Recent Estimates of Neutron Star Masses and Radii and Limits on Properties of Quark Matter EOSs

J. M. Lattimer

STONY BROOK UNIVERSITY

Collaborators: E. Brown (MSU), K. Hebeler (OSU), D. Page (UNAM), C.J. Pethick (NORDITA), M. Prakash (Ohio U), A. Steiner (INT), A. Schwenk (TU Darmstadt), Y. Lim (Daegu Univ., Korea)

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Outline

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  - Maximally Compact Equation of State
  - Radius Limits from the Neutron Star Maximum Mass and Causality
  - Constraints on Quark Matter in Hybrid Star Interiors
- The Neutron Star Radius and the Nuclear Symmetry Energy
- Nuclear Experimental Constraints on the Symmetry Energy
  - Binding Energies
  - Heavy ion Collisions
  - Neutron Skin Thicknesses
  - Dipole Polarizabilities and Giant Dipole Resonances
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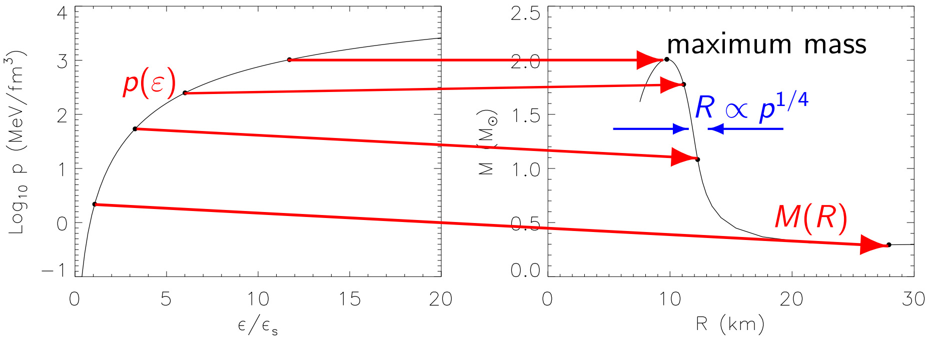
Neutron Star Structure

Tolman-Oppenheimer-Volkov equations

$$\frac{dp}{dr} = -\frac{G}{c^4}\frac{(mc^2 + 4\pi pr^3)(\varepsilon + p)}{r(r - 2Gm/c^2)}$$

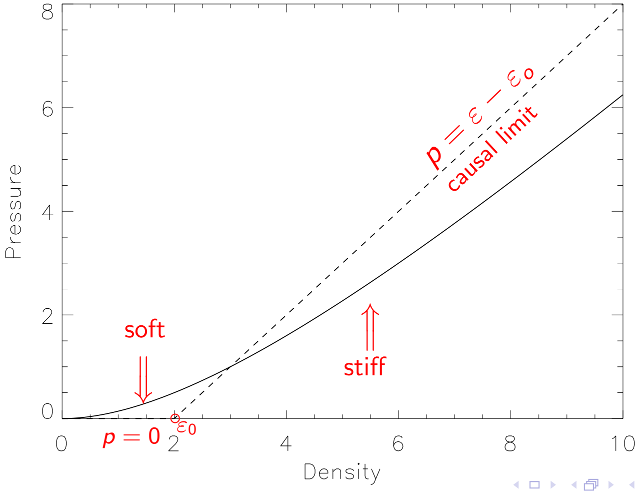
$$\frac{dm}{dr} = 4\pi\frac{\varepsilon}{c^2}r^2$$



Equation of State ← Observations

Extremal Properties of Neutron Stars

- The most compact and massive configurations occur when the low-density equation of state is "soft" and the high-density equation of state is "stiff" (Koranda, Stergioulas & Friedman 1997).



ε_0 is the only EOS parameter

The TOV solutions scale with ε_0

$w = \varepsilon/\varepsilon_0$
$y = p/\varepsilon_0$
$x = r\sqrt{G\varepsilon_0}/c^2$
$z = m\sqrt{G^3\varepsilon_0}/c^2$

Extremal Properties of Neutron Stars

The maximum mass configuration is achieved when
$x_R = 0.2404$, $w_c = 3.034$, $y_c = 2.034$, $z_R = 0.08513$.

A useful reference density is the nuclear saturation density (interior density of normal nuclei):
$\rho_s = 2.7 \times 10^{14}$ g cm^{-3}, $n_s = 0.16$ baryons fm^{-3}, $\varepsilon_s = 150$ MeV fm^{-3}

- $M_{\rm max} = 4.1\ (\varepsilon_s/\varepsilon_0)^{1/2} M_\odot$ (Rhoades & Ruffini 1974)
- $M_{B,{\rm max}} = 5.41\ (m_B c^2/\mu_o)(\varepsilon_s/\varepsilon_0)^{1/2} M_\odot$
- $R_{\rm min} = 2.82\ GM/c^2 = 4.3\ (M/M_\odot)\ {\rm km}$
- $\mu_{b,{\rm max}} = 2.09$ GeV
- $\varepsilon_{c,{\rm max}} = 3.034\ \varepsilon_0 \simeq 51\ (M_\odot/M_{\rm largest})^2\ \varepsilon_s$
- $p_{c,{\rm max}} = 2.034\ \varepsilon_0 \simeq 34\ (M_\odot/M_{\rm largest})^2\ \varepsilon_s$
- $n_{B,{\rm max}} \simeq 38\ (M_\odot/M_{\rm largest})^2\ n_s$
- ${\rm BE}_{\rm max} = 0.34\ M$
- $P_{\rm min} = 0.74\ (M_\odot/M_{\rm sph})^{1/2}(R_{\rm sph}/10\ {\rm km})^{3/2}\ {\rm ms} = 0.20\ (M_{\rm sph,max}/M_\odot)\ {\rm ms}$

Causality and the Maximum Mass

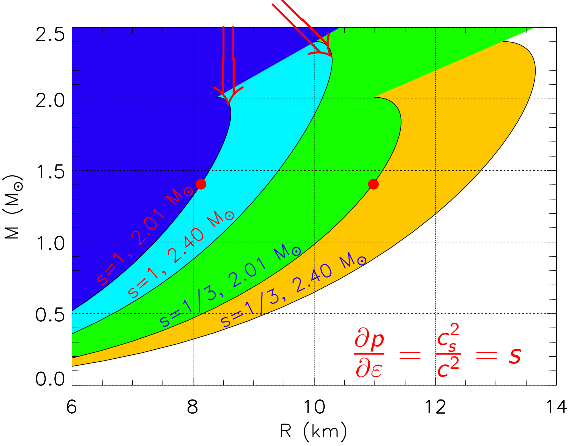
A lower limit to the maximum mass sets a lower limit to the radius for a given mass.

Similarly, a precise (M, R) measurement sets an upper limit to the maximum mass.

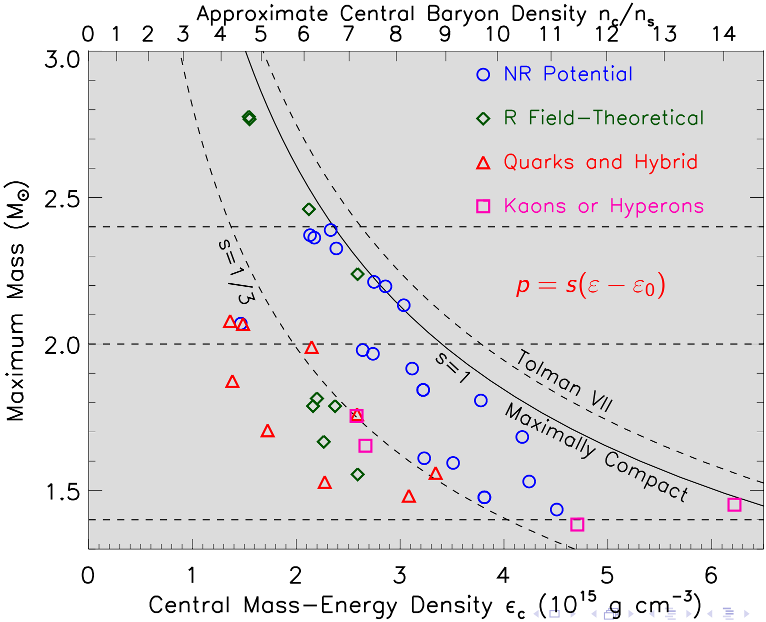
$1.4M_\odot$ stars must have $R > 8.15M_\odot$.

$1.4M_\odot$ strange quark matter stars (and likely hybrid quark/hadron stars) must have $R > 11$ km.

$M - R$ curves for maximally compact EOS



Maximum Energy Density in Neutron Stars



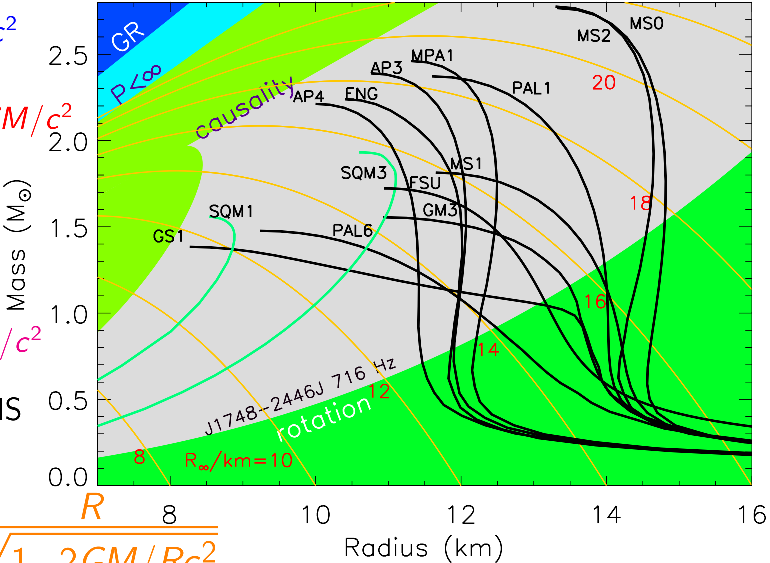
Mass-Radius Diagram and Theoretical Constraints

GR:
$R > 2GM/c^2$

$P < \infty$:
$R > (9/4)GM/c^2$

causality:
$R \gtrsim 2.9GM/c^2$

— normal NS
— SQS



— $R_\infty = \dfrac{R}{\sqrt{1-2GM/Rc^2}}$

Neutron Star Structure

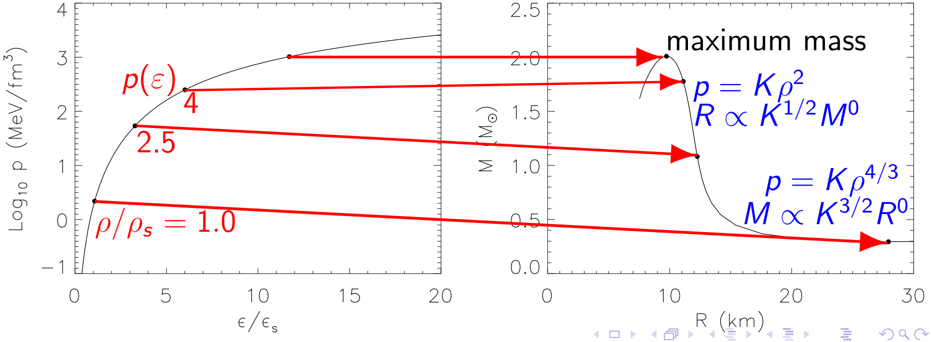
Newtonian Gravity:

$$\frac{dp}{dr} = -\frac{Gm\rho}{r^2}; \qquad \frac{dm}{dr} = 4\pi\rho r^2; \qquad \rho c^2 = \varepsilon$$

Newtonian Polytrope:

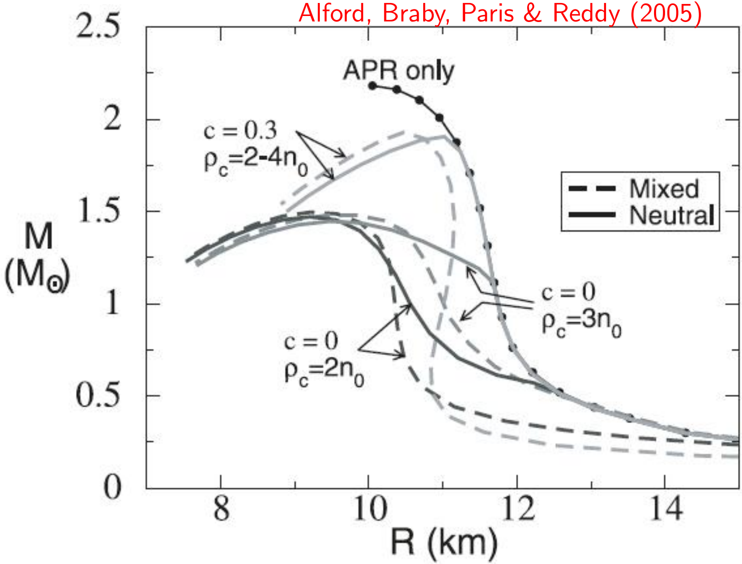
$$p = K\rho^\gamma; \qquad M \propto K^{1/(2-\gamma)} R^{(4-3\gamma)/(2-\gamma)}$$

$$\rho < \rho_s: \ \gamma \simeq \frac{4}{3}; \qquad \rho > \rho_s: \ \gamma \simeq 2$$

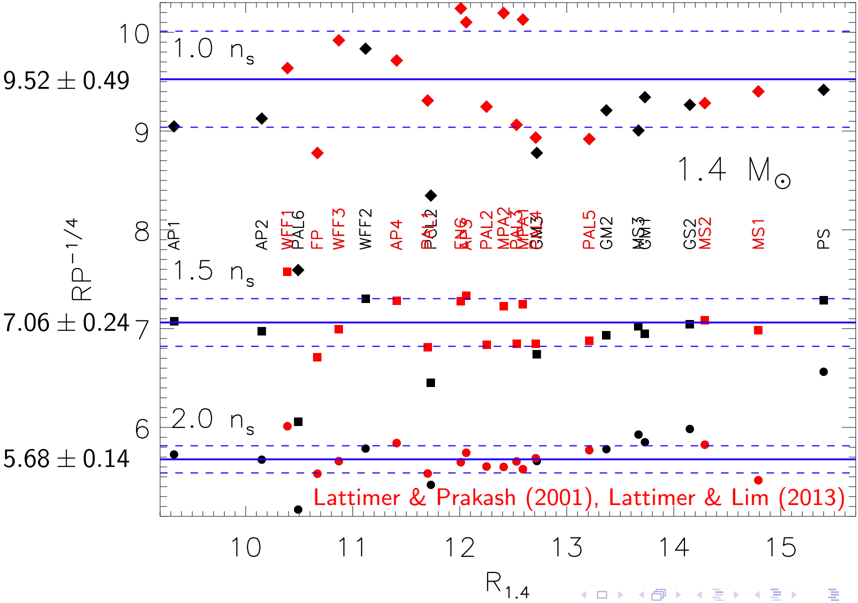


Hybrid Stars

Alford, Braby, Paris & Reddy (2005)



The Radius – Pressure Correlation



Nuclear Symmetry Energy

Defined as the difference between energies of pure neutron matter ($x = 0$) and symmetric ($x = 1/2$) nuclear matter.

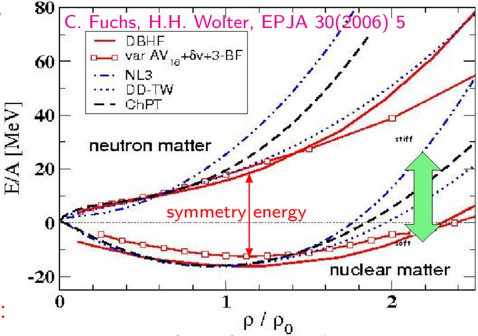
$$S(\rho) = E(\rho, x = 0) - E(\rho, x = 1/2)$$

Expanding around the saturation density (ρ_s) and symmetric matter ($x = 1/2$)

$$E(\rho, x) = E(\rho, 1/2) + (1 - 2x)^2 S_2(\rho) + \ldots$$

$$S_2(\rho) = \mathbf{S_v} + \frac{\mathbf{L}}{3}\frac{\rho - \rho_s}{\rho_s} + \ldots$$

$$\mathbf{S_v} \simeq 31 \text{ MeV}, \quad \mathbf{L} \simeq 50 \text{ MeV}$$



Connections to pure neutron matter:

$$E(\rho_s, 0) \approx S_v + E(\rho_s, 1/2) \equiv S_v - B, \qquad p(\rho_s, 0) = L\rho_s/3$$

Neutron star matter (in beta equilibrium):

$$\frac{\partial(E + E_e)}{\partial x} = 0, \quad p(\rho_s, x_\beta) \simeq \frac{L\rho_s}{3}\left[1 - \left(\frac{4S_v}{\hbar c}\right)^3 \frac{4 - 3S_v/L}{3\pi^2\rho_s}\right]$$

Determining Symmetry Parameters from Nuclear Masses

From liquid drop model:

$$E_{\rm sym}(N,Z) = (S_v A - S_s A^{2/3}) I^2$$

$$\chi^2 = \sum_i (E_{ex,i} - E_{sym,i})^2 / \mathcal{N}\sigma_D^2$$

$$\chi_{vv} = \frac{2}{\mathcal{N}\sigma_D^2}\sum_i I_i^4 A_i^2$$

$$\chi_{ss} = \frac{2}{\mathcal{N}\sigma_D^2}\sum_i I_i^4 A_i^{4/3}$$

$$\chi_{vs} = \frac{2}{\mathcal{N}\sigma_D^2}\sum_i I_i^4 A_i^{5/3}$$

$$\sigma_{S_v} = \sqrt{\frac{2\chi_{ss}}{\chi_{vv}\chi_{ss} - \chi_{sv}^2}} \simeq 2.3\,\sigma_D$$

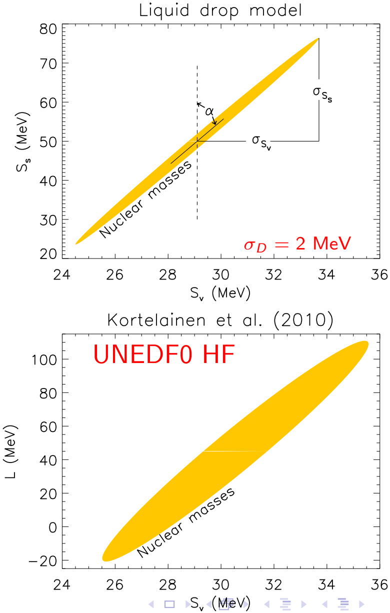
$$\sigma_{S_s} = \sqrt{\frac{2\chi_{vv}}{\chi_{vv}\chi_{ss} - \chi_{sv}^2}} \simeq 13.2\,\sigma_D$$

$$\alpha = \frac{1}{2}\tan^{-1}\frac{2\chi_{vs}}{\chi_{vv} - \chi_{ss}} \simeq 9^\circ.8$$

$$r_{vs} = -\frac{\chi_{vs}}{\sqrt{\chi_{vv}\chi_{ss}}} \simeq 0.997$$

Liquid droplet model:

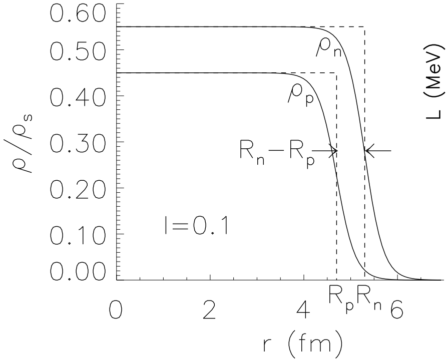
$$E_{\rm sym}(N,Z) = \frac{S_v A I^2}{1 + (S_s/S_v) A^{-1/3}}$$

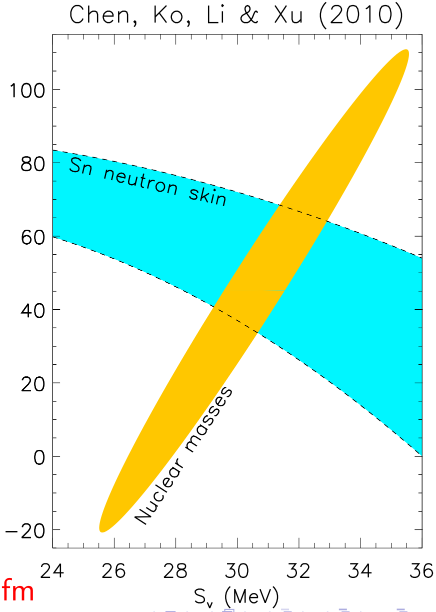


Neutron Skin Thickness

$$\frac{R_n - R_p}{r_o} \simeq \sqrt{\frac{4}{15}} \frac{S_s I}{S_v + S_s A^{-1/3}}$$

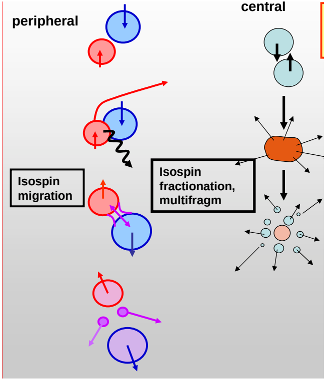
$$r_o = \left(\frac{3}{4\pi\rho_s}\right)^{1/3} \simeq 1.14 \text{ fm}$$





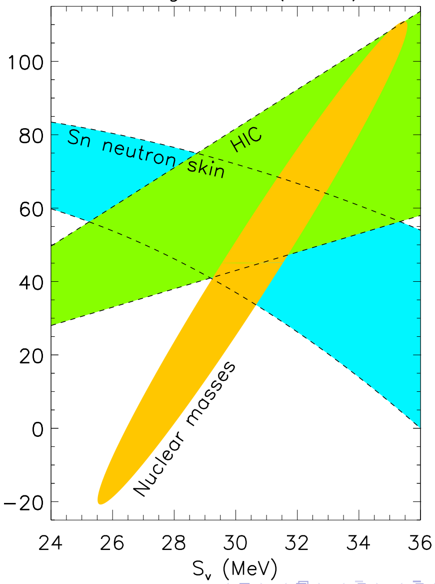
$$(R_n - R_p)_{208} = 0.175 \pm 0.020 \text{ fm}$$

Heavy Ion Collisions

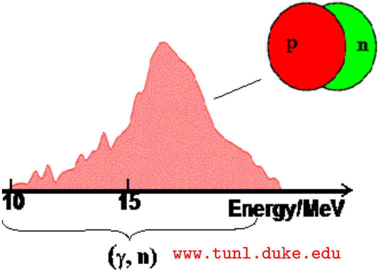


Wolter, NuSYM11

Tsang et al. (2009)

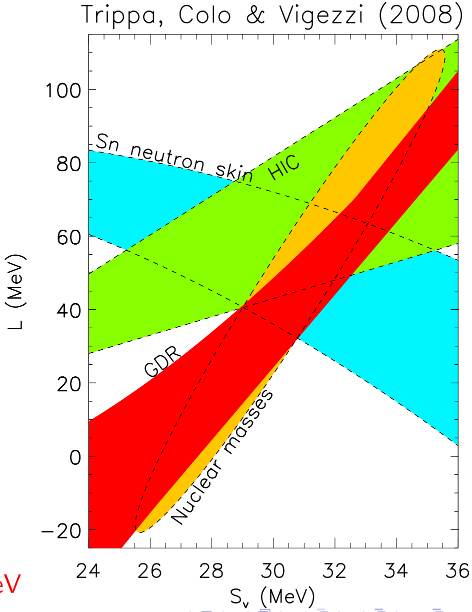


Giant Dipole Resonances



$$E_{-1} \propto \sqrt{\frac{S_v}{1+\frac{5S_s}{3S_v}A^{-1/3}}}$$

23.3 MeV$< S_2(0.1\ \mathrm{fm}^{-3}) <$24.9 MeV



Dipole Polarizability

The linear response, or dynamic polarizability, of a nuclear system excited from its ground state to an excited state, due to an external oscillating dipolar field.

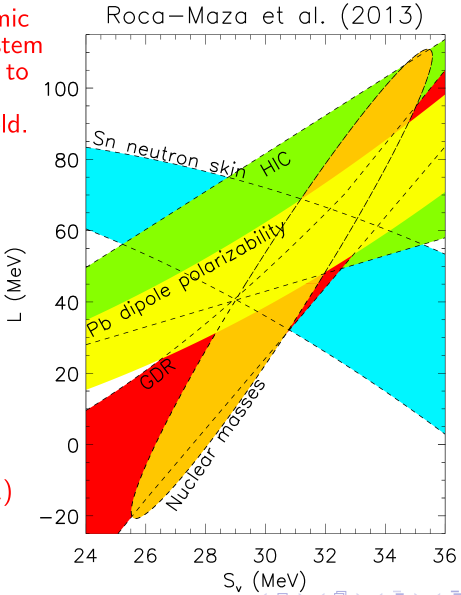
Liquid droplet model:

$$\alpha_D = \frac{AR^2}{20S_v}\left(1 + \frac{5}{3}\frac{S_s}{S_v}A^{-1/3}\right)$$

Roca-Maza et al. found an empirical correlation between $\alpha_D S_v$ and $R_n - R_p$

Data from Tamii et al. (2011)

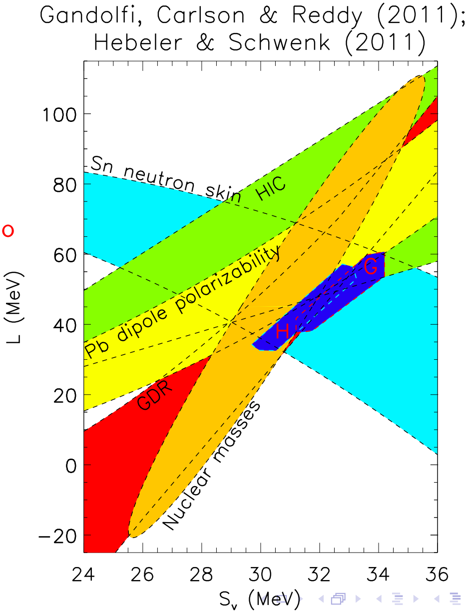
$$\alpha_{D,208} = 20.1 \pm 0.6 \text{ fm}^2$$



Theoretical Neutron Matter Calculations

H&S: Chiral Lagrangian

GC&R: Quantum Monte Carlo



Binary Mass Measurements

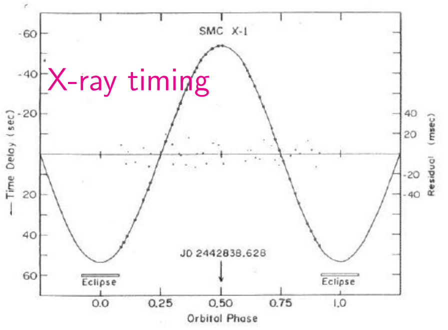
Mass function

$$f(M_1) = \frac{P(v_2 \sin i)^3}{2\pi G} = \frac{(M_1 \sin i)^3}{(M_1+M_2)^2} < M_1$$

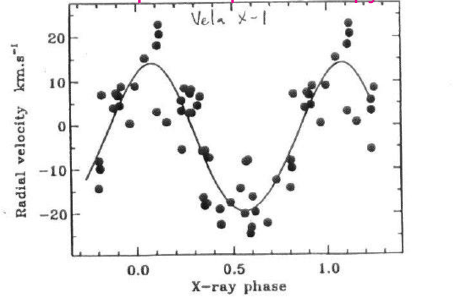
$$f(M_2) = \frac{P(v_1 \sin i)^3}{2\pi G} = \frac{(M_2 \sin i)^3}{(M_1+M_2)^2} < M_2$$

In an X-ray binary, $v_{optical}$ has the largest uncertainties. In some cases $\sin i \sim 1$ if eclipses are observed. If no eclipses observed, limits to i can be made based on the estimated radius of the optical star.

X-ray timing



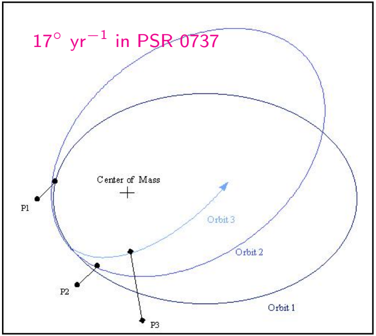
Optical spectroscopy



Pulsar Mass Measurements

Mass functions for pulsars are precisely measured.

In some cases, the rate of periastron advance and the Einstein gravitational redshift/time dilation term are known:



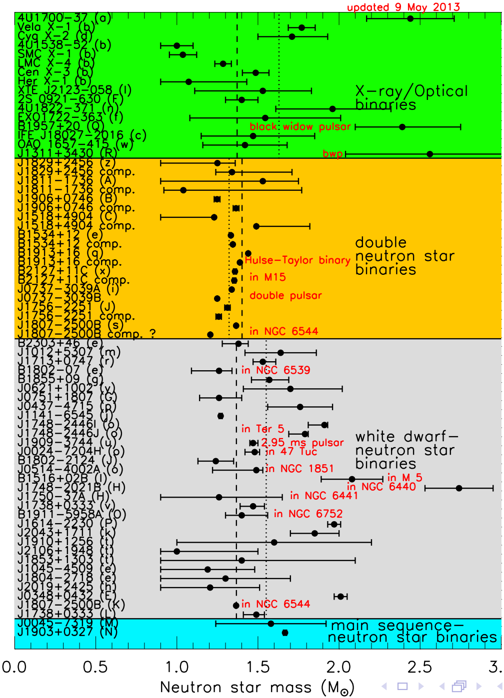
$$\dot{\omega} = \frac{3}{1-e^2}\left(\frac{2\pi}{P}\right)^{5/3}\left(\frac{GM}{c^2}\right)^{2/3}$$

$$\gamma = \left(\frac{P}{2\pi}\right)^{1/3} e M_2 (2M_2 + M_1)\left(\frac{G}{M^2 c^2}\right)^{2/3}$$

Gravitational radiation leads to orbit decay:

$$\dot{P} = -\frac{192\pi}{5c^5}\left(\frac{2\pi G}{P}\right)^{5/3}(1-e^2)^{-7/2}\left(1+\frac{73}{24}e^2+\frac{37}{96}e^4\right)\frac{M_1 M_2}{M^{1/2}}$$

In some cases, can also constrain Shapiro time delay, $r(M_2, e, \sin i)$ is magnitude and $s = \sin i$ is shape parameter.



vanKerkwijk 2010

Romani et al. 2012

Although simple average mass of w.d. companions is 0.23 $M_\odot$ larger, weighted average is 0.04 $M_\odot$ smaller

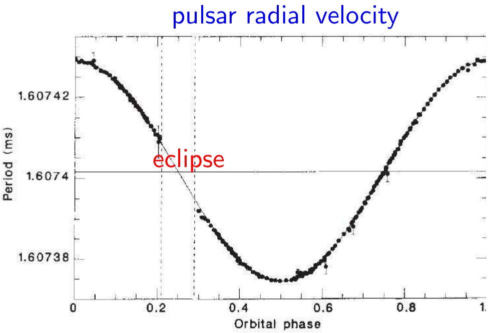
Demorest et al. 2010

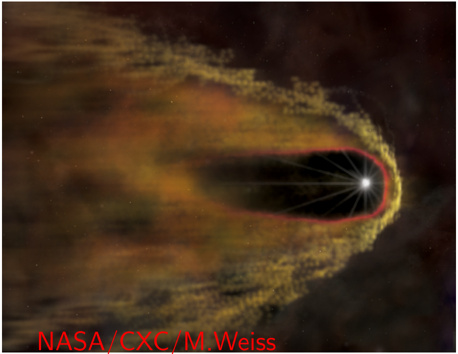
Antoniadis et al. 2013

Champion et al. 2008

Black Widow Pulsar PSR B1957+20

1.6ms pulsar in circular 9.17h orbit with a $\sim 0.03\ M_\odot$ companion.
Pulsar is eclipsed for 50-60 minutes each orbit; eclipsing object has a volume much larger than the companion or its Roche lobe.
It is believed the companion is ablated by the pulsar leading to mass loss and an eclipsing plasma cloud. Companion nearly fills its Roche lobe.
Ablation by pulsar leads to eventual disappearance of companion.
The optical light curve does not represent the center of mass of the companion, but the motion of its irradiated hot spot.





Black Widow Pulsar PSR B1957+20

$K_i = 2\pi \frac{a_i \sin i}{P}$

$q = \frac{M_P}{M_*} = \frac{a_*}{a_P} = \frac{K_*}{K_P}$

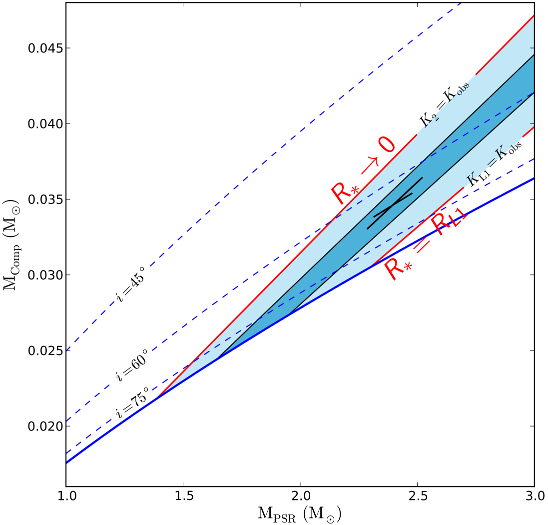
$K_* = K_{obs}\left(1 + \frac{R_*}{a_*}\right)$

Radiated area of companion has a smaller orbit than the center of mass.

$M_P = q(1+q)^2 \frac{P}{2\pi G}\left(\frac{K_P}{\sin i}\right)^3$

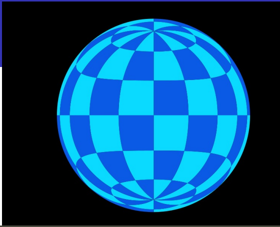
Modeling of light curve shape suggests that

$M_P > 1.8$ M$_\odot$, $\sin i < 66^\circ$



van Kerkwijk 2010

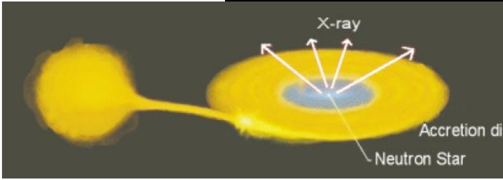
Simultaneous Mass/Radius Measurements



- Measurements of flux $F_\infty = \left(\frac{R_\infty}{D}\right)^2 \sigma T_{\rm eff}^4$ and color temperature $T_c \propto \lambda_{\rm max}^{-1}$ yield an apparent angular size (pseudo-BB):

$$\frac{R_\infty}{D} = \frac{R}{D}\frac{1}{\sqrt{1-2GM/Rc^2}}$$

- Observational uncertainties include distance D, interstellar absorption N_H, atmospheric composition

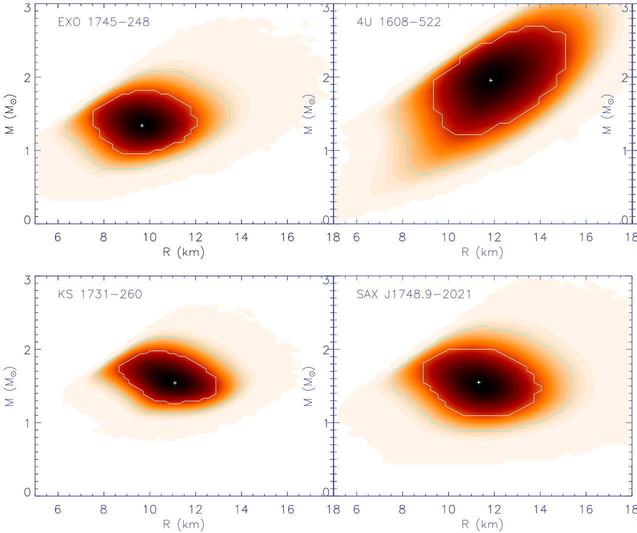


Best chances for accurate radius measurement:

- Nearby isolated neutron stars with parallax (uncertain atmosphere)
- Quiescent low-mass X-ray binaries (QLMXBs) in globular clusters (reliable distances, low B H-atmospheres)
- Bursting sources (XRBs) with peak fluxes close to Eddington limit (where gravity balances radiation pressure)

$$F_{\rm Edd} = \frac{cGM}{\kappa D^2}\sqrt{1-2GM/Rc^2}$$

$M - R$ PRE Burst Estimates



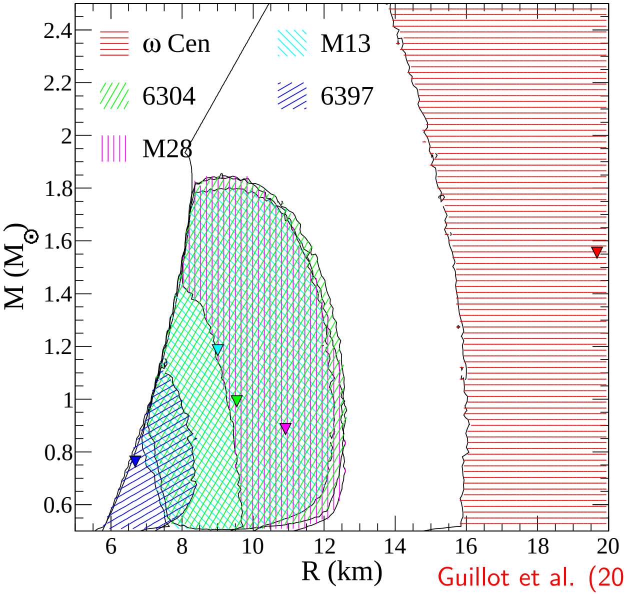
$F_{Edd,\infty}, (R_\infty/D)^2 f_c^{-4}, D$ from Ozel et al.

$z_{ph} = 0$

Revised uncertainties

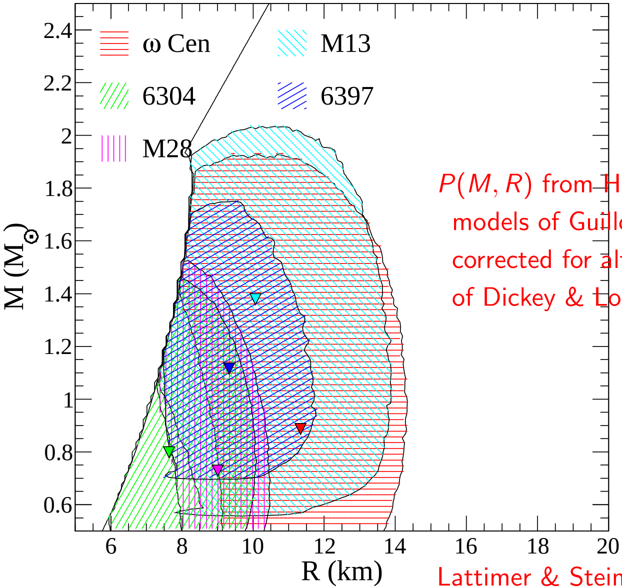
Lattimer & Steiner (2013)

$M - R$ QLMXB Estimates



Guillot et al. (2013)

$M - R$ QLMXB Estimates

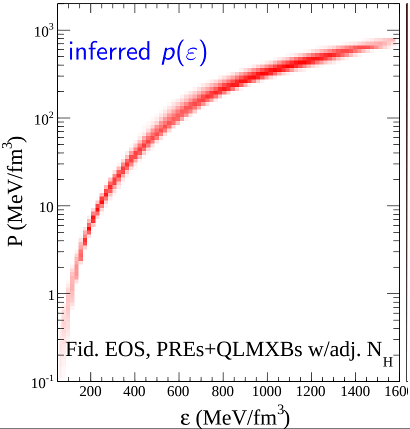


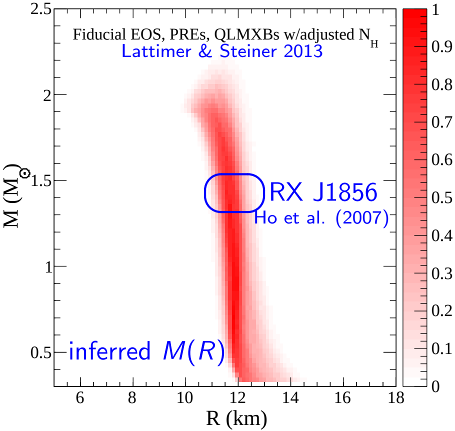
$P(M, R)$ from H atmosphere models of Guillot et al. (2013), corrected for alternate N_H values of Dickey & Lockman (1990)

Lattimer & Steiner (2013)

Bayesian TOV Inversion

- $\varepsilon < 0.5\varepsilon_0$: Known crustal EOS
- $0.5\varepsilon_0 < \varepsilon < \varepsilon_1$: EOS parametrized by K, K', S_v, γ
- Polytropic EOS: $\varepsilon_1 < \varepsilon < \varepsilon_2$: n_1; $\varepsilon > \varepsilon_2$: n_2
- EOS parameters $K, K', S_v, \gamma, \varepsilon_1, n_1, \varepsilon_2, n_2$ uniformly distributed
- $M_{\rm max} \geq 1.97$ $M_\odot$, causality enforced
- All 10 stars equally weighted



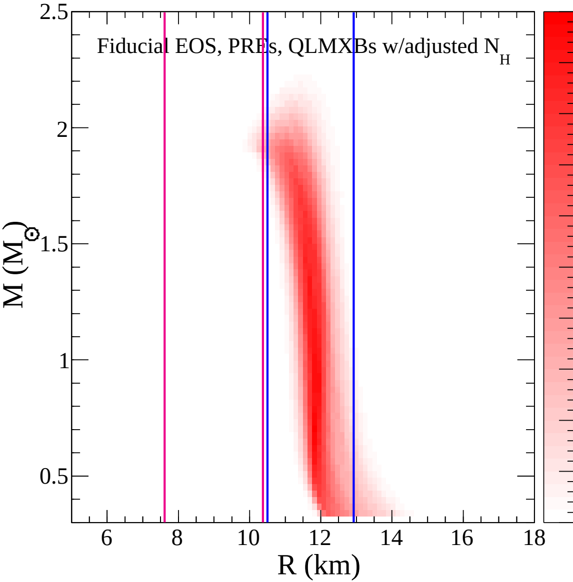


Astronomy vs. Astronomy vs. Physics

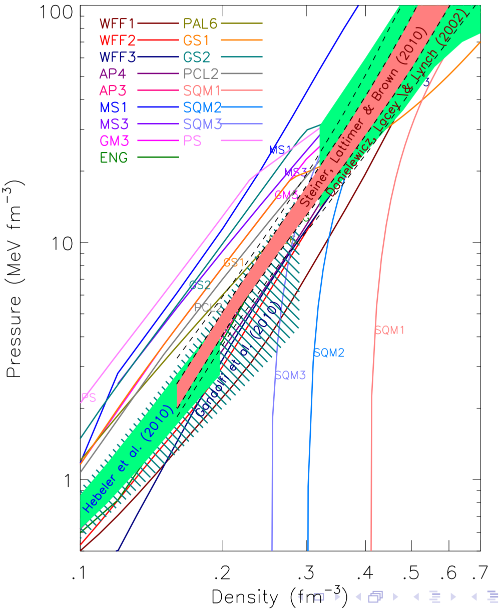
Guillot et al. (2013): Bayesian analysis assuming neutron stars of every mass have the same radius.
$R = 9.1^{+1.3}_{-1.5}$ km.

Lattimer & Steiner (2013): Bayesian analysis assuming TOV relates M and R, known crust EOS, causality maximum mass $> 2M_{\odot}$.

Lattimer & Lim (2013): Nuclear experiments imply
29 MeV $< S_v <$ 33 MeV
40 MeV $< L <$ 65 MeV
which is equivalent to
$R_{1.4} = 11.7 \pm 1.2$ km

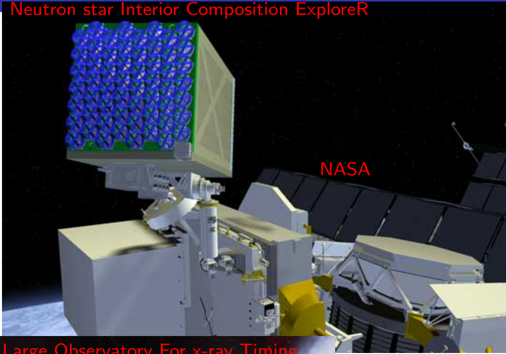


Consistency with Neutron Matter and Heavy-Ion Collisions



Additional Proposed Radius and Mass Constraints

- Pulse profiles
  Hot or cold regions on rotating neutron stars alter pulse shapes: NICER and LOFT will enable timing and spectroscopy of thermal and non-thermal emissions. Light curve modeling $\rightarrow M/R$; phase-resolved spectroscopy $\rightarrow R$.
- Moment of inertia
  Spin-orbit coupling of ultra-relativistic binary pulsars (e.g., PSR 0737+3039) vary i and contribute to $\dot{\omega}$: $I \propto MR^2$.
- Supernova neutrinos
  Millions of neutrinos detected from a Galactic supernova will measure BE$= m_B N - M$, $< E_\nu >$, τ_ν.
- QPOs from accreting sources
  ISCO and crustal oscillations



Constraints from Observations of Gravitational Radiation

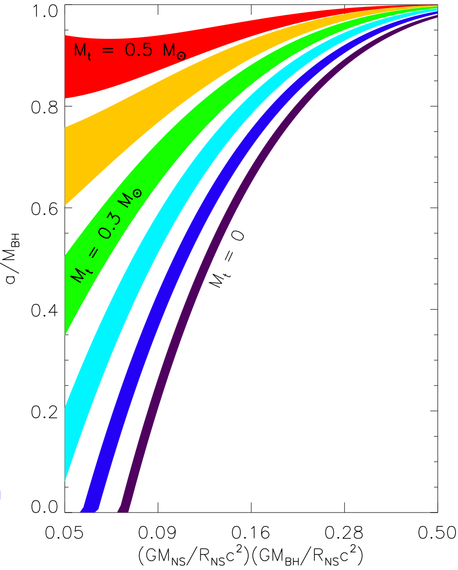
Mergers:

Chirp mass $\mathcal{M} = (M_1 M_2)^{3/5} M^{-1/5}$ and tidal deformability $\lambda \propto R^5$ (Love number) are potentially measurable during inspiral.

$\bar{\lambda} \equiv \lambda M^{-5}$ is related to $\bar{I} \equiv I M^{-3}$ by an EOS-independent relation (Yagi & Yunes 2013). Both $\bar{\lambda}$ and $\bar{I}$ are also related to M/R in a relatively EOS-independent way (Lattimer & Lim 2013).

- Neutron star - neutron star: $M_{\rm crit}$ for prompt black hole formation, $f_{\rm peak}$ depends on R.
- Black hole - neutron star: $f_{\rm tidal\ disruption}$ depends on $R, a, M_{\rm BH}$. Disc mass depends on $a/M_{\rm BH}$ and on $M_{\rm NS} M_{\rm BH} R^{-2}$.

Rotating neutron stars: r-modes



Conclusions

- Nuclear experiments set reasonably tight constraints on symmetry energy parameters and the symmetry energy behavior near the nuclear saturation density.
- Theoretical calculations of pure neutron matter predict very similar symmetry constraints.
- These constraints predict neutron star radii $R_{1.4}$ in the range 11.7 ± 1.2 km.
- Combined astronomical observations of photospheric radius expansion X-ray bursts and quiescent sources in globular clusters suggest $R_{1.4} \sim 12.1 \pm 0.6$ km.
- The nearby isolated neutron star RX J1856-3754 appears to have a radius near 12 km, assuming a solid surface with thin H atmosphere (Ho et al. 2007).
- The observation of a 1.97 $M_\odot$ neutron star, together with the radius constraints, implies the EOS above the saturation density is relatively stiff.