Radial oscillations in neutral and charged compact stars

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Outline

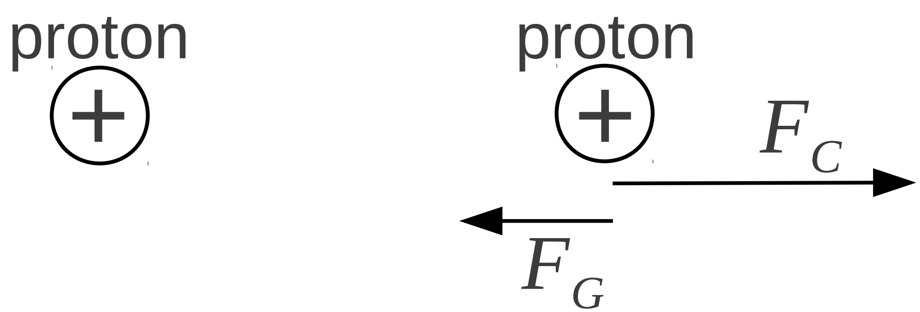
Why care about charge in compact stars?

- even if the global charge is zero, there can be separation of charges inside / freedom to arrange charge within

- charge might prevent gravitational collapse and support supermassive stars

- charged balls might be natural candidates to form extremal black holes

How much charge allowed?



$$\frac{F_C}{F_G} = \frac{e^2}{G\, m_p^2} \approx 10^{36}$$

Number of baryons in one neutron star: $N_B \approx 3 \cdot 10^{57}$

Number of net unit charges allowed to build “reasonable” charged compact stars: $N_c < 10^{-18} N_B$

assumption: EOS for charged compact stars calculated at charge neutrality (only 1 independent chemical potential)

Spheres in GR

	Neutral	Charged
Equilibrium	Tolman (1939) Oppenheimer, Volkoff (1939)	Bekenstein (1971)
Radial eigenmodes	Chandrasekhar (1964)	**3 papers -** **3 different equations** Glazer (1979)

3 papers

Glazer 1979

$$\sigma^2 e^{\lambda-\nu}(P+\epsilon)\xi = \left[\frac{4P'}{r} + \frac{8\eta(P'-\eta')}{r(P+\epsilon)} - \frac{16\eta^2}{r^2(P+\epsilon)} - \frac{(P'-\eta')^2}{P+\epsilon} + \frac{8\pi G}{c^4} e^{\lambda}(P+\epsilon)(P+\eta)\right]\xi$$
$$- e^{-(\lambda+2\nu)/2}\frac{d}{dr}\left[e^{(\lambda+3\nu)/2}\frac{\gamma P}{r^2}\frac{d}{dr}(r^2 e^{-\nu/2}\xi)\right],$$

Anninos, Rothman 2001

$$(F\zeta')' + (H + \omega^2 W)\zeta = 0$$
$$\begin{aligned}
F &= \gamma p_0 r^{-2} e^{\Lambda_0 + 3\Phi_0}, \\
W &= r^{-2}(\rho_0 + p_0) e^{3\Lambda_0 + \Phi_0}, \\
H &= r^{-2} e^{\Lambda_0 + 3\Phi_0}\Big[4\pi r(\rho_0 + p_0) e^{2\Lambda_0}\left(-p_0' + \tfrac{Q_0 Q_0'}{4\pi r^4} + \tfrac{1}{r}(\rho_0 + p_0)\right) \\
&\quad + 2p_0'\Phi_0' + p_0'' + \Phi_0'\rho_0' - \tfrac{2p_0'}{r} - \tfrac{1}{4\pi r^4}\left(Q_0'^2 + Q_0 Q_0'' + Q_0 Q_0'(\Phi_0' - \tfrac{2}{r})\right)\Big]
\end{aligned}$$

de Felice, Siming, Yunqiang 1999

$$(F\zeta')' + (H + \omega^2 W)\zeta = 0$$
$$F = \gamma p_i r^{-2} \mathrm{e}^{\lambda_i/2 + 3\eta_i/2}$$
$$H = \mathrm{e}^{\lambda_i/2+3\eta_i/2}\left[\frac{(\mathrm{p}_i' - (\mathrm{Q}_i\mathrm{Q}_i')/(4\pi \mathrm{r}^4))^2}{\mathrm{r}^2(\rho_i + \mathrm{p}_i)} + \frac{4}{\mathrm{r}^3}\left(\frac{\mathrm{Q}_i\mathrm{Q}_i'}{4\pi\mathrm{r}^4} - \mathrm{p}_i'\right) - 8\pi\mathrm{p}_i(\rho_i + \mathrm{p}_i)\frac{\mathrm{e}^{\lambda_i}}{\mathrm{r}^2}\right.$$
$$\left. - \frac{\mathrm{e}^{\lambda_i}}{\mathrm{r}^6} Q_i^2(\rho_i + p_i) + \frac{Q_i Q_i''}{4\pi r^6} - \frac{Q_i Q_i'}{\pi r^7}\right]$$
$$W = (\rho_i + p_i) r^{-2} \mathrm{e}^{3\lambda_i/2 + \eta_i/2}$$

prescription to find oscillation equation

- time-dependent spherically symmetric metric

- equations: $G_{\mu\nu} = 8\pi T_{\mu\nu}$ $T^{\mu\nu}{}_{;\nu} = 0$ $(nu^{\mu})_{;\mu} = 0$

$$\partial_{\mu}\left[\sqrt{-g}F^{\nu\mu}\right] = 4\pi\sqrt{-g}\,j^{\nu}$$

- decompose variables: $A(r,t) = A_0(r) + \delta A(r,t)$

- linearize nonlinear equations

- subtract equilibrium equations from time-dependent equations and get perturbations: $\delta A(r,t)$

- substitute perturbations in $T^{\mu\, r}{}_{;\,\mu} = 0$ and get pulsation equation

Derivation of oscillation equation 1

$$ds^2 = -e^{2\Phi}dt^2 + e^{2\Lambda}dr^2 + r^2 d\theta^2 + r^2 sin^2\theta d\phi^2$$

$$G_0{}^0 = -e^{-2\Lambda}\left[2r^{-1}\Lambda' - \left(1 - e^{2\Lambda}\right)r^{-2}\right]$$

$$G_1{}^1 = e^{-2\Lambda}\left[2r^{-1}\Phi' + r^{-2}\right] - r^{-2}$$

$$G_2{}^2 = e^{-2\Lambda}\left[\Phi'' - \Phi'\Lambda' + \Phi'^2 + r^{-1}\left(\Phi' - \Lambda'\right)\right]$$
$$+ e^{-2\Phi}\left[\dot{\Phi}\dot{\Lambda} - \ddot{\Lambda} - \dot{\Lambda}^2\right]$$

$$G_0{}^1 = 2r^{-1}e^{-2\Lambda}\dot{\Lambda}$$

$$T_\mu{}^\nu = (\rho + P)\,u_\mu u^\nu + Pg_\mu{}^\nu + \frac{1}{4\pi}\left[F_{\mu\alpha}F^{\alpha\nu} - \frac{1}{4}g_\mu{}^\nu F^{\beta\gamma}F_{\beta\gamma}\right]$$

Derivation of oscillation equation 2

$$\delta\Lambda = -(\Phi_0{}' + \Lambda_0{}')\,\xi$$

$$\delta\rho = -\xi\rho_0{}' - (\rho_0 + P_0)\,\frac{e^{\Phi_0}}{r^2}\left(r^2 e^{-\Phi_0}\xi\right)'$$

$$\delta\Phi' = 4\pi r e^{2\Lambda_0}\delta P + 2\Phi_0{}'\delta\Lambda + r^{-1}\delta\Lambda - \frac{Q_0\delta Q e^{2\Lambda_0}}{r^3}$$

$$\delta P = \frac{dP_0}{d\rho_0}\delta\rho = -\xi P_0{}' - \frac{\gamma P_0 e^{\Phi_0}}{r^2}\left(r^2 e^{-\Phi_0}\xi\right)'$$

Energy-momentum conservation:

$$e^{2\Lambda_0 - 2\Phi_0}(\rho_0 + P_0)\,\dot{v} + \delta P' + \frac{Q_0 Q_0{}'\xi'}{4\pi r^4} + \frac{Q_0 Q_0{}''\xi}{4\pi r^4}$$

$$+ \frac{Q_0{}'^2\xi}{4\pi r^4} + \Phi_0{}'(\delta\rho + \delta P) + (\rho_0 + P_0)\,\delta\Phi' = 0$$

The oscillation equation

Chandrasekhar 1964

$$\sigma^2 e^{\lambda_0-\nu_0}(p_0+\epsilon_0)\,\xi = \frac{4}{r}\frac{dp_0}{dr}\xi - e^{-(\lambda_0+2\nu_0)/2}\frac{d}{dr}\left[e^{(\lambda_0+3\nu_0)/2}\frac{\gamma p_0}{r^2}\frac{d}{dr}\left(r^2 e^{-\nu_0/2}\xi\right)\right] + \frac{8\pi G}{c^4}e^{\lambda_0}p_0(p_0+\epsilon_0)\,\xi - \frac{1}{p_0+\epsilon_0}\left(\frac{dp_0}{dr}\right)^2\xi\,.$$

AB 2013

$$\omega^2 e^{2\Lambda_0-2\Phi_0}(\rho_0+P_0)\,\xi = -e^{-\Lambda_0-2\Phi_0}\left[e^{\Lambda_0+3\Phi_0}\frac{\gamma P_0}{r^2}\left(r^2 e^{-\Phi_0}\xi\right)'\right]' - (\rho_0+P_0)\,\Phi_0'^2\xi + 4r^{-1}\xi P_0' + 8\pi(\rho_0+P_0)\,\xi e^{2\Lambda_0}P_0 + (\rho_0+P_0)\,r^{-4}\xi e^{2\Lambda_0}Q_0^2$$

← CHARGE TERM

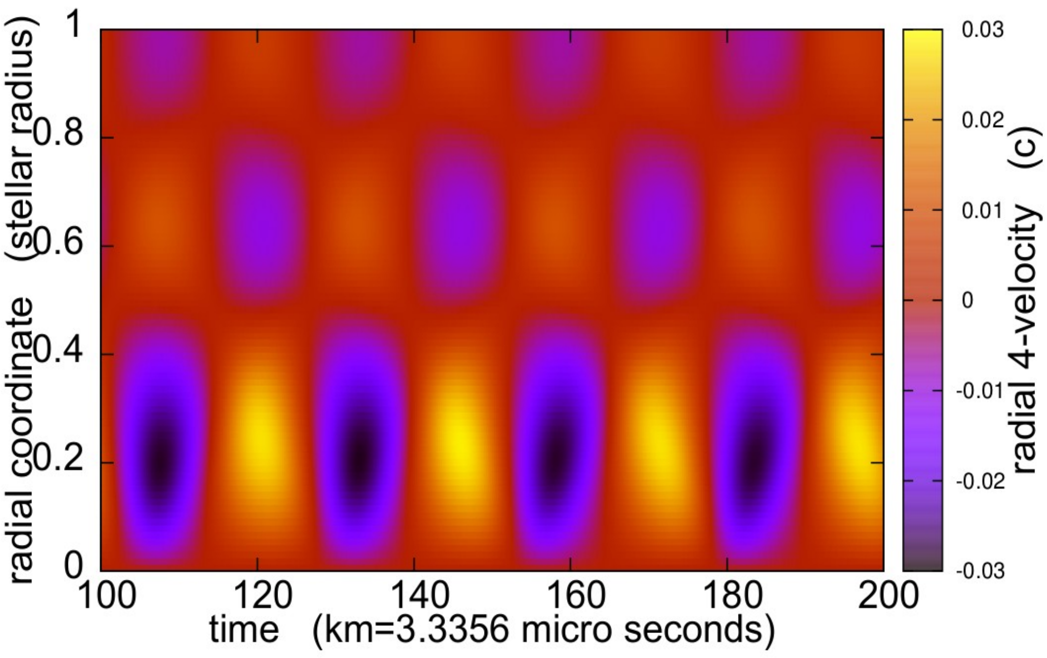
Radial oscillations

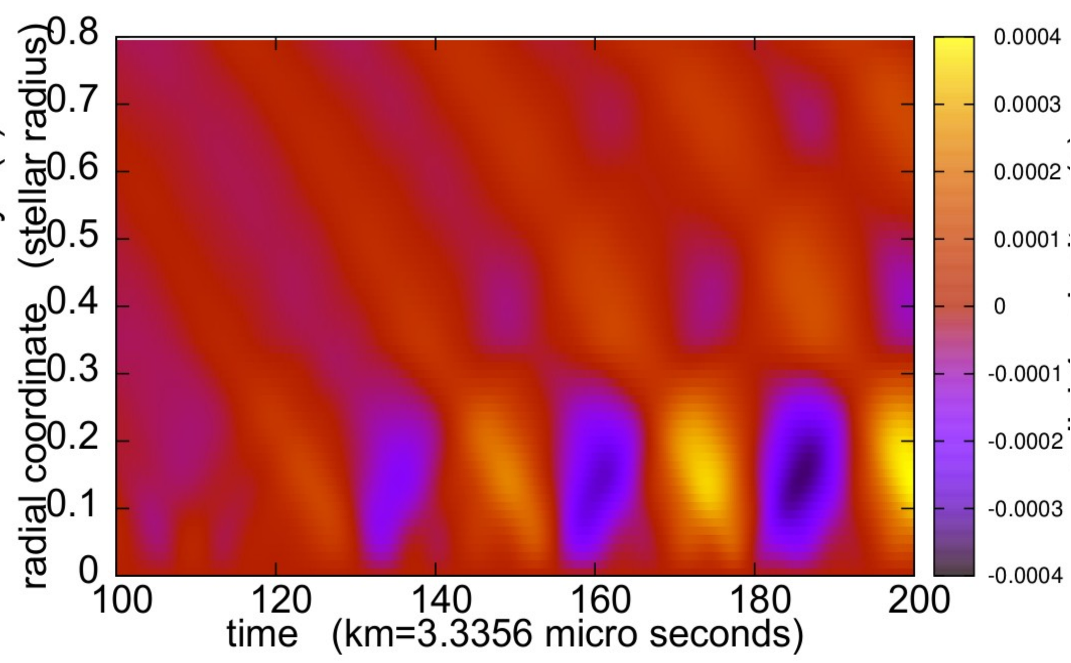
- spherical symmetry preserved

- type of oscillation described by number of nodes

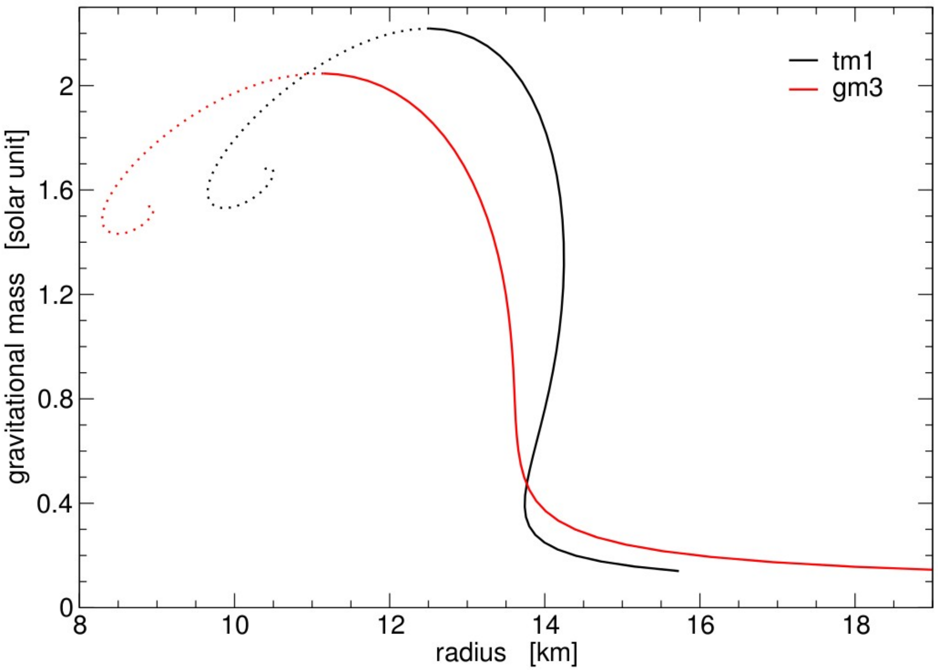
- Sturm-Liouville equation as in Newtonian gravity

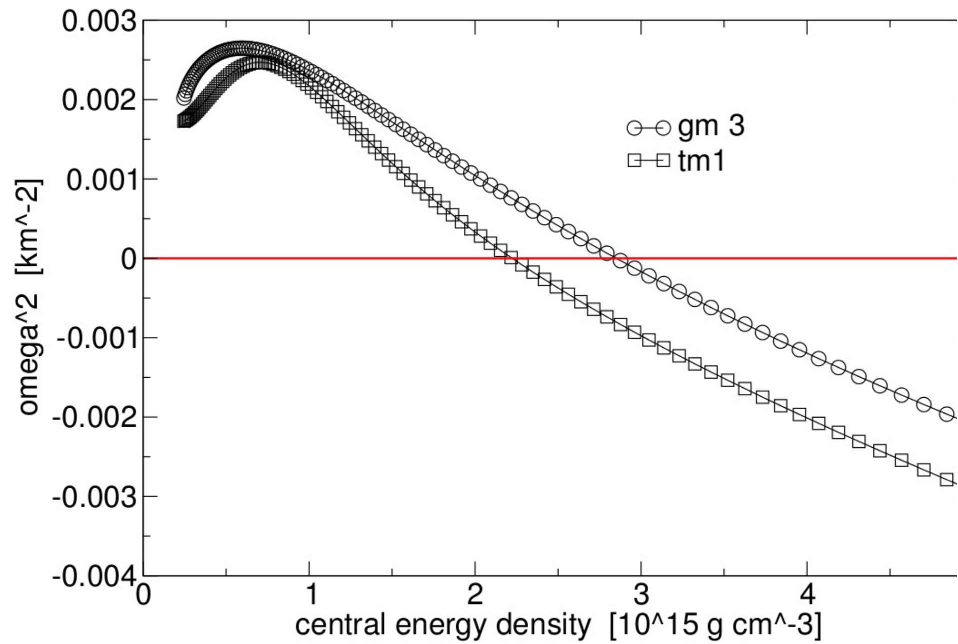
- discrete set of frequencies given by boundary conditions: $\xi(r=0)=0,\ \Delta P(r=R)=0$





Soft vs. hard EOS



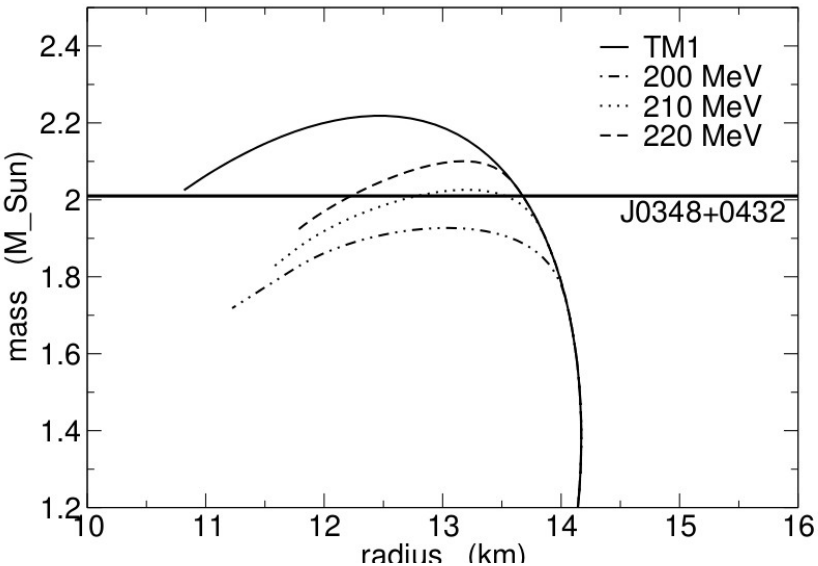


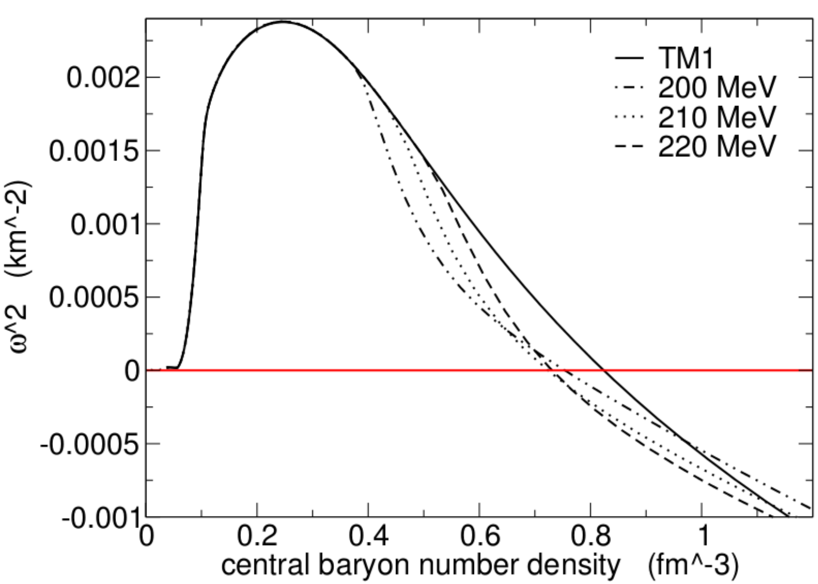
Neutral hybrid stars – Gibbs constr.

$$\Omega_{\mathrm{QM}} = \sum_{i=u,d,s,e} \Omega_i + \frac{3\mu^4}{4\pi^2}(1 - a_4) + B_{\mathrm{eff}}$$

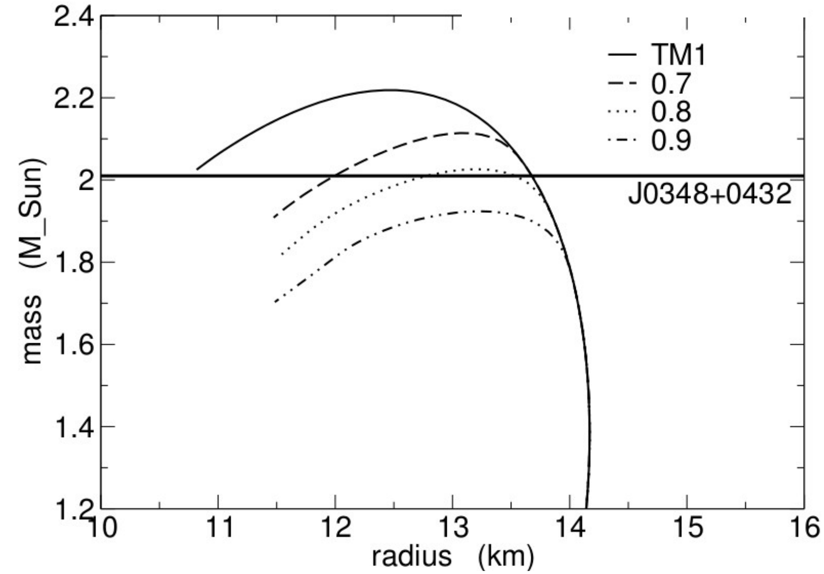
Weissenborn et al. Astrophys. J. 740, L14 (2011)

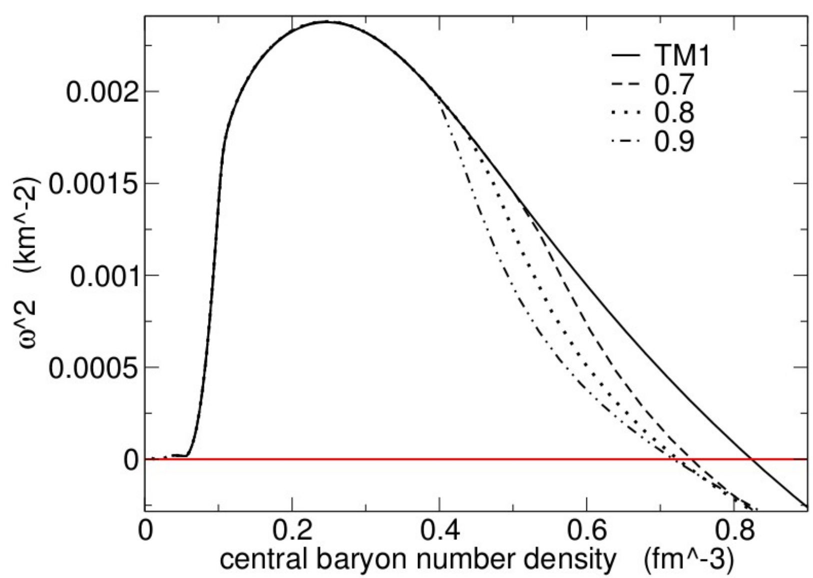
$m_s = 100\ MeV, a_4 = 0.8$





$m_s = 100\ MeV\ \ B^{\frac{1}{4}} = 210\ MeV$





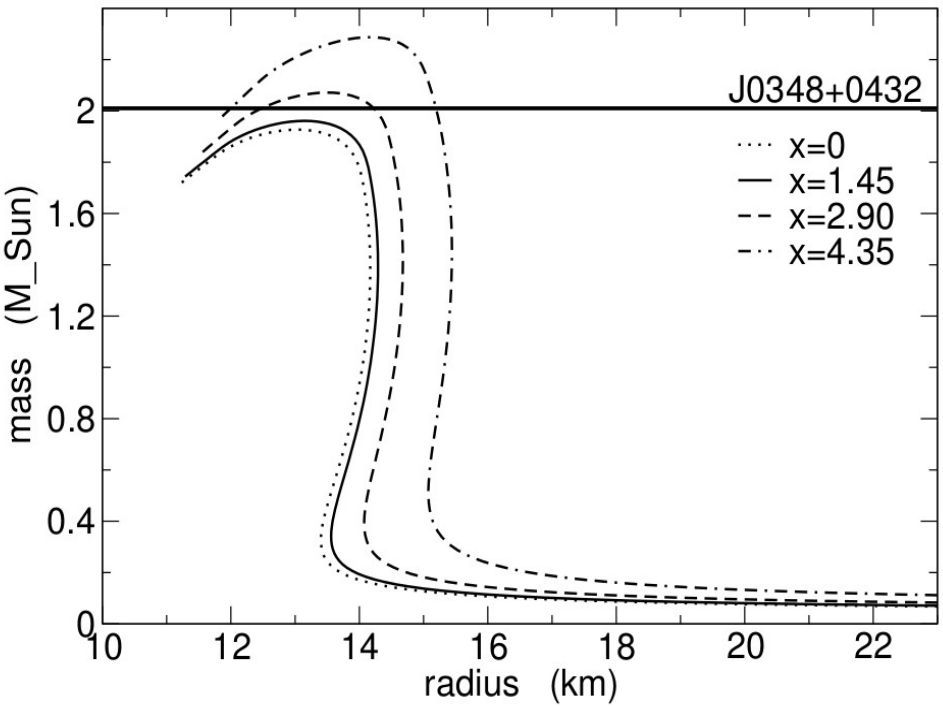
Charged hybrid stars – Gibbs constr.

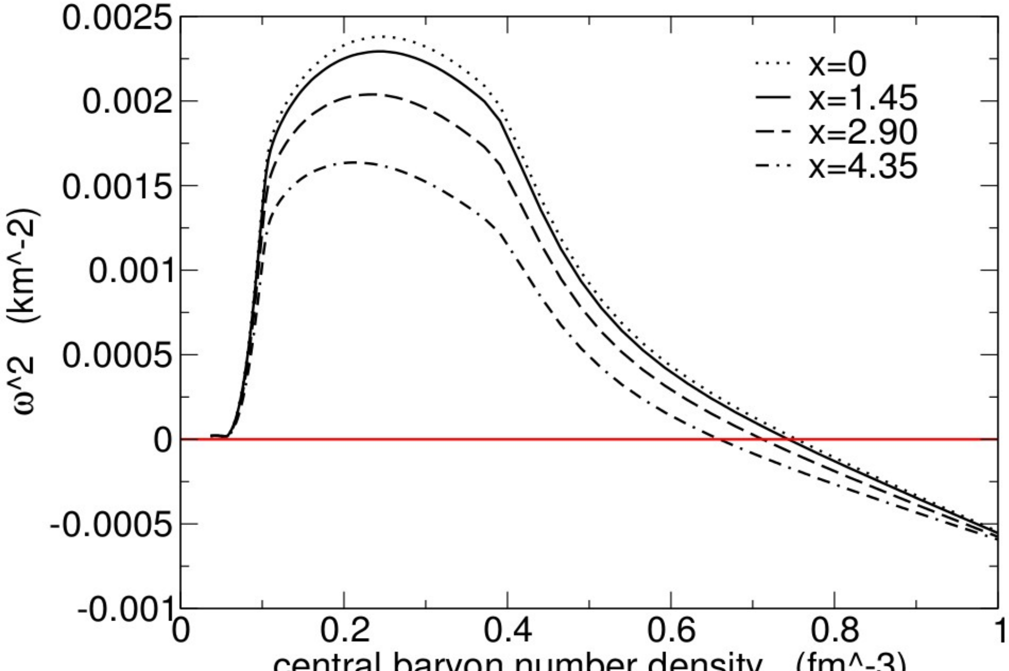
Hadronic phase: relativistic mean-field model, TM1 parameter set

Quark phase: MIT bag model, $m_s = 100\,MeV$, $a_4 = 0.8$, $B^{1/4} = 200\,MeV$

$$x = 10^{19} \frac{N_c}{N_b}$$

M. Alford, M. Braby, M. Paris, S. Reddy, ApJ, 629, 969, (2005)



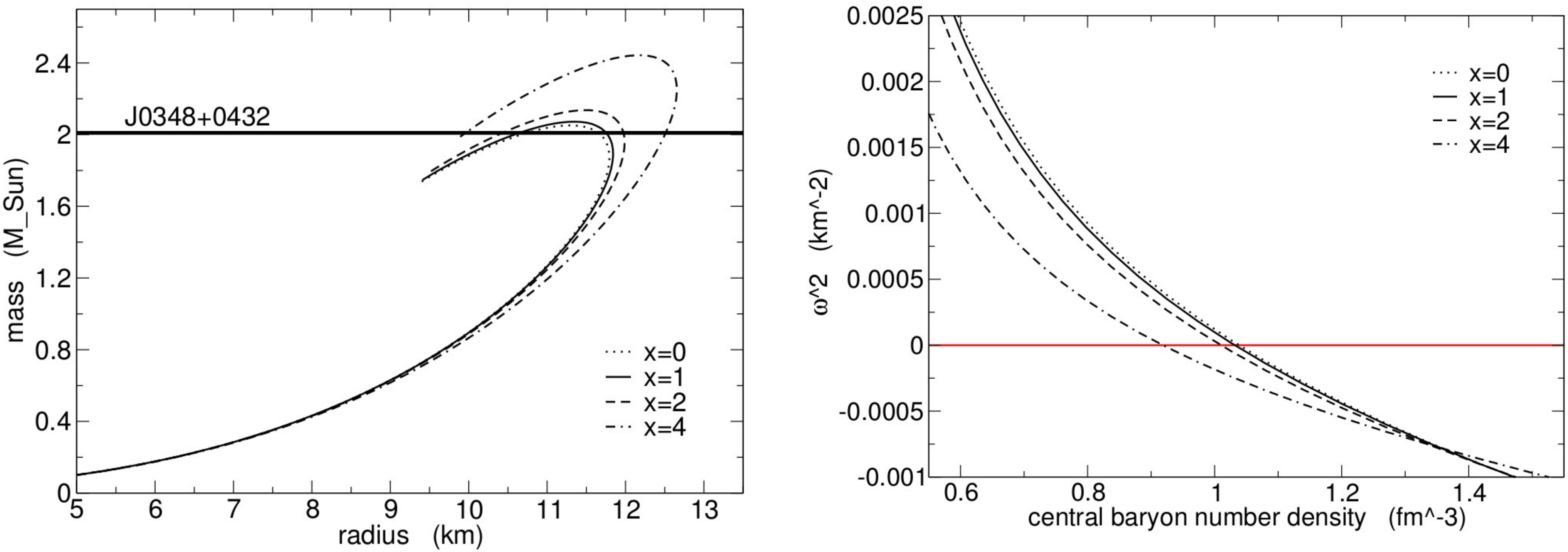


Charged strange stars

Quark phase: MIT bag model, $m_s = 100\,MeV$, $a_4 = 1.0$, $B^{1/4} = 140\,MeV$

$$x = 10^{19}\frac{N_c}{N_b}$$

M. Alford, M. Braby, M. Paris, S. Reddy, ApJ, 629, 969, (2005)



Results

- generalization of Chandrasekhar's equation to charge

- enhancement of masses of hybrid and strange stars due to Coulomb repulsion

- both Coulomb interaction and deconfinement lead to lower frequencies of radial eigenmodes at given central density

- A naïve application to stars with sharp density discontinuity contradicts with the *static stability criterion*.

- We constructed hollow spheres in hydrostatic equilibrium with positive pressure gradient.

Outlook

- investigate properties of charged hollow spheres/ introduce viscosity