Minijet thermalization and jet transport coefficients in QCD kinetic theory

Fabian Zhou

Institut für Theoretische Physik
Universität Heidelberg

EMMI Workshop Darmstadt, 17.07.26

with K. Boguslavski, F. Lindenbauer, A. Mazeliauskas and A. Takacs, JHEP, 2510.25669

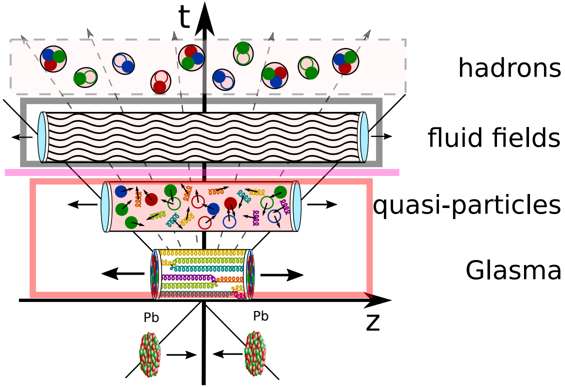
with J. Brewer and A. Mazeliauskas, JHEP, 2402.09298

Overview

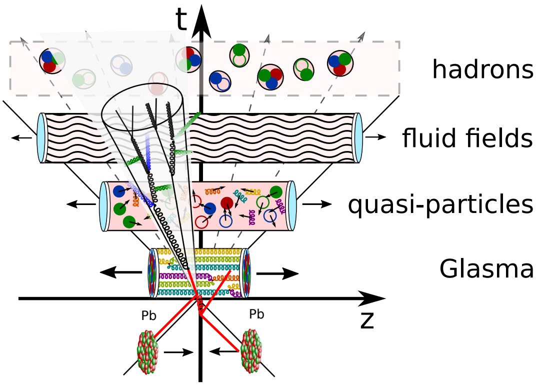
Hydrodynamics

Initial state at τ_{hydro}

Pre-equilibrium



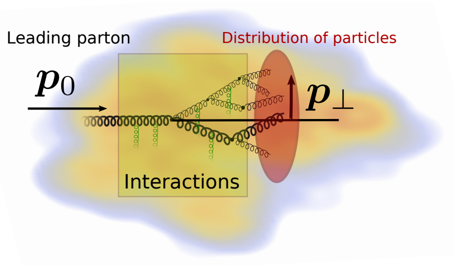
Overview



- study minijet thermalisation with kinetic theory

Overview

- Jet transport coefficient $\hat{q} \to$ medium property



1 bring together transport coefficients and minijet evolution

2 estimate thermalisation time of a jet

Outline

Linearised kinetic theory

Transport coefficients and minijet evolution

Equilibration time

Expanding QGP

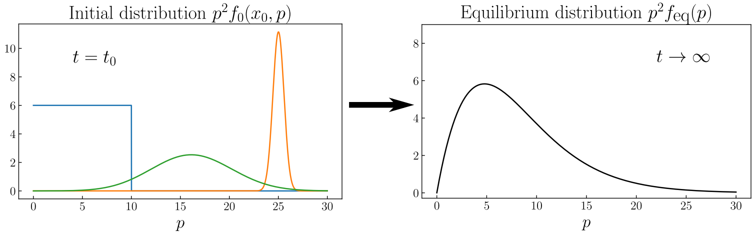
Linearised kinetic theory

Framework

- Effective Kinetic Theory for high temperature gauge theories AMY (2003), JHEP 0209353
- weakly coupled quasi-particle picture, $\lambda = 4\pi\alpha_s N_c$

  $\rightarrow$phase space distribution $f(\tau, \boldsymbol{x}, \boldsymbol{p})$ $\rightarrow$ homogeneous system

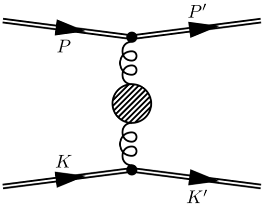
$$\partial_\tau f(\tau, \boldsymbol{p}) = -C[f]$$



Out-of-equilibrium initial state is transported to equilibrium

Collision kernel

$C_{2\leftrightarrow 2}$

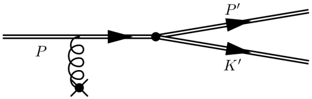


- small momentum transfer $q = |\boldsymbol{p}' - \boldsymbol{p}| \ll 1$

$\boxed{\text{isotropic HTL screening}}$

Boguslavski, Lindenbauer, PRD 2407.09605

$C_{1\leftrightarrow 2}$



- medium induced radiation of gluons
- $g \to q\bar{q}$ splittings
- LO: strictly collinear

Linearised EKT

- thermal background + linear perturbation

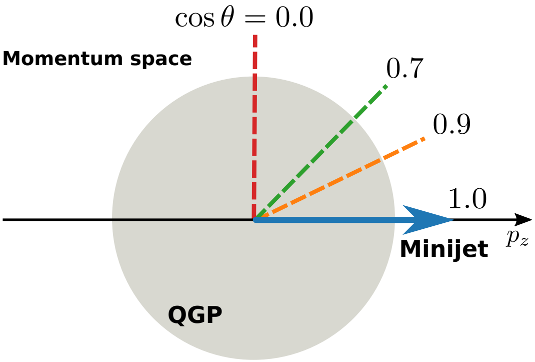
$$f(t,\boldsymbol{p}) = f_{\rm eq}(p) + \delta f_{\rm jet}(t,\boldsymbol{p})$$

- linearized Boltzmann eq.

$$\partial_t \delta f = -\delta C[\delta f, f_{\rm eq}]$$

Mehtar-Tani, Schlichting, Soudi, JHEP 2209.10569
Barrera, HP2024
Brewer, Mazeliauskas, FZ, JHEP 2402.09298

- jet-like initial conditions



Initial conditions

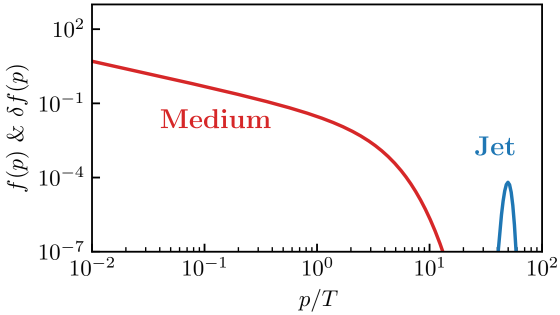
- jet evolution

$$\partial_t \delta f = -\delta C[\delta f, f_{\rm eq}]$$

- initial jet distribution

$$\delta f_{\rm Jet}(t_0, \boldsymbol{p}) \propto \delta(\boldsymbol{p} - \boldsymbol{p}_0)$$

$$\boldsymbol{p}_0 = (0, 0, E)$$



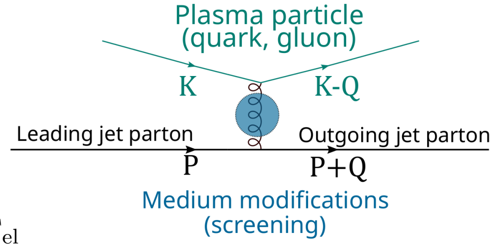
"single" parton in a thermal brick

$$\boxed{\text{transport coefficients + momentum broadening of } \delta f_{\rm jet}(t_0, \boldsymbol{p})}$$

Boguslavski, Lindenbauer, Mazeliauskas, Takacs, FZ, JHEP, 2510.25669

Transport coefficients and minijet evolution

Transport coefficient $\hat{q}$ - Literature



Lindenbauer et al., PRD 2312.00447

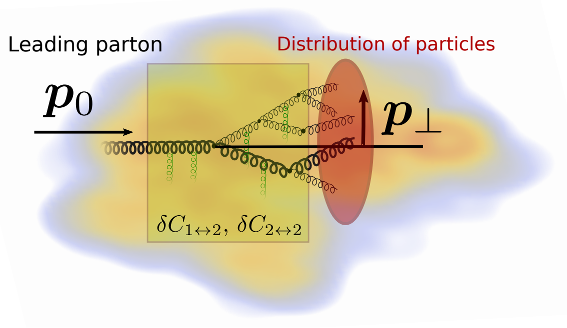
- conventionally

$$\hat{q} = \int d^2\boldsymbol{q}_\perp \boldsymbol{q}_\perp^2 \frac{d\Gamma_{\mathrm{el}}}{d^2\boldsymbol{q}_\perp}$$

- broadening of 1 parton with energy p_0

$$\hat{q}(p_0) = \frac{1}{2p_0} \int d\Omega |\mathcal{M}|^2 f_{\mathrm{eq}}(k)(1 + f_{\mathrm{eq}}(k')) \, \boldsymbol{p}'^{\,2}_\perp$$

Transport coefficient $\hat{q}$ - This work



$$\hat{q} = \frac{d}{dt}\langle p_T^2\rangle = \frac{d}{dt}\left(\int_{\boldsymbol{p}} p_T^2 \delta f_{\text{Jet}}(t,\boldsymbol{p})\right)$$

- broadening of the "jet distribution"

$$\hat{q} = -\int_{\boldsymbol{p}} p_T^2 \delta C[\delta f, f_{\text{eq}}]$$

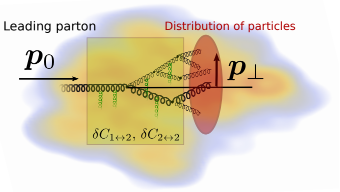
- **single hit approximation** $\delta f = \delta f^{(0)} + \delta f^{(1)} + \ldots$

$$\hat{q}^{(1)}(p_0) = \frac{1}{2p_0}\int d\Omega |\mathcal{M}|^2 f_{\text{eq}}(k)(1+f_{\text{eq}}(k'))\,(\boldsymbol{p}_\perp'^{\,2} + \underbrace{\boldsymbol{k}_\perp'^{\,2} - \boldsymbol{k}_\perp^2}_{\text{additional terms}})$$

cf. AMY, JHEP 0010177

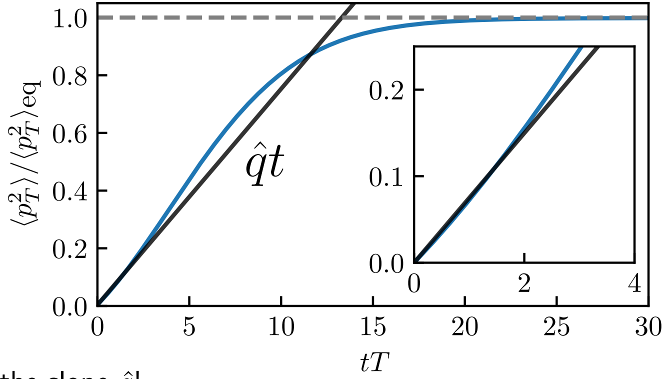
$\boxed{\text{EKT takes into account effects from the recoiling medium}}$

Momentum broadening from minijet evolution



- transverse momentum

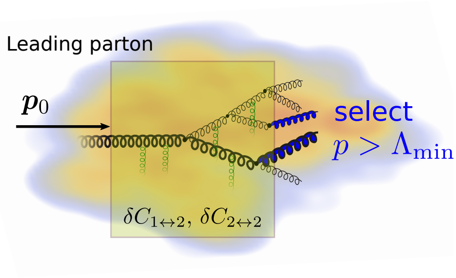
$$\langle p_T^2\rangle = \int_{\boldsymbol{p}} p_T^2 \delta f_{\text{Jet}}(t,\boldsymbol{p}) \approx \hat{q}t$$

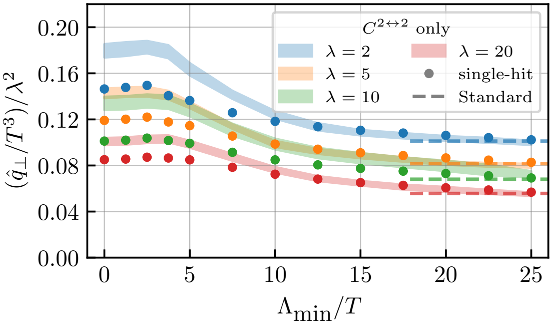


- **Extraction** of the slope $\hat{q}$!

Medium effects

- exclude soft momenta using lower momentum cutoff $\Lambda_{\min}$
- single-hit $\hat{q}^{(1)}$ vs extraction from minijet: $\hat{q} = \frac{d}{dt}\langle p_T^2 \rangle_{p>\Lambda_{\min}}$



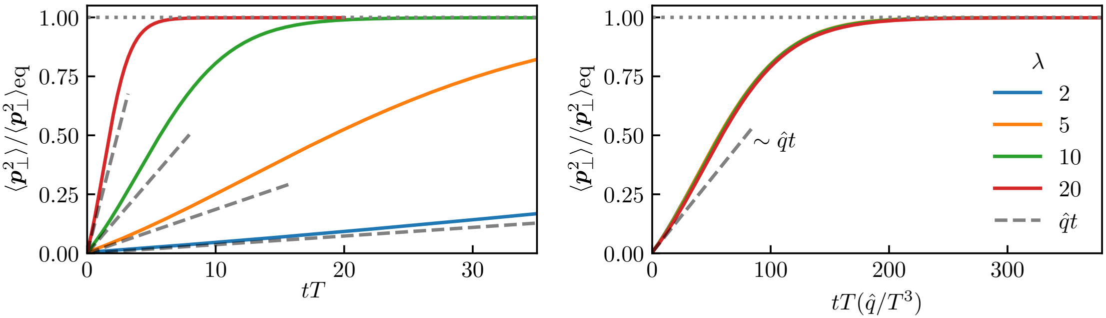


$\Lambda_{\min}$ removes contributions from the **broadening of the medium**!

Equilibration time

Scaling of transverse broadening

- time evolution of $\langle \boldsymbol{p}_\perp^2 \rangle$

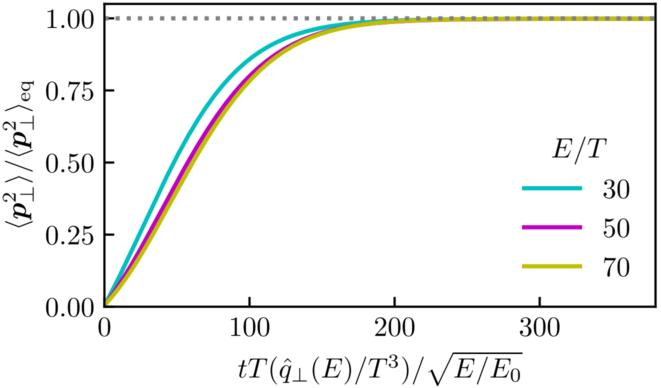


- collapse for different couplings λ!
- coupling dependence controlled by $\hat{q}$, "early time quantity"

Energy dependence

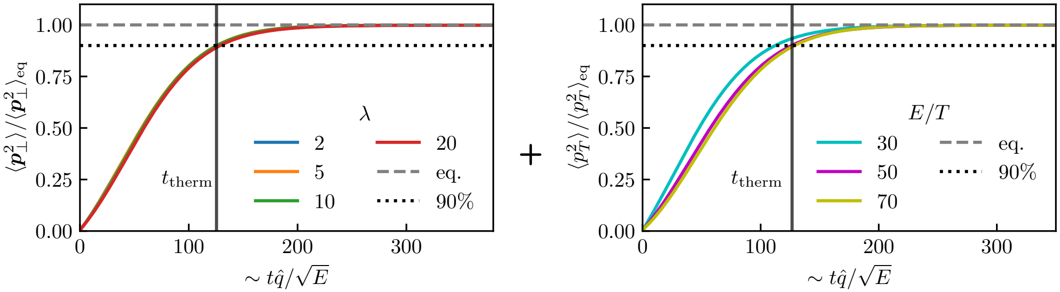
- **Equilibration** controlled by timescale of the first hard splitting $\sim \sqrt{E}$

  isotropic perturbation: Kurkela, Lu, PRL 1405.6318



- Collapse for different jet energies E!

Thermalisation time of minijets



Scaling with $\hat{q}$ and $\sqrt{E}$

$$t_{\rm therm} = 16.43\,{\rm fm}\left(\frac{T}{0.3\,{\rm GeV}}\right)^{-2}\left(\frac{\hat{q}/T^3}{6}\right)^{-1}\left(\frac{E}{15\,{\rm GeV}}\right)^{1/2}$$

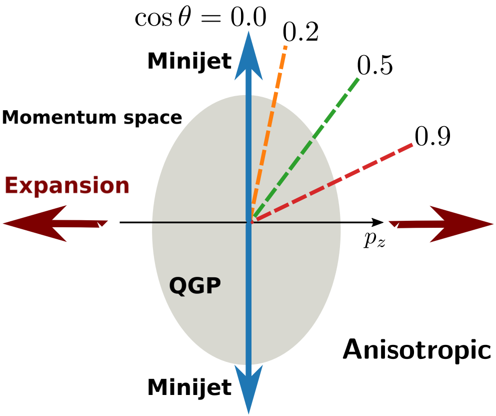
Expanding QGP

Non-equilibrium medium

$$\left(\partial_\tau - \frac{p_z}{\tau}\partial_{p_z}\right)\delta f(\tau, \mathbf{p}) = -\delta C[\delta f, \bar{f}]$$

$$\left(\partial_\tau - \frac{p_z}{\tau}\partial_{p_z}\right)\bar{f}(\tau, \mathbf{p}) = -C[\bar{f}]$$

Kurkela and Zhu, PRL 1506.06647
Kurkela and Mazeliauskas,
PRD 1811.03068, PRL 1811.03040



- non-thermal initial condition

$$\bar{f}(\tau_0, \mathbf{p}) \propto \frac{1}{\lambda}\exp\left(-\frac{2}{3}\frac{p_\perp^2 + \xi^2 p_z^2}{Q^2}\right)$$

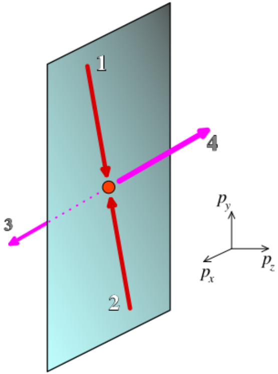
$\boxed{\bar{f}(\tau, \mathbf{p})\text{: approach to hydrodynamics around } \sim 1\ \mathrm{fm}}$

Early times - Out-of-plane scatterings

- $f \gg 1 \to$ classical approximation
- initial background suppressed in p_z-direction $\to f(\tau_0, \boldsymbol{p}_{3,4}) \approx 0$
- elastic kernel $C_{2\leftrightarrow 2}[f](\mathbf{p})$

$$\partial_t f_4 \sim f_1 f_2 (f_3 + f_4) - f_3 f_4 (f_1 + f_2) \quad \text{classical}$$

$$\underline{+ f_1 f_2} - f_3 f_4 \qquad \text{quantum}$$

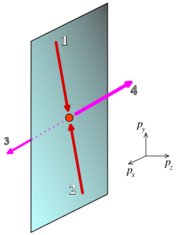


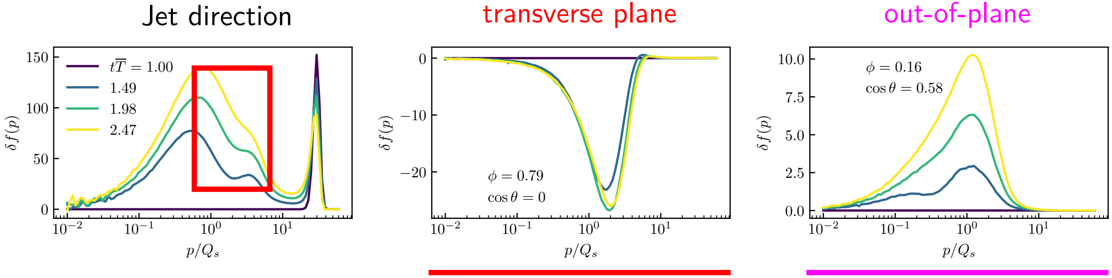
Wu et al.,
JHEP 1506.05580

$\boxed{\text{Quantum terms responsible for isotropisation!}}$

Jet distribution $\delta f(\mathbf{p})$ in 3 directions

- early times $\tau \gtrsim \tau_0$
- particles are scattered out of the transverse plane

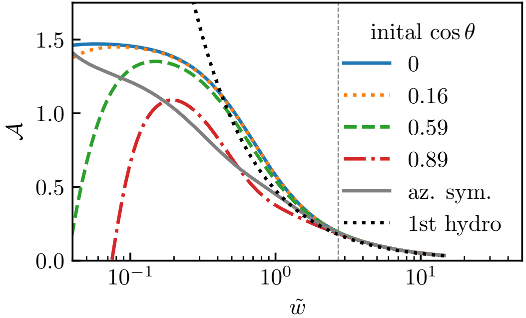


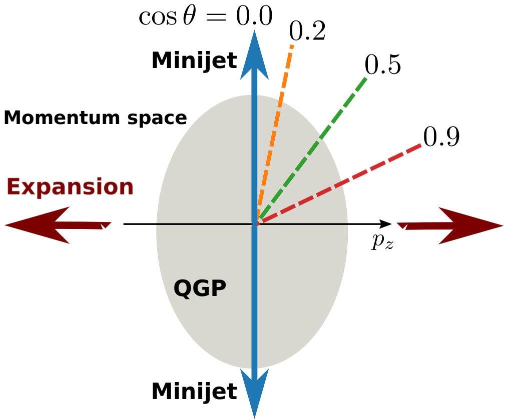


Depletion in the transverse plane $\leftrightarrow$ **Isotropisation**

Hydrodynamisation

- jets with different initial $\cos\theta$
- anisotropy $\mathcal{A} = \frac{\delta P_T - \delta P_L}{\delta e/3}$

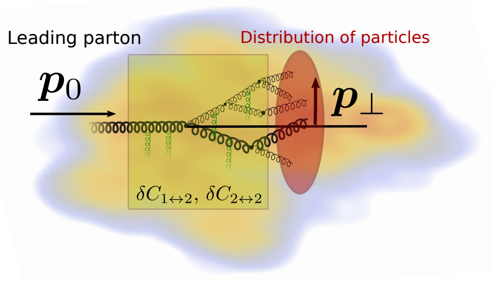




Indistinguishability around $\tilde{w}_{\text{hydro}} \approx 2.7$ $\boxed{\rightarrow \textbf{Hydrodynamisation}}$

Summary

Minijets as linear perturbations



Transport coefficients:

- cutoff Λ_{min} exhibits effects of the medium

Equilibration:

- universal evolution by scaling with $\hat{q}$ and $\sqrt{E}$
- $t_{\text{therm}} \sim 16\,\text{fm}$ for a jet of $15\,\text{GeV}$ (weakly coupled picture)
- jet perturbation becomes part of the hydrodynamical background

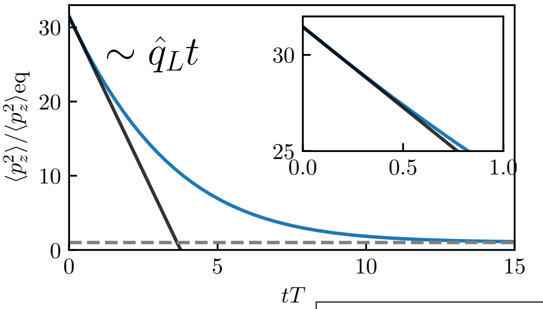
Outlook:

- non-equilibrium $\hat{q}_{ij}(\tau)$ Lindenbauer et al., PLB 2303.12595, PRD, 2312.0044

Momentum broadening from minijet evolution

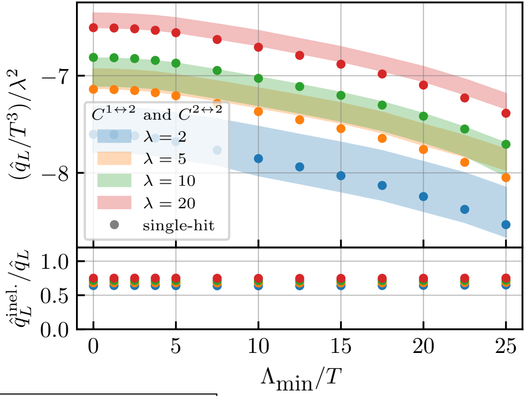
- longitudinal broadening $\langle p_z^2 \rangle$

$$\langle p_z^2 \rangle = \int p_z^2 \delta f_{\mathrm{Jet}}(t, \boldsymbol{p}) d^3\boldsymbol{p}$$
$$\approx \langle p_z^2 \rangle_0 + \hat{q}_L t$$



- extract $\hat{q}_L$

$$\hat{q}_L = \frac{d}{dt} \langle p_z^2 \rangle_{p > \Lambda_{\mathrm{min}}}$$

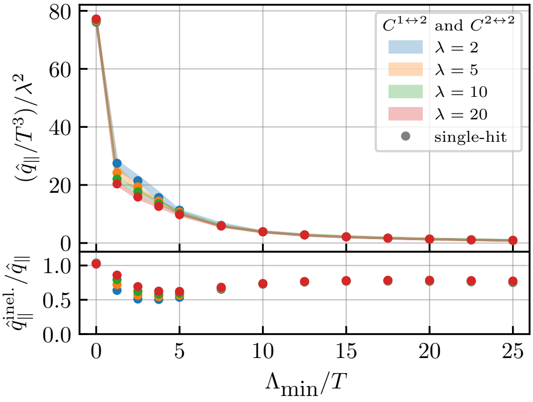


Important contributions from $C_{1\leftrightarrow 2}$!

Further transport coefficients Ghiglieri, Teaney, Int.J.Mod.Phys.E, 1502.03730

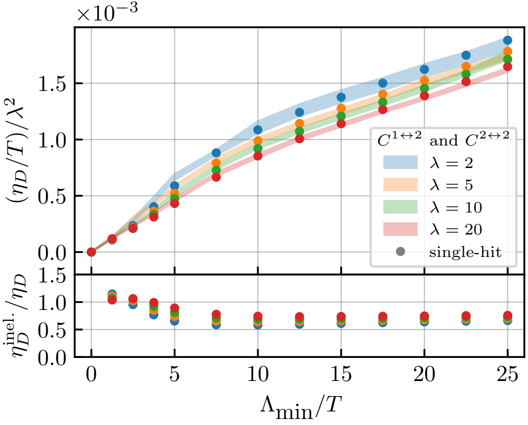
- relative momentum broadening

$$\hat{q}_{\parallel} = \frac{d}{dt}\langle (p_z - \langle p_z \rangle)^2 \rangle_{p>\Lambda_{\min}}$$



- drag coefficient

$$\eta_D = -\frac{1}{\langle p_z \rangle}\frac{d}{dt}\langle p_z \rangle_{p>\Lambda_{\min}}$$



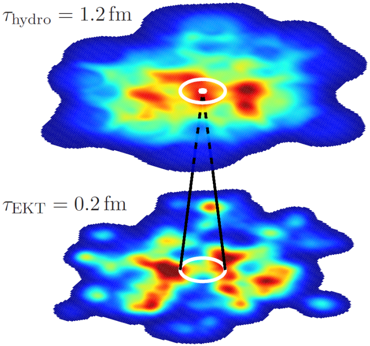
Expanding QGP

- longitudinal expansion
- approximate boost invariance
- homogeneity in the transverse plane

$$\left(\partial_\tau - \frac{p_z}{\tau}\partial_{p_z}\right) f(\tau, \boldsymbol{p}) = -C[f]$$

- leading order elastic and inelastic scattering processes

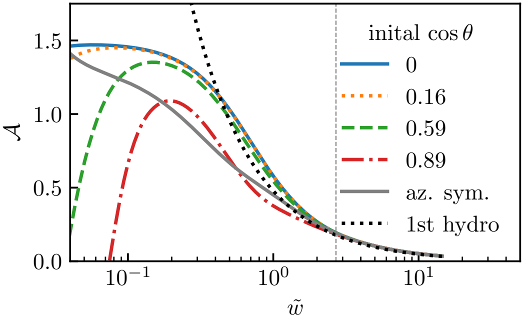
$$C[f](\boldsymbol{p}) = C_{2\leftrightarrow 2}[f](\boldsymbol{p}) + C_{1\leftrightarrow 2}[f](\boldsymbol{p})$$

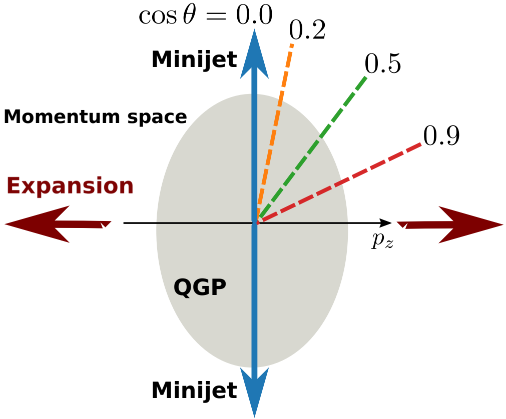


KøMPøST,
PRC 1805.00961

Minijet hydrodynamisation

- scaled time $\tilde{w} = \tau T(\tau)/(4\pi\eta/s)$



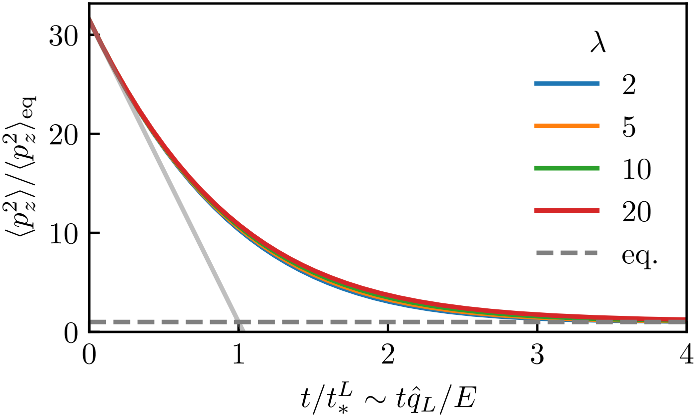


- timescale of minijet quenching

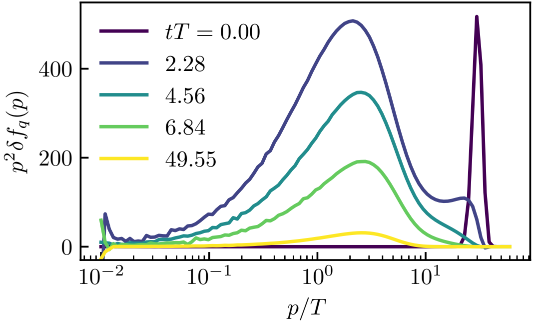
$$\tau_{\mathrm{mjh}} = 5.1\,\mathrm{fm}\left(\frac{4\pi\eta/s}{2}\right)^{3/2}\left(\frac{E}{31\,\mathrm{GeV}}\right)^{1/2}$$

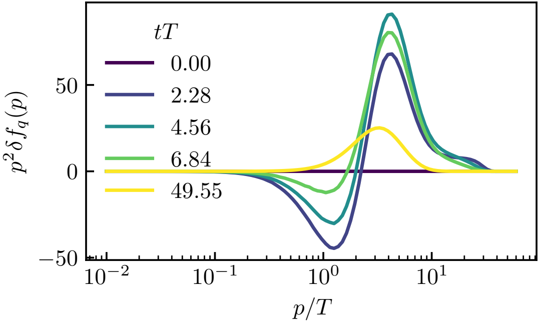
Longitudinal broadening

- longitudinal timescale $t_*^L = \frac{\langle p_z^2 \rangle_{\mathrm{eq}} - \langle p_z^2 \rangle_0}{\hat{q}_L}$



Quark perturbation





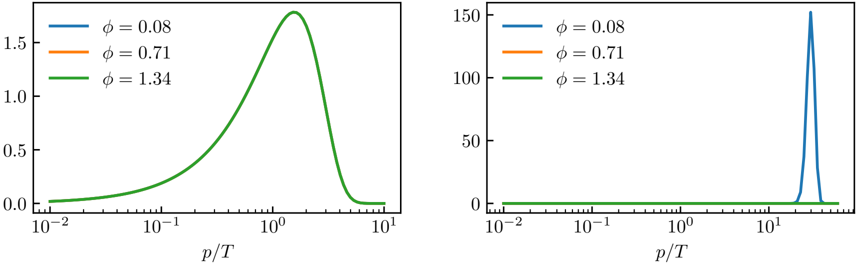
- early times: depletion of quarks around $p \sim T$

  $\rightarrow$ hard gluon scatters off of the soft background

Comparison with background-like perturbation

- azimuthal symmetric (ϕ): $\delta f^{\mathrm{az}}_{\mathrm{sym}}(t_0, \boldsymbol{p}) = \epsilon \bar{f}(t_0, \boldsymbol{p})$

$$\bar{f} + \delta f^{\mathrm{az}}_{\mathrm{sym}} = (1+\epsilon)\bar{f}$$

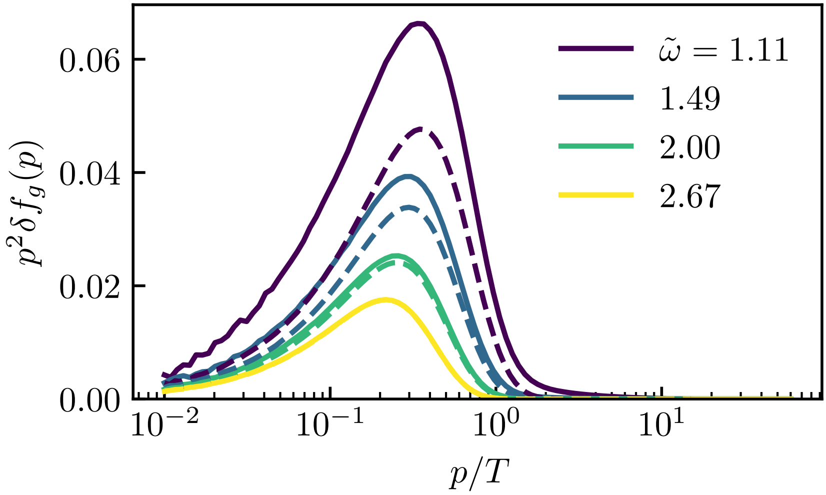


(a) $p^2 \delta f^{\mathrm{az}}_{\mathrm{sym}}(t_0, p, \phi)$ (b) $p^2 \delta f_{\mathrm{Jet}}(t_0, p, \phi)$

- Hydrodynamization occurs, if both perturbations are indistinguishable

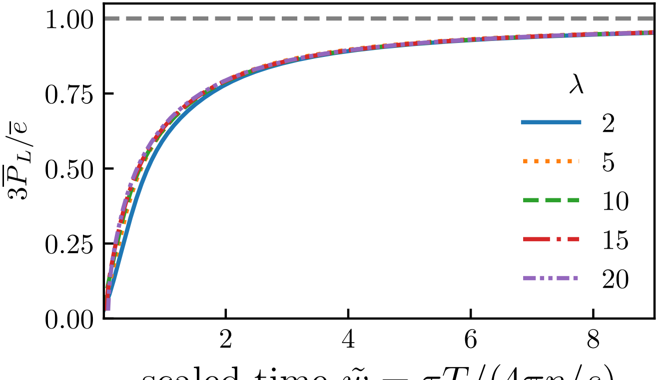
Hydrodynamisation

- anisotropy $\mathcal{A} = \frac{\delta P_T - \delta P_L}{\delta e/3}$



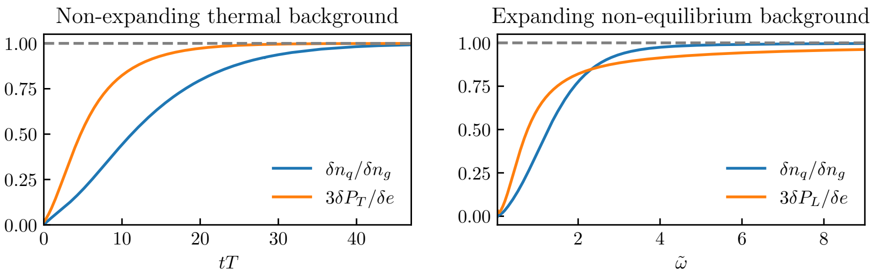
Pressure equilibration background

- energy momentum tensor $T^{\mu\nu} = \int_{\boldsymbol{p}} \frac{p^\mu p^\nu}{p} f(\tau, \boldsymbol{p})$
- $T^{\mu\nu}_{\mathrm{eq}} = \mathrm{diag}(e, P, P, P)$ with $P = e/3$
- Background longitudinal pressure $\overline{P}_L = \overline{T}^{zz}$



Chemical equilibration

- compare with kinetic equilibrium (isotropy of pressure)



- in chemical equilibrium more fermionic degrees of freedom
- chem. equilibration not affected by expansion

Backup

Equations of motion from 2PI effective action

$$\left[iG^{-1,\mu\gamma}_{0,ac}(x;\mathcal{A}) + \Pi^{(0),\mu\gamma}_{ac}(x)\right]\rho^{cb}_{\gamma\nu}(x,y)$$
$$= -\int_{y^0}^{x^0} dz \Pi^{(\rho),\mu\gamma}_{ac}(x,z)\rho^{cb}_{\gamma\nu}(z,y) \tag{1}$$
$$\left[iG^{-1,\mu\gamma}_{0,ac}(x;\mathcal{A}) + \Pi^{(0),\mu\gamma}_{ac}(x)\right]F^{cb}_{\gamma\nu}(x,y)$$
$$= -\int_{t_0}^{x^0} dz \Pi^{(\rho),\mu\gamma}_{ac}(x,z)F^{cb}_{\gamma\nu}(z,y) + \int_{t_0}^{y^0} dz \Pi^{(F),\mu\gamma}_{ac}(x,z)\rho^{cb}_{\gamma\nu}(z,y) \tag{2}$$

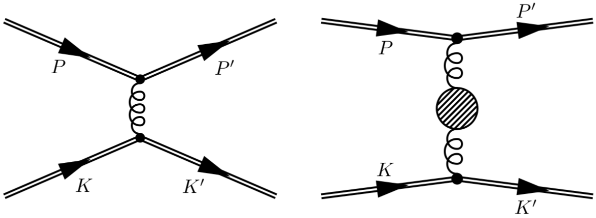
Backup

- local homogeneity $\rightarrow$ relative coordinate $s^\mu = x^\mu - y^\mu$ and center coordinate $X^\mu = \frac{1}{2}(x^\mu + y^\mu)$
- gradient expansion in X^μ
- to lowest order, spectral function ρ is on shell
  $\rightarrow$ quasi-particle picture
- non-equilibrium distribution function $f(X,p)$:

$$F(X,p) = -i\left[\frac{1}{2} \pm f(X,p)\right]\rho(X,p)$$

$$\Rightarrow p^\mu \partial_\mu f(X,p) = -C[f]$$

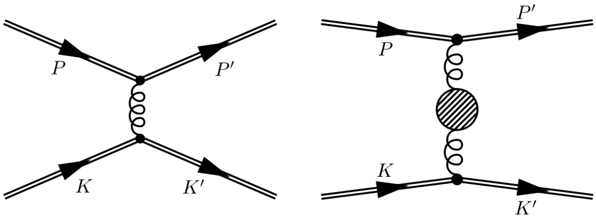
$2 \leftrightarrow 2$



Hard (left) and soft (right) medium regulated scattering

$$C_{2\leftrightarrow 2}[f](\boldsymbol{p}) = \frac{1}{4p\nu} \int_{\boldsymbol{k},\boldsymbol{p}',\boldsymbol{k}'} (2\pi)^4 \delta^{(4)} \left(p^\mu + k^\mu - p'^\mu - k'^\mu\right)$$
$$\times \left|\mathcal{M}\right|^2 \{\underbrace{f_{\boldsymbol{p}} f_{\boldsymbol{k}} \left(1 \pm f_{\boldsymbol{p}'}\right)\left(1 \pm f_{\boldsymbol{k}'}\right)}_{\text{loss}} - \underbrace{f_{\boldsymbol{p}'} f_{\boldsymbol{k}'} \left(1 \pm f_{\boldsymbol{p}}\right)\left(1 \pm f_{\boldsymbol{k}}\right)}_{\text{gain}}\} \qquad (3)$$

$2 \leftrightarrow 2$



Hard (left) and soft (right) medium regulated scattering

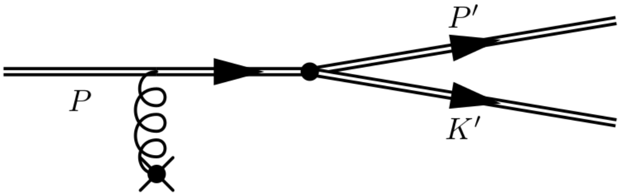
$$|\mathcal{M}|^2 = 2\lambda^2\nu\left(9 + \frac{(s-t)^2}{u^2} + \frac{(u-s)^2}{t^2} + \frac{(t-u)^2}{s^2}\right)$$

- small momentum transfer $q = |\boldsymbol{p}' - \boldsymbol{p}| \ll 1$ regulated by

$$\frac{1}{q^2} \to \frac{1}{q^2 + m_{\text{eff}}^2}$$

$$m_{\text{eff}}^2 = 2g^2 \int \frac{d^3\boldsymbol{p}}{(2\pi)^3 p}\left[N_c f_{\boldsymbol{p}}^g + N_f f_{\boldsymbol{p}}^q\right]$$

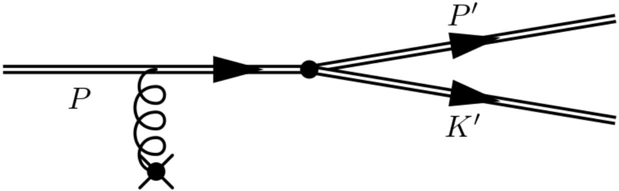
$1 \leftrightarrow 2$



effective $1 \leftrightarrow 2$ process

$$C_{1\leftrightarrow 2}[f](\boldsymbol{p}) = \frac{1}{2}\frac{1}{\nu}(2\pi)^3 \int_{\tilde{\boldsymbol{p}},\boldsymbol{p}',\boldsymbol{k}'} (2\pi)^4 \delta^{(4)}\left(\tilde{p}^{\mu} - p'^{\mu} - k'^{\mu}\right)$$
$$\times \left[\delta^{(3)}(\boldsymbol{p}-\tilde{\boldsymbol{p}}) - \delta^{(3)}\left(\boldsymbol{p}-\boldsymbol{p}'\right) - \delta^{(3)}\left(\boldsymbol{p}-\boldsymbol{k}'\right)\right]$$
$$\times \gamma\{\underbrace{f_{\boldsymbol{p}}\left(1 \pm f_{\tilde{\boldsymbol{p}}'}\right)\left(1 \pm f_{\boldsymbol{k}'}\right)}_{\text{loss}} - \underbrace{f_{\boldsymbol{p}'} f_{\boldsymbol{k}'}\left(1 \pm f_{\tilde{\boldsymbol{p}}}\right)}_{\text{gain}}\} \tag{4}$$

$1 \leftrightarrow 2$



effective $1 \leftrightarrow 2$ process

- LO $\rightarrow$ strictly collinear
- medium induced radiation of gluons
- $N + 1 \leftrightarrow N + 2$ effectively $1 \leftrightarrow 2$

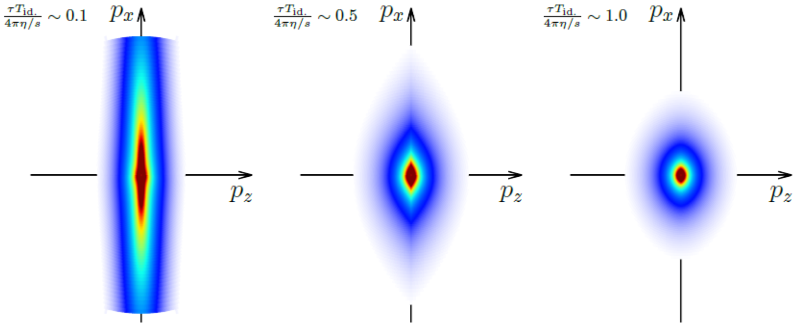
$1 \leftrightarrow 2$

Parton going through the medium, Figure from [**Jasmine**]

- hard parton receiving multiple kicks
- formation time $\tau_f \sim E$
- BH: $l_f \ll l_{\mathrm{mfp}}$, independent emissions
- LPM: $l_f \sim l_{\mathrm{mfp}}$, destructive interference $\rightarrow$ suppression

Bottom-up thermalization

Baier, Müller, Schiff, Son (2001)

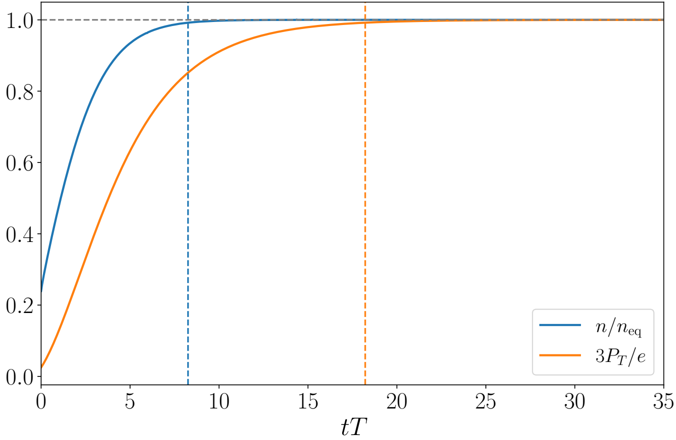


Isotropization of the distribution, Figure from [**Kurkela_2019**]

- 1: overoccupied system getting more anisotropic
- 2: population of soft gluons
- 3: inverse energy cascade

Radiation vs. elastic scattering

- particle number $n = \int_{\boldsymbol{p}} f(\tau, \boldsymbol{p}) \to C_{1\leftrightarrow 2}[f]$
- transverse pressure $P_T = \frac{1}{2}\int_{\boldsymbol{p}} p_\perp^2/p\, f(\tau, \boldsymbol{p}) \to C_{2\leftrightarrow 2}[f]$



Equilibrium distribution

- equilibrated jet $\rightarrow$ change in temperature δT and velocity δu^z

$$\delta f_{\mathrm{eq}}(\boldsymbol{p}) = \left(\delta T \partial_T + \delta u^z \partial_{u^z}\right) n_{\mathrm{BE}}\left(p_\mu u^\mu / T\right)\Big|_{u^z=0}$$

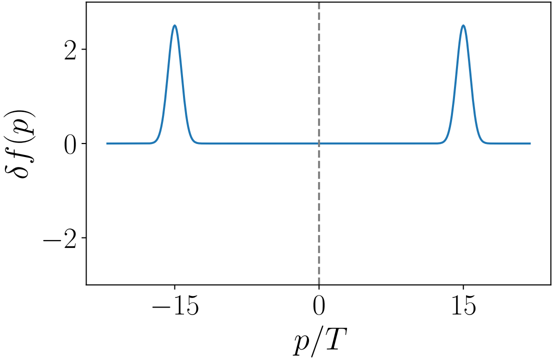
$$\delta f_{\mathrm{eq}}(p,\theta) = \left(\delta u^z \cos\theta + \frac{\delta T}{T}\right) F(p/T)$$

- both contributions can be disentangled

Equilibrium distribution

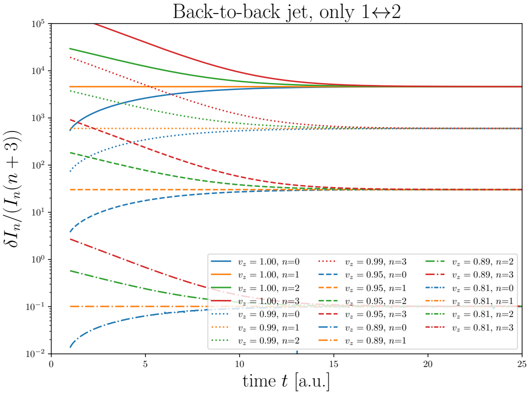
- back-to-back jet conserves net momentum:

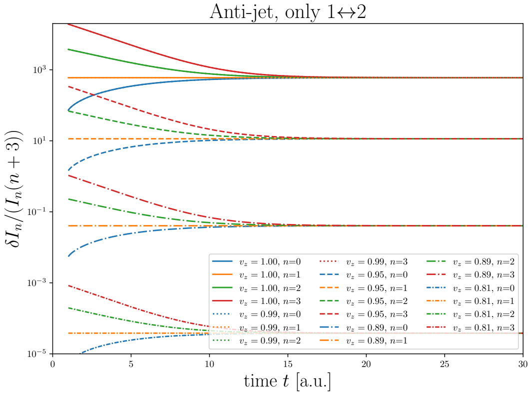
$$\delta f_{\mathrm{eq}}(p,\theta) = \left(\cancel{\delta u^z \cos\theta} + \frac{\delta T}{T}\right) F(p/T)$$



Initial condition: back-to-back jet

Only $1 \leftrightarrow 2$





- similar timescales of equilibration
  $\Rightarrow$ 2↔2 contribute more to equilibration of the anti-jet

Moments of δf

- angular effective temperature

$$I_n(\theta) \equiv 4\pi \int \frac{p^2 dp}{(2\pi)^3} p^n f(p,\theta) = \mathcal{N}_n \times T(\theta)^{n+3} \tag{5}$$

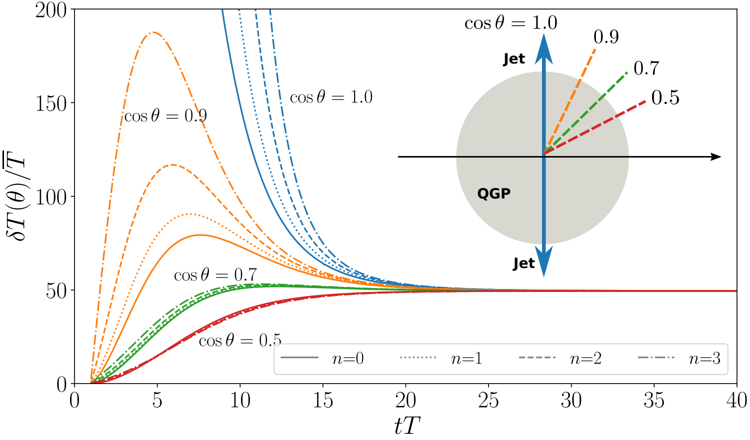
$$T(\theta) = \overline{T} + \delta T(\theta)$$

- temperature perturbation

$$\frac{\delta T(\theta)}{\overline{T}} = \frac{\delta I_n(\theta)}{(n+3)\bar{I}_n(\theta)}$$

- look at time evolution!

Moments of δf



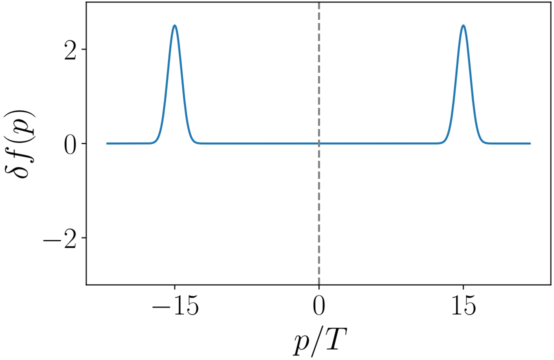
Moments of the back-to-back jet

- different moments agree before different angles do!

Equilibrium distribution

- back-to-back jet conserves net momentum:

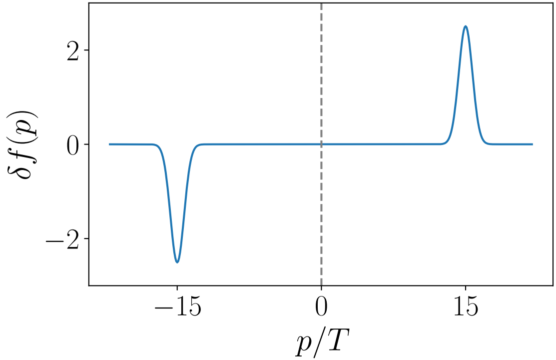
$$\delta f_{\text{eq}}(p,\theta) = \left(\cancel{\delta u^z \cos\theta} + \frac{\delta T}{T}\right) F(p/T)$$



Initial condition: back-to-back jet

Anti-jet

- introduce anti-jet $\to$ no energy deposited, **only** net momentum

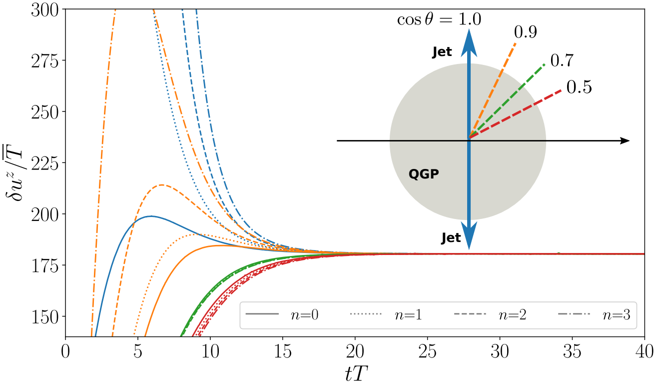


Particle and "hole"

$$\delta f_{\mathrm{eq}}(p,\theta) = \left(\delta u^z \cos\theta + \cancel{\frac{\delta T}{T}}\right) F(p/T)$$

- allows us to study the build up of δu^z

Moments of δf_{anti}



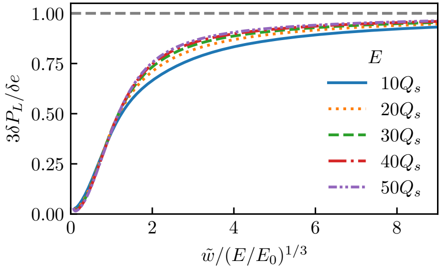
Collapse of different θ much earlier!

$$\delta f_{\mathrm{eq}}(p,\theta) = (\delta u^z \cos\theta)\, F(p/T)$$

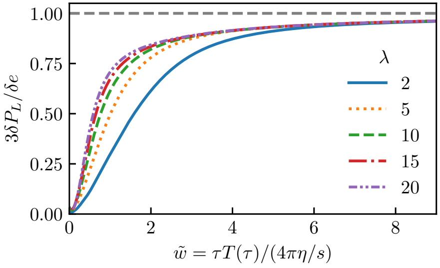
- θ-dependence $\rightarrow$ faster reached by elastic processes
- single jet: velocity builds up faster than temperature

Pressure equilibration

- scaled time $\tilde{\omega} = \tau/\tau_R$ with $\tau_R = \frac{4\pi\eta/s}{T(\tau)}$
- effective temperature from $e(\tau) = \nu_{\text{eff}} \frac{\pi^2}{30} T(\tau)^4$



(c) Scaling with jet energy E



(d) coupling λ