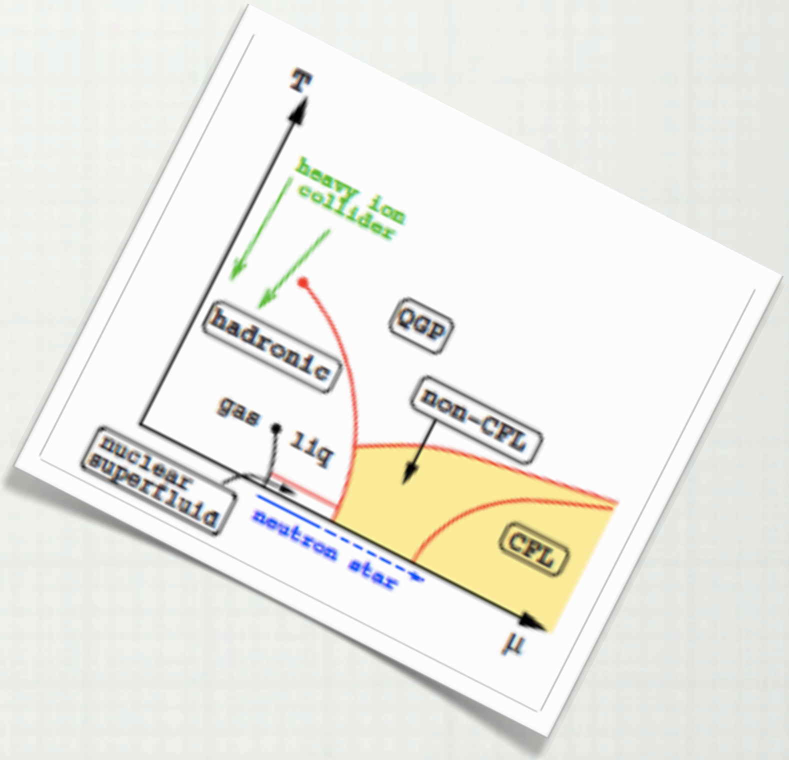
QUARK MATTER IN COMPACT STARS: INDICATIONS FROM X-RAY AND RADIO PULSARS



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IN COLLABORATION WITH
MARK ALFORD

EMMI TASK FORCE MEETING, 7.10.2013. FIAS FRANKFURT

INTRODUCTION

COMPACT STARS

☐ PULSARS ARE COMPACT OBJECTS $M > M_\odot$ @ $R < 15\mathrm{km}$

☐ INTERIOR DENSE ENOUGH THAT THE COULD CONTAIN MANY NOVEL FORMS OF MATTER:

☐ TO LEARN ABOUT THEM REQUIRES TO CONNECT OBSERVATIONAL ASPECTS TO THE MICROSCOPIC PROPERTIES OF MATTER

Hyperons?

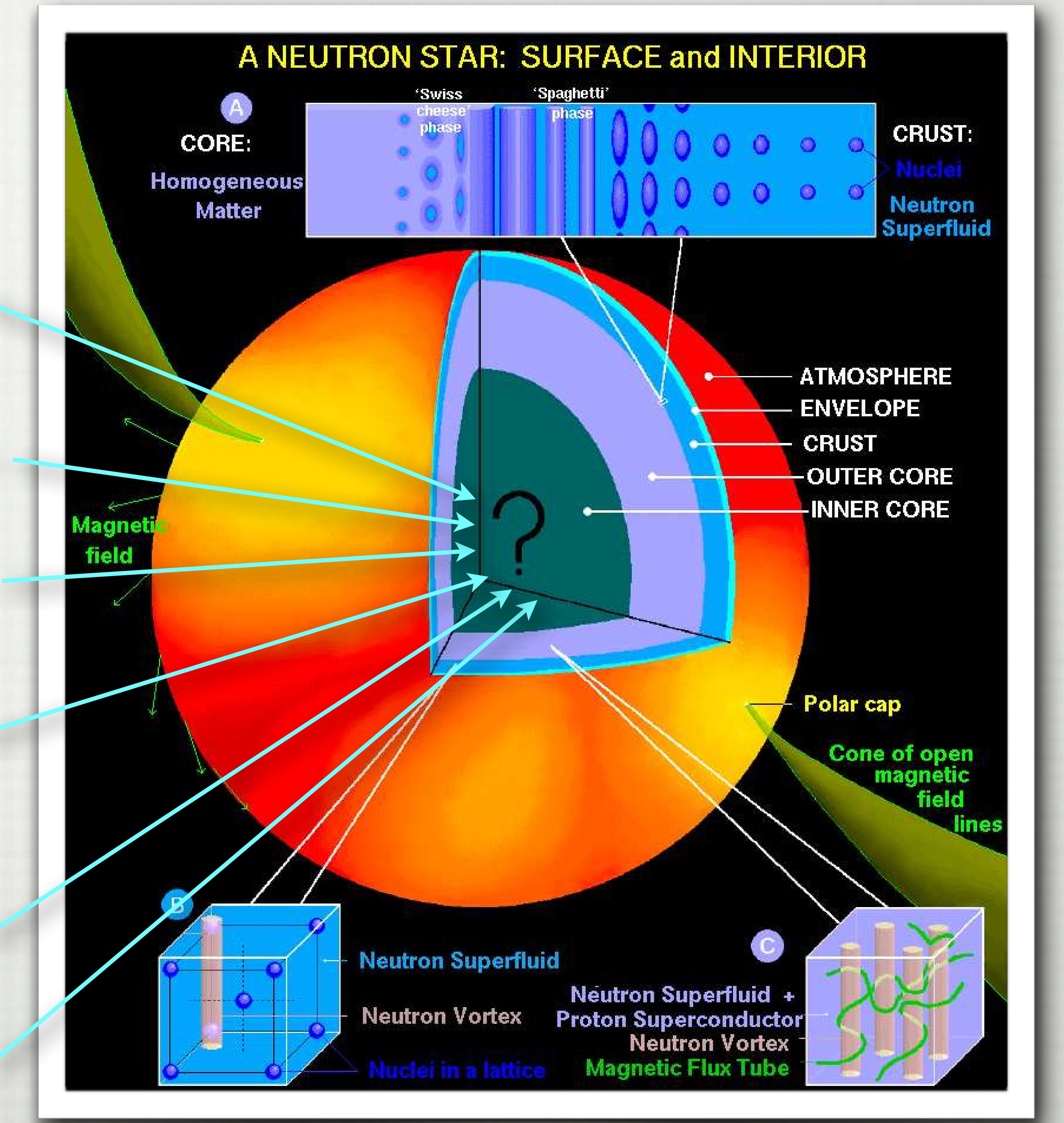
Meson condensates?

Mixed phases?

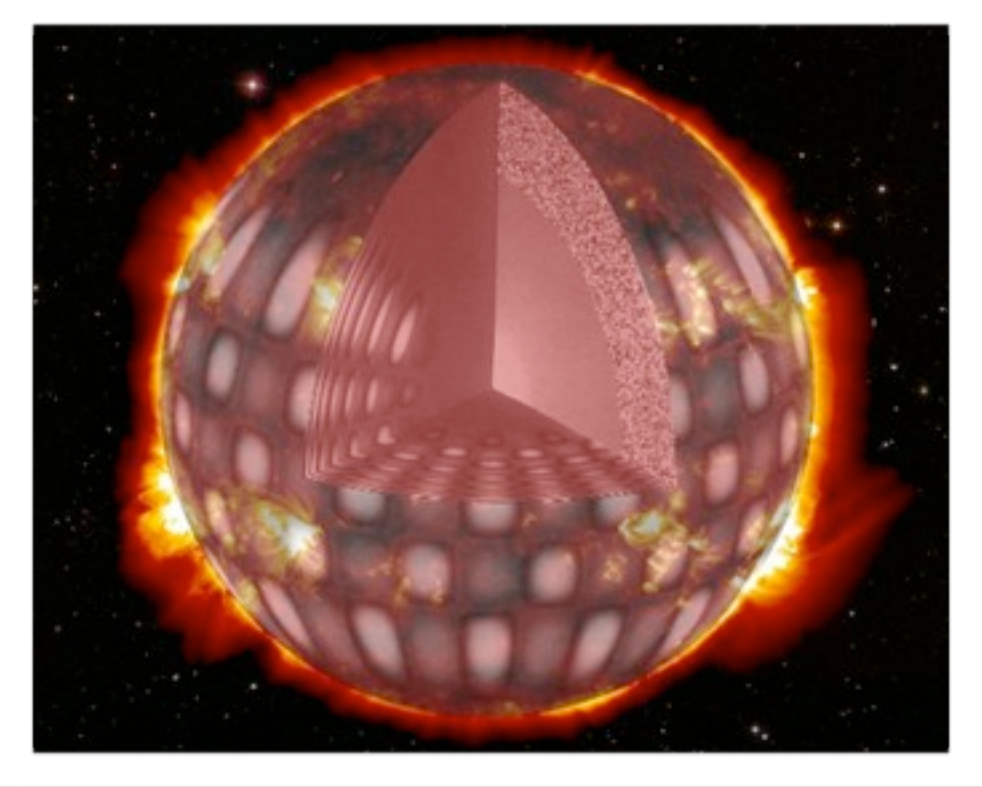
Quark matter?

Anisotropy / Inhomogeneity?

Color super-conductivity?



"SEEING INSIDE A COMPACT STAR ..."



- ▷ ELECTROMAGNETIC RADIATION ORIGINATES FROM THE SURFACE - CONNECTION TO THE INTERIOR VERY INDIRECT

- ☐ YET, ONE CAN USE SIMILAR METHODS WE USE TO LEARN ABOUT THE INTERIOR OF THE EARTH OR THE SUN: "SEISMOLOGY"

- ☐ WHEN NON-AXISYMMETRIC OSCILLATIONS ARE NOT DAMPED AWAY THEY EMIT GRAVITATIONAL WAVES ...
  - ★ DIRECT DETECTION VIA GRAVITATIONAL WAVE DETECTORS
  - ☆ INDIRECT DETECTION VIA THE SPIN DATA OF PULSARS

- ▷ STAR OSCILLATIONS ARE DAMPED BY VISCOSITY WHICH IS INDUCED BY PARTICLE INTERACTIONS AND THEREBY DIRECTLY LINK MACROSCOPIC OBSERVABLES TO THE MICROPHYSICS OF DENSE MATTER

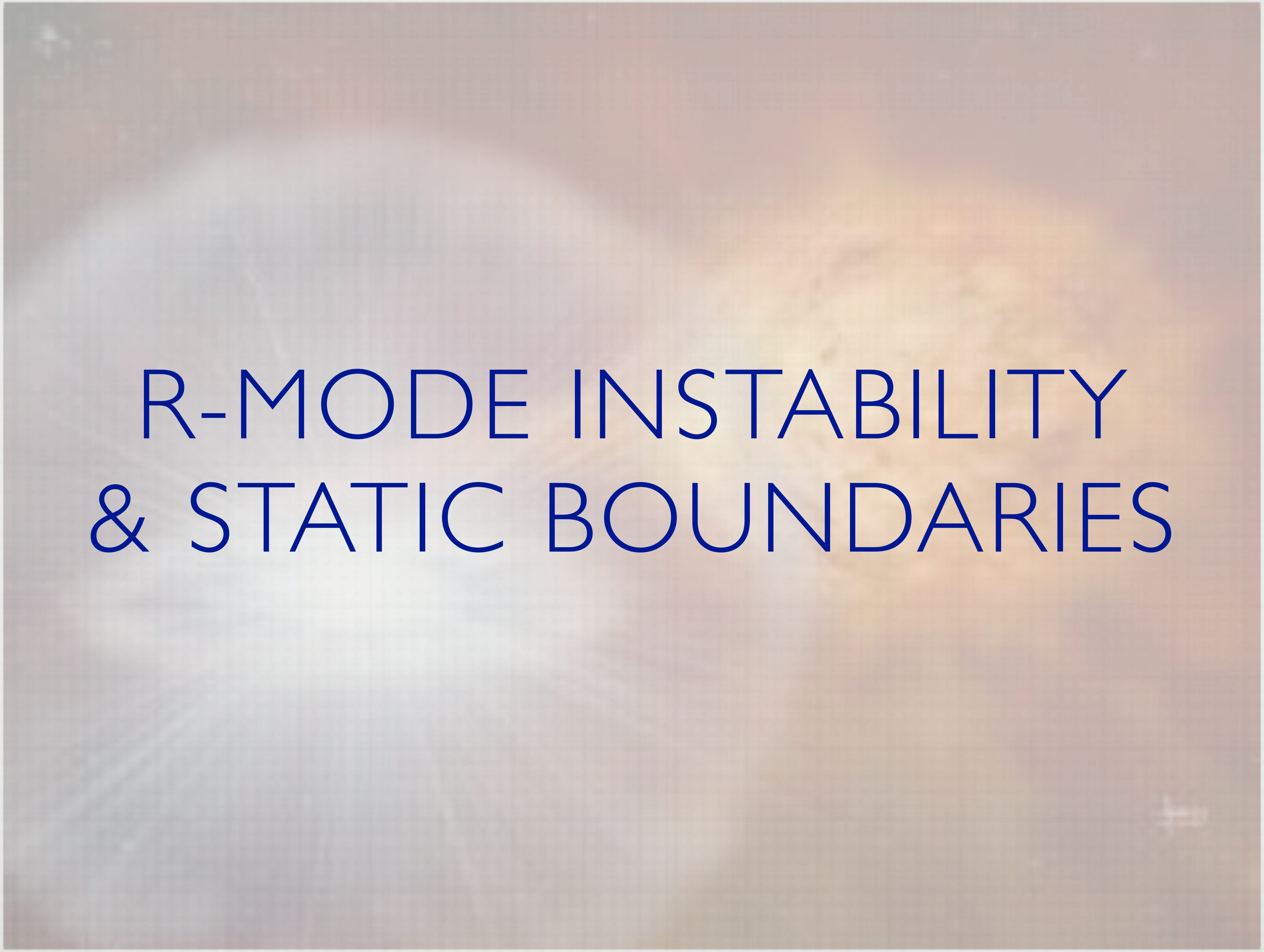
CONNECTION TO SPINDOWN AND PULSAR TIMING DATA

- ☐ GRAVITATIONAL WAVES EMITTED BY STAR OSCILLATIONS WOULD GENERALLY QUICKLY SPIN DOWN A FAST SPINNING STAR
- ☐ BUT MANY FAST ("MILLISECOND") PULSARS ARE OBSERVED - THEY CAN BE GROUPED INTO TWO CLASSES:
  - MS X-RAY PULSARS IN (LOW MASS) BINARIES (LMXB) CURRENTLY ACCRETE FROM A COMPANION WHICH ALLOWS A TEMPERATURE MEASUREMENT
    - T'S INVOLVE MODELING AND ARE UNCERTAIN

  - MS RADIO PULSARS ARE VERY OLD AND DON'T ACCRETE ANY MORE (NO HIGH ENERGY EMISSION) BUT FEATURE EXTREMELY STABLE TIMING DATA
    - ☆ SOME OF THE MOST PRECISE DATA IN PHYSICS!

- ☐ FAST PULSARS ARE A PUZZLE WHEN MODES BECOME UNSTABLE

R-MODE INSTABILITY & STATIC BOUNDARIES



R-MODE OSCILLATIONS

- ☐ R-MODE: EIGENMODE OF A ROTATING STAR WHICH IS UNSTABLE AGAINST GRAVITATIONAL WAVE EMISSION

  N. ANDERSSON, ASTROPHYS. J. 502 (1998) 708,
  L. LINDBLOM, ET. AL., PRL 80 (1998) 4843

- ☐ LARGE AMPLITUDE R-MODES COULD CAUSE A QUICK SPINDOWN

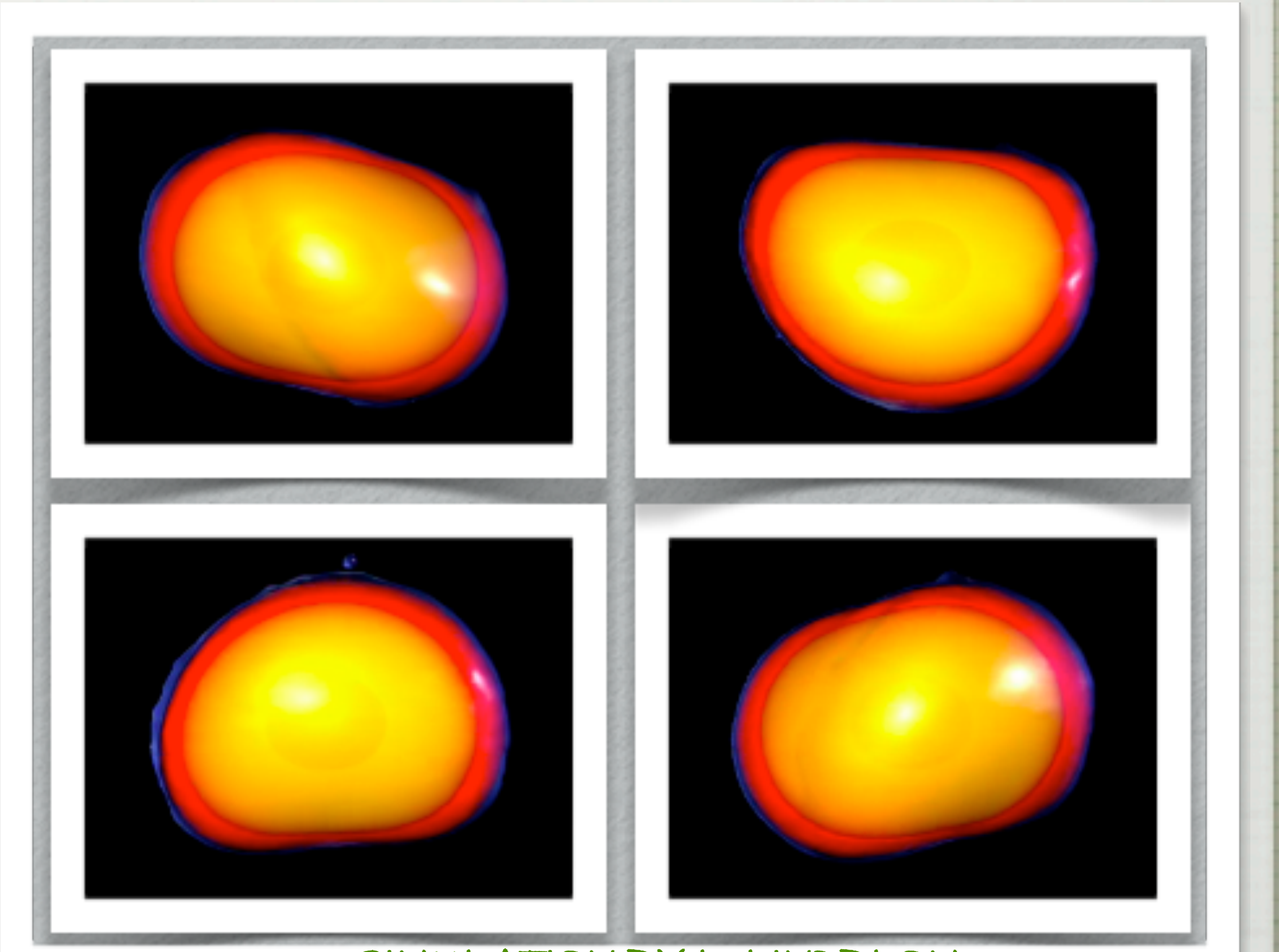
  B. J. OWEN, ET. AL.,
  PHYS. REV. D 58 (1998) 084020

- ☐ BUT R-MODE GROWTH HAS TO BE STOPPED BY SOME NON-LINEAR DAMPING MECHANISM, E.G.
  - ☐ NON-LINEAR VISCOUS DAMPING

    M. ALFORD, S. MAHMOODIFAR AND K.S.,
    PRD 85 (2012) 044051
  - ☐ NON-LINEAR HYDRO EFFECTS - LARGE $\alpha = O(1)$

    L. LINDBLOM, ET. AL., PRL 86 (2001) 1152,
    W. KASTAUN, ARXIV:1109.4839
  - ☐ MODE-COUPLING - SMALL $\alpha \ll 1$

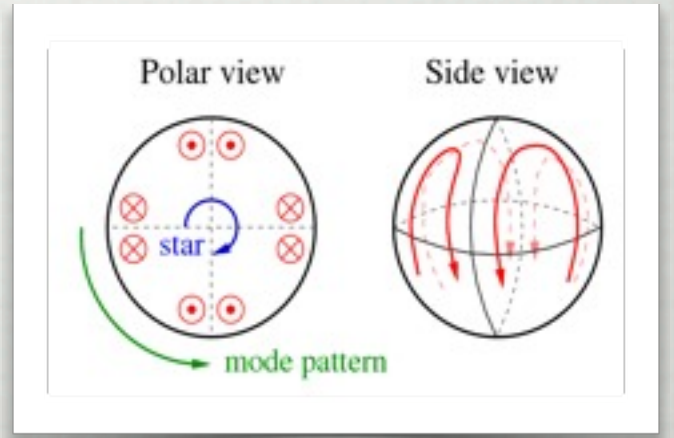
    P. ARRAS, ET. AL., ASTROPHYS. J. 591 (2003) 1129,
    R. BONDARESCU, ET. AL., PRD 79 (2009) 104003



SIMULATION BY L. LINDBLOM

VELOCITY OSCILLATION:

$$\delta\vec{v} = \alpha R\Omega \left(\frac{r}{R}\right)^{l} \vec{Y}^{B}_{ll}\, \mathrm{e}^{i\omega t}$$



“EFFECTIVE THEORY OF PULSARS”

☐ OBSERVABLE MACROSCOPIC PROPERTIES DEPEND ONLY ON QUANTITIES THAT ARE INTEGRATED OVER THE ENTIRE STAR:

$I = \tilde{I} M R^2$ (MOMENT OF INERTIA)

$$P_G = \frac{32\pi (m-1)^{2m} (m+2)^{2m+2}}{((2m+1)!!)^2 (m+1)^{2m+2}} \tilde{J}_m{}^2 G M^2 R^{2m+2} \alpha^2 \Omega^{2m+4}$$

(POWER RADIATED IN GRAVITATIONAL WAVES)

$$P_S = -\frac{(m-1)(2m+1) \tilde{S}_m \Lambda_{\mathrm{QCD}}^{3+\sigma} R^3 \alpha^2 \Omega^2}{T^\sigma}$$

$$P_B = -\frac{16m}{(2m+3)(m+1)^5 \kappa^2} \frac{\Lambda_{QCD}^{9-\delta} \tilde{V}_m R^8 \alpha^2 \Omega^4 T^\delta}{\Lambda_{EW}^4 \tilde{J}_m}$$

(DISSIPATED POWER DUE TO SHEAR / BULK VISCOSITY)

$C_V = 4\pi \Lambda_{QCD}^{3-\upsilon} R^3 \tilde{C}_V T^\upsilon$ (SPECIFIC HEAT)

$L_\nu = 4\pi R^3 \Lambda_{EW}^4 \Lambda_{QCD}^{1-\theta} \tilde{L} T^\theta$ (NEUTRINO LUMINOSITY)

CONNECTION BETWEEN MACRO AND MICROSCOPIC PROPERTIES

☐ OBSERVABLE MACROSCOPIC PROPERTIES DEPEND ONLY ON QUANTITIES THAT ARE INTEGRATED OVER THE ENTIRE STAR:

$$I = \tilde{I} M R^2$$

$$P_G = \frac{32\pi (m-1)^{2m} (m+2)^{2m+2}}{((2m+1)!!)^2 (m+1)^{2m+2}} \tilde{J}_m{}^2 G M^2 R^{2m+2} \alpha^2 \Omega^{2m+4}$$

$$P_S = -(m-1)(2m+1)\, \tilde{S}_m \frac{\Lambda_{\mathrm{QCD}}^{3+\sigma} R^3 \alpha^2 \Omega^2}{T^\sigma}$$

$$P_B = -\frac{16m}{(2m+3)(m+1)^5 \kappa^2} \tilde{V}_m \frac{\Lambda_{QCD}^{9-\delta} R^8 \alpha^2 \Omega^4 T^\delta}{\Lambda_{EW}^4 \tilde{J}_m}$$

$$C_V = 4\pi \Lambda_{QCD}^{3-\upsilon} R^3 \tilde{C}_V T^\upsilon$$

$$L_\nu = 4\pi R^3 \Lambda_{EW}^4 \Lambda_{QCD}^{1-\theta} \tilde{L} T^\theta$$

WITH ...

RIGOROUSLY BOUNDED WITHIN A FACTOR 2

$$\tilde{I} \equiv \frac{8\pi}{3MR^2} \int_0^R dr\, r^4 \rho$$

$$\tilde{J}_m \equiv \frac{1}{MR^{2m}} \int_0^R dr\, r^{2m+2} \rho$$

$$\tilde{S}_m \equiv \frac{1}{R^{2m+1} \Lambda_{QCD}^{3+\sigma}} \int_{R_i}^{R_o} dr\, r^{2m} \tilde{\eta}$$

$$\tilde{V}_m \equiv \frac{\Lambda_{EW}^4}{R^3 \Lambda_{QCD}^{9-\delta}} \int_{R_i}^{R_o} dr\, r^2 A^2 C^2 \tilde{\Gamma} \left(\delta\Sigma_m\right)^2$$

$$\tilde{C}_V \equiv \frac{1}{R^3 \Lambda_{QCD}^{3-\upsilon}} \int_{R_i}^{R_o} dr\, r^2 \tilde{c}_V$$

$$\tilde{L} \equiv \frac{1}{R^3 \Lambda_{EW}^4 \Lambda_{QCD}^{1-\theta}} \int_{R_i}^{R_o} dr\, r^2 \tilde{\epsilon}$$

- POWER LAWS IN α, Ω, T AND COMPLETE INFORMATION ON THE INTERIOR DEPENDS ON A FEW DIMENSIONLESS CONSTANTS

R-MODE
DENSITY
FLUCTUATION

INSTABILIY REGIONS AND SEMI-ANALYTIC RESULTS

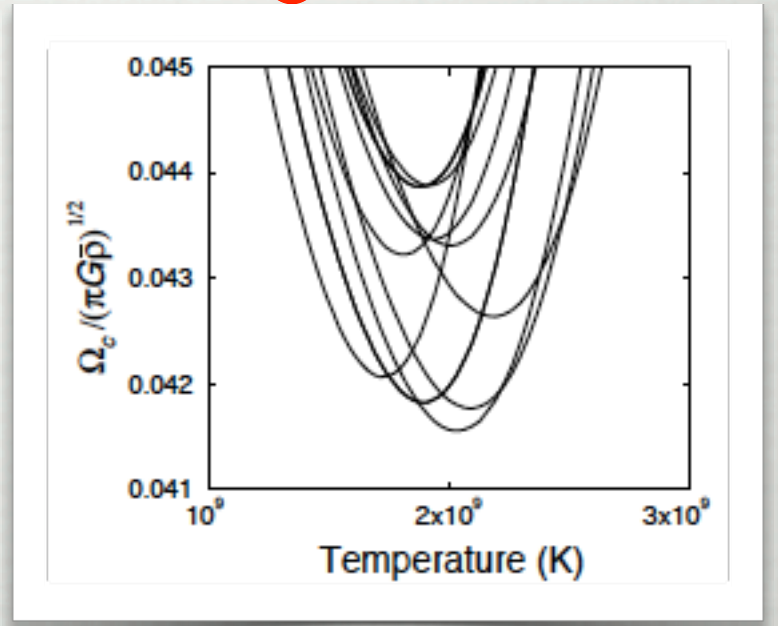
- ☐ R-MODES UNSTABLE FOR $P_G \geq P_V$
  - ☐ "ET" ALLOWS SEMI-ANALYTIC RESULT: $\Omega_{ib}(T) = \left(\hat{D} T^{\delta} \lambda^{\Delta} / \hat{G}\right)^{1/(8-\psi)}$
- ☐ SIMILAR RESULT FOR MINIMUM OF THE INSTABILITY REGION:

$$\Omega_{min} \approx \left(\left(\frac{m\,(m+1)^{2m-1}\,((2m+1)!!)^2}{4\pi\,(2m+3)\,(m+2)^{2m+2}\,(m-1)^{2m}} \right)^{\sigma} \left(\frac{((2m+1)!!)^2\,(2m+1)\,(m+1)^{2m+2}}{16\pi\,(m-1)^{2m-1}\,(m+2)^{2m+2}} \right)^{\delta} \frac{\tilde{V}_m^{\sigma} \tilde{S}_m^{\delta} \Lambda_{QCD}^{3\delta+9\sigma}}{\tilde{J}_m^{2(\delta+\sigma)} \Lambda_{EW}^{4\sigma} G^{\delta+\sigma} R^{2m(\delta+\sigma)-\delta-5\sigma} M^{2(\delta+\sigma)}} \right)^{\frac{1}{2m(\delta+\sigma)+2\delta}}$$

- ☐ LOWEST R-MODE OF A NEUTRON STAR WITH MODIFIED URCA:

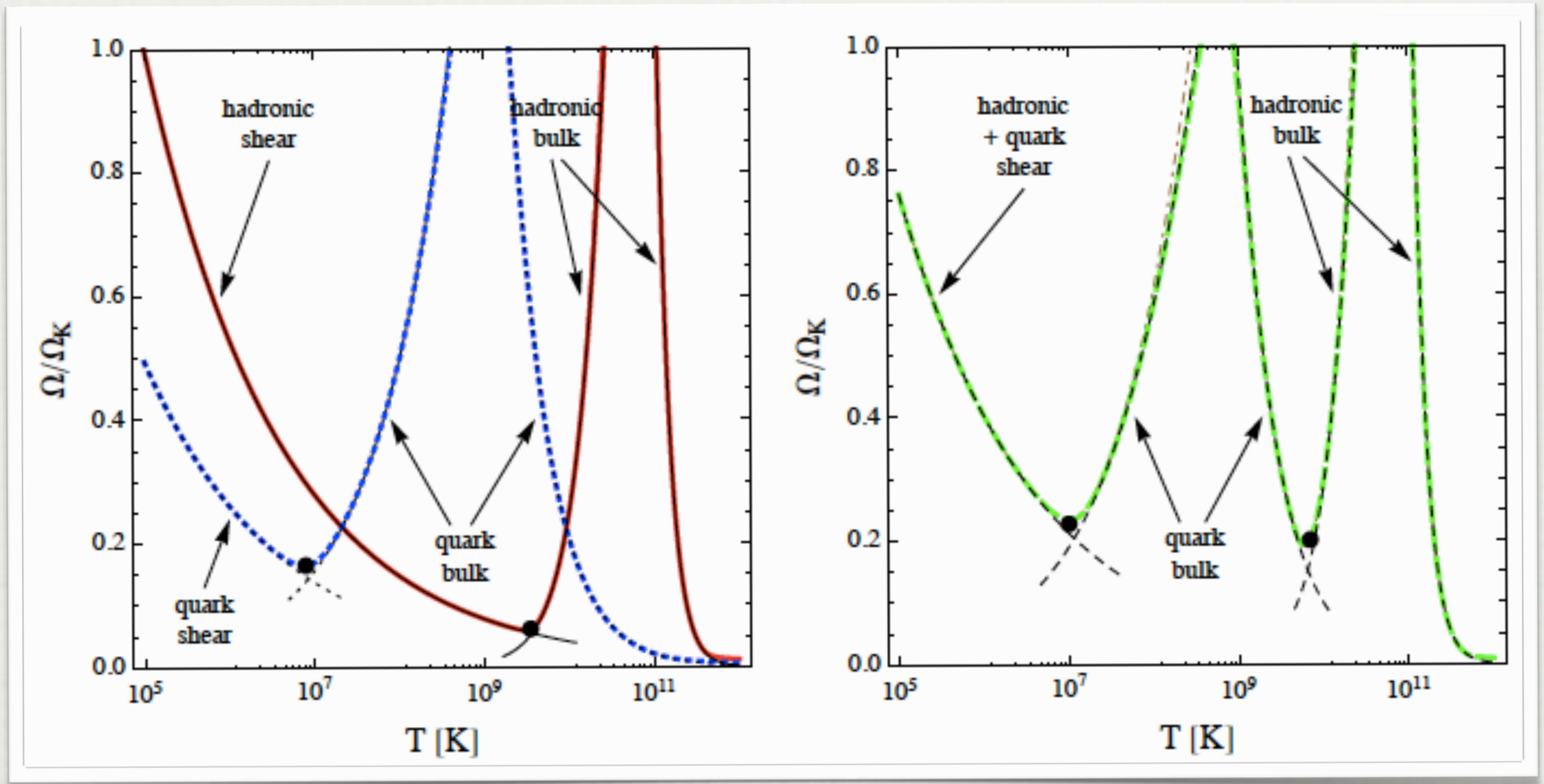
$$\Omega_{min}^{(NS)} \approx 1.06 \frac{\Lambda_{QCD}^{\frac{99}{128}} \tilde{S}^{\boxed{\frac{9}{64}}} \tilde{V}^{\boxed{\frac{5}{128}}}}{R^{\frac{49}{128}} \tilde{J}^{\frac{23}{64}} G^{\frac{23}{128}} M^{\frac{23}{64}} \Lambda_{EW}^{\frac{5}{32}}}, \quad T_{min}^{(NS)} \approx 1.89 \frac{\tilde{S}^{\boxed{\frac{3}{32}}} \Lambda_{QCD}^{\frac{1}{64}} \Lambda_{EW}^{\frac{9}{16}} G^{\frac{3}{64}} \tilde{J}^{\frac{3}{32}} M^{\frac{3}{32}}}{\tilde{V}^{\boxed{\frac{9}{64}}} R^{\frac{27}{64}}}$$

  - ☐ EXTREMELY LOW POWERS OF $\tilde{S}$ & $\tilde{V}$!
  - ☐ INSENSITIVE TO DETAILS WITHIN A CLASS ...
  - ☐ ... BUT NOT TO DIFFERENT CLASSES (δ & σ)
- ☐ GENERALIZATION OF A PREVIOUS RESULT



L. LINDBLOM, ET. AL., PRL 80 (1998) 4843

ANALYTIC VS. NUMERIC RESULTS FOR THE INSTABILITY REGION



M. ALFORD, S. MAHMOODIFAR AND K.S., PRD 85 (2012) 024007

☐ VERY GOOD AGREEMENT BETWEEN THE SEMI-ANALYTIC AND NUMERIC RESULTS

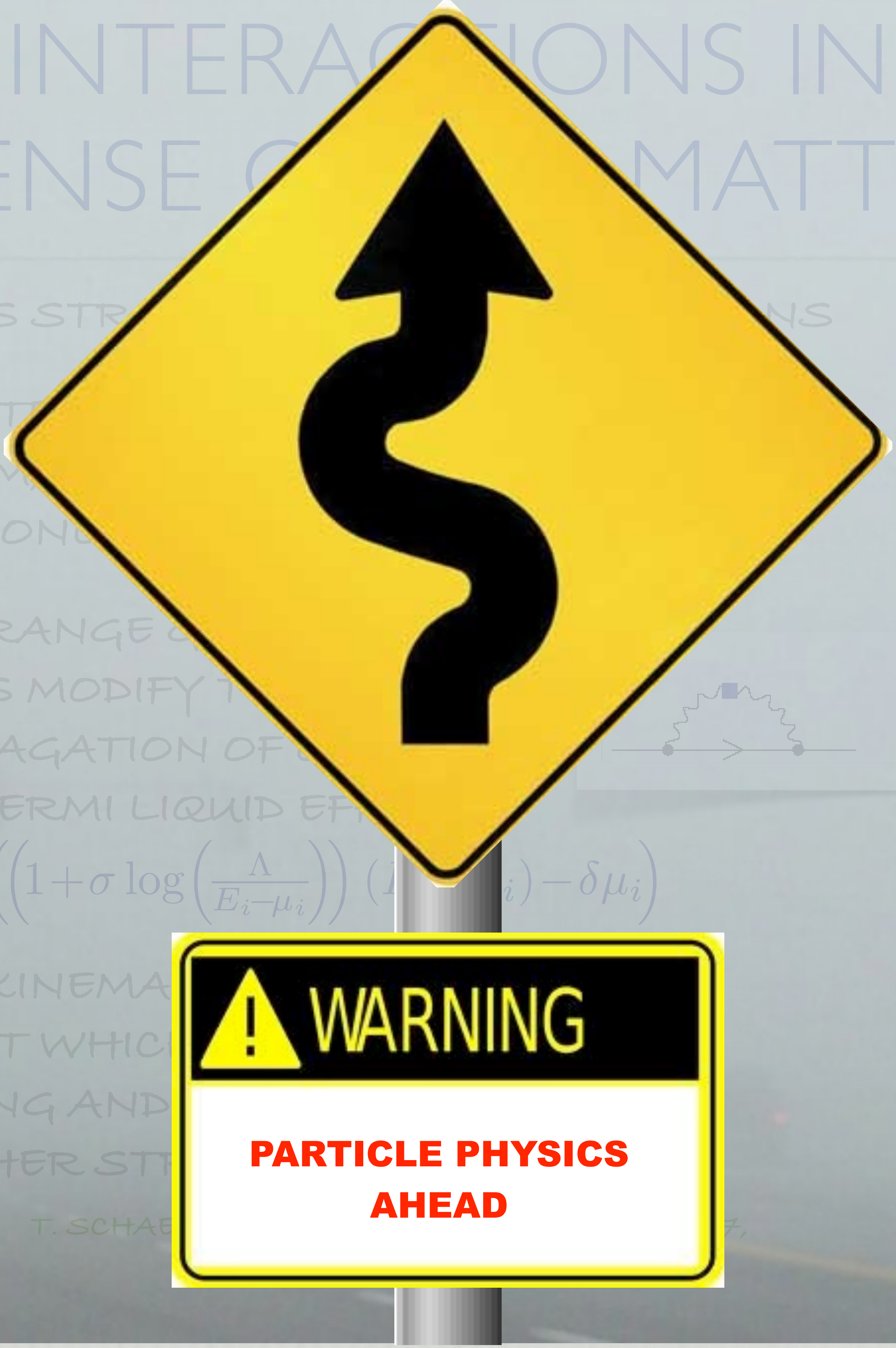
☐ ANALYTIC EXPRESSIONS COVER THE BASICALLY ENTIRE INSTABILITY BOUNDARY

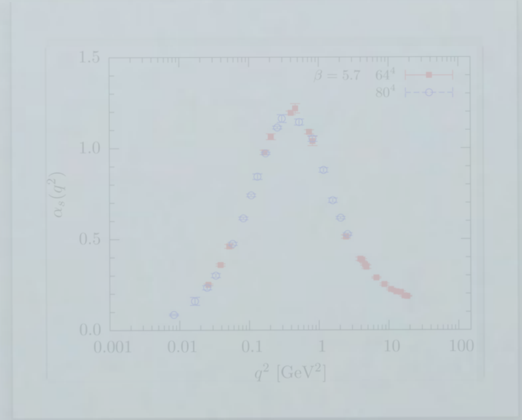
INTERA IONS IN
DENSE MATTER

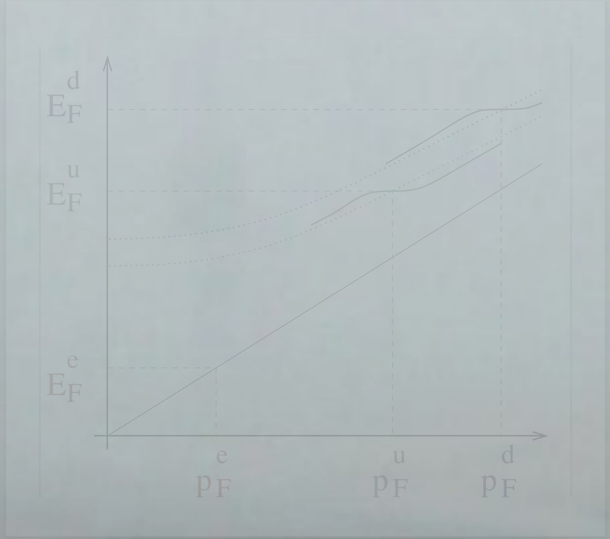
- ☐ QCD FEATURES STR NS
- ☐ IN DENSE MATT
  BY THE MEDIUM
  GLUONS ARE ON
- ☐ THESE LONG-RANGE
  INTERACTIONS MODIFY T
  ENERGY PROPAGATION OF
  INDUCE NON-FERMI LIQUID EF

$$p_i \approx p_{Fi} + v_{Fi}^{-1}\Big(\Big(1+\sigma\log\Big(\frac{\Lambda}{E_i-\mu_i}\Big)\Big)\ (\quad i)-\delta\mu_i\Big)$$

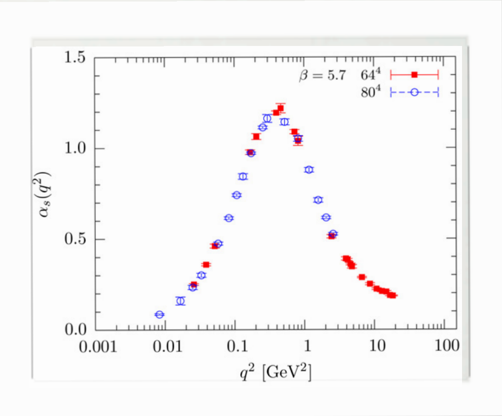
- ☐ PARAMETRIC KINEMA
  ENHANCEMENT WHIC
  WEAK COUPLING AND
  DOMINATE OTHER ST
  CORRECTIONS T. SCHAE 7.



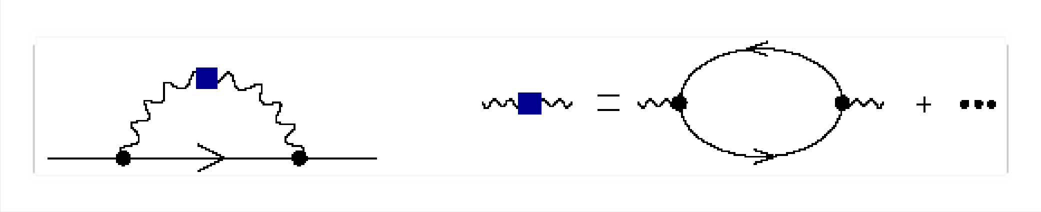




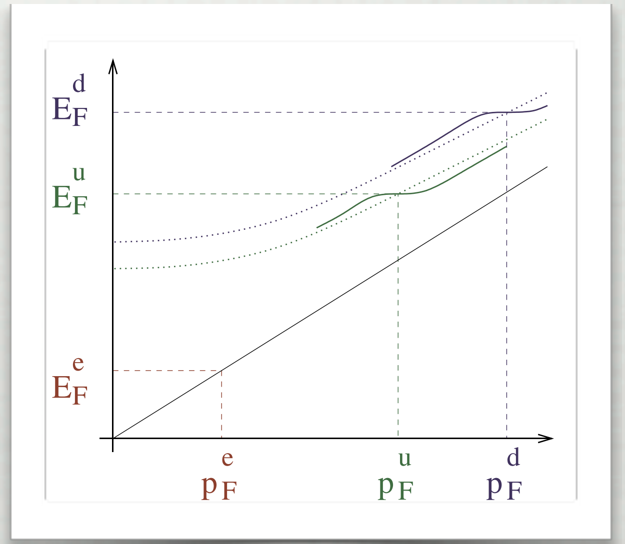
INTERACTIONS IN DENSE QUARK MATTER



- ☐ QCD FEATURES STRONG GLUONIC INTERACTIONS
- ☐ IN DENSE MATTER THEY ARE MOSTLY SCREENED BY THE MEDIUM, BUT TRANSVERSE SPACE-LIKE GLUONS ARE ONLY DYNAMICALLY DAMPED
- ☐ THESE LONG-RANGE GLUONIC INTERACTIONS MODIFY THE LOW ENERGY PROPAGATION OF QUARKS INDUCE NON-FERMI LIQUID EFFECTS:



$$p_i \approx p_{Fi} + v_{Fi}^{-1}\left(\left(1+\sigma \log\left(\frac{\Lambda}{E_i-\mu_i}\right)\right)(E_i-\mu_i)-\delta\mu_i\right)$$



- ☐ PARAMETRIC KINEMATIC LOW-ENERGY ENHANCEMENT WHICH IS EVEN PRESENT AT WEAK COUPLING AND SHOULD THEREBY DOMINATE OTHER STRONG INTERACTION CORRECTIONS

T. SCHAEFER AND K.S., PRD 70 (2004) 054007, PRL 97 (2006) 092301

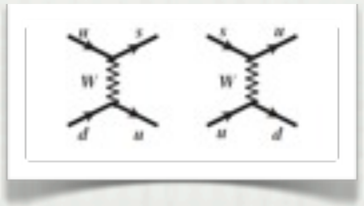
NON-FERMI LIQUID ENHANCEMENT

☐ THE STRONG INCREASE OF THE DENSITY OF STATES NEAR THE FERMI SURFACE LEADS TO A LOGARITHMIC ENHANCEMENT OF MATERIAL PROPERTIES BY FACTORS OF $\lambda(T) \approx 1 + \frac{4\alpha_s}{9\pi}\log\left(\frac{\Lambda}{T}\right)$ WITH $T \ll \Lambda = O(\mu)$

☐ SPECIFIC HEAT AND NEUTRINO EMISSIVITY ARE MODERATELY ENHANCED: $c_V \sim T\lambda(T)$, $\epsilon \sim T^6(\lambda(T))^2$

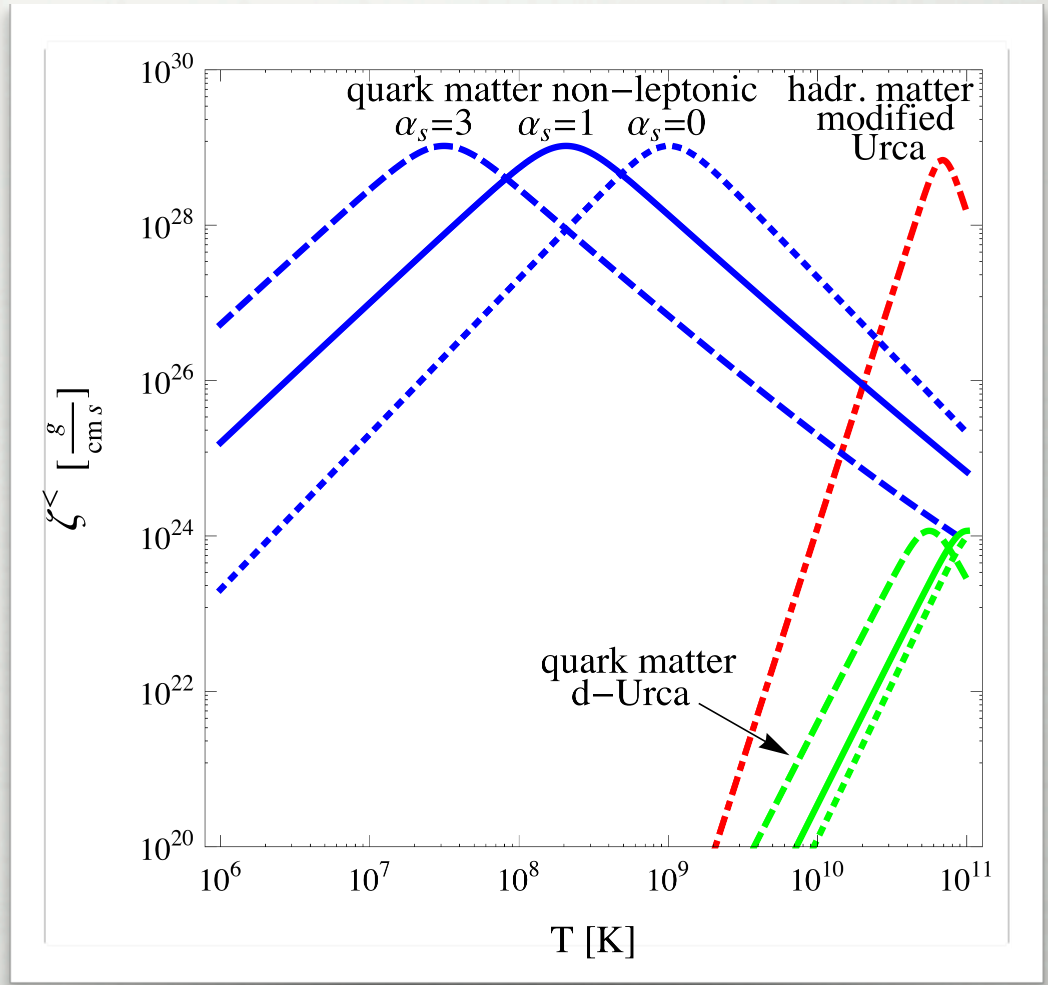
A. GERHOLD, ET.AL., PRD 70 (2004) 105015;
T. SCHAEFER AND K.S., PRD 70 (2004) 114037

☐ STRONG ENHANCEMENT OF NON-LEPTONIC RATE ...



$$\Gamma_{nl}^{(\leftrightarrow)} \approx -\frac{64G_F^2\sin^2\theta_c\cos^2\theta_c}{5\pi^3}\mu_q^5T^2(\lambda(T))^4\mu_\Delta$$

☐ ... SIGNIFICANTLY SHIFTS RESONANT MAXIMA OF THE BULK VISCOSITY



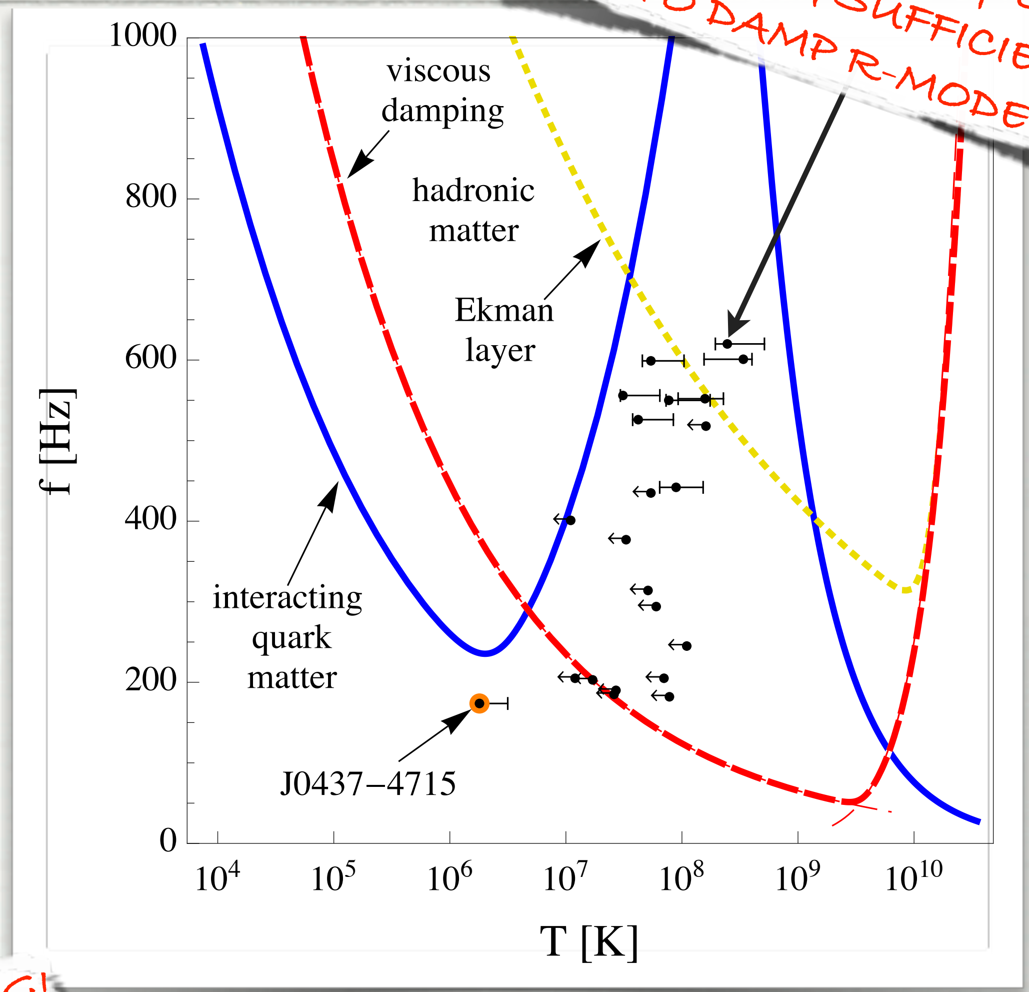
K.S.., ARXIV:1212.5242

☐ POTENTIALLY OBSERVABLE IMPACT OF NFL EFFECTS OUTSIDE THE LABORATORY AND AT HUGE TEMPERATURES!

G. STEWART, REV. MOD. PHYS. 73, 797 (2001)

STATIC INSTABILITY REGIONS VS. THERMAL X-RAY DATA

- ☐ R-MODES ARE UNSTABLE AT SMALL AMPLITUDE IF THE DAMPING IS NOT SUFFICIENT
- ☐ BOUNDARY GIVEN BY $P_G = P_D|_{\alpha \to 0}$
- ☐ REQUIRES TEMPERATURE MEASUREMENTS WHICH ARE ONLY AVAILABLE FOR A FEW LOW MASS X-RAY BINARIES
- ☐ TWO SCENARIOS TO EXPLAIN DATA:
  NO R-MODE: COMPLETELY DAMPED
  TINY R-MODE: UNSTABLE, INTERESTING!
  BUT SATURATED AT SMALL α_{sat}
  BORING TRIVIAL CASE

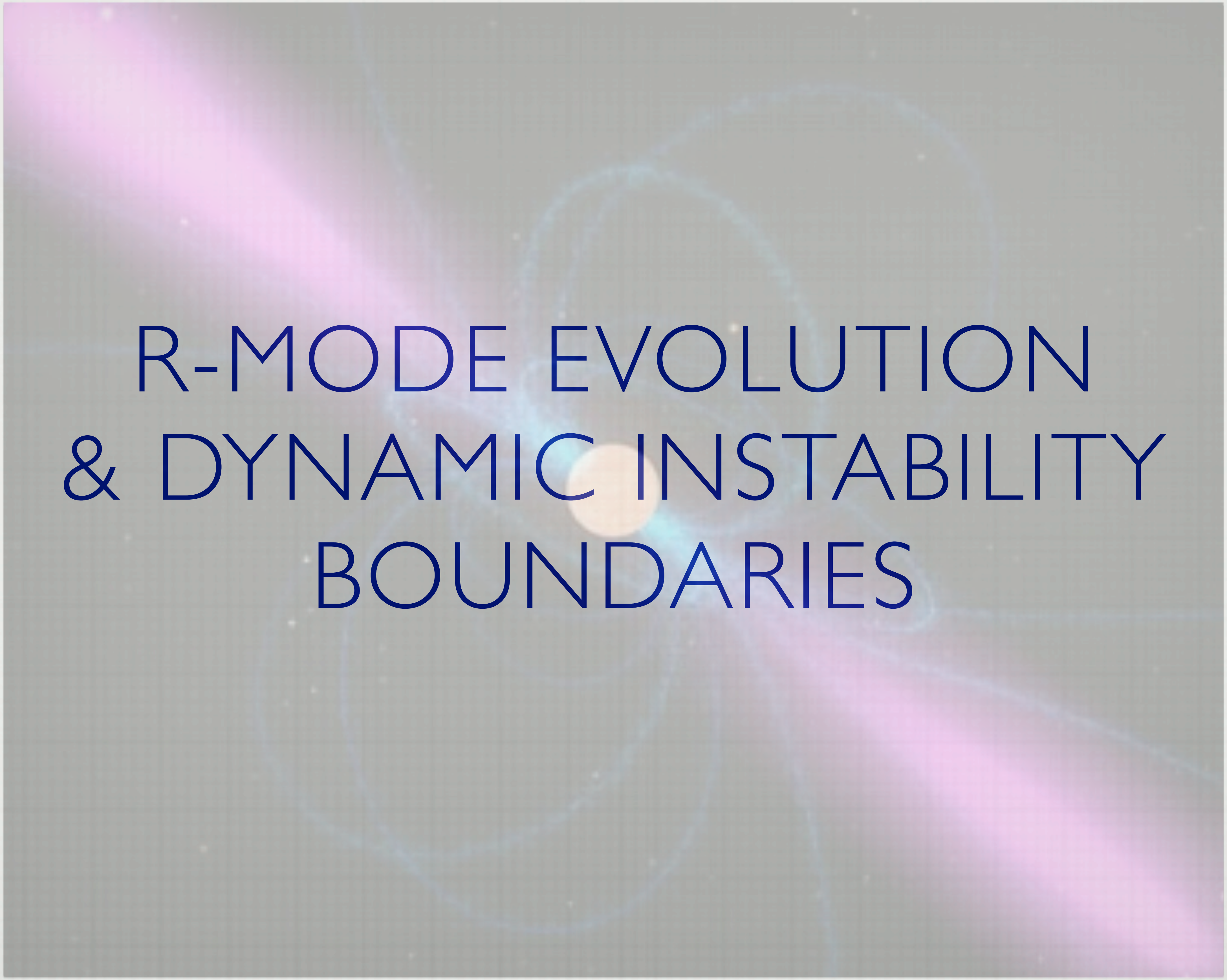


NOT EVEN MAXIMUM (THIN) BOUNDARY LAYER DAMPING SUFFICIENT TO DAMP R-MODE!

K. SCHWENZER, ARXIV:1212.5242

ANALYTIC RESULT: $\Omega_{ib}(T) = \left(\hat{D} T^{\delta} \lambda^{\Delta} / \hat{G}\right)^{1/(8-\psi)}$

- ☐ MANY SOURCES ARE CLEARLY WITHIN THE INSTABILITY REGION FOR NEUTRON STARS WITH STANDARD DAMPING (TINY RM REQUIRED)
- ☐ QUARK MATTER WITH NFL-INTERACTIONS FULLY DAMPS MODE (NO RM)



R-MODE EVOLUTION & DYNAMIC INSTABILITY BOUNDARIES

R-MODE EVOLUTION EQUATIONS

- THE PULSAR EVOLUTION IS OBTAINED FROM GLOBAL CONSERVATION EQUATIONS:

$$\frac{d\alpha}{dt} = -\alpha\left(\frac{1}{\tau_G} + \frac{1}{\tau_V}\left(\frac{1-Q\alpha^2}{1+Q\alpha^2}\right)\right) \approx -\alpha\left(\frac{1}{\tau_G} + \frac{1}{\tau_V}\right)$$

$$\frac{d\Omega}{dt} = -\frac{2\Omega Q\alpha^2}{\tau_V}\frac{1}{1+Q\alpha^2} \approx -\frac{2\Omega Q\alpha^2}{\tau_V}$$

$$\frac{dT}{dt} = -\frac{1}{C_V}\left(L_\nu - P_V\right)$$

B. OWEN, ET. AL., PRD 58 (1998) 084020,
W. HO AND D. LAI, ASTROPHYS. J. 543 (2000) 386

- GENERAL BOUNDS SHOW THAT $Q \equiv 3\tilde{J}/(2\tilde{I}) < 81/(112\pi) \approx 0.23$ SO THAT THE APPROXIMATE FORMS HOLD FOR PHYSICAL AMPLITUDES

- ONLY GRAVITATIONAL WAVE EMISSION EXPLICITLY CONSIDERED - BUT OTHER (ELECTROMAGNETIC) SPINDOWN MECHANISMS PRESENT

- SINCE THE R-MODE IS UNSTABLE IT REQUIRES A VISCOUS SATURATION MECHANISM WHICH IS ASSUMED TO OPERATE AND TO STOP THE EXPONENTIAL GROWTH AT A FINITE AMPLITUDE $\alpha_{sat}(T,\Omega)$

QUALITATIVE ASPECTS OF THE EVOLUTION

☐ TO ANALYZE THE QUALITATIVE FORM OF THE EVOLUTION IT IS USEFUL TO DEFINE CHARACTERISTIC EVOLUTION TIME SCALES

$$\tau_\alpha \equiv -\alpha \left(\frac{d\alpha}{dt}\right)^{-1} , \quad \tau_\Omega \equiv -\Omega \left(\frac{d\Omega}{dt}\right)^{-1} , \quad \tau_T \equiv -T \left(\frac{dT}{dt}\right)^{-1}$$

☐ THE AMPLITUDE RISES QUICKLY ON TIME SCALES $\tau_\alpha \approx \tau_G = O(s)$ AND SATURATES AT $\alpha_{\rm sat} = \hat\alpha_{\rm sat} T^\beta \Omega^\gamma$

☐ QUESTION: WHAT IS FASTER - THERMAL EVOLUTION OR SPINDOWN?

☐ HEATING IS VERY STRONG AND DOMINATES OVER THE SPINDOWN FOR:

$$\frac{\Omega}{\Omega_K} > \frac{27}{8\pi}\sqrt{\frac{\tilde{C}_V}{\tilde{I}(1-Q\alpha_{sat}^2)}\frac{R}{R_S}\frac{\Lambda_{QCD}^4}{\bar\rho}\left(\frac{T}{\Lambda_{QCD}}\right)^{\upsilon+1}} \xrightarrow[\upsilon=1]{} O(1)\frac{T}{\Lambda_{QCD}}$$

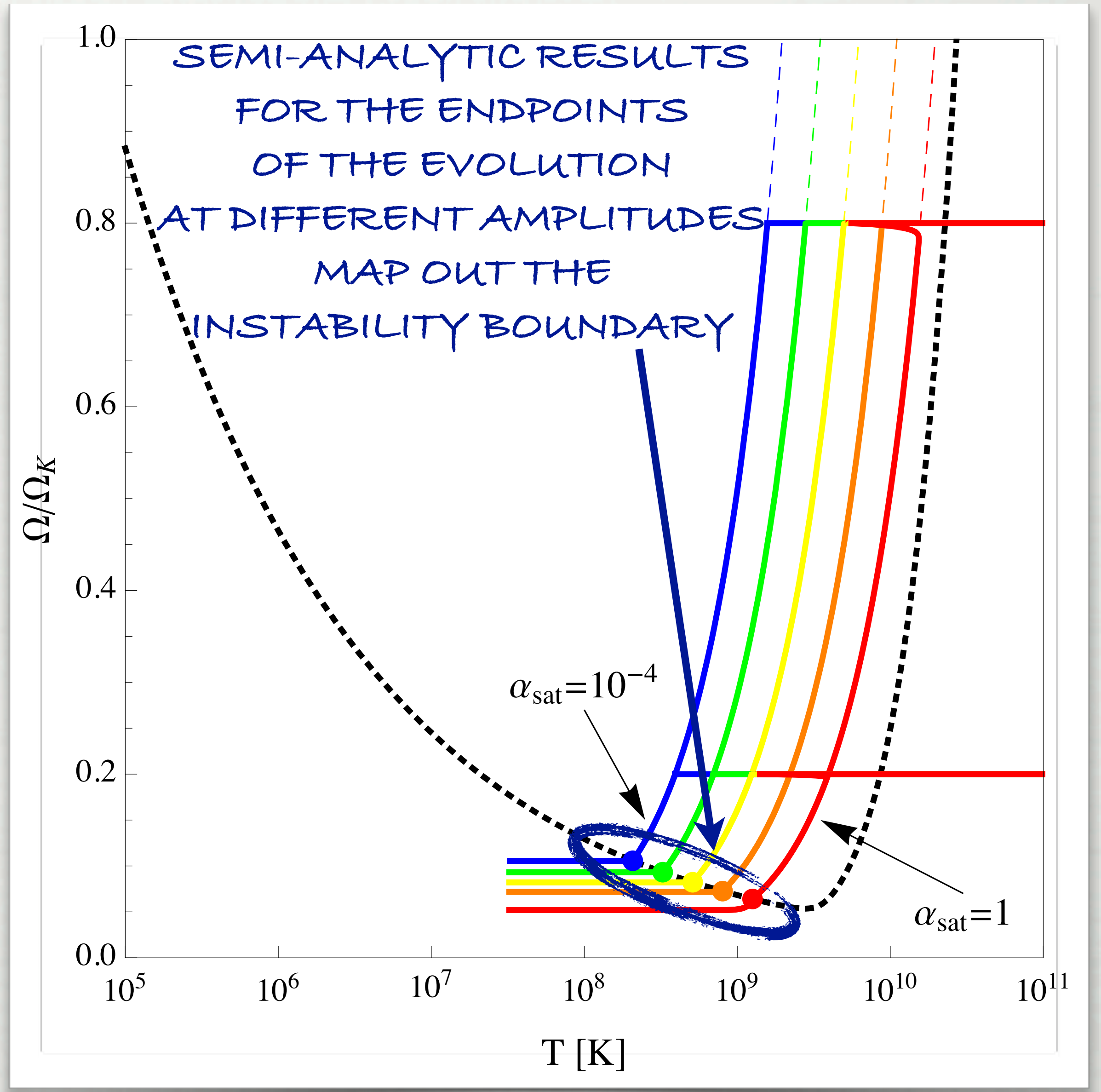
▶ THERMAL EVOLUTION IS ALWAYS FASTER FOR ANY STAR COMPOSITION!

☐ STEADY STATE IS REACHED WHERE NEUTRINO COOLING AND VISCOUS HEATING BALANCE:

$$\Omega_{hc}(T) = \left(\frac{3^8 5^2}{2^{15}}\frac{\tilde{L}\Lambda_{QCD}^{9-\theta}T^{\theta-2\beta}}{\tilde{J}^2\Lambda_{EW}^4 G M^2 R^3 \hat\alpha_{sat}^2}\right)^{\frac{1}{8+2\gamma}}$$

SPINDOWN OF YOUNG NEUTRON STARS

- THERMAL STEADY-STATE FULLY DETERMINES THE EVOLUTION
- NUMERIC AND SEMI-ANALYTIC RESULTS COMPARE FAVORABLY
- EVOLUTION HAS A REMARKABLE “MEMORY LOSS” TO THE INITIAL CONDITIONS, I.E. THE ...
    - INITIAL AMPLITUDE AND TEMP.
    - AND EVEN INITIAL FREQUENCY
- STRONG INSENSITIVITY OF THE FINAL FREQUENCY TO UNKNOWN MICROSCOPIC PARAMTERS & $\alpha_{\rm sat}$:



SEMI-ANALYTIC RESULTS FOR THE ENDPOINTS OF THE EVOLUTION AT DIFFERENT AMPLITUDES MAP OUT THE INSTABILITY BOUNDARY

M. ALFORD AND K.S., ARXIV:1210.6091

$$f_f^{(NS)} \approx 61.4\,\mathrm{Hz}\,\frac{\Delta\tilde{S}^{\frac{3}{23}}\,\Delta\tilde{L}^{\frac{5}{184}}}{\Delta\tilde{J}^{\frac{29}{92}}\,\alpha_{\rm sat}^{\frac{5}{92}}}\left(\frac{1.4\,M_\odot}{M}\right)^{\frac{29}{92}}\left(\frac{11.5\,\mathrm{km}}{R}\right)^{\frac{87}{184}} \approx (61.4 \pm 9.4)\,\mathrm{Hz}\;\alpha_{\rm sat}^{-\frac{5}{92}}$$

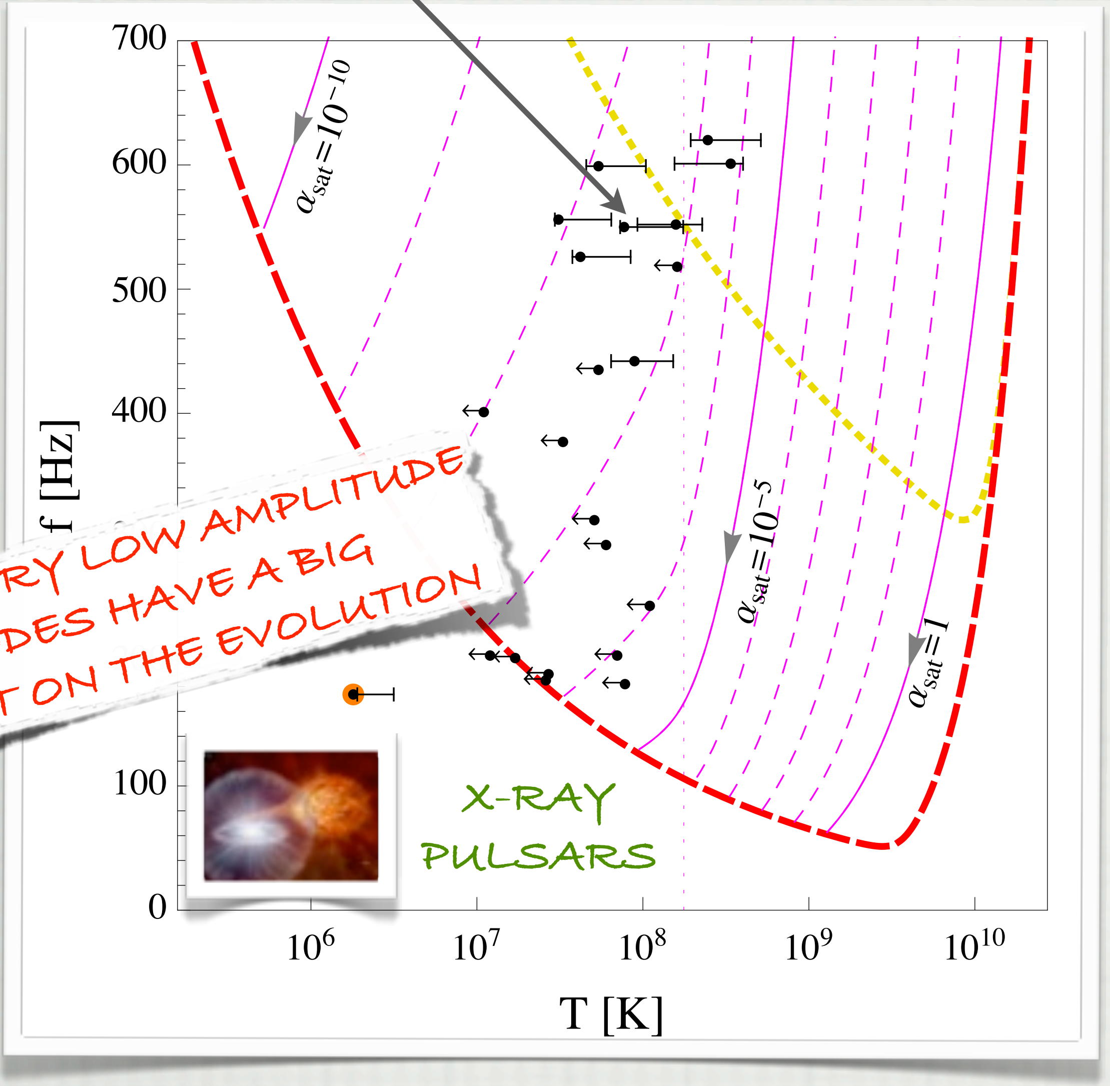
PULSAR EVOLUTION & R-MODE INSTABILITY

TEMPERATURES HAVE LARGE UNCERTAINTIES

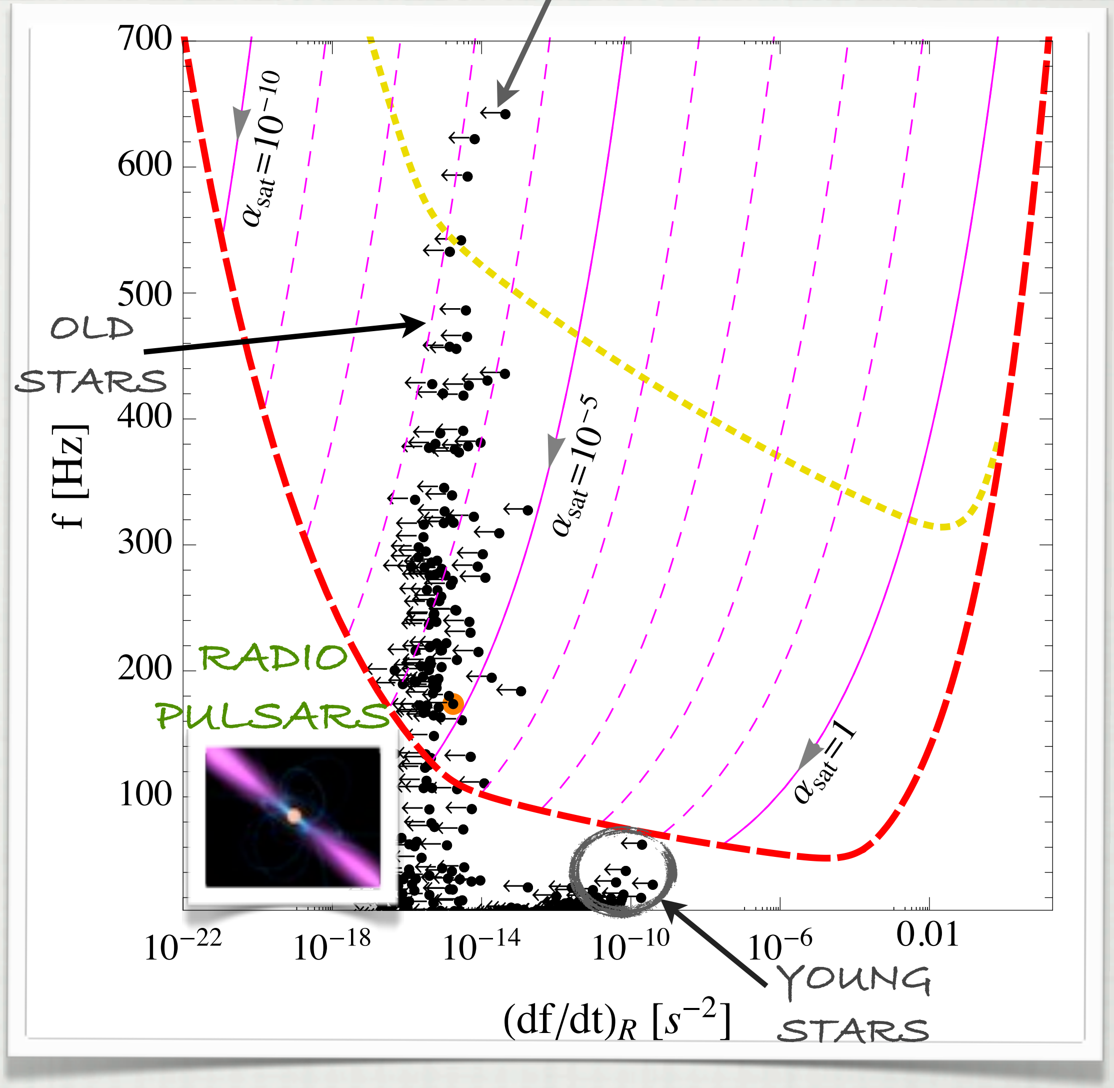
HASKELL, ET. AL., MNRAS 424 (2012) 93

OBSERVED SPINDOWN RATES ARE UPPER LIMITS FOR R-MODE CONTRIBUTION

MANCHESTER, ET. AL., ASTRO-PH/0412641



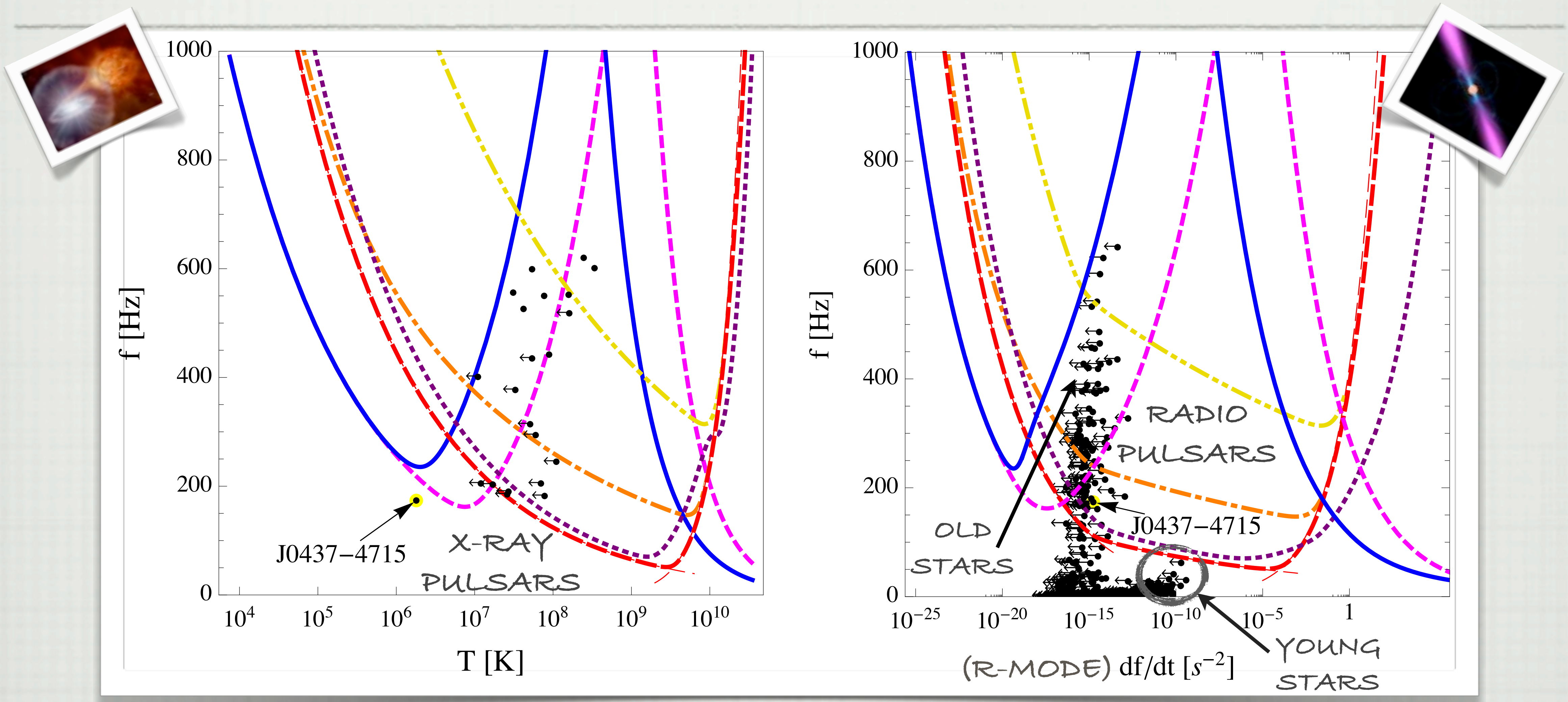
EVEN VERY LOW AMPLITUDE MODES HAVE A BIG IMPACT ON THE EVOLUTION



- ☆ "DYNAMIC" R-MODE INSTABILITY REGIONS ALLOW TO CONNECT TO TIMING DATA - INDEPENDENT OF SATURATION MECHANISM

- ⚠ DATA IMPLIES THAT $\alpha_{\text{sat}} \lesssim O(10^{-8})$ BUT ALL PROPOSED R-MODE SATURATION MECHANISMS CAN ONLY SATURATE AT $\alpha_{\text{sat}} \gtrsim O(10^{-6})$

R-MODE INSTABILITY REGIONS VS. THERMAL X-RAY & RADIO TIMING DATA



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☐ INSTABILITY BOUNDARIES IN TIMING PARAMETER SPACE ... INDEPENDENT OF SATURATION: $\Omega_{ib}\left(\dot{\Omega}\right) = \left(\hat{D}^{\theta} I^{\delta} |\dot{\Omega}|^{\delta} / \left(3^{\delta} \hat{G}^{\theta} \hat{L}^{\delta}\right)\right)^{1/((8-\psi)\theta-\delta)}$

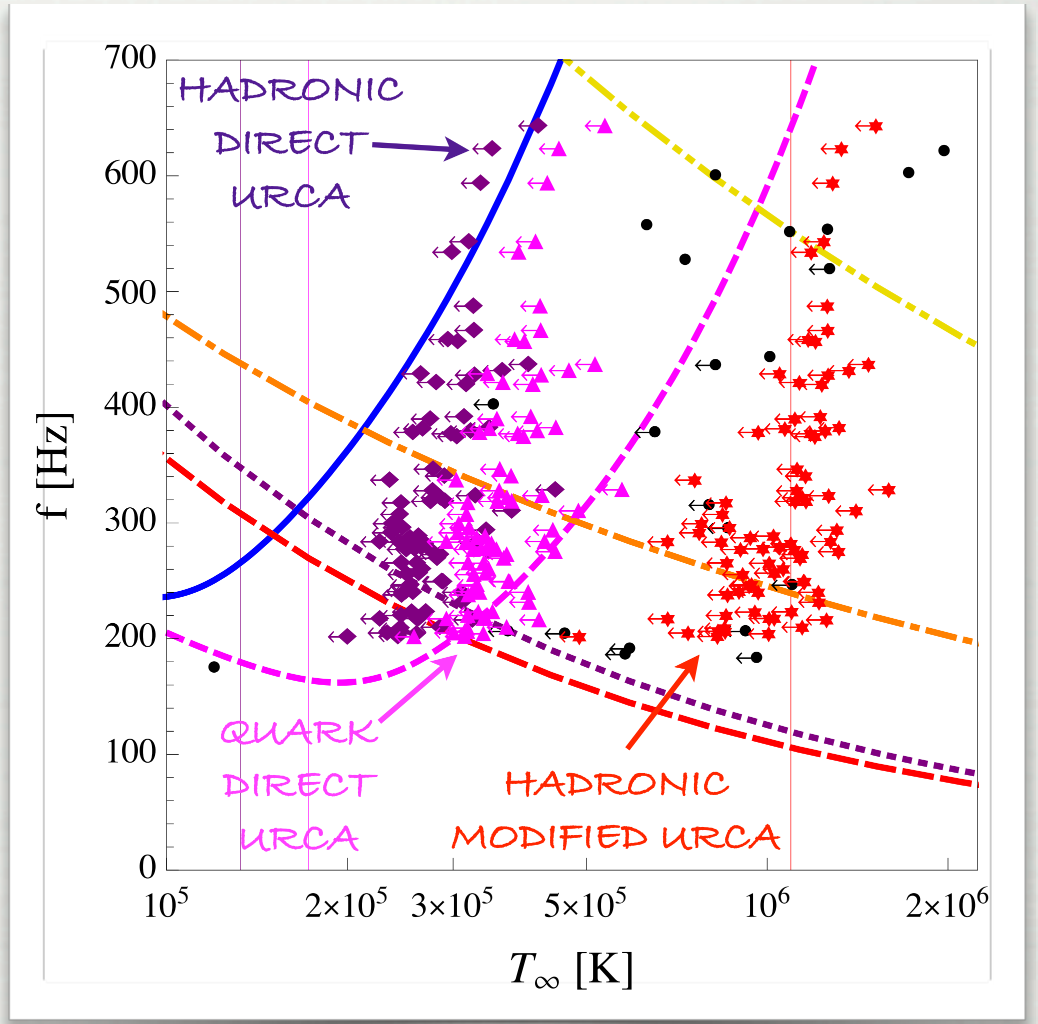
☐ INTERACTING (NFL) QUARK MATTER CONSISTENT WITH BOTH X-RAY AND RADIO DATA (NO R-MODE SCENARIO)!

“R-MODE TEMPERATURES“

☐ THE CONNECTION BETWEEN THE SPINDOWN CURVES ALLOWS TO DETERMINE THE R-MODE TEMPERATURE OF A STAR WITH SATURATED R-MODE OSCILLATIONS (SCENARIO (II)) FOR GIVEN TIMING DATA

$$T_{rm} = \left(I\Omega\dot{\Omega}/\left(3\hat{L}\right)\right)^{1/\theta}$$

☐ LIKEWISE INDEPENDENT OF THE SATURATION MECHANISM ... BUT DEPENDS ON THE COOLING



☐ THESE ARE ONLY UPPER TEMPERATURE BOUNDS SINCE THE OBSERVED SPINDOWN RATE CAN ALSO STEM FROM ELECTROMAGNETIC RADIATION

☐ MEASUREMENTS OF TEMPERATURES OF FAST PULSARS WOULD ALLOW US TO TEST IF SATURATED TINY R-MODES CAN BE PRESENT

CONCLUSION

☐ SEMI-ANALYTIC SOLUTION OF THE PULSAR EVOLUTION IS SURPRISINGLY INSENSITIVE TO MICROPHYSICAL DETAILS BUT CAN CLEARLY DISTINGUISH BETWEEN DIFFERENT CLASSES OF DENSE MATTER ...

... WHICH ALLOWS TO CONNECT THEORETICAL RESULTS TO EXTENSIVE AND PRECISE PULSAR TIMING DATA

☐ PURE NEUTRON STARS CANNOT DAMP R-MODES IN LMXBS AND CANNOT EXPLAIN THE RADIO PULSAR DATA FOR ALL PROPOSED R-MODE SATURATION MECHANISMS

☐ INTERACTING QUARK MATTER CAN GIVE A SIMULTANEOUS EXPLANATION FOR BOTH THE OBSERVED X-RAY AND RADIO PULSAR TIMING DATA