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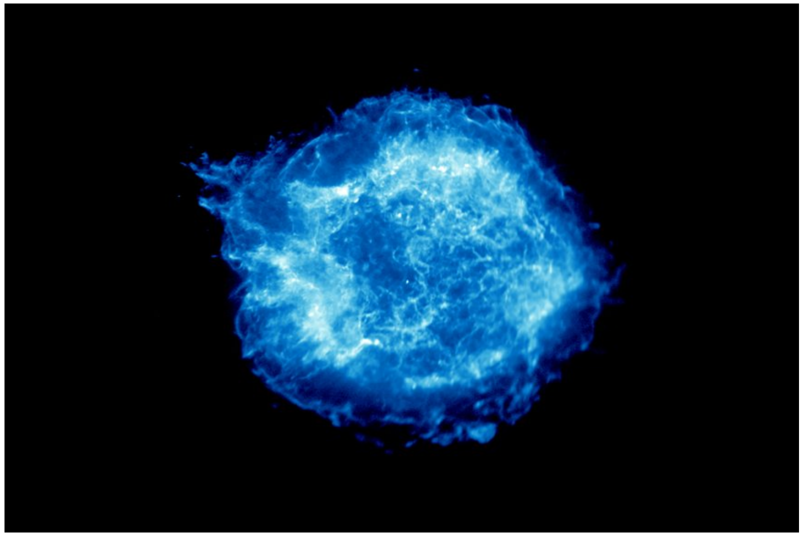
Role reversal in first and second sound in a relativistic superfluid

- two-fluid picture from field theory
  M.G. Alford, S.K. Mallavarapu, A. Schmitt, S. Stetina, PRD 87, 065001 (2013)
- critical velocity and sound modes for all temperatures with 2PI
  M.G. Alford, S.K. Mallavarapu, A. Schmitt, S. Stetina, in preparation

- **Superfluid hydrodynamics: relevance for compact stars**

  - r-mode instability
  - pulsar glitches
  - precession
  - asteroseismology
  - superfluid turbulence (?)



Cas A, Chandra X-Ray Observatory

- **Superfluidity in dense matter**

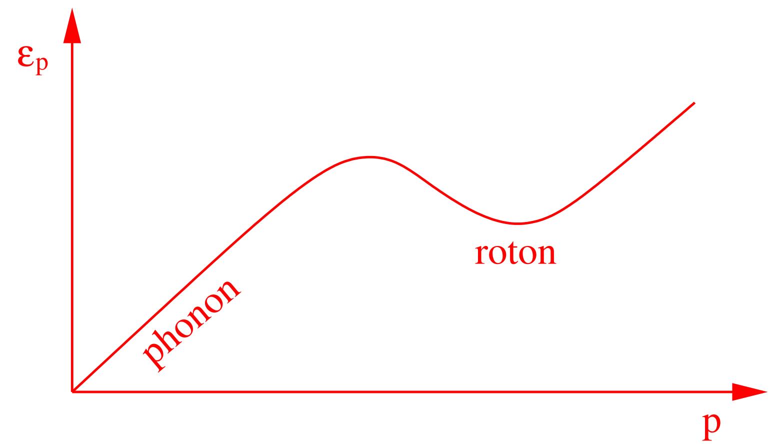
Nuclear matter	Quark matter
neutrons ($T_c \lesssim 10\,\mathrm{keV}$)	color-flavor locked phase ($T_c \sim 10\,\mathrm{MeV}$)
hyperons	color-spin locked phase ($T_c \sim 10\,\mathrm{keV}$)

• Two-fluid picture of a superfluid (liquid helium)

London, Tisza (1938); Landau (1941)

relativistic: Khalatnikov, Lebedev (1982); Carter (1989)

- "superfluid component": condensate, carries no entropy
- "normal component": excitations (Goldstone mode), carries entropy



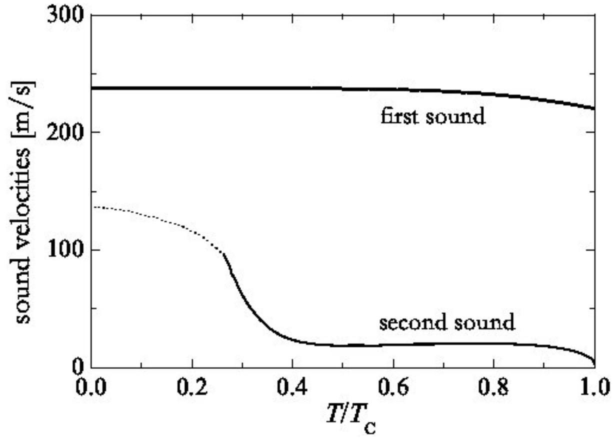
$\boxed{\text{Hydrodynamic eqs.} \Rightarrow \textbf{two sound modes}}$

1st sound	**2nd sound**
in-phase oscillation	out-of-phase oscillation
(primarily) density wave	(primarily) entropy wave

• First and second sound in non-relativistic systems

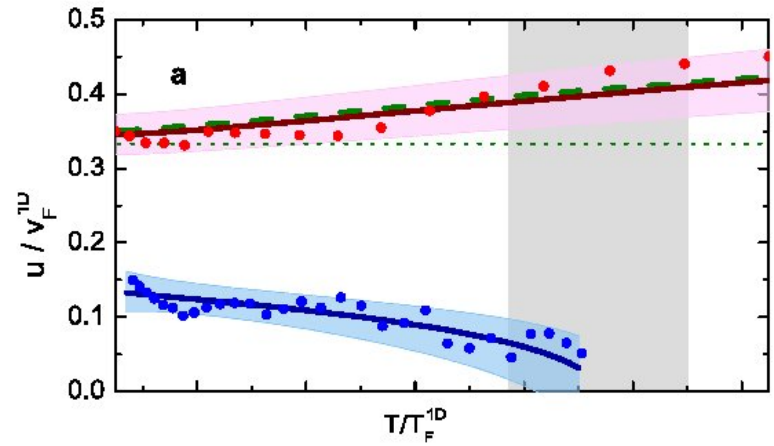
liquid helium

K.R. Atkins *et al.* (1953)



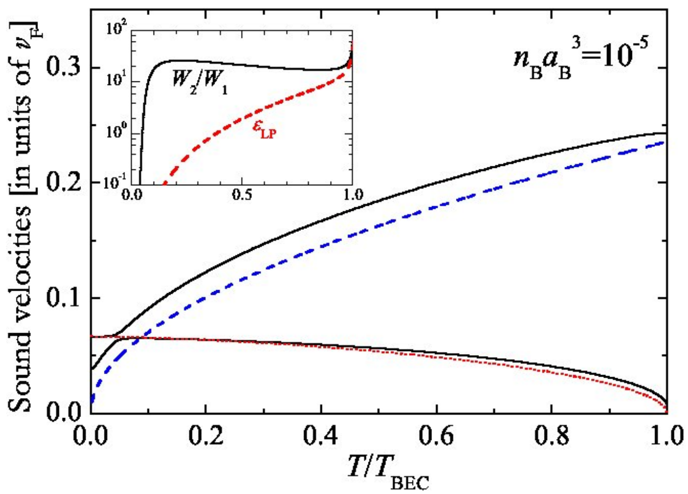
ultracold fermionic gas (exp.)

L.A. Sidorenkov *et al.*, Nature 498, 78 (2013)



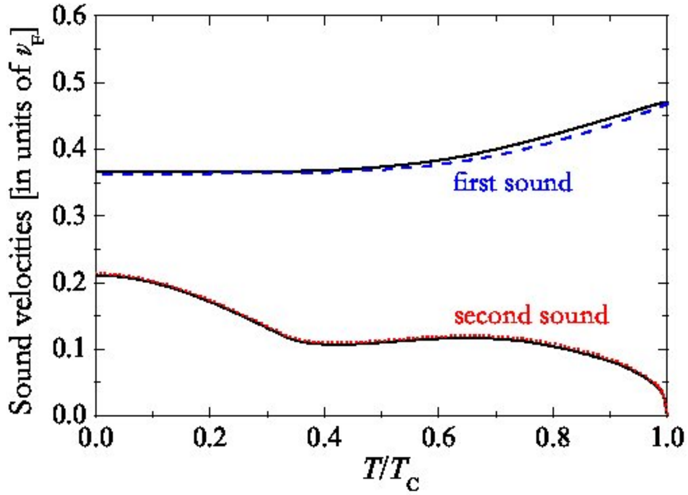
weakly interacting Bose gas

H.Hu, *et al.*, New Journ.Phys. 12, 043040 (2010)



unitary Fermi gas

E. Taylor *et al.*, PRA 80, 053601 (2009)



- **Goals**

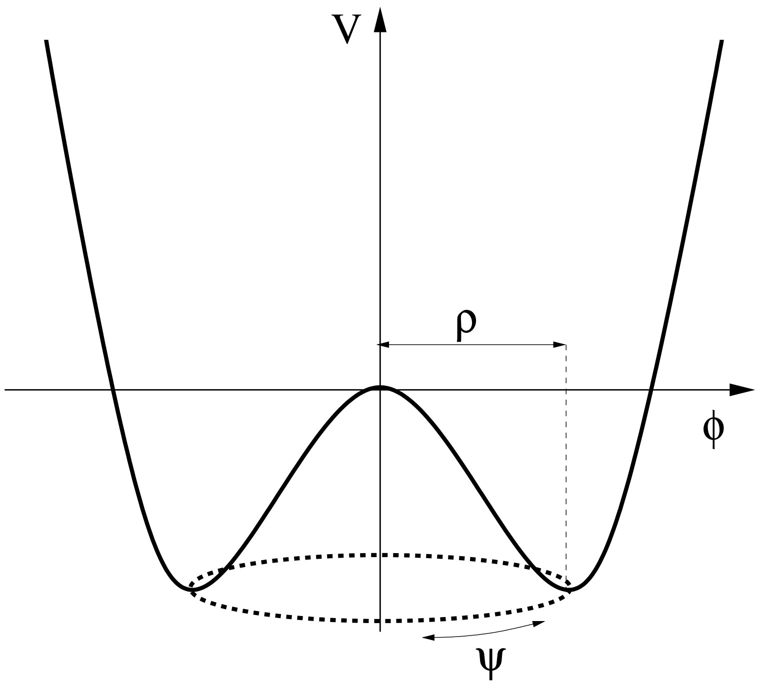
How does the two-fluid picture
arise from a microscopic theory?

Compute sound modes in a relativistic superfluid
(and in the presence of a superflow)

• Bose condensation and superfluid velocity (page 1/2)

- starting point: φ^4 model

$$\boxed{\mathcal{L} = (\partial\varphi)^2 - m^2|\varphi|^2 - \lambda|\varphi|^4}$$



- $\varphi \to \phi + \varphi$, condensate $\phi = \frac{\rho}{\sqrt{2}} e^{i\psi}$
- first step: no fluctuations ($T = 0$)
- minimize $V(\rho) = -\mathcal{L}$

$$\boxed{\rho^2 = \frac{(\partial\psi)^2 - m^2}{\lambda}}$$

(assumption: ρ, $\partial\psi$ const.)

• Bose condensation and superfluid velocity (page 2/2)

• "translation" at zero temperature (single fluid!) $(m = 0)$

	Field theory	**Hydrodynamics**
j^μ	$\frac{(\partial\psi)^2}{\lambda}\partial^\mu\psi$	nv^μ
$T^{\mu\nu}$	$-g^{\mu\nu}\mathcal{L} + \frac{(\partial\psi)^2}{\lambda}\partial^\mu\psi\partial^\nu\psi$	$(\epsilon + P)v^\mu v^\nu - g^{\mu\nu}P$

• With $\epsilon + P = \mu n$:

$$P = \frac{(\partial\psi)^4}{4\lambda}, \quad \epsilon = \frac{3(\partial\psi)^4}{4\lambda}$$

$$\mu = |\partial\psi|, \quad n = \frac{|\partial\psi|^3}{\lambda}$$

• **superfluid velocity**

$$\boxed{v^\mu = \frac{\partial^\mu\psi}{\mu}}$$

$\Rightarrow$ irrotationality of superfluid, $\nabla \times \vec{v} = 0$

• Relativistic two-fluid formalism (page 1/2)

- write stress-energy tensor as

$$\boxed{T^{\mu\nu} = -g^{\mu\nu}\Psi + j^\mu \partial^\nu \psi + s^\mu \Theta^\nu}$$

- **"generalized pressure"** Ψ:
  - $\Psi = P_\perp$ in superfluid and normal-fluid rest frames,
  - Ψ depends on momenta $\partial^\mu \psi$, Θ^μ

$$\Psi = \Psi[(\partial\psi)^2, \Theta^2, \partial\psi \cdot \Theta]$$

- **"generalized energy density"** $\Lambda \equiv -\Psi + j \cdot \partial\psi + s \cdot \Theta$
  - Λ is Legendre transform of Ψ,
  - Λ depends on currents j^μ, s^μ

$$\Lambda = \Lambda[j^2, s^2, j \cdot s]$$

• Relativistic two-fluid formalism (page 2/2)

$$j^\mu = \frac{\partial\Psi}{\partial(\partial_\mu\psi)} = \mathcal{B}\,\partial^\mu\psi + \mathcal{A}\,\Theta^\mu$$

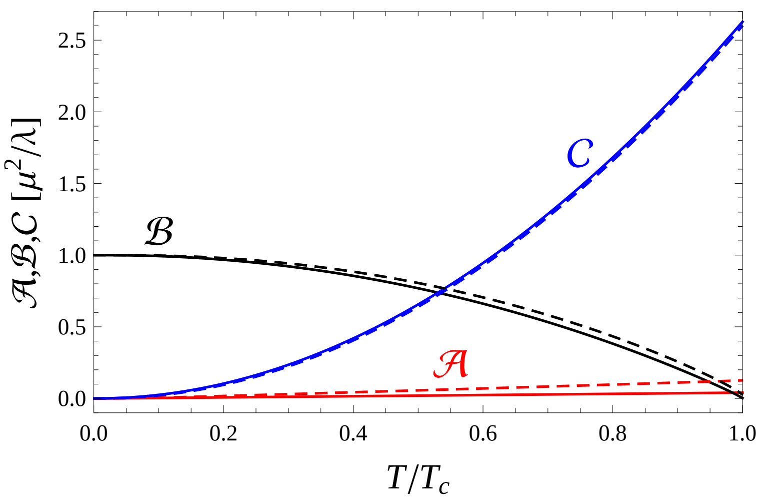
$$s^\mu = \frac{\partial\Psi}{\partial\Theta_\mu} = \mathcal{A}\,\partial^\mu\psi + \mathcal{C}\,\Theta^\mu$$

$$\mathcal{B} = 2\frac{\partial\Psi}{\partial(\partial\psi)^2}\,, \quad \mathcal{C} = 2\frac{\partial\Psi}{\partial\Theta^2}$$

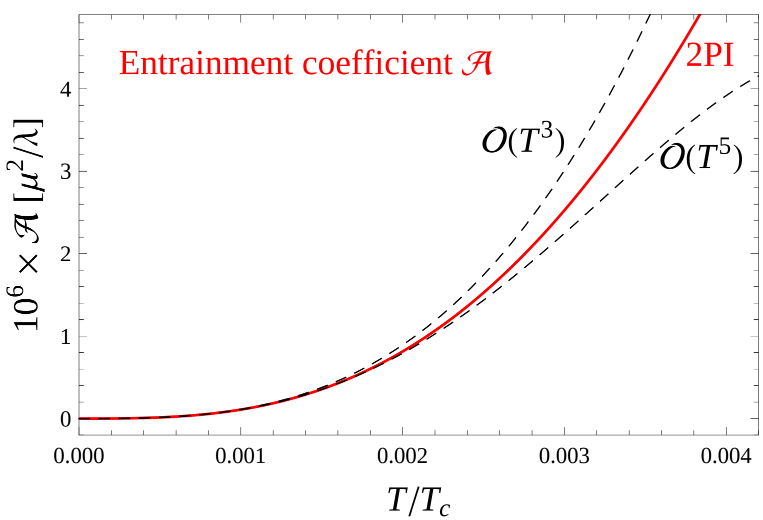
$$\mathcal{A} = \frac{\partial\Psi}{\partial(\partial\psi\cdot\Theta)}$$

“entrainment coefficient”

• compute $\mathcal{A}$, $\mathcal{B}$, $\mathcal{C}$ from microscopic physics



all temperatures



(very) small temperatures

• Microscopic calculation

- effective action density in the 2PI formalism (CJT)

$$\boxed{\Gamma[\rho, S] = -U(\rho) - \frac{1}{2}\mathrm{Tr}\ln S^{-1} - \frac{1}{2}\mathrm{Tr}[S_0^{-1}(\rho)S - 1] - V_2[\rho, S]}$$

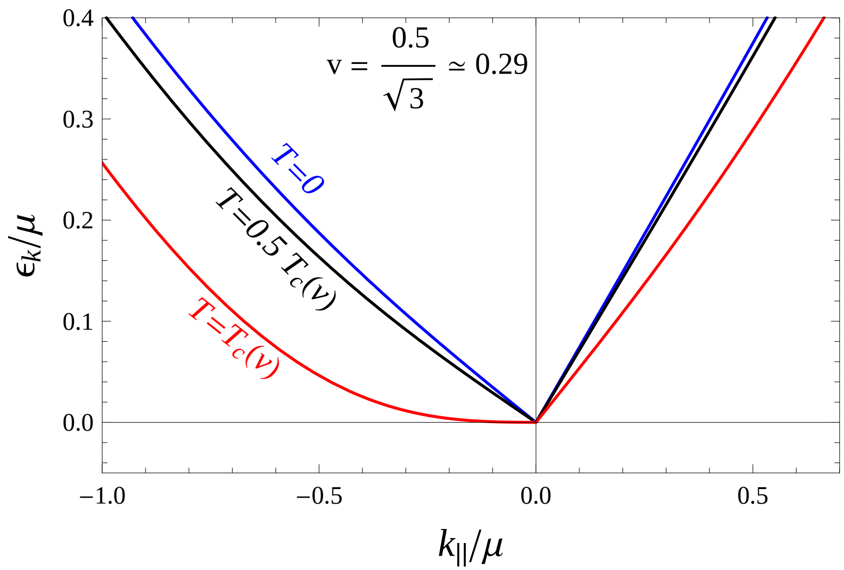
  - $V_2[\rho, S]$: two-loop two-particle irreducible (2PI) diagrams
  - use Hartree approximation
  - impose Goldstone theorem as external constraint
  - solve self-consistency equations for condensate ρ and M, δM

- microscopic calculation done in normal-fluid rest frame
- identify effective action density with generalized pressure

$$\boxed{\Gamma[\mu, T, \nabla\psi] = \Psi}$$

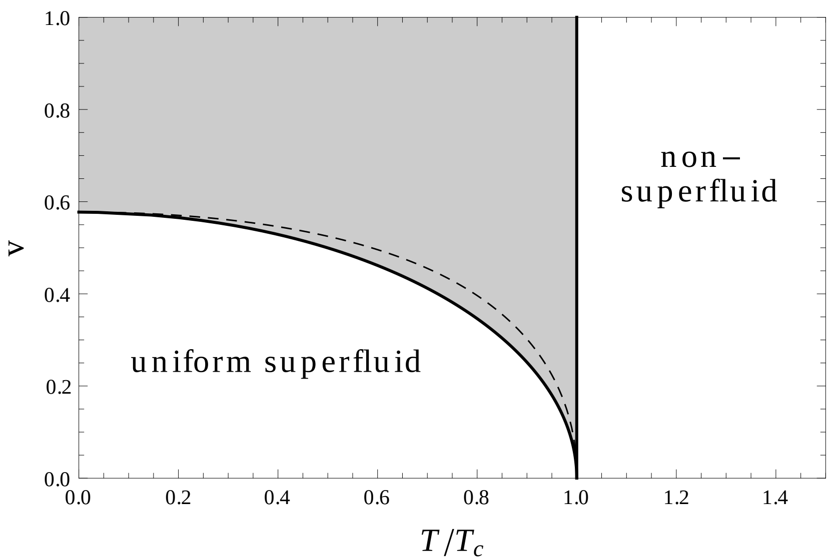
- compute $\mathcal{A}$, $\mathcal{B}$, $\mathcal{C}$, sound velocities etc.

• Results I: critical velocity



- instability at $v = v_c$
- negative energies in Goldstone dispersion $\epsilon_{\mathbf{k}}(\mathbf{v}) < 0$

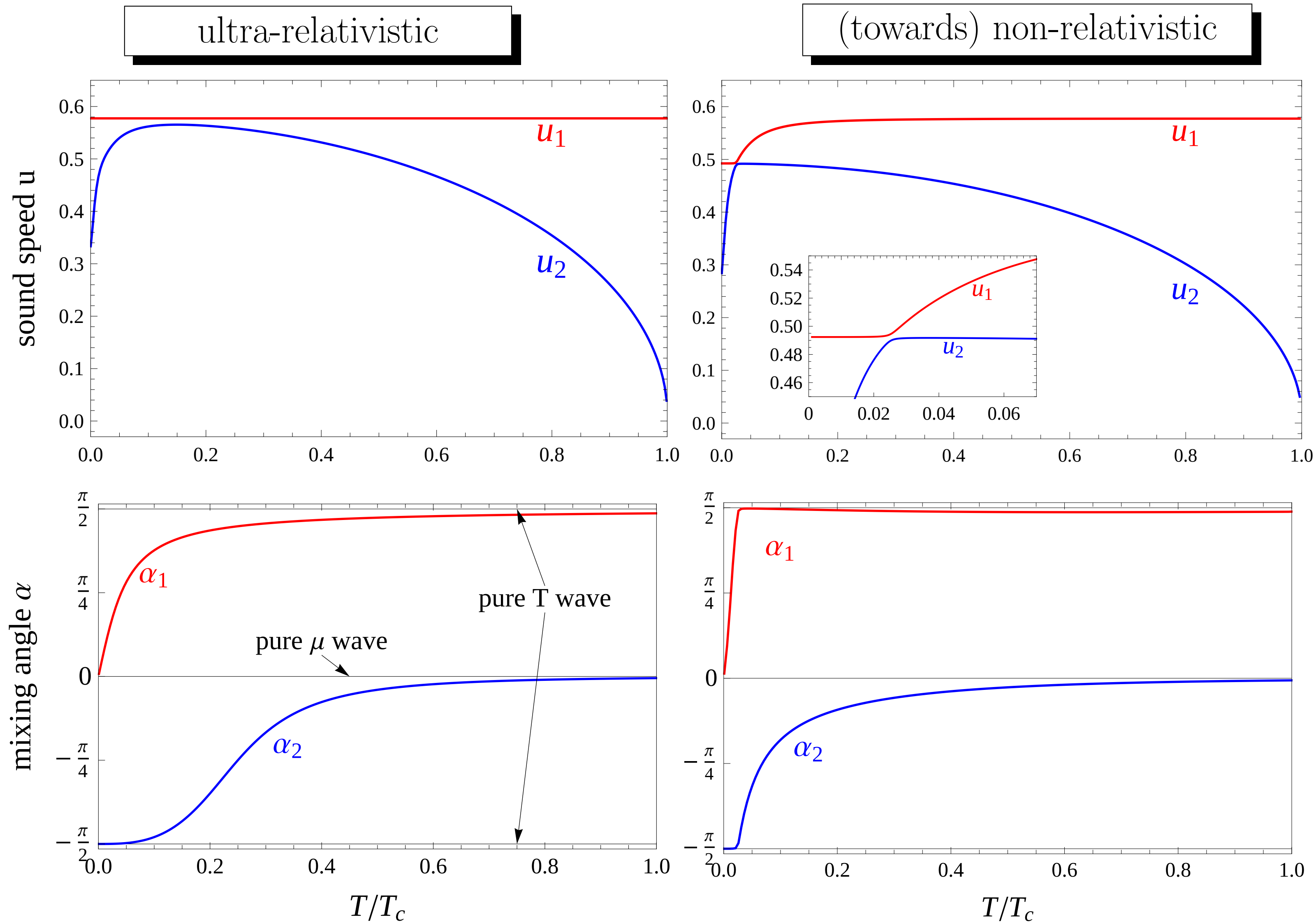
- generalization to Landau's original argument $\epsilon_{\mathbf{k}} - \mathbf{k} \cdot \mathbf{v} < 0$



- dashed line: without backreaction of condensate

- shaded region: superfluid turbulence?

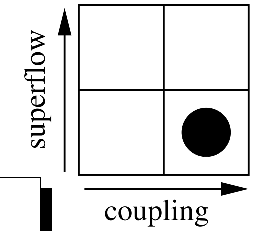
• Results II: sound speeds and mixing angle

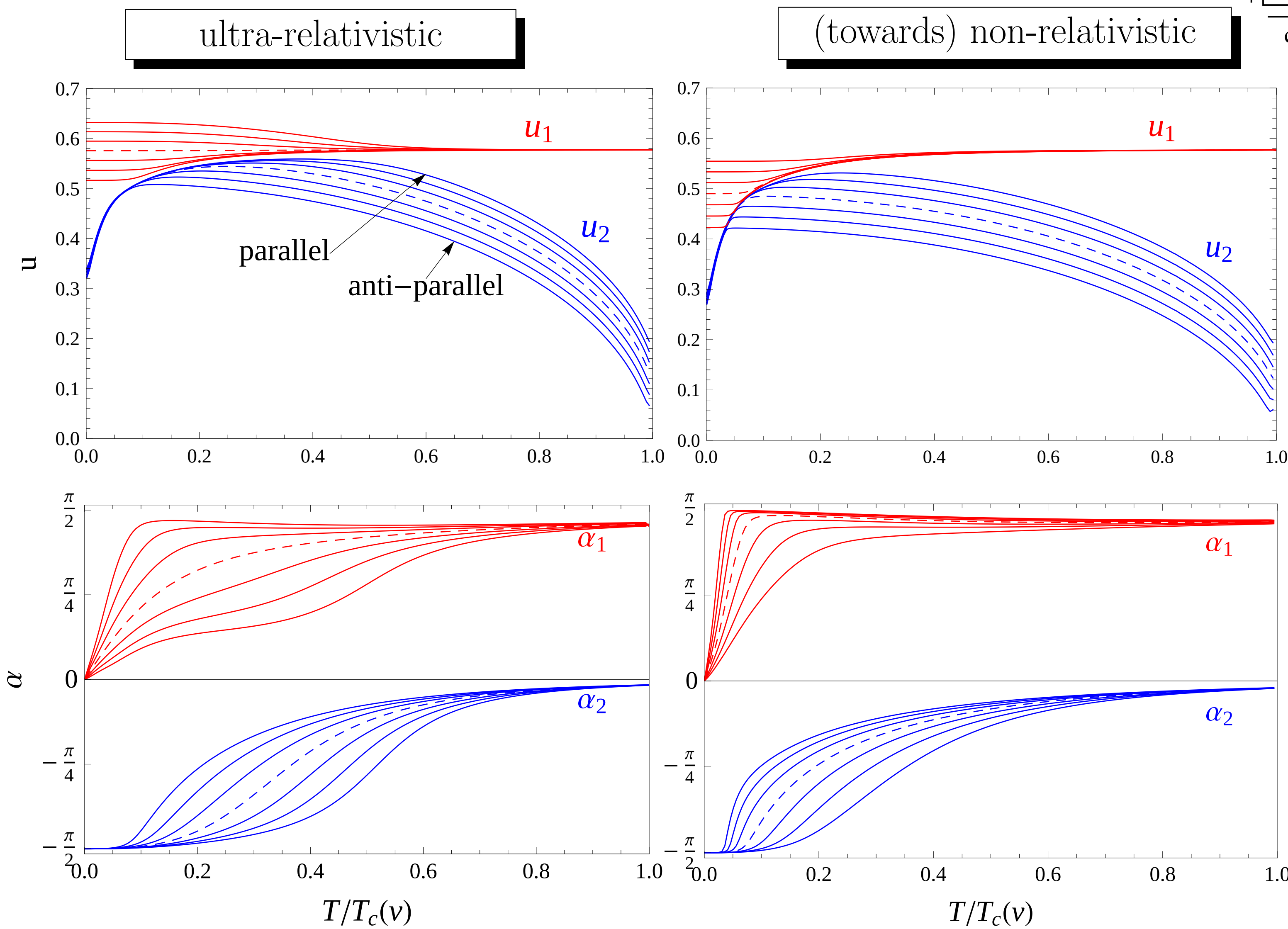


$$\alpha = \arctan \frac{\delta T}{\delta \mu}$$

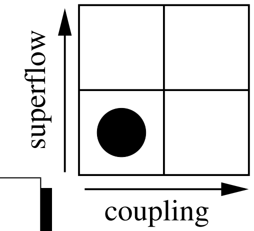
role reversal of first and second sound!

• Sound speeds and mixing angle with superflow



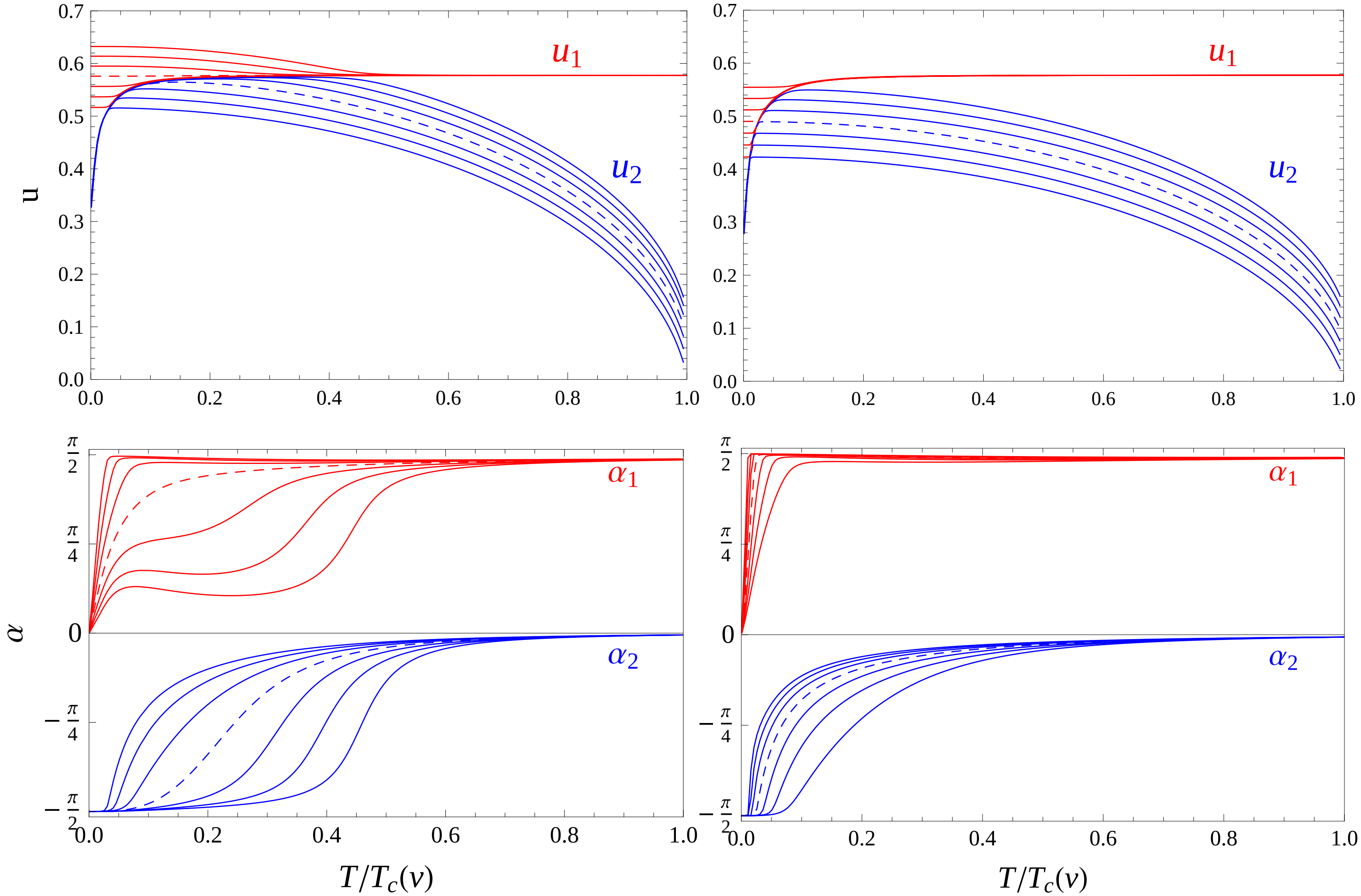


• Sound speeds and mixing angle with superflow



ultra-relativistic

(towards) non-relativistic



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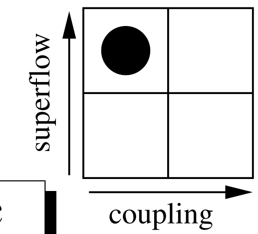

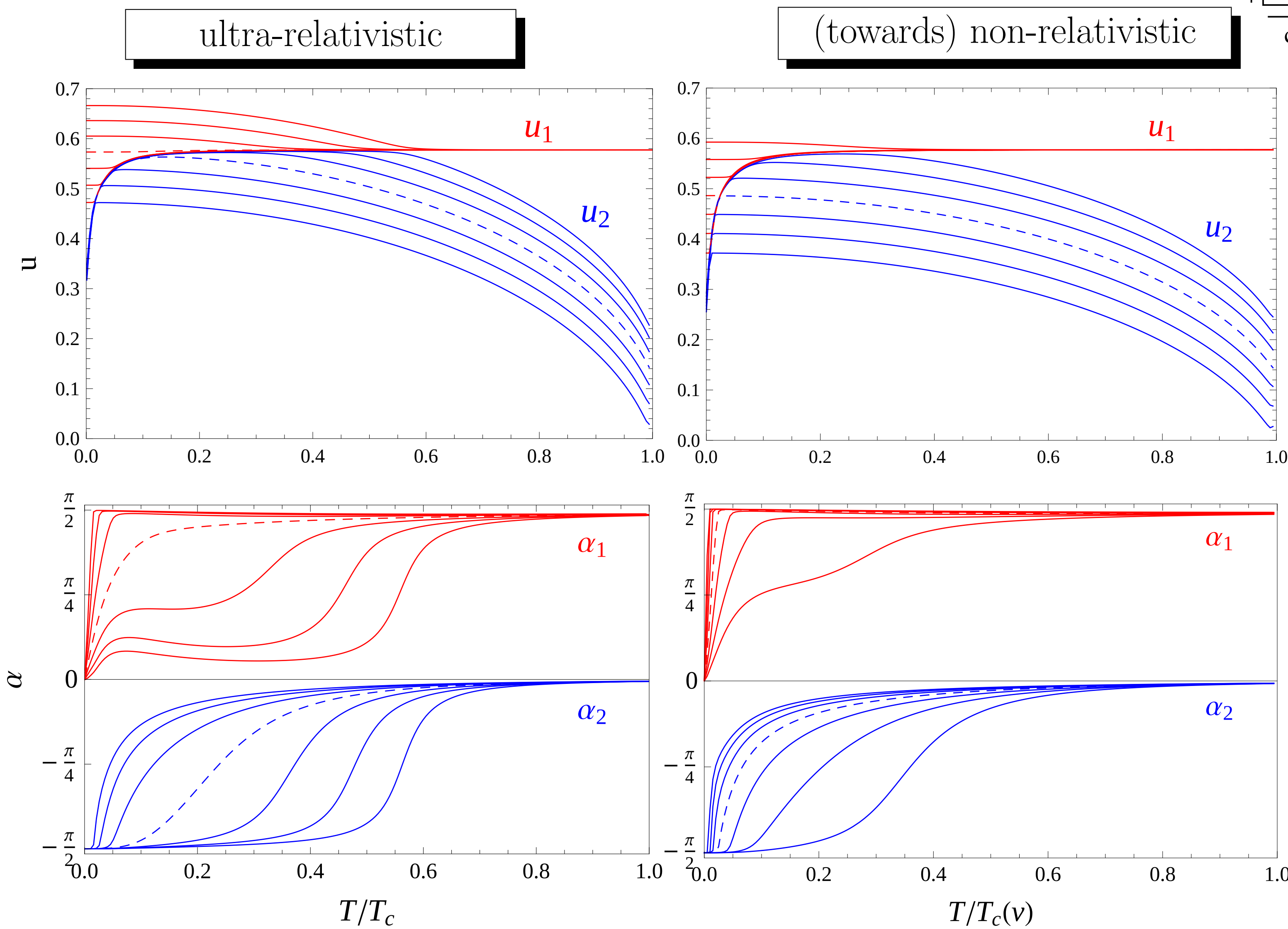

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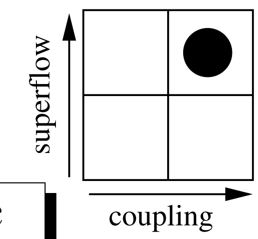

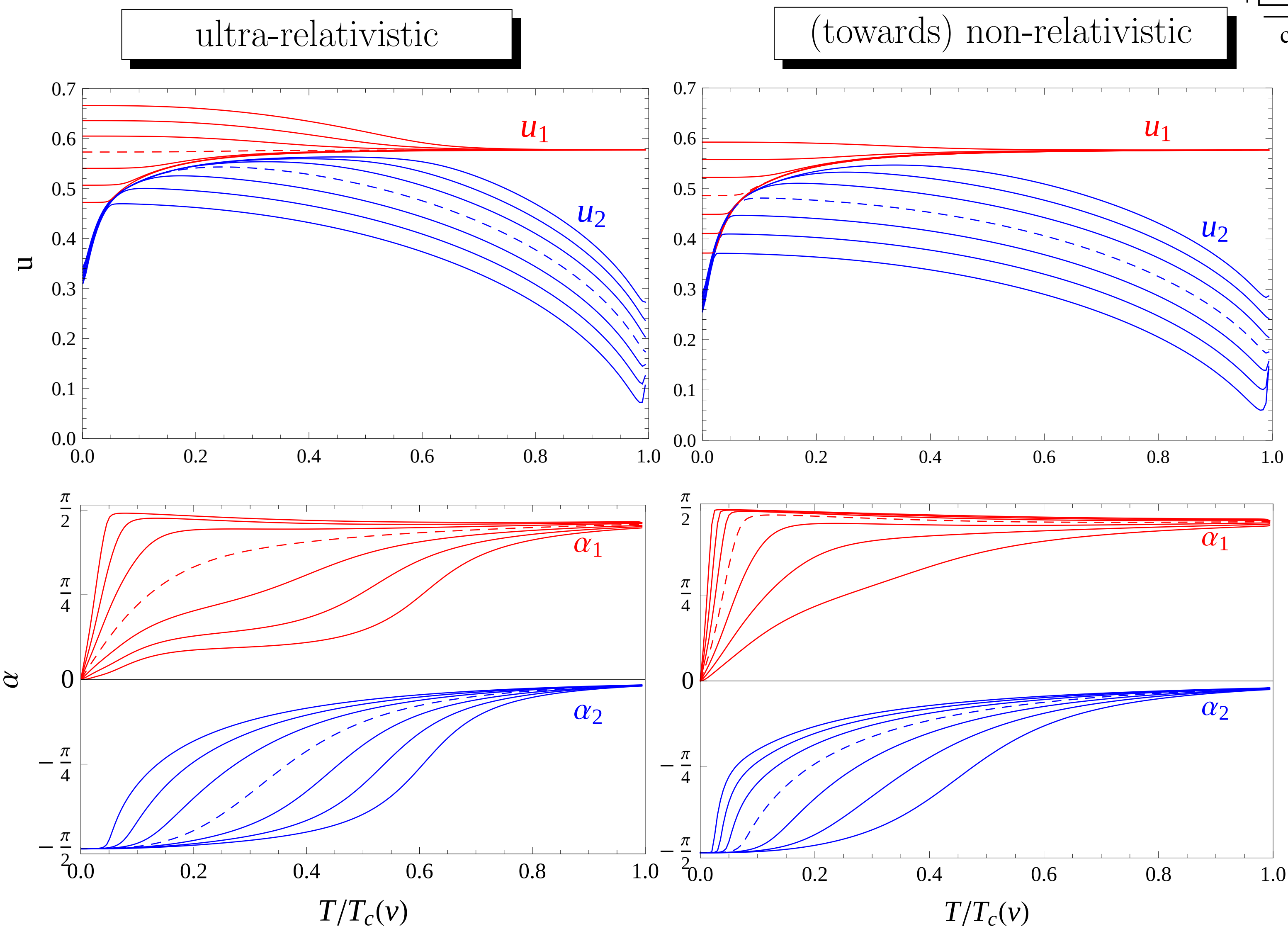

• Summary/Outlook

- hydrodynamics of relativistic superfluid

  dissipationless, uniform, (weakly coupled)

  – microscopic input for two-fluid formalism

  – low-T approximation & results for all $T \leq T_c$ within 2PI formalism

  – critical velocity including backreaction of condensate

  – sound modes (with superflow): role reversal;
  continuously connect ultra-relativistic and non-relativistic cases

- start from fermionic theory
- hydrodynamics of CFL-K^0
- behavior beyond critical velocity
- predictions for ^{4}He or ultracold gases?
- apply to compact stars

  neutron superfluid & ion lattice: N. Chamel, D. Page and S. Reddy, PRC 87, 035803 (2013)