# Maximum mass of hybrid stars and microscopic stability of strange cores

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# Short summary of Warsaw talk (PH)

## Hyperon stars (B)

- $M_{\max} > 2M_{\odot}$ possible
- rather fine tuning of parameters required
- strong repulsion at high density needed

Dexheimer & Schramm (2008)
Bednarek et al. (2012)
Weissenborn et al. (2011)
Bonanno & Sedrakian (2012)

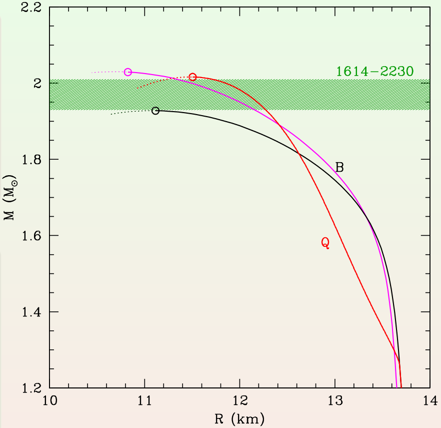
## Hybrid stars (Q)

- interpolation of the quark matter EOS by a linear function $P = a(\rho - \rho_0)c^2$
  very good fit to the advanced calculations

  Agrawal (2010)
  Blaschke et al. (2010)

- first order phase transition $\mathrm{B} \to \mathrm{Q}$ at $n_{\mathrm{b}} \simeq 2 \div 2.5 n_0$
- $M_{\max} > 2M_{\odot}$ possible for $a > 0.4$

Zdunik & Haensel (2013)




BM15 model *Bednarek et al. 2012*

BM165 model *Bednarek et al. 2012*

BM15+Q model

# Re-confinement

## Problem - stability of quark phase

Thermodynamically stable phase - minimum of $\mu_b$ at given $P$.

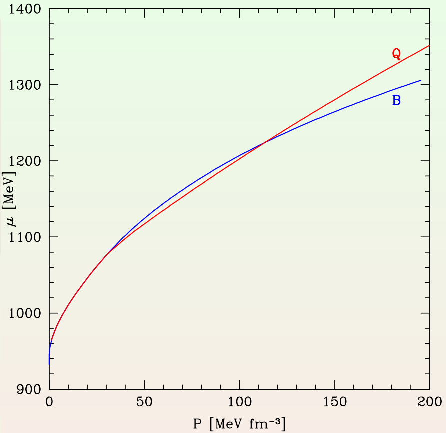
- large maximum mass - stiff EOS.Q
- stiff EOS - significant increase of $\mu^{\mathrm{Q}}(P)$
- at $P_{\mathrm{recon}}$, $\mu^{\mathrm{Q}}(P)$ crosses $\mu^{\mathrm{B}}(P)$
- above $P_{\mathrm{recon}}$ quark matter thermodynamically unstable

## Consequences for $M_{\max}$

- $M_{\max}$ similar to the values without Q phase

## Solution

- the point particle model of baryons is unjustified at high densities.
- the finite-size correction for hadrons (the excluded volume approximation) in the baryon phase
- result - re-confinement prohibited.




BM15+Q model
TM1 - baryon matter, RMF
Quark matter: $\lambda = \frac{\rho_s}{\rho_B} = 1.2$
$a = 0.42 \; v_{\mathrm{sound}} = \frac{2}{3}c$

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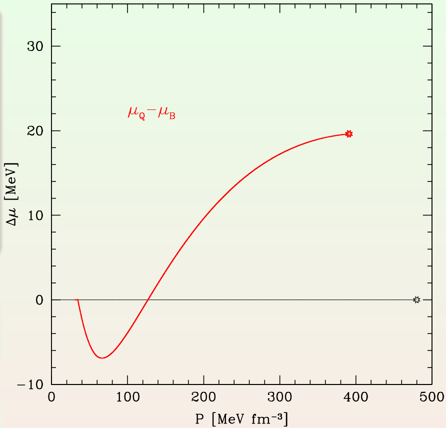


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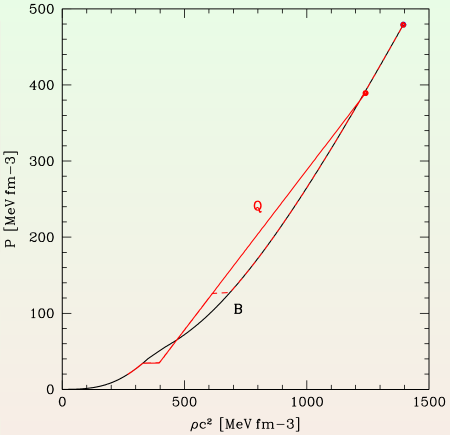


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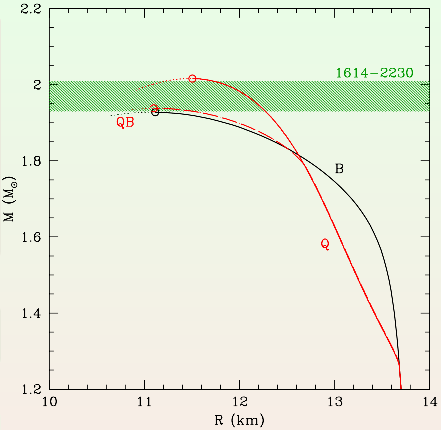


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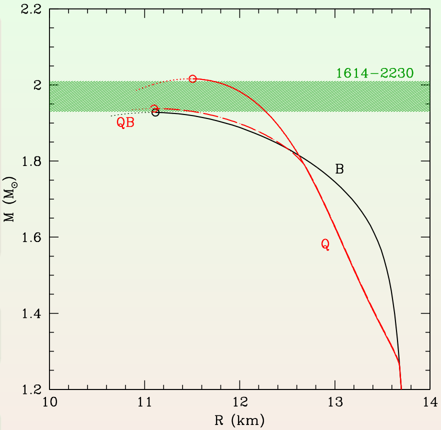


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Non-relativistic: *Rischke et al. 1991*; **relativistic**: *Zhang Q.-R. 1995*

$B = n, p, \Lambda, \ldots$ **RMF** $\quad \widetilde{m}_B = \left(m^\star_B\right)_{\rm Dirac} = m_B - g_{\sigma B}\sigma - \ldots$

**Two coordinate systems (reference frames)**

(1) $\mathcal{C}_{\rm sys}$: matter (many-body system) macroscopically at rest
(2) $\mathcal{C}_p(\boldsymbol{p}=0)$: attached to a baryon $B$ (baryon momentum $\boldsymbol{p}$ in $\mathcal{C}_{\rm sys}$)

"Excluded volume" (i.e., not available) is due to "hard-core repulsion" of baryons of radius $r_B = r_0$.
Hard-core repulsion has range $2r_0 \Longrightarrow$ excluded volume per one baryon measured in $\mathcal{C}_p(\boldsymbol{p}=0)$ is

$$\boxed{\upsilon_0 = \frac{16}{3}\pi r_0^3}$$

$\mathcal{C}_p(\boldsymbol{p}=0)$: excluded volume $\upsilon_0$ (assumed same for all baryons)
$\mathcal{C}_{\rm sys}$: excluded volume $\upsilon_p = \upsilon_0 \frac{\widetilde{m}_B}{\sqrt{p^2+\widetilde{m}_B^2}}$

Calculate statistical average $\langle\upsilon_p\rangle$ using distribution function $f_B(p) = \Theta(p_B - p)$ in $\mathcal{C}_{\rm sys}$

Same as for point baryons: $N_{\rm b} = \sum_B N_B, \;\; n_{\rm b} = N_{\rm b}/V = \sum_B n_B$

$\langle v_p \rangle_B = \frac{v_0}{\pi^2} \int_0^{p_B} \frac{p^2 \widetilde{m}_B}{\sqrt{p^2 + \widetilde{m}_B^2}} {\rm d}p$ $\quad \boxed{\sum_B n_B \langle v_p \rangle_B = v_0 n_{\rm s}}$ Lorenz scalar

hard-spheres in $V \Longrightarrow$ point baryons in $V - V_{\rm excl}$

$\boxed{n_B = (1 - n_{\rm s} v_0) \frac{p_B^3}{3\pi^2}}$ $p_B > p_B^{(0)}$ $\quad \mathcal{E} = \mathcal{E}_{\rm b} + \mathcal{E}_e + \mathcal{E}_\mu$

baryon contribution to $\mathcal{E}$ modified by *excl-vol* effects $\boxed{\mathcal{E}_{\rm b} = (1 - n_{\rm s} v_0) \mathcal{E}_{\rm b}^{(0)}(\{p_B\})}$

auxiliary function: $\widetilde{\mathcal{E}} = \mathcal{E} + \lambda_1 \left(\sum_B n_B - n_{\rm b}\right) + \lambda_2 \left(\sum_B q_B n_B - n_e - n_\mu\right)$

$$\frac{\partial \widetilde{\mathcal{E}}}{\partial p_i} = 0 \; , \;\; i = n, p, \Lambda, \ldots, e, \mu$$

solution $\boxed{\{p_B\}, p_e, p_\mu}$ is close to point-baryons one ($v_0 = 0$) $\boxed{\{p_B^{(0)}\}, p_e^{(0)}, p_\mu^{(0)}}$

# Consequences

Necessary condition - excluded volume is a small correction to the dense matter model.

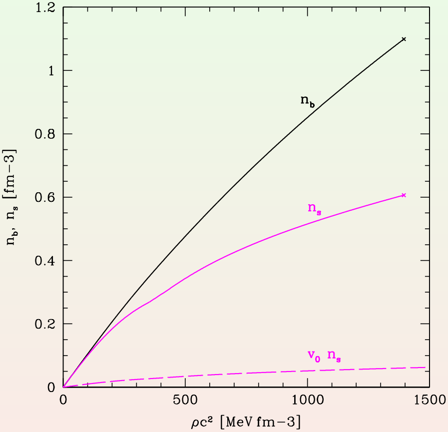
$$\frac{1}{\upsilon_0} \gg n_{\rm s}(P) < n_{\rm b}(P)$$

$n_{c(}M_{\rm max}) \simeq 1{\rm fm}^{-3} \Rightarrow \upsilon_0 \ll 1{\rm fm}^3$

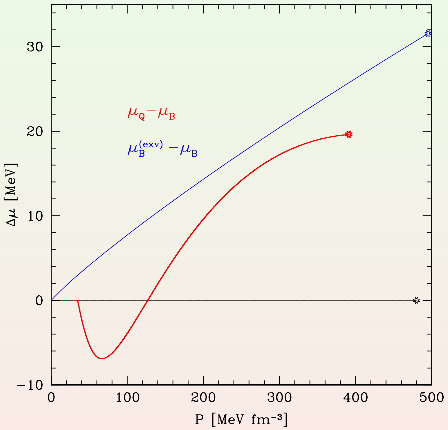
- decrease of baryon number density at given pressure
- decrease of mass-energy density at given pressure
- result - for given $P$: $n_{\rm b} \simeq n_{\rm b}^{(0)}(P)(1 - \upsilon_0 n_{\rm s}^{(0)}(P))$

$\Downarrow$

**stiffening of the EOS**



- $\mu_Q > \mu_B$ at high pressures
  Q ustable
- inclusion of the excluded volume removes this instability
  $v_0 = 0.1\mathrm{fm}^3$
  BUT
- EOS.B phase stiffer



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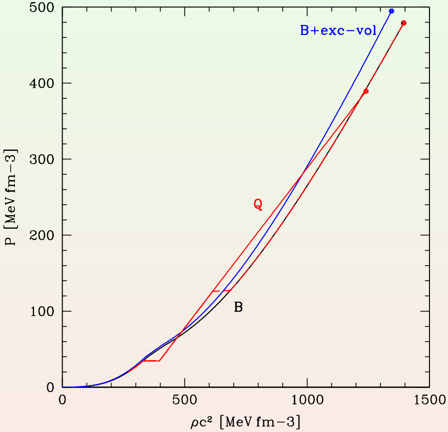

# Excluded volume and maximum mass

- $\mu_Q > \mu_B$ at high pressures (Q ustable)
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- with excluded volume EOS.B stiffer
- maximum mass for EOS.B larger
- to fulfill simultanously 2 conditions:
  $\mu_Q < \mu_B$ and $M_{max}(Q) > 2M_\odot$
  we obtain
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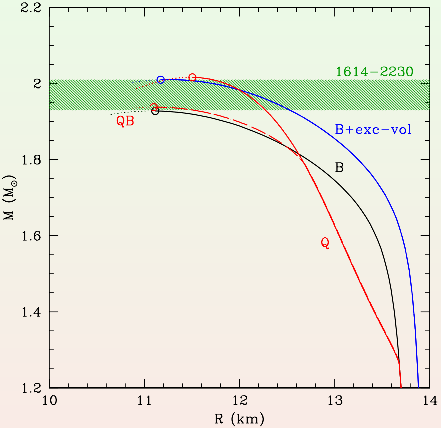

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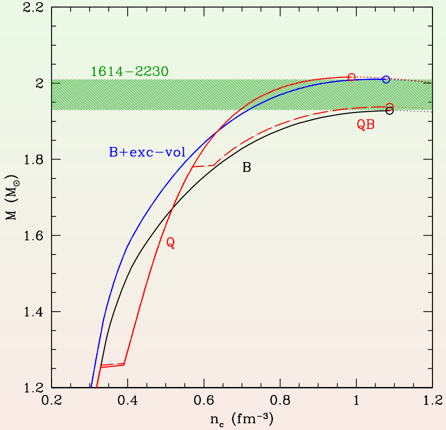

# Conclusions

**Problem**

**EOS.B**

$M_{\max}(\mathrm{B})$ too small

**EOS.Q too stiff**

Q matter unstable at high densities
reconfinement
$M_{\max}(\mathrm{QB}) < M_{\mathrm{obs}}$

**EOS.B stiffer**

increase of $M_{\max}(\mathrm{B}) > M_{\mathrm{obs}}$

**Solution**

**EOS.Q**

Q matter stiff to fulfill $M_{\max} > M_{\mathrm{obs}}$

**EOS.B + exc-Vol**

Inclusion of the excluded volume for $\mathrm{B}$ matter
increase of $\mu_{\mathrm{B}}$
microscopic stability of Q matter $\mu_{\mathrm{Q}} < \mu_{\mathrm{B}}$

**Conclusion**

Similar maximum masses for confined (baryon star) and deconfined quark cores (hybrid star) if quark matter instability removed by exluded volume effect.

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