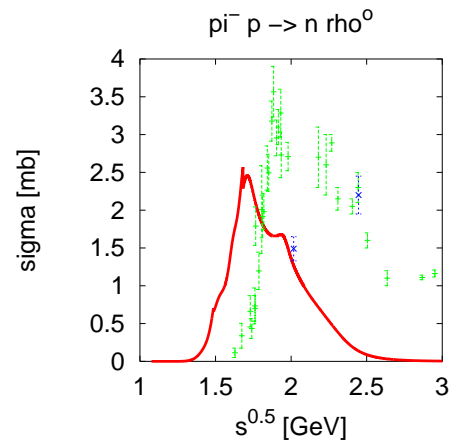
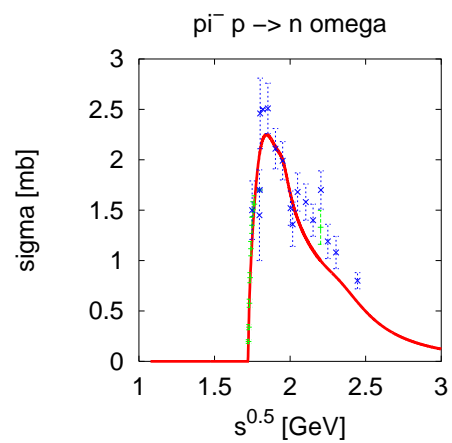
Resonance production cross section $NN \rightarrow NR$ is given by the fit of

$$\sigma_{NN \rightarrow NM} = \sum_R \sigma_{NN \rightarrow NR} \frac{\Gamma_{R \rightarrow NM}}{\Gamma_{tot}}$$

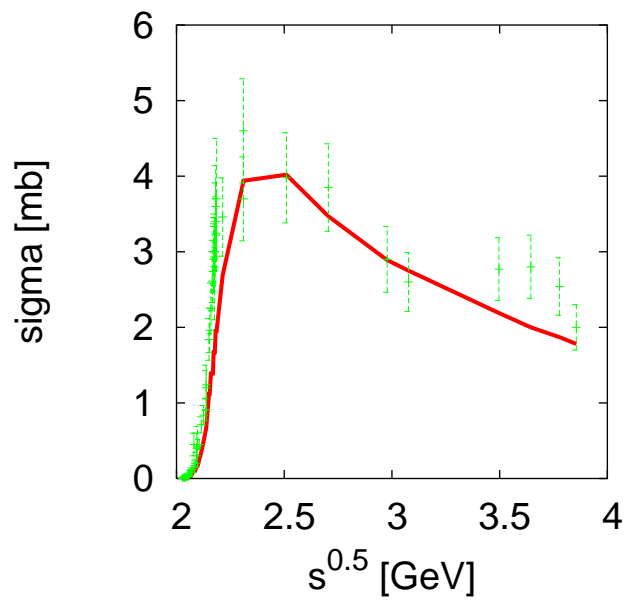
27 baryons, 6 mesons. Fit is done by the Minuit package (CERN)



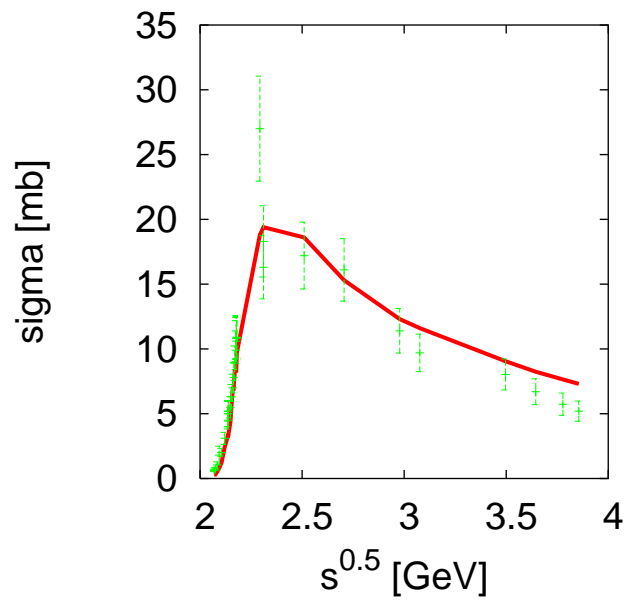




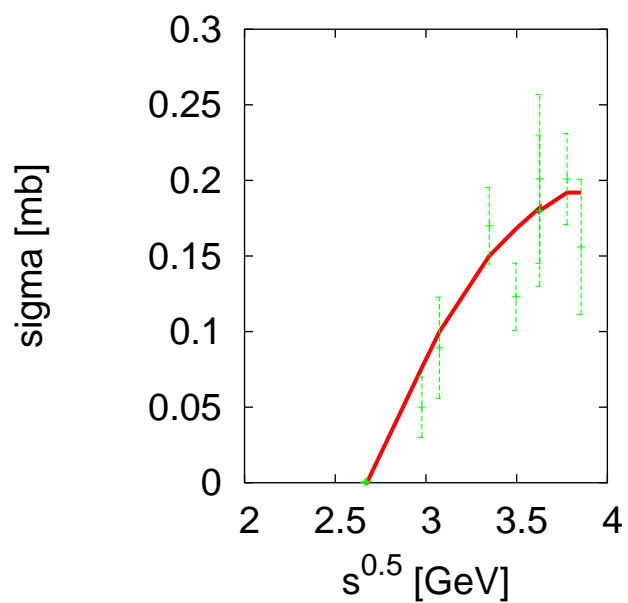
pp→pp pi⁰



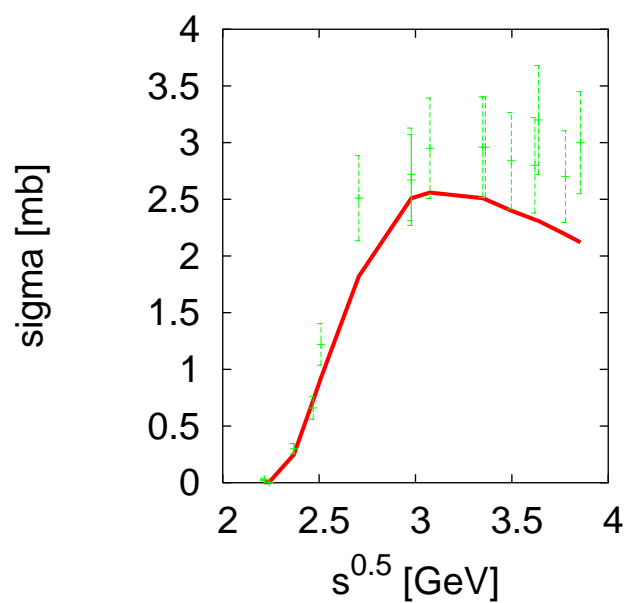
pp→pn pi⁺



pp→pp omega



pp→pp pi⁺ pi⁻



Kadanoff-Baym Equation

Schwinger-Dyson equation:

$$G = G_0 + G_0 \Sigma G$$

$$G^{11}(1, 2) = G^T(1, 2) = \langle T(\phi(1)\phi(2)) \rangle$$

$$G^{22}(1, 2) = G^{AT}(1, 2) = \langle \tilde{T}(\phi(1)\phi(2)) \rangle$$

$$G^{21}(1, 2) = G^<(1, 2) = \langle \phi(2)\phi(1) \rangle$$

$$G^{12}(1, 2) = G^>(1, 2) = \langle \phi(1)\phi(2) \rangle$$

$$G^r(1, 2) = \theta(t_1 - t_2)(G^>(x_1, t_1; x_2, t_2) - G^<(x_1, t_1; x_2, t_2))$$

$$G^a(1, 2) = \theta(t_2 - t_1)(G^<(x_1, t_1; x_2, t_2) - G^>(x_1, t_1; x_2, t_2))$$

After some manipulation: *Kadanoff-Baym equation*:

$$(i\hbar\partial_{t_1} - H_0(1))G^<(1, 2) = \int d^3\Sigma^r(1, 3)G^<(3, 2) + \int d^3\Sigma^<(1, 3)G^a(3, 2)$$

$$(i\hbar\partial_{t_1} - H_0(1))G^r(1, 2) = \delta^4(1, 2) + \int d^3\Sigma^r(1, 3)G^r(3, 2)$$

Wigner-transformation

- Retarded propagator is not a distribution function
- Wigner transform:

$$r = x_1 - x_2 \quad , \quad R = x_1 + x_2$$

R (center of mass) dependence of propagators and selfenergies are weaker than the r dependence

$$G^r(R, P) = \int d^4r G^r(X + r, X - r)$$

- Gradient expansion in r . Neglect all terms with more than one derivative in R

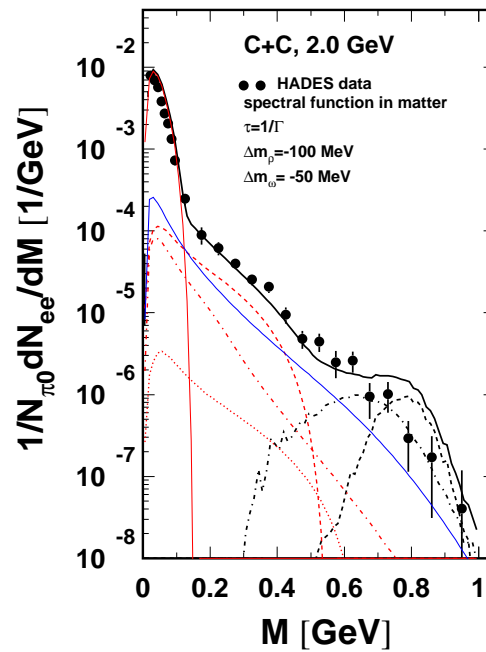
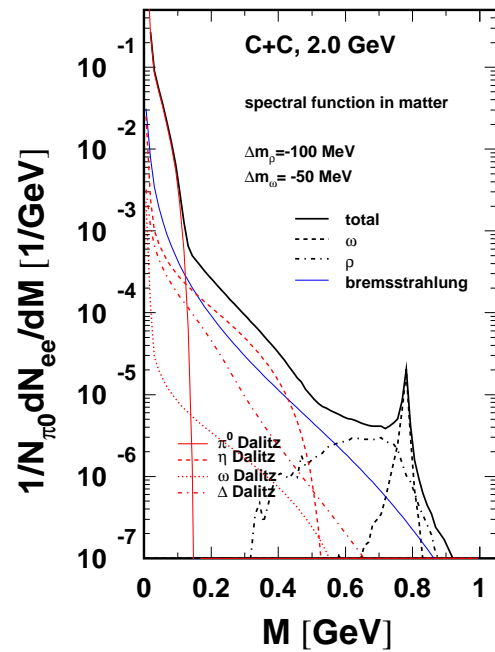
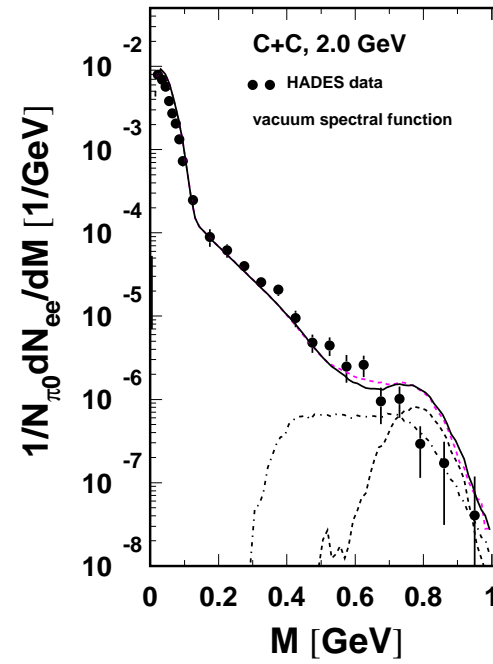
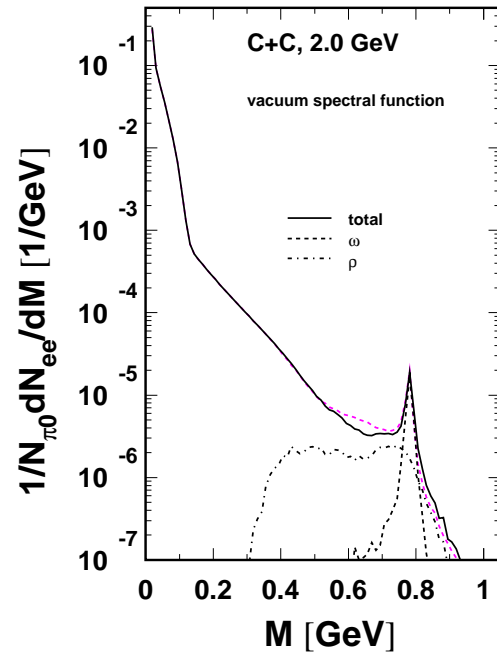
- transport equation for $F_\alpha = iG^<(R, P) = f_\alpha(x, p, t)A_\alpha$

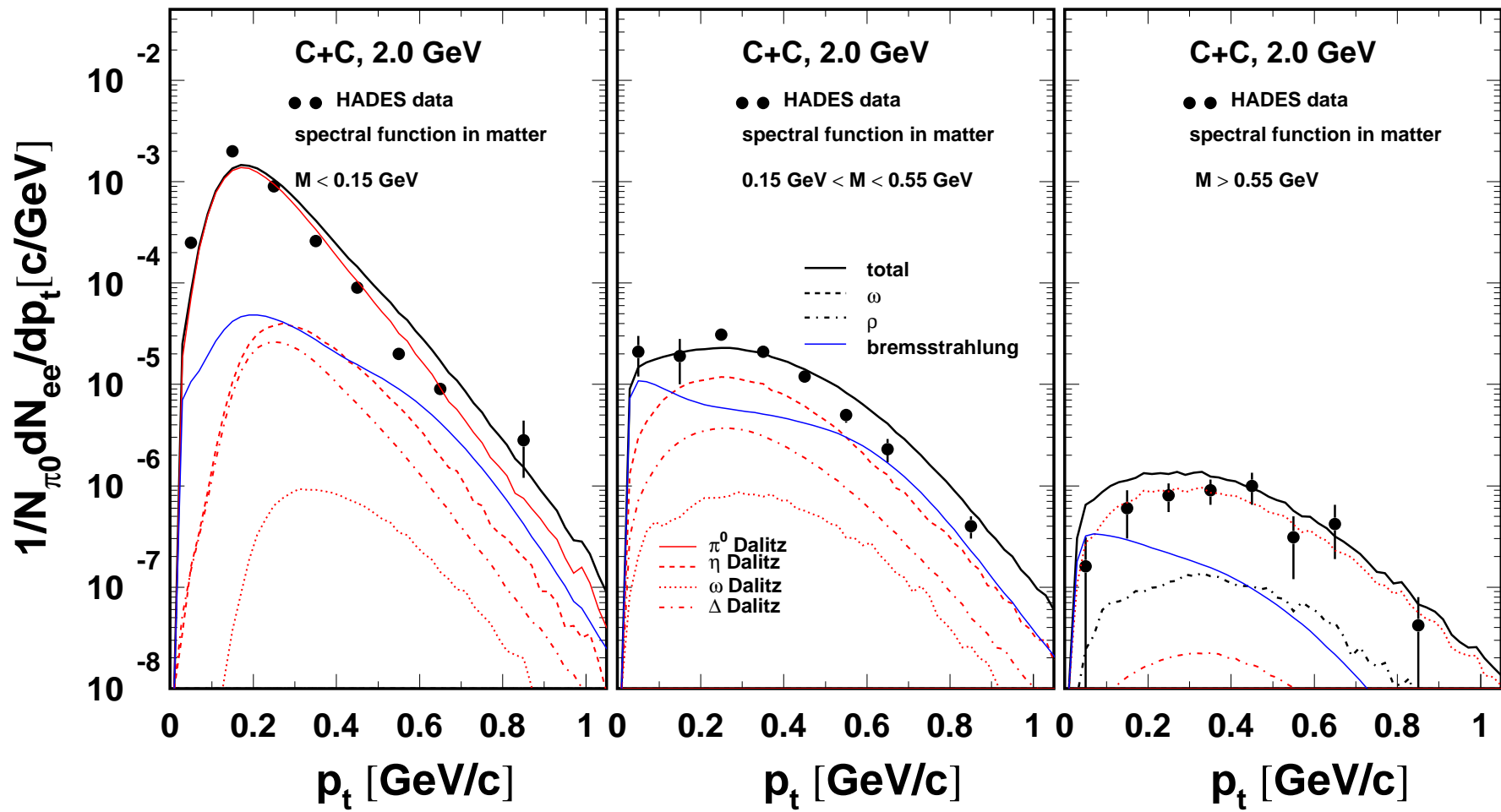
$$A(p) = -2ImG^r = \frac{\hat{\Gamma}}{(E^2 - \mathbf{p}^2 - m_0^2 - \text{Re}\Sigma^r)^2 + \frac{1}{4}\hat{\Gamma}^2},$$

Cassing, Juchem (2000) and Leupold (2000)

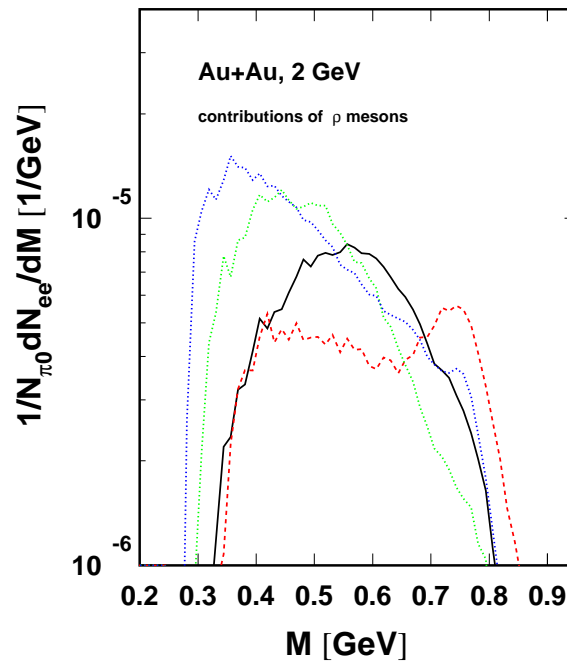
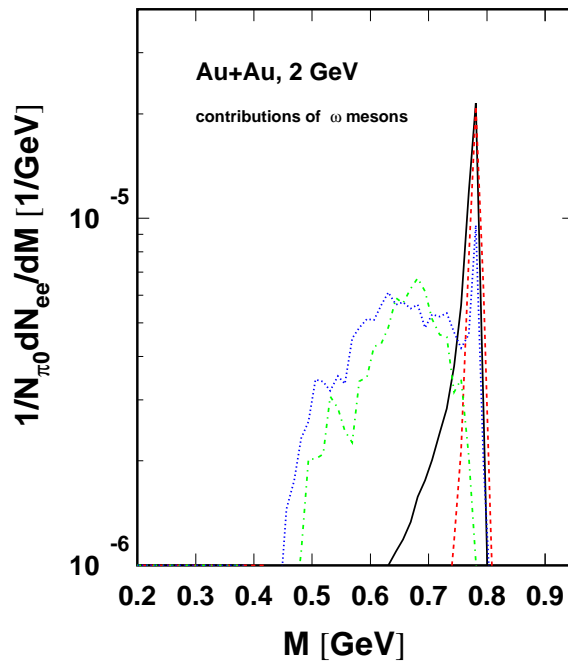
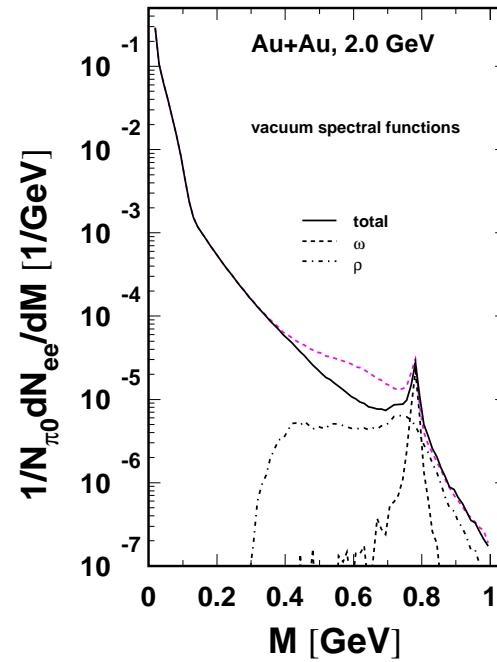
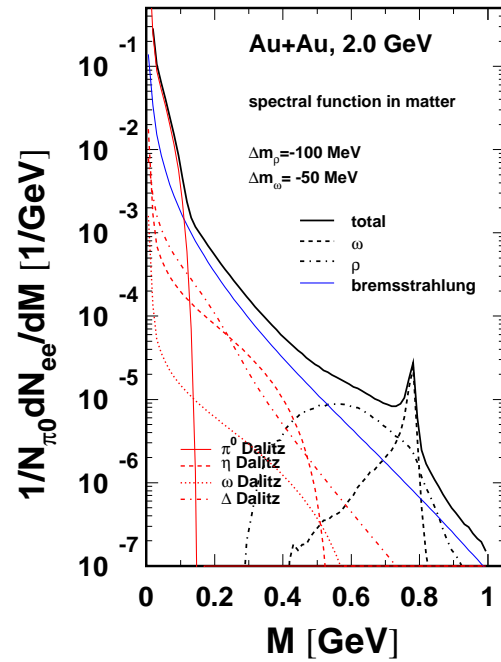
- testparticle approximation

C + C 2 GeV





Au + Au 2 GeV



Vacuum

Matter

Static

C + C 1 AGeV

