

Probing QCD media with projected energy correlators

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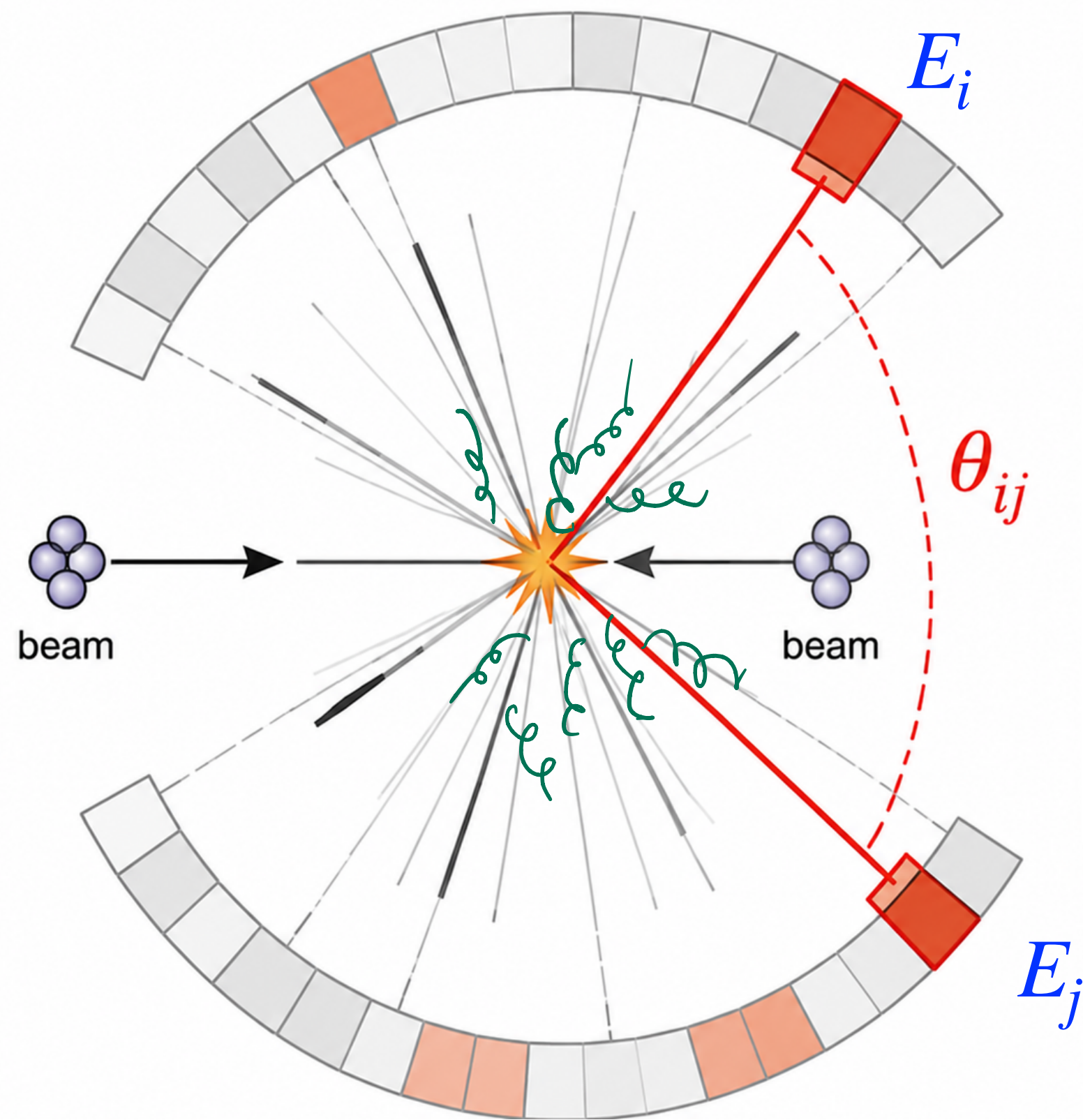


EMMI Workshop: Probing the early stages with jets, July 13-17, 2026

Energy-energy correlator

Measure correlations of energy flow as a function of angular separation between particles

Two point energy correlator, EEC



$$\frac{d\Sigma_{\text{EEC}}}{d\chi} \simeq \sum_{ij} \int \frac{d\sigma}{Q^2} \underbrace{E_i E_j}_{\text{Energy weights}} \underbrace{\delta(\chi - \theta_{ij})}_{\text{Angular measurement}}$$

$\theta_{ij} \longrightarrow$ angular separation between particles

Probe evolution of QCD radiation, DGLAP

IRC safe, well defined in QFT

Perturbatively calculable

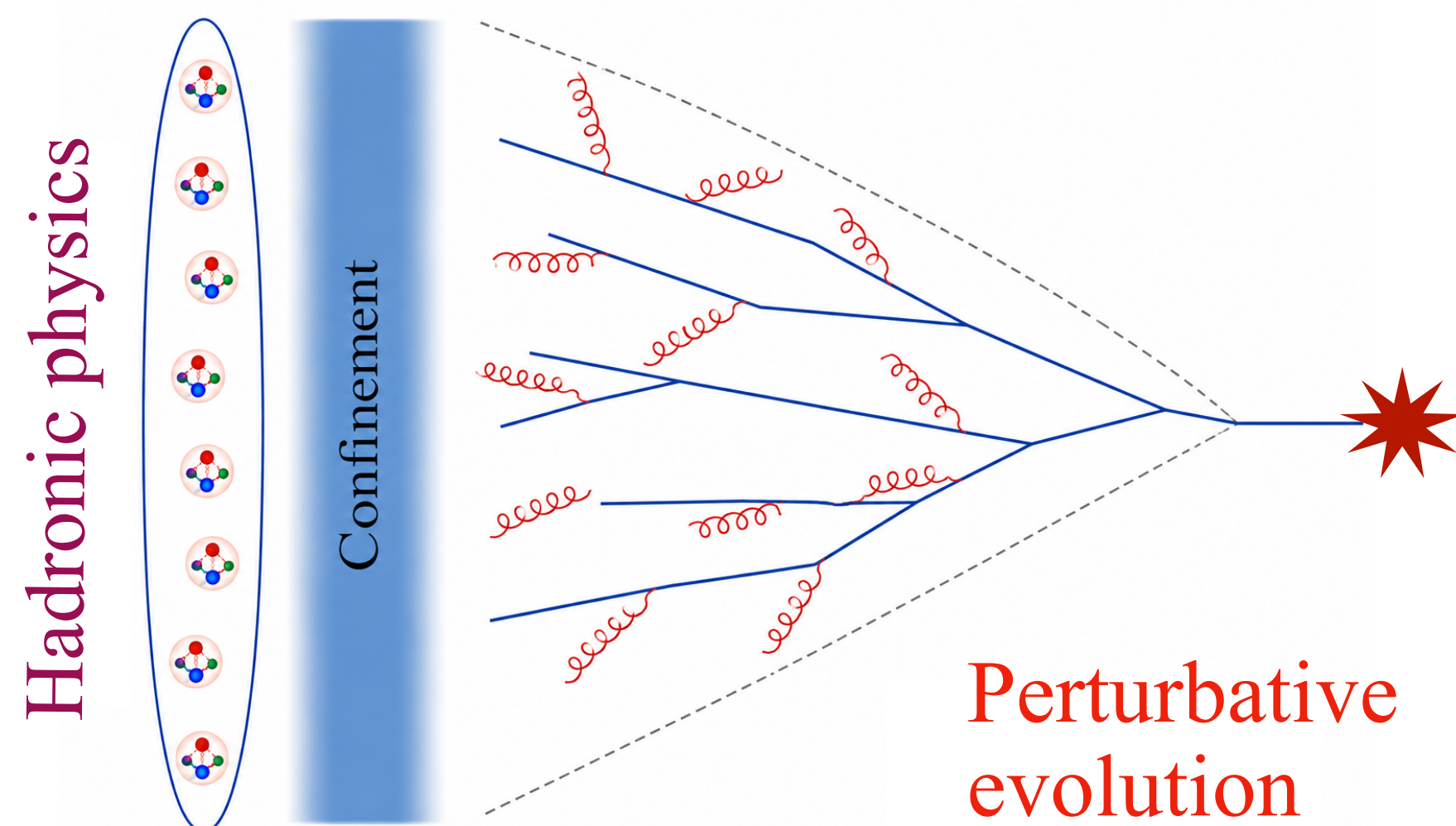
Sensitive to confinement and medium modifications in HICs

For jets: i, j are particles inside the jet

Notations: χ, θ, R_L are same variable

Why Energy correlators

Single measurement images the entire evolution from perturbative production to non-perturbative hadronization

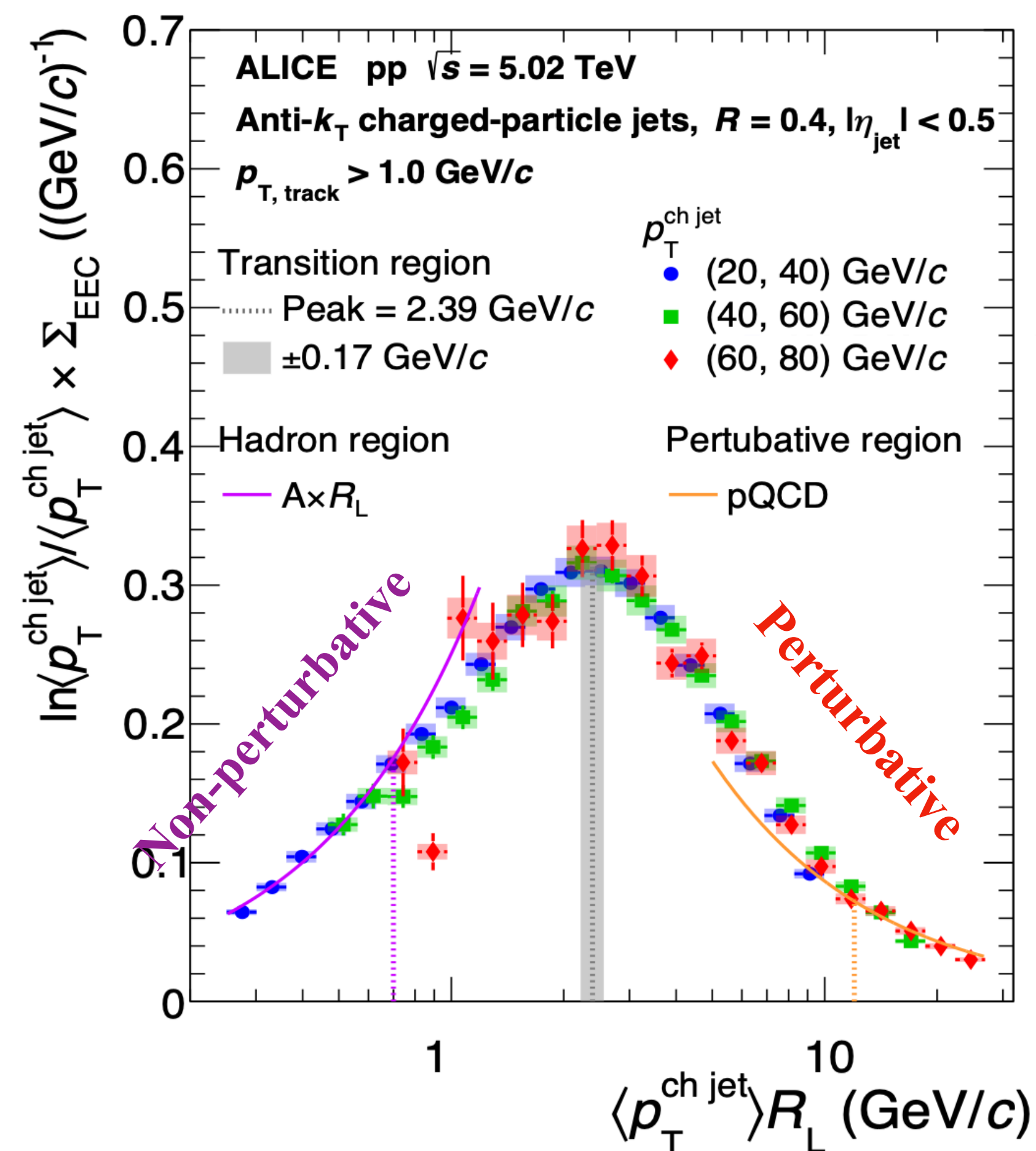


Two distinct scaling behaviors

Access to anomalous dimension

Collinear limit $\frac{d\Sigma_{\text{EEC}}^{\text{pert}}}{d\chi} \sim \chi^{-1+\gamma(3)}$

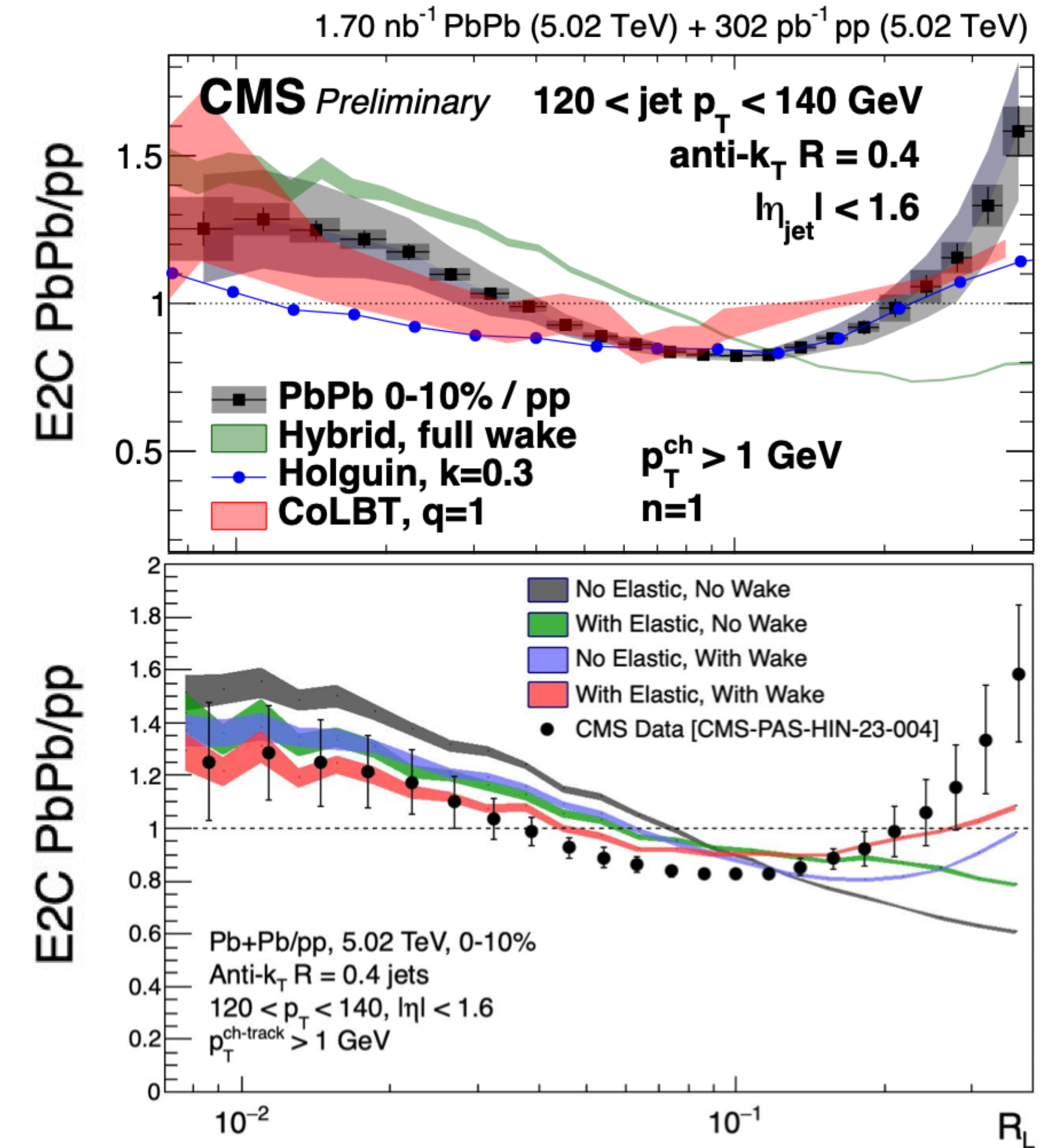
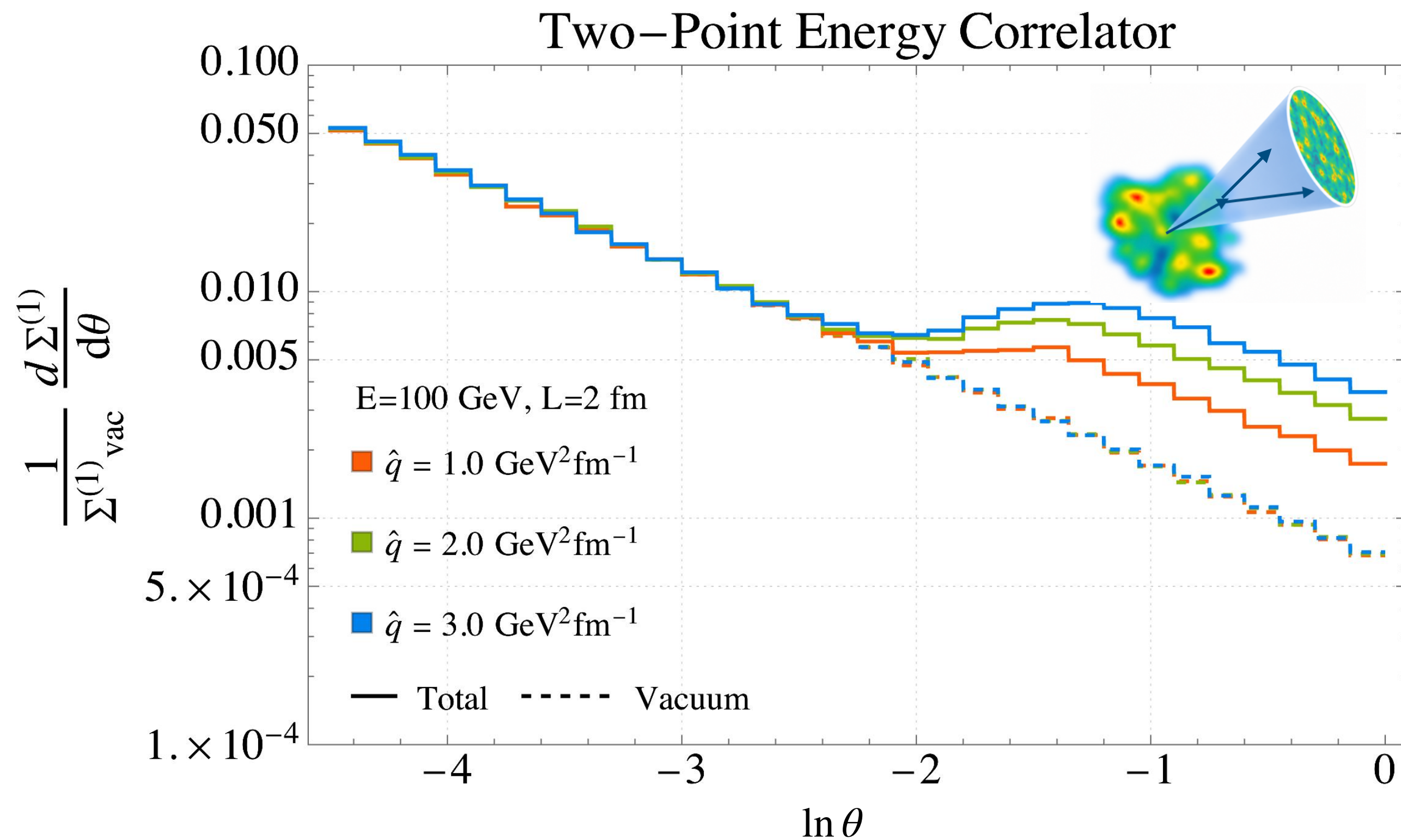
$$\gamma(3) = - \int_0^1 dz z^2 P_{ij}(z)$$



Energy correlators in HICs

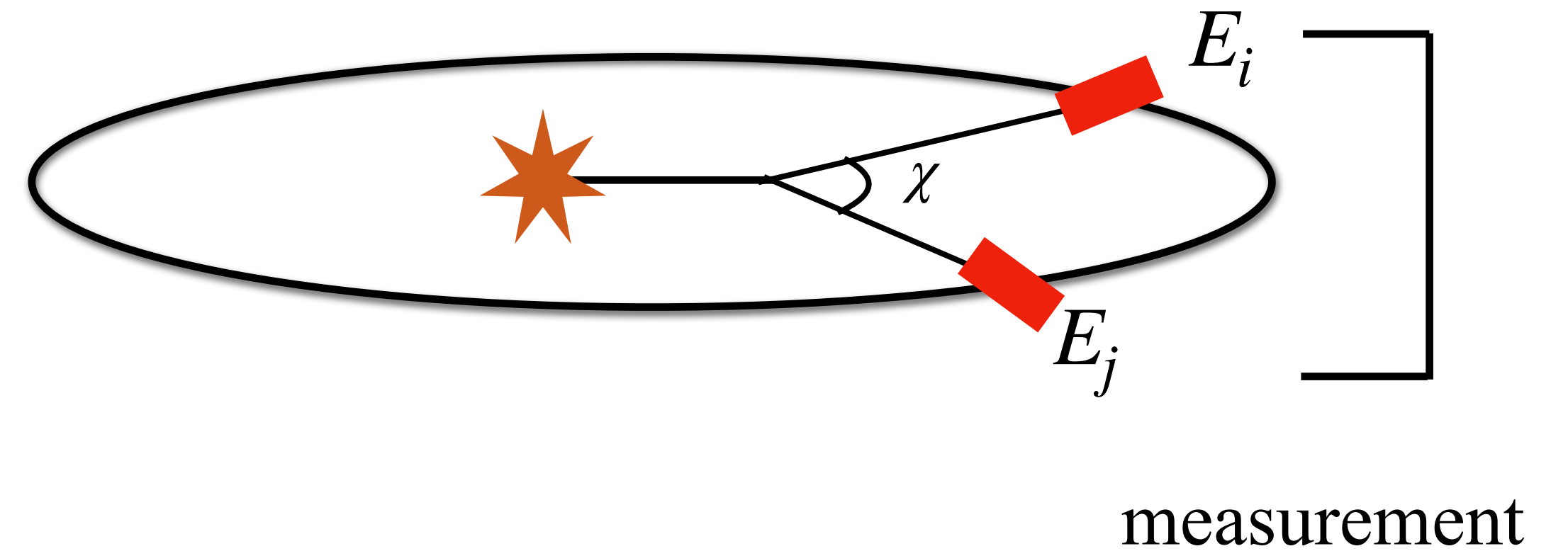
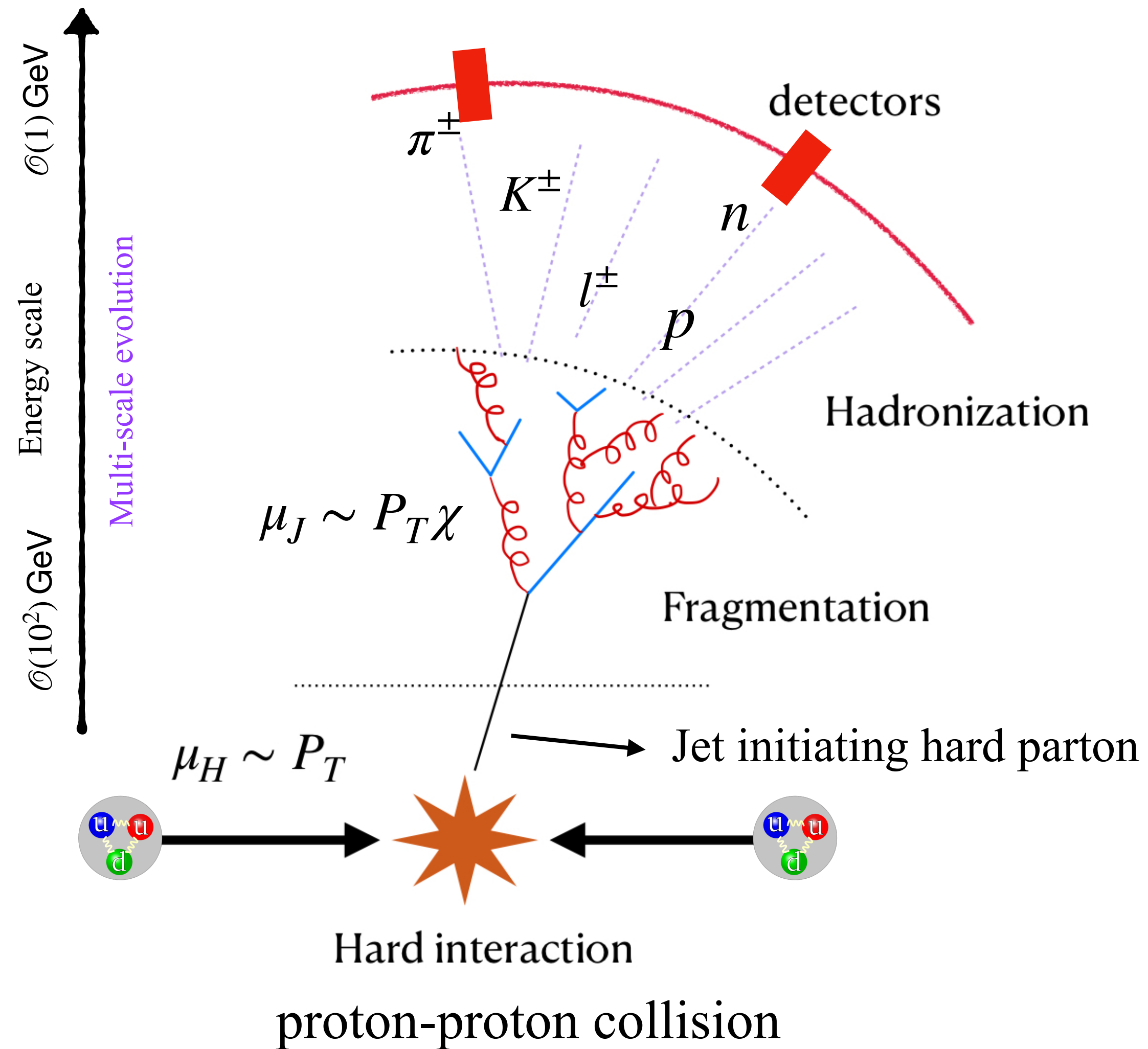
Medium induced dynamics is imprinted on final state distributions

Large angle: medium induced + medium response



I will focus on **factorization approach** for a general **class** of energy correlators

Factorization approach in pp collision



- ➔ E_i and E_j are energies of two particles
- ➔ In pp collision only two scales

$$\mu_H \sim p_T \quad \mu_J \sim p_T \chi$$
- ➔ For $\chi \ll 1$ two scales are widely separated

$$\mu_H \gg \mu_J$$

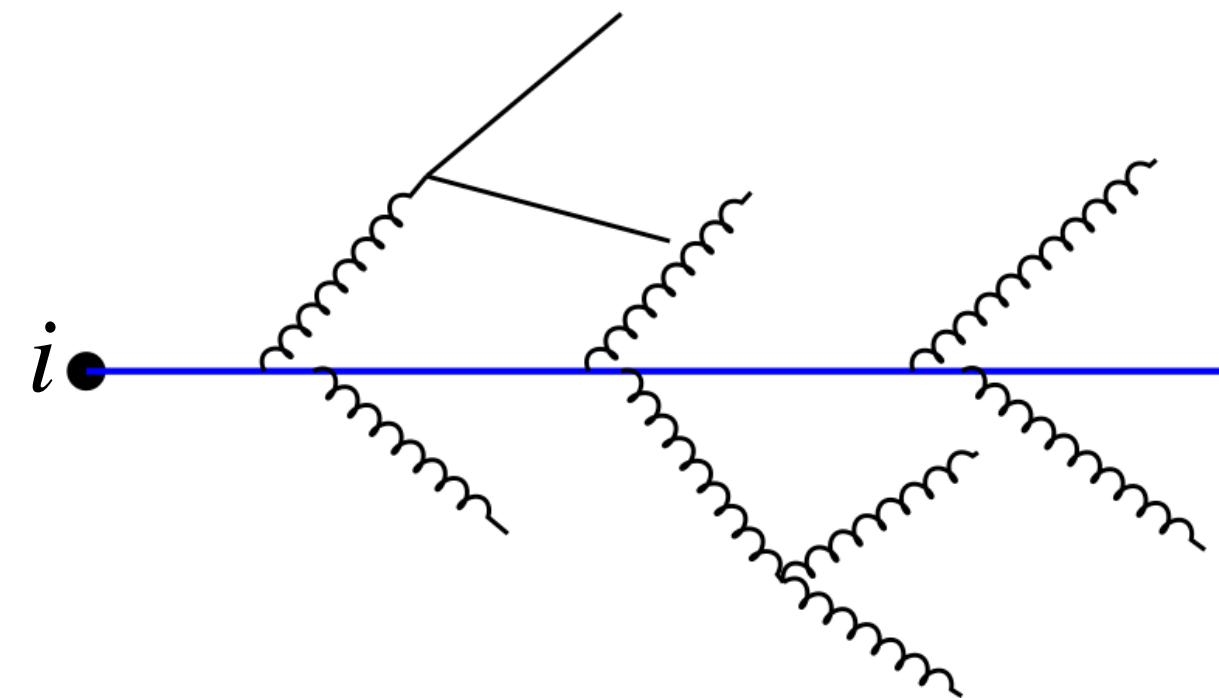
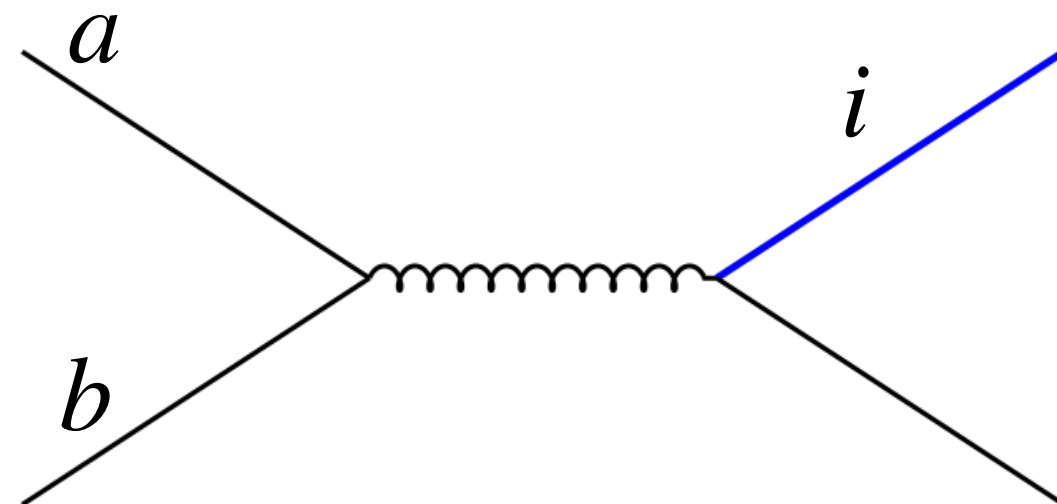
Factorization approach in pp collision

Differential cross section can be factorized; collinear limit $\chi \ll 1$

$$\frac{d\Sigma}{d\chi} = \sum_{a,b,i} \int dx x^2 H_{ab \rightarrow i}(Q, \mu) J_i(Q, \chi, \mu)$$

hard function

Jet function



Production mechanism

Subsequent evolution of jet

Hard function:

production of jet initiating parton, **does not** depend on final state measurements

Jet function:

subsequent evolution, **depends** on **measurement** on final state particles

DGLAP evolution

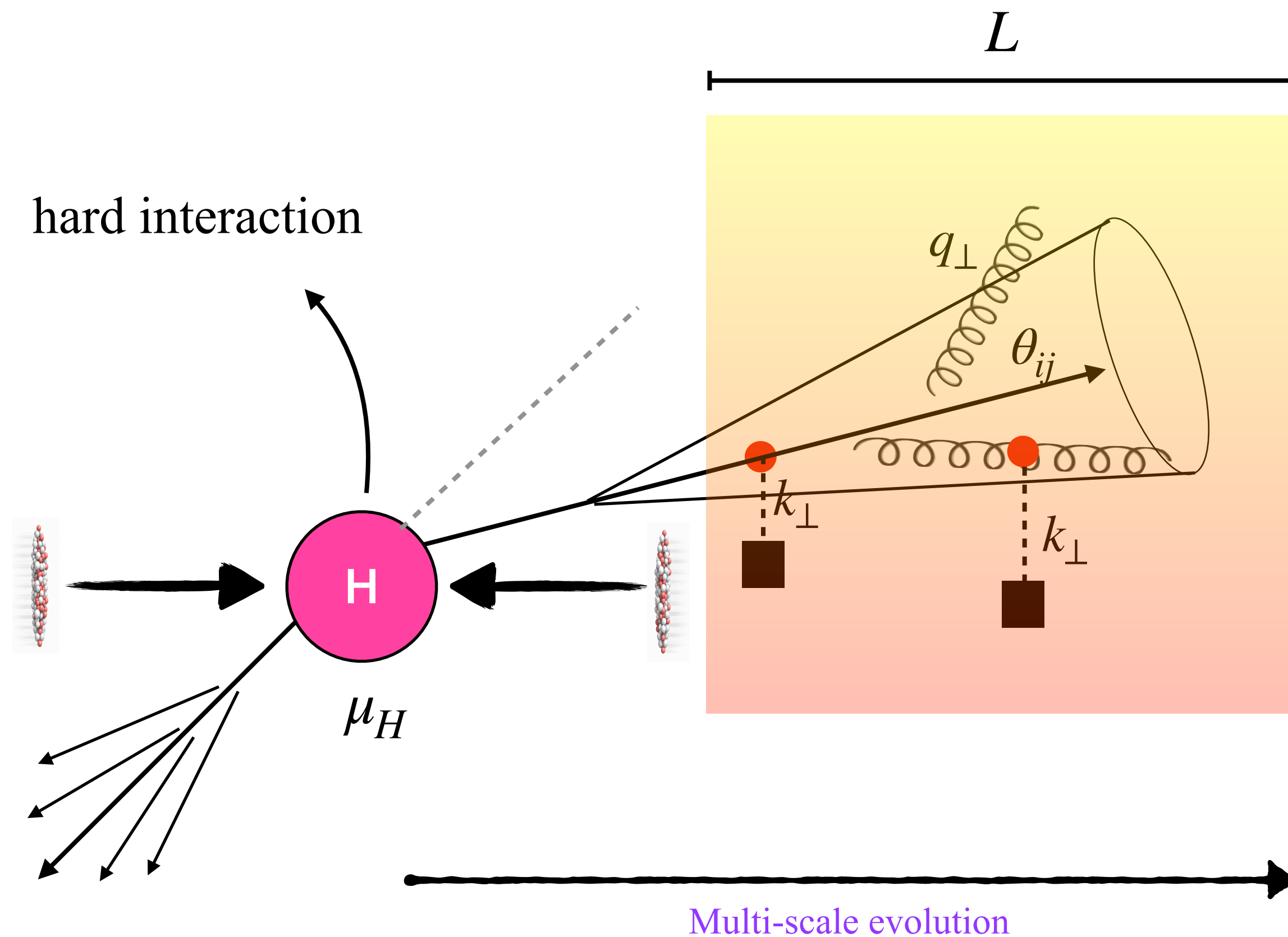
$$\frac{dJ_i(\mu)}{d \ln \mu} = 2 \gamma(3, \alpha_s(\mu)) J_i(\mu)$$

$$J(\chi) \sim \chi^{-1+\gamma(3)}$$

leading log accuracy

Jet in a medium

Jet encounters **various** direct and indirect **scales** introduced by the medium



$l_{\text{mfp}} \rightarrow$ mean free path

$T \sim m_D \rightarrow$ medium temperature

$\tau_f \sim \frac{E_g}{q_\perp^2} \rightarrow$ formation time

Direct
scales

$\hat{q} \rightarrow$ jet quenching parameter

$\theta_c \sim \frac{1}{\sqrt{\hat{q}L^3}} \rightarrow$ critical angle

emergent
scales

Scale hierarchy: $\mu_H \gg \mu_J, T \sim m_D \geq \Lambda_{QCD}$

$g \sim \mathcal{O}(1)$, so T and m_D are not separate scales

We will use these **scales** as a **guide** for the EFT

Soft Collinear Effective Theory (SCET)

SCET 1410.1892

EFT of QCD designed to study **collinear** and **soft** radiations

Collinear: boosted along jet direction

$$A^\mu = A_n^\mu + A_s^\mu \quad \psi = \xi_n + \psi_s$$

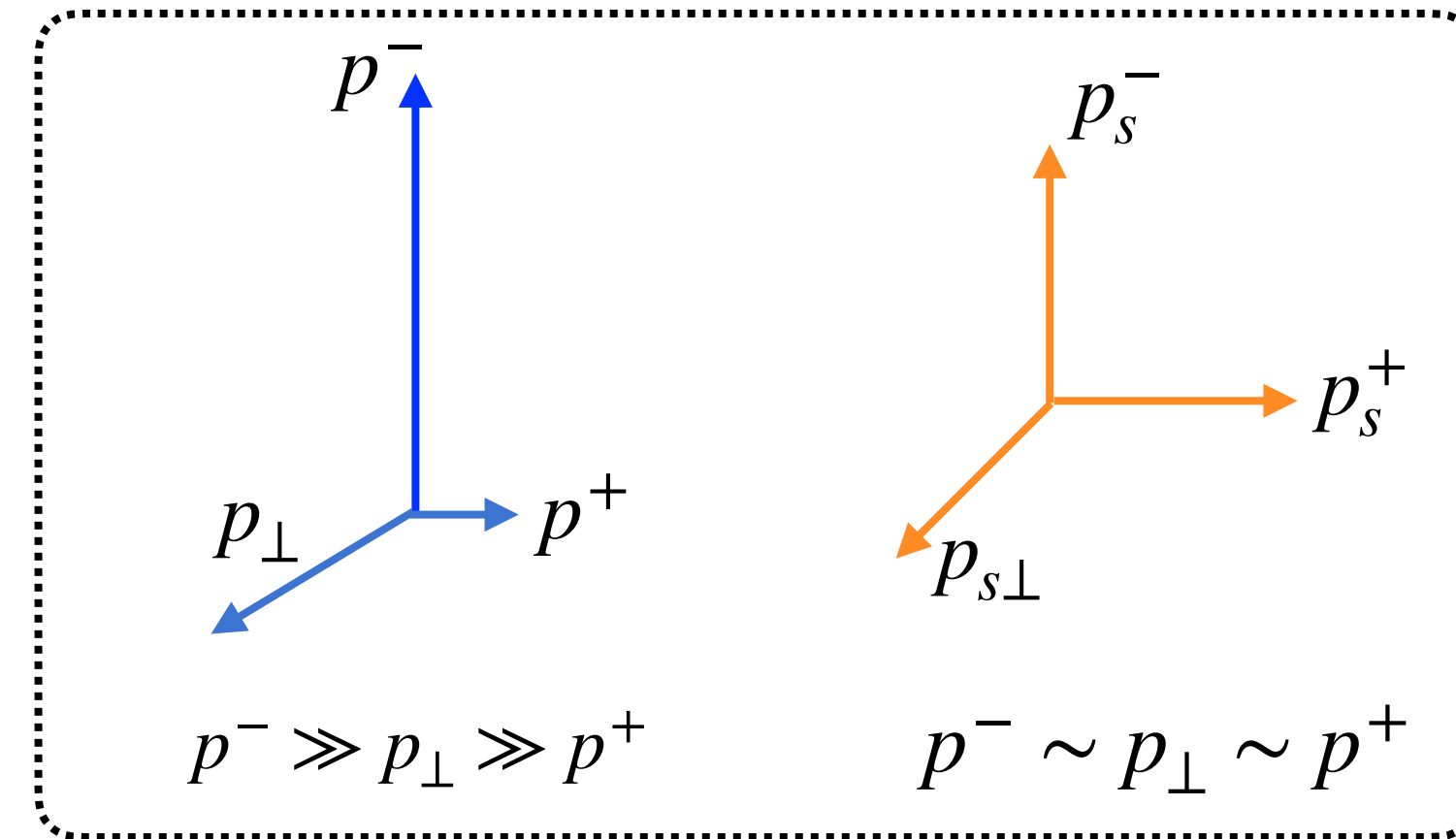
$$\mathcal{L}_n^0 = \bar{\xi}_n \left(in \cdot D + iD_\perp \frac{1}{i\bar{n} \cdot D} iD_\perp \right) \frac{\bar{n}}{2} \xi_n$$

Soft : no preferred direction

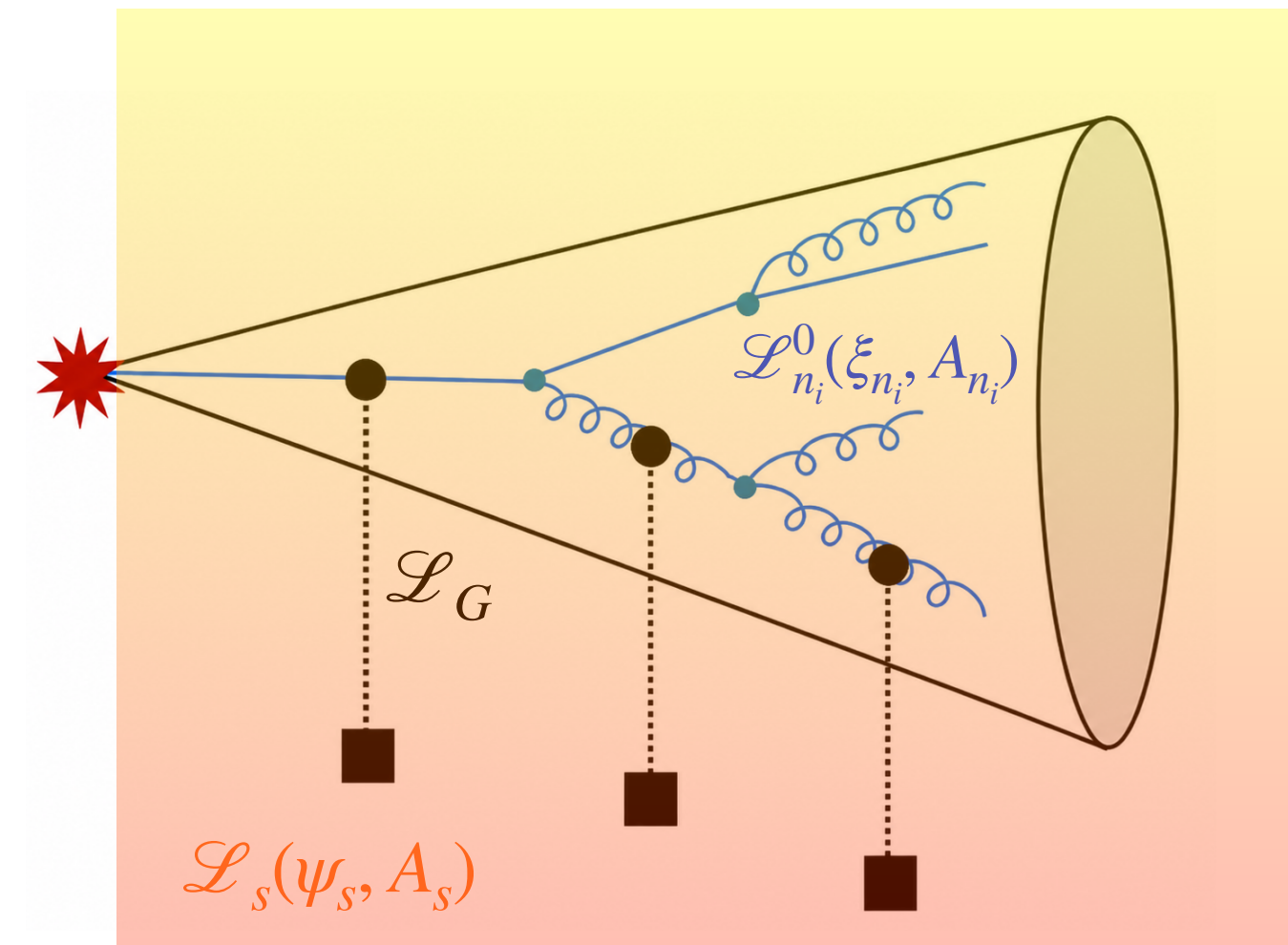
$$\mathcal{L}_{\text{SCET}}^0 = \mathcal{L}_s(\psi_s, A_s) + \sum_{n_i} \mathcal{L}_{n_i}^0(\xi_{n_i}, A_{n_i}) + \mathcal{L}_G(\xi_{n_i}, A_{n_i}, \psi_s, A_s)$$

↓
SCET building blocks

- All collinear emission $\mathcal{L}_n^0$
- Medium interactions $\mathcal{L}_s$ (same as full QCD Lagrangian)
- Jet and medium interactions $\mathcal{L}_G$



$$\begin{aligned} n^\mu &= (1, 0, 0, 1) \\ \bar{n}^\mu &= (1, 0, 0, -1) \\ p^\mu &= (p^-, p^+, \vec{p}_\perp) \\ p^+ &= n \cdot p \\ p^- &= \bar{n} \cdot p \end{aligned}$$



Jet-medium interaction

Glauber operators in EFT

Glauber operators are made up of **bilinears** in each sector and are **Gauge invariant**

$$\boxed{\mathcal{L}_G(x) = \sum_{ij=q,g} C_{ij} O_{ns}^{ij}(x)}$$

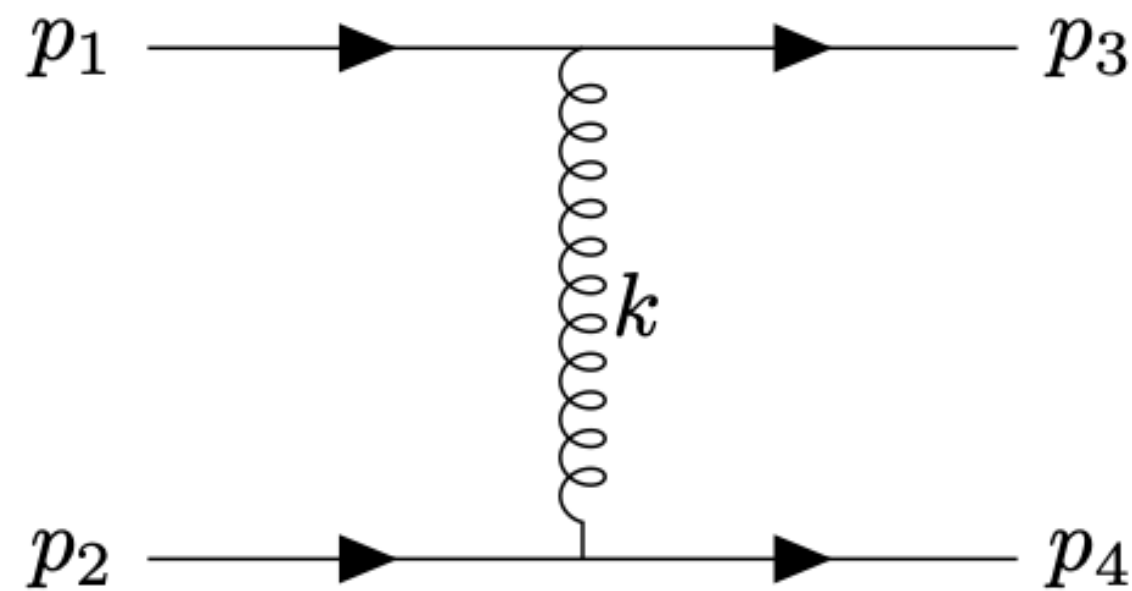
$$\mathcal{O}_{ns}^{qg} = O_n^{qA} \frac{1}{\mathbb{P}_\perp^2} O_s^{gA} \quad \mathcal{O}_{ns}^{qq} = O_n^{qA} \frac{1}{\mathbb{P}_\perp^2} O_s^{qA}$$

$$\mathcal{O}_n^{qA} = \bar{\chi}_n T^A \frac{\vec{n}}{2} \chi_n \quad \mathcal{O}_s^{qA} = \bar{\psi}_s T^A \frac{\vec{n}}{2} \psi_s \quad \chi_n = W_n \xi_n$$

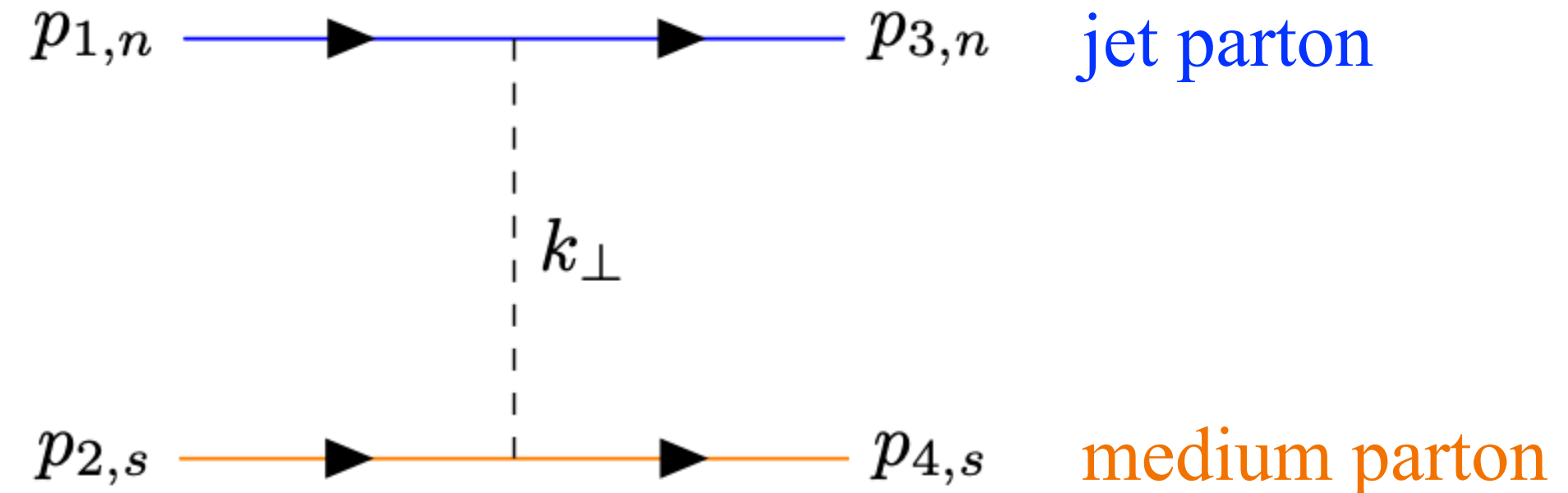
$$W_n(x) = \text{P exp} \left[ig \int_{-\infty}^0 ds \vec{n} \cdot A_n(x + s\vec{n}) \right]$$

$\mathbb{P}_\perp$ is transverse momentum operators that pulls out Glauber gluon momentum from medium operators

C_{ij} is Glauber Wilson coefficient fixed by the matching between full QCD and EFT



Full QCD diagram



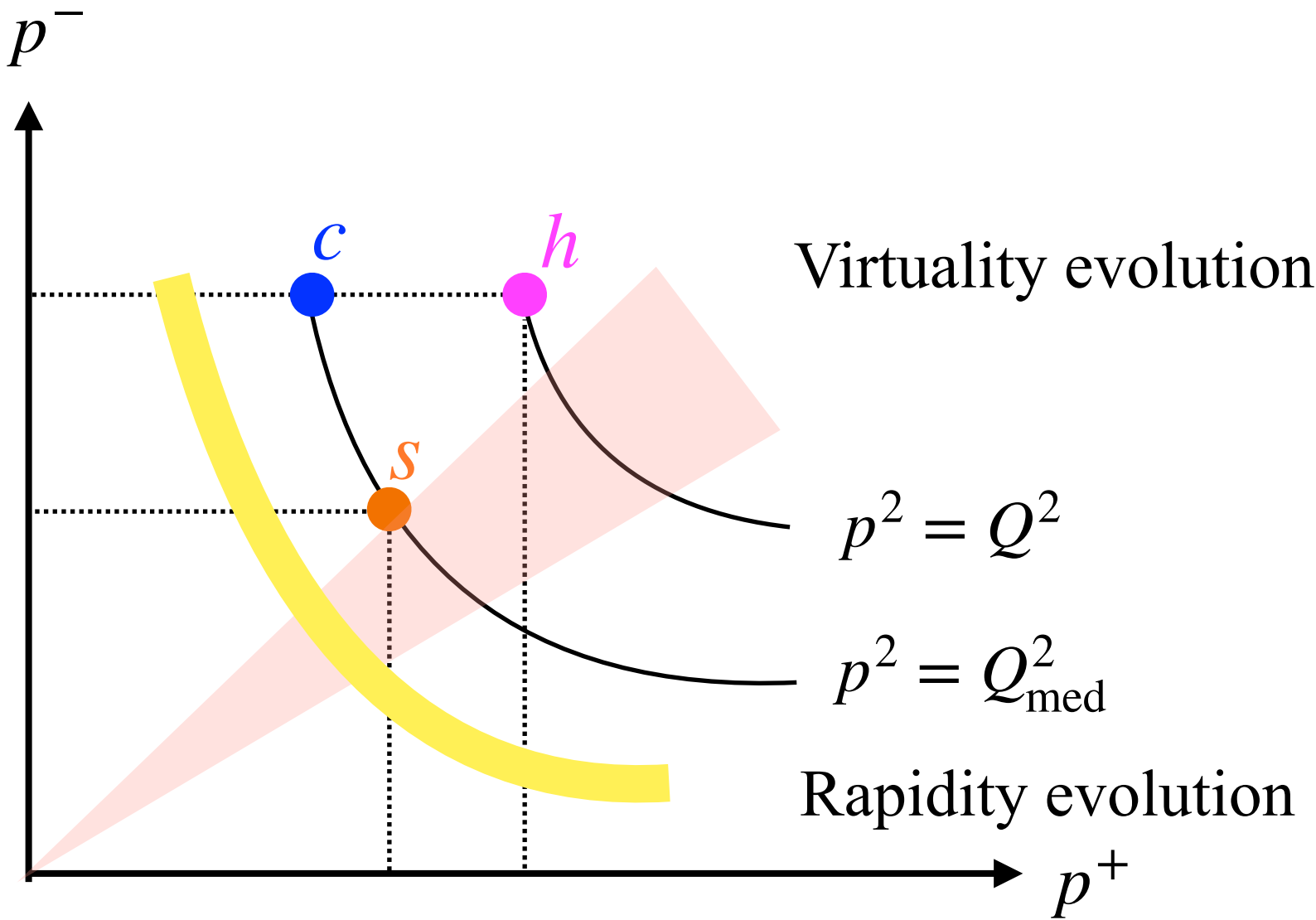
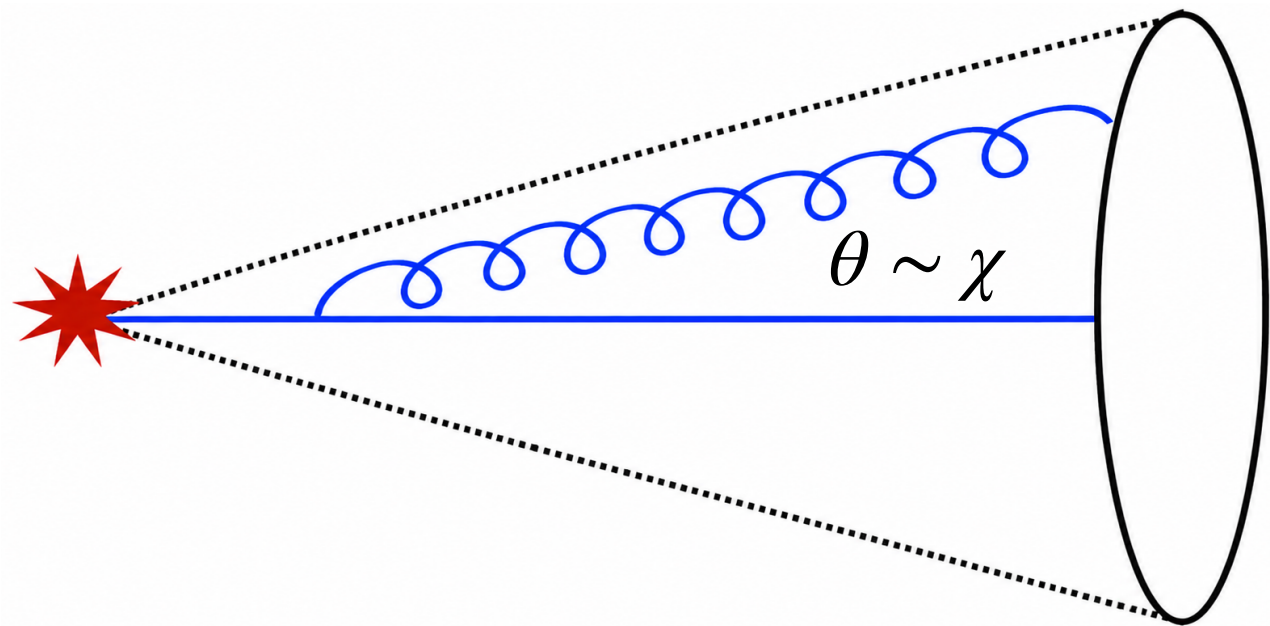
EFT diagram

EFT landscape

At least four momentum modes to describe jet evolution in a medium

Mode	Momentum scaling $p = (p^-, p^+, p_\perp)$	Describes	Scale
Hard	$p_h \sim Q(1, 1, 1)$	hard production	$Q \sim \mathcal{O}(p_T)$ production scale
Collinear	$p_c \sim Q(1, \lambda^2, \lambda)$	jet evolution	$p_{c\perp} = Q\lambda = Q\chi$
Soft	$p_s \sim Q(\lambda, \lambda, \lambda)$	medium dynamics	$Q\lambda = T \approx Q_{\text{med}}$
Glauber	$p_G \sim Q(\lambda, \lambda^2, \lambda)$	jet-medium interaction	off-shell modes

Scaling of c mode is fixed by the measurement $p_{c\perp} = Q\lambda = Q\chi$



Jet as open quantum system

Jet propagating is **non-equilibrium** evolution in a **thermal bath**

treat jet as open quantum system

1. Factorized initial density matrix; **decoupled**

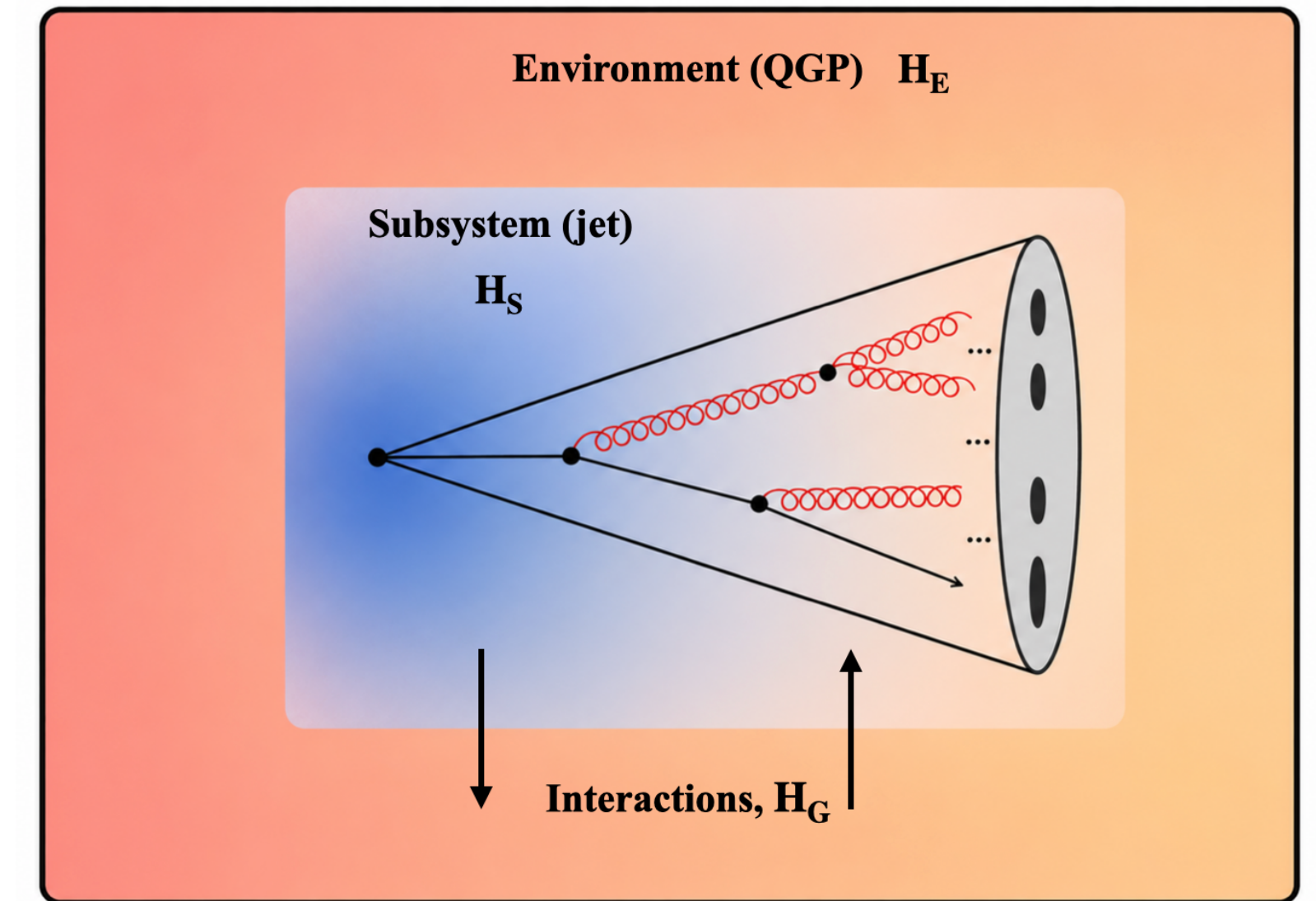
$$\rho(0) = |e^+e^-\rangle\langle e^+e^-| \otimes \rho_E(0) \quad \rho_E = \frac{e^{-\beta H_E}}{\text{Tr}[e^{-\beta H_E}]}$$

2. Time evolution of the jet is defined through system density matrix evolution

$$\rho(t) = e^{-iHt} \rho(0) e^{iHt}$$

3. Compute the jet observable ($\mathcal{M}$ - Two point Energy correlator)

$$\frac{d\sigma}{d\chi} = \lim_{t \rightarrow \infty} \text{Tr}_E[\rho(t) \mathcal{M}] \quad \longrightarrow \quad \text{Trace out the bath}$$



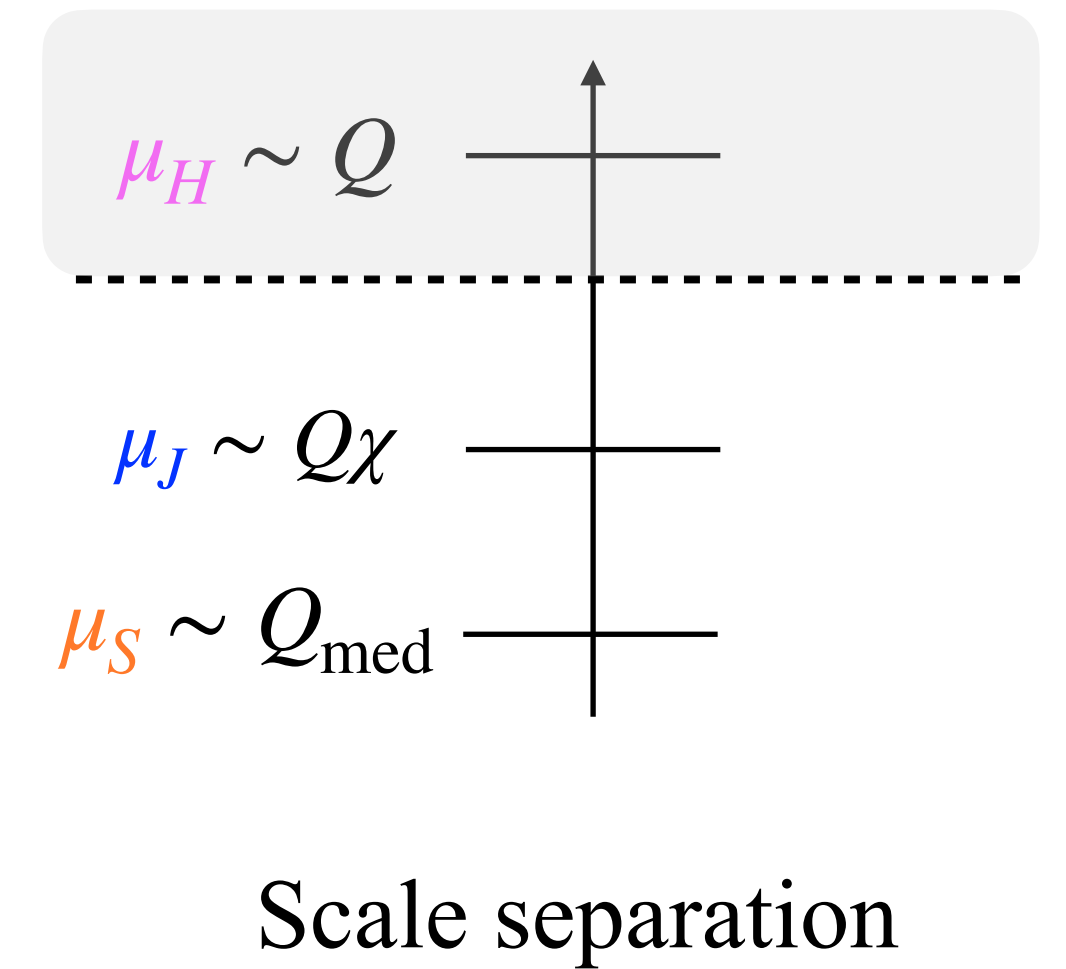
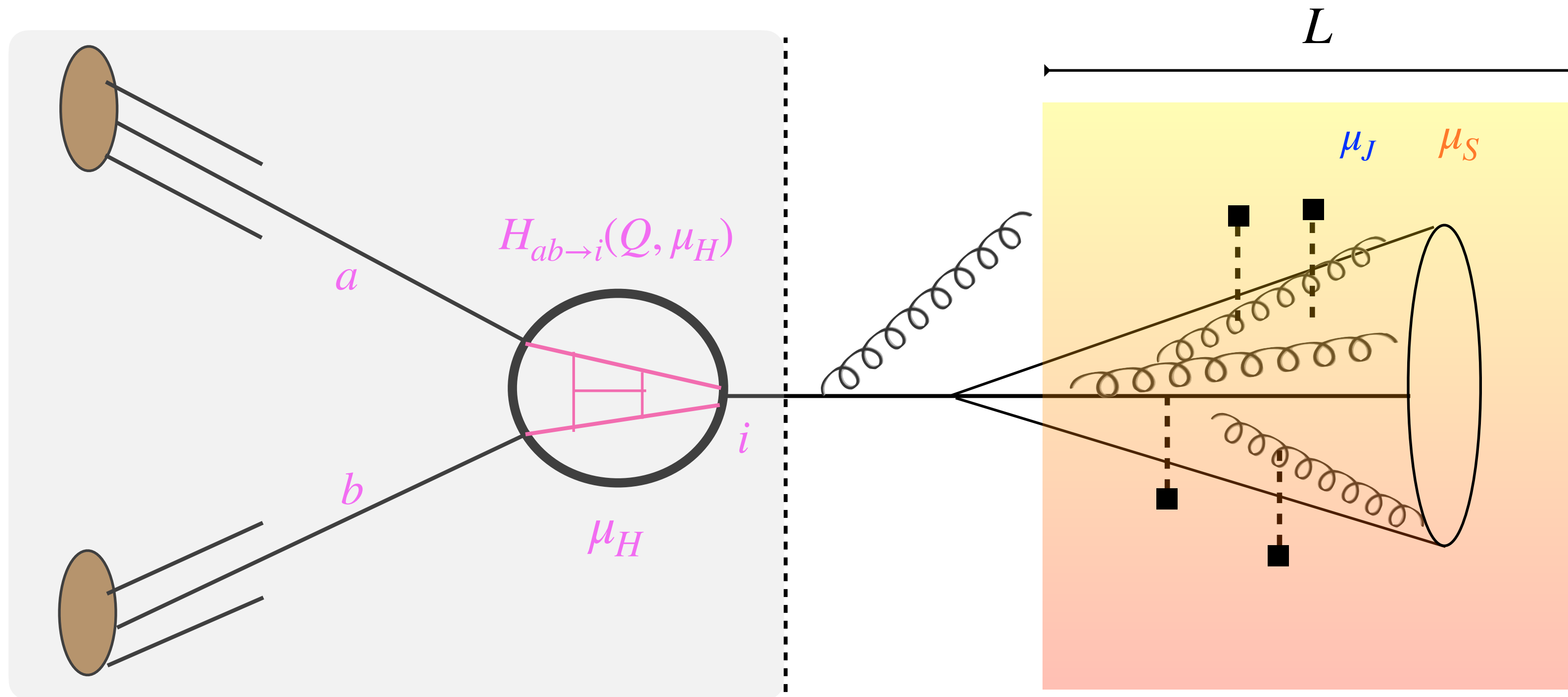
$$H = H_S + H_E + H_G + C(Q) l^\mu j_\mu \equiv H_0 + \mathcal{O}_H$$

↓
Hard interaction

$$j^\mu = \bar{\chi}_n \gamma^\mu \chi_n$$

Factorizing hard function

OPE factorizes hard production of the jet



$$\frac{d\sigma}{d\chi} = \sum_{i \in \{q, \bar{q}, g\}} \int dx x^2 H_{ab \rightarrow i}(Q, \mu) J_i(L, \chi; \mu_J, \mu_S)$$

Hard function is universal

Measurement factorization

$$J_q(\chi) = \frac{1}{2N_c} \sum_X \text{Tr} \left[\rho_E(0) \frac{\bar{n}}{2} e^{iH_{ns}t} \underbrace{\bar{\mathbf{T}} \left\{ e^{-i \int_0^t dt' H_{G,I}(t')} \chi_{n,I}(0) \right\}}_{\text{Glauber interaction}} \mathcal{M} |X\rangle \langle X| \underbrace{\mathbf{T} \left\{ e^{-i \int_0^t dt' H_{G,I}(t')} \bar{\chi}_{n,I}(0) \right\}}_{\text{Glauber interaction}} e^{-iH_{ns}t} \right]$$

Measurement function acts on state $|X\rangle$ which composed of both collinear and soft Fock spaces

$$|X\rangle = |X_n\rangle \otimes |X_s\rangle$$

$$\mathcal{M} = \hat{E}(\chi) |X\rangle = \frac{1}{Q} \sum_{i \in \{X_n, X_s\}} \left(E_{i,n} \delta(\chi - \theta_{n,i}) + E_{i,s} \delta(\chi - \theta_{s,i}) \right) |X_n\rangle |X_s\rangle$$

► Soft contributions to the measurement are power **suppressed**

► **Glauber** being off-shell modes **do not** contribute to the measurement

► **Only** collinear modes contribute to the measurement at the final state

Power suppressed contributions
in the EFT

$$E_{i,n} \gg E_{i,s}$$

Separating vacuum jet dynamics

Expanding the jet function in Glauber Hamiltonian gives **vacuum jet** function at **leading order**

$$J_q(\chi, \mu) = J_q^{(0)} + J_q^{(2)} + J_q^{(4)} + \dots = \sum_{i=0}^{\infty} J_q^{(i)}(\chi, \mu)$$

$$J_q^{(0)}(\chi, \mu) = \frac{1}{2N_c} \sum_{X_n} \text{Tr} \left[\frac{\bar{n}}{2} \langle 0 | \chi_n(0) \mathcal{M} | X_n \rangle \langle X_n | \bar{\chi}_n(0) | 0 \rangle \right]$$

$i = 0$, vacuum, **no Glauber insertions**

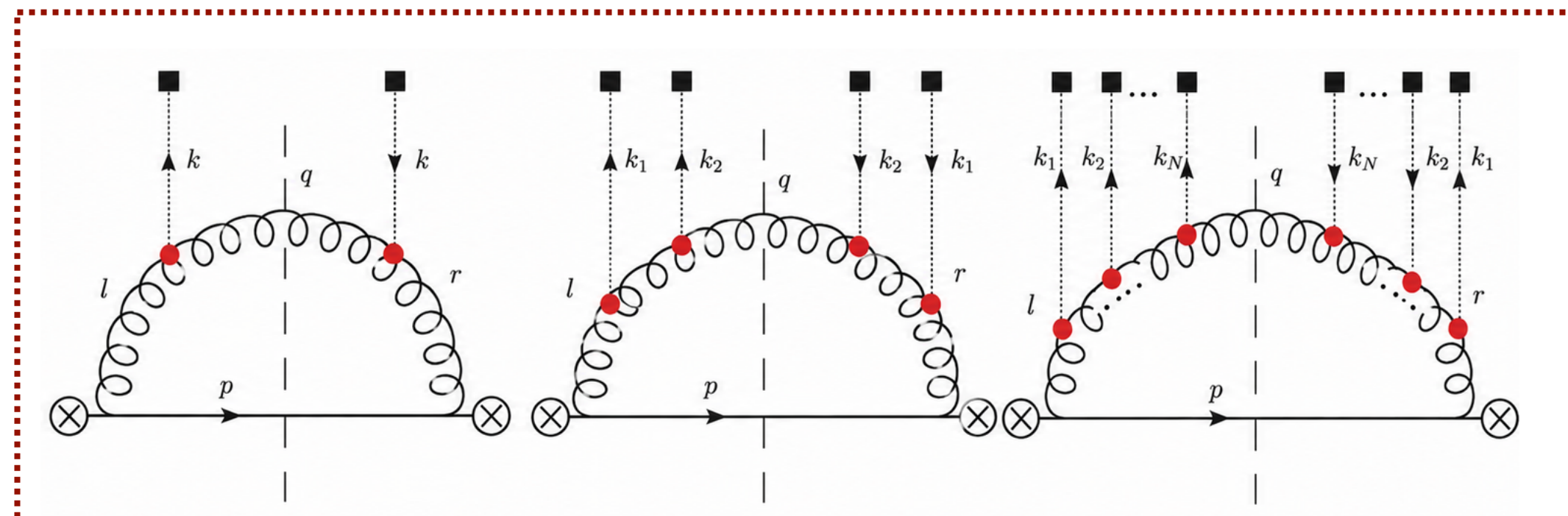
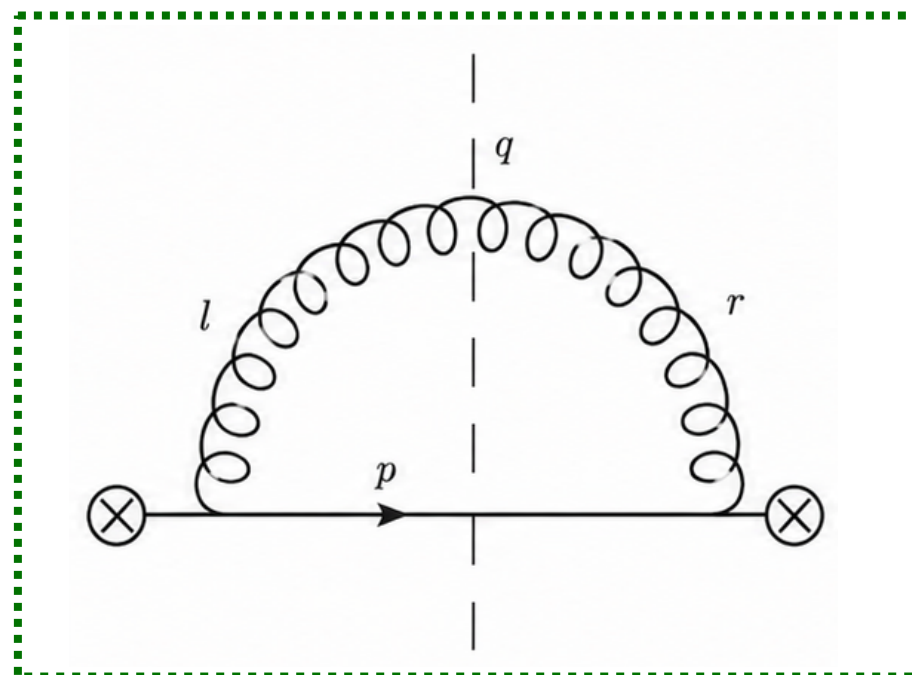
$i = 2$, single scattering, **dilute medium**

$i \geq 4$, multiple scattering, **dense medium**

For $J_q^{(0)}$ soft function becomes identity; no medium

$$\text{Tr}[\rho(0)_E X_s \rangle \langle X_s |] = 1$$

$$J_q^{(N)} \equiv \int \prod_{i=1}^a d^4 x_i \prod_{j=1}^b d^4 y_j \text{Tr} \left[\rho_B \frac{\bar{n}}{2} \mathbf{T} \{ O_{ns}^{qq}(x_1) \dots O_{ns}^{qq}(x_a) \} \chi_{n,I}(0) | X \rangle \langle X | \bar{\mathbf{T}} \{ O_{ns}^{qq}(y_1) \dots O_{ns}^{qq}(y_b) \} \bar{\chi}_{n,I}(0) \right]$$

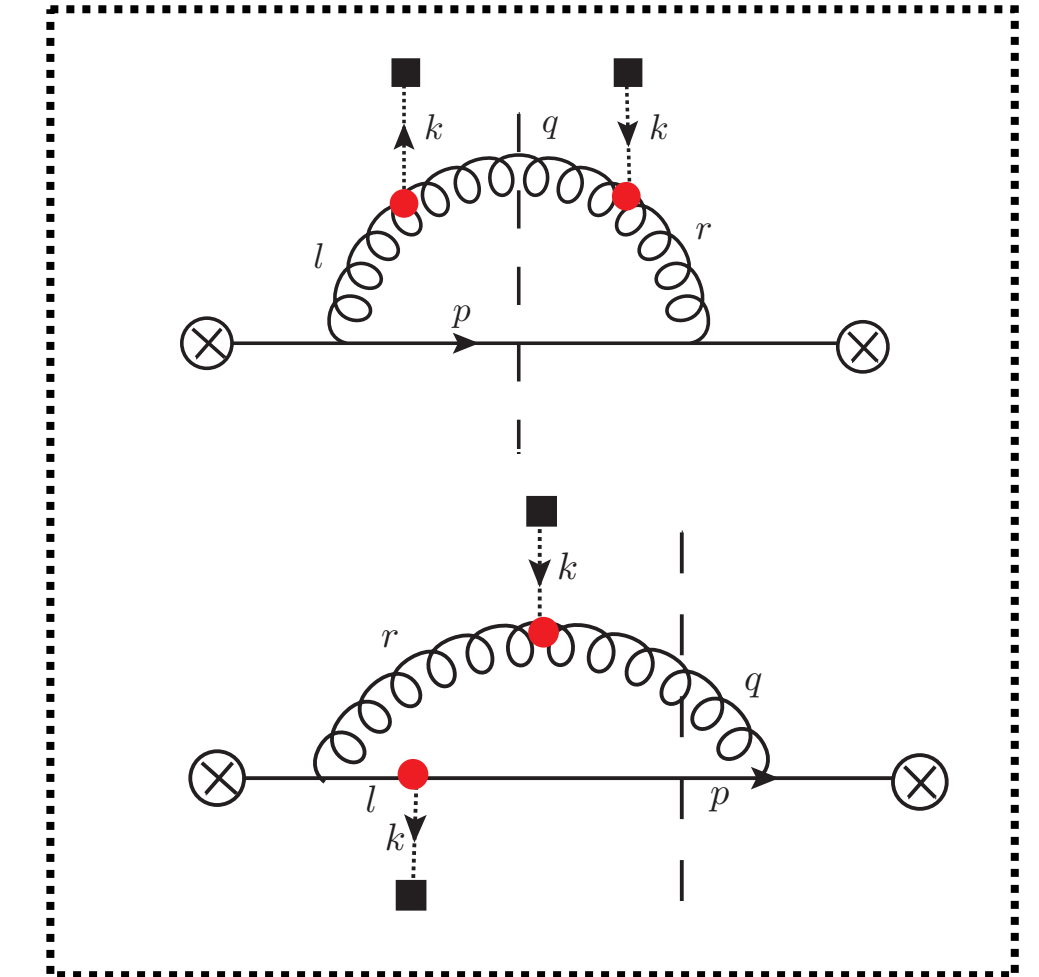


Medium induced jet dynamics

In the dilute limit, EFT recovers GLV results

$$J_q^{(2)}(\chi, L) = L \int \frac{d^2 k_\perp}{(2\pi)^2} [\mathbf{J}_{q,R}^{(2)}(\chi, k_\perp, L) - \mathbf{J}_{q,V}^{(2)}(\chi, k_\perp, L)] \mathbf{B}(k_\perp) + \mathcal{O}(H_G^4)$$

$$\mathbf{J}_q^{(2)}(\chi, \omega, k_\perp) \simeq g^2 L \int \frac{dz}{z} \int \frac{d^2 q_\perp}{(2\pi)^2} \underbrace{\frac{\vec{q}_\perp \cdot \vec{k}_\perp}{\vec{q}_\perp^2 \vec{k}_\perp^2}}_{\vec{k} = \vec{q}_\perp - \vec{k}_\perp} \underbrace{\left(1 - \frac{z\omega}{\vec{k}_\perp^2 L} \sin\left[\frac{L\vec{k}^2}{z\omega}\right]\right)}_{\text{LPM}} 2z \delta\left(\chi^2 - \frac{q_\perp^2}{z^2 \omega^2}\right)$$



Medium function ($\mathbf{B}(k_\perp)$) is thermal correlators of soft bilinear operators

$$B_E(K) = \int_0^{\frac{1}{T}} d\tau \int d^3x e^{iK \cdot X} \langle \hat{O}^{qA}(X) \hat{O}^{qB}(0) \rangle$$

For multiple scatterings medium function becomes correlations of Wilson lines

$$\hat{O}^{qA}(x) = \frac{1}{\mathbb{P}_\perp^2} O_s^{qA}(x)$$

$$B(x_\perp) = \langle W^\dagger(0) \rho(0)_E W(x_\perp) \rangle \quad W(x) = \mathbf{P} \exp \left[-ig \int_0^L d(\vec{n} \cdot r) \hat{O}_s^{qA}(x + \vec{n} \cdot r) \right]$$

In progress

Can be computed on lattice?

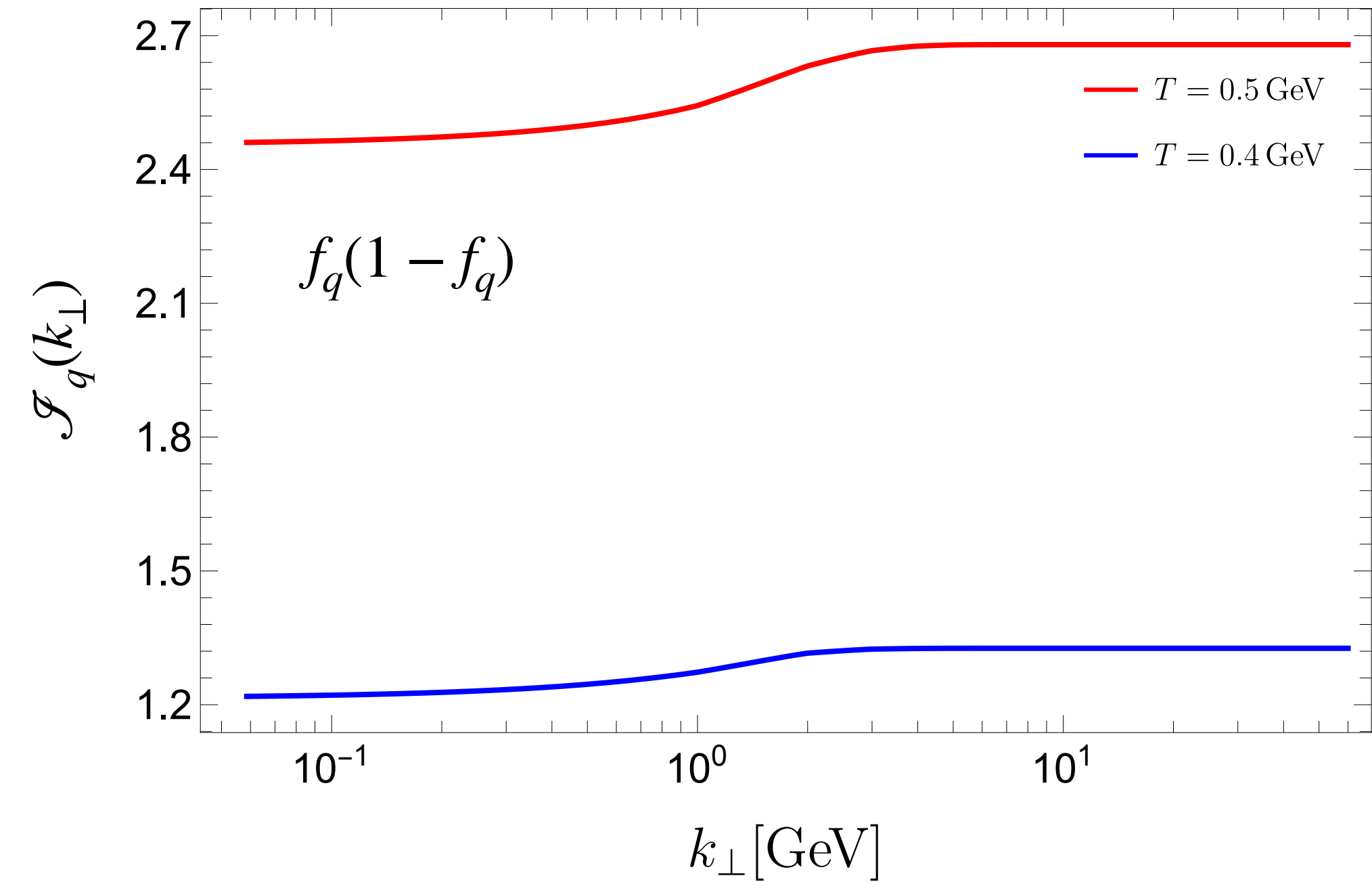
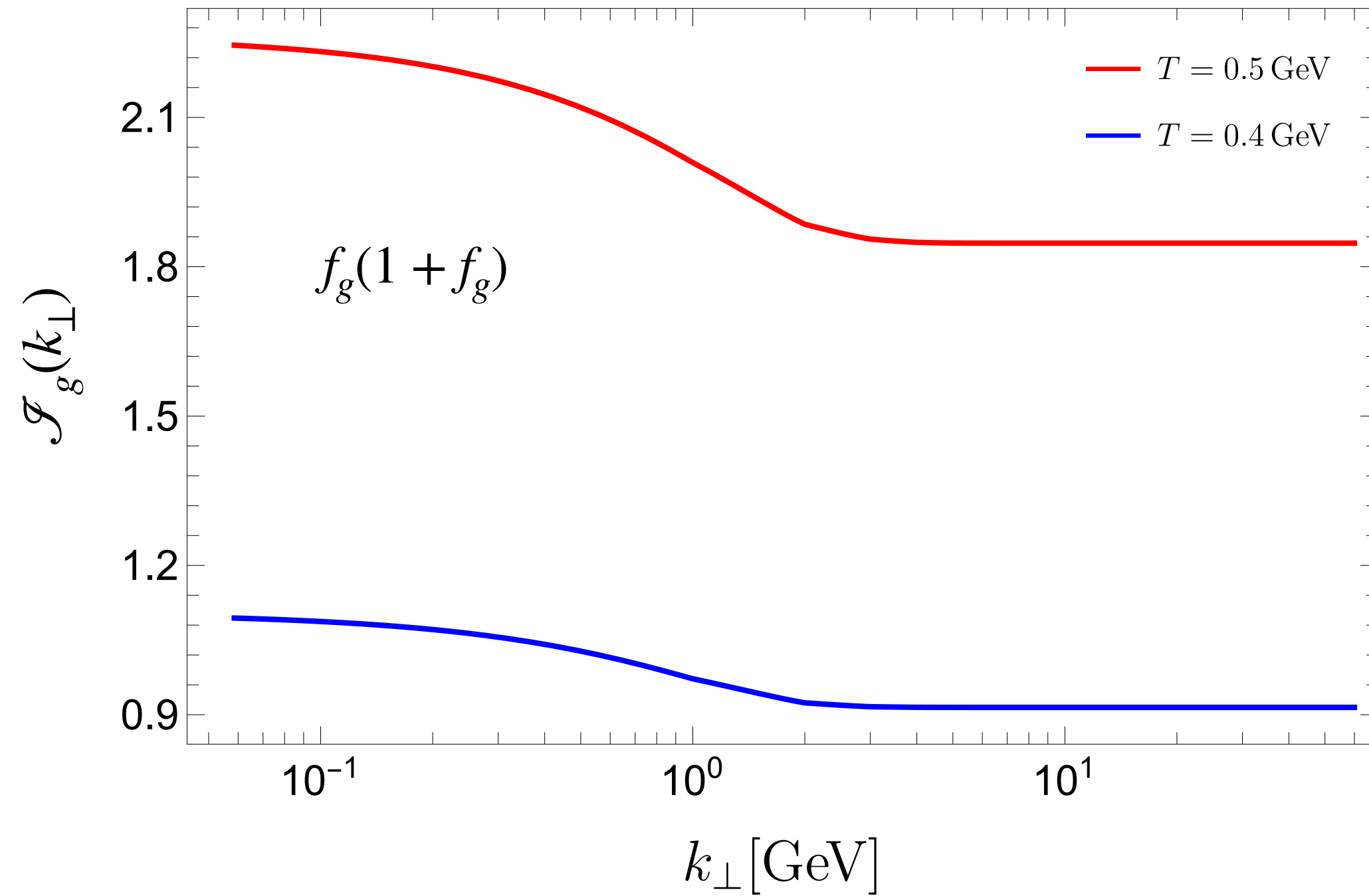
Budhraj, Yong and **BS**

LO medium function

At leading order small $k_\perp$ dependence in the numerator term

$$\mathbf{B}_{\text{LO}}(k_\perp) \approx \frac{\mathcal{J}(k_\perp)}{k_\perp^4} \equiv \frac{\mathcal{J}(k_\perp)}{(k_\perp^2 + m_D^2)^2}$$

$$\mathcal{J}(k_\perp) = \mathcal{J}_g(k_\perp) + \mathcal{J}_q(k_\perp)$$



Higher order effects in medium correlator?

In progress

Budhraja, Yong and BS

Resummed EEC jet function

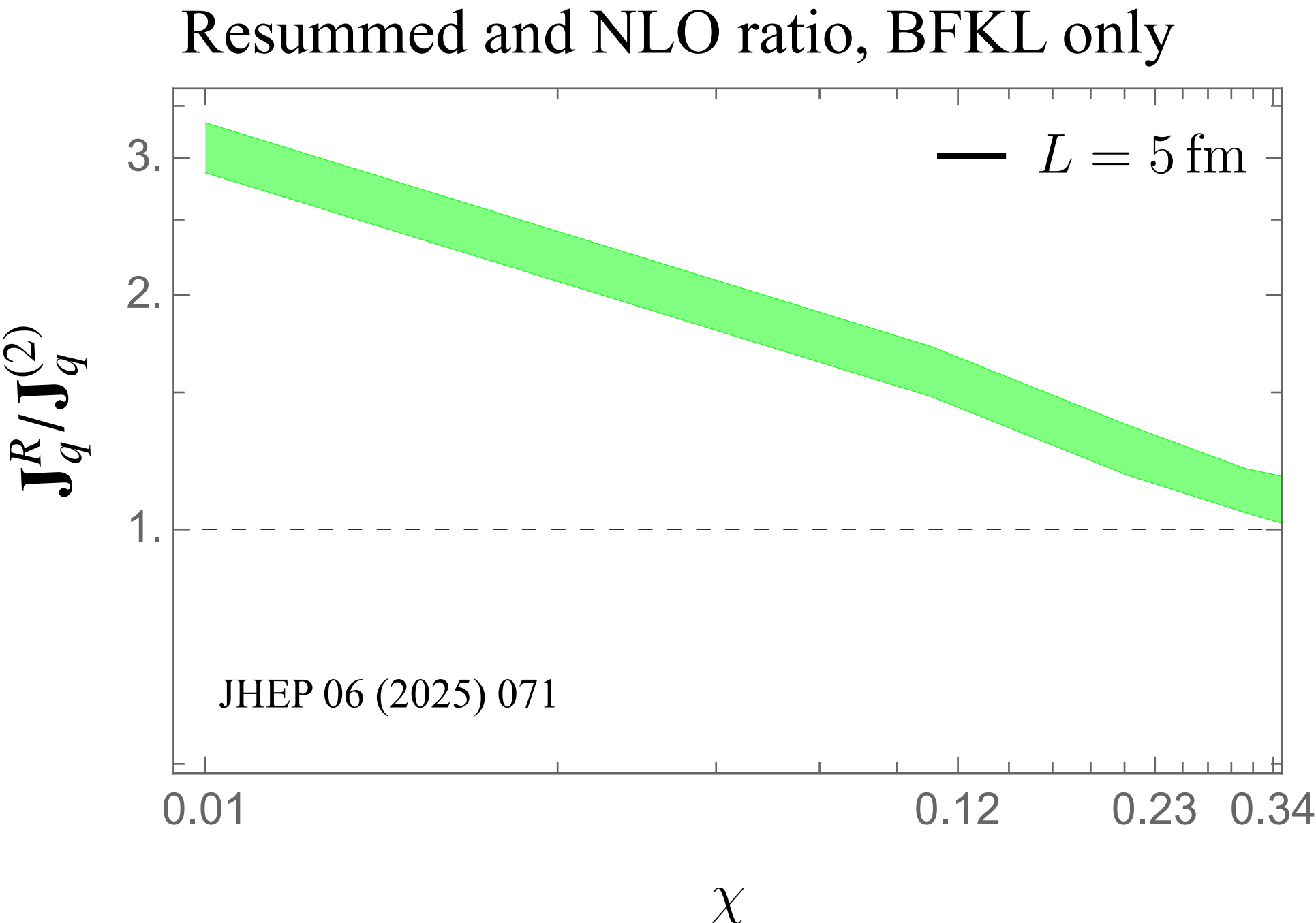
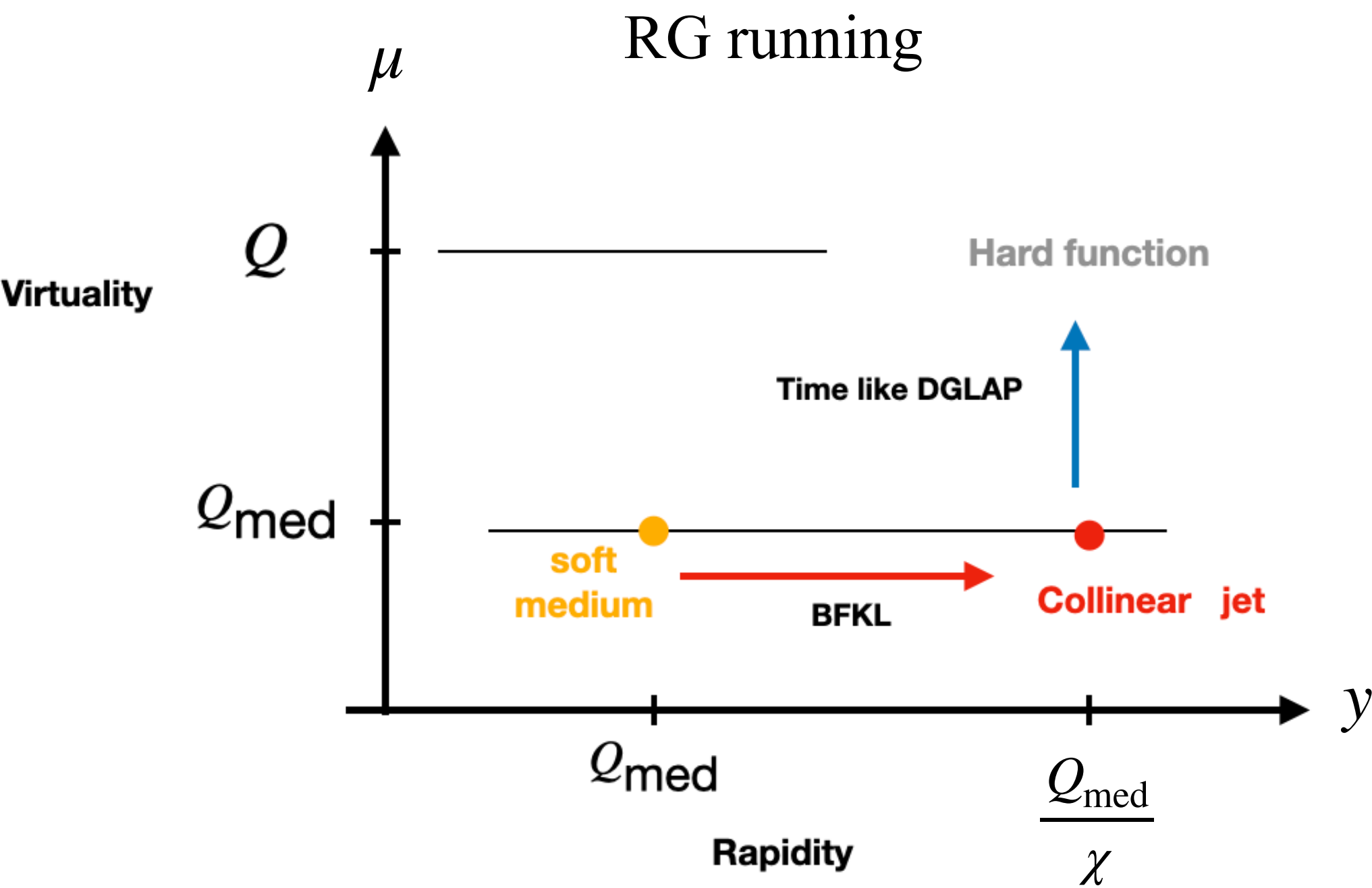
Medium and jet function obey **BFKL evolution** equation

$$\frac{d\Sigma}{d\chi} \sim H \otimes \left(J^0(\chi) + J_q^{(2)}(\chi) \otimes \mathbf{B} \right) \qquad \gamma_{J_q^{(2)}} = -\gamma_B$$

$$\mathbf{J}_q^R(k_\perp, \mu, y_f) = \int d^2 l_\perp \mathbf{J}_q^{(2)}(l_\perp, \mu, y_0) \int \frac{d\xi}{2\pi} k_\perp^{-1+2i\xi} l_\perp^{-1-2i\xi} e^{in(\phi_k-\phi_l)} e^{-\frac{\alpha_s(\mu)N_c}{\pi}\Phi(n,r) \log \frac{y_f}{y_0}}$$

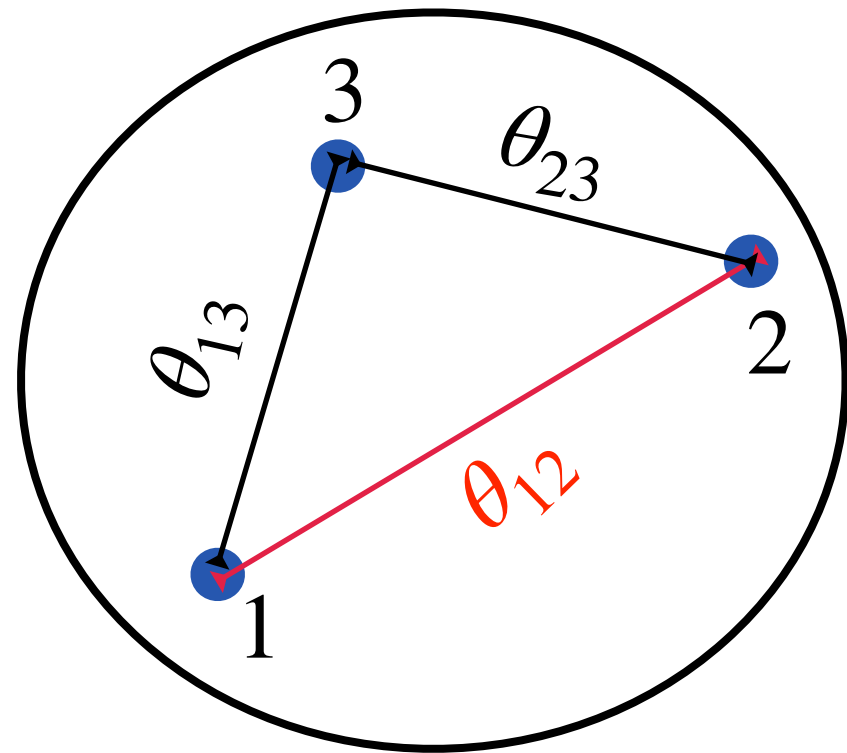
$$y_0 \sim \frac{Q_{\text{med}}}{\chi} \qquad y_f \sim Q_{\text{med}}$$

Resums $\sim \alpha_s \log \chi$ terms which are relevant in small χ limit



Generalizing energy correlators

N-point projected energy correlators are defined by tracking particles with maximum angular separation



$$\frac{d\Sigma^{[N]}}{d\chi} = \sum_M \int d\sigma_M \sum_{i_1, \dots, i_N=1}^M \frac{E_{i_1} \dots E_{i_N}}{Q^N} \delta(\chi - \theta_{\max}(i_1, \dots, i_N)) \quad \theta_{\max}(i_1, \dots, i_N) \equiv \max_{a < b} \theta_{i_a i_b}$$

Energy weights
of correlators

$$\frac{E_{i_1}^N}{Q^N} \delta(\chi)$$

Single particle

$$\left[\frac{(E_{i_1} + E_{i_2})^N}{Q^N} - \frac{E_{i_1}^N}{Q^N} - \frac{E_{i_2}^N}{Q^N} \right] \delta(\chi - \theta_{i_1 i_2})$$

Two particles

These are **binomial** expansions and can be **analytically continued** to **non-integer** values $N \rightarrow \nu$

► For IRC safe $\nu > 0$

$$\frac{d\Sigma^{[\nu]}}{d\chi} = \int d\sigma_X \left[\sum_{a=1,2} W_1^{[\nu]}(i_a) \delta(\chi) + \sum_{i_1 < i_2} W_2^{[\nu]}(i_1, i_2) \delta(\chi - \theta_{12}) \right]$$

$\nu = 2$ gives standard energy-energy correlator (EEC)

Energy weights

$$W_1^{[\nu]}(i_a) = \frac{E_{i_a}^\nu}{Q^\nu}$$

$$W_2^{[\nu]}(i_1, i_2) = \frac{(E_{i_1} + E_{i_2})^\nu}{Q^\nu} - \sum_{a=1,2} W_1^{[\nu]}(i_a)$$

Factorization for ν -correlators

Analytic continuation **does not** change the power counting of the modes

Factorization structure

$$\frac{d\Sigma^{[\nu]}}{d\chi} = \sum_{i \in \{q, \bar{q}, g\}} \int dx x^\nu H_{ab \rightarrow i}(Q, \mu) J_i^{[\nu]}(Q, \chi, \mu)$$

Measurement function

$$\mathcal{M}^{[\nu]} = \sum_{a=1,2} W_1^{[\nu]}(i_a) \delta(\chi) + W_2^{[\nu]}(i_1, i_2) \delta(\chi - \theta_{12})$$

weights hard and soft particles differently

Soft limit simplifications of measurement function

$$\text{For } \nu = 2 \quad \mathcal{M}^{[2]} = -2z [\delta(\chi) - \delta(\chi - \theta)]$$

$$\text{For } \nu < 1 \quad \mathcal{M}^{[\nu]} = z^\nu [\delta(\chi) - \delta(\chi - \theta)]$$

Jet function contains medium correlator and captures both vacuum and medium induced jet dynamics

$$J_q^{[\nu]}(\chi) = \frac{1}{2N_c} \sum_X \text{Tr} \left[\rho_E(0) \frac{\bar{n}}{2} e^{iH_{ns}t} \underbrace{\bar{\mathbf{T}} \left\{ e^{-i \int_0^t dt' H_{G,I}(t')} \chi_{n,I}(0) \right\}}_{\text{Glauber interaction}} \mathcal{M}^{[\nu]} |X\rangle \langle X| \underbrace{\mathbf{T} \left\{ e^{-i \int_0^t dt' H_{G,I}(t')} \bar{\chi}_{n,I}(0) \right\}}_{\text{Glauber interaction}} e^{-iH_{ns}t} \right]$$

ν -correlators in dilute medium

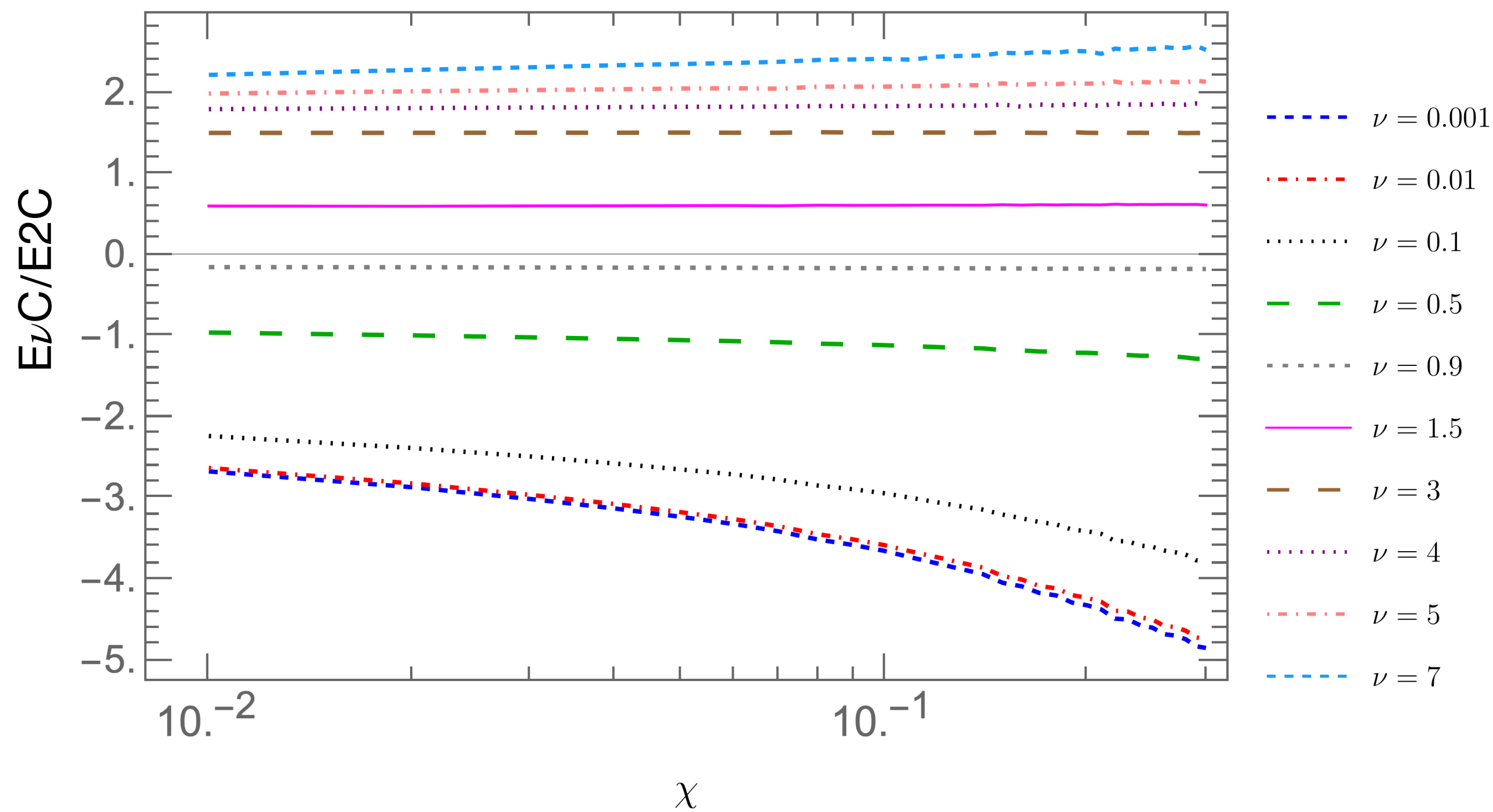
For small ν values correlators **saturate** at small ν values

BFKL resummation **does not** include ν dependent anomalous dimensions

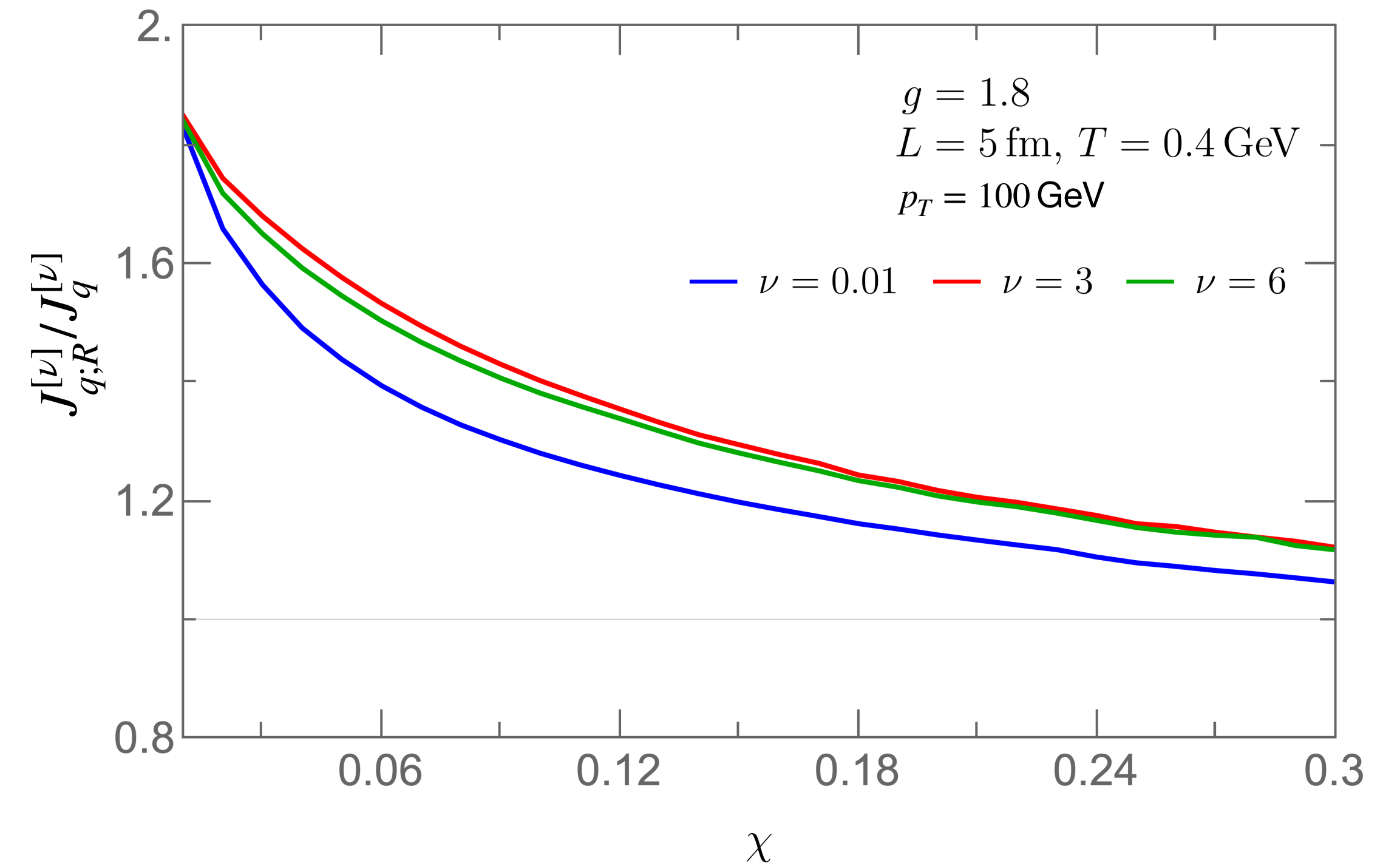
Parameters

$$L = 5 \text{ fm} \quad T = 0.4 \text{ GeV} \quad \alpha_s = 0.3$$

Ratio with two point EC



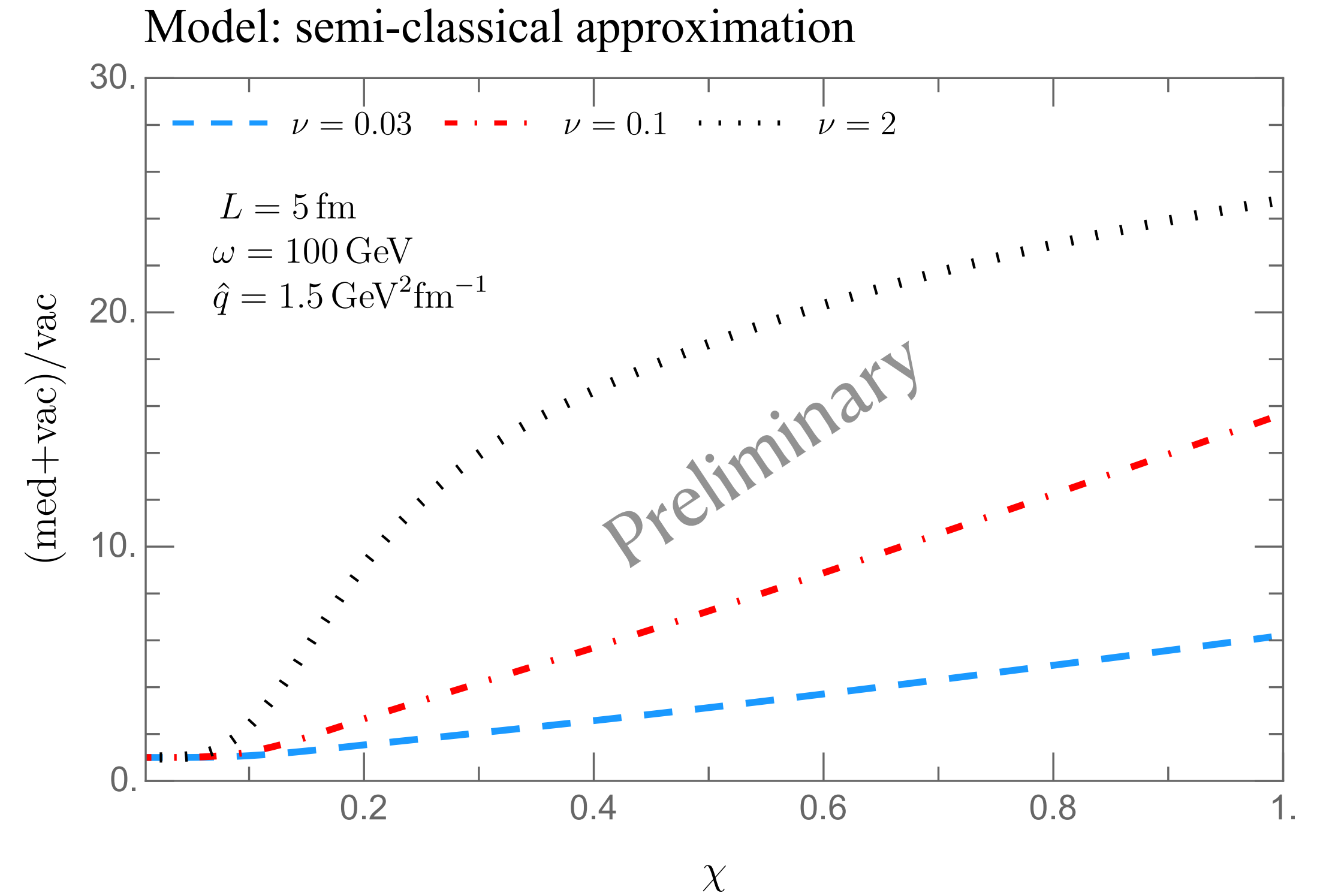
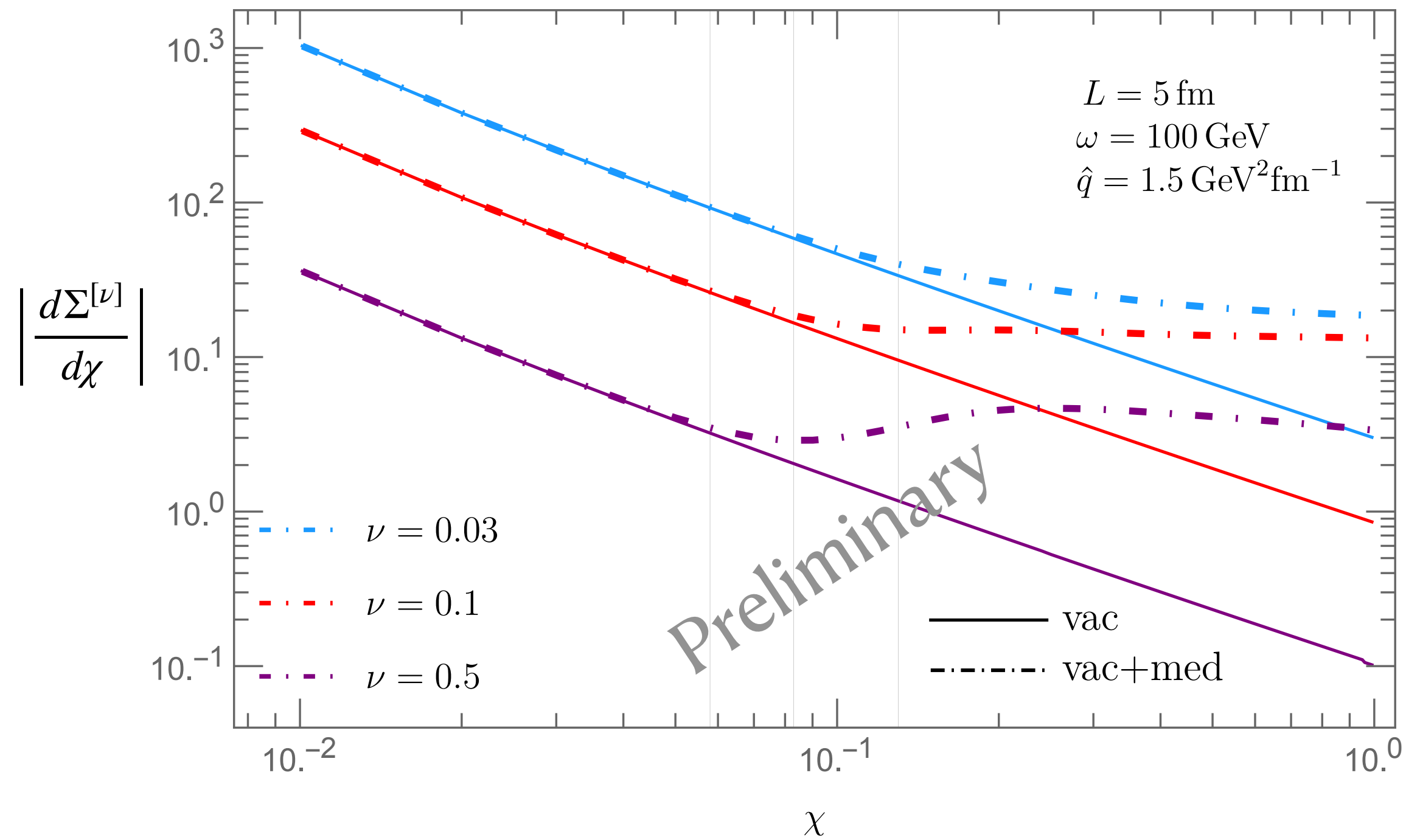
Ratio of resummed and fixed order, BFKL only



ν -correlators in dense medium

In progress
Andrey, Liliana and BS

Medium-induced emission contribution is **suppressed** for smaller ν valued correlators



$$\frac{d\Sigma_{\text{vac}}^{[\nu]}}{d\chi} \sim \frac{\hat{\gamma}(1+\nu)}{\chi^{1-\gamma(1+\nu)}} \quad \hat{\gamma}_{gq} \sim \frac{1}{\nu}$$

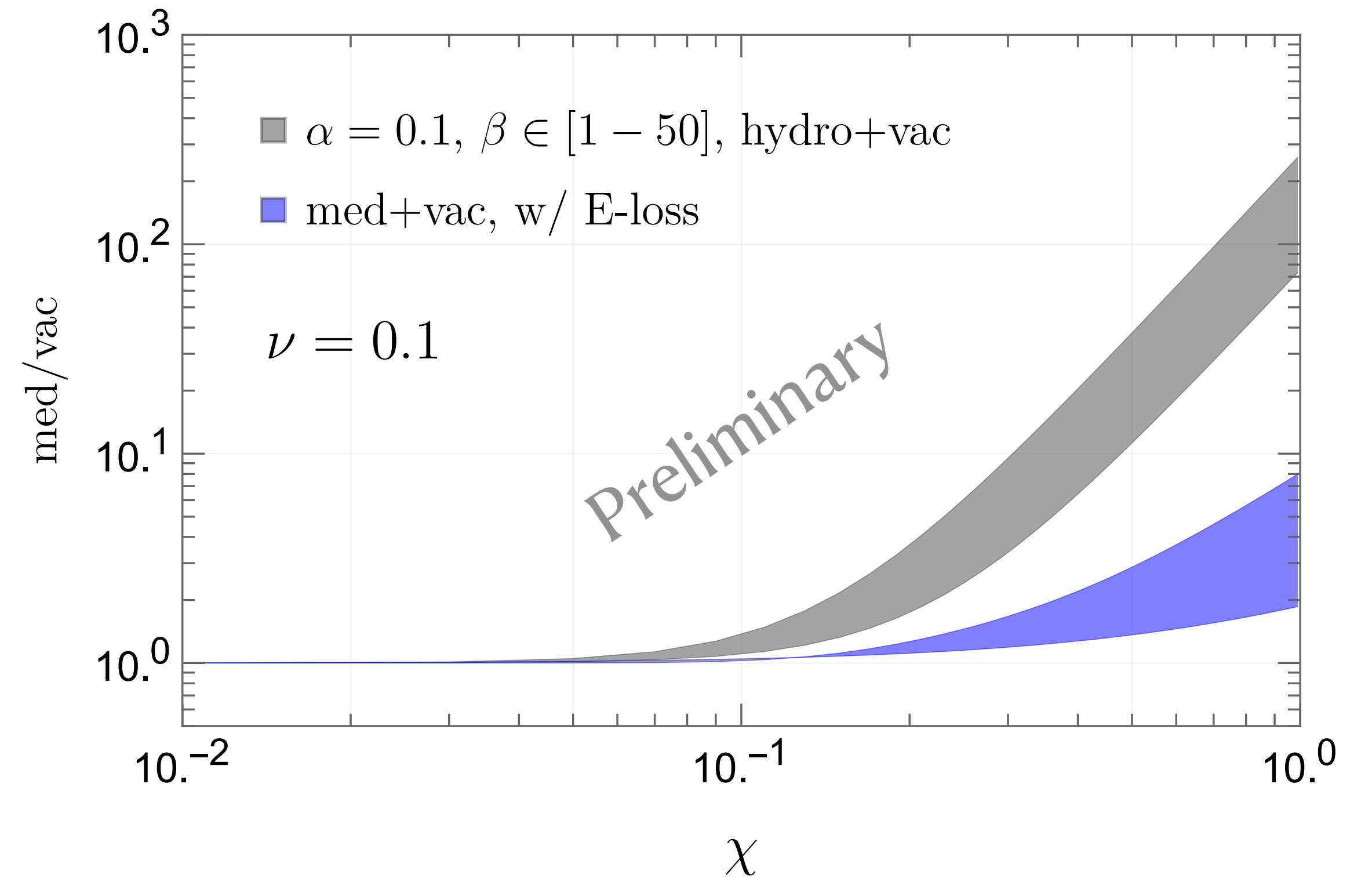
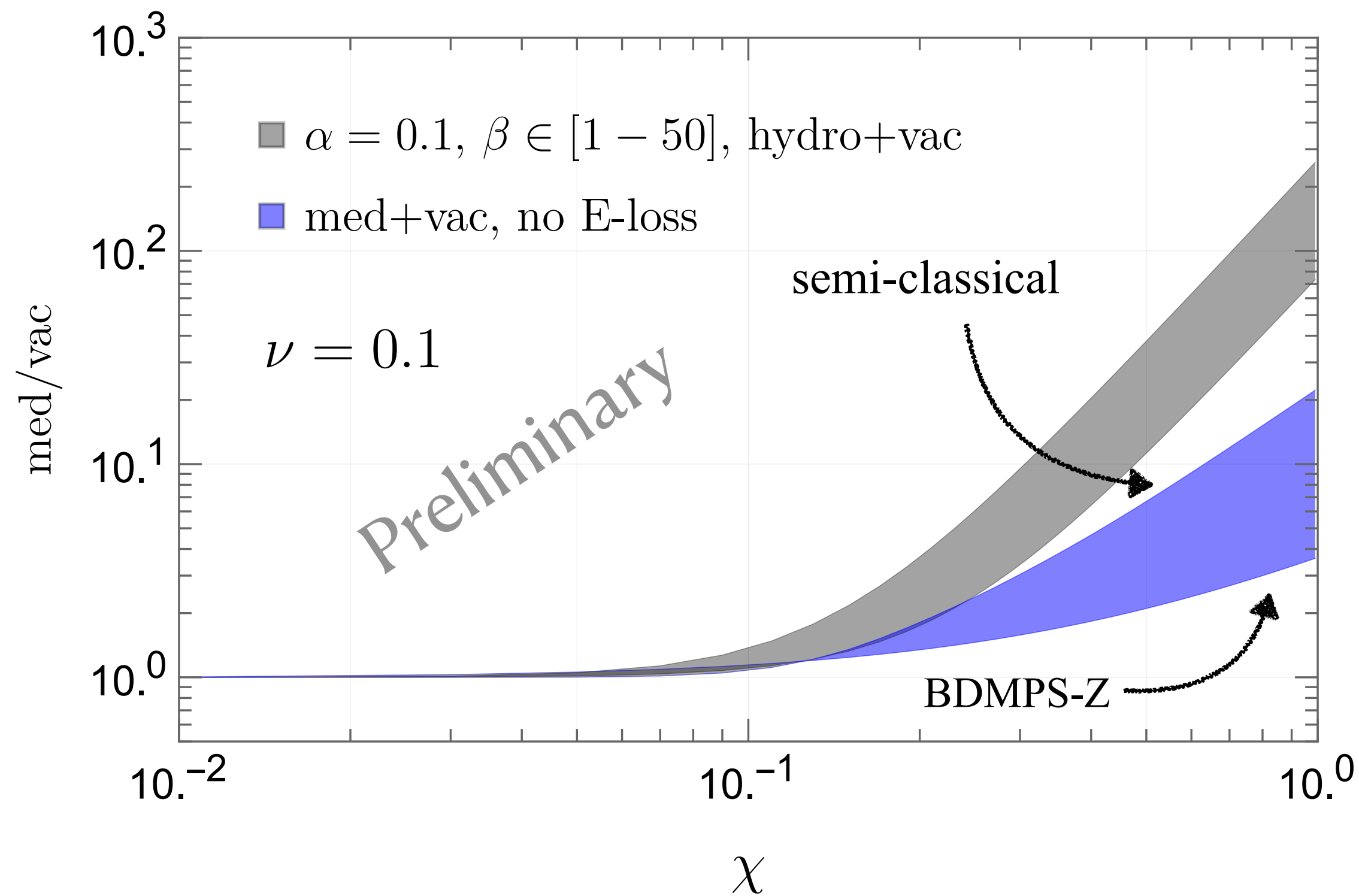
$$\frac{d\Sigma_{\text{med}}^{[\nu]}}{d\chi} \sim \mathbf{c}(\chi) + \mathcal{O}(\nu) \quad \text{Consistent behavior in EFT, BDMPS-Z, harmonic oscillator approximation}$$

Medium response to ν -correlators

In progress
Andrey, Liliana and BS

For medium response, we define the final differential distribution of ν -correlators through $f(\mathbf{p})$

$$\frac{1}{\sin \chi} \frac{d\Sigma^{(\nu)}}{d\chi} = \int_{\mathbf{p}_1, \mathbf{p}_2} f(\mathbf{p}_1) f(\mathbf{p}_2) \frac{(E_1 + E_2)^\nu - E_1^\nu - E_2^\nu}{p_T^\nu} \delta(\hat{\mathbf{p}}_1 \cdot \hat{\mathbf{p}}_2 - \cos \chi) \Theta(\chi > 0)$$



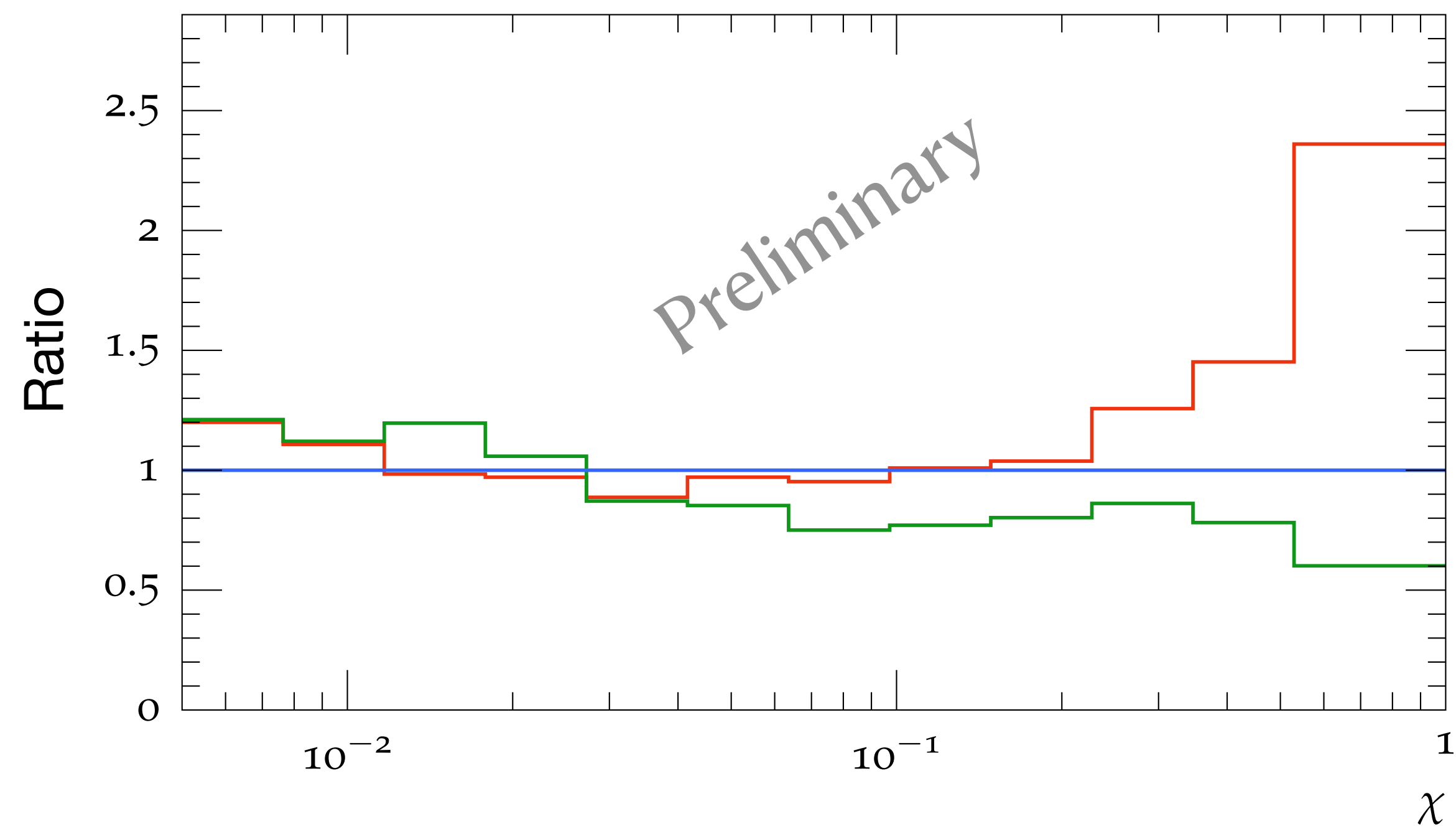
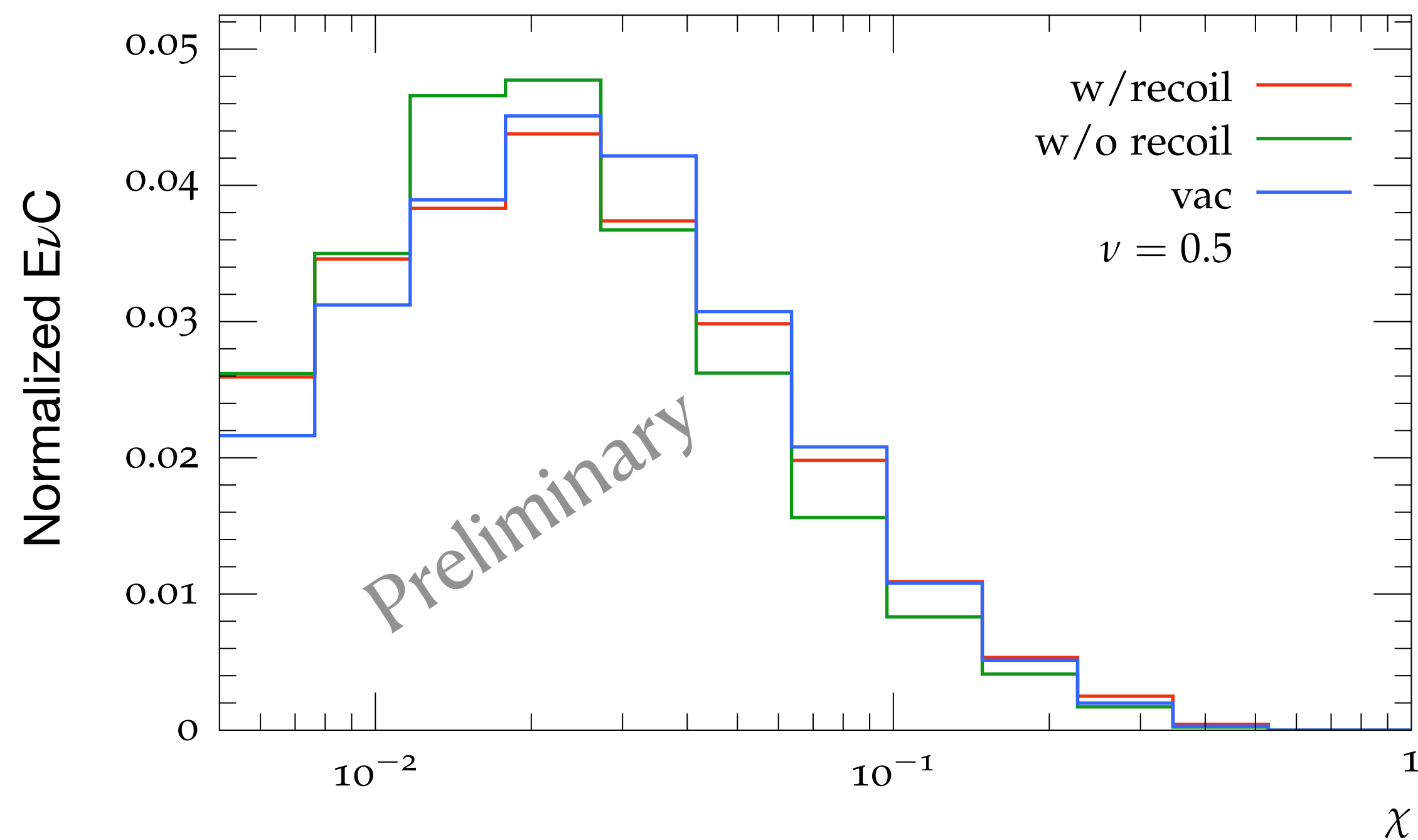
$$f(p, \cos \theta) = \frac{(2\pi)^3}{p^2} \left[\alpha e^{-\frac{(p-p_{ch})^2}{e_h^2}} e^{-\frac{\theta^2}{R^2}} \Theta(p - p_{\min}) + \beta e^{-\frac{p}{T}} \right] \Theta(R - |\theta|)$$

$$\int \frac{d^3 p}{(2\pi)^3} f(\theta) = N \quad (\text{Number of particles})$$

JEWEL simulations

In progress
Andrey, Liliana and BS

$p_T^J \in [100 - 120] \text{ GeV}$ $|\eta|^J < 2$ $R = 0.4$ $T_i = 0.45 \text{ GeV}$ Centrality = $[0 - 10] \%$



ν correlators seem to be interesting, can it used to quantify various medium-induced effects?

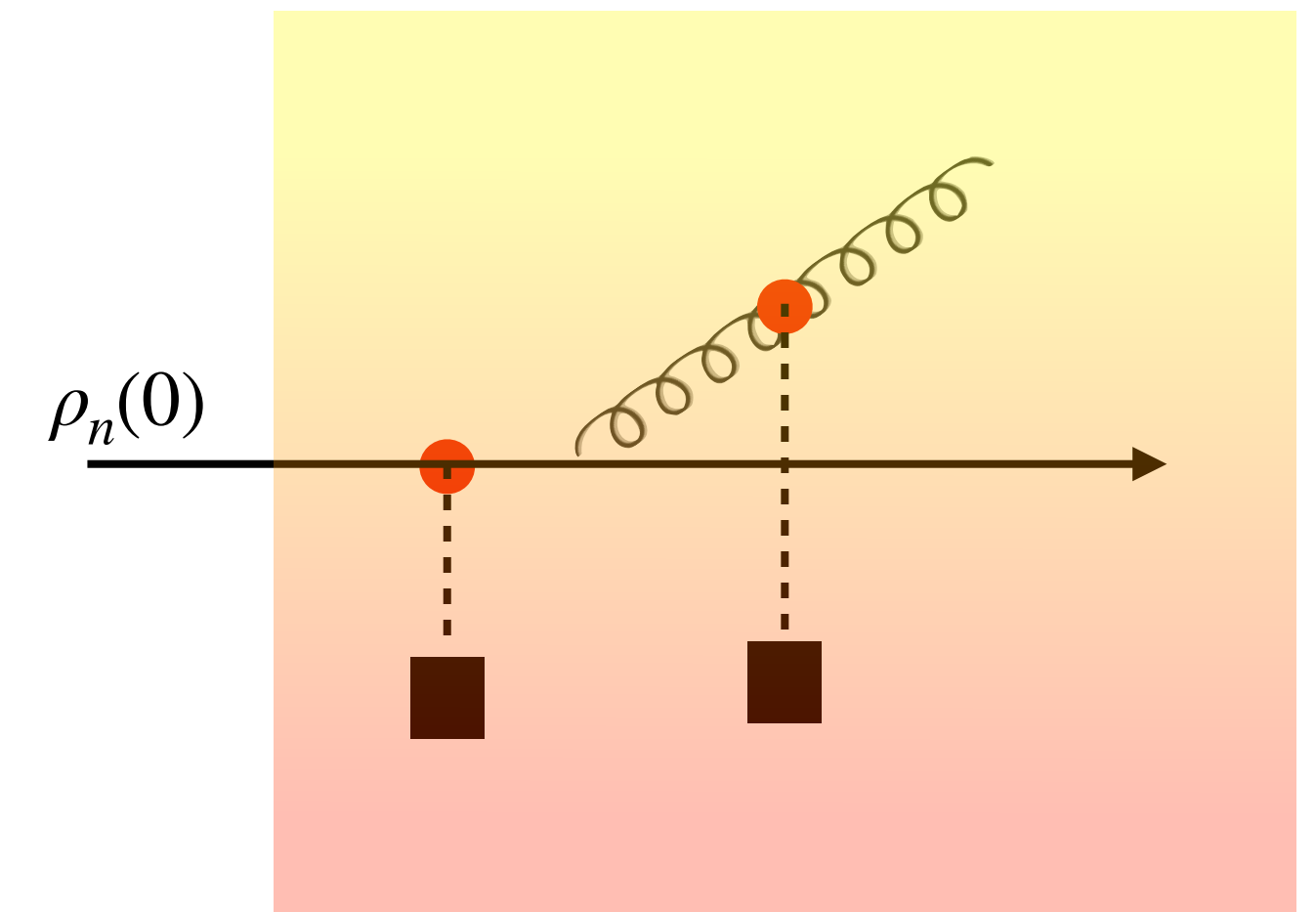
Summary and outlook

Ongoing work

Liliana and BS

1. EFT provides a systematically improvable framework and enables resummations of large logarithms
2. Higher order calculations of medium-induced jet function will provide better accuracy
3. ν -correlators could be useful in identifying various medium-induced effects in jets
4. EFT + open quantum system $\rightarrow$ time evolution of jets

$$H = H_n + H_s + H_G, \quad \rho(0) = \rho_n(0) \otimes \rho_E, \quad \rho_n(0) = |X_n\rangle\langle X_n|$$



Markovian limit $\tau_E \ll \tau_f < L$, Lindbladian evolution

$$\dot{\rho}_n(t) = -i[H_n + H_{LS}, \rho_n(t)] + \sum_i \left(L_i \rho_n(t) L_i^\dagger - \frac{1}{2} \{L_i^\dagger L_i, \rho_n(t)\} \right)$$

$$L_i \equiv \sqrt{\gamma^A} A^A(\omega)$$

Medium correlator

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Medium model

Semi-classical approximation

$$\frac{d\Sigma_q^{[\nu]}}{d\chi} = \sum_{i,j\in J} \int dz\, d\theta_{ij} \left(1 + \frac{\alpha_{s,\text{med}}}{\alpha_s(\mu)} F_{\text{med}}(z, \theta) \right) \frac{dP_{\text{vac}}}{dz\, d\theta} \mathcal{M}^{[\nu]}(z, \theta_{ij}, \chi)$$

$$F_{\text{med}}(z, \theta) = \frac{2}{\tau_f} \int_0^L dt \left[\frac{4}{f_1 \hat{q} \tau_f \theta^2 t^2} \left\{ 1 - e^{-\frac{f_1 \hat{q} (L-t) \theta^2 t^2}{4}} \right\} \cos\left(\frac{t}{\tau_f}\right) - \sin\left(\frac{t}{\tau_f}\right) \right] e^{-\frac{1}{12} \hat{q} f_2 \theta^2 t^3}$$

BDMPS-Z limit

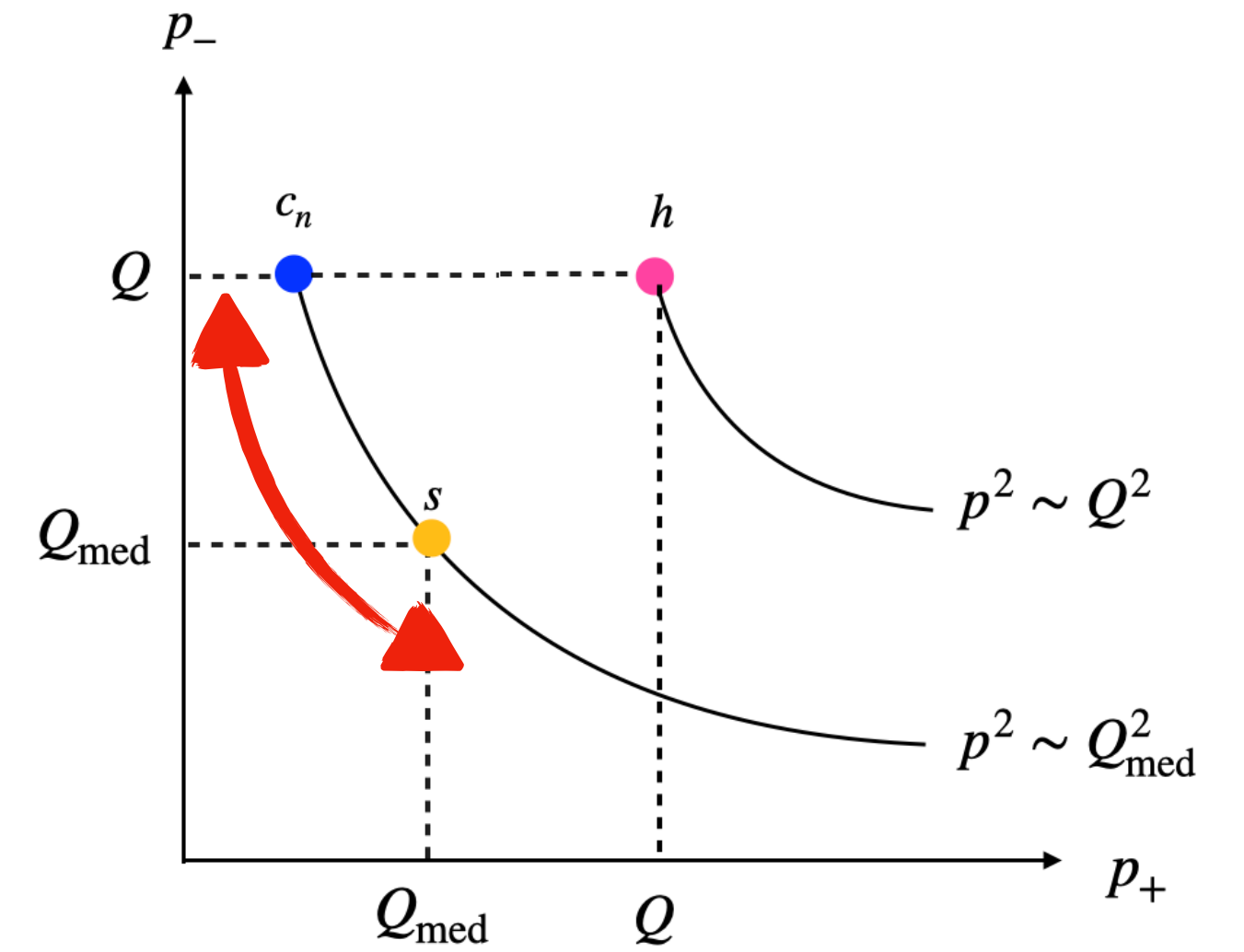
$$\frac{d\mathcal{P}}{dzd\theta} = \frac{\alpha_{s,med}}{L} \sqrt{\frac{8\omega_c}{z^3\omega}} \Theta(\omega_c - z\omega) (z^2\omega^2\theta) \int_0^L \frac{dt\, e^{-\frac{z^2\omega^2\theta^2}{\hat{q}(L-t)}}}{\hat{q}(L-t)}$$

BFKL evolution

- Medium function has **BFKL anomalous dimension**

$$y \frac{d\mathbf{J}_q^{(2)}(k_\perp, \nu)}{dy} = - \frac{\alpha_s(\mu) N_c}{\pi^2} \int d^2 l_\perp \left[\frac{\mathbf{J}_q^{(2)}(l_\perp, y)}{(\vec{l}_\perp - \vec{k}_\perp)^2} - \frac{k_\perp^2 \mathbf{J}_q^{(2)}(k_\perp, y)}{2l_\perp^2 (\vec{l}_\perp - \vec{k}_\perp)^2} \right]$$

y is rapidity scale which appears at NNLO for energy correlators



Scales are fixed by the corresponding jet and medium functions in perturbation theory