Open quantum system approach to the transverse momentum broadening of a colour dipole

Pietro Benzoni (Subatech, Nantes)

based on

F. Arleo, PB, P. Caucal, P. B. Gossiaux, 2606.07744

15th July 2026, EMMI Workshop: Probing the early stages with jets

Jets as QGP probes

- Study of **jet interaction with the QCD medium (QGP)**:
  - BDMPS-Z effect: momentum broadening and **medium-induced radiation**
  - **colour (de)coherence** → (anti-)angular ordering [Chiara’s talk Tuesday]

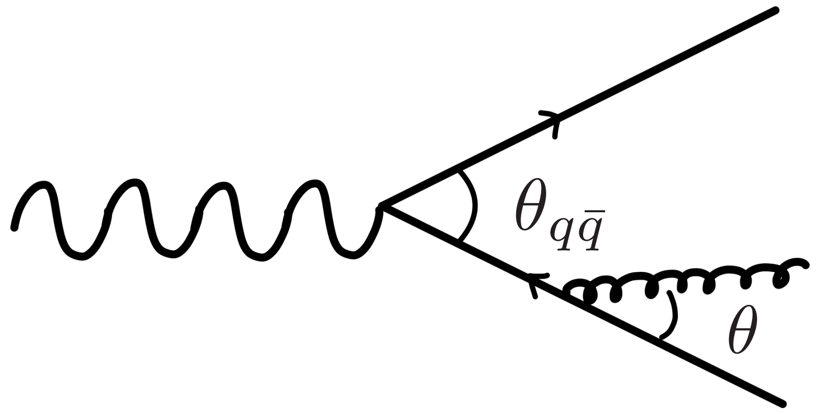
Jets as QGP probes

- Study of **jet interaction with the QCD medium (QGP)**:
  - BDMPS-Z effect: momentum broadening and **medium-induced radiation**
  - **colour (de)coherence** → (anti-)angular ordering [Chiara's talk Tuesday]

Colour coherence (vacuum)

Colour decoherence (in-medium)

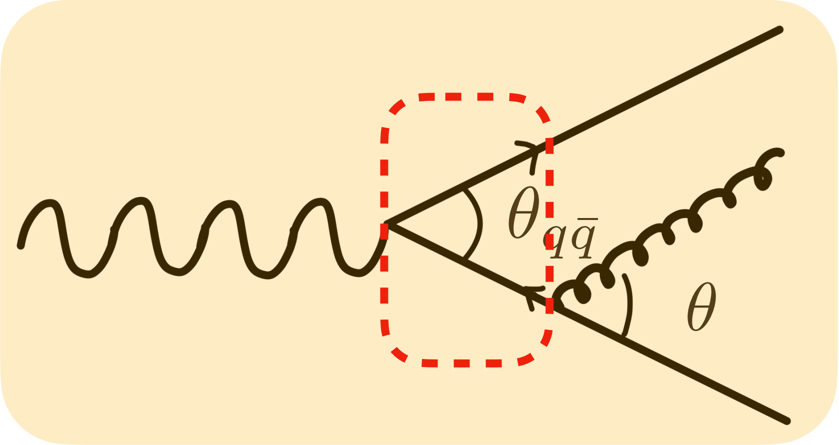
Soft (low energy) gluon radiation:



$\theta < \theta_{q\bar{q}}$

otherwise, the gluon
resolves the full $q\bar{q}$ dipole
⟶ **suppressed**

Dokshitzer, Khoze, Mueller, Troyan (1991)



radiation **also** for $\theta > \theta_{q\bar{q}}$

⟶ **enhancement** of
large angle radiation

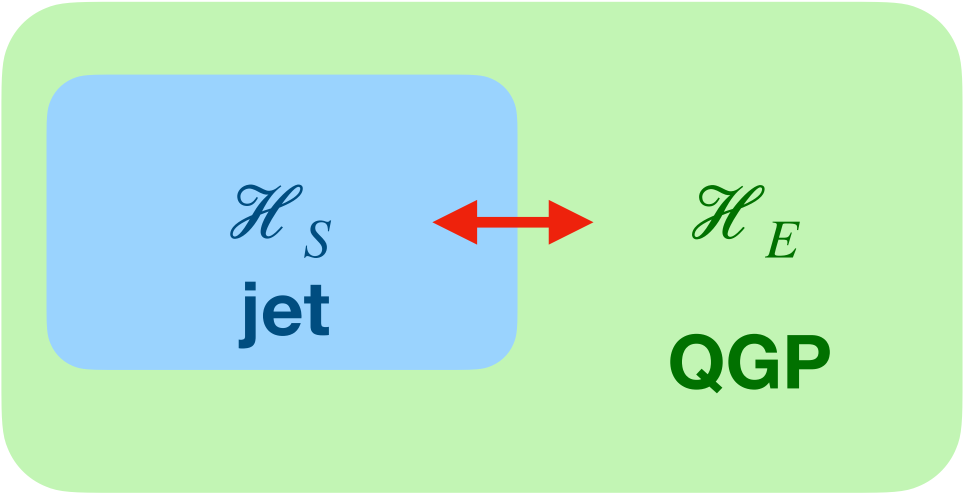
Mehtar-Tani, Salgado, Tywoniuk (2011-2013)
Casalderrey-Solana, Iancu (2011)

the dipole survival probability plays a role in the radiation spectrum

⟶ let's study the colour dipole in a framework where decoherence is naturally encoded

Jets as Open Quantum Systems

- We can study these phenomena with a suitable framework: **open quantum systems**!



$\mathscr{H}_S$ ↔ $\mathscr{H}_E$

jet

QGP

$$\rho = \rho_S \otimes \rho_E$$

- Coupling with an environment modifies the subsystem and “steals” quantum correlations from it

→ quantify **momentum broadening** and **decoherence**

e.g. spin state decoherence

$$|\psi\rangle = \frac{|\uparrow\rangle + |\downarrow\rangle}{\sqrt{2}}$$

$$\rho = |\psi\rangle \langle\psi|$$

probabilities

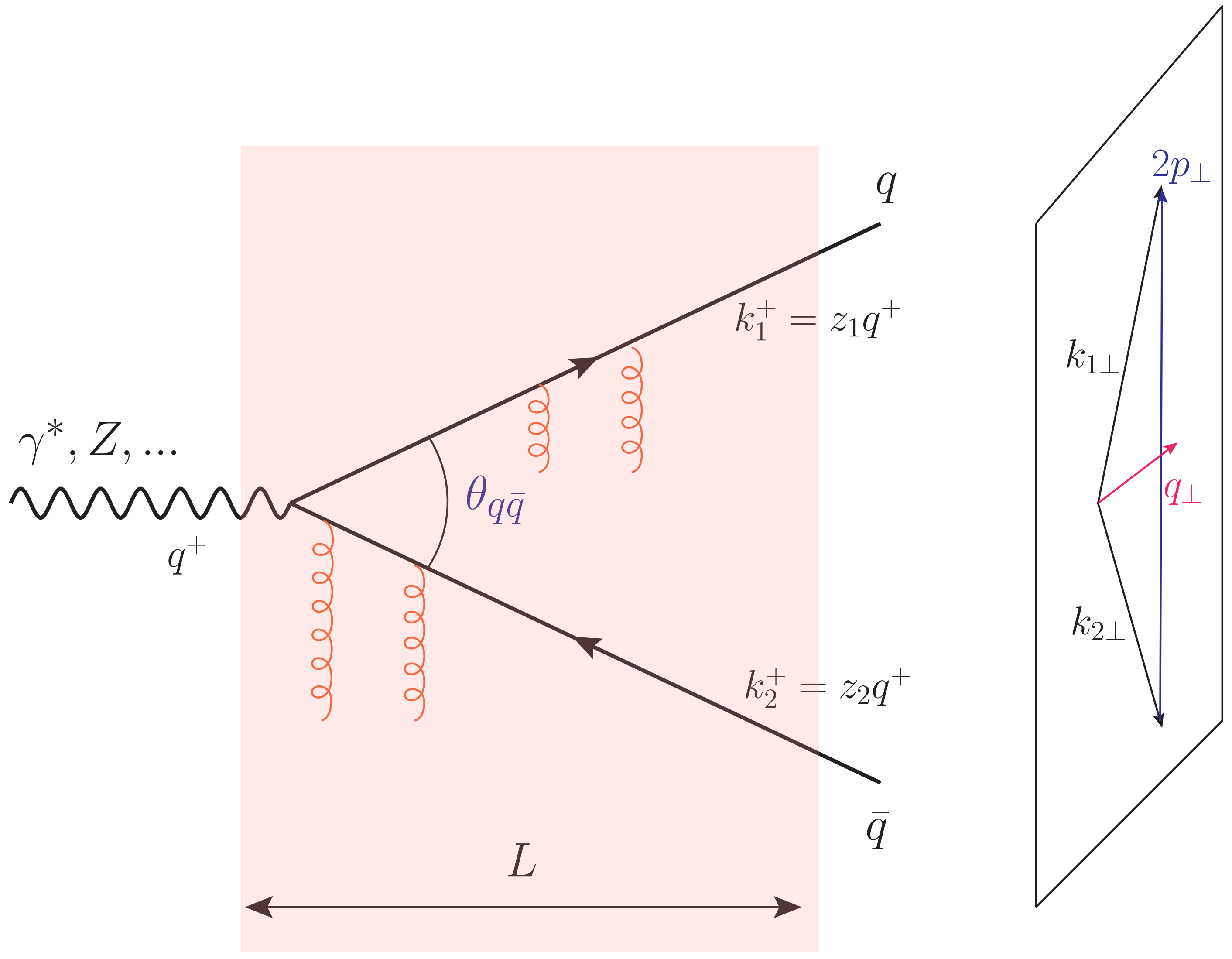
coherences

$$\rho(t=0) = \frac{1}{2} \begin{array}{c} \begin{array}{cc} |\uparrow\rangle & |\downarrow\rangle \end{array} \\ \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix} \end{array} \begin{array}{c} \langle\uparrow| \\ \langle\downarrow| \end{array} \xrightarrow[\text{stochastic magnetic field}]{} \rho(t \to \infty) = \frac{1}{2} \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

A simple jet: a $q\bar{q}$ dipole

- As a **first step** we can concentrate on the case of a boosted $q\bar{q}$ **dipole**

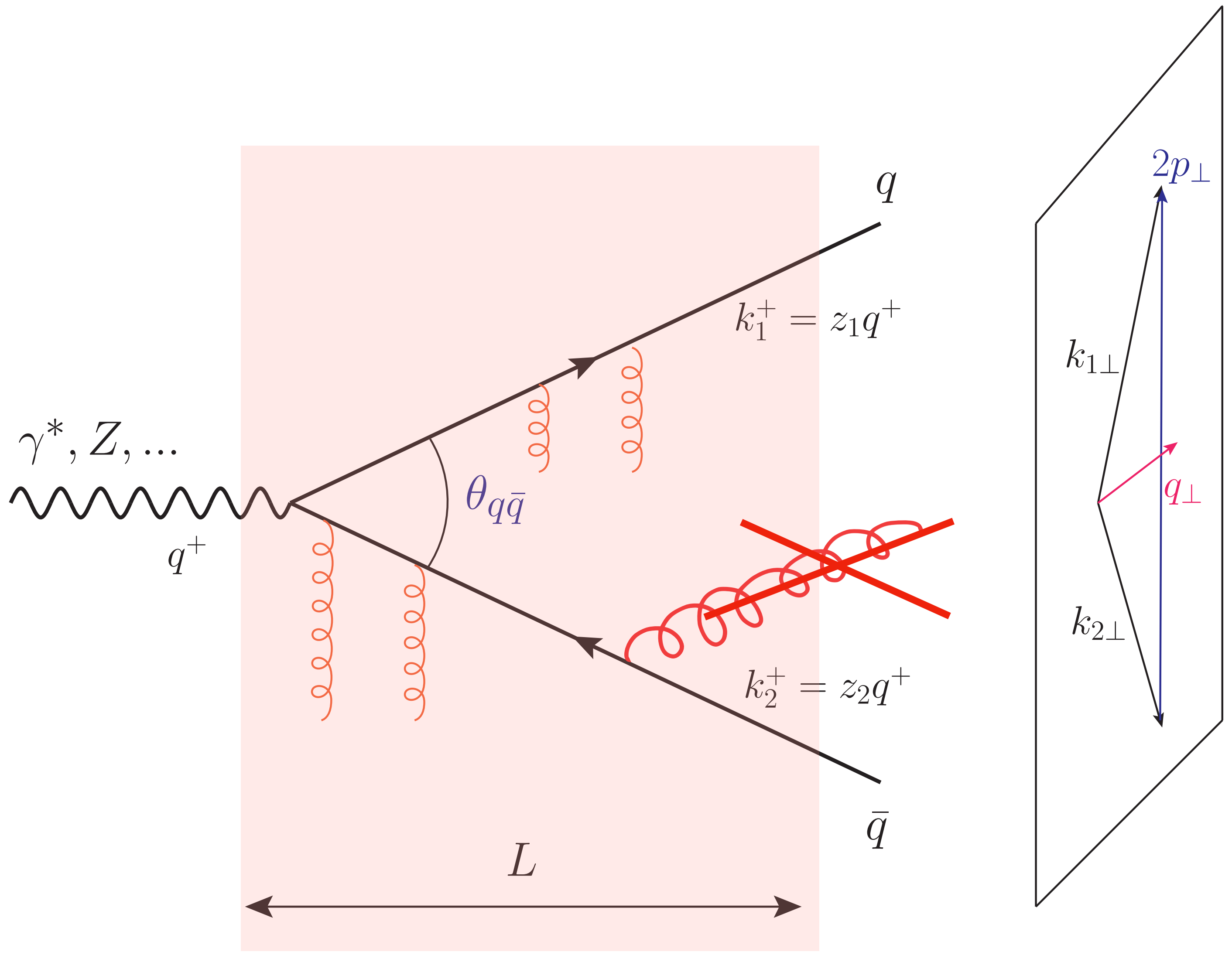
  in a dense QCD medium (QGP)



A simple jet: a $q\bar{q}$ dipole

- As a **first step** we can concentrate on the case of a boosted $q\bar{q}$ **dipole**

  in a dense QCD medium (QGP)

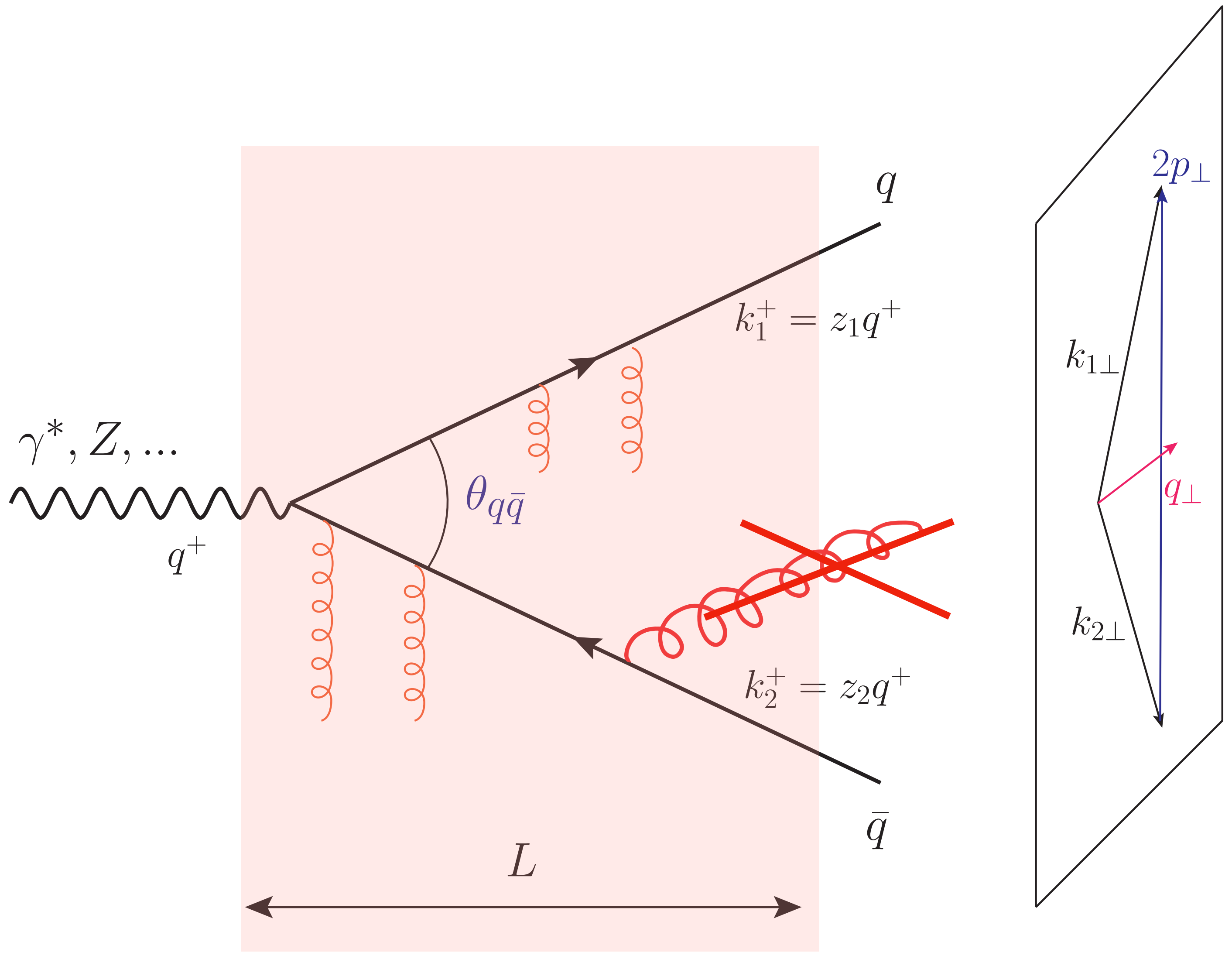


No radiation emitted from the dipole (yet)

A simple jet: a $q\bar{q}$ dipole

- As a **first step** we can concentrate on the case of a boosted $q\bar{q}$ **dipole**

  in a dense QCD medium (QGP)



No radiation emitted from the dipole (yet)

- Learning from…

[Peter's talk Friday]

  - **Heavy quark-antiquark** (quarkonium) OQS

    Blaizot, Escobedo (2018)

  - **Single quark jet** OQS

    Barata, Blaizot, Mehtar-Tani (2023)


heavy mass

high energy

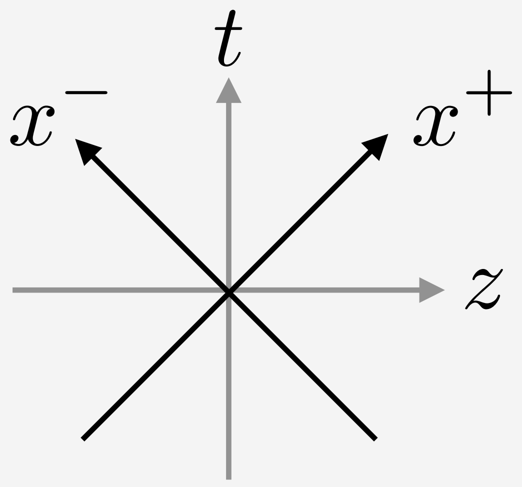
non-relativistic description

NRQCD

in 3+1 dimensions

in 2+1 dimensions

Light-cone coordinates:



$$x^+ = \frac{t+z}{\sqrt{2}} \propto t \simeq z$$

$$x^- = \frac{t-z}{\sqrt{2}} \simeq 0$$

A simple jet: a $q\bar{q}$ dipole

- We consider a kinematical configuration which satisfies the following hierarchy:

$$\frac{1}{E} = \frac{1}{k_1^+} + \frac{1}{k_1^+}$$
$$q^+ = k_1^+ + k_2^+$$

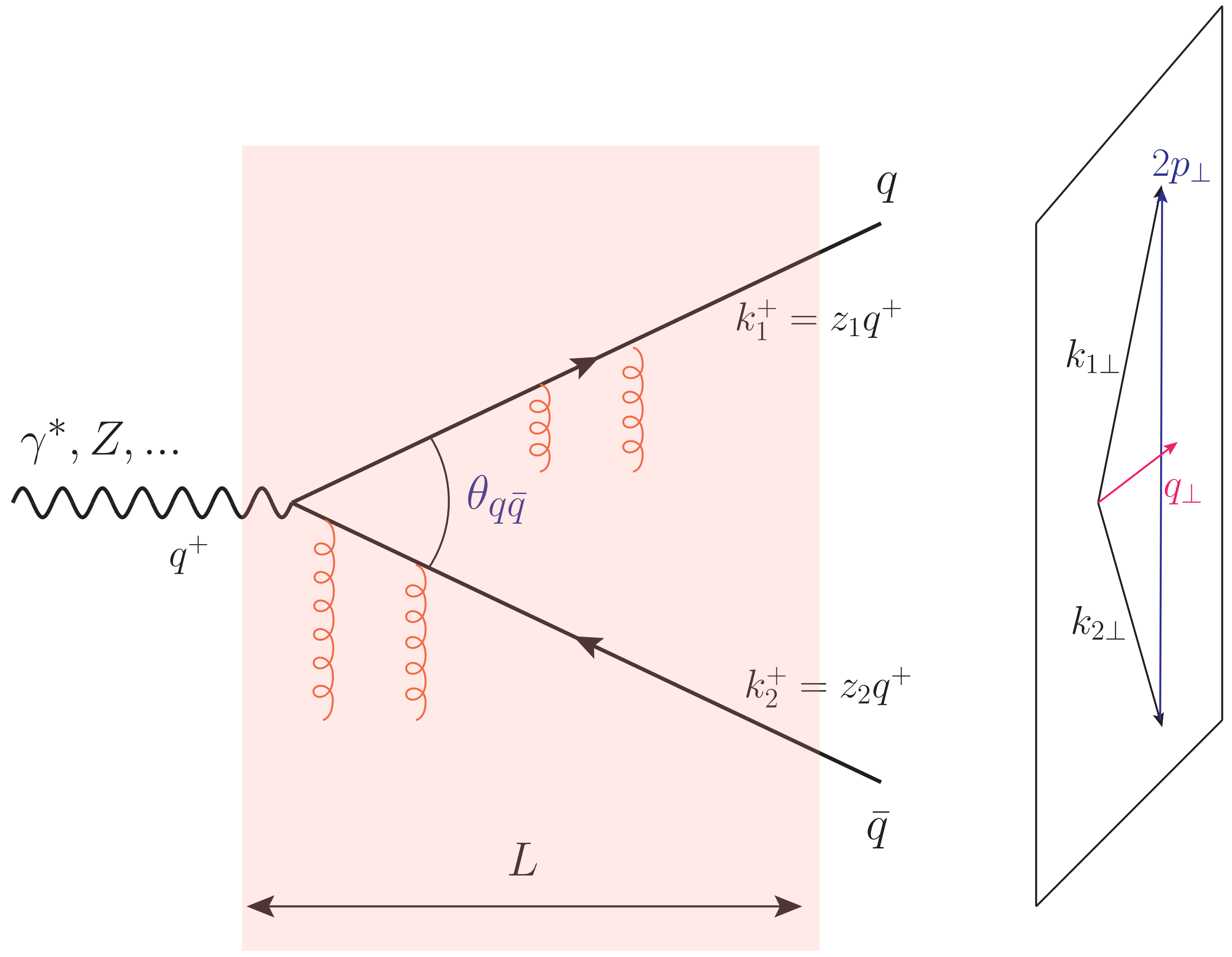
boosted ⌐ back-to-back

$$E \sim q^+ \gg \boldsymbol{p}_\perp \gg \boldsymbol{q}_\perp \sim Q_s = \sqrt{\hat{q}L}$$

longitudinal momentum

relative transverse momentum

center-of-mass transverse momentum



⟶ meaningful for (e.g. LHC) phenomenology: $\theta_{q\bar{q}}^2 = \frac{2\boldsymbol{p}_\perp^2}{E^2} \lesssim 0.1 \div 0.4$

$E \sim 100 \div 1000$ GeV

$\boldsymbol{p}_\perp \gtrsim 5 \div 50$ GeV

$Q_s \sim 5$ GeV

- Dipole production: gluons (octet), vector bosons e.g. γ^*, W, … (singlet)

Derivation of the master equation

- **Hamiltonian** in 2D transverse space:

$$H = \underbrace{\left(\frac{\hat{p}^2_{\perp,q}}{2k_1^+} + \frac{\hat{p}^2_{\perp,\bar{q}}}{2k_2^+}\right) \otimes \mathbb{1}_E}_{\text{kinetic term}} - \underbrace{g \int \mathrm{d}^2\boldsymbol{x}_\perp \; \hat{n}^a(\boldsymbol{x}_\perp) \otimes A_a^-(t,\boldsymbol{x}_\perp)}_{\text{interaction}} + \underbrace{\mathbb{1}_S \otimes H_{\mathrm{pl}}}_{\text{QGP}}$$

number density: $$\hat{n}^a(\boldsymbol{x}_\perp) = \underbrace{\delta(\boldsymbol{x}_\perp - \hat{x}_{\perp,q})t^a \otimes \mathbb{1}_{\bar{q}}}_{\text{quark}} - \underbrace{\mathbb{1}_q \otimes \delta(\boldsymbol{x}_\perp - \hat{x}_{\perp,\bar{q}})\tilde{t}^a}_{\text{antiquark}}$$

Derivation of the master equation

- **Hamiltonian** in 2D transverse space:

$$H = \overbrace{\left(\frac{\hat{p}^2_{\perp,q}}{2k_1^+} + \frac{\hat{p}^2_{\perp,\bar{q}}}{2k_2^+}\right) \otimes \mathbb{1}_E}^{\text{kinetic term}} - \overbrace{g \int \mathrm{d}^2\boldsymbol{x}_\perp \; \hat{n}^a(\boldsymbol{x}_\perp) \otimes A_a^-(t, \boldsymbol{x}_\perp)}^{\text{interaction}} + \overbrace{\mathbb{1}_S \otimes H_{\mathrm{pl}}}^{\text{QGP}}$$

number density: $\hat{n}^a(\boldsymbol{x}_\perp) = \underbrace{\delta(\boldsymbol{x}_\perp - \hat{x}_{\perp,q})t^a \otimes \mathbb{1}_{\bar{q}}}_{\text{quark}} - \underbrace{\mathbb{1}_q \otimes \delta(\boldsymbol{x}_\perp - \hat{x}_{\perp,\bar{q}})\tilde{t}^a}_{\text{antiquark}}$

(unitary)

$$\frac{\mathrm{d}\rho}{\mathrm{d}t} = -i\,[H, \rho]$$

Von Neumann's equation

Partial trace over environment

$$\mathrm{Tr}_E\,(\cdot)$$

$$\rho = \underbrace{\rho_S}_{\text{system}} \otimes \underbrace{\cancel{\rho_E}}_{\text{environment}}$$

thermal equilibrium state

$$\rho_E = \frac{1}{Z} e^{-\beta H_{\mathrm{pl}}}$$

(non-unitary)

$$\frac{\mathrm{d}\rho_S}{\mathrm{d}t} = -i\,[H_0, \rho_S] + \mathcal{D}(\rho_S)$$

Linblad's equation

describes the interactions with the medium

→ in the **high-energy limit**: mainly **collisions**

Derivation of the master equation

- **Lindblad's equation:**

$$\frac{\mathrm{d}\rho_S}{\mathrm{d}t} = -i\left[H_0,\rho_S\right] + \mathcal{D}(\rho_S)$$

QGP colour field correlator

$$W(\boldsymbol{r}_\perp) \equiv \mathfrak{Im}(\Delta(\omega=0,\boldsymbol{r}_\perp))$$

$$-i\delta^{ab}\Delta(\omega,\boldsymbol{r}_\perp) = \int_{-\infty}^{\infty}\mathrm{d}t\, e^{i\omega t}\langle T\left(A_a^-(t,\boldsymbol{r}_\perp)A_b^-(0,\boldsymbol{0}_\perp)\right)\rangle_{\rm pl}$$

$$\frac{\mathrm{d}\rho(t)}{\mathrm{d}t} + i[H_0,\rho(t)] = \frac{g^2}{2}\int_{\boldsymbol{x}_\perp,\boldsymbol{x}'_\perp}\left(\{\hat{n}^a(\boldsymbol{x}_\perp)\hat{n}^a(\boldsymbol{x}'_\perp),\rho(t)\} - 2\hat{n}^a(\boldsymbol{x}'_\perp)\rho(t)\hat{n}^a(\boldsymbol{x}_\perp)\right) W(\boldsymbol{x}_\perp-\boldsymbol{x}'_\perp) +$$

$$+\ \cancel{\text{friction term}}\ + \dots$$

boosted regime

number density: $\hat{n}^a(\boldsymbol{x}_\perp) = \underbrace{\delta(\boldsymbol{x}_\perp - \hat{x}_{\perp,q})t^a \otimes \mathbb{1}_{\bar{q}}}_{\text{quark}} - \underbrace{\mathbb{1}_q \otimes \delta(\boldsymbol{x}_\perp - \hat{x}_{\perp,\bar{q}})\tilde{t}^a}_{\text{antiquark}}$

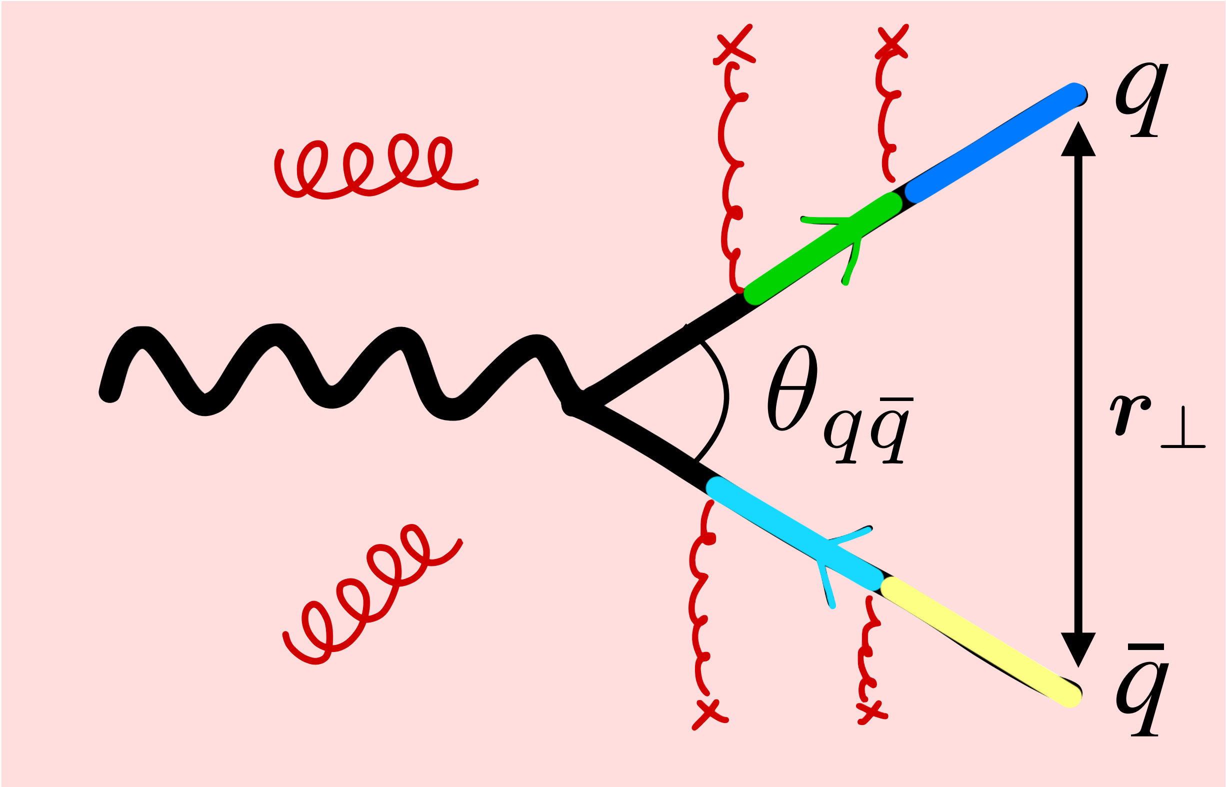
- We recover a *quarkonia*-like (2D) master equation

Colour structure

$$\rho = \rho_s \left|s\right\rangle \left\langle s\right| + \rho_o \sum_{c=1}^{N_c^2-1} \left|o^c\right\rangle \left\langle o^c\right|$$

singlet octet

local $\Longrightarrow$ global colour symmetry:
no singlet – octet interference $\left|s\right\rangle \left\langle o^c\right|$



ρ_s

$\Updownarrow$

ρ_o

$$\theta_c = \frac{2}{\sqrt{\hat{q}L^3}}$$

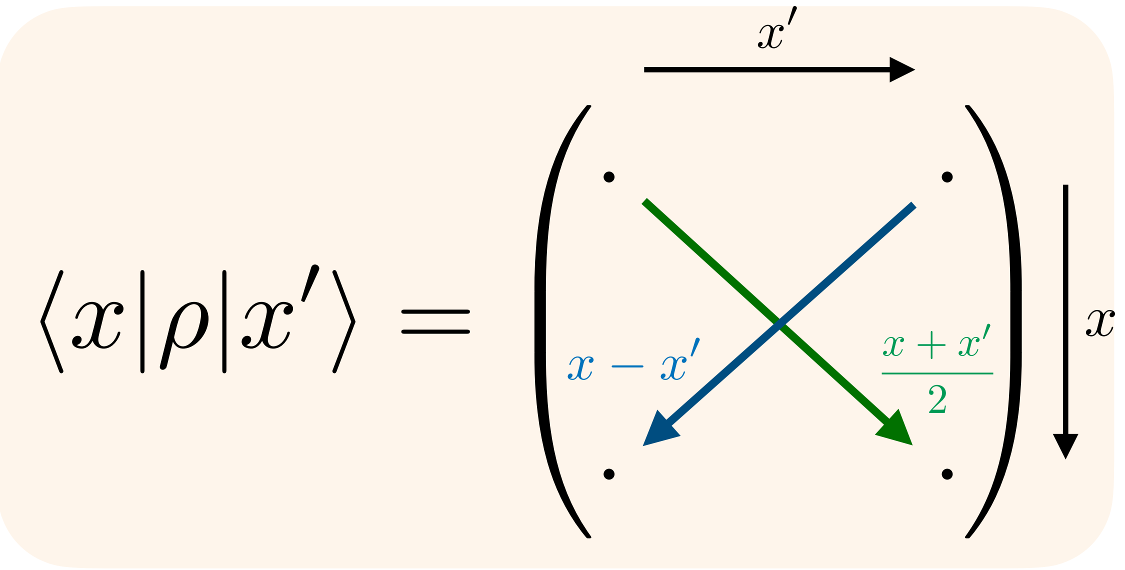
resolution angle
of the medium

Derivation of the master equation

Wigner function and variables

- We want to study the evolution of the matrix elements

$$\langle \boldsymbol{r}_{1\perp}\boldsymbol{r}_{2\perp}|\rho|{\boldsymbol{r}_{1\perp}}'{\boldsymbol{r}_{2\perp}}'\rangle = \rho(\boldsymbol{r}_{1\perp}\boldsymbol{r}_{2\perp};{\boldsymbol{r}_{1\perp}}'{\boldsymbol{r}_{2\perp}}')$$



- Introduce new variables

	classical	quantum		classical	quantum
relative	$\boldsymbol{r}_\perp = \dfrac{\boldsymbol{r}_{1\perp} - \boldsymbol{r}_{2\perp} + (')}{2}$	$\boldsymbol{\mathcal{R}}_\perp = \boldsymbol{r}_{1\perp} - \boldsymbol{r}_{2\perp} - (')$	◀·····▶	$\boldsymbol{p}_\perp$	$\boldsymbol{\mathcal{P}}_\perp$
	relative position		Fourier-conjugate	relative momentum	
CoM	$\boldsymbol{b}_\perp$	$\boldsymbol{\mathcal{B}}_\perp$	◀·····▶	$\boldsymbol{q}_\perp$	$\boldsymbol{\mathcal{Q}}_\perp$
	impact parameter			momentum imbalance	

- Wigner function:

$$\rho(\boldsymbol{r}_\perp,\boldsymbol{\mathcal{R}}_\perp,\boldsymbol{b}_\perp,\boldsymbol{\mathcal{B}}_\perp) \xrightarrow{\text{partial Fourier transforms}} \boxed{\rho(\boldsymbol{r}_\perp,\boldsymbol{p}_\perp,\boldsymbol{b}_\perp,\boldsymbol{q}_\perp)}$$

Derivation of the master equation

$$\frac{1}{E} = \frac{1}{k_1^+} + \frac{1}{k_1^+}$$

$$q^+ = k_1^+ + k_2^+$$

- Master equation

$$\frac{\partial}{\partial t}\begin{pmatrix}\rho_s \\ \rho_o\end{pmatrix} - i\left(\frac{1}{E}\nabla_{\boldsymbol{r}_\perp}\cdot\nabla_{\boldsymbol{\mathcal{R}}_\perp} + \frac{1}{q^+}\nabla_{\boldsymbol{b}_\perp}\cdot\nabla_{\boldsymbol{\mathcal{B}}_\perp}\right)\begin{pmatrix}\rho_s \\ \rho_o\end{pmatrix} = g^2\begin{pmatrix}-C_F\Gamma_c & C_F\left(\Gamma_b - \Gamma_a\right) \\ \frac{\Gamma_b-\Gamma_a}{2N_c} & -C_F\Gamma_a + \frac{\Gamma_a+\Gamma_c-2\Gamma_b}{2N_c}\end{pmatrix}\begin{pmatrix}\rho_s \\ \rho_o\end{pmatrix}$$

with $\Gamma_i \equiv W_i - W(\mathbf{0}_\perp)$ and $W_a \equiv W_{11'} + W_{22'}$ $W_b \equiv W_{21'} + W_{12'}$ $W_c \equiv W_{12} + W_{1'2'}$

Derivation of the master equation

$$\frac{1}{E} = \frac{1}{k_1^+} + \frac{1}{k_1^+}$$

$$q^+ = k_1^+ + k_2^+$$

- Master equation

$$\frac{\partial}{\partial t}\begin{pmatrix}\rho_s\\ \rho_o\end{pmatrix} - i\left(\frac{1}{E}\nabla_{\boldsymbol{r}_\perp}\cdot\nabla_{\boldsymbol{\mathcal{R}}_\perp} + \frac{1}{q^+}\nabla_{\boldsymbol{b}_\perp}\cdot\nabla_{\boldsymbol{\mathcal{B}}_\perp}\right)\begin{pmatrix}\rho_s\\ \rho_o\end{pmatrix} = g^2\begin{pmatrix} -C_F\Gamma_c & C_F\left(\Gamma_b-\Gamma_a\right)\\ \frac{\Gamma_b-\Gamma_a}{2N_c} & -C_F\Gamma_a+\frac{\Gamma_a+\Gamma_c-2\Gamma_b}{2N_c}\end{pmatrix}\begin{pmatrix}\rho_s\\ \rho_o\end{pmatrix}$$

with $\Gamma_i \equiv W_i - W(\mathbf{0}_\perp)$ and $W_a \equiv W_{11'} + W_{22'}$ $W_b \equiv W_{21'} + W_{12'}$ $W_c \equiv W_{12} + W_{1'2'}$

- **Harmonic approximation** of the potential:

$\Gamma = \Gamma(\boldsymbol{r}_\perp, \boldsymbol{b}_\perp, \boldsymbol{\mathcal{R}}_\perp, \boldsymbol{\mathcal{B}}_\perp)$ → harmonic approximation $\xrightarrow{\text{Wigner transform}}$ $\propto \hat{q}\,\hat{\mathcal{O}}\left[\boldsymbol{r}_\perp^2, \nabla^2_{\boldsymbol{p}_\perp}, \nabla^2_{\boldsymbol{q}_\perp}\right]$

dependence cancels

(Thermal field theory ard Hard Thermal Loop resummation)

jet quenching parameter

$\boldsymbol{r}_\perp$ relative position

$\boldsymbol{b}_\perp$ impact parameter

$\boldsymbol{p}_\perp$ relative } transverse momentum

$\boldsymbol{q}_\perp$ center-of-mass

Derivation of the master equation

$$\frac{1}{E}=\frac{1}{k_1^+}+\frac{1}{k_1^+}$$

$$q^+=k_1^++k_2^+$$

- Master equation

$$\frac{\partial}{\partial t}\begin{pmatrix}\rho_s\\ \rho_o\end{pmatrix}-i\left(\frac{1}{E}\nabla_{\boldsymbol{r}_\perp}\cdot\nabla_{\boldsymbol{\mathcal{R}}_\perp}+\frac{1}{q^+}\nabla_{\boldsymbol{b}_\perp}\cdot\nabla_{\boldsymbol{\mathcal{B}}_\perp}\right)\begin{pmatrix}\rho_s\\ \rho_o\end{pmatrix}=g^2\begin{pmatrix}-C_F\Gamma_c & C_F\left(\Gamma_b-\Gamma_a\right)\\ \frac{\Gamma_b-\Gamma_a}{2N_c} & -C_F\Gamma_a+\frac{\Gamma_a+\Gamma_c-2\Gamma_b}{2N_c}\end{pmatrix}\begin{pmatrix}\rho_s\\ \rho_o\end{pmatrix}$$

with $\Gamma_i\equiv W_i-W(\boldsymbol{0}_\perp)$ and $W_a\equiv W_{11'}+W_{22'}$ $W_b\equiv W_{21'}+W_{12'}$ $W_c\equiv W_{12}+W_{1'2'}$

- **Harmonic approximation** of the potential:

$\Gamma=\Gamma(\boldsymbol{r}_\perp,\boldsymbol{b}_\perp,\boldsymbol{\mathcal{R}}_\perp,\boldsymbol{\mathcal{B}}_\perp)$ ⟶ harmonic approximation $\xrightarrow{\text{Wigner transform}}$ $\propto\hat{q}\,\hat{\mathcal{O}}\left[\boldsymbol{r}_\perp^2,\nabla_{\boldsymbol{p}_\perp}^2,\nabla_{\boldsymbol{q}_\perp}^2\right]$

dependence cancels

(Thermal field theory ard Hard Thermal Loop resummation)

jet quenching parameter

$\boldsymbol{r}_\perp$ relative position

$\boldsymbol{b}_\perp$ impact parameter

$\boldsymbol{p}_\perp$ relative } transverse momentum

$\boldsymbol{q}_\perp$ center-of-mass

- Symmetric splitting $k_1^+=k_2^+$, drops mixed operators $\nabla_{\boldsymbol{q}_\perp}\cdot\nabla_{\boldsymbol{p}_\perp}$

Master equation

Colour factors:

N_c # of colours

$C_F = \frac{N_c^2 - 1}{2N_c}$

- We have a **master equation** for the Wigner function:

$$\frac{\partial}{\partial t}\begin{pmatrix}\rho_s\\ \rho_o\end{pmatrix} + \Bigg(\overset{\text{kinetic term}}{\frac{1}{E}\boldsymbol{p}_\perp\cdot\nabla_{\boldsymbol{r}_\perp} + \frac{1}{q^+}\boldsymbol{q}_\perp\cdot\nabla_{\boldsymbol{b}_\perp}}\Bigg)\begin{pmatrix}\rho_s\\ \rho_o\end{pmatrix} = \frac{\overset{\text{jet quenching parameter}}{\hat{q}}}{4C_F}\Bigg\{\overset{\text{CoM diffusion}}{\begin{pmatrix}0 & 0\\ 0 & N_c\end{pmatrix}\nabla^2_{\boldsymbol{q}_\perp}} + \overset{\text{colour transitions}}{\begin{pmatrix}-2C_F & 2C_F\\ \frac{1}{N_c} & -\frac{1}{N_c}\end{pmatrix}\boldsymbol{r}^2_\perp}$$

$$+ \underset{\text{relative diffusion}}{\begin{pmatrix}\frac{C_F}{2} & \frac{C_F}{2}\\ \frac{1}{4N_c} & \frac{C_F}{2} - \frac{1}{2N_c}\end{pmatrix}\nabla^2_{\boldsymbol{p}_\perp}}\Bigg\}\begin{pmatrix}\rho_s\\ \rho_o\end{pmatrix}$$

$$\frac{1}{E} = \frac{1}{k_1^+} + \frac{1}{k_1^+}$$

$$q^+ = k_1^+ + k_2^+$$

quantum diffusion:

It causes _diffusion_ in $\boldsymbol{p}_\perp, \boldsymbol{q}_\perp$
and _decoherence_

- A “two species” Fokker-Planck -like equation: any analytical **solutions**?

Master equation

Colour factors:

N_c # of colours

$C_F = \frac{N_c^2 - 1}{2N_c}$

- We have a **master equation** for the Wigner function:

$$\frac{\partial}{\partial t}\begin{pmatrix}\rho_s\\ \rho_o\end{pmatrix} + \overbrace{\left(\frac{1}{E}\boldsymbol{p}_\perp\cdot\nabla_{\boldsymbol{r}_\perp} + \frac{1}{q^+}\boldsymbol{q}_\perp\cdot\nabla_{\boldsymbol{b}_\perp}\right)}^{\text{kinetic term}}\begin{pmatrix}\rho_s\\ \rho_o\end{pmatrix} = \frac{\overset{\text{jet quenching parameter}}{\hat{q}}}{4C_F}\Bigg\{\overbrace{\begin{pmatrix}0 & 0\\ 0 & N_c\end{pmatrix}\nabla^2_{\boldsymbol{q}_\perp}}^{\text{CoM diffusion}} + \overbrace{\begin{pmatrix}-2C_F & 2C_F\\ \frac{1}{N_c} & -\frac{1}{N_c}\end{pmatrix}\boldsymbol{r}^2_\perp}^{\text{colour transitions}}$$

$$+ \underbrace{\begin{pmatrix}\frac{C_F}{2} & \frac{C_F}{2}\\ \frac{1}{4N_c} & \frac{C_F}{2} - \frac{1}{2N_c}\end{pmatrix}\nabla^2_{\boldsymbol{p}_\perp}}_{\text{relative diffusion}}\Bigg\}\begin{pmatrix}\rho_s\\ \rho_o\end{pmatrix}$$

$$\frac{1}{E} = \frac{1}{k_1^+} + \frac{1}{k_1^+}$$

$$q^+ = k_1^+ + k_2^+$$

quantum diffusion:

It causes *diffusion* in $\boldsymbol{p}_\perp, \boldsymbol{q}_\perp$ and *decoherence*

rescaling of variables

→ **relative diffusion** operator **scales** as

$$\kappa = \frac{2\theta_s}{\theta_c} = \frac{Q_s}{p_\perp} \times \frac{\theta_{q\bar{q}}}{\theta_c}$$

small in $E \gg \boldsymbol{p}_\perp \gg \boldsymbol{q}_\perp$ regime

- A "two species" Fokker-Planck -like equation: any analytical **solutions**?

Master equation: colour transitions

- We take the $\mathcal{O}(1)$ in the relative diffusion term $\kappa^2 \nabla^2_{\bar{\boldsymbol{p}}_\perp}$
- We **integrate out** the center-of-mass degrees of freedom $\boldsymbol{q}_\perp$ and $\boldsymbol{b}_\perp$
  ⟶ *quasi*-factorization (see later for the *quasi*)
- Straight-line motion and **colour transitions**:

Colour factors:

N_c # of colours

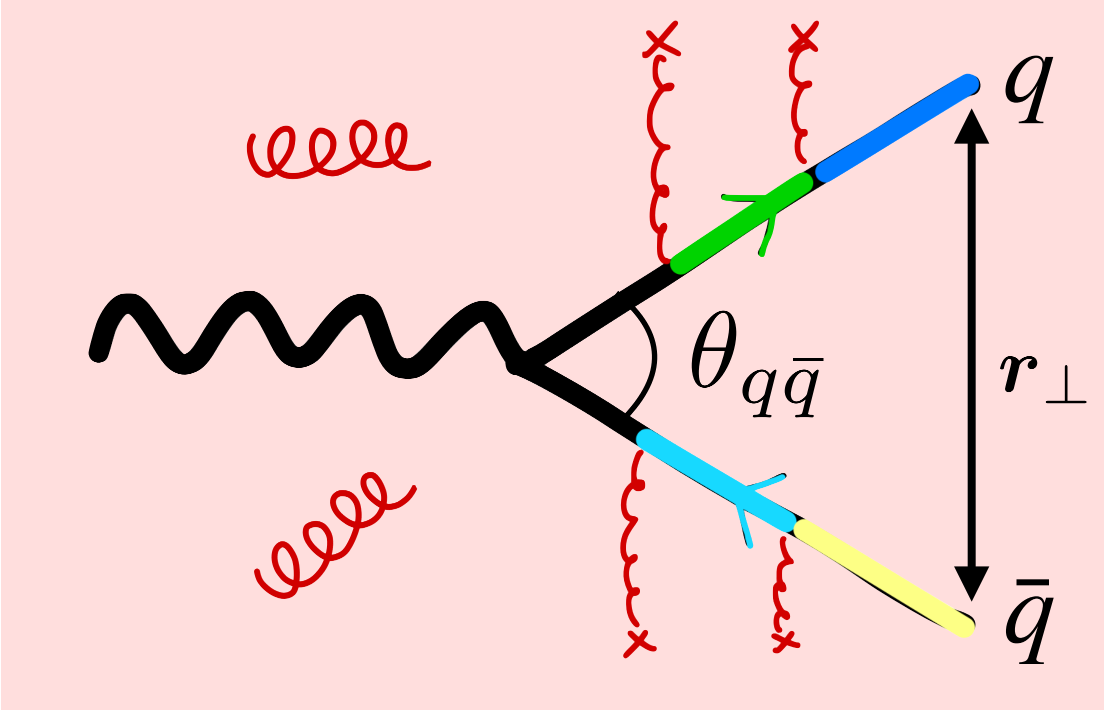
$C_F = \frac{N_c^2 - 1}{2N_c}$

kinetic term **colour transitions**

$$\frac{\partial}{\partial t}\begin{pmatrix}\rho_s \\ \rho_o\end{pmatrix} + \frac{1}{E}\boldsymbol{p}_\perp \cdot \nabla_{\boldsymbol{r}_\perp}\begin{pmatrix}\rho_s \\ \rho_o\end{pmatrix} = \frac{\hat{q}\boldsymbol{r}_\perp^2}{4C_F}\begin{pmatrix}-2C_F & 2C_F \\ \frac{1}{N_c} & -\frac{1}{N_c}\end{pmatrix}\begin{pmatrix}\rho_s \\ \rho_o\end{pmatrix}$$

$$\frac{1}{E} = \frac{1}{k_1^+} + \frac{1}{k_2^+}$$

effective longitudinal momentum



classical rate equations

Solutions: colour transitions

- **Analytical solution** with the method of characteristics: 1st order PDE into an ODE
- **Initial** Wigner function (colour **singlet**):

$$\rho(\boldsymbol{p}_\perp,\boldsymbol{r}_\perp;\boldsymbol{q}_\perp,\boldsymbol{b}_\perp;0) = \underbrace{\rho_R(\boldsymbol{p}_\perp,\boldsymbol{r}_\perp;0)}_{\text{relative}}\underbrace{\rho_T(\boldsymbol{q}_\perp,\boldsymbol{b}_\perp;0)}_{\text{center-of-mass}}$$

hard factor: production of the $q\bar{q}$ pair

$$\rho_R(\boldsymbol{p}_\perp,\boldsymbol{r}_\perp;0) = H(\boldsymbol{p}_\perp)\delta(\boldsymbol{r}_\perp)$$

Solutions: colour transitions

- **Analytical solution** with the method of characteristics: 1st order PDE into an ODE
- **Initial** Wigner function (colour **singlet**):

$$\rho(\boldsymbol{p}_\perp, \boldsymbol{r}_\perp; \boldsymbol{q}_\perp, \boldsymbol{b}_\perp; 0) = \underbrace{\rho_R(\boldsymbol{p}_\perp, \boldsymbol{r}_\perp; 0)}_{\text{relative}}\underbrace{\rho_T(\boldsymbol{q}_\perp, \boldsymbol{b}_\perp; 0)}_{\text{center-of-mass}}$$

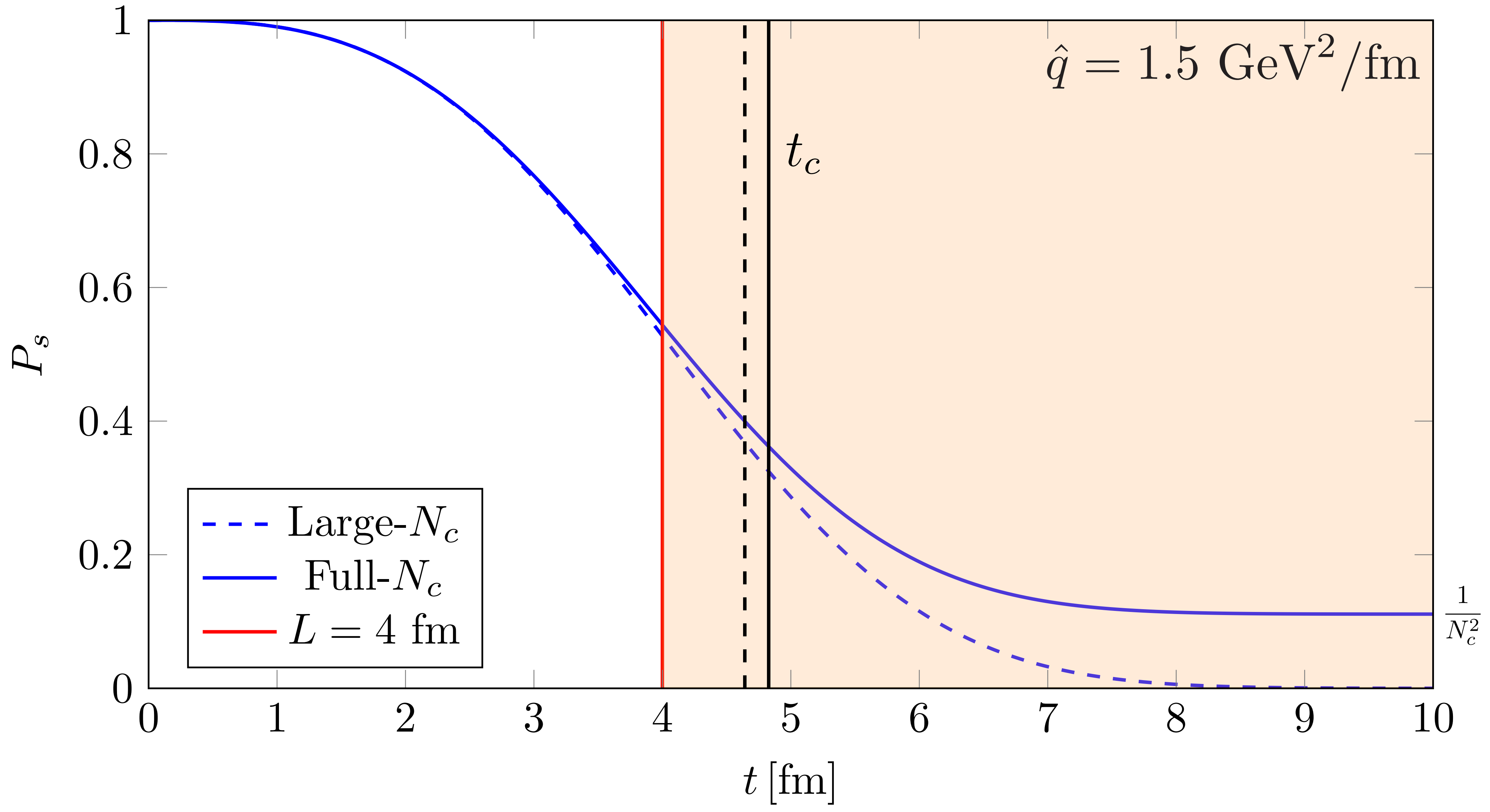
hard factor: production of the $q\bar{q}$ pair

$$\rho_R(\boldsymbol{p}_\perp, \boldsymbol{r}_\perp; 0) = H(\boldsymbol{p}_\perp)\delta(\boldsymbol{r}_\perp)$$

- Singlet **survival probability**:

$$\rho_s(t) = \frac{1}{N_c^2} + \frac{N_c^2-1}{N_c^2}\exp\left(-\frac{N_c \hat{q}_F \theta_{q\bar{q}}^2 t^3}{24 C_F}\right) \xrightarrow{\text{large } N_c} P_s(t) = \exp\left(-\frac{\hat{q}\theta_{q\bar{q}}^2}{12}t^3\right)$$

$$\boxed{t_c = \left(\frac{4}{\hat{q}\theta_{q\bar{q}}^2}\right)^{\frac{1}{3}}} \qquad \theta_{q\bar{q}}^2 = \frac{2\boldsymbol{p}_\perp^2}{E^2}$$



Solutions: colour transitions

- **Analytical solution** with the method of characteristics: 1st order PDE into an ODE
- **Initial** Wigner function (colour **singlet**):

$$\rho(\boldsymbol{p}_\perp, \boldsymbol{r}_\perp; \boldsymbol{q}_\perp, \boldsymbol{b}_\perp; 0) = \underbrace{\rho_R(\boldsymbol{p}_\perp, \boldsymbol{r}_\perp; 0)}_{\text{relative}}\underbrace{\rho_T(\boldsymbol{q}_\perp, \boldsymbol{b}_\perp; 0)}_{\text{center-of-mass}}$$

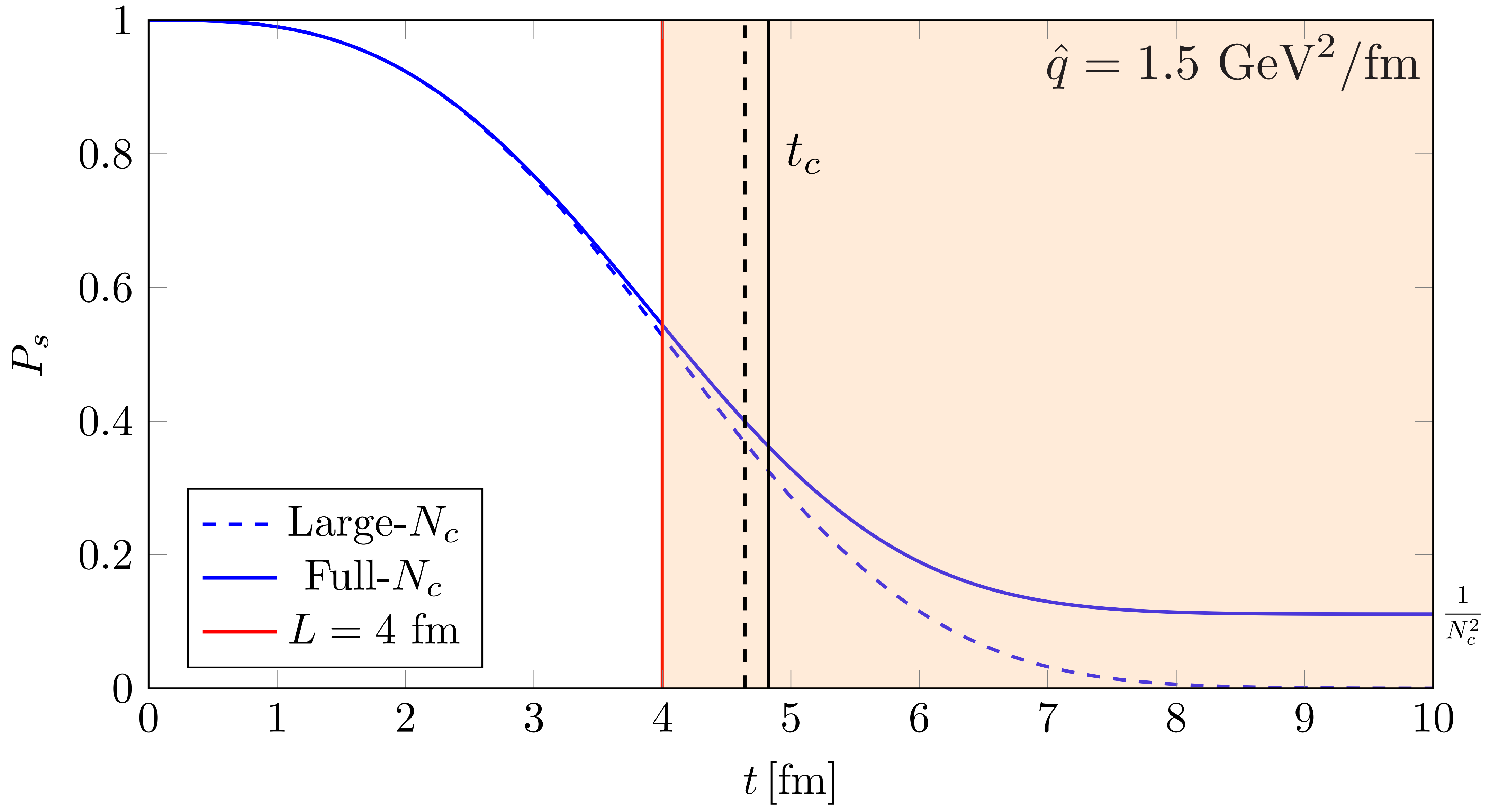
hard factor: production of the $q\bar{q}$ pair

$$\rho_R(\boldsymbol{p}_\perp, \boldsymbol{r}_\perp; 0) = H(\boldsymbol{p}_\perp)\delta(\boldsymbol{r}_\perp)$$

- Singlet **survival probability**:

$$\rho_s(t) = \frac{1}{N_c^2} + \frac{N_c^2 - 1}{N_c^2}\exp\left(-\frac{N_c \hat{q}_F \theta_{q\bar{q}}^2 t^3}{24 C_F}\right) \xrightarrow{\text{large } N_c} P_s(t) = \exp\left(-\frac{\hat{q}\theta_{q\bar{q}}^2}{12}t^3\right)$$

$$\boxed{t_c = \left(\frac{4}{\hat{q}\theta_{q\bar{q}}^2}\right)^{\frac{1}{3}}} \qquad \theta_{q\bar{q}}^2 = \frac{2\boldsymbol{p}_\perp^2}{E^2}$$



… or rewritten as

$$P_s(t) = \exp\left(-\frac{\theta_{q\bar{q}}^2}{\theta_c^2(t)}\right)$$

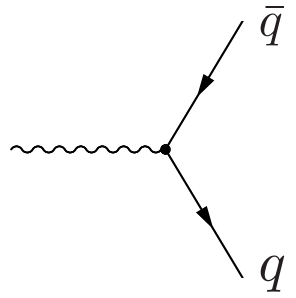
$$\boxed{\theta_c^2 = \frac{4}{\hat{q}t^3}}$$

Solutions: colour decoherence

- We want to investigate the **quantum** behaviour of **colour decoherence**:

Solutions: colour decoherence

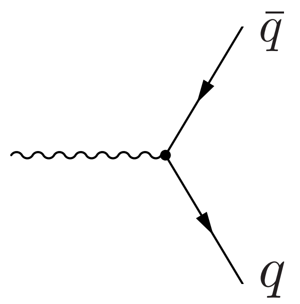
- We want to investigate the **quantum** behaviour of **colour decoherence**:
  - Initial colour **singlet** state: $|\psi(0)\rangle = \frac{|r\bar{r}\rangle + |b\bar{b}\rangle + |g\bar{g}\rangle}{\sqrt{3}}$



$$q \in \{|r\rangle\,, |g\rangle\,, |b\rangle\}$$
$$\bar{q} \in \{|\bar{r}\rangle\,, |\bar{g}\rangle\,, |\bar{b}\rangle\}$$

Solutions: colour decoherence

- We want to investigate the **quantum** behaviour of **colour decoherence**:
  - Initial colour **singlet** state: $|\psi(0)\rangle = \frac{|r\bar{r}\rangle + |b\bar{b}\rangle + |g\bar{g}\rangle}{\sqrt{3}}$



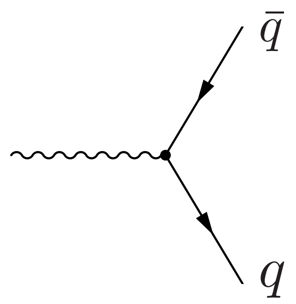
$$q \in \left\{ |{\color{red}r}\rangle , |{\color{green}g}\rangle , |{\color{blue}b}\rangle \right\}$$
$$\bar{q} \in \left\{ |{\color{cyan}\bar{r}}\rangle , |{\color{magenta}\bar{g}}\rangle , |{\color{yellow}\bar{b}}\rangle \right\}$$

$$\rightarrow \quad \hat{\rho}(0) = \begin{pmatrix} |r\bar{r}\rangle\langle r\bar{r}| & & {\color{red}|r\bar{r}\rangle\langle b\bar{b}|} & & {\color{red}|r\bar{r}\rangle\langle g\bar{g}|} \\ & 0 & & 0 & \\ {\color{red}|b\bar{b}\rangle\langle r\bar{r}|} & & |b\bar{b}\rangle\langle b\bar{b}| & & {\color{red}|b\bar{b}\rangle\langle g\bar{g}|} \\ & 0 & & 0 & \\ {\color{red}|g\bar{g}\rangle\langle r\bar{r}|} & & {\color{red}|g\bar{g}\rangle\langle b\bar{b}|} & & |g\bar{g}\rangle\langle g\bar{g}| \end{pmatrix}$$

Solutions: colour decoherence

- We want to investigate the **quantum** behaviour of **colour decoherence**:

  - Initial colour **singlet** state: $|\psi(0)\rangle = \frac{|r\bar{r}\rangle + |b\bar{b}\rangle + |g\bar{g}\rangle}{\sqrt{3}}$



$$q \in \{|{\color{red}r}\rangle, |{\color{green}g}\rangle, |{\color{blue}b}\rangle\}$$
$$\bar{q} \in \{|{\color{cyan}\bar{r}}\rangle, |{\color{magenta}\bar{g}}\rangle, |{\color{yellow}\bar{b}}\rangle\}$$

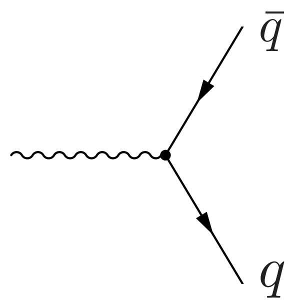
$$\rightarrow \quad \hat{\rho}(0) = \begin{pmatrix} |r\bar{r}\rangle\langle r\bar{r}| & & {\color{red}|r\bar{r}\rangle\langle b\bar{b}|} & & {\color{red}|r\bar{r}\rangle\langle g\bar{g}|} \\ & 0 & & 0 & \\ {\color{red}|b\bar{b}\rangle\langle r\bar{r}|} & & |b\bar{b}\rangle\langle b\bar{b}| & & {\color{red}|b\bar{b}\rangle\langle g\bar{g}|} \\ & 0 & & 0 & \\ {\color{red}|g\bar{g}\rangle\langle r\bar{r}|} & & {\color{red}|g\bar{g}\rangle\langle b\bar{b}|} & & |g\bar{g}\rangle\langle g\bar{g}| \end{pmatrix} \xrightarrow[\substack{\text{time}\\ \text{evolution}\\ t \to \infty}]{t_c = \left(\frac{4}{\hat{q}\theta_{q\bar{q}}^2}\right)^{\frac{1}{3}}} \begin{pmatrix} |r\bar{r}\rangle\langle r\bar{r}| & & & 0 & & \\ & |r\bar{g}\rangle\langle r\bar{g}| & & & & \\ & & |r\bar{b}\rangle\langle r\bar{b}| & & & \\ & & & |b\bar{r}\rangle\langle b\bar{r}| & & \\ & 0 & & & \ddots & \\ & & & & & \ddots \end{pmatrix}$$

  - Colour **decoherence**: suppression of the off-diagonal elements (coherences)

  - Colour **equilibration**: total randomization of the colour of the dipole

Solutions: colour decoherence

- We want to investigate the **quantum** behaviour of **colour decoherence**:
  - Initial colour **singlet** state: $|\psi(0)\rangle = \frac{|r\bar{r}\rangle + |b\bar{b}\rangle + |g\bar{g}\rangle}{\sqrt{3}}$

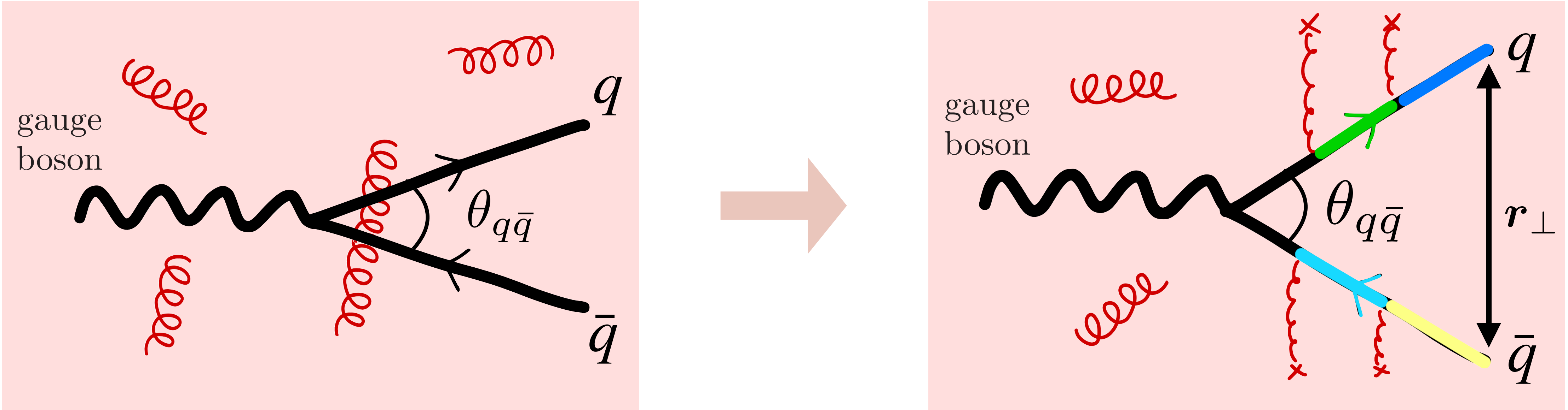


$$q \in \{|r\rangle, |g\rangle, |b\rangle\}$$
$$\bar{q} \in \{|\bar{r}\rangle, |\bar{g}\rangle, |\bar{b}\rangle\}$$

$$\to \hat{\rho}(0) = \begin{pmatrix} |r\bar{r}\rangle\langle r\bar{r}| & & |r\bar{r}\rangle\langle b\bar{b}| & & |r\bar{r}\rangle\langle g\bar{g}| \\ & 0 & & 0 & \\ |b\bar{b}\rangle\langle r\bar{r}| & & |b\bar{b}\rangle\langle b\bar{b}| & & |b\bar{b}\rangle\langle g\bar{g}| \\ & 0 & & 0 & \\ |g\bar{g}\rangle\langle r\bar{r}| & & |g\bar{g}\rangle\langle b\bar{b}| & & |g\bar{g}\rangle\langle g\bar{g}| \end{pmatrix} \xrightarrow[\substack{\text{time}\\ \text{evolution}\\ t \to \infty}]{t_c = \left(\frac{4}{\hat{q}\theta^2_{q\bar{q}}}\right)^{\frac{1}{3}}} \begin{pmatrix} |r\bar{r}\rangle\langle r\bar{r}| & & & & 0 & \\ & |r\bar{g}\rangle\langle r\bar{g}| & & & & \\ & & |r\bar{b}\rangle\langle r\bar{b}| & & & \\ & & & |b\bar{r}\rangle\langle b\bar{r}| & & \\ & 0 & & & \ddots & \\ & & & & & \ddots \end{pmatrix}$$

- Colour **decoherence**: suppression of the off-diagonal elements (coherences)
- Colour **equilibration**: total randomization of the colour of the dipole

- Physical picture:



Master equation: center-of-mass diffusion

- Let’s now consider the **motion of the center-of-mass** too:

$$\frac{\partial}{\partial t}\begin{pmatrix}\rho_s\\ \rho_o\end{pmatrix} + \Bigg(\overset{\text{kinetic term}}{\frac{1}{E}\boldsymbol{p}_\perp \cdot \nabla_{\boldsymbol{r}_\perp} + \frac{1}{q^+}\boldsymbol{q}_\perp \cdot \nabla_{\boldsymbol{b}_\perp}}\Bigg)\begin{pmatrix}\rho_s\\ \rho_o\end{pmatrix} = \frac{\hat{q}}{4C_F}\left\{\overset{\text{CoM diffusion}}{\begin{pmatrix}0 & 0\\ 0 & N_c\end{pmatrix}\nabla^2_{\boldsymbol{q}_\perp}} + \overset{\text{colour transitions}}{\begin{pmatrix}-2C_F & 2C_F\\ \frac{1}{N_c} & -\frac{1}{N_c}\end{pmatrix}\boldsymbol{r}^2_\perp}\right\}\begin{pmatrix}\rho_s\\ \rho_o\end{pmatrix}$$

$$\frac{1}{E} = \frac{1}{k_1^+} + \frac{1}{k_1^+}\,; \quad q^+ = k_1^+ + k_2^+$$

Master equation: center-of-mass diffusion

- Let’s now consider the **motion of the center-of-mass** too:

$$\frac{\partial}{\partial t}\begin{pmatrix}\rho_s\\ \rho_o\end{pmatrix} + \left(\overset{\textbf{kinetic term}}{\frac{1}{E}\boldsymbol{p}_\perp\cdot\nabla_{\boldsymbol{r}_\perp} + \frac{1}{q^+}\boldsymbol{q}_\perp\cdot\nabla_{\boldsymbol{b}_\perp}}\right)\begin{pmatrix}\rho_s\\ \rho_o\end{pmatrix} = \frac{\hat{q}}{4C_F}\left\{\overset{\textbf{CoM diffusion}}{\begin{pmatrix}0 & 0\\ 0 & N_c\end{pmatrix}\nabla^2_{\boldsymbol{q}_\perp}} + \overset{\textbf{colour transitions}}{\begin{pmatrix}-2C_F & 2C_F\\ \frac{1}{N_c} & -\frac{1}{N_c}\end{pmatrix}\boldsymbol{r}^2_\perp}\right\}\begin{pmatrix}\rho_s\\ \rho_o\end{pmatrix}$$

$$\frac{1}{E} = \frac{1}{k_1^+} + \frac{1}{k_1^+}\,;\quad q^+ = k_1^+ + k_2^+$$

- Solve with method of characteristics in Fourier space $\boldsymbol{q}_\perp \to \boldsymbol{\mathcal{B}}_\perp$

$$\boldsymbol{b}_\perp \to \boldsymbol{\mathcal{Q}}_\perp$$

Master equation: center-of-mass diffusion

- Let’s now consider the **motion of the center-of-mass** too:

$$\frac{\partial}{\partial t}\begin{pmatrix}\rho_s\\ \rho_o\end{pmatrix} + \left(\overbrace{\frac{1}{E}\boldsymbol{p}_\perp\cdot\nabla_{\boldsymbol{r}_\perp} + \frac{1}{q^+}\boldsymbol{q}_\perp\cdot\nabla_{\boldsymbol{b}_\perp}}^{\text{kinetic term}}\right)\begin{pmatrix}\rho_s\\ \rho_o\end{pmatrix} = \frac{\hat{q}}{4C_F}\left\{\overbrace{\begin{pmatrix}0&0\\0&N_c\end{pmatrix}\nabla^2_{\boldsymbol{q}_\perp}}^{\text{CoM diffusion}} + \overbrace{\begin{pmatrix}-2C_F & 2C_F\\ \frac{1}{N_c} & -\frac{1}{N_c}\end{pmatrix}\boldsymbol{r}^2_\perp}^{\text{colour transitions}}\right\}\begin{pmatrix}\rho_s\\ \rho_o\end{pmatrix}$$

$$\frac{1}{E} = \frac{1}{k_1^+} + \frac{1}{k_1^+}\,;\quad q^+ = k_1^+ + k_2^+$$

- Solve with method of characteristics in Fourier space $\boldsymbol{q}_\perp \to \boldsymbol{\mathcal{B}}_\perp$

$\boldsymbol{b}_\perp \to \boldsymbol{\mathcal{Q}}_\perp$

- **Initial** Wigner function (colour **singlet**):

$$\rho(\boldsymbol{p}_\perp,\boldsymbol{r}_\perp;\boldsymbol{q}_\perp,\boldsymbol{b}_\perp;0) = \underbrace{\rho_R(\boldsymbol{p}_\perp,\boldsymbol{r}_\perp;0)}_{\text{relative}}\underbrace{\rho_T(\boldsymbol{q}_\perp,\boldsymbol{b}_\perp;0)}_{\text{center-of-mass}}$$

hard factor: production of the $q\bar{q}$ pair

$$\rho_R(\boldsymbol{p}_\perp,\boldsymbol{r}_\perp;0) = \underset{\uparrow}{H(\boldsymbol{p}_\perp)}\delta(\boldsymbol{r}_\perp)$$

$$\rho_T(\boldsymbol{q}_\perp,\boldsymbol{b}_\perp;0) = \frac{1}{\pi^2}e^{-\boldsymbol{q}_\perp^2/\Lambda^2-\Lambda^2\boldsymbol{b}_\perp^2}$$

Master equation: center-of-mass diffusion

- Let’s now consider the **motion of the center-of-mass** too:

$$\frac{\partial}{\partial t}\begin{pmatrix}\rho_s\\ \rho_o\end{pmatrix}+\Big(\overbrace{\frac{1}{E}\boldsymbol{p}_\perp\cdot\nabla_{\boldsymbol{r}_\perp}+\frac{1}{q^+}\boldsymbol{q}_\perp\cdot\nabla_{\boldsymbol{b}_\perp}}^{\text{kinetic term}}\Big)\begin{pmatrix}\rho_s\\ \rho_o\end{pmatrix}=\frac{\hat{q}}{4C_F}\left\{\overbrace{\begin{pmatrix}0 & 0\\ 0 & N_c\end{pmatrix}\nabla^2_{\boldsymbol{q}_\perp}}^{\text{CoM diffusion}}+\overbrace{\begin{pmatrix}-2C_F & 2C_F\\ \frac{1}{N_c} & -\frac{1}{N_c}\end{pmatrix}\boldsymbol{r}^2_\perp}^{\text{colour transitions}}\right\}\begin{pmatrix}\rho_s\\ \rho_o\end{pmatrix}$$

$$\frac{1}{E}=\frac{1}{k_1^+}+\frac{1}{k_1^+};\quad q^+=k_1^++k_2^+$$

- Solve with method of characteristics in Fourier space $\boldsymbol{q}_\perp\to\boldsymbol{\mathcal{B}}_\perp$
$\boldsymbol{b}_\perp\to\boldsymbol{\mathcal{Q}}_\perp$
- **Initial** Wigner function (colour **singlet**):

$$\rho(\boldsymbol{p}_\perp,\boldsymbol{r}_\perp;\boldsymbol{q}_\perp,\boldsymbol{b}_\perp;0)=\underbrace{\rho_R(\boldsymbol{p}_\perp,\boldsymbol{r}_\perp;0)}_{\text{relative}}\underbrace{\rho_T(\boldsymbol{q}_\perp,\boldsymbol{b}_\perp;0)}_{\text{center-of-mass}}$$

hard factor: production of the $q\bar{q}$ pair

$$\rho_R(\boldsymbol{p}_\perp,\boldsymbol{r}_\perp;0)=H(\boldsymbol{p}_\perp)\delta(\boldsymbol{r}_\perp)$$

$$\rho_T(\boldsymbol{q}_\perp,\boldsymbol{b}_\perp;0)=\frac{1}{\pi^2}e^{-\boldsymbol{q}^2_\perp/\Lambda^2-\Lambda^2\boldsymbol{b}^2_\perp}$$

- By integrating over the coordinates $\boldsymbol{r}_\perp$ and $\boldsymbol{b}_\perp$: **broken** factorization

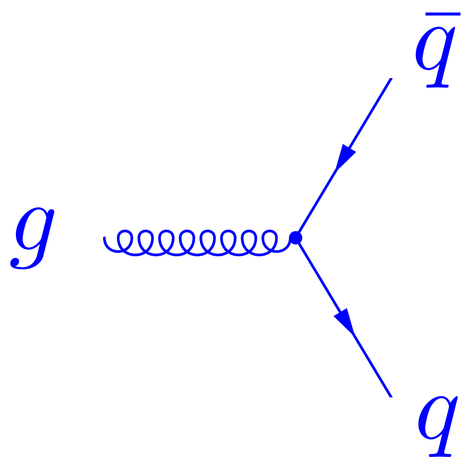
$$(\Lambda\to 0)\qquad\frac{\mathrm{d}P}{\mathrm{d}^2\boldsymbol{p}_\perp\mathrm{d}^2\boldsymbol{q}_\perp}=H(\boldsymbol{p}_\perp)\times\frac{1}{\pi}\left\{\underbrace{e^{-\frac{\hat{q}\theta^2_{q\bar{q}}t^3}{12}}\delta(\boldsymbol{q}_\perp)}_{\text{singlet}}+\underbrace{\int_0^t\mathrm{d}s\,\frac{\theta^2_{q\bar{q}}(t-s)^2}{8s}e^{-\frac{\hat{q}\theta^2_{q\bar{q}}(t-s)^3}{12}}e^{-\frac{\boldsymbol{q}^2_\perp}{2\hat{q}s}}}_{\text{octet}}\right\}\qquad\theta^2_{q\bar{q}}=\frac{2\boldsymbol{p}^2_\perp}{E^2}$$

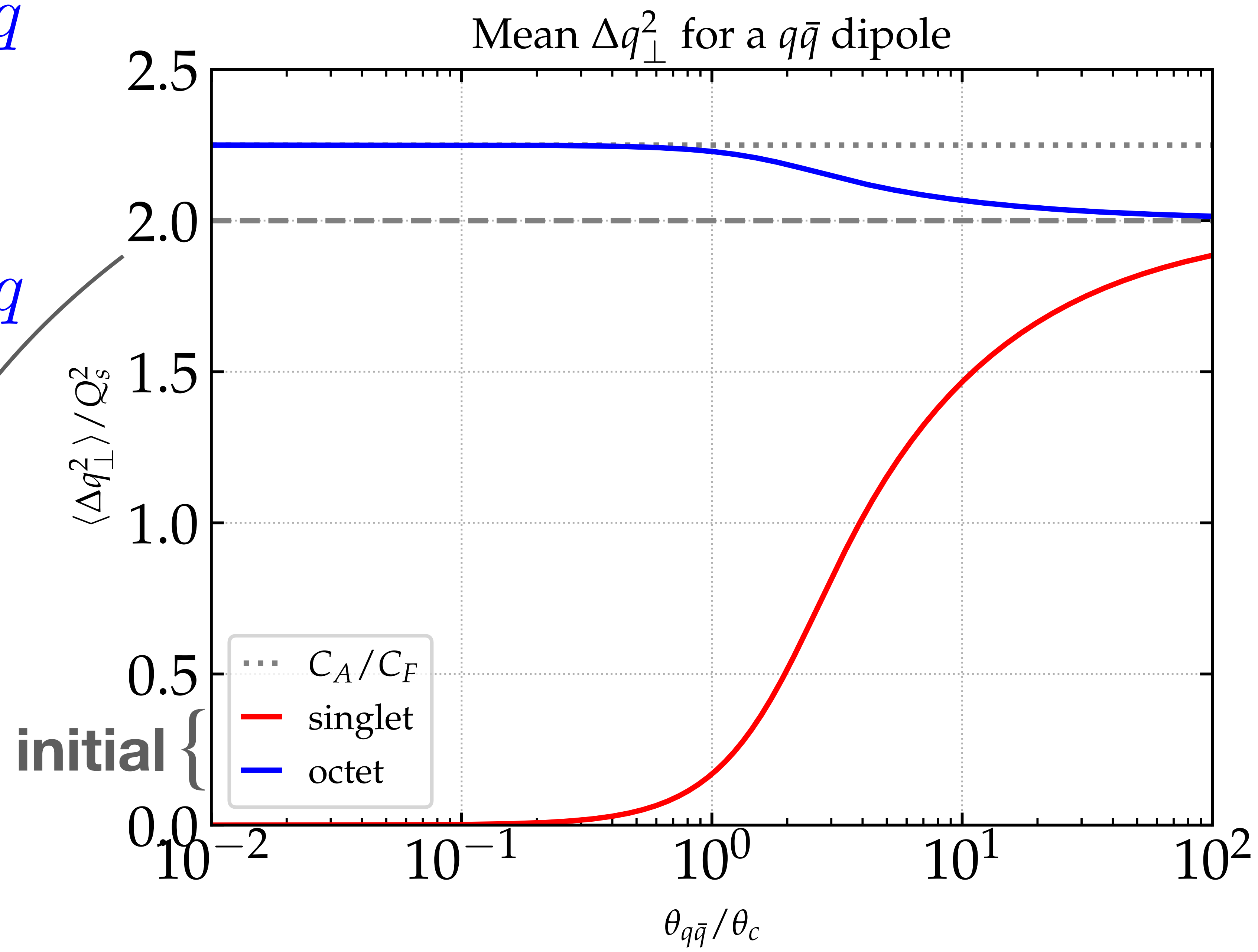
Solutions: transverse momentum broadening

- We compute the **CoM transverse momentum distribution**:

$$t = L$$

$$Q_s^2 = \hat{q}L$$

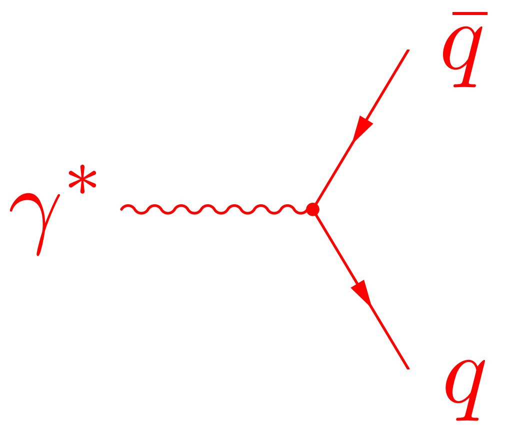




$$\langle \Delta \boldsymbol{q}_\perp^2 \rangle \big|_{\text{incoh.}} = \langle \Delta \boldsymbol{k_{1\perp}}^2 \rangle + \langle \Delta \boldsymbol{k_{2\perp}}^2 \rangle = 2Q_s^2$$

$$\theta_{q\bar{q}}^2 = \frac{2\boldsymbol{p}_\perp^2}{E^2}$$

$$\theta_c^2 = \frac{4}{\hat{q}L^3}$$

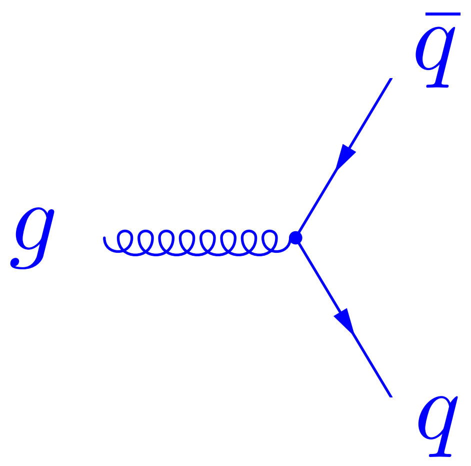


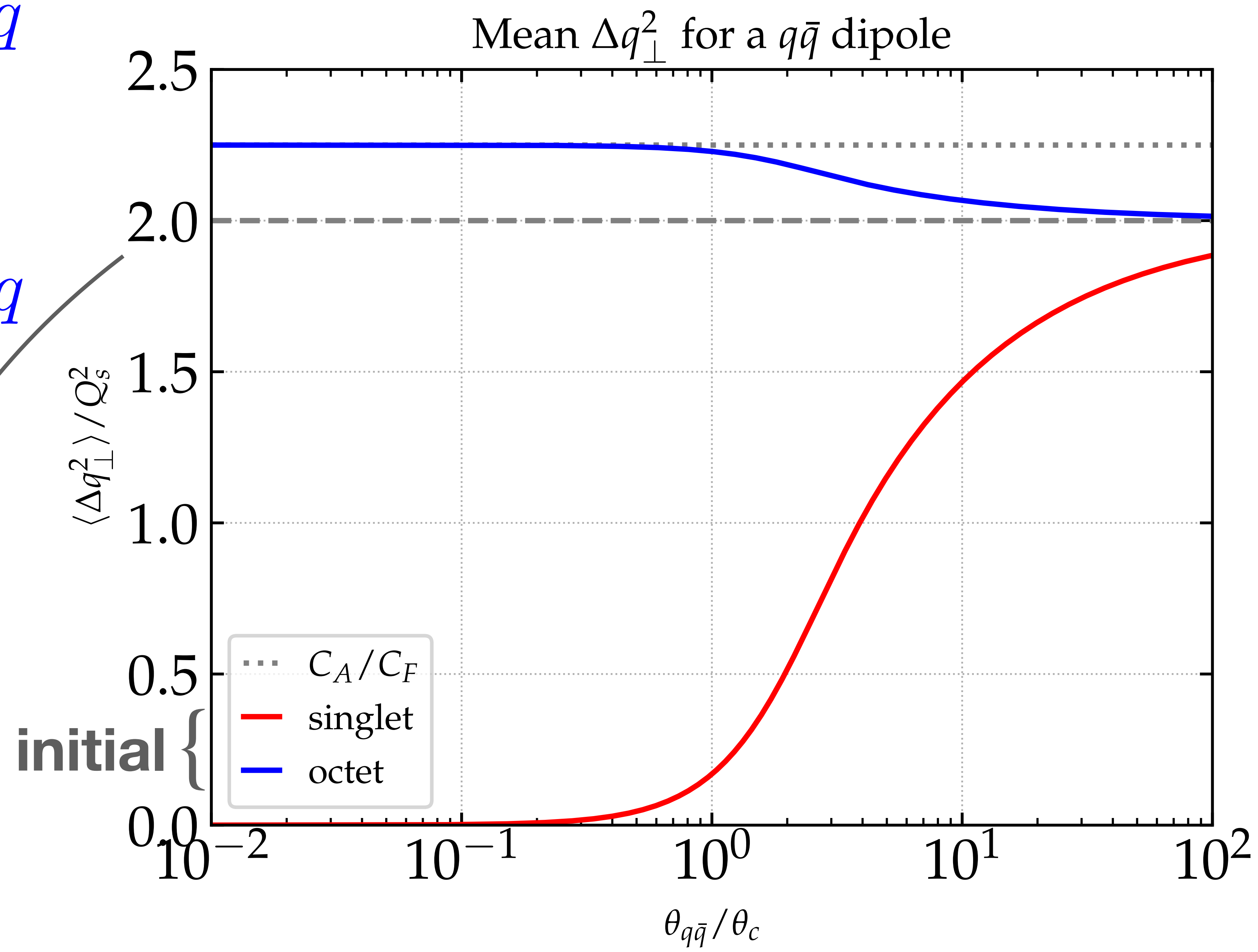
Solutions: transverse momentum broadening

- We compute the **CoM transverse momentum distribution**:

$$t = L$$

$$Q_s^2 = \hat{q}L$$

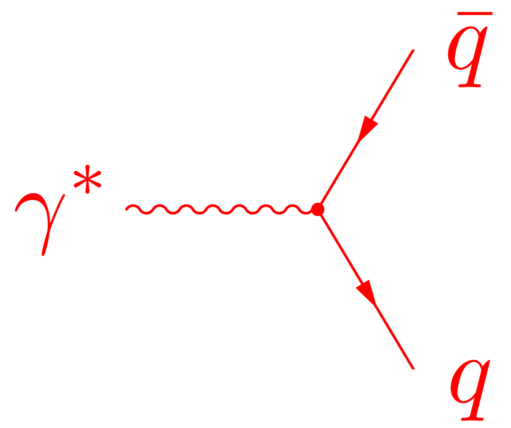




$$\langle \Delta \boldsymbol{q}_\perp^2 \rangle \big|_{\text{incoh.}} = \langle \Delta \boldsymbol{k_1}_\perp{}^2 \rangle + \langle \Delta \boldsymbol{k_2}_\perp{}^2 \rangle = 2Q_s^2$$

$$\theta_{q\bar{q}}^2 = \frac{2\boldsymbol{p}_\perp^2}{E^2}$$

$$\theta_c^2 = \frac{4}{\hat{q}L^3}$$



- **Non-trivial check** of known *Wilson lines*-like calculations with OQSs
  Dominguez, Milhano, Salgado, Tywoniuk, Vila (2019)
  Isaksen, Tywoniuk (2023)

- **Clarify hypothesis** behind the same results
  - initial factorization
  - neglect diffusion term

- Possible **pheno** applications: even higher Casimirs
  Cougoulic, Peigné (2017)
  Zakharov (2018)

Solutions: kinematical decoherence

- Initial colour **singlet** state

$$\rho_T(\boldsymbol{q}_\perp, \boldsymbol{b}_\perp; 0) = \frac{1}{\pi^2} e^{-\boldsymbol{q}_\perp^2/\Lambda^2 - \Lambda^2 \boldsymbol{b}_\perp^2}$$

(Gaussian with width Λ)

$$\rho_T(\boldsymbol{q}_\perp, \boldsymbol{\mathcal{Q}}_\perp; t) \longrightarrow \quad q'_\perp = |\boldsymbol{q}_\perp - \boldsymbol{\mathcal{Q}}_\perp/2| \qquad q_\perp = |\boldsymbol{q}_\perp + \boldsymbol{\mathcal{Q}}_\perp/2|$$

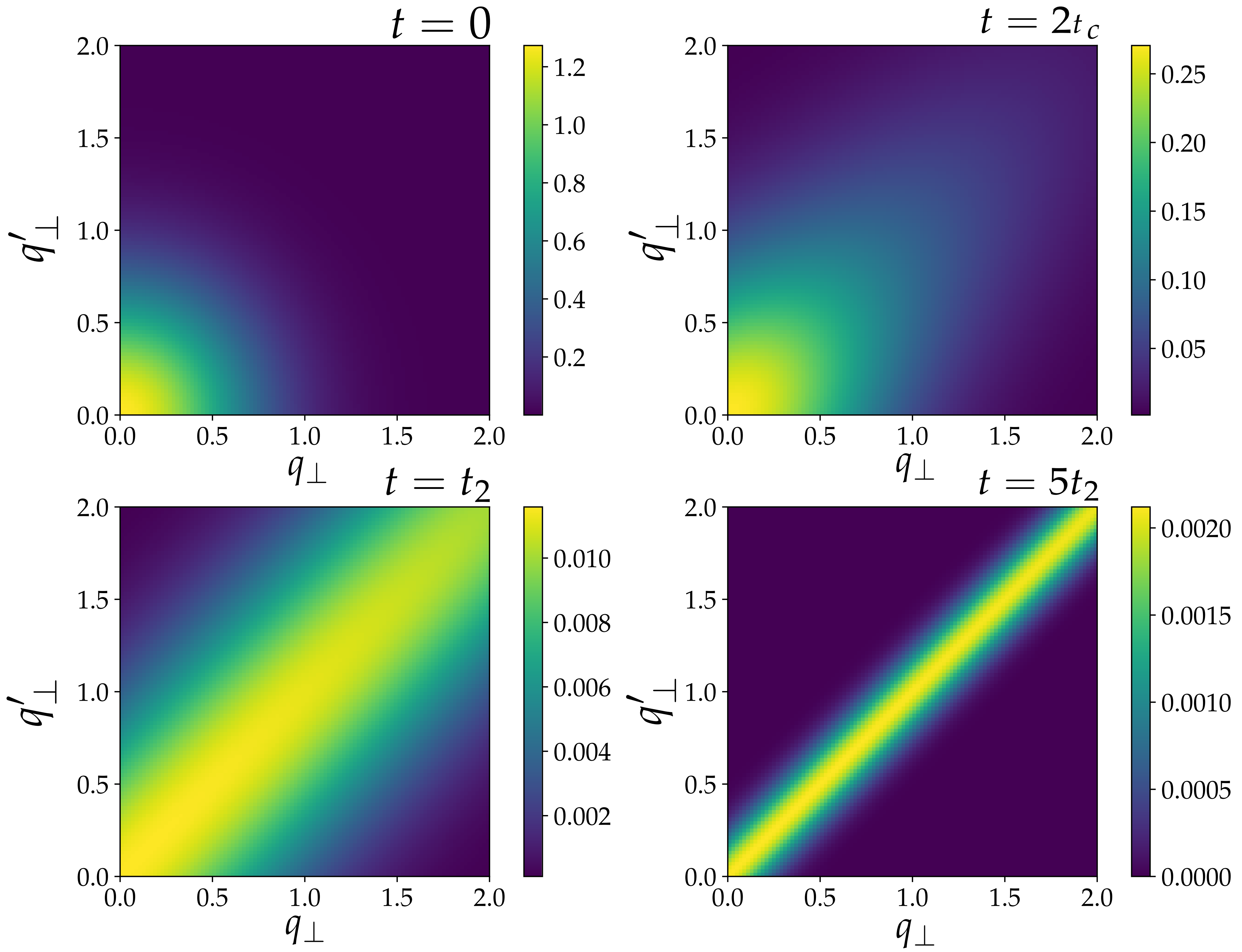
Numbers:

$$\Lambda = 500 \text{ MeV}$$

$$\hat{q} = 1.5 \text{ GeV}^2/\text{fm}$$

$$q^+ = 50 \text{ GeV}$$

$$\theta_{q\bar{q}} = 0.4$$



$$t_c = \left(\frac{4}{\hat{q}\theta_{q\bar{q}}^2}\right)^{\frac{1}{3}}$$

$$\sim 0.87 \text{ fm}$$

colour decoherence occurs (**inside** the medium)

$L = 4$ fm medium length

$$t_2 = \left(\frac{(q^+)^2}{\hat{q}\Lambda^2}\right)^{1/3}$$

$$\sim 10 \text{ fm}$$

kinematical decoherence occurs (**outside** the medium)

Master equation: perturbative relative diffusion

- Perturbative scheme for the **relative transverse momentum** diffusion term:

kinetic term

Rescaled master equation

large-N_c

$$\frac{\partial}{\partial \bar{t}}\begin{pmatrix}\rho_s\\ \tilde{\rho}_o\end{pmatrix} + \left(\bar{\boldsymbol{p}}_\perp \cdot \nabla_{\bar{\boldsymbol{r}}_\perp} + \bar{\boldsymbol{q}}_\perp \cdot \nabla_{\bar{\boldsymbol{b}}_\perp}\right)\begin{pmatrix}\rho_s\\ \tilde{\rho}_o\end{pmatrix}$$

relative diffusion

$$= \frac{1}{2}\left\{\begin{pmatrix}0 & 0\\ 0 & 1\end{pmatrix}\nabla^2_{\bar{\boldsymbol{q}}_\perp} + \begin{pmatrix}-1 & 0\\ 1 & 0\end{pmatrix}\bar{\boldsymbol{r}}^2_\perp + \frac{\kappa^2}{4}\begin{pmatrix}1 & 0\\ 1 & 1\end{pmatrix}\nabla^2_{\bar{\boldsymbol{p}}_\perp}\right\}\begin{pmatrix}\rho_s\\ \tilde{\rho}_o\end{pmatrix}$$

$$\kappa = \frac{2\theta_s}{\theta_c} = \frac{Q_s}{p_\perp} \times \frac{\theta_{q\bar{q}}}{\theta_c}$$

CoM diffusion **colour transitions**

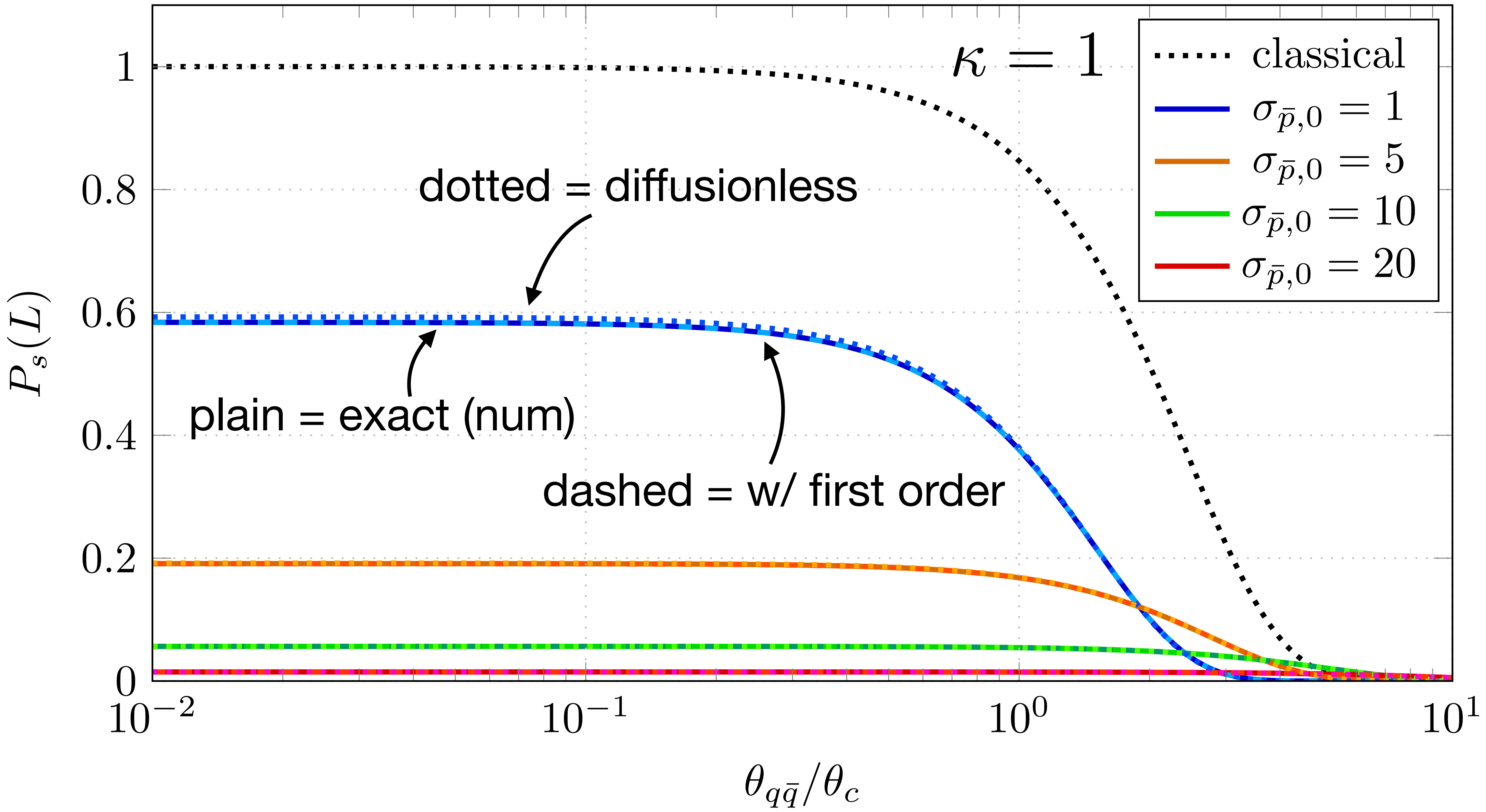
- Analytical solution:
  - **Gaussian** ansatz (scan of widths) → plug into the equation → closed system of equations for the 0th, 1st, 2nd moments
  - Perturbative expansion

$$y(t) = \sum_{i=0}^{+\infty} y^{(i)}(t)\kappa^{2i}$$

up to $\mathcal{O}(\kappa^2)$

Master equation: perturbative relative diffusion

- Singlet **survival probability:**



in our kinematical configuration

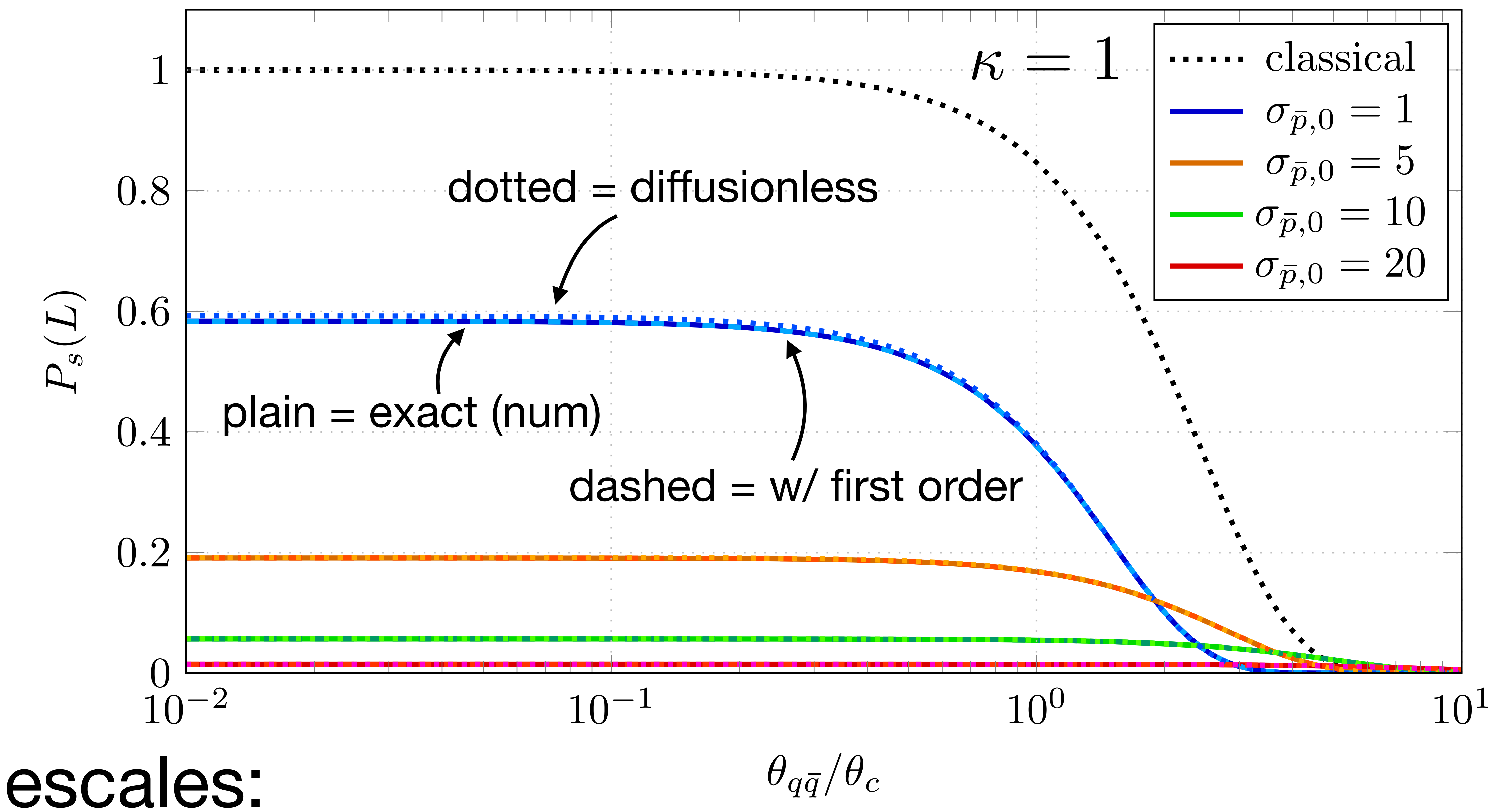
$$E \gg \boldsymbol{p}_\perp \gg \boldsymbol{q}_\perp$$

neglecting the relative diffusion term is a **good** approximation

checked also for higher moments (variances,…)

Master equation: perturbative relative diffusion

- Singlet **survival probability:**



dotted = diffusionless

plain = exact (num)

dashed = w/ first order

in our kinematical configuration

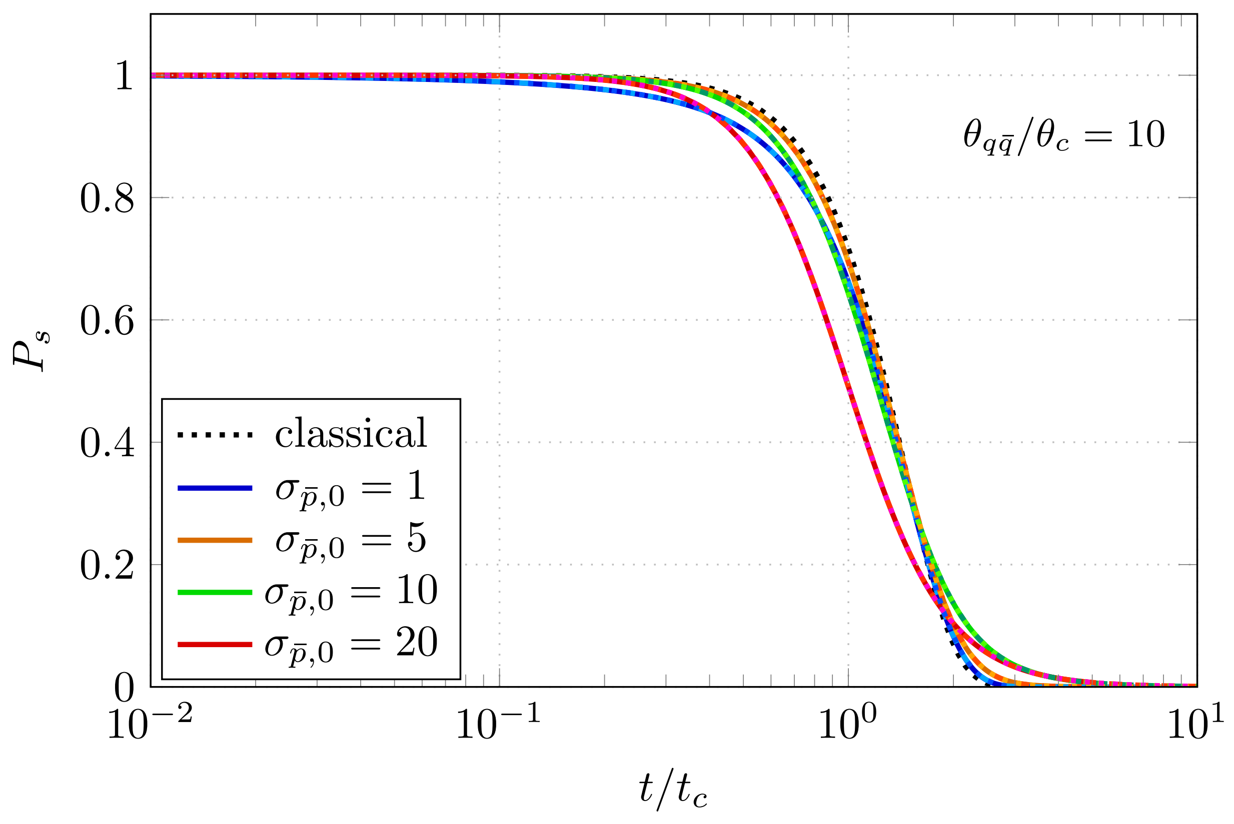
$$E \gg \boldsymbol{p}_\perp \gg \boldsymbol{q}_\perp$$

neglecting the relative diffusion term is a **good** approximation

checked also for higher moments (variances,…)

- Interplay of two timescales:

wide dipole

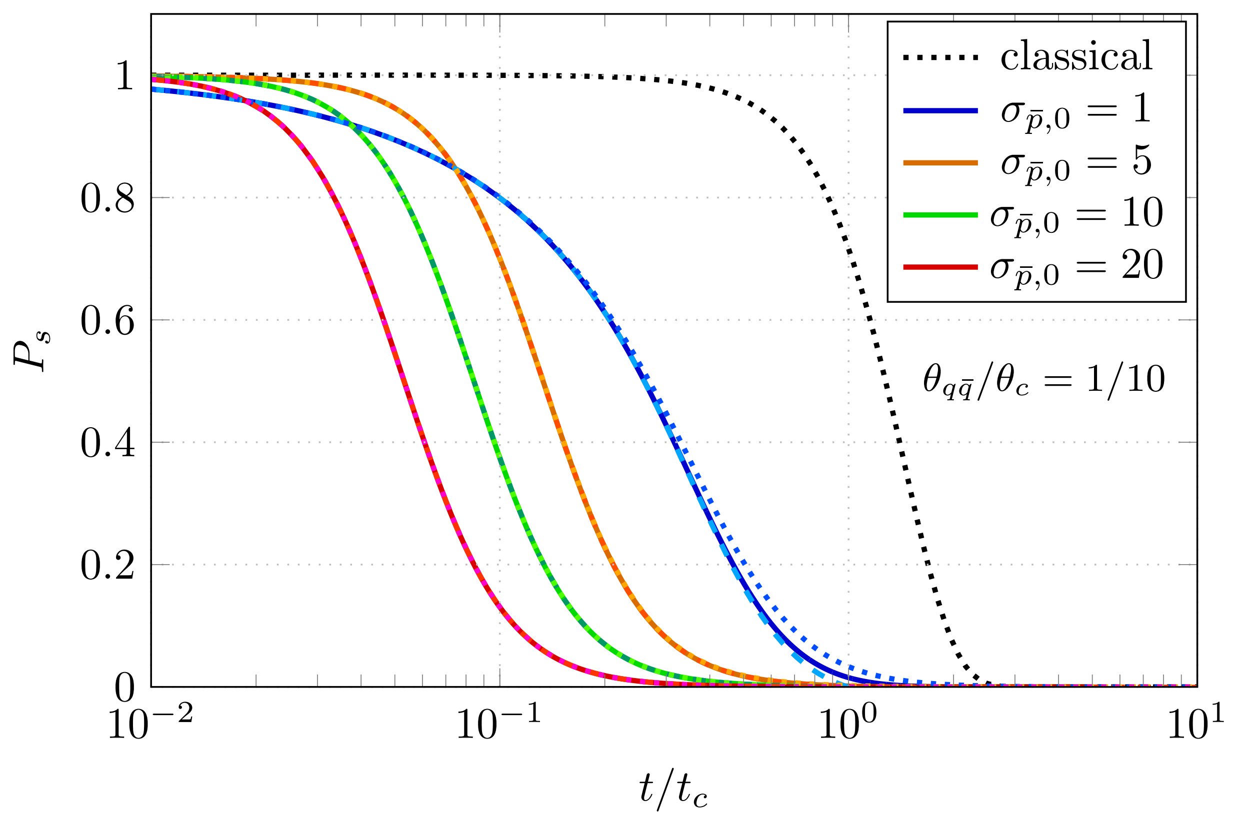


$$t_c = \left(\frac{4}{\hat{q}\theta_{q\bar{q}}^2}\right)^{\frac{1}{3}}$$

vs

$$t_2' = \left(\frac{12E^2}{\hat{q}\sigma_{p,0}^2}\right)^{\frac{1}{3}}$$

narrow dipole



Summary

- We have derived a **master equation** for the $q\bar{q}$ evolution with the **OQS formalism**
  - The collisions with the medium cause **colour decoherence**
    - **colour transitions** (singlet-octet)
    - **diagonalization** of the density matrix in the colour-anticolour basis
    - **colour equilibration**
  - the **broadening** of the center-of-mass transverse momentum, breaking **factorisation**

    → interest also in future application with **higher Casimirs**, e.g. gg dipole
  - **kinematical decoherence** in momentum space (if not already outside the QGP)
  - **perturbatively** solved the **full** master equation equipped of the relative transverse momentum diffusion term: **mild** corrections
- Work in progress to apply this framework also on radiation-related phenomena

Summary

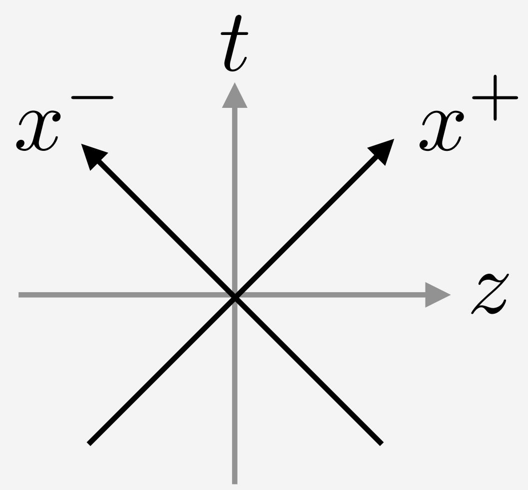
- We have derived a **master equation** for the $q\bar{q}$ evolution with the **OQS formalism**
  - The collisions with the medium cause **colour decoherence**
    - **colour transitions** (singlet-octet)
    - **diagonalization** of the density matrix in the colour-anticolour basis
    - **colour equilibration**
  - the **broadening** of the center-of-mass transverse momentum, breaking **factorisation**
    → interest also in future application with **higher Casimirs**, e.g. gg dipole
  - **kinematical decoherence** in momentum space (if not already outside the QGP)
  - **perturbatively** solved the **full** master equation equipped of the relative transverse momentum diffusion term: **mild** corrections
- Work in progress to apply this framework also on radiation-related phenomena

Thank you for your attention!

Backups

Modelling of the QGP: colour field

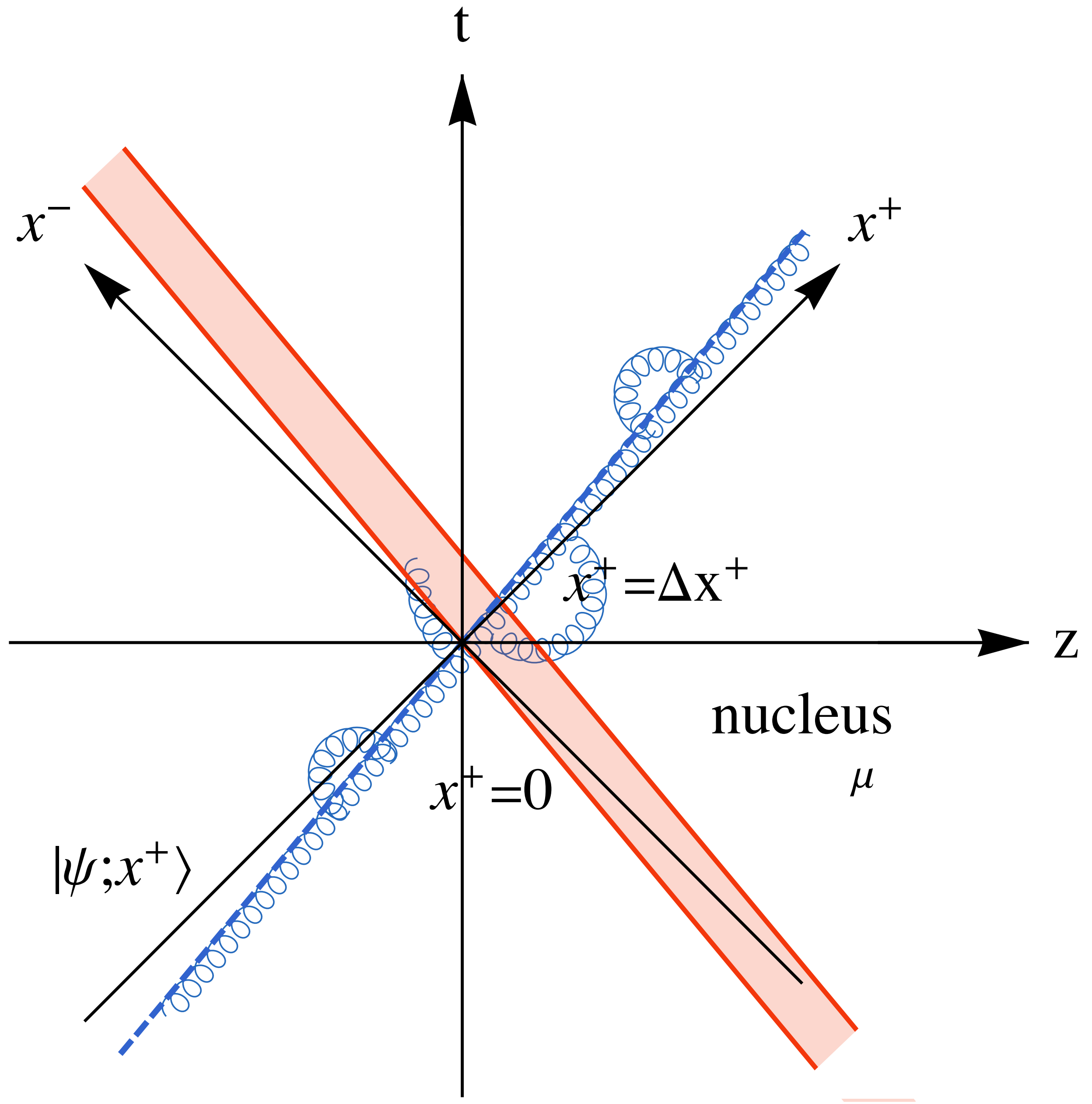
Light-cone coordinates:



$$x^+ = \frac{t+z}{\sqrt{2}} \propto t \simeq z$$

$$x^- = \frac{t-z}{\sqrt{2}} \simeq 0$$

- $A^+ = 0$ light-cone gauge
- $A^\mu \simeq A^\mu \delta^{\mu -}$ as one can see from boost of static colour field
- $A^- = A^-(x^+, \boldsymbol{x}_\perp)$



Non-relativistic Hamiltonian in (2+1)-dim.

- High energy gluon inside a medium A^-:

$$S_{\rm YM}[a^\mu + A^\mu] = -\frac{1}{4}\int d^4x\, {\rm tr} F_{\mu\nu}F^{\mu\nu}$$

  - we want to find the **propagator** from the **quadratic term** of the action (kinetic term):

$$S_0[a^\mu; A^-] = \int d^4x \left[a_\mu \left(G^{\mu\nu}\right)^{-1} a_\nu\right]$$

$$S_0[a^\mu; A^-] = \frac{1}{2}\int d^4x \left[a^i\left(-D^2\right)a^i + \cancel{\left(\partial^+\tilde{a}^-\right)^2}\right]$$

can be shown that it is sub-leading

$$\sim \frac{1}{(p^+)^2} \sim \frac{1}{E^2}$$

$$\boxed{G^{ij} = -\delta^{ij} G}$$

diagonal: **scalar**

$$D^2 = g_{\mu\nu}D^\mu D^\nu = 2\partial^+ D^- - \nabla_\perp^2$$

$$\boxed{\left[\underbrace{i\partial_+ + \frac{\nabla_\perp^2}{2p^+} + gA^-(x)}_{\text{hamiltonian}}\right]\mathcal{G}(x^+, x_\perp; y^+, y_\perp; p^+) = i\delta(x^+ - y^+)\delta^{(2)}(x_\perp - y_\perp)}$$

$$G(x^+, x_\perp; y^+, y_\perp; p^+) \equiv \frac{1}{2p^+}\mathcal{G}(x^+, x_\perp; y^+, y_\perp; p^+)$$

Derivation of the master equation

- From the Von Neumann equation we have carried out the following approximations:

$$i\frac{d\rho}{dt} = [H, \rho]$$

Von Neumann equation

  - Double iteration of the equation by integrating it and insert the solution back

$$\frac{d}{dt}\rho^I(t) = -ig\left[H^I(t), \rho^I(0)\right] - g^2\int_0^t dt'\left[H^I(t), \left[H^I(t'), \rho^I(t')\right]\right]$$

partial trace ——

  - **Born** approximation: the state remains factorized at all times (**weak coupling**) $\rho(t) = \rho_S(t)\otimes\rho_E$

  - **Markov** approximation: we impose **locality in time** thanks to the **weak coupling** g

→ **Redfield** equation: $$\frac{d}{dt}\rho_S^I(t) = -g^2\int_0^t dt'\mathrm{Tr}_E\left[H^I(t), \left[H^I(t'), \rho_S^I(\mathbf{t})\otimes\rho_E\right]\right]$$

  - Locality is not enough for **Markovianity**: the equation still depends on the initial condition at $t' = 0$

Derivation of the master equation

- If we assume that the **environment correlation time** τ_E is much smaller than the **system relaxation time** τ_R than, the information on the specific initial state at $t' = 0$ becomes irrelevant

  → we extend the integration domain from $t' \in [0,t]$ to $[-\infty, t]$. Then, after a change of variables we obtain

$$\frac{d}{dt}\rho_S^I(t) = -g^2 \int_0^{+\infty} dt' \mathrm{Tr}_E \left[H^I(t), \left[H^I(t-t'), \rho_S^I(t) \otimes \rho_E\right]\right]$$

- Now, one has just to substitute the factorized structure of the Hamiltonian and keep within the partial trace just the environment contributions

  → two-point correlators of the environment part of the interaction

Derivation of the master equation

$$\frac{d\rho_S}{dt} + i[\mathcal{H}_0, \rho_S] = \frac{1}{2}\int_{\boldsymbol{r}'_\perp \boldsymbol{r}_\perp} [\{\hat{n}^a(\boldsymbol{r}_\perp)\hat{n}^a(\boldsymbol{r}'_\perp), \rho_S\} - 2\hat{n}^a(\boldsymbol{r}'_\perp)\rho_S \hat{n}^a(\boldsymbol{r}_\perp)]\, W(\boldsymbol{r}_\perp - \boldsymbol{r}'_\perp) +$$

$$+\cancel{\text{friction term}} + \ldots$$

high-energy limit

this approximation is related to the **Quantum Brownian Regime**:

in the Schrödinger picture, the number density operators evolve with an evolution operator

$$U_0(\tau) = e^{-iH_0\tau} \simeq 1 - i\cancel{H_0}\tau$$

for $H_0\tau \ll 1$

$$H_0\tau \lesssim \frac{p_\perp^2}{E}\frac{1}{m_D} \sim \frac{\tau_E}{t_f} \ll 1$$

larger for quarkonia where E is the mass M

- Imaginary potential:

$$W(\boldsymbol{r}_\perp) \equiv \mathfrak{Im}(\Delta(\omega = 0, \boldsymbol{r}_\perp))$$

$$\Delta(\omega, \boldsymbol{r}_\perp) = \int_{-\infty}^{\infty} \mathrm{d}t e^{i\omega t}\Delta(t, \boldsymbol{r}_\perp)$$

$$\left\langle T\left(A_a^-(t, \boldsymbol{r}_\perp) A_b^-(0, \boldsymbol{0}_\perp)\right)\right\rangle_{\text{pl}} \equiv -i\delta_{ab}\Delta(t, \boldsymbol{r}_\perp)$$

- in quarkonia the *real* component (screening) is relevant too…

Harmonic approximation

$$\Gamma(\boldsymbol{x}_\perp) = \int \frac{\mathrm{d}^2\boldsymbol{k}_\perp}{(2\pi)^2} \left(1 - e^{i\boldsymbol{k}_\perp\cdot\boldsymbol{x}_\perp}\right) \frac{m_D^2 T}{\boldsymbol{k}_\perp^2(\boldsymbol{k}_\perp^2 + m_D^2)}$$

$$g^2 C_F \Gamma(\boldsymbol{x}_\perp) = \frac{\hat{q}_0}{4}\boldsymbol{x}_\perp^2 \ln\left(\frac{4e^{2-2\gamma_E}}{\boldsymbol{x}_\perp^2 m_D^2}\right) + O(|\boldsymbol{x}_\perp|^4 m_D^4) \qquad \text{for} \quad |\boldsymbol{x}_\perp| \lesssim 1/m_D$$

Redefining $\hat{q} \equiv \hat{q}_0 \ln(4e^{2-2\gamma_E} Q^2/m_D^2)$

$$g^2 C_F \Gamma(\boldsymbol{x}_\perp) = \frac{\hat{q}}{4}\boldsymbol{x}_\perp^2 + \frac{\hat{q}_0}{4}\boldsymbol{x}_\perp^2 \ln\left(\frac{1}{\boldsymbol{x}_\perp^2 Q^2}\right) + O(|\boldsymbol{x}_\perp|^4 m_D^4)$$

Initial state

- Motivation of relative - c.o.m factorisation:

probability of producing a $q\bar{q}$ dipole in vacuum from a virtual photon

$$\frac{\mathrm{d}P}{\mathrm{d}^2\boldsymbol{p}_\perp \mathrm{d}^2\boldsymbol{q}_\perp} = \frac{\alpha_{\mathrm{em}} N_c}{4\pi} \frac{(z_1^2 + z_2^2)}{\boldsymbol{p}_\perp^2} \times \delta(\boldsymbol{q}_\perp) \qquad E \gg p_\perp \gg q_\perp$$

at LO in α_{em} and α_s

Perturbative scheme: rescaling the m. eq.

$$\frac{\partial}{\partial t}\begin{pmatrix}\rho_s\\ \rho_o\end{pmatrix} + \left(\frac{1}{E}\boldsymbol{p}_\perp\cdot\nabla_{\boldsymbol{r}_\perp} + \frac{1}{q^+}\boldsymbol{q}_\perp\cdot\nabla_{\boldsymbol{b}_\perp}\right)\begin{pmatrix}\rho_s\\ \rho_o\end{pmatrix}$$

$$= \frac{\hat{q}}{4C_F}\left\{\begin{pmatrix}0 & 0\\ 0 & N_c\end{pmatrix}\nabla^2_{\boldsymbol{q}_\perp} + \begin{pmatrix}-2C_F & 2C_F\\ \frac{1}{N_c} & -\frac{1}{N_c}\end{pmatrix}\boldsymbol{r}^2_\perp + \begin{pmatrix}\frac{C_F}{2} & \frac{C_F}{2}\\ \frac{1}{4N_c} & \frac{C_F}{2}-\frac{1}{2N_c}\end{pmatrix}\nabla^2_{\boldsymbol{p}_\perp}\right\}\begin{pmatrix}\rho_s\\ \rho_o\end{pmatrix}$$

$$t \sim L$$

$$\bar{t} = \frac{t}{L}$$

$$\theta_{q\bar{q}} \sim \theta_c$$

$$\boldsymbol{p}_\perp \sim \frac{E}{\sqrt{\hat{q}L^3}}$$

$$\bar{\boldsymbol{p}}_\perp = \boldsymbol{p}_\perp\frac{\sqrt{\hat{q}L^3}}{E}$$

$$\boldsymbol{q}_\perp \sim \sqrt{\hat{q}L}$$

$$\bar{\boldsymbol{q}}_\perp = \frac{\boldsymbol{q}_\perp}{\sqrt{\hat{q}L}}$$

$$\frac{L}{E}\boldsymbol{p}_\perp\cdot\nabla_{\boldsymbol{r}_\perp} \sim \mathcal{O}(1)$$

$$\boldsymbol{r}_\perp \sim \frac{1}{\sqrt{\hat{q}L}}$$

$$\bar{\boldsymbol{r}}_\perp = \boldsymbol{r}_\perp\sqrt{\hat{q}L}$$

$$\frac{L}{q^+}\boldsymbol{q}_\perp\cdot\nabla_{\boldsymbol{b}_\perp} \sim \mathcal{O}(1)$$

$$\boldsymbol{b}_\perp \sim \frac{\sqrt{\hat{q}L^3}}{q^+}$$

$$\bar{\boldsymbol{b}}_\perp = \boldsymbol{b}_\perp\frac{q^+}{\sqrt{\hat{q}L^3}}$$

$$\frac{\partial}{\partial \bar{t}}\begin{pmatrix}\rho_s\\ \rho_o\end{pmatrix} + \left(\bar{\boldsymbol{p}}_\perp\cdot\nabla_{\bar{\boldsymbol{r}}_\perp} + \bar{\boldsymbol{q}}_\perp\cdot\nabla_{\bar{\boldsymbol{b}}_\perp}\right)\begin{pmatrix}\rho_s\\ \rho_o\end{pmatrix}$$

$$= \frac{1}{4C_F}\left\{\begin{pmatrix}0 & 0\\ 0 & N_c\end{pmatrix}\nabla^2_{\bar{\boldsymbol{q}}_\perp} + \begin{pmatrix}-2C_F & 2C_F\\ \frac{1}{N_c} & -\frac{1}{N_c}\end{pmatrix}\bar{\boldsymbol{r}}^2_\perp + \kappa^2\begin{pmatrix}\frac{C_F}{2} & \frac{C_F}{2}\\ \frac{1}{4N_c} & \frac{C_F}{2}-\frac{1}{2N_c}\end{pmatrix}\nabla^2_{\bar{\boldsymbol{p}}_\perp}\right\}\begin{pmatrix}\rho_s\\ \rho_o\end{pmatrix}$$

$$\kappa = \frac{2\theta_s}{\theta_c} = \frac{Q_s}{p_\perp}\times\frac{\theta_{q\bar{q}}}{\theta_c}$$

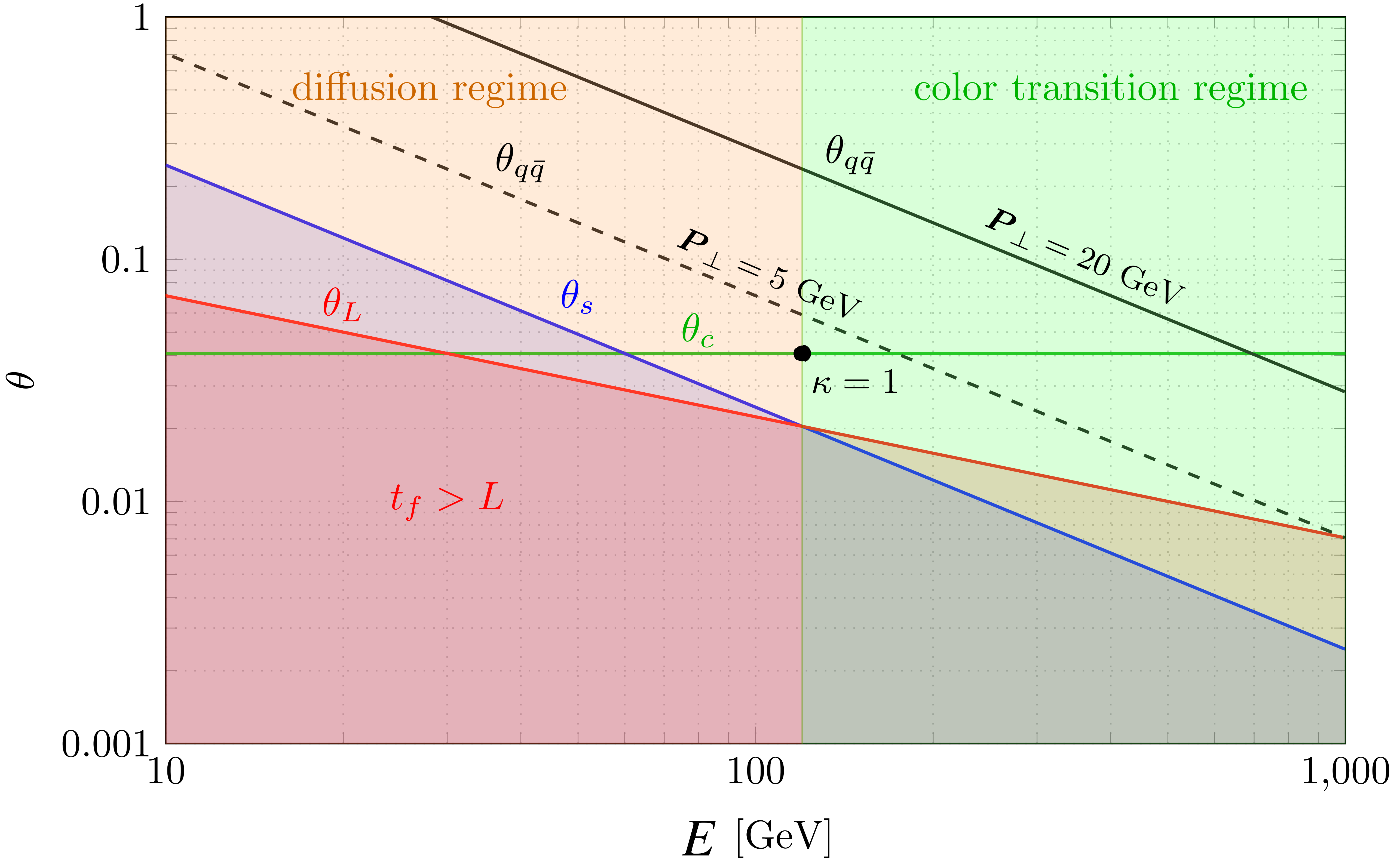
Smallness of the perturbative parameter κ

- Scales hierarchy: $E \gg \boldsymbol{P}_\perp \gg \boldsymbol{q}_\perp \sim Q_s \sim \sqrt{\hat{q}L}$ → transverse momentum gained by a single parton in a QGP with length L

  $(1 \gg \theta_{q\bar{q}} \gg \theta_s)$

- Where does the diffusion term $\nabla^2_{\boldsymbol{p}_\perp}$ become small?

perturbative constant $\kappa = \dfrac{2\theta_s}{\theta_c} = \dfrac{Q_s}{p_\perp} \times \dfrac{\theta_{q\bar{q}}}{\theta_c}$



with

$$\theta_{q\bar{q}} = \frac{\sqrt{2}\boldsymbol{P}_\perp}{E}$$ opening angle

$$\theta_c = \frac{2}{\sqrt{\hat{q}L^3}}$$

$$\theta_s = \frac{\sqrt{\hat{q}L}}{E}$$

$$\theta_L = \frac{1}{\sqrt{EL}}$$

Master equation: perturbative relative diffusion

- Gaussian ansatz:

$$\rho_{R,s}(\bar{\boldsymbol{p}}_\perp, \bar{\boldsymbol{r}}_\perp; \bar{t}) = \frac{\alpha(0)\beta(0)}{\pi^2} \exp\left[-\alpha(\bar{t})\bar{\boldsymbol{r}}_\perp^2 - \beta(\bar{t})\bar{\boldsymbol{p}}_\perp^2 - \delta(\bar{t}) + \gamma(\bar{t})\bar{\boldsymbol{p}}_\perp \cdot \bar{\boldsymbol{r}}_\perp \right.$$
$$\left. + \boldsymbol{\epsilon}_\perp(\bar{t}) \cdot \bar{\boldsymbol{p}}_\perp + \boldsymbol{\zeta}_\perp(\bar{t}) \cdot \bar{\boldsymbol{r}}_\perp\right]$$

$$\dot{\alpha} = \frac{1}{2} - \frac{1}{8}\kappa^2\gamma^2\,, \quad \dot{\beta} - \gamma = -\frac{1}{2}\kappa^2\beta^2\,, \quad \dot{\gamma} - 2\alpha = -\frac{\kappa^2}{2}\gamma\beta\,, \quad \dot{\delta} = \frac{1}{2}\kappa^2\beta - \frac{1}{8}\kappa^2|\boldsymbol{\epsilon}_\perp|^2\,,$$
$$\dot{\boldsymbol{\epsilon}}_\perp = -\boldsymbol{\zeta}_\perp - \frac{1}{2}\kappa^2\beta\boldsymbol{\epsilon}_\perp\,, \quad \dot{\boldsymbol{\zeta}}_\perp = \frac{1}{4}\kappa^2\gamma\boldsymbol{\epsilon}_\perp$$

$$\rho_{R,s}(\bar{\boldsymbol{p}}_\perp, \bar{\boldsymbol{r}}_\perp; \bar{t}) = \frac{1}{\pi^2\sigma^2_{\bar{p},0}\sigma^2_{\bar{r},0}} \exp\left[-\frac{\sigma^2_{\bar{p}}\delta\bar{\boldsymbol{r}}_\perp^2 + \sigma^2_{\bar{r}}\delta\bar{\boldsymbol{p}}_\perp^2 - 2c_{\bar{r},\bar{p}}\delta\bar{\boldsymbol{r}}_\perp \cdot \delta\bar{\boldsymbol{p}}_\perp}{\sigma^2_{\bar{r}}\sigma^2_{\bar{p}} - c^2_{\bar{r},\bar{p}}} - \tilde{\delta}\right]$$

$$\delta\bar{\boldsymbol{r}}_\perp = \bar{\boldsymbol{r}}_\perp - \langle\bar{\boldsymbol{r}}_\perp\rangle \qquad \delta\bar{\boldsymbol{p}}_\perp = \bar{\boldsymbol{p}}_\perp - \langle\bar{\boldsymbol{p}}_\perp\rangle \qquad \sigma^2_{\bar{r}} = \langle\delta\bar{\boldsymbol{r}}^2_\perp\rangle \qquad \sigma^2_{\bar{p}} = \langle\delta\bar{\boldsymbol{p}}^2_\perp\rangle \qquad c_{\bar{r},\bar{p}} = \langle\delta\bar{\boldsymbol{p}}_\perp \cdot \delta\bar{\boldsymbol{r}}_\perp\rangle$$

Master equation: perturbative relative diffusion

- Zeroth moment (singlet survival probability)

$$\frac{\mathrm{d}P_s}{\mathrm{d}\bar{t}} = -\frac{\langle \bar{\boldsymbol{r}}_\perp \rangle^2 + \sigma_{\bar{r}}^2}{2} P_s$$

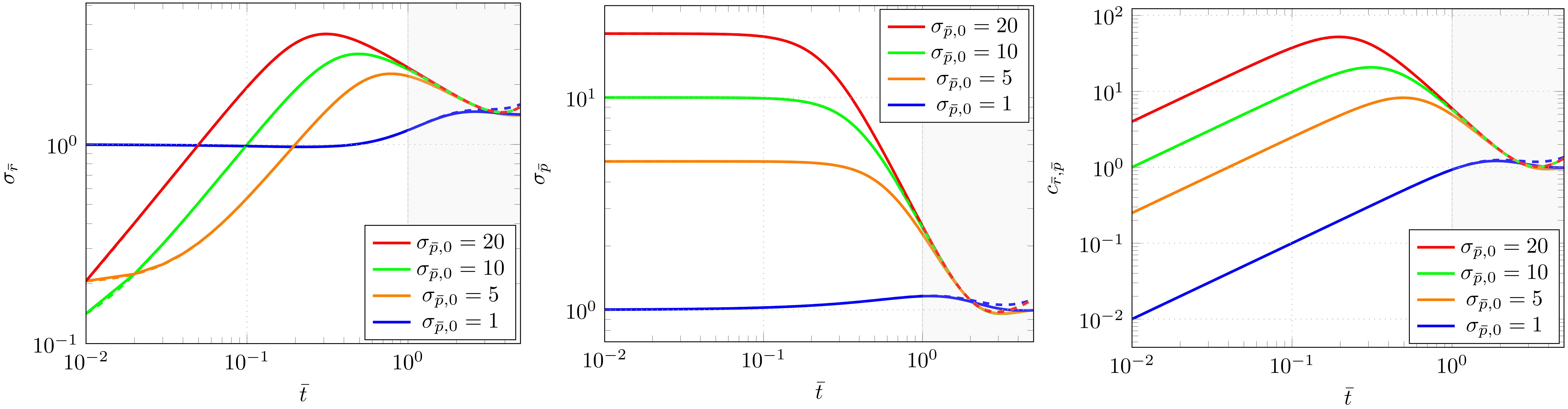
- First moments

$$\frac{\mathrm{d}\langle \bar{\boldsymbol{r}}_\perp \rangle}{\mathrm{d}\bar{t}} = \langle \bar{\boldsymbol{p}}_\perp \rangle - \frac{\sigma_{\bar{r}}^2}{2} \langle \bar{\boldsymbol{r}}_\perp \rangle, \qquad \frac{\mathrm{d}\langle \bar{\boldsymbol{p}}_\perp \rangle}{\mathrm{d}\bar{t}} = -\frac{1}{2} c_{\bar{r},\bar{p}} \langle \bar{\boldsymbol{r}}_\perp \rangle$$

- Second moments

$$\frac{\mathrm{d}\sigma_{\bar{r}}^2}{\mathrm{d}\bar{t}} = 2c_{\bar{r},\bar{p}} - \frac{\sigma_{\bar{r}}^4}{2}, \quad \frac{\mathrm{d}\sigma_{\bar{p}}^2}{\mathrm{d}\bar{t}} = \frac{\kappa^2}{2} - \frac{c_{\bar{r},\bar{p}}^2}{2} \qquad \frac{\mathrm{d}c_{\bar{r},\bar{p}}}{\mathrm{d}\bar{t}} = \sigma_{\bar{p}}^2 - \frac{\sigma_{\bar{r}}^2\, c_{\bar{r},\bar{p}}}{2}$$

Master equation: perturbative relative diffusion



In-medium gluon radiation project

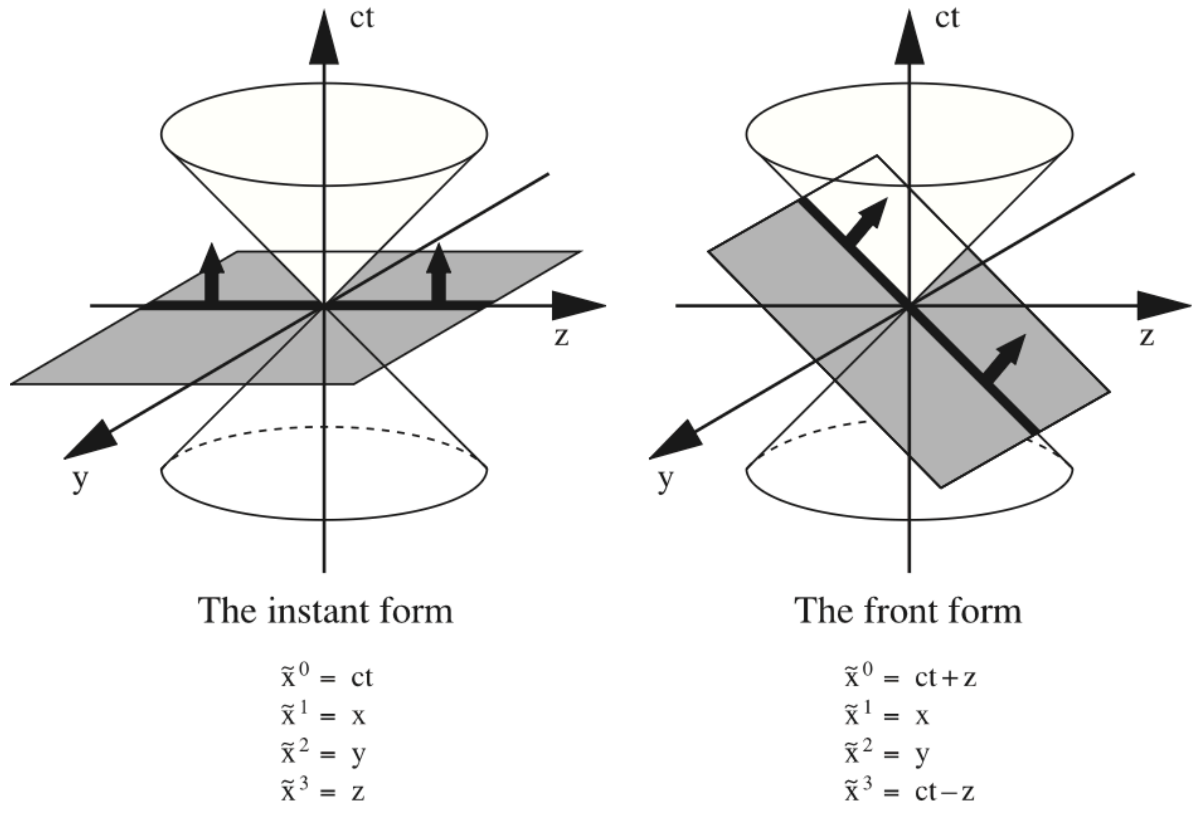
- We want to apply the OQS framework to study the effects of **in-medium gluon radiation**
- Generalisation w.r.t. the first part of the project:

2-particles Hilbert space
no creation/annihilation

⟶

1- and 2- particles Fock space
($q \to qg$, $g \to gg$), and more…
possibility to **radiate**: splittings (and annihilations)

- In **light-cone quantisation**: vacuum is the same at each front x^+, no need of asymptotic states: real-(light-cone)time evolution



The instant form
$\bar{x}^0 = ct$
$\bar{x}^1 = x$
$\bar{x}^2 = y$
$\bar{x}^3 = z$

The front form
$\bar{x}^0 = ct + z$
$\bar{x}^1 = x$
$\bar{x}^2 = y$
$\bar{x}^3 = ct - z$

  - Applications in QCD [Brodsky et al., Phys.Rept. 301 (1998) 299-486]

jet quenching in QGP and cold nuclear matter [Li et al., Phys.Rev.D 112 (2025) 1, 016009]
Phys.Rev.D 108 (2023) 3, 3
Phys.Rev.D 104 (2021) 5, 056014

and glasma [Avramescu et al., 2605.10413]

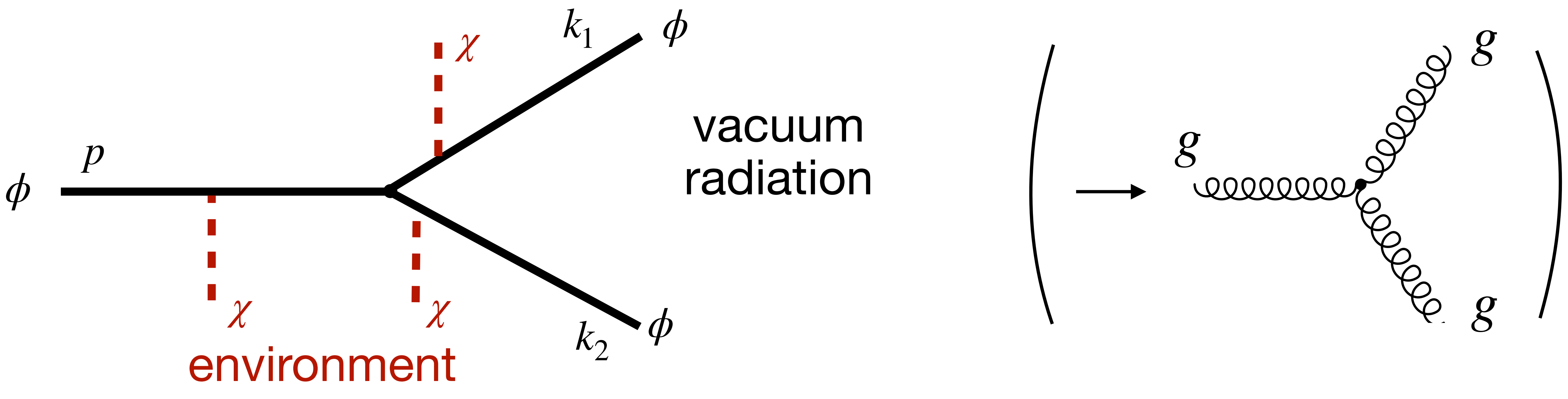
we can contribute with **OQS** !

with quantum computing [Barata et al., 2604.11616]
[Qian et al., Phys.Rev.D 111 (2025) 9, 9]
[Barata et al., Phys.Rev.D 108 (2023) 5, 056023]

Towards a master equation with radiation

- First steps with a massive scalar ϕ^3 toy model: with **light-cone quantisation**

$$H_{\text{int}} = \int \mathrm{d}^3x \, \{g\phi^3 \otimes \mathbb{1}_E + \mu\phi^2 \otimes \chi\}$$



  - **vacuum radiation:** ϕ^3 splittings → ‣ transitions between states with a different number of particles

  - **in-medium modifications:** $\phi^2 \otimes \chi$ scatterings → ‣ modifications to vacuum radiation
    - ‣ not directly producing transitions
    - ‣ match with the previous “radiation free” cases (single particle, dipole,…)

Towards a master equation with radiation

- Von Neumann's equation → partial trace

vacuum radiation

$$\frac{\mathrm{d}\rho^I(x)}{\mathrm{d}x^+} = -ig\int_{x^-\boldsymbol{x}_\perp}\left[\phi^3(x),\rho^I(x)\right] - \int_{x^-\boldsymbol{x}_\perp}\int_{x'^-\boldsymbol{x}'_\perp}\int_0^{x^+}\mathrm{d}x'^+\Big\{g^2\left[\phi^3(x),\left[\phi^3(x'),\rho^I(x)\right]\right]$$

$$+\,\mu^2\Big(\left[\phi^2(x),\phi^2(x')\rho^I(x)\right]\langle\chi(x)\chi(x')\rangle_{\rho_E} + \left[\rho^I(x)\phi^2(x'),\phi^2(x)\right]\langle\chi(x')\chi(x)\rangle_{\rho_E}\Big)\Big\}$$

in-medium modifications

same structure as for the $q\bar{q}$ dipole

↓

$$\langle\mathcal{F}|\cdot|\mathcal{F}'\rangle$$

with

$$|\mathcal{F}\rangle = \left[\prod_{n=1}^{N} a^\dagger(k_n^+,\boldsymbol{k}_\perp)\right]|0\rangle$$

n-particle Fock state

↓

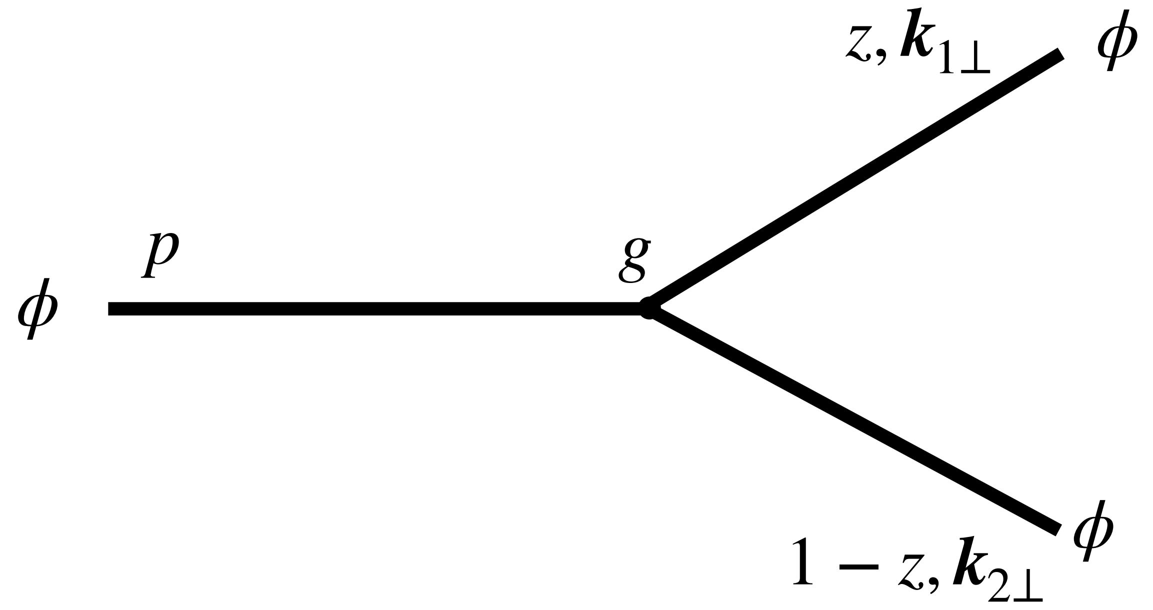
- truncate Fock space up to 2-particles states (extendable)

$$\langle 1,\{k\}|\cdot|1,\{k'\}\rangle \qquad \langle 1,\{k\}|\cdot|2,\{k_1,k_2\}\rangle \qquad \langle 2,\{k_1,k_2\}|\cdot|2,\{k_1',k_2'\}\rangle$$

- evaluate matrix elements of fields (automated in Mathematica)

Vacuum radiation: splittings $g\phi^3 \otimes \mathbb{1}_E$

$$\frac{\mathrm{d}\rho^I(x)}{\mathrm{d}x^+} = -ig\int_{x^-\boldsymbol{x}_\perp}\left[\phi^3(x),\rho^I(x)\right] - \int_{x^-\boldsymbol{x}_\perp}\int_{x'^-\boldsymbol{x}'_\perp}\int_0^{x^+}\mathrm{d}x'^+\left\{g^2\left[\phi^3(x),\left[\phi^3(x'),\rho^I(x)\right]\right]\right. + \dots$$

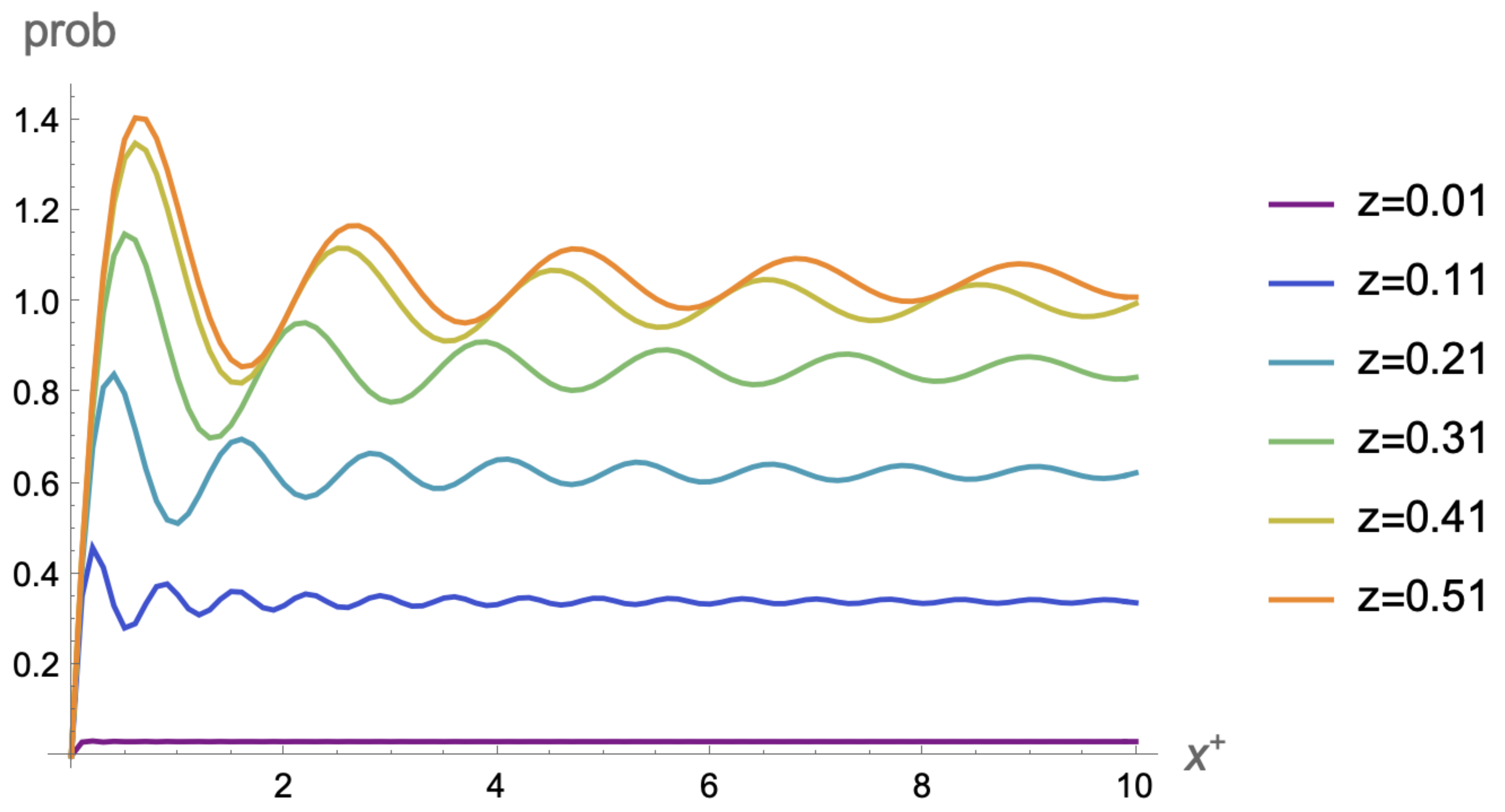


- 2-particles probability: $\langle 2,\{k_1,k_2\}|\,\rho\,|2,\{k_1,k_2\}\rangle$

obtained by applying a **“strict” perturbation** expansion

⟶ iterate by inserting in the equation the solution

$$\rho(x^+) = \rho(0) + O(g)$$



- 1-particle + 2-particles probability $= 1$

In-medium scatterings $\mu\phi^2 \otimes \chi$

$$\frac{\mathrm{d}\rho^I(x)}{\mathrm{d}x^+} = \ldots + \mu^2\Big(\left[\phi^2(x), \phi^2(x')\rho^I(x)\right] \langle\chi(x)\chi(x')\rangle_{\rho_E} + \left[\rho^I(x)\phi^2(x'), \phi^2(x)\right] \langle\chi(x')\chi(x)\rangle_{\rho_E}\Big)\Big\}$$

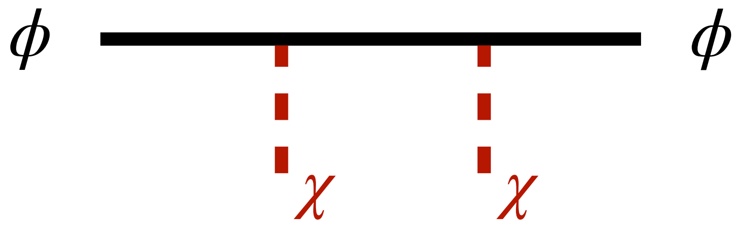
- Medium correlator: $\langle\chi(x^+, \boldsymbol{x}_\perp)\chi(x'^+, \boldsymbol{x}'_\perp)\rangle_{\rho_E} = \delta(x^+ - x'^+)\Gamma(\boldsymbol{x}_\perp - \boldsymbol{x}'_\perp)$ (Markovianity)

In-medium scatterings $\mu\phi^2 \otimes \chi$

$$\frac{\mathrm{d}\rho^I(x)}{\mathrm{d}x^+} = \ldots + \mu^2\Big(\left[\phi^2(x), \phi^2(x')\rho^I(x)\right] \langle\chi(x)\chi(x')\rangle_{\rho_E} + \left[\rho^I(x)\phi^2(x'), \phi^2(x)\right] \langle\chi(x')\chi(x)\rangle_{\rho_E}\Big)\Big\}$$

- Medium correlator: $\langle\chi(x^+, \boldsymbol{x}_\perp)\chi(x'^+, \boldsymbol{x}'_\perp)\rangle_{\rho_E} = \delta(x^+ - x'^+)\Gamma(\boldsymbol{x}_\perp - \boldsymbol{x}'_\perp)$ (Markovianity)

$$\langle 1, \{k\}| \cdot |1, \{k'\}\rangle$$



suitable change of variables

$$\frac{\mathrm{d}}{\mathrm{d}x^+}\rho_{1,1}(\boldsymbol{l}_\perp, \boldsymbol{x}_\perp, x^+) + \frac{\boldsymbol{l}_\perp \cdot \partial_{\boldsymbol{x}_\perp}}{2k^+}\rho_{1,1}(\boldsymbol{l}_\perp, \boldsymbol{x}_\perp, x^+) =$$

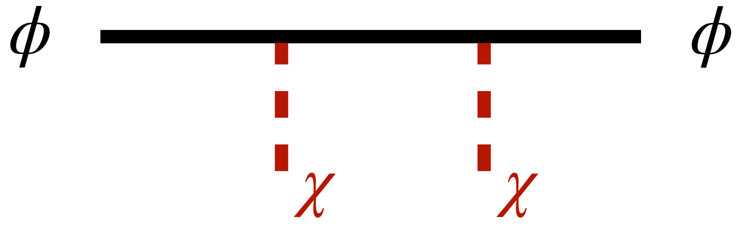
$$= 2\mu^2 \int_{\boldsymbol{q}_\perp} \gamma(\boldsymbol{q}_\perp)\left[e^{i\boldsymbol{q}_\perp \cdot \boldsymbol{x}_\perp} - 1\right]\rho_{1,1}(\boldsymbol{l}_\perp, \boldsymbol{x}_\perp, x^+),$$

In-medium scatterings $\mu\phi^2 \otimes \chi$

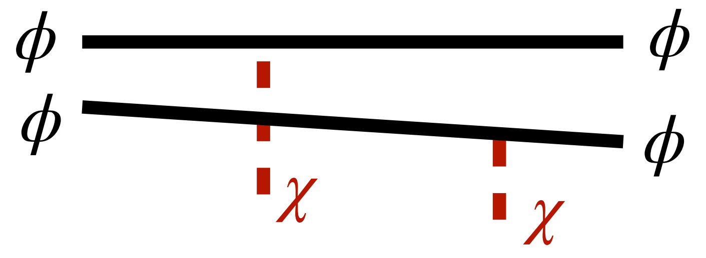
$$\frac{\mathrm{d}\rho^I(x)}{\mathrm{d}x^+} = \ldots + \mu^2\Big(\left[\phi^2(x), \phi^2(x')\rho^I(x)\right] \langle\chi(x)\chi(x')\rangle_{\rho_E} + \left[\rho^I(x)\phi^2(x'), \phi^2(x)\right] \langle\chi(x')\chi(x)\rangle_{\rho_E} \Big)\Big\}$$

- Medium correlator: $\langle\chi(x^+, \boldsymbol{x}_\perp)\chi(x'^+, \boldsymbol{x}'_\perp)\rangle_{\rho_E} = \delta(x^+ - x'^+)\Gamma(\boldsymbol{x}_\perp - \boldsymbol{x}'_\perp)$ (Markovianity)

$\langle 1, \{k\}| \cdot |1, \{k'\}\rangle$



$\langle 2, \{k_1, k_2\}| \cdot |2, \{k'_1, k'_2\}\rangle$



suitable change of variables

$$\frac{\mathrm{d}}{\mathrm{d}x^+}\rho_{1,1}(\boldsymbol{l}_\perp, \boldsymbol{x}_\perp, x^+) + \frac{\boldsymbol{l}_\perp \cdot \partial_{\boldsymbol{x}_\perp}}{2k^+}\rho_{1,1}(\boldsymbol{l}_\perp, \boldsymbol{x}_\perp, x^+) =$$
$$= 2\mu^2 \int_{\boldsymbol{q}_\perp} \gamma(\boldsymbol{q}_\perp)\left[e^{i\boldsymbol{q}_\perp \cdot \boldsymbol{x}_\perp} - 1\right]\rho_{1,1}(\boldsymbol{l}_\perp, \boldsymbol{x}_\perp, x^+),$$

$$\frac{\mathrm{d}}{\mathrm{d}x^+}\rho_{2,2}(\boldsymbol{p}_\perp, \boldsymbol{r}_\perp; \boldsymbol{q}_\perp, \boldsymbol{b}_\perp, x^+) + \left(\frac{1}{E}\boldsymbol{p}_\perp \cdot \nabla_{\boldsymbol{r}_\perp} + \frac{1}{q^+}\boldsymbol{q}_\perp \cdot \nabla_{\boldsymbol{b}_\perp}\right)\rho_{2,2}(\boldsymbol{p}_\perp, \boldsymbol{r}_\perp; \boldsymbol{q}_\perp, \boldsymbol{b}_\perp; x^+) =$$
$$= 128\mu^2\left\{\left(\frac{3}{2}(z_1^2 + z_2^2) - z_1 z_2 - \frac{1}{2}\right)\nabla^2_{\boldsymbol{p}_\perp} + 4\nabla^2_{\boldsymbol{q}_\perp} - 4(z_1 - z_2)\nabla_{\boldsymbol{q}_\perp} \cdot \nabla_{\boldsymbol{p}_\perp}\right\}\cdot$$
$$\times\, \rho_{2,2}\left(\boldsymbol{p}_\perp, \boldsymbol{r}_\perp; \boldsymbol{q}_\perp, \boldsymbol{b}_\perp; x^+\right)$$

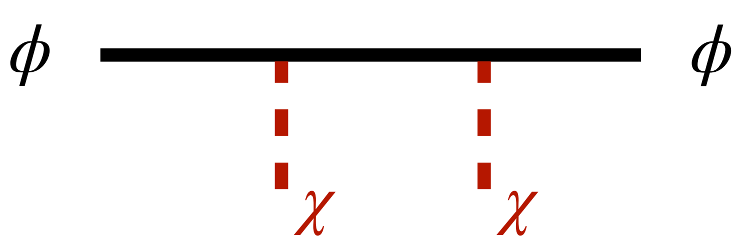
(with harmonic approximation)

In-medium scatterings $\mu\phi^2 \otimes \chi$

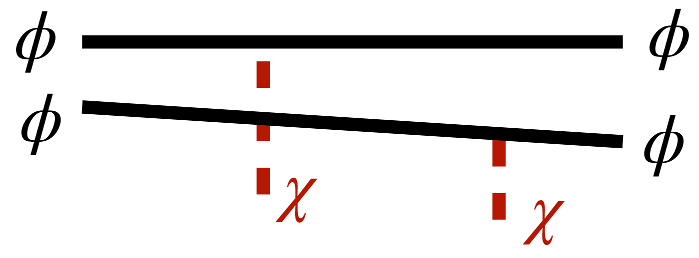
$$\frac{\mathrm{d}\rho^I(x)}{\mathrm{d}x^+} = \ldots + \mu^2\Big(\left[\phi^2(x), \phi^2(x')\rho^I(x)\right] \langle\chi(x)\chi(x')\rangle_{\rho_E} + \left[\rho^I(x)\phi^2(x'), \phi^2(x)\right] \langle\chi(x')\chi(x)\rangle_{\rho_E} \Big)\Big\}$$

- Medium correlator: $\langle\chi(x^+, \boldsymbol{x}_\perp)\chi(x'^+, \boldsymbol{x}'_\perp)\rangle_{\rho_E} = \delta(x^+ - x'^+)\Gamma(\boldsymbol{x}_\perp - \boldsymbol{x}'_\perp)$ (Markovianity)

$\langle 1, \{k\}| \cdot |1, \{k'\}\rangle$



$\langle 2, \{k_1, k_2\}| \cdot |2, \{k'_1, k'_2\}\rangle$



suitable change of variables

$$\frac{\mathrm{d}}{\mathrm{d}x^+}\rho_{1,1}(\boldsymbol{l}_\perp, \boldsymbol{x}_\perp, x^+) + \frac{\boldsymbol{l}_\perp \cdot \partial_{\boldsymbol{x}_\perp}}{2k^+}\rho_{1,1}(\boldsymbol{l}_\perp, \boldsymbol{x}_\perp, x^+) =$$
$$= 2\mu^2 \int_{\boldsymbol{q}_\perp} \gamma(\boldsymbol{q}_\perp) \left[e^{i\boldsymbol{q}_\perp \cdot \boldsymbol{x}_\perp} - 1\right] \rho_{1,1}(\boldsymbol{l}_\perp, \boldsymbol{x}_\perp, x^+),$$

$$\frac{\mathrm{d}}{\mathrm{d}x^+}\rho_{2,2}(\boldsymbol{p}_\perp, \boldsymbol{r}_\perp; \boldsymbol{q}_\perp, \boldsymbol{b}_\perp, x^+) + \left(\frac{1}{E}\boldsymbol{p}_\perp \cdot \nabla_{\boldsymbol{r}_\perp} + \frac{1}{q^+}\boldsymbol{q}_\perp \cdot \nabla_{\boldsymbol{b}_\perp}\right) \rho_{2,2}(\boldsymbol{p}_\perp, \boldsymbol{r}_\perp; \boldsymbol{q}_\perp, \boldsymbol{b}_\perp; x^+) =$$
$$= 128\mu^2 \left\{ \left(\frac{3}{2}(z_1^2 + z_2^2) - z_1 z_2 - \frac{1}{2}\right) \nabla^2_{\boldsymbol{p}_\perp} + 4\nabla^2_{\boldsymbol{q}_\perp} - 4(z_1 - z_2)\nabla_{\boldsymbol{q}_\perp} \cdot \nabla_{\boldsymbol{p}_\perp} \right\} \cdot$$
$$\times \rho_{2,2}\left(\boldsymbol{p}_\perp, \boldsymbol{r}_\perp; \boldsymbol{q}_\perp, \boldsymbol{b}_\perp; x^+\right)$$

(with harmonic approximation)

colourless limit of previously investigated cases:

single quark master equation
[Barata et al., Phys.Rev.D 108 (2023) 1, 014039]

quark-antiquark dipole master equation
[our $q\bar{q}$ project]