Perturbative QCD at finite density: The EoS of deconfined quark matter

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Topics of the talk

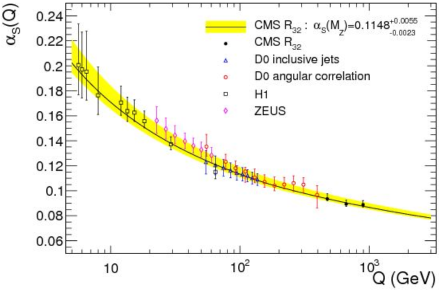
1. Weak coupling methods in thermal field theory
   - Basic concepts, uses and limitations
2. High temperature quark gluon plasma
   - Scale hierarchies and effective theory approach
   - EoS at zero and moderate quark number density
3. Cold quark matter
   - Differences to the high T system
   - Three-loop EoS with nonzero strange quark mass

Weak coupling methods: Basics

Goal: Diagrammatic evaluation of grand potential of deconfined QCD matter in (resummed) expansion in α_s

$$\Omega(T,\{\mu_f\},\{m_f\}) = -T\log\int\mathcal{D}\bar{\psi}\mathcal{D}\psi\mathcal{D}A_\mu e^{-\int_0^\beta d\tau\int d^3x\mathcal{L}_{\rm QCD}}$$

$$\mathcal{L}_{\rm QCD} = \frac{1}{4}F^a_{\mu\nu}F^a_{\mu\nu}+\sum_f\bar{\psi}_f(\gamma_\mu D_\mu+m_f-\mu_f\gamma_0)\psi_f$$



Weak coupling methods: Uses

Truly first principles approach, useful in several parts of the QCD phase diagram:

1. High T, $\mu = 0$:
   - Connect lattice region to asymptotically high T
   - Provide qualitative understanding of plasma properties
2. High T, $0 < \mu \lesssim T$:
   - No sign problem — $\mu = 0$ results straightforwardly extendable to finite density, with improved convergence
   - Test lattice predictions: Quark number susceptibilities (QNS)
3. $T \approx 0$, high μ:
   - Predict EoS at asymptotically high μ and attempt to constrain nuclear matter EoSs somewhat above saturation density
   - No nonperturbative first principles alternative here

Analytic and extremely versatile method, with built-in error estimation

Weak coupling methods: Limitations

QCD is a strongly interacting theory $\Rightarrow$ Phenomenologically interesting physics rarely weakly coupled

- When available, nonperturbative approaches always preferable

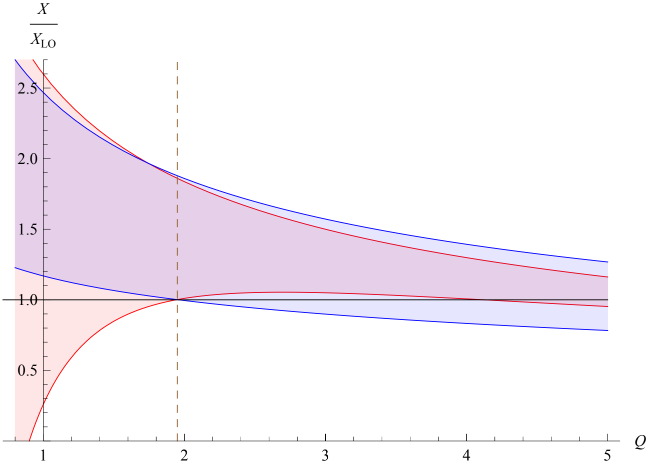
Perturbative approach may miss important effects even at asymptotically weak coupling

- Cf. color superconductivity with gap $\Delta \sim e^{-\#/g}$

When truncated to finite loop order, results exhibit residual dependence on *renormalization scale* $\bar{\Lambda}$

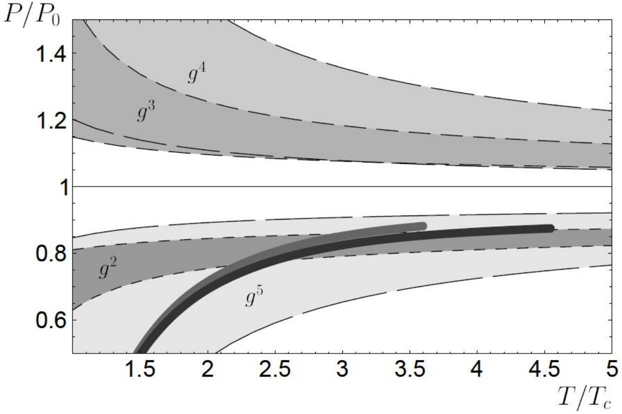
- Convergence only when $\bar{\Lambda}$ dependence *decreases order by order* (with the exception of trivial LO)
- Varying $\bar{\Lambda}$ provides natural estimate on systematic uncertainties

Weak coupling methods: Limitations



High temperatures: Dimensional reduction

- Perturbative EoS notoriously badly convergent due to static gluonic (IR) sector ($q_0 = 0,\ q \lesssim gT$)



High temperatures: Dimensional reduction

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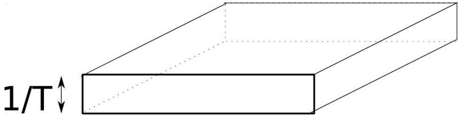
  - Resolution: Effective description via dimensional reduction



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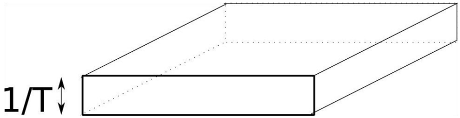

- Result: 3d eff. thy for static dof's (Kajantie et al; Braaten, Nieto):

$$\begin{aligned}\mathcal{L}_{\rm EQCD} &= g_{\rm E}^{-2}\left\{\frac{1}{2}\,{\rm Tr}\,F_{ij}^2 + {\rm Tr}\big[(D_iA_0)^2\big] + m_{\rm E}^2\,{\rm Tr}\,(A_0^2)\right. \\ &+ \left. i\zeta\,{\rm Tr}\,(A_0^3) + \lambda_{\rm E}\,{\rm Tr}\,(A_0^4)\right\} + \delta\mathcal{L}_{\rm E}, \\ & g_{\rm E} \equiv \sqrt{T}g,\ m_{\rm E} \sim gT,\ \zeta \sim g^3,\ \lambda_{\rm E} \sim g^4\end{aligned}$$

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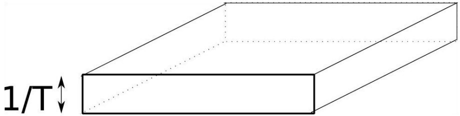
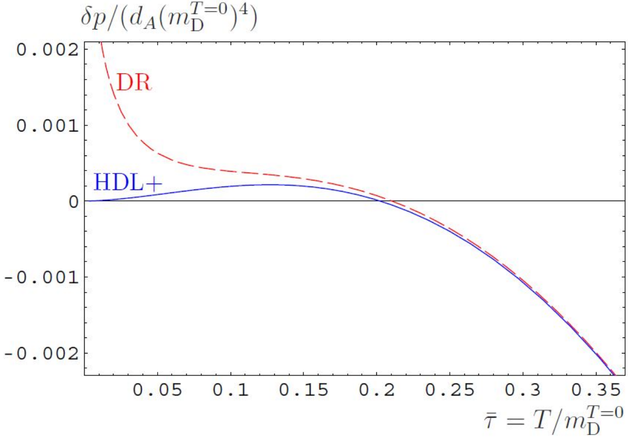


- EQCD valuable tool in reorganizing perturbation theory
  - All IR sensitive contributions through partition function of 3d theory

$$p_{\rm QCD}(T,\mu) \;\; = \;\; p_{\rm E}(T,\mu) + \frac{T}{V}\ln\int \mathcal{D}A_i^a\,\mathcal{D}A_0^a\,\exp\left\{-S_{\rm E}\right\}$$

- Setup essentially unchanged at nonzero μ
  - One new operator ($\mathrm{Tr}\,(A_0^3)$) generated in $\mathcal{L}_{\rm E}$
  - DR seen to work until $\mu \sim T/g$ (Ipp, Kajantie, Rebhan, AV)

High temperatures: Dimensional reduction



High temperatures: Results

Status of perturbation theory at high temperature:

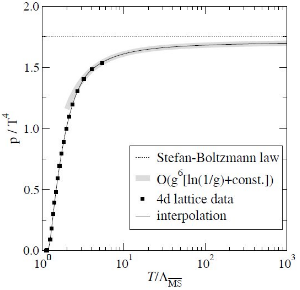
- Using EQCD, pressure computed to partial g^6 order (Kajantie, Laine, Rummukainen, Schröder; ...)
- Results generalized to nonzero density (AV)
- HTLpt worked out up to three-loop order at $\mu = 0$ and $\mu \neq 0$ (Andersen, Hague, Mustafa, Strickland, Su)

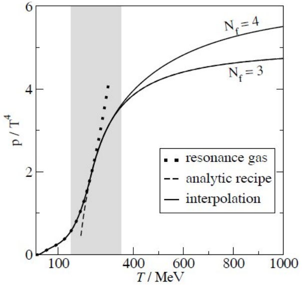
For direct comparison with lattice results at nonzero μ, natural observables: Quark number susceptibilities (QNS)

$$\chi_{ijk} \equiv -\frac{\partial^n \Omega(T, \{\mu_f\}, \{m_f\})}{\partial \mu_u^i \partial \mu_d^j \partial \mu_s^k}\bigg|_{\mu_f=0}, \quad n = i + j + k$$

High temperatures: Results

Fitting one unknown ($\mathcal{O}(g^6)$) parameter to hadron resonance gas results and keeping EQCD parameters unexpanded gives an almost perfect match with lattice results (Laine, Schröder)

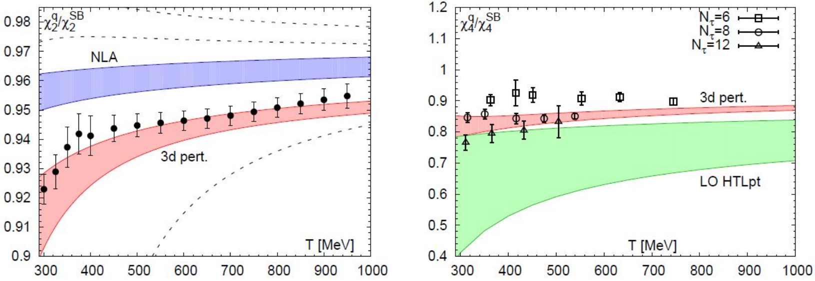




Note: $\mathcal{O}(30\%)$ correct estimate of T_c for deconfinement transition!

High temperatures: Results

Same strategy works beautifully also for the QNS, this time with no fitted constants (Mogliacci, Andersen, Strickland, Su, AV)



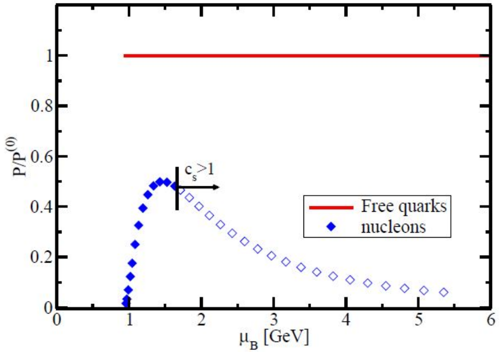
Bazavov et al, 1309.2317

Cold quark matter: Expectations

- Results based on 3d resummations fail in strict $T = 0$ limit, as IR divergent sector now four-dimensional
  - Problems enter one order higher, but no effective theory description known $\Rightarrow$ Need explicit diagrammatic resummations
- Naive expectation: Somewhat better convergence than at $\mu = 0$ due to absence of purely gluonic contributions
  - However: $\pi T \sim \mu_q = \mu_B/3$
- Ground state color superconducting
  - Difficult to handle quantitatively; contribution to EoS strongly suppressed at high densities

$$p = p_{\rm pert} + \# \times \frac{\Delta^2 \mu_{\rm B}^2}{3\pi^2}$$

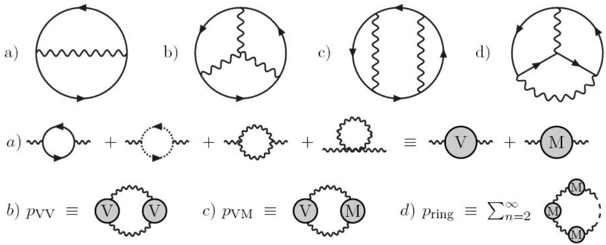
Cold quark matter: The calculation



Challenge: Interpolate between nuclear matter EoS and Stefan-Boltzmann limit

$$p_{\rm SB}(\mu_f, m_s = 0) \quad = \quad \frac{1}{4\pi^2}\left(\mu_u^2 + \mu_d^2 + \mu_s^2\right)$$

Cold quark matter: The calculation

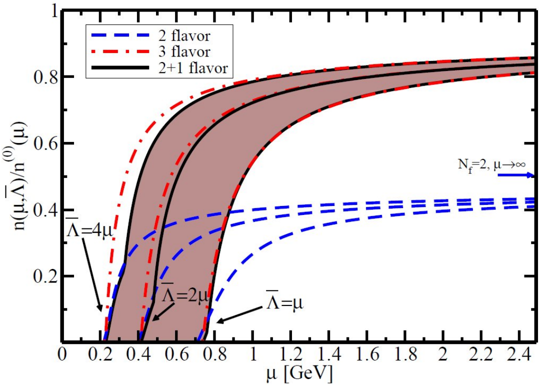


State of the art: Three loops, keeping strange quark mass nonzero (Kurkela, Romatschke, AV)

$$p_{\text{pert}}(\mu_f, m_s) = p_{\text{SB}}(\mu_f, m_s) + p_1(\mu_f, m_s)\alpha_s + p_2(\mu_f, m_s)\alpha_s^2 + \mathcal{O}(\alpha_s^3)$$

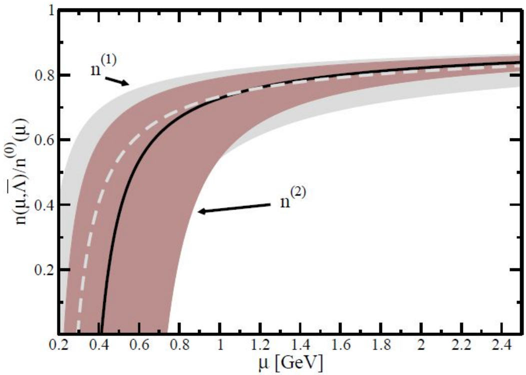
- $p_1(\mu_f, m_s)$, $p_2(\mu_f, 0)$: Freedman, McLerran (1977); Baluni (1978)
- $p_2(\mu_f, m_s)$: Kurkela, Romatschke, AV (2009)

Cold quark matter: Results



- Smooth interpolation of n_{total} between 2 and 3 flavor cases
- Bands from variation of $\bar{\Lambda}$ by factor of 2 around $\bar{\Lambda} = 2\mu_s$

Cold quark matter: Results



- Apparent convergence around $\mu_s \approx 900$ MeV
- Pressure from integration of quark number density — integration constant undetermined!

Cold quark matter EoS: Uses

Main use of the result: Replace $p_{\mathrm{SB}}(\mu_f)$ in MIT bag model EoS

$$p(\mu_f, m_s) = p_{\mathrm{pert}}(\mu_f, m_s) - B\,,$$

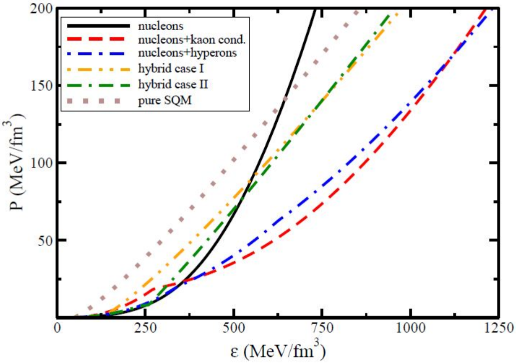
leading to:

- Significantly stiffer QM EoS at low density
- Realistic uncertainty estimates from renormalization scale dependence!

Applications in:

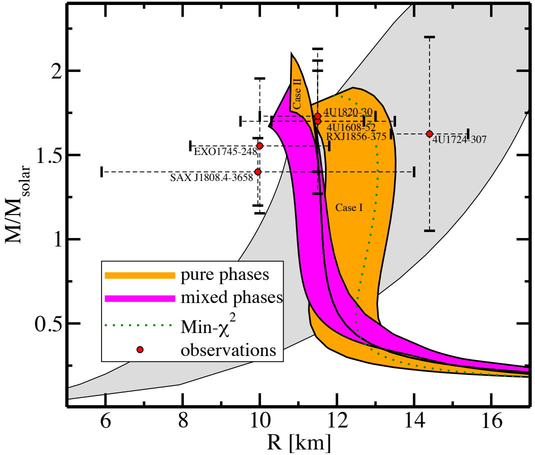
- *M*-*R* relation of quark stars
- Thermodynamically consistent matching with nuclear EoSs $\Rightarrow$ Hybrid star EoSs
- Constrainting phenomenological models of quark matter

Cold quark matter EoS: Uses



Kurkela, Romatschke, AV

Cold quark matter EoS: Uses



Kurkela, Romatschke, AV, Wu

Conclusion

1. Instead of MIT bag model EoS, use your most trusted perturbative/model/whatever pressure *and* a bag constant!
   - Otherwise, end up arbitrarily hiding systematic uncertainties
2. 3-loop cold quark matter EoS available at
   - arXiv:0912.1856 (Phys.Rev. D81 (2010) 105021)
   - http://theory.physics.helsinki.fi/∼aekurkel/neutron/