Compact-star matter from NJL models and QCD

TECHNISCHE UNIVERSITÄT DARMSTADT

Michael Buballa

EMMI Rapid Reaction Task Force Meeting on
Quark Matter in Compact Stars
FIAS, Frankfurt, October 7-9, 2013

Introduction

TECHNISCHE
UNIVERSITÄT
DARMSTADT

- central question of this meeting:

  Is there deconfined quark matter in compact stars?

Introduction

TECHNISCHE UNIVERSITÄT DARMSTADT

- central question of this meeting:

  Is there deconfined quark matter in compact stars?

- If we could derive the exact EoS of QCD, there would be no discussion:

  Just plug it into the TOV equation and check!

Introduction

- central question of this meeting:

  Is there deconfined quark matter in compact stars?
- If we could derive the exact EoS of QCD, there would be no discussion:

  Just plug it into the TOV equation and check!
- problem: We don't know it ...

Introduction

TECHNISCHE UNIVERSITÄT DARMSTADT

- central question of this meeting:

  Is there deconfined quark matter in compact stars?
- If we could derive the exact EoS of QCD, there would be no discussion:

  Just plug it into the TOV equation and check!
- problem: We don't know it ...
- perturbative QCD ($\rightarrow$ A. Vuorinen)
  - not applicable at nuclear-matter density (confined phase)
  - probably not even at central densities of compact stars

Introduction

TECHNISCHE UNIVERSITÄT DARMSTADT

- central question of this meeting:

  Is there deconfined quark matter in compact stars?

- If we could derive the exact EoS of QCD, there would be no discussion:

  Just plug it into the TOV equation and check!

- problem: We don't know it ...

- perturbative QCD ($\rightarrow$ A. Vuorinen)
  - not applicable at nuclear-matter density (confined phase)
  - probably not even at central densities of compact stars

- hadronic EoSs
  - microscopic or phenomenological input
  - well constrained around nuclear-matter density
  - range of validity at higher densities?

Hybrid Equations of State

TECHNISCHE UNIVERSITÄT DARMSTADT

- standard approach: hybrid equations of state
  - construct a phase transition from independent hadronic and quark-matter EoSs
  - general problem: relative normalization of the pressure

Hybrid Equations of State

TECHNISCHE UNIVERSITÄT DARMSTADT

- standard approach: hybrid equations of state
  - construct a phase transition from independent hadronic and quark-matter EoSs
  - general problem: relative normalization of the pressure
- prototype: hadronic EoS + bag model

Hybrid Equations of State

TECHNISCHE UNIVERSITÄT DARMSTADT

- standard approach: hybrid equations of state
  - construct a phase transition from independent hadronic and quark-matter EoSs
  - general problem: relative normalization of the pressure
- prototype: hadronic EoS + bag model
  - most simple (too simple?) version: $p_{bm} = p_{ideal} - B$
  - phase transition: $p_{hadronic}(\mu_c) = p_{bm}(\mu_c) \quad \Rightarrow \quad \mu_c$ is very sensitive to B

Hybrid Equations of State

TECHNISCHE UNIVERSITÄT DARMSTADT

- standard approach: hybrid equations of state
  - construct a phase transition from independent hadronic and quark-matter EoSs
  - general problem: relative normalization of the pressure
- prototype: hadronic EoS + bag model
  - most simple (too simple?) version: $p_{bm} = p_{ideal} - B$
  - phase transition: $p_{hadronic}(\mu_c) = p_{bm}(\mu_c)$ $\Rightarrow$ μ_c is very sensitive to B
- Interpretation of the bag constant:

  pressure difference between non-trivial and perturbative vacuum

Hybrid Equations of State

- standard approach: hybrid equations of state
  - construct a phase transition from independent hadronic and quark-matter EoSs
  - general problem: relative normalization of the pressure
- prototype: hadronic EoS + bag model
  - most simple (too simple?) version: $p_{bm} = p_{ideal} - B$
  - phase transition: $p_{hadronic}(\mu_c) = p_{bm}(\mu_c)$ $\Rightarrow$ μ_c is very sensitive to B
- Interpretation of the bag constant:

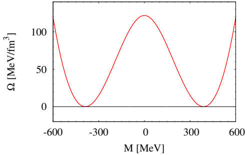
  pressure difference between non-trivial and perturbative vacuum
- What's its value?
  - original MIT fit to hadron spectra: $\sim$ 60 MeV/fm^3
  - T_c fit at $\mu = 0$ with a pion gas: $\sim$ 400 MeV/fm^3
  - QCD vacuum energy (from gluon condensate): $\sim$ 500 MeV/fm^3

NJL model

- quarks interacting by contact terms
  - e.g. standard NJL Lagrangian $\mathcal{L} = \bar{q}(i\partial\!\!\!/ - m)q + G\left[(\bar{q}q)^2 + (\bar{q}i\gamma_5\vec{\tau}q)^2\right]$

NJL model

- quarks interacting by contact terms
  - e.g. standard NJL Lagrangian $\mathcal{L} = \bar{q}(i\partial\!\!\!/ - m)q + G\left[(\bar{q}q)^2 + (\bar{q}i\gamma_5\vec{\tau}q)^2\right]$
- main features:
  - chiral symmetry, spontaneously broken in vacuum, restoration at large T or μ
  - dynamically generated bag pressure
    $\rightarrow$ B not an input parameter!
  - color superconductivity easily included
  - T and μ dependent dynamical quark masses, pairing gaps, bag pressure



NJL model: problems

TECHNISCHE UNIVERSITÄT DARMSTADT

- It's only a model ...
  (does not agree with QCD at asymptotic densities)
- not renormalizable ($\rightarrow$ cutoff dependent results, cutoff artifacts)
- no confinement
  (less severe in the deconfined phase;
  partial fix at nonzero T by coupling to Polyakov loops)
- symmetries do not uniquely fix the interaction
  $\rightarrow$ (infinitely) many interaction terms and model parameters
- T and μ dependence of the effective couplings unknown and usually neglected (in principle countained in higher-order n-point interactions)

How can we make predictions?

TECHNISCHE UNIVERSITÄT DARMSTADT

How can we make predictions?

- standard procedure:
  - choose a certain set of interaction terms
  - fix the parameters in vacuum
  - assume that they do not change in medium and that other terms are irrelevant

How can we make predictions?

TECHNISCHE UNIVERSITÄT DARMSTADT

- standard procedure:
  - choose a certain set of interaction terms
  - fix the parameters in vacuum
  - assume that they do not change in medium and that other terms are irrelevant
- alternative:
  - vary interactions and parameters as much as possible
    and look for common features
  - not really systematic, why not simply parametrize the EoS?

How can we make predictions?

- standard procedure:
  - choose a certain set of interaction terms
  - fix the parameters in vacuum
  - assume that they do not change in medium and that other terms are irrelevant
- alternative:
  - vary interactions and parameters as much as possible
    and look for common features
  - not really systematic, why not simply parametrize the EoS?
- intermediate:
  - fix some of the parameters and vary others

Example

TECHNISCHE UNIVERSITÄT DARMSTADT

- 3-flavor NJL model with $q\bar{q}$ and qq interactions:

$$
\begin{aligned}
\mathcal{L} \;=\; & \bar{q}_f(i\partial\!\!\!/ - m_f)q_f + G\left\{(\bar{q}\tau^a q)^2 + (\bar{q}i\gamma_5\tau^a q)^2\right\} \\
& - K\left\{\det{}_f\left(\bar{q}(1+\gamma_5)q\right) + \det{}_f\left(\bar{q}(1-\gamma_5)q\right)\right\} \\
& + H\,(\bar{q}\,i\gamma_5\tau_A\lambda_{A'}\,C\bar{q}^T)(q^T C\,i\gamma_5\tau_A\lambda_{A'}\,q)
\end{aligned}
$$

  - $m_{u,d}$, m_s, G, K, Λ fitted to pseudoscalar meson spectrum
  - H unclear, argued to be $\mathcal{O}(G)$

Example

- 3-flavor NJL model with $q\bar{q}$ and qq interactions:

$$\begin{aligned}\mathcal{L} \;=\; & \bar{q}_f(i\partial\!\!\!/ - m_f)q_f + G\left\{(\bar{q}\tau^a q)^2 + (\bar{q}i\gamma_5\tau^a q)^2\right\} \\ & - K\left\{\det{}_f\left(\bar{q}(1+\gamma_5)q\right) + \det{}_f\left(\bar{q}(1-\gamma_5)q\right)\right\} \\ & + H\,(\bar{q}\,i\gamma_5\tau_A\lambda_{A'}\,C\bar{q}^T)(q^T C\,i\gamma_5\tau_A\lambda_{A'}\,q)\end{aligned}$$

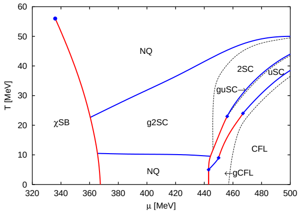
  - $m_{u,d}$, m_s, G, K, Λ fitted to pseudoscalar meson spectrum
  - H unclear, argued to be $\mathcal{O}(G)$

- phase diagram for $H = 0.75G$

  [Rüster et al., PRD (2005)]

  - phases at $T = 0$:

    vacuum $\rightarrow$ NQ $\rightarrow$ $gCFL$ $\rightarrow$ CFL



Example

- 3-flavor NJL model with $q\bar{q}$ and qq interactions:

$$\begin{aligned}\mathcal{L} \;=\; & \bar{q}_f(i\partial\!\!\!/ - m_f)q_f + G\left\{(\bar{q}\tau^a q)^2 + (\bar{q}i\gamma_5\tau^a q)^2\right\} \\ & - K\left\{\det_f\left(\bar{q}(1+\gamma_5)q\right) + \det_f\left(\bar{q}(1-\gamma_5)q\right)\right\} \\ & + H\,(\bar{q}\,i\gamma_5\tau_A\lambda_{A'}\,C\bar{q}^T)(q^T C\,i\gamma_5\tau_A\lambda_{A'}\,q)\end{aligned}$$

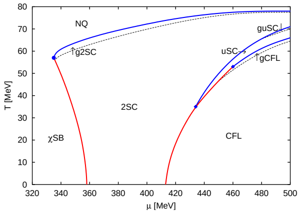
  - $m_{u,d}$, m_s, G, K, Λ fitted to pseudoscalar meson spectrum
  - H unclear, argued to be $\mathcal{O}(G)$

- phase diagram for $H = G$

  [Rüster et al., PRD (2005)]

  - phases at $T = 0$:

    vacuum $\rightarrow$ *2SC* $\rightarrow$ *CFL*



Hybrid stars

[Baldo et al., PLB (2003), MB et al., PLB (2004)]

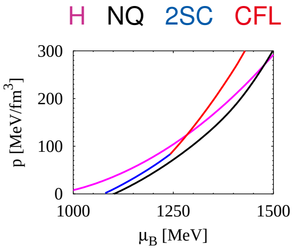
TECHNISCHE UNIVERSITÄT DARMSTADT

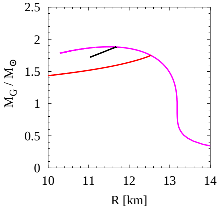
- different hadronic EoSs $\otimes$ different NJL parametrizations ($H = 0$, $H = G$)
- construct phase transition (Maxwell construction)
- solve TOV equation

Hybrid stars

[Baldo et al., PLB (2003), MB et al., PLB (2004)]

- different hadronic EoSs $\otimes$ different NJL parametrizations ($H = 0$, $H = G$)
- construct phase transition (Maxwell construction)
- solve TOV equation
- typical result:
  - quark matter can compete with hadrons only if strange quarks are present
  - phase transition to quark matter renders star unstable





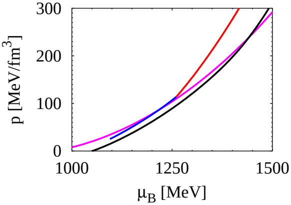
Hybrid stars

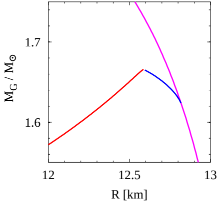
[Baldo et al., PLB (2003), MB et al., PLB (2004)]

TECHNISCHE UNIVERSITÄT DARMSTADT

- different hadronic EoSs $\otimes$ different NJL parametrizations ($H = 0$, $H = G$)
- construct phase transition (Maxwell construction)
- solve TOV equation
- typical result:
  - quark matter can compete with hadrons only if strange quarks are present
  - phase transition to quark matter renders star unstable
- one exception:
  - stable hybrid star with 2SC core
  - $M_{max} = 1.66 M_\odot$

H NQ 2SC CFL





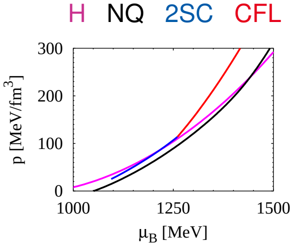
Hybrid stars

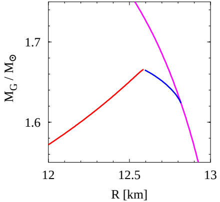
[Baldo et al., PLB (2003), MB et al., PLB (2004)]

TECHNISCHE UNIVERSITÄT DARMSTADT

- different hadronic EoSs $\otimes$ different NJL parametrizations ($H = 0$, $H = G$)
- construct phase transition (Maxwell construction)
- solve TOV equation
- typical result:
  - quark matter can compete with hadrons only if strange quarks are present
  - phase transition to quark matter renders star unstable
- one exception:
  - stable hybrid star with 2SC core
  - $M_{max} = 1.66 M_\odot$

excluded by $M = 2M_\odot$ measurement !





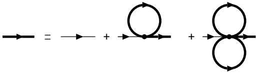
Discussion 1: dressed quark masses

TECHNISCHE UNIVERSITÄT DARMSTADT

Discussion 1: dressed quark masses

TECHNISCHE UNIVERSITÄT DARMSTADT

- dressed strange-quark mass in NJL:



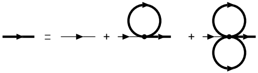
$$M_s = m_s - 4G\langle\bar{s}s\rangle + 2K\langle\bar{u}u\rangle\langle\bar{d}d\rangle$$

$$\to m_s - 4G\langle\bar{s}s\rangle \qquad \text{for } \langle\bar{u}u\rangle \approx \langle\bar{d}d\rangle \approx 0$$

Discussion 1: dressed quark masses

TECHNISCHE UNIVERSITÄT DARMSTADT

- dressed strange-quark mass in NJL:



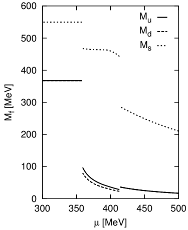
$$M_s = m_s - 4G\langle\bar{s}s\rangle + 2K\langle\bar{u}u\rangle\langle\bar{d}d\rangle$$

$$\to m_s - 4G\langle\bar{s}s\rangle \qquad \text{for } \langle\bar{u}u\rangle \approx \langle\bar{d}d\rangle \approx 0$$

$\Rightarrow$ rather large M_s at intermediate μ

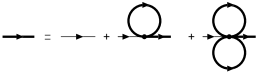
$\Rightarrow$ strange-quark contribution to pressure small

$\Rightarrow$ late phase transition



Discussion 1: dressed quark masses

- dressed strange-quark mass in NJL:



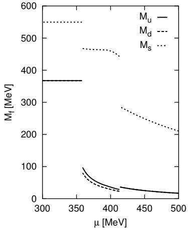
$$M_s = m_s - 4G\langle\bar{s}s\rangle + 2K\langle\bar{u}u\rangle\langle\bar{d}d\rangle$$

$$\to m_s - 4G\langle\bar{s}s\rangle \qquad \text{for } \langle\bar{u}u\rangle \approx \langle\bar{d}d\rangle \approx 0$$

$\Rightarrow$ rather large M_s at intermediate μ bug or feature ?

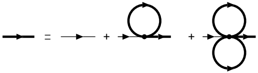
$\Rightarrow$ strange-quark contribution to pressure small

$\Rightarrow$ late phase transition



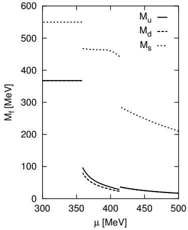
Discussion 1: dressed quark masses

- dressed strange-quark mass in NJL:



$$M_s = m_s - 4G\langle\bar{s}s\rangle + 2K\langle\bar{u}u\rangle\langle\bar{d}d\rangle$$

$$\to m_s - 4G\langle\bar{s}s\rangle \qquad \text{for } \langle\bar{u}u\rangle \approx \langle\bar{d}d\rangle \approx 0$$

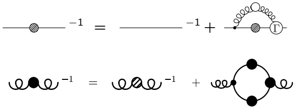


$\Rightarrow$ rather large M_s at intermediate μ bug or feature ?

$\Rightarrow$ strange-quark contribution to pressure small

$\Rightarrow$ late phase transition

- QCD beyond rainbow-ladder:
  - gluons screened by light quarks
  - M_s smaller ?



Discussion 2: effective bag pressure

TECHNISCHE UNIVERSITÄT DARMSTADT

Discussion 2: effective bag pressure

- To normalize the NJL pressure, the chirally broken vacuum solution is identified with $p = 0$.

Discussion 2: effective bag pressure

TECHNISCHE UNIVERSITÄT DARMSTADT

- To normalize the NJL pressure, the chirally broken vacuum solution is identified with $p = 0$.

$\rightarrow$ We believe in the NJL model in vacuum and at high density (quark phase), but not at low density (hadronic phase).

Does this make sense?

Discussion 2: effective bag pressure

TECHNISCHE UNIVERSITÄT DARMSTADT

- To normalize the NJL pressure, the chirally broken vacuum solution is identified with $p = 0$.

$\rightarrow$ We believe in the NJL model in vacuum and at high density (quark phase), but not at low density (hadronic phase).

Does this make sense?

- Alternative pressure normalization [Pagliara & Schaffner-Bielich, PRD (2008)]:
  - introduce additional (negative) bag constant by hand such that $\mu_c^{h \to q} = \mu_c^{\chi,NJL}$
  - maximal shift which does not lead to a deconfinement phase transition into a chirally broken quark phase
  - any smaller shift would be reasonable as well

Discussion 2: effective bag pressure

- To normalize the NJL pressure, the chirally broken vacuum solution is identified with $p = 0$.

$\rightarrow$ We believe in the NJL model in vacuum and at high density (quark phase), but not at low density (hadronic phase).

Does this make sense?

- Alternative pressure normalization [Pagliara & Schaffner-Bielich, PRD (2008)]:
  - introduce additional (negative) bag constant by hand such that $\mu_c^{h\to q} = \mu_c^{\chi,NJL}$
  - maximal shift which does not lead to a deconfinement phase transition into a chirally broken quark phase
  - any smaller shift would be reasonable as well

- If we don't believe in the NJL vacuum pressure,
  why do we believe in the parameter fit and the resulting $\mu_c^{\chi,NJL}$?

Unified description

- only convincing way out:
  unified description of quark and hadronic phase in a single framework

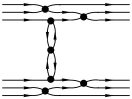
Unified description

TECHNISCHE
UNIVERSITÄT
DARMSTADT

- only convincing way out:

  unified description of quark and hadronic phase in a single framework

- here: construct nucleons and nuclear matter in NJL?
  - Fadeev + $q\bar{q}$-meson exchange $\rightarrow$ far beyond mean field

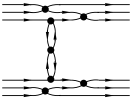


Unified description

- only convincing way out:

  unified description of quark and hadronic phase in a single framework

- here: construct nucleons and nuclear matter in NJL?
  - Fadeev + $q\bar{q}$-meson exchange $\rightarrow$ far beyond mean field



- first steps in this direction in the literature

  [Rezaeian & Pirner, NPA (2006); Lawley, Bentz & Thomas, JPG (2006); Wang, Wang & Rischke PLB (2011)]

- should be pursued further !

Discussion 3: vector interactions

Discussion 3: vector interactions

[Klähn et al., PLB (2007)]:

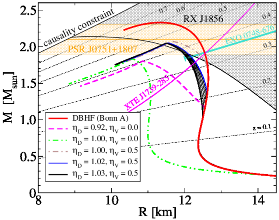
- introduce repulsive vector interaction ($G_V = G/2$):
  stiffens EoS but reduces the pressure
- compensate with strong quark-quark interaction

Discussion 3: vector interactions

[Klähn et al., PLB (2007)]:

- introduce repulsive vector interaction ($G_V = G/2$): stiffens EoS but reduces the pressure
- compensate with strong quark-quark interaction

$\rightarrow$ $M > 2M_\odot$ possible!

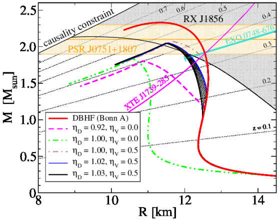


Discussion 3: vector interactions

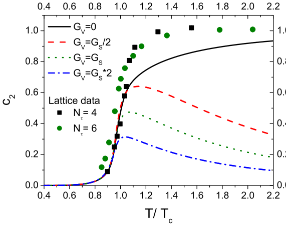
[Klähn et al., PLB (2007)]:

- introduce repulsive vector interaction ($G_V = G/2$): stiffens EoS but reduces the pressure
- compensate with strong quark-quark interaction

$\rightarrow \quad M > 2M_\odot$ possible!



- $G_V = G/2$ is incompatible with lattice data at $\mu = 0$ [Steinheimer & Schramm, PLB (2011)]

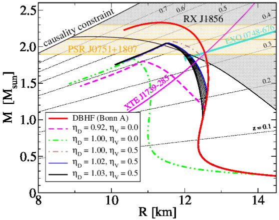


Discussion 3: vector interactions

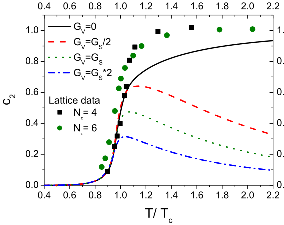
[Klähn et al., PLB (2007)]:

- introduce repulsive vector interaction ($G_V = G/2$): stiffens EoS but reduces the pressure
- compensate with strong quark-quark interaction

$\rightarrow$ $M > 2M_\odot$ possible!



- $G_V = G/2$ is incompatible with lattice data at $\mu = 0$ [Steinheimer & Schramm, PLB (2011)]



- μ-dependent G_V?

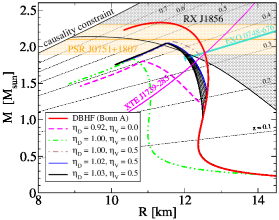
Discussion 3: vector interactions

TECHNISCHE UNIVERSITÄT DARMSTADT

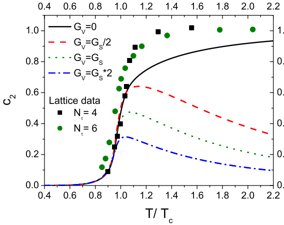
[Klähn et al., PLB (2007)]:

- introduce repulsive vector interaction ($G_V = G/2$): stiffens EoS but reduces the pressure
- compensate with strong quark-quark interaction

$\rightarrow$ $M > 2M_\odot$ possible!



- $G_V = G/2$ is incompatible with lattice data at $\mu = 0$ [Steinheimer & Schramm, PLB (2011)]
- μ-dependent G_V?
- possible, but then we loose all predictive power ...



Towards deriving the EoS from QCD

Towards deriving the EoS from QCD

- Lattice calculations at large baryon density:
  - standard methods do not work (sign problem)
  - several new ideas, but not yet realistic

Towards deriving the EoS from QCD

TECHNISCHE UNIVERSITÄT DARMSTADT

- Lattice calculations at large baryon density:
  - standard methods do not work (sign problem)
  - several new ideas, but not yet realistic
- Dyson-Schwinger equations:

  - in principle exact
  - input for the quark DSE:

    dressed gluon propagator and dressed quark-gluon vertex

    ⇒ truncations needed, but systematically amendable

Towards deriving the EoS from QCD

TECHNISCHE
UNIVERSITÄT
DARMSTADT

- Lattice calculations at large baryon density:
  - standard methods do not work (sign problem)
  - several new ideas, but not yet realistic
- Dyson-Schwinger equations:

  - in principle exact
  - input for the quark DSE:

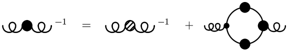
    dressed gluon propagator and dressed quark-gluon vertex

    $\Rightarrow$ truncations needed, but systematically amendable

  - pressure (CJT formalism): $p \equiv \Gamma[\mathcal{S}] = \mathrm{Tr}\ln \mathcal{S}^{-1} - \mathrm{Tr}\,(1 - Z_2 \mathcal{S}_0^{-1}\mathcal{S}) + \Gamma_2[\mathcal{S}]$
    - numerically very demanding (integrals quartically divergent)
    - not applicable for all truncations

Dyson-Schwinger equations: results

- truncation:

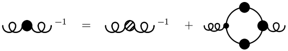


  - gluon: lattice Yang-Mills propagator + quark corrections
    - simplified scheme: polarization loop with bare quarks (HTL-HDL)
    - improved scheme: with selfconsistently dressed quarks
  - vertex model with infrared enhancement and perturbative ultraviolet behavior

Dyson-Schwinger equations: results

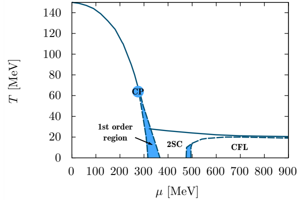
TECHNISCHE UNIVERSITÄT DARMSTADT

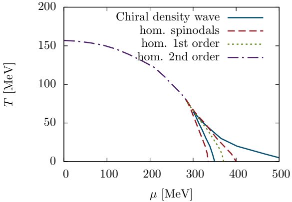
- truncation:



  - gluon: lattice Yang-Mills propagator + quark corrections
    - simplified scheme: polarization loop with bare quarks (HTL-HDL)
    - improved scheme: with selfconsistently dressed quarks
  - vertex model with infrared enhancement and perturbative ultraviolet behavior

- recent achievements: [D. Müller et al., 2013]
  phase diagrams with color superconducting and inhomogeneous phases





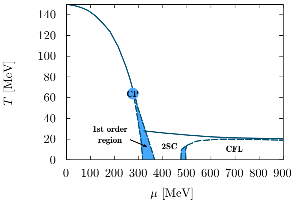
Dyson-Schwinger equations: challenges

TECHNISCHE UNIVERSITÄT DARMSTADT

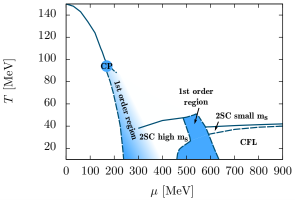
Dyson-Schwinger equations: challenges

- truncations not yet converged:

simplified scheme



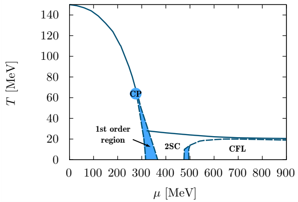
improved scheme



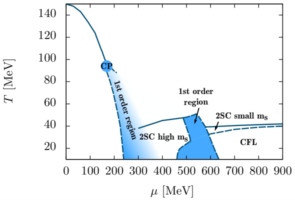
Dyson-Schwinger equations: challenges

- truncations not yet converged:

simplified scheme



improved scheme

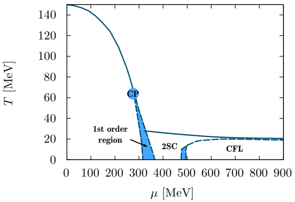


- no satisfactory pressure yet ($\rightarrow$ no EoS):
  - simplified scheme: numerical difficulties
  - improved scheme: DSE not derivable from an effective action

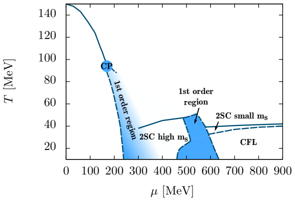
Dyson-Schwinger equations: challenges

- truncations not yet converged:

simplified scheme



improved scheme



- no satisfactory pressure yet ($\rightarrow$ no EoS):
  - simplified scheme: numerical difficulties
  - improved scheme: DSE not derivable from an effective action
- present truncations do not yet include baryonic degrees of freedom!

(obviously even more difficult than in NJL ...)