Non-equilibrium Evolution of Heavy Quarkonium in Medium

EMMI Workshop: Probing the Early Stages with Jets

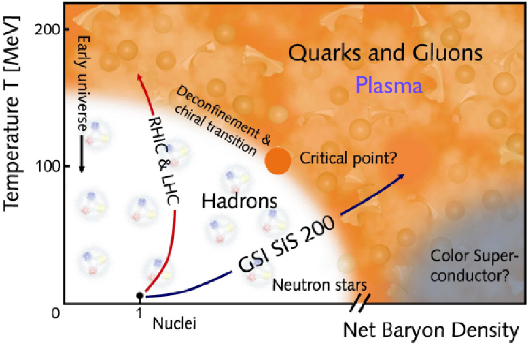
Peter Vander Griend

University of Kentucky and Fermilab

17 July 2026

Confinement

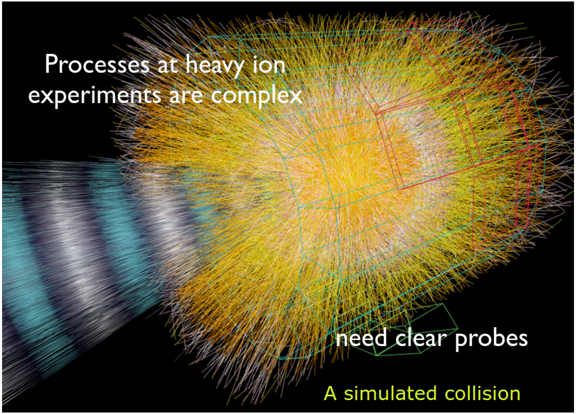
QCD phenomenologically rich



Nuclear Matter

confinement: key open question for strongly coupled QCD but also early universe, astrophysics, cosmology, etc.

Experiment: Heavy Ion Collisions



complicated environment and massive amounts of data

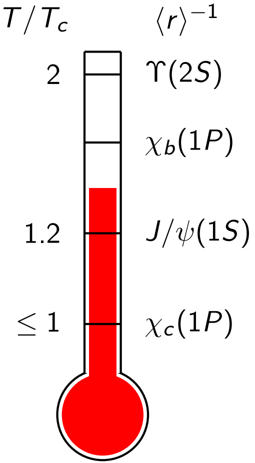
in 5 week run in October 2023, ALICE had 12 billion collisions, up to 770 gigabytes of data/second, 47.7 petabytes of data

https://alice-collaboration.web.cern.ch/2023$_Dec_heavy_ion$

$Q\bar{Q}$ as Probes of Medium

- formed early $M^{-1} \sim 0.1$ fm
- multiscale system sensitive to medium
- dilepton signal a clear experimental signature
- expected to dissociate due to color screening

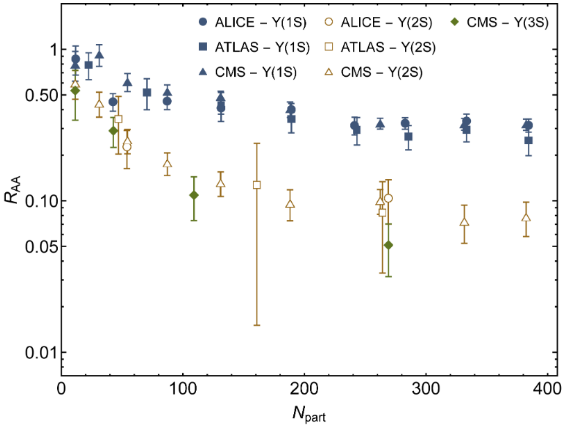
$$V(r) \sim -\alpha_s \frac{e^{-m_D r}}{r}$$



Phys. Lett. B178, 416 (1986) (Matsui, Satz)

Experimental Results

Nuclear Modification Factor R_{AA}



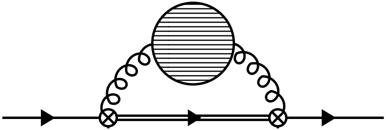
$$R_{AA} < 1 \implies Q\bar{Q} \text{ suppression}$$

PLB 790 (2019) 270 (CMS), PLB 822 (2021) 136579 (ALICE), PRC 107 (2023) 054912 (ATLAS)

Beyond the Static Potential

Matsui, Satz picture statically screened potential
lattice and EFT technology allowed for calculations of potential

Landau Damping: innelastic parton scattering



JHEP 03 (2007) 054 (Laine, Philipsen, Romatschke, Tassler); Phys.Rev.A 79 (2009) 063623 (Escobedo, Soto); JHEP 05 (2013) 130 (Brambilla, Escobedo, Ghiglieri, Vairo)

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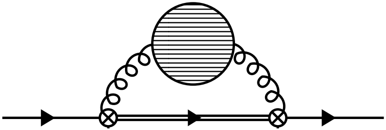

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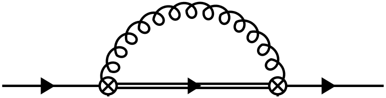
Color Transitions: gluo-dissociation



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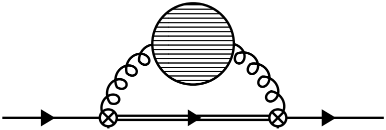

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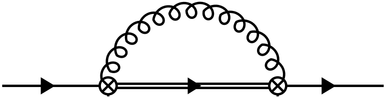

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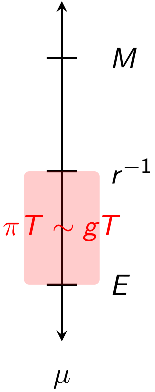
key results: $\mathrm{Im}[V] \neq 0$ and $\mathrm{Im}[V] > \mathrm{Re}[V]$ in relevant region
$\Longrightarrow$ other mechanisms of dissociation

Modern Approach: EFT + OQS

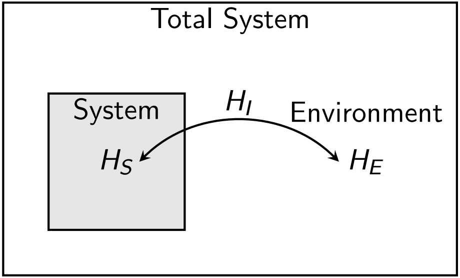
- **non-equilibrium**, **quantum** process in a **strongly-coupled** medium
- need non-Abelian, quantum (QCD) **master equation**
- **EFT** $\Longrightarrow$ multiscale (small) probe of strongly coupled QGP

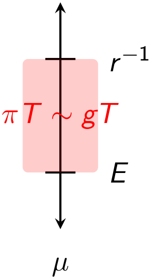
$$M \gg r^{-1} \gg T \sim gT, \Lambda_{\rm QCD}, E$$

- **OQS** $\Longrightarrow$ nonequilibrium



Open Quantum Systems





time scales:

$\boxed{\text{environment: } \tau_E \sim (\pi T)^{-1}}$, $\boxed{\text{relaxation: } \tau_R \sim r^{-2}(\pi T)^{-3}}$,

$\boxed{\text{system: } \tau_S \sim E^{-1}}$

regimes: $\tau_E \ll \tau_R, \tau_S \iff$ Quantum Brownian motion

$\implies$ **Markov approximation** (locality in time)

$\implies$ **Born approximation** (trace over environment $\mathrm{Tr}_E[\mathcal{O}\,\rho_E] = \langle \mathcal{O} \rangle$)

Open Quantum Systems

Also used for jets in medium!

See talk by Pietro Benzoni (and references) on Wednesday!

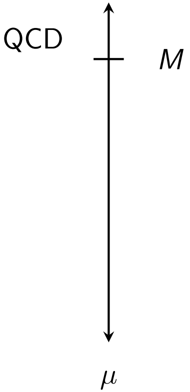
See talk by Balbeer Singh (and references) on Thursday!

system: $\tau_S \sim E^{-1}$

$\Longrightarrow$ **Markov approximation** (locality in time)

$\Longrightarrow$ **Born approximation** (trace over environment $\mathrm{Tr}_E[\mathcal{O}\,\rho_E] = \langle \mathcal{O} \rangle$)

potential Non-Relativistic QCD



- QCD:
  $\Longrightarrow$ model is the Standard Model

Phys.Lett.B 167 (1986) 437-442 (Caswell, Lepage), Phys.Rev.D 51 (1995) 1125-1171 (Bodwin, Braaten, Lepage), Nucl.Phys.B 566 (2000) 275, Rev.Mod.Phys. 77 (2005) (Brambilla, Pineda, Soto, Vairo)

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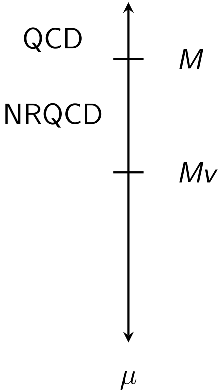

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  $\Longrightarrow$ non-relativistic treatment

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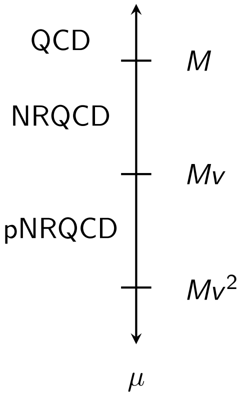

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- pNRQCD: NR, heavy-heavy system
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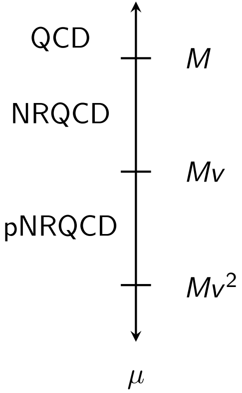

- QCD:
  $\Longrightarrow$ model is the Standard Model
- NRQCD: small velocity
  $\Longrightarrow$ non-relativistic treatment
- pNRQCD: NR, heavy-heavy system
  $\Longrightarrow$ QM treatment
  - $h_s = \frac{\mathbf{p}^2}{M} - \frac{C_F \alpha_s(a_0^{-1})}{r}$
  - $h_o = \frac{\mathbf{p}^2}{M} + \frac{\alpha_s(a_0^{-1})}{2N_c r}$

$$\mathcal{L}_{\mathrm{pNRQCD}} = \mathrm{Tr}\Big[S^\dagger \left(i\partial_0 - h_s\right) S + O^\dagger (iD_0 - h_o) O + O^\dagger \mathbf{r}\cdot g\,\mathbf{E}\,S$$
$$+ S^\dagger \mathbf{r}\cdot g\,\mathbf{E}\,O + \frac{1}{2} O^\dagger \left\{\mathbf{r}\cdot g\,\mathbf{E}\,,\ O\right\}\Big]$$

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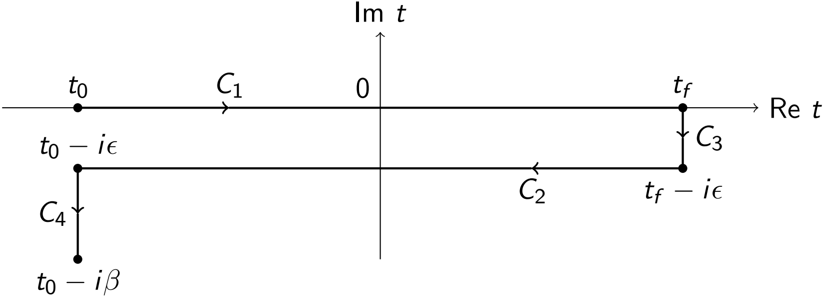
Real Time Calculations at Finite Temperature

Schwinger-Keldysh Contour: $e^{-\beta H}$ as time evolution



density matrix: transport from t_0 to $t_0 - i\beta$

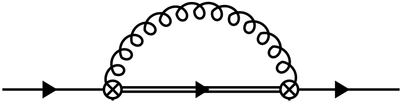
$\implies$ **octet sector**: $\rho_E^{ab} \propto U^{ab}(t_0 - i\beta, t_0)$

$\implies$ ensures **gauge invariance, conservation of color** charge

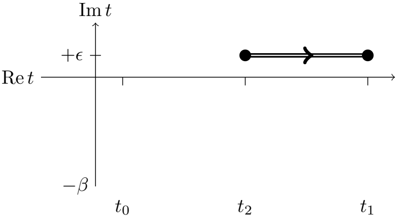
$\implies$ **expt. values**: $\mathrm{Tr}\left[O^{a\dagger}\mathcal{O}^{ab}O^{b}\rho\right] \propto \left\langle U^{ba}(t_0 - i\beta, t_0)\mathcal{O}^{ab}\right\rangle$

Dissociation

singlet-octet transition



gauge structure



process determined by correlator

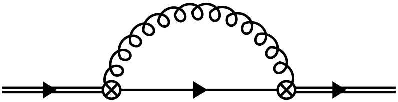
$$\boxed{D^{>}_{so}(t) = \left\langle E^a_i(t) U^{ab}(t,0) E^b_i(0) \right\rangle}$$

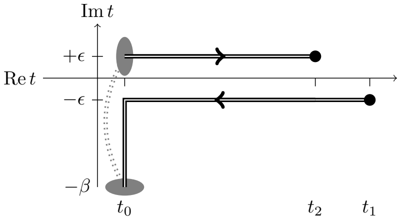
Phys. Rev. D 97 (2018) 7, 074009 (Brambilla, Escobedo, Soto, Vairo)

Recombination

gauge structure

octet-singlet transitions





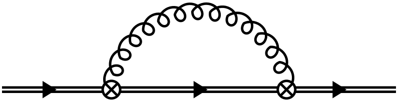
process determined by correlator

$$\boxed{D^{>}_{os}(t) = \frac{\left\langle U^{bc}(t_0 - i\beta, t_0) U^{cd}(t_0, t) E^d_i(t) E^e_i(0) U^{eb}(0, t_0) \right\rangle}{\left\langle U^{aa}(t_0 - i\beta, t_0) \right\rangle}}$$

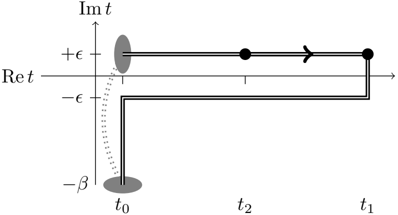
JHEP 02 (2021) 062 (Yao, Mehen)

Octet Evolution

octet-octet transitions



gauge structure



process determined by correlator

$$\boxed{D^{>}_{oo}(t) = \frac{\left\langle U^{bc}(t_0 - i\beta, t_0) U^{cd}(t_0, t) E_i^{df}(t) U^{fg}(t, 0) E_i^{gj}(0) U^{jb}(0, t_0) \right\rangle}{\langle U^{aa}(t_0 - i\beta, t_0)\rangle}}$$

where $E_i^{df} = E_i^e d^{def}$, $E_i^{gj} = E_i^h d^{ghj}$

arXiv:26xx.xxxxx (Brambilla, Escobedo, Panayiotou, Strickland, Vairo, **PVG**)

Transport Coefficients

need integrated correlators

$$\frac{g^2}{6N_c}\int_0^\infty dt\, D_{uv}^{>\dagger}(t) = \kappa_{uv} + i\gamma_{uv}$$

- define transport coefficients
- κ_{so}: **heavy quarkonium momentum diffusion coefficient** $\implies$ analogous to κ **heavy quark momentum diffusion coefficient**
- well known quantity(ies) with **lattice measurements** and **perturbative calculations**
- $\kappa_{so} \sim \Gamma$ and extractable from unquenched lattice measurements
- γ_{so} is dispersive counterpart (correction to mass $\sim \delta M$)

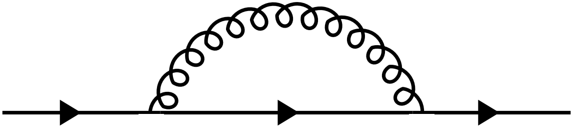
Momentum Diffusion Coefficients

Heavy Quark

in-medium diffusion (Brownian motion $\iff$ Langevin equation) of **fundamental object** through medium

$$\kappa = \frac{g^2}{3N_c}\mathrm{Re}\left[\int_0^\infty dt\ \langle \mathrm{Tr}\left[U(t_0,t)E_i(t)U(t,0)E_i(0)U(0,t_0)\right]\rangle\right]$$

where U is a Wilson line in the **fundamental** representation



(where single line here represents heavy quark)

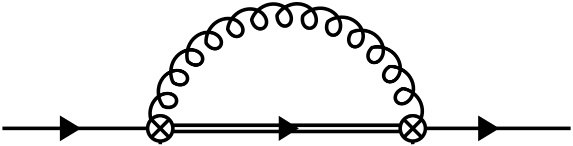
JHEP 04 (2007) 039 (Casalderrey-Solana, Teaney)

Momentum Diffusion Coefficients

Heavy Quarkonium

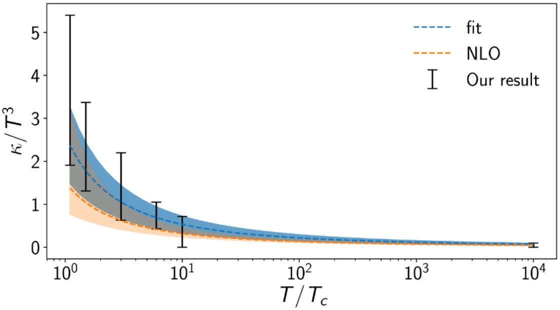
in-medium diffusion of **singlet state (transitioning to octet)** through medium

$$\kappa_{so} = \frac{g^2}{6N_c}\mathrm{Re}\left[\int_0^\infty dt\,\left\langle E_i^a(t)U^{ab}(t,0)E_i^b(0)\right]\right\rangle$$



Lattice Measurements

Heavy Quark Momentum Diffusion Coefficient



Lattice measurement of heavy quark momentum diffusion coefficient κ and perturbative calculations.

Phys.Rev.D 102 (2020) 7, 074503 (Brambilla, Leino, Petreczky, Vairo)

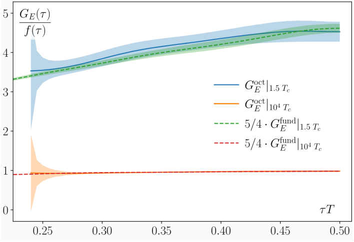
Heavy Quarkonium

Correlators Include Wilson Lines

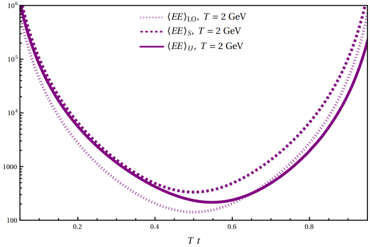
$\implies$ not 2-point functions

$\implies$ nonstandard analytic properties of spectral functions

Lattice Measurements



Euclidean Pert. Theory



Phys.Rev.D 112 (2025) 7, 074509 (TUMQCD); JHEP 08 (2025) 219 (Brambilla, Panayiotou, Säppi, Vairo)

Transport Coefficients: Summary

coefficient	role	physics	calculations
κ_{so}	heavy quarkonium momentum diffusion	width	lattice, pert. theory
$\kappa_{o\{s,o\}}$	octet momentum diffusion	transition rates	lattice, pert. theory
γ_{so}	heavy quarkonium dispersion	mass correction	lattice, pert. theory
$\gamma_{o,\{s,o\}}$	octet dispersion	correction to energies	lattice, pert. theory

past pheno studies without lattice or pert. results

$\Longrightarrow$ use single κ (either heavy quark κ or κ_{so})

$\Longrightarrow$ use single γ (γ_{so})

Pheno Studies

single κ and γ $\implies$ **Lindbald equation**

$$\frac{\mathrm{d}\rho(t)}{\mathrm{d}t} = -i[H,\rho(t)] + \sum_n \left(C_i^n \rho(t) C_i^{n\dagger} - \frac{1}{2}\left\{ C_i^{n\dagger} C_i^n, \rho(t) \right\} \right)$$

H encodes **corrections to energy** ($\propto \gamma$)

$$\rho = \begin{pmatrix} \rho_s & 0 \\ 0 & \rho_o \end{pmatrix}, \; H = \begin{pmatrix} h_s & 0 \\ 0 & h_o \end{pmatrix} + \frac{r^2}{2}\gamma \begin{pmatrix} 1 & 0 \\ 0 & \frac{N_c^2-2}{2(N_c^2-1)} \end{pmatrix}$$

C_i^n encode **rates of medium interaction and transitions** ($\propto \kappa$)

$$C_i^0 = \sqrt{\frac{\kappa}{N_c^2-1}}\, r_i \begin{pmatrix} 0 & 1 \\ \sqrt{N_c^2-1)} & 0 \end{pmatrix}, \; C_i^1 = \sqrt{\frac{(N_c^2-4)\kappa}{2(N_c^2-1)}}\, r_i \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}$$

can systematically extend to higher order in $E/(\pi T)$

Phys.Rev.D 97 (2018) 7, 074009 (Brambilla, Escobedo, Soto, Vairo)

Pheno Studies

single κ and γ $\implies$ **Lindbald equation**

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can systematically extend to higher order in $E/(\pi T)$
Work on all orders solution (Brambilla, Lin, Magorsch, Vairo)

Phys.Rev.D 97 (2018) 7, 074009 (Brambilla, Escobedo, Soto, Vairo)

Numerical Solution

spherical symmetry $\Longrightarrow$ discretize on 1D lattice

Numerical Solution

spherical symmetry $\implies$ discretize on 1D lattice

Direct solution: Expensive

color, radial position, angular momentum: $\rho = \rho_c \otimes \rho_r \otimes \rho_\ell$

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spherical symmetry $\implies$ discretize on 1D lattice

Direct solution: Expensive

color, radial position, angular momentum: $\rho = \rho_c \otimes \rho_r \otimes \rho_\ell$

$$
\begin{array}{r}
\ell = 0\left\{\vphantom{\rho_c \otimes \rho_r}\right. \\
\ell = 1\left\{\vphantom{\rho_c \otimes \rho_r}\right. \\
\ell = 2\left\{\vphantom{\rho_c \otimes \rho_r}\right. \\
\vdots
\end{array}
\begin{pmatrix}
\rho_c \otimes \rho_r & & & \\
 & \rho_c \otimes \rho_r & & \\
 & & \rho_c \otimes \rho_r & \\
 & & & \ddots
\end{pmatrix}
$$

blows up for large lattice and w/o truncation in ℓ

Numerical Solution: Monte Carlo

Quantum Trajectories Algorithm

evolve wave function $|\psi\rangle$ with non-Hermitian $H_{eff} = H - i\Gamma$

$\implies$ reduction in norm $\sim$ probability of interaction

$\implies$ Monte Carlo sample to implement medium interaction $C_n |\psi\rangle$

couple to state of the art hydrodynamics code

Necessary to Solve Lindblad Equation

$\implies$ **no cutoff in angular momentum**

$\implies$ **vast reduction in memory requirement** ($|\psi\rangle$ vs ρ)

$\implies$ **samples independent (embarrassingly parallelizable)**

JHEP 05 (2021) 136 (Brambilla, Escobedo, Strickland, Vairo, **PVG**, Weber), Comput.Phys.Commun. 273 (2022) 108266 (Ba Omar, Escobedo, Islam, Strickland, Thapa, **PVG**, Weber)

Nuclear Modification Factor

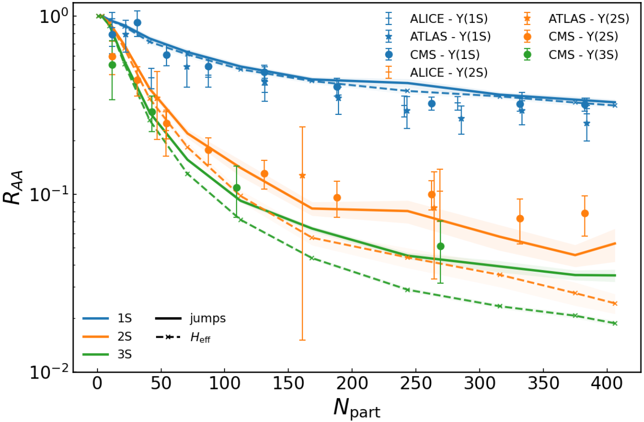

Figure: R_{AA} of $\Upsilon(1S)$, $\Upsilon(2S)$, and $\Upsilon(3S)$ vs. centrality compared to experimental measurements. Dashed lines are dissociation only; solid lines include correlated recombination.

R_{AA} vs. p_T

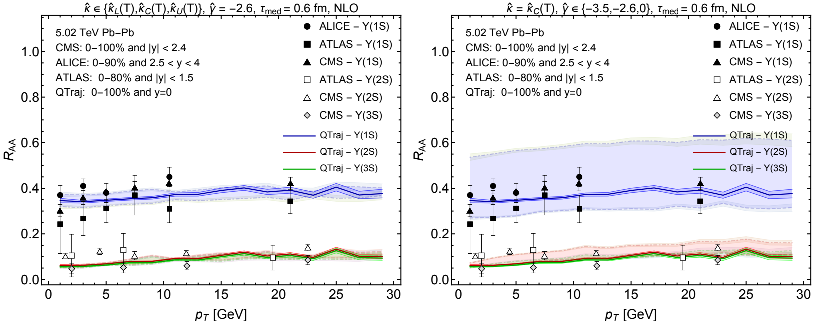

Figure: R_{AA} for the $\Upsilon(1S)$, $\Upsilon(2S)$, and $\Upsilon(3S)$ as a function of p_T compared to experimental measurements. NLO in E/T.

JHEP 08 (2022) 303 (Brambilla, Escobedo, Islam, Strickland, Tiwari, Vairo, **PVG**)

$v_2[\Upsilon(1S)]$ vs. Centrality

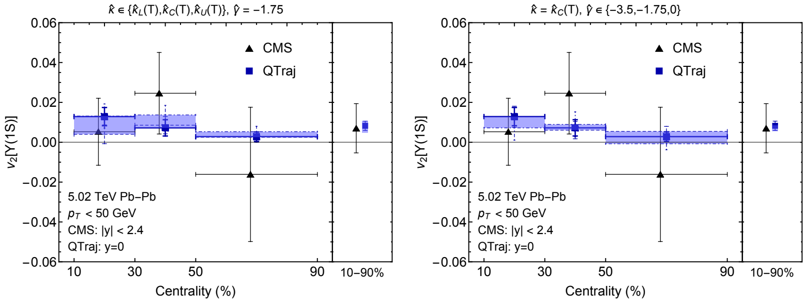

Figure: The elliptic flow v_2 of the $\Upsilon(1S)$ as a function of centrality compared to experimental measurements. LO in E/T.

Phys. Rev. D 104 (2021) 9, 094049 (Brambilla, Escobedo, Strickland, Vairo, **PVG**, Weber)

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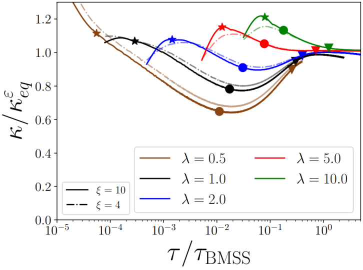

Figure: The non-equilibrium heavy quark momentum diffusion coefficient.

Phys.Rev.D 109 (2024) 1, 014025 (Boguslavski, Kurkela, Lappi, Lindenbauer, Peuron)

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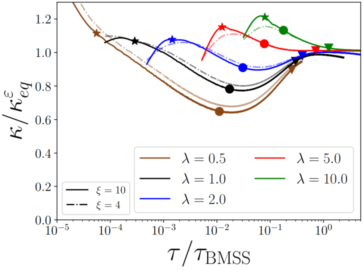

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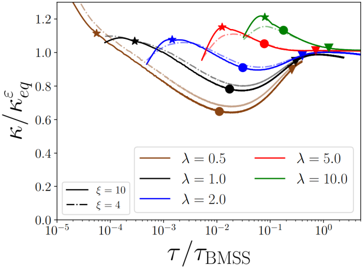

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Probing early stages?

$\Longrightarrow$ **other observables?**

$\rightarrow$ energy correlators

$\Longrightarrow$ **other systems?**

$\rightarrow$ smaller ions

Phys.Rev.D 109 (2024) 1, 014025 (Boguslavski, Kurkela, Lappi, Lindenbauer, Peuron)

Conclusions and Outlook

Conclusions

- $Q\bar{Q}$ suppression signal of deconfinement and QGP formation
- incredibly complicated system: with **OQS** and **EFT**
  - $Q\bar{Q}$ as multiscale probe of strongly coupled medium
  - non-equilibrium, quantum evolution
  - systematically derive QCD master equation
    $\Longrightarrow$ get observables
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Outlook: Early Stages

- **OQS** and **EFT** ((p)NRQCD, SCET) for jet evolution
- **smaller systems** perhaps more sensitive to early stages
- other $Q\bar{Q}$ observables (**energy correlators**) may be sensitive

Thank you!

Backup Slides

R_{AA} vs. Centrality

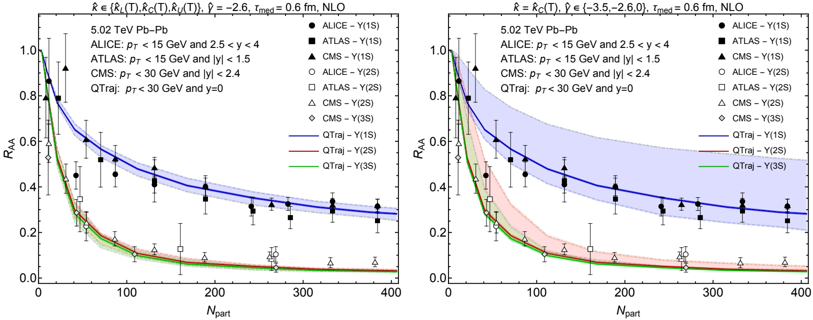


Figure: R_{AA} of $\Upsilon(1S)$, $\Upsilon(2S)$, and $\Upsilon(3S)$ vs. centrality compared to experimental measurements. Variation in κ (left) and γ (right).

JHEP 08 (2022) 303 (Brambilla, Escobedo, Islam, Strickland, Tiwari, Vairo, PVG)

Double Ratio 2S vs. Centrality

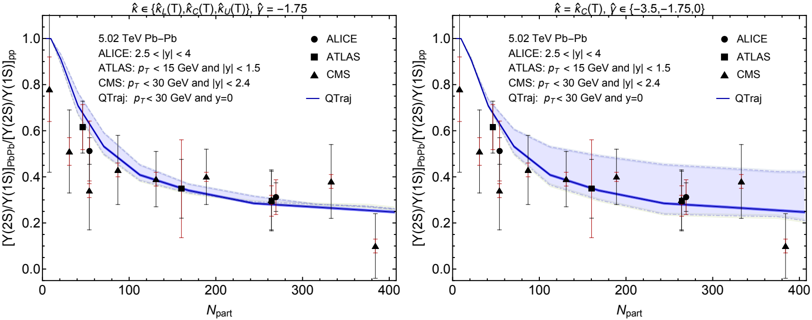


Figure: Double ratio of $R_{AA}(2S)$ to $R_{AA}(1S)$ as a function of centrality compared to experimental measurements. The left panel shows variation of $\hat{\kappa} \in \{\kappa_L(T), \kappa_C(T), \kappa_U(T)\}$ and the right panel shows variation of $\hat{\gamma}$ in the range $-3.5 \leq \hat{\gamma} \leq 0$. In both panels, the solid line corresponds to $\hat{\kappa} = \hat{\kappa}_C(T)$ and the best fit value of $\hat{\gamma} = -1.75$. LO in E/T.

Phys. Rev. D 104 (2021) 9, 094049 (Brambilla, Escobedo, Strickland, Vairo, PVG, Weber)

Double Ratio 3S vs. Centrality

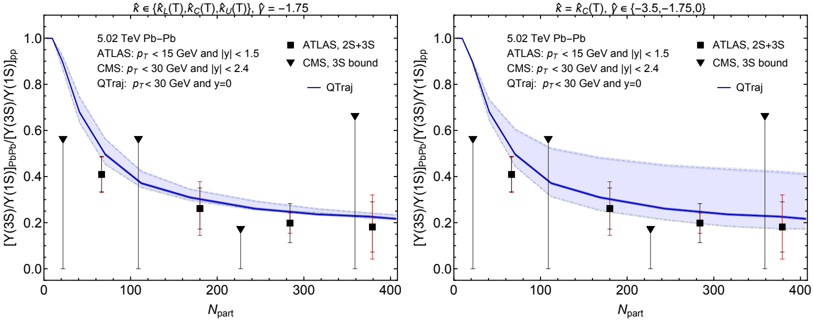


Figure: Double ratio of $R_{AA}(3S)$ to $R_{AA}(1S)$ as a function of centrality compared to experimental measurements. LO in E/T.

Phys. Rev. D 104 (2021) 9, 094049 (Brambilla, Escobedo, Strickland, Vairo, PVG, Weber)

Double Ratio $2S$ vs. p_T

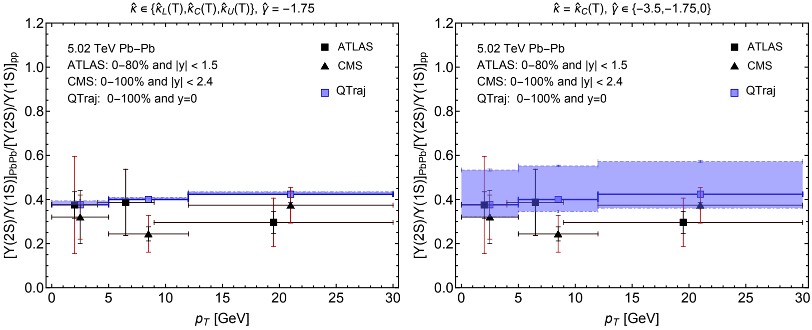


Figure: Double ratio of $R_{AA}(2S)$ to $R_{AA}(1S)$ as a function of p_T compared to experimental measurements. LO in E/T.

Phys. Rev. D 104 (2021) 9, 094049 (Brambilla, Escobedo, Strickland, Vairo, PVG, Weber)

$v_2[\Upsilon(1S)]$ vs. p_T

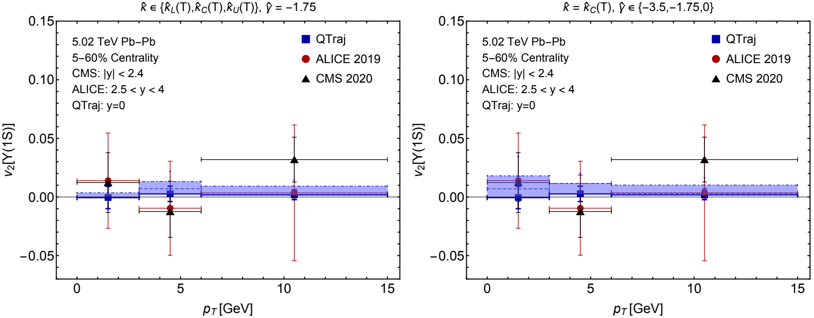


Figure: The elliptic flow v_2 of the $\Upsilon(1S)$ as a function of p_T compared to experimental measurements. LO in E/T.

Phys. Rev. D 104 (2021) 9, 094049 (Brambilla, Escobedo, Strickland, Vairo, PVG, Weber)

$v_2[\Upsilon(2,3S)]$ vs. Centrality

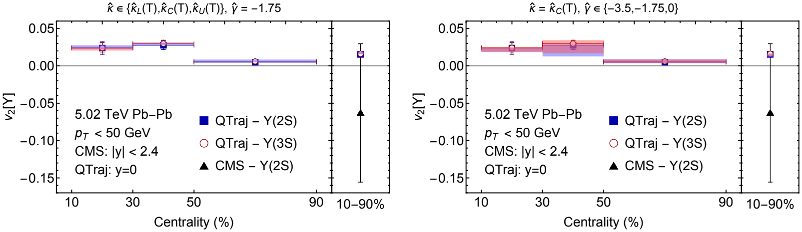


Figure: The elliptic flow v_2 of the $\Upsilon(2S)$ and $\Upsilon(3S)$ as a function of centrality compared to experimental measurements. LO in E/T.

Phys. Rev. D 104 (2021) 9, 094049 (Brambilla, Escobedo, Strickland, Vairo, PVG, Weber)

QTraj Implementation

1. **initialize** wave function $|\psi(t_0)\rangle$
2. **generate random number** $0 < r_1 < 1$, evolve with H_{eff} until

$$|| \, e^{-i \int_{t_0}^{t} dt' H_{\text{eff}}(t')} |\psi(t_0)\rangle \, ||^2 \leq r_1,$$

   and initiate a quantum jump
3. **quantum jump**
   - 3.1 **color**: if singlet, jump to octet; if octet, generate random number $0 < r_2 < 1$ and jump to singlet if r_2 less than the branching fraction to singlet; otherwise, remain in octet
   - 3.2 **angular momentum**: generate random number $0 < r_3 < 1$; if $r_3 < l/(2l+1)$, $l \to l-1$; otherwise, $l \to l+1$.
   - 3.3 **normalize**: multiply wavefunction by r and normalize
4. Continue from step 2.

Comput. Phys. Commun. 273 (2022) 108266 (Ba Omar, et. al.)

Simulation Parameters

initial condition

smeared Gaussian:
$\psi_\ell \propto r^\ell e^{-r^2/c^2}$

hydro

$3+1$D dissipative relativistic hydro with realistic equation of state fit to LQCD

physical trajectories

position sampled from overlap profile $N_{AA}^{\mathrm{bin}}(x, y, b)$, p_T from E_T^{-4}, $\phi \in [0, 2\pi)$

lattice parameters

sites	4096
length	80 GeV^{-1}
spacing	0.0195 GeV^{-1}
time spacing	0.001 GeV^{-1}

evolution

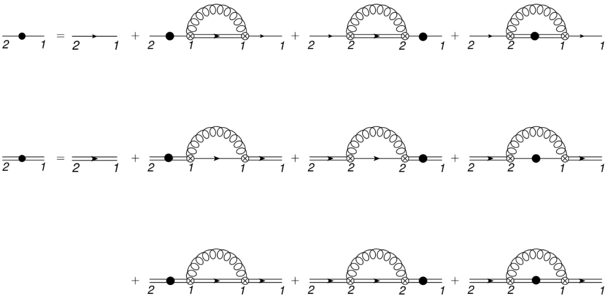
vacuum: $0 < t < 0.6$ fm
medium: 0.6 fm until $T = T_f = 250$ MeV (LO) or $T_f = 190$ MeV (NLO)

Evolution Equations

coupled set of evolution equations:

$$\frac{\mathrm{d}\rho_s(t)}{\mathrm{d}t} = -\, i\,[h_s, \rho_s(t)] - \Sigma_s \rho_s(t) - \rho_s(t)\Sigma_s^\dagger + \Xi_{so}(\rho_o(t))$$

$$\frac{\mathrm{d}\rho_o(t)}{\mathrm{d}t} = -\, i\,[h_o, \rho_o(t)] - \Sigma_o \rho_o(t) - \rho_o(t)\Sigma_o^\dagger + \Xi_{os}(\rho_s(t)) + \Xi_{oo}(\rho_o(t))$$



Phys. Rev. D 97 (2018) 7, 074009 (Brambilla, Escobedo, Soto, Vairo)

Master Equation I

evolution equations can be rewritten as master equation

$$\frac{\mathrm{d}\rho(t)}{\mathrm{d}t} = -i[H,\rho(t)] + \sum_{n,m} h_{nm}\left(L_i^n \rho(t) L_i^{m\dagger} - \frac{1}{2}\left\{L_i^{m\dagger} L_i^n, \rho(t)\right\}\right),$$

H encodes **corrections to mass/energy**

$$\rho(t) = \begin{pmatrix} \rho_s(t) & 0 \\ 0 & \rho_o(t) \end{pmatrix}, \quad H = \begin{pmatrix} h_s + \mathrm{Im}(\Sigma_s) & 0 \\ 0 & h_o + \mathrm{Im}(\Sigma_o) \end{pmatrix}$$

Master Equation II

evolution equations can be rewritten as master equation

$$\frac{\mathrm{d}\rho(t)}{\mathrm{d}t} = -i[H, \rho(t)] + \sum_{n,m} h_{nm} \left(L_i^n \rho(t) L_i^{m\dagger} - \frac{1}{2} \left\{ L_i^{m\dagger} L_i^n, \rho(t) \right\} \right)$$

L_i^n encode **rates of medium interaction**

$$h_{nm} = \mathrm{diag}(\sigma_1, \sigma_1, \sigma_1), \quad \sigma_1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

$$L_i^0 = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} r_i, \quad L_i^2 = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}, \quad L_i^4 = \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix}$$

$$L_i^1 \propto \begin{pmatrix} 0 & 0 \\ 0 & A_i^{oo\dagger} \end{pmatrix}, \quad L_i^3 \propto \begin{pmatrix} 0 & A_i^{os\dagger} \\ 0 & 0 \end{pmatrix}, \quad L_i^5 \propto \begin{pmatrix} 0 & 0 \\ A_i^{so\dagger} & 0 \end{pmatrix}$$

$$\boxed{L_i^n \propto A_i^{uv} = \frac{g^2}{6N_c} \int_0^\infty \mathrm{d}t\, e^{-ih_u t} r^i e^{ih_v t} D_{uv}^{>\dagger}(t)}$$

H_{eff} Evolution

- evolve wavefunction with H_{eff}

$$|\psi(t+\delta t)\rangle = (1 - iH_{eff}\delta t)|\psi(t)\rangle$$

- H_{eff} evolution preserves quantum numbers of the state and decreases its norm

$$\begin{aligned}\langle\psi(t+\delta t)|\psi(t+\delta t)\rangle &\approx 1 - i\langle\psi(t)|(H_{eff} - H_{eff}^{\dagger})|\psi(t)\rangle\delta t \\ &= 1 - \delta p\end{aligned}$$

  where

$$\delta p = \sum_n \langle\psi(t)|C_n^{\dagger}C_n|\psi(t)\rangle\delta t = \sum_n \delta p_n$$

- decrease in norm related to probability a change of quantum numbers, implemented by $C_n|\psi(t)\rangle$, occurs

Monte Carlo

(normalized) evolution of state

$$|\tilde{\psi}(t+\delta t)\rangle = \begin{cases} \frac{|\psi(t+\delta t)\rangle}{\sqrt{1-\delta p}} & \text{with probability } 1-\delta p \\ \frac{C_n|\psi(t)\rangle}{\sqrt{\delta p_n/\delta t}} & \text{with probability } \delta p \end{cases}$$

i.e., with probability $1-\delta p$, the state evolves as governed by H_{eff}, and with probability δp, is acted on by the collapse operator C_n

simulation

- generate a random number $0 < r_1 < 1$
- evolve state with H_{eff} until norm squared $< r_1$
- generate additional random number(s) to determine which collapse operator C_n to apply

Equivalence of Evolution and Convergence

equivalence of evolution

$$
\begin{aligned}
\rho(t+\delta t) =& (1-\delta p)\frac{|\psi(t+\delta t)\rangle}{\sqrt{1-\delta p}}\frac{\langle\psi(t+\delta t)|}{\sqrt{1-\delta p}} \\
&+ \delta p \sum_n \frac{\delta p_n}{\delta p}\frac{C_n|\psi(t)\rangle}{\sqrt{\delta p_n/\delta t}}\frac{\langle\psi(t)|C_n^\dagger}{\sqrt{\delta p_n/\delta t}} \\
=& \rho(t) - i[H_{eff}\rho(t) - \rho(t)H_{eff}^\dagger]\delta t + \sum_n C_n\rho(t)C_n^\dagger\delta t,
\end{aligned}
$$

as given by Lindblad equation

convergence

- calculate expectation values using evolved state
- evolve many states and average to converge to result of directly solving the Lindblad equation

Singlet Density Matrix

Singlet sector:

density matrix is proportional to exponential of Hamiltonian from t_0 to $t_0 - i\beta$

$$\rho_E = Z_s^{-1} \exp\left[-i \int_{t_0}^{t_0 - i\beta} dt\, H_\ell(t)\right]$$

trace normalized to unity

$$Z_s = \mathrm{Tr}\left\{\exp\left[-i \int_{t_0}^{t_0 - i\beta} dt\, H_\ell(t)\right]\right\}$$

this defines in-medium expectation values

$$\langle \mathcal{O} \rangle = \mathrm{Tr}\left[\mathcal{O}\, \rho_E\right]$$

Octet Density Matrix

Octet sector:

density matrix is proportional to exponential of Hamiltonian from t_0 to $t_0 - i\beta$

$$\begin{aligned}\rho_E^{ab} =& Z_o^{-1}\exp\left[-i\int_{t_0}^{t_0-i\beta} dt\ \left(H_\ell(t) - g T^c_{\mathrm{adj}} A_0^c(t)\right)\right]^{ab} \\ =& Z_o^{-1} Z_s\, \rho_E\, U^{ab}(t_0 - i\beta, t_0)\end{aligned}$$

where $T^c_{\mathrm{adj}} = i f^{abc}$ and U^{ab} is a Wilson line in the adjoint representation

$$U^{ab}(t_0, t_0 - i\beta) = \exp\left[-ig\int_{t_0}^{t_0-i\beta} dt\, A_0^c(t)\, T^c_{\mathrm{adj}}\right]^{ab}$$

trace normalized to unity

$$Z_o = \langle U^{aa}(t_0 - i\beta, t_0)\rangle\, Z_s$$

Expectation values of Octet Operators

How to take the expectation value of a non-gauge-invariant operator $\mathcal{O}^{ab}$?

expectation value well defined for the operator $O^{a\dagger}\mathcal{O}^{ab}O^{b}$

$$\mathrm{Tr}\left[O^{a\dagger}(\mathbf{r}')\mathcal{O}^{ab}O^{b}(\mathbf{r})\rho\right] = \rho_{o}(\mathbf{r},\mathbf{r}')\,\mathrm{Tr}\left[\mathcal{O}^{ab}\rho_{E}^{ba}\right]$$

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plugging in definition of ρ_{E}^{ba}

$$\mathrm{Tr}\left[\mathcal{O}^{ab}\rho_{E}^{ba}\right] = \left\langle U^{bc}(t_0 - i\beta, t_0)\mathcal{O}^{cb}\right\rangle \left\langle U^{aa}(t_0 - i\beta, t_0)\right\rangle^{-1}$$

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expectation values of octet operators $\mathcal{O}^{ab}$ contain a Wilson line $U^{ba}(t_0 - i\beta, t_0)$ (ensuring gauge invariance) and are normalized by expectation value of Polyakov loop expectation