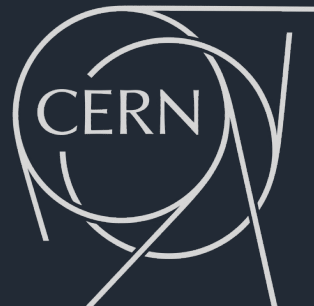


Deriving a parton shower for jet thermalization in QCD plasmas

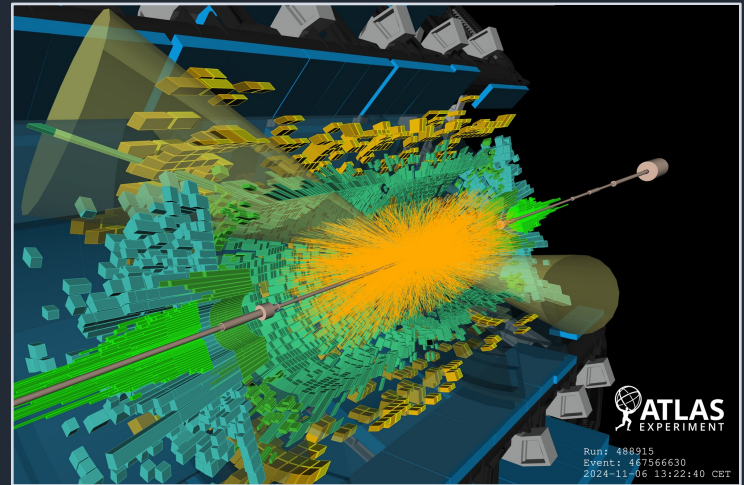
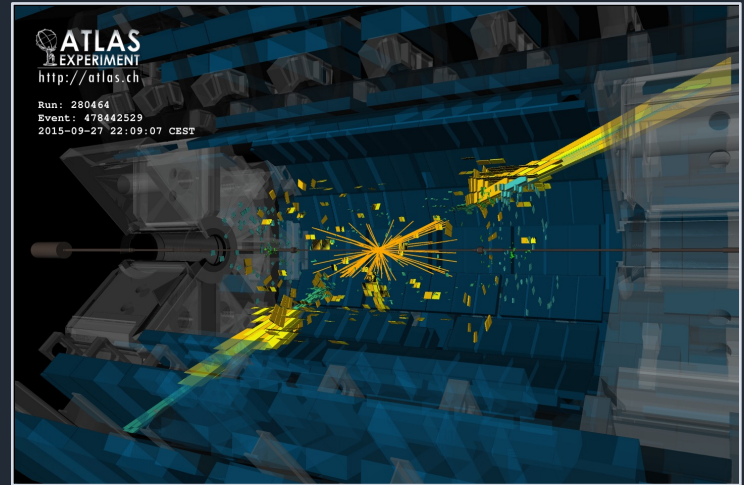
[I. Soudi, AT [2510.25837](#), [2607.xxxx](#)]

Adam Takacs

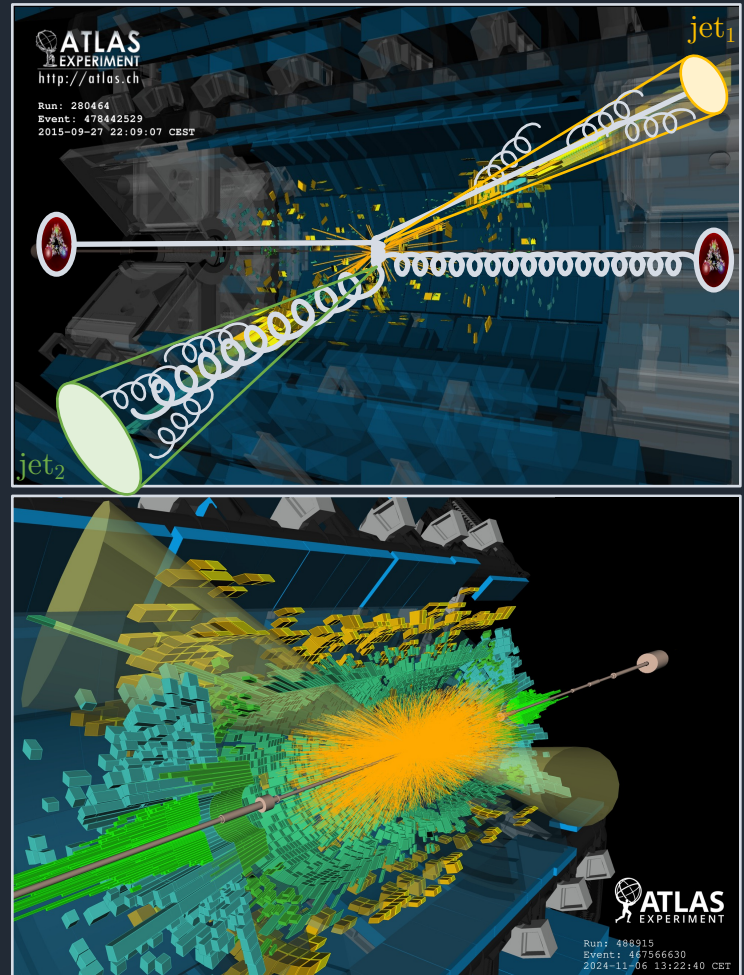
CERN TH,
University of Bergen



1. Jets in heavy-ion collisions



1. Jets in heavy-ion collisions



Scattering amplitudes in the QGP

- Separate fields to hard and bkg. ($q = q_h + q_0$, $A = A_h + A_0$)

$$\begin{aligned}\mathcal{L}_{QCD}(q, A) &= \mathcal{L}(q_h, A_h) + \mathcal{L}(q_0, A_0) + \mathcal{L}_{int}(q_h, A_h, q_0, A_0) \\ &\approx \mathcal{L}(q_h, A_h) + g\bar{q}_h\hat{J}q_h + gA_h\hat{J}A_h\end{aligned}$$

← integrating out soft
degrees of freedom

- Dressed propagators (and vertices):

$$\begin{aligned}\text{Feynman diagram 1} &= \text{Feynman diagram 2} + \text{Feynman diagram 3} + \text{Feynman diagram 4} + \dots \\ \text{Feynman diagram 5} &= \text{Feynman diagram 6} + \text{Feynman diagram 7} + \text{Feynman diagram 8} + \dots\end{aligned}$$

effective,
in-medium
Feynman rules

- Models for the background $\langle \hat{J}(x^\mu)\hat{J}^+(y^\mu) \rangle$:

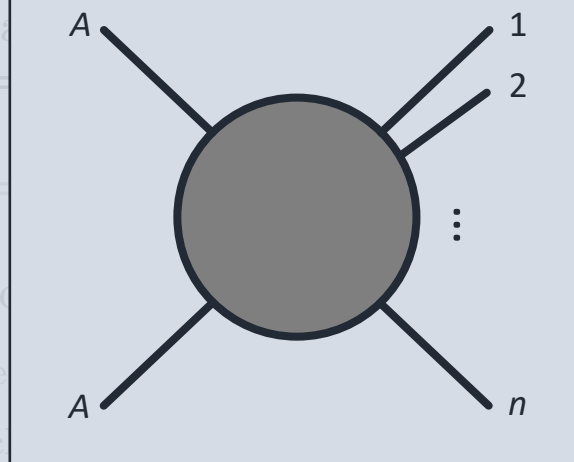
- High-temperature plasma ($T \gg \Lambda_{QCD}$)
- Color charge distribution
- Non-perturbative “kernel”

Scattering amplitudes in the QGP

- Separate fields to hard and bkg. ($q = q_h + q_0$, $A = A_h + A_0$)

$$\mathcal{L}_{QCD}(q, A) = \mathcal{L}(q_h, A_h) + \mathcal{L}(q_0, A_0) + \mathcal{L}_{int}(q_h, A_h, q_0, A_0)$$

Goal: $\langle 1, 2 \dots n | AA \rangle$



← integrating out soft degrees of freedom

effective,
in-medium
Feynman rules

- Dressed propagators (a)



- Models for the background

- High-temperature
- Color charge
- Non-perturbative “kernel”

Scattering amplitudes in the QGP

- Separate fields to hard and bkg. ($q = q_h + q_0$, $A = A_h + A_0$)

$$\begin{aligned}\mathcal{L}_{QCD}(q, A) &= \mathcal{L}(q_h, A_h) + \mathcal{L}(q_0, A_0) + \mathcal{L}_{int}(q_h, A_h, q_0, A_0) \\ &\approx \mathcal{L}(q_h, A_h) + g\bar{q}_h\hat{J}q_h + gA_h\hat{J}A_h\end{aligned}$$

← integrating out soft
degrees of freedom

- Dressed propagators (and vertices):

effective,
in-medium
Feynman rules

- Models for the background $\langle \hat{J}(x^\mu) \hat{J}^+(y^\mu) \rangle$:
 - High-temperature plasma ($T \gg \Lambda_{QCD}$)
 - Color charge distribution
 - Non-perturbative “kernel”

Example: jets in large medium

[Blaizot, Dominguez, Iancu, Mehtar-Tani]
[Casalderrey-Solana, Mehtar-Tani, Tywoniuk, Salgado]
[Caucal, Iancu, Mueller, Soyez]

- Broadening: (in vacuum straight line $\delta_{ij}\delta(p-p')$)

$$\left| \text{diagram} \right|^2 \approx G_{ij}(p', p, L) \quad \text{changing color and momentum}$$

The diagram shows a horizontal line with three vertical wavy lines (representing gluons) attached to it. The first and third wavy lines are crossed out with an 'X'. The second wavy line is not crossed out.

- Evolution equation for mom. distr:

$$\partial_t \mathcal{P}(p, t) = \int_{q_\perp} [C(q_\perp) \mathcal{P}(p - q_\perp, t) - C(q_\perp) \mathcal{P}(p, t)]$$

Annotations for the equation:

- "time" points to ∂_t
- mom. distribution points to $\mathcal{P}$
- scattering rate points to $C(q_\perp)$
- virtual term points to the minus sign and the $\mathcal{P}(p, t)$ term
- diffusion approx. $q \ll p$ points to $\approx \hat{q} \partial_p^2 \mathcal{P}(p, t)$

- Radiation: medium-induced (+ vacuum)

$$\left| \text{diagram} \right|^2 \Rightarrow \frac{d\sigma}{dz d\vartheta} \approx 2\bar{\alpha}_s \sqrt{\frac{Q_{med}}{z^3 E}} B(\vartheta)$$

Annotations for the equation:

- Medium ($k_t \sim Q_{med}, z \ll Q_{med}/E$): soft & ~~collin~~ pole points to Q_{med}

- Wide-angle medium-induced cascade:



- Cascade evolution:

$$\partial_t D(x, t) = \int dz [\Gamma(z, \frac{x}{z}) D(\frac{x}{z}, t) - \frac{1}{2} \Gamma(z, p) D(x, t)]$$

Annotations for the equation:

- "time" points to ∂_t
- FF points to $\Gamma(z, \frac{x}{z})$
- BDMPS-Z splitting rate points to $\Gamma(z, \frac{x}{z})$
- virtual term points to the minus sign and the $D(x, t)$ term

Example: jets in large medium

[Blaizot, Dominguez, Iancu, Mehtar-Tani]
[Casalderrey-Solana, Mehtar-Tani, Tywoniuk, Salgado]
[Caucal, Iancu, Mueller, Soyez]

- Broadening: (in vacuum straight line $\delta_{ij}\delta(p-p')$)

$$\left| \text{diagram} \right|^2 \approx G_{ij}(p', p, L) \quad \text{changing color and momentum}$$

The diagram shows a horizontal line with three vertical wavy lines (representing gluons) attached to it. The first and third wavy lines are crossed out with an 'X'. The second wavy line is not crossed out.

- Evolution equation for mom. distr:

$$\partial_t \mathcal{P}(\mathbf{p}, t) = \int_{\mathbf{q}_\perp} [C(\mathbf{q}_\perp) \mathcal{P}(\mathbf{p} - \mathbf{q}_\perp, t) - C(\mathbf{q}_\perp) \mathcal{P}(\mathbf{p}, t)]$$

Annotations for the equation:

- "time" points to ∂_t
- mom. distribution points to $\mathcal{P}$
- scattering rate points to C
- virtual term points to the minus sign and the second C
- diffusion approx. $q \ll p$ points to $\hat{q} \partial_p^2 \mathcal{P}(\mathbf{p}, t)$

- Radiation: medium-induced (+ vacuum)

$$\left| \text{diagram} \right|^2 \Rightarrow \frac{d\sigma}{dzd\vartheta} \approx 2\bar{\alpha}_s \sqrt{\frac{Q_{med}}{z^3 E}} B(\vartheta)$$

Medium ($k_t \sim Q_{med}, z \ll Q_{med}/E$): soft & ~~collin~~ pole

The diagram shows a horizontal line with three vertical wavy lines (representing gluons) attached to it. The first and third wavy lines are crossed out with an 'X'. The second wavy line is not crossed out.

- Wide-angle medium-induced cascade:

$$\left| \text{diagram} \right|^2 + \left| \text{diagram} \right|^2 + \dots \Rightarrow \text{diagram}$$

The diagram shows a horizontal line with three vertical wavy lines (representing gluons) attached to it. The first and third wavy lines are crossed out with an 'X'. The second wavy line is not crossed out.

(+ mixing with jet evolution!)

- Cascade evolution:

$$\partial_t D(x, t) = \int dz [\Gamma(z, \frac{x}{z}) D(\frac{x}{z}, t) - \frac{1}{2} \Gamma(z, p) D(x, t)]$$

Annotations for the equation:

- "time" points to ∂_t
- FF points to Γ
- BDMPS-Z splitting rate points to Γ
- virtual term points to the minus sign and the second Γ

Example: jets in large medium

[Blaizot, Dominguez, Iancu, Mehtar-Tani]
[Casalderrey-Solana, Mehtar-Tani, Tywoniuk, Salgado]
[Caucal, Iancu, Mueller, Soyez]

- Broadening: (in vacuum straight line $\delta_{ij}\delta(p-p')$)

$$\left| \text{diagram} \right|^2 \approx G_{ij}(p', p, L) \quad \text{changing color and momentum}$$

The diagram shows a horizontal line with three vertical wavy lines (representing gluons) attached to it. The wavy lines are labeled with 'x' and 'o' symbols.

- Evolution equation for mom. distr:

$$\partial_t \mathcal{P}(\mathbf{p}, t) = \int_{\mathbf{q}_\perp} [C(\mathbf{q}_\perp) \mathcal{P}(\mathbf{p} - \mathbf{q}_\perp, t) - C(\mathbf{q}_\perp) \mathcal{P}(\mathbf{p}, t)]$$

Annotations for the equation:

- "time" points to ∂_t
- mom. distribution points to $\mathcal{P}$
- scattering rate points to C
- virtual term points to the bracketed term
- diffusion approx. $q \ll p$ points to $\hat{q} \partial_p^2 \mathcal{P}(\mathbf{p}, t)$

The diagram shows a horizontal line with a vertical wavy line (representing a gluon) attached to it. The wavy line is labeled with 'x' and 'o' symbols.

- Radiation: medium-induced (+ vacuum)

$$\left| \text{diagram} \right|^2 \Rightarrow \frac{d\sigma}{dzd\vartheta} \approx 2\bar{\alpha}_s \sqrt{\frac{Q_{med}}{z^3 E}} B(\vartheta)$$

Medium ($k_t \sim Q_{med}, z \ll Q_{med}/E$): soft & coll. pole

The diagram shows a horizontal line with a vertical wavy line (representing a gluon) attached to it. The wavy line is labeled with 'x' and 'o' symbols.

- Wide-angle medium-induced cascade:



- Cascade evolution:

$$\partial_t D(x, t) = \int dz [\Gamma(z, \frac{x}{z}) D(\frac{x}{z}, t) - \frac{1}{2} \Gamma(z, p) D(x, t)]$$

Annotations for the equation:

- "time" points to ∂_t
- FF points to D
- BDMPS-Z splitting rate points to Γ
- virtual term points to the bracketed term

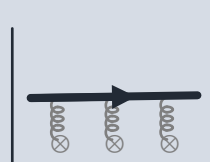
The diagram shows a horizontal line with a vertical wavy line (representing a gluon) attached to it. The wavy line is labeled with 'x' and 'o' symbols.

(+ mixing with jet evolution!)

Example: jets in large medium

[Blaizot, Dominguez, Iancu, Mehtar-Tani]
[Casalderrey-Solana, Mehtar-Tani, Tywoniuk, Salgado]
[Caucal, Iancu, Mueller, Soyez]

- Broadening: (in vacuum straight line $\delta_{ij}\delta(p-p')$)

$$\left| \text{diagram} \right|^2 \approx G_{ij}(p', p, L) \quad \text{changing color and momentum}$$


- Evolution equation for mom. distr:

$$\partial_t \mathcal{P}(\mathbf{p}, t) = \int_{q_\perp} [C(\mathbf{q}_\perp) \mathcal{P}(\mathbf{p} - \mathbf{q}_\perp, t) - C(\mathbf{q}_\perp) \mathcal{P}(\mathbf{p}, t)]$$

“time” mom. distribution scattering rate

virtual term

$$\approx \hat{q} \partial_p^2 \mathcal{P}(\mathbf{p}, t)$$

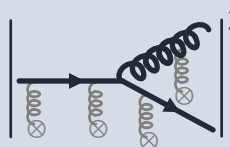
diffusion approx. $q \ll p$



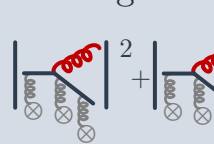

- Radiation: medium-induced (+ vacuum)

$$\left| \text{diagram} \right|^2 \Rightarrow \frac{d\sigma}{dzd\vartheta} \approx 2\bar{\alpha}_s \sqrt{\frac{Q_{med}}{z^3 E}} B(\vartheta)$$

Medium ($k_t \sim Q_{med}, z \ll Q_{med}/E$):
soft & ~~collin.~~ pole



- Wide-angle medium-induced **cascade**:

$$\left| \text{diagram} \right|^2 + \left| \text{diagram} \right|^2 + \dots \Rightarrow \text{cascade diagram}$$


(+ mixing with jet evolution!)

- Cascade evolution:

$$\partial_t D(x, t) = \int dz [\Gamma(z, \frac{x}{z}) D(\frac{x}{z}, t) - \frac{1}{2} \Gamma(z, p) D(x, t)]$$

“time” FF BDMPs-Z splitting rate virtual term





Example: jets in large medium

[Blaizot, Dominguez, Iancu, Mehtar-Tani]
[Casalderrey-Solana, Mehtar-Tani, Tywoniuk, Salgado]
[Caucal, Iancu, Mueller, Soyez]

- Broadening: (in vacuum straight line $\delta_{ij}\delta(p-p')$)

$$\left| \text{diagram} \right|^2 \approx G_{ij}(p', p, L)$$

changing color and momentum

- Evolution equation for mom. distr:

$$\partial_t \mathcal{P}(\mathbf{p}, t) = \int_{\mathbf{q}_\perp} [C(\mathbf{q}_\perp) \mathcal{P}(\mathbf{p} - \mathbf{q}_\perp, t) - C(\mathbf{q}_\perp) \mathcal{P}(\mathbf{p}, t)]$$

“time” mom. distribution scattering rate

virtual term

$$\approx \hat{q} \partial_p^2 \mathcal{P}(\mathbf{p}, t)$$

diffusion approx. $q \ll p$

- Radiation: medium-induced (+ vacuum)

$$\left| \text{diagram} \right|^2 \Rightarrow \frac{d\sigma}{dzd\vartheta} \approx 2\bar{\alpha}_s \sqrt{\frac{Q_{med}}{z^3 E}} B(\vartheta)$$

Medium ($k_t \sim Q_{med}, z \ll Q_{med}/E$):
soft & ~~collin.~~ pole

- Wide-angle medium-induced **cascade**:

$$\left| \text{diagram} \right|^2 + \left| \text{diagram} \right|^2 + \dots \Rightarrow \text{diagram}$$

(+ mixing with jet evolution!)

- Cascade** evolution:

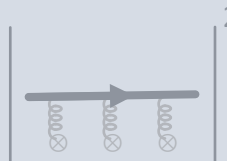
$$\partial_t D(x, t) = \int dz \left[\Gamma(z, \frac{x}{z}) D(\frac{x}{z}, t) - \frac{1}{2} \Gamma(z, p) D(x, t) \right]$$

“time” FF BDMPS-Z splitting rate virtual term

Example: jets in large medium

[Blaizot, Dominguez, Iancu, Mehtar-Tani]
[Casalderrey-Solana, Mehtar-Tani, Tywoniuk, Salgado]
[Caucal, Iancu, Mueller, Soyez]

- Broadening: (in vacuum straight line $\delta_{ij}\delta(p-p')$)



$$\approx G_{ij}(p', p, L)$$

changing color and momentum

- Evolution equation for mom. distr.

$$\partial_t \mathcal{P}(p, t) = \int_{q_1} [C(q_1, p, t) - \mathcal{P}(p, t) C(q_1, p, t)]$$

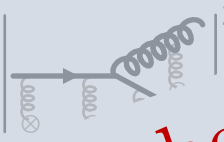
“time” mom. distribution scattering 1

virtual term

diffusion approx. $q \ll p$ $\approx \hat{q} \partial_p^2 \mathcal{P}$

Jet evolution in the QGP

- Radiation: medium-induced (+ vacuum)



$$\sqrt{\frac{Q_{med}}{z^3 E}} B(\vartheta)$$

Medium ($k_t \sim Q_{med}, z \ll Q_{med}/E$):
soft & ~~collin.~~ pole

- Wide-angle medium-induced cascade:



(+ mixing with jet evolution!)

$\mathcal{P}(x, t)$

term

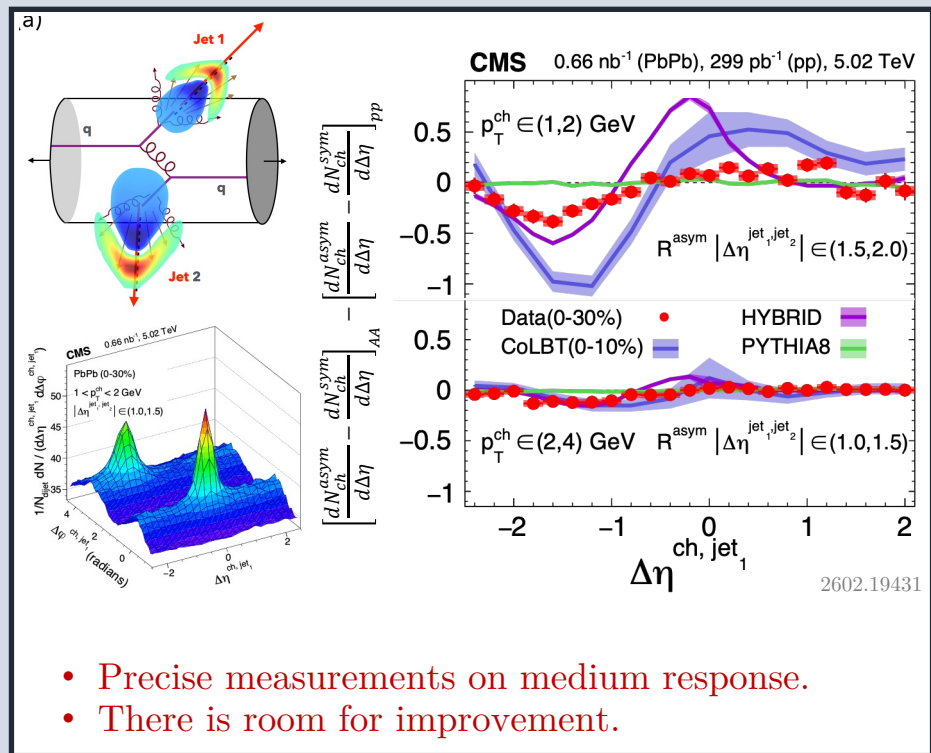
- Successful in pheno (R_{AA} , jet substr)
- Does not thermalize!**
- Not applicable for observables sensitive to med. resp.

2. Jet thermalization and Medium response



[Wikipedia]

Recent experimental progression



Medium response

In-medium jet evolution

Strongly coupled

Falling string + metric perturbation evolution
in AdS/CFT

[Chesler, Yaffe]

In practice: instantaneous freeze-out/hydro

[Hybrid 2010.01140]

$$\partial_\nu T^{\mu\nu} = \Delta J^\mu$$

Weakly coupled

QCD-EKT: thermalization from first principles.

[Arnold,Morre,Yaffe]

[Baier,Mueller,Schiff,Son]

$$(\partial_t + v \cdot \nabla_x) \delta f(t, x, p) = \delta \mathcal{C}^{2 \leftrightarrow 2} + \delta \mathcal{C}^{1 \leftrightarrow 2}$$

How EKT is related to jet quenching?

Minijet quenching in AMY-EKT: [Mehtar-Tani,Schlichting,Soudi]

[Brewer,Mazeliauskas,Zhou]

In practice: few scatterings + hydro.

[JEWEL, LBT, Hybrid (w Moliere)]

Medium response

In-medium jet evolution

Strongly coupled

Falling string + metric perturbation evolution
in AdS/CFT

In practice: instant

∂_v

Weakly coupled

QCD-EKT: thermalization from first principles.

[Arnold,Morre,Yaffe]
[Schiff,Son]
→2

**In this talk: (i) AMY-EKT $\leftrightarrow$ jet quenching,
(ii) EKT $\leftrightarrow$ parton showers,
(iii) EKT parton shower $\rightarrow$ med. resp**

[Gyng,Soudi]
[Drewer,Mazeliauskas,Zhou]

In practice: few scatterings + hydro.

[JEWEL, LBT, Hybrid (w Moliere)]

(i) Linearized kinetic theory

- lin. Boltzmann eq: $f(t, \mathbf{x}, \mathbf{p}) = n(\mathbf{p}) + \delta f(t, \mathbf{x}, \mathbf{p})$

$$\left(\partial_t + \frac{\mathbf{p}}{p} \nabla_{\mathbf{x}}\right) \delta f(t, \mathbf{x}, \mathbf{p}) = \delta \mathcal{C}^{1 \leftrightarrow 2}[n, \delta f] + \delta \mathcal{C}^{2 \leftrightarrow 2}[n, \delta f].$$

- 1 $\leftrightarrow$ 2 collisions:

$$\delta \mathcal{C}^{1 \leftrightarrow 2}[n, \delta f] = \int dz \left[\underbrace{\frac{1}{z^3} \Gamma(z, \frac{p}{z})}_{\text{splitting rate}} \underbrace{\delta \mathcal{F}\left(\frac{p}{z}, p, \frac{zp}{z}\right)}_{\text{statistical factors (detailed balance)}} - \frac{1}{2} \Gamma(z, p) \delta \mathcal{F}(p; zp, \bar{z}p) \right]$$

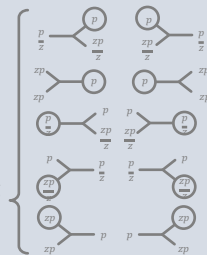
$$\begin{aligned} \delta \mathcal{F}(p, k, p') &= \delta f(p) [(1 + n(k))(1 + n(p')) - n(k)n(p')] \\ &\quad - \delta f(k) [n(p') (1 + n(p)) - (1 + n(p'))n(p)] \\ &\quad - \delta f(p') [n(k) (1 + n(p)) - (1 + n(k))n(p)]. \end{aligned}$$

- 2 $\leftrightarrow$ 2 collisions:

$$\delta \mathcal{C}^{2 \leftrightarrow 2}[n, \delta f] = \int d\Omega \underbrace{|\mathcal{M}|^2}_{\text{scattering mx}} \underbrace{\delta \mathcal{F}(p_1, p_2; p_3 p_4)}_{\text{statistical factors (detailed balance)}}$$

$$\begin{aligned} \delta \mathcal{F} &= \delta f_3 [n_4 n_1 n_2 \mp n_4 n_1 n_2] \\ &\quad + \delta f_4 [n_3 n_1 n_2 \mp n_3 n_1 n_2] \\ &\quad - \delta f_2 [n_1 n_3 n_4 \mp n_1 n_3 n_4] \\ &\quad - \delta f_1 [n_2 n_3 n_4 \mp n_2 n_3 n_4]. \end{aligned}$$

- Two type of splitting
- Merging (holes)



- Single type of scatt.
- Recoils, holes.



(i) Linearized kinetic theory (high-energy limit $p \gg T$)

- lin. Boltzmann eq:

$$\left(\partial_t + \frac{p}{p} \nabla_x\right) \delta f(t, \mathbf{x}, \mathbf{p}) \approx \delta C^{1 \leftrightarrow 2}[n, \delta f] + \delta C^{2 \leftrightarrow 2}[n, \delta f].$$

- 1 $\leftrightarrow$ 2 collisions: (high energy limit of Γ = BDMPS-Z in the HO limit)

$$\delta C^{1 \leftrightarrow 2}[n, \delta f] \approx \int dz \left[\frac{1}{z^3} \Gamma(z, \frac{p}{z}) \delta f(t, \frac{p}{z}) - \frac{1}{2} \Gamma(z, p) \delta f(t, p) \right]$$

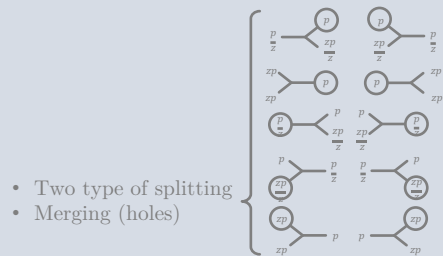
Agrees with the BDMPS-Z medium-induced cascade!

- 2 $\leftrightarrow$ 2 collisions: (high-energy + small angle limit of $C(q_\perp)$ = BDMPS-Z in the HO limit)

$$\delta C^{2 \leftrightarrow 2}[n, \delta f] \approx \int_{q_\perp} dq_\perp C(q_\perp) [\delta f(p + q_\perp) - \delta f(p)]$$

Agrees with the BDMPS-Z broadening evolution!

(i) AMY-EKT $\leftrightarrow$ jet quenching!

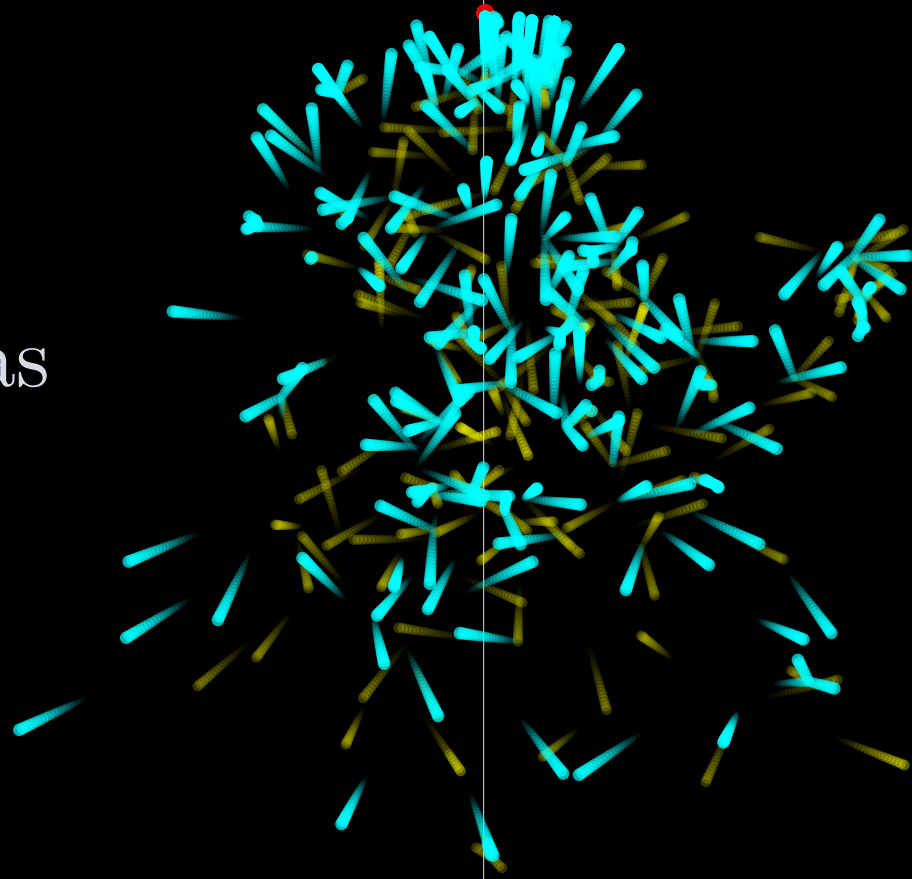


- Two type of splitting
- Merging (holes)

- Single type of scatt.
- Recoils, holes.



3. Kinetic theories as parton showers



Linearized Boltzmann eq. as a parton shower

$$\left(\partial_t + \frac{\mathbf{p}}{p} \nabla_{\mathbf{x}}\right) \delta f(t, \mathbf{x}, \mathbf{p}) = \delta C[n, \delta f].$$

- Homogeneous case:

1. Separate “real” and “virtual” collisions: $\delta C = \delta C^r[n, \delta f] - \delta C^v[n] \delta f$.

2. Integrate the Boltzmann eq: where $\Delta(t) = \exp \left[\int^t dt' \delta C^v[n] \right]$

$$\delta f(t) = \Delta(t) \delta f_0 + \int^t dt' \frac{\Delta(t)}{\Delta(t')} \delta C^r[n, \delta f].$$

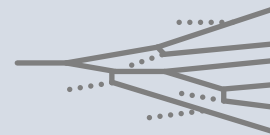
4. Iterate: $\delta f_0 = \delta(p - p_0)$.

5. Histogram the particles: $\delta f(t, \mathbf{x}, \mathbf{p}) = \frac{1}{N_{\text{ev}}} \frac{dN}{dp d\mathbf{x}}$

(ii) kinetic theories $\leftrightarrow$ parton showers!

- Inhomogeneous case is trivial: free streaming between collisions.

(iii) inhomogeneous perturbations



```
Initial condition:#[ID, p, x, weight]
[21 (100,0,0,100) (0,0,0,0) 1]

Output:
[[21 (1.5,0.0,0.00,1.5) (0.1,0.0,0.0,0.1) 1],
 [21 (2.8,0.0,0.0,2.8) (2.0,0.0,0.0,2.0) 1],
 [21 (5.0,0.0,0.0,5.0) (9.4,0.0,0.0,9.4) 1],
 [21 (1.7,1.1,0.6,1.1) (9.7,0.0,0.0,9.7) -1],
 [21 (82,-0.9,-0.2,82) (9.7,0.0,0.0,9.7) 1],
 [21 (1.0,0.4,0.2,0.9) (12.0,1.4,0.5,12.0) 1],
 [21 (2.2,0.0,0.0,2.2) (15.0,0.0,0.0,15.0) 1],
 [21 (3.3,0.0,0.0,3.3) (15.0,0.0,0.0,15.0) 1],
 [21 (1.5,0.7,0.2,1.3) (15.5,2.6,1.0,14.8) 1],
 [21 (2.1,0.9,0.3,1.8) (15.5,2.6,1.0,14.8) 1]]
```

Linearized Boltzmann eq. as a parton shower

$$\left(\partial_t + \frac{p}{p} \nabla_x\right) \delta f(t, \mathbf{x}, \mathbf{p}) = \delta C[n, \delta f].$$

- Homogeneous case:

1. Separate “real” and “virtual” collisions: $\delta C = \delta C^r[n, \delta f] - \delta C^v[n] \delta f$.

2. Integrate the Boltzmann eq: where $\Delta(t) = \exp \left[\int^t dt' \delta C^v[n] \right]$

$$\delta f(t) = \Delta(t) \delta f_0 + \int^t dt' \frac{\Delta(t)}{\Delta(t')} \delta C^r[n, \delta f].$$

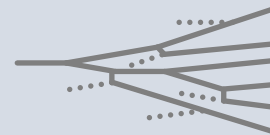
4. Iterate: $\delta f_0 = \delta(p - p_0)$.

5. Histogram the particles: $\delta f(t, x, p) = \frac{1}{N_{\text{ev}}} \frac{dN}{dp dx}$

(ii) kinetic theories $\leftrightarrow$ parton showers!

- Inhomogeneous case is trivial: free streaming between collisions.

(iii) inhomogeneous perturbations



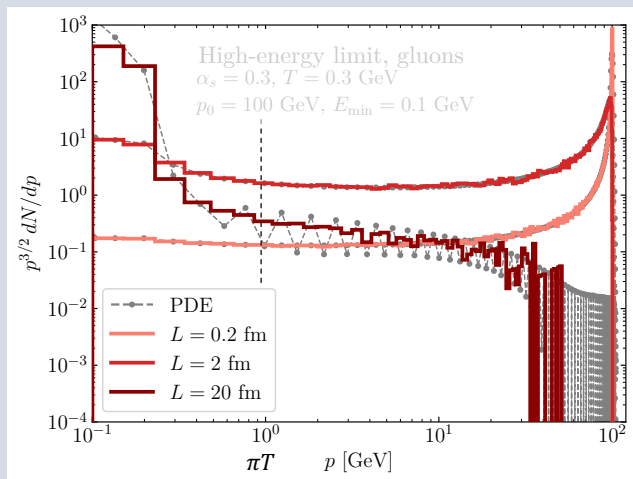
```
Initial condition:#[ID, p, x, weight]
[21 (100,0,0,100) (0,0,0,0) 1]

Output:
[[21 (1.5,0.0,0,0,1.5) (0.1,0.0,00,0.1) 1],
 [21 (2.8,0.0,0.0,2.8) (2.0,0.0,00,2.0) 1],
 [21 (5.0,0.0,0.0,5.0) (9.4,0.0,0.0,9.4) 1],
 [21 (1.7,1.1,0.6,1.1) (9.7,0.0,0.0,9.7) -1],
 [21 (82,-0.9,-0.2,82) (9.7,0.0,0.0,9.7) 1],
 [21 (1.0,0.4,0.2,0.9) (12.0,1.4,0.5,12.0) 1],
 [21 (2.2,0.0,0.0,2.2) (15.0,0.0,0.0,15.0) 1],
 [21 (3.3,0.0,0.0,3.3) (15.0,0.0,0.0,15.0) 1],
 [21 (1.5,0.7,0.2,1.3) (15.5,2.6,1.0,14.8) 1],
 [21 (2.1,0.9,0.3,1.8) (15.5,2.6,1.0,14.8) 1]]
```

Numerical solution (gluons)

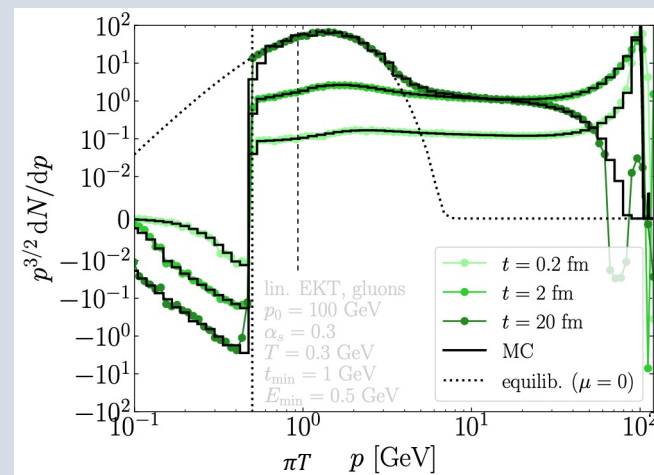
see also [Mehtar-Tani,Schlichting,Soudi]
[Brewer,Mazeliauskas,Zhou]

High-energy limit



- Parton shower = diff. eq. solver
- Cascade doesn't thermalize!

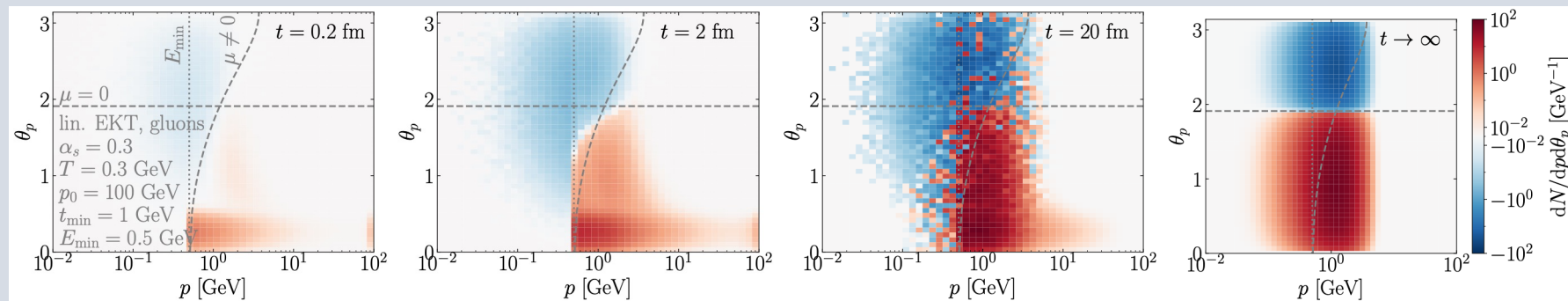
Full EKT



- Parton shower = diff. eq. solver
- Thermalization!
- Mixture of $\mu = 0$ and $\mu = 0$ equilibrium.

Numerical solution(gluons)

see also [Mehtar-Tani,Schlichting,Soudi]
[Brewer,Mazeliauskas,Zhou]



1. Collinear radiation of soft gluons.
 2. Broadening of soft fragments
 3. $\mu \neq 0$ equilibrium $\rightarrow \mu = 0$ equilibrium.
- (+) and (-) medium response.
 - Soft and hard medium response.
 - Boosted equilibrium only at very late times.

Outlook: Why all the fuss?

Applications

lin. Boltzmann eq:

$$\left(\partial_t + \frac{\mathbf{p}}{p} \nabla_x\right) \delta f(t, \mathbf{x}, \mathbf{p}) = \delta C[n, \delta f].$$

1. New phenomenological applications of EKT!

Interface with jets, hadronization, etc.

2. Multi-particle correlations in EKT.

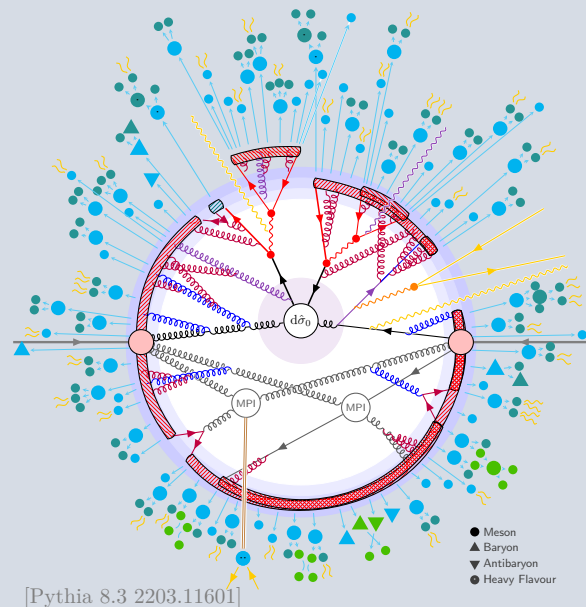
Parton-shower is an n -particle cross-section generator:

Unexplored topic in EKT, great relevance in EEC.

3. Incorporate inhomogeneities.

Previously neglected! (parton shower speedup)

Hydrodynamization: $\delta f(p, x) \rightarrow \delta f_{eq}(p, \delta T(t, x), \delta u(t, x))$



Applications

lin. Boltzmann eq:

$$\left(\partial_t + \frac{\mathbf{p}}{p} \nabla_x\right) \delta f(t, \mathbf{x}, \mathbf{p}) = \delta C[n, \delta f].$$

1. New phenomenological applications of EKT!

Interface with jets, hadronization, etc.

2. Multi-particle correlations in EKT.

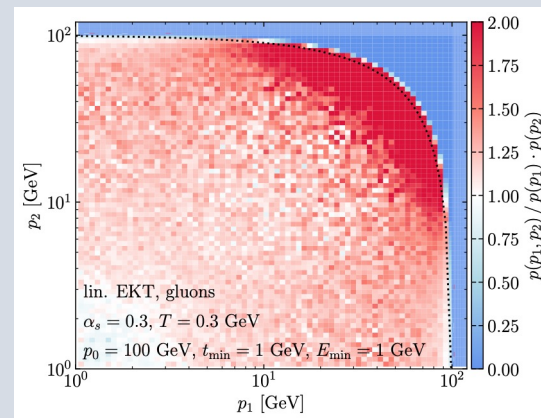
Parton-shower is an n -particle cross-section generator: $\delta f(p_1, p_2, \dots) = \frac{1}{N} \frac{dN}{dp_1 dp_2 \dots}$

Unexplored topic in EKT, great relevance in EEC.

3. Incorporate inhomogeneities.

Previously neglected! (parton shower speedup)

Hydrodynamization: $\delta f(p, x) \rightarrow \delta f_{eq}(p, \delta T(t, x), \delta u(t, x))$



Correlations beyond molecular chaos!

Applications

lin. Boltzmann eq:

$$\left(\partial_t + \frac{\mathbf{p}}{p} \nabla_x\right) \delta f(t, \mathbf{x}, \mathbf{p}) = \delta C[n, \delta f].$$

1. New phenomenological applications of EKT!

Interface with jets, hadronization, etc.

2. Multi-particle correlations in EKT.

Parton-shower is an n -particle cross-section generator:

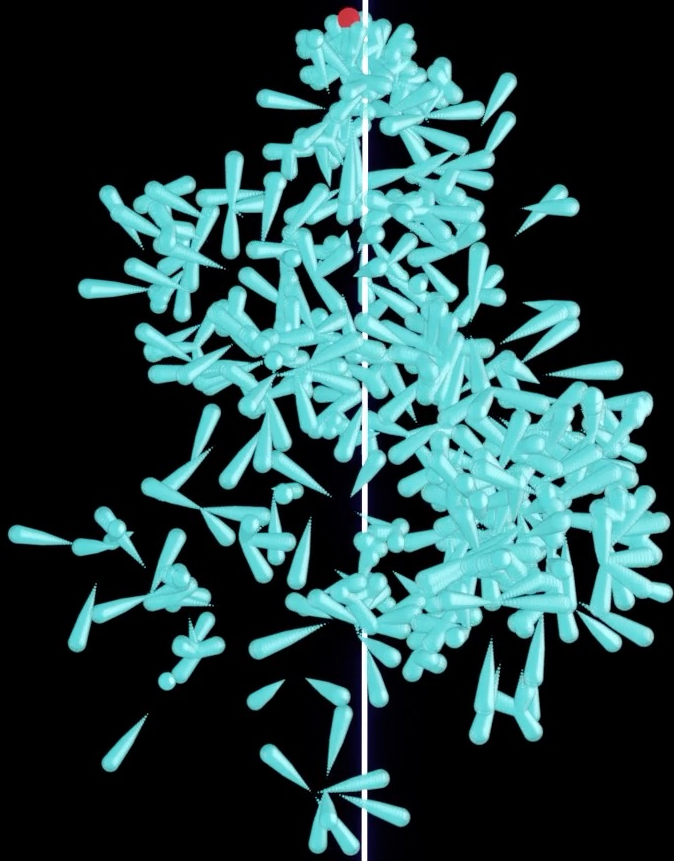
Unexplored topic in EKT, great relevance in EEC.

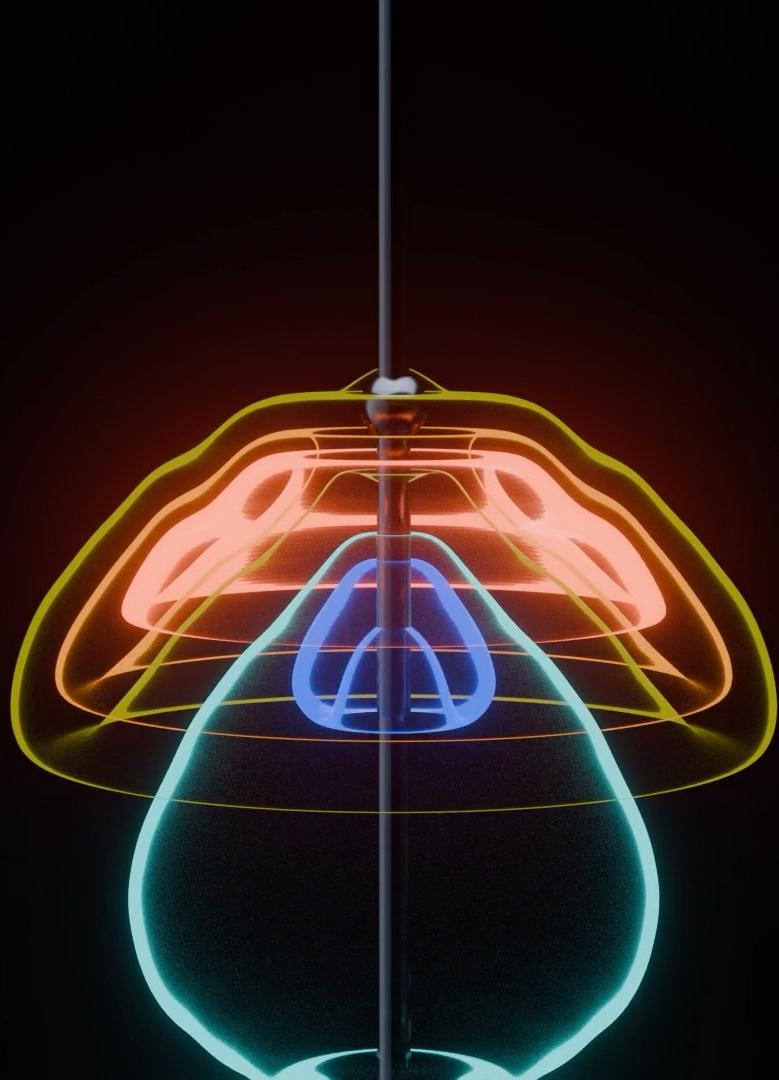
3. Incorporate inhomogeneities.

Previously neglected! (parton shower speedup)

Hydrodynamization: $\delta f(p, x) \rightarrow \delta f_{eq}(p, \delta T(t, x), \delta u(t, x))$

$$\delta f(x) = \frac{1}{N} \frac{dN}{dx}$$





Summary:

A decade of jet substructure in heavy ion collisions

Dec 7 – 18, 2026
CERN

QCD@LHC 2026

28 September 2026 to 2 October 2026
University of Oxford
Europe/London timezone

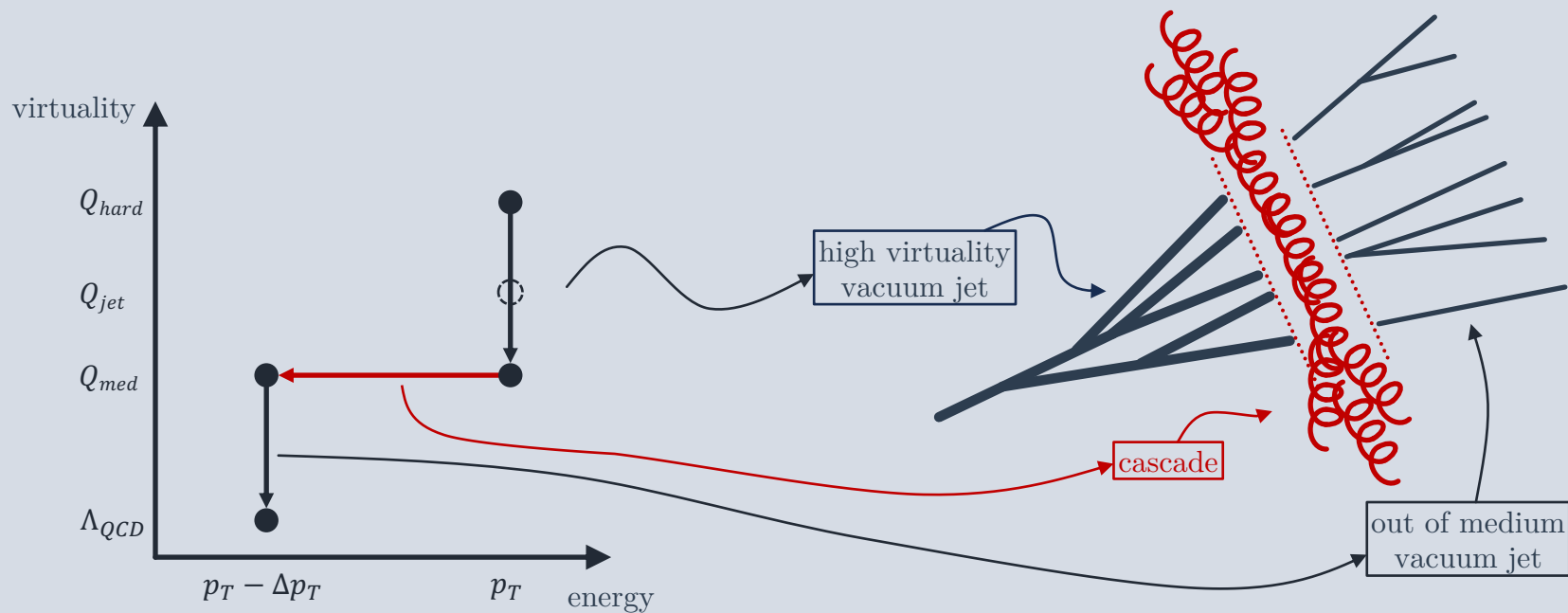
1. Perturbative grounds of jet quenching
2. AMY-EKT $\leftrightarrow$ parton energy loss and broadening
3. Lin. Boltzmann eq. $\leftrightarrow$ parton shower
4. New opportunities for EKT:
 - Jet thermalization,
 - Multi-particle correlations, hydrodynamization, fluctuations
 - Including in phenomenological workflows



Thank you for your attention!

Jet evolution in QGP

[Casalderrey-Solana, Mehtar-Tani, Tywoniuk, Salgado]
[Caucal, Iancu, Mueller, Soyez]

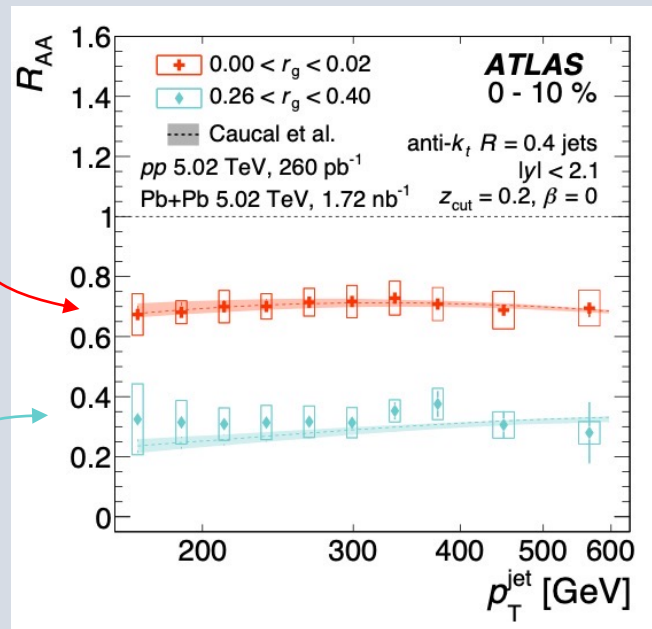
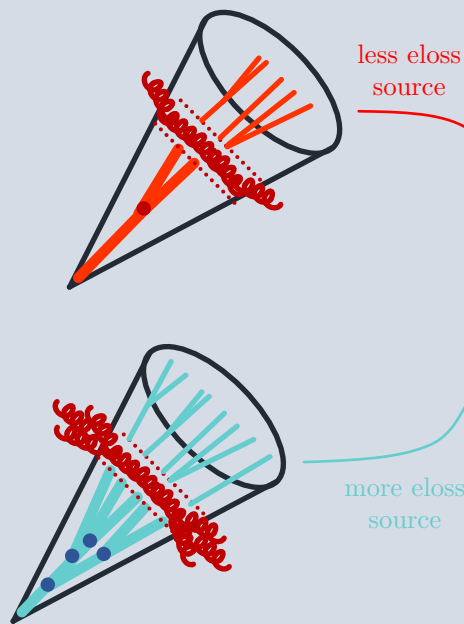


Experimental tests

[Caucal,Soto-Ontoso,AT, PRD105(2022)114046] → [ALICE PRL135(2025)031901]
 [AT in PRD110(2024)014015] → [CMS 2602.09271]

Jet spectrum for different substructure

$$R_{AA} \sim \frac{d\sigma_{AA}^{jet}/dp_T}{d\sigma_{pp}^{jet}/dp_T}$$



[ATLAS PRC107(2023)054909]