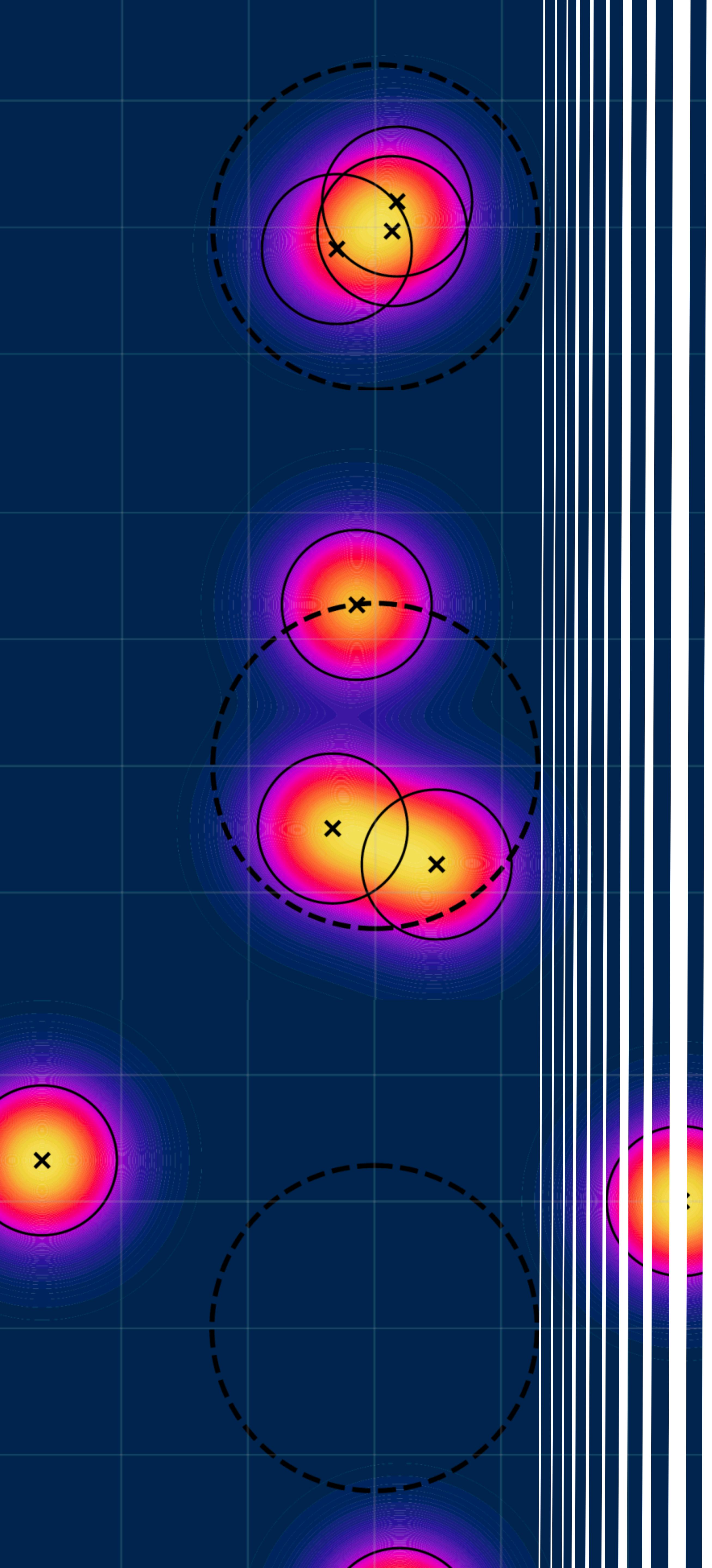

Bielefeld
June 2026

Oscar Garcia-Montero

Instituto Galego de Física de Altas Enerxías
Universidade de Santiago de Compostela

Further questions/comments:

Work from O. G-M, 2608.XXXXX

And partially based on various works *in collaboration with*
***H. Elfner, J. Zhu and* S. Schlichting**, including
Phys.Rev.C 109 (2024) 4, 044916, *Phys.Rev.D* 111 (2025) 7, 076029,
and O.G-M., Zhu, 2608.XXXX. among some others

Rede CIGUS Centros de investigación do Sistema Universitario de Galicia
Cofinanciado pola Unión Europea
Fondos Europeos
Xacobeo 2027
MINISTERIO DE CIENCIA, INNOVACIÓN Y UNIVERSIDADES
AGENCIA ESTATAL DE INVESTIGACIÓN
EXCELENCIA MARÍA DE MAEZTU

INITIAL CONDITIONS

From the Hot QCD White Paper, a list of current, pressing avenues on the initial states

- Initial condition: $\sqrt{s_{NN}}$ dependence and longitudinal structure
- B,Q,S Charge deposition and the search for the CP
- From medium to small systems
- Better quantification of ICs for large systems
- Initial condition for hard probes

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- Better quantification of ICs for large systems
- Initial condition for hard probes

Seemingly different,
But in reality, intimately related!!

QGP LONGITUDINAL STRUCTURE

$\sqrt{s_{NN}}$ dependence

longitudinal structure

INITIAL STATE
+PRE-EQUILIBRIUM

small systems

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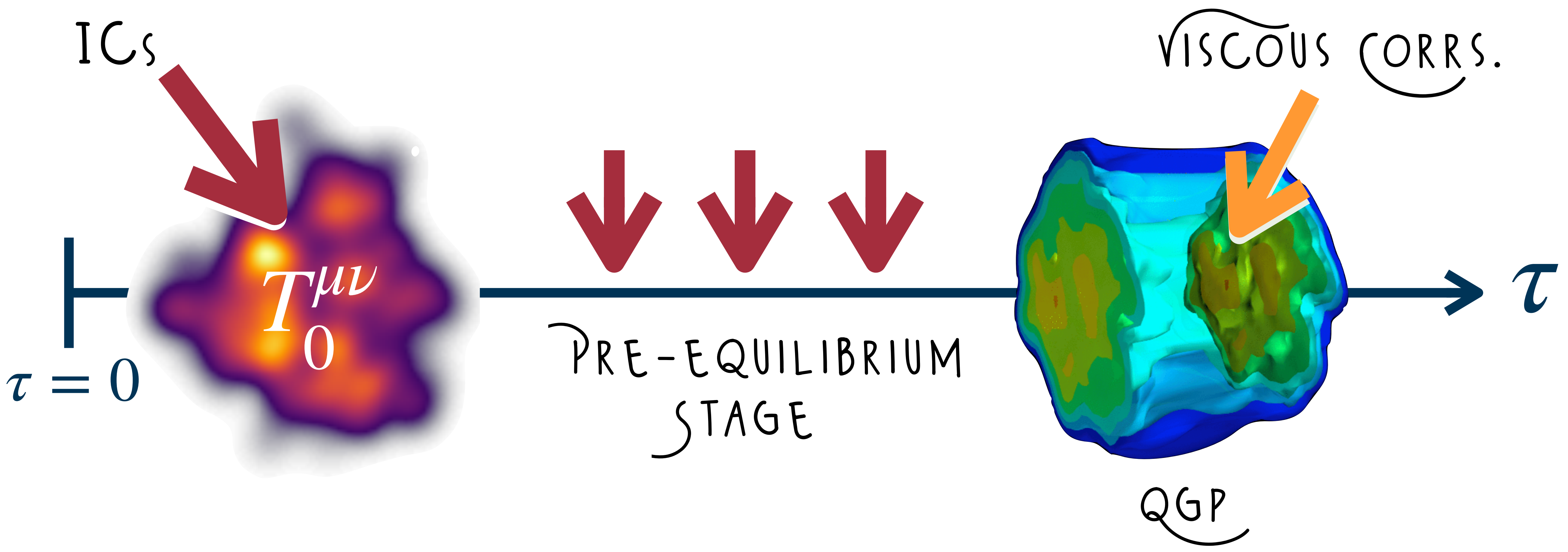

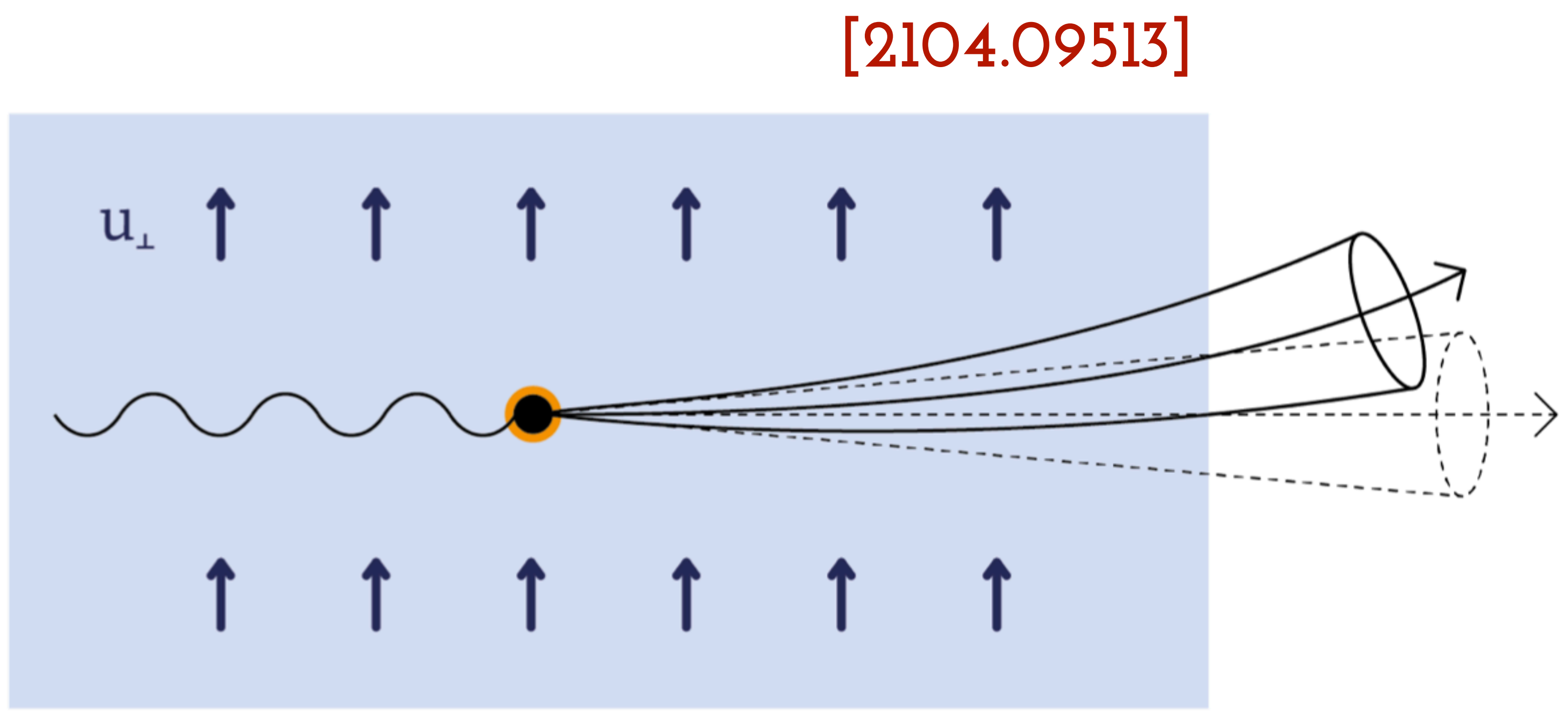

[See Jo's talk. Mon. 10am]

By momentum conservation we know that transverse velocity is created through gradients

$$\frac{1}{\tau}\partial_\tau\left(\tau T^{\tau i}\right) = -\partial_j T^{ji}$$

Additionally, transverse anistropies in the energy-stress tensor accumulate in the shear tensor

$$T^{ij} - P\delta^{ij} \to \pi^{ij}$$

$$\delta\hat{q}^{ij} = \boxed{\alpha\,\pi^{ij}} + \boxed{\beta\,\delta^{ij}\,\hat{n}^a\hat{n}^b\pi_{ab}}$$
$$\boxed{+\gamma\,\left(\hat{n}^i\pi^{ja}\hat{n}_a + \hat{n}^j\pi^{ia}\hat{n}_a + \hat{n}^i\hat{n}^j\hat{n}^a\hat{n}^b\pi_{ab}\right)}$$

arXiv:2605.12791 [hep-ph]

[See Isabella's talk. Tue. 9am]

METHODS: STATE OF THE ART

DoFs/motivation behind the energy and charge deposition

LARGE-X

Collinear fact.
Described by PDFs

GEOMETRICAL

Effective description
Often parametrical

LOW-X

Overoccupied
Color fields

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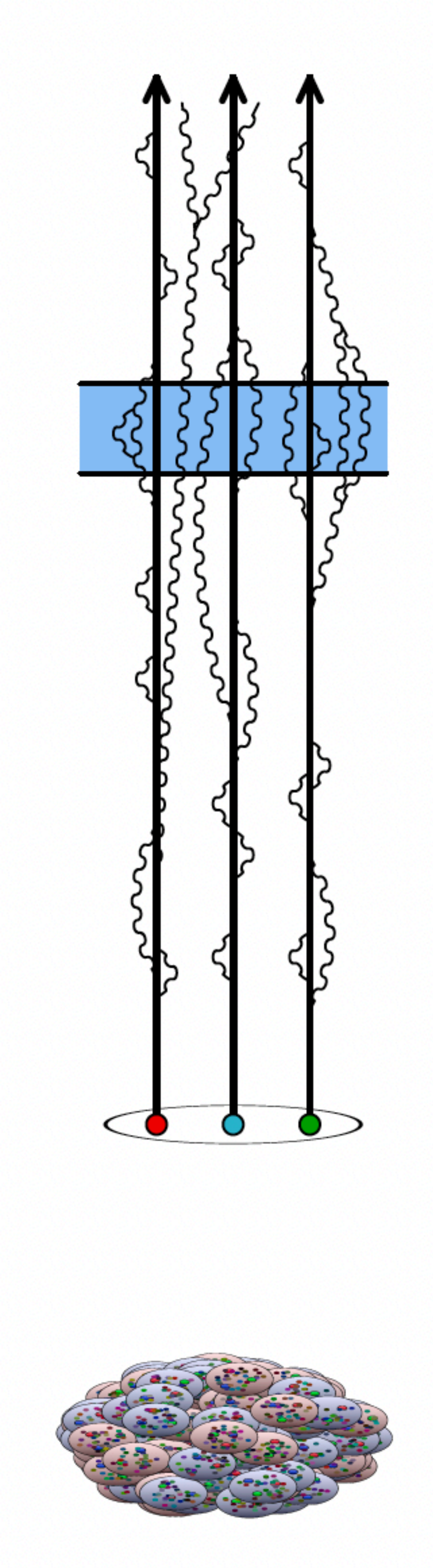
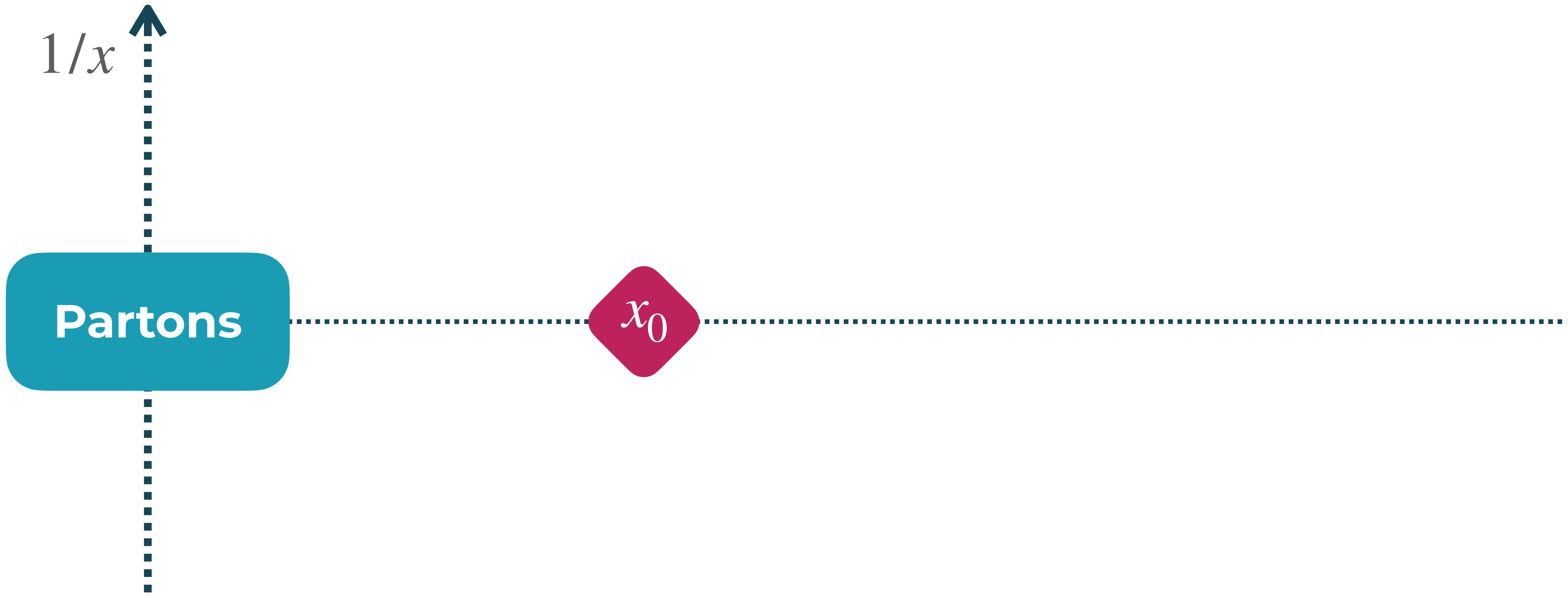
CGC

THE COLOR GLASS CONDENSATE





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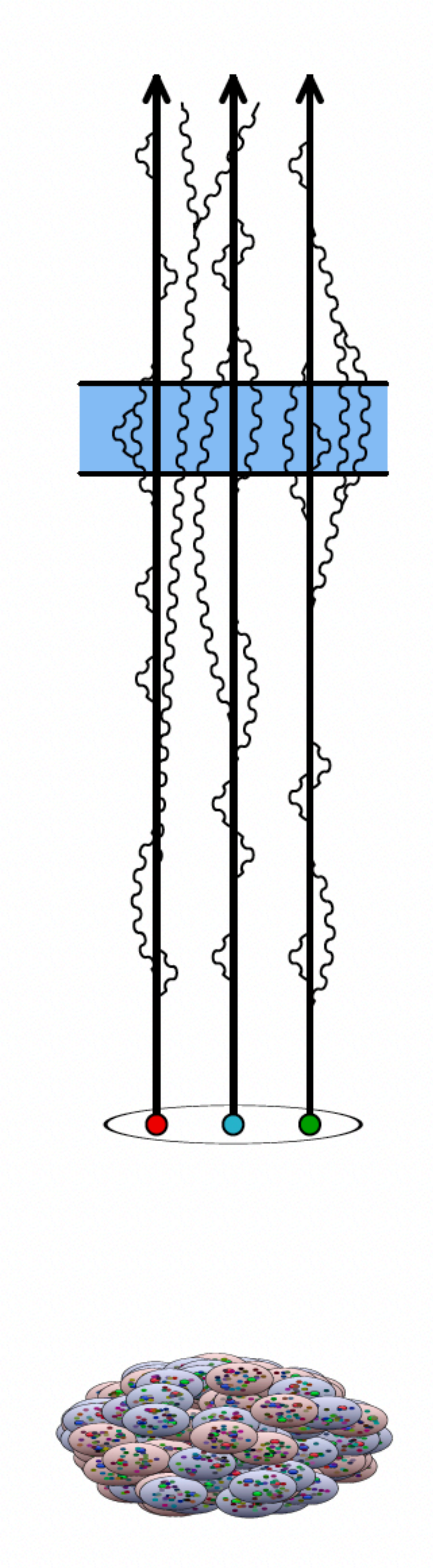


$1/x$

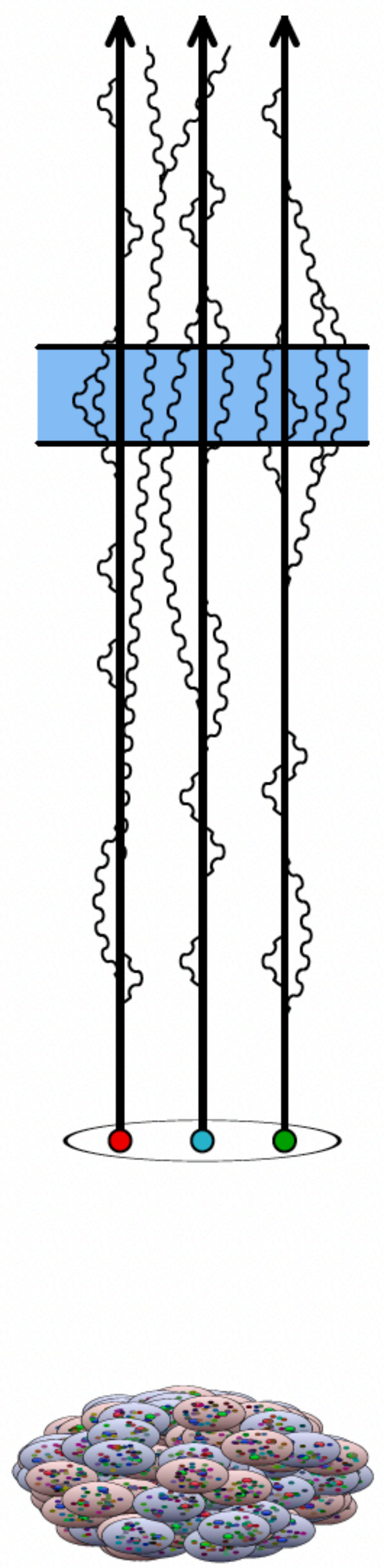
Partons

x_0

Hard Partons $J^{-}(x) \sim \rho(x^{+}, \boldsymbol{x}_{\perp})$

Static color sources sampled from distribution $W[x_0; \rho]$

THE COLOR GLASS CONDENSATE



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Soft Partons

Macroscopic Field

$A(x)$

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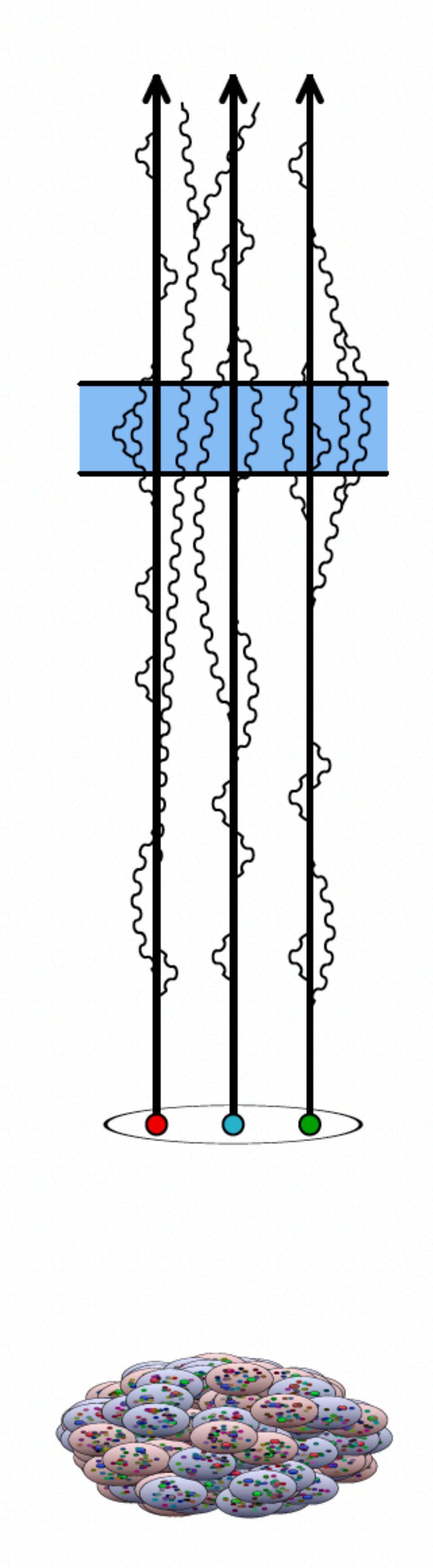

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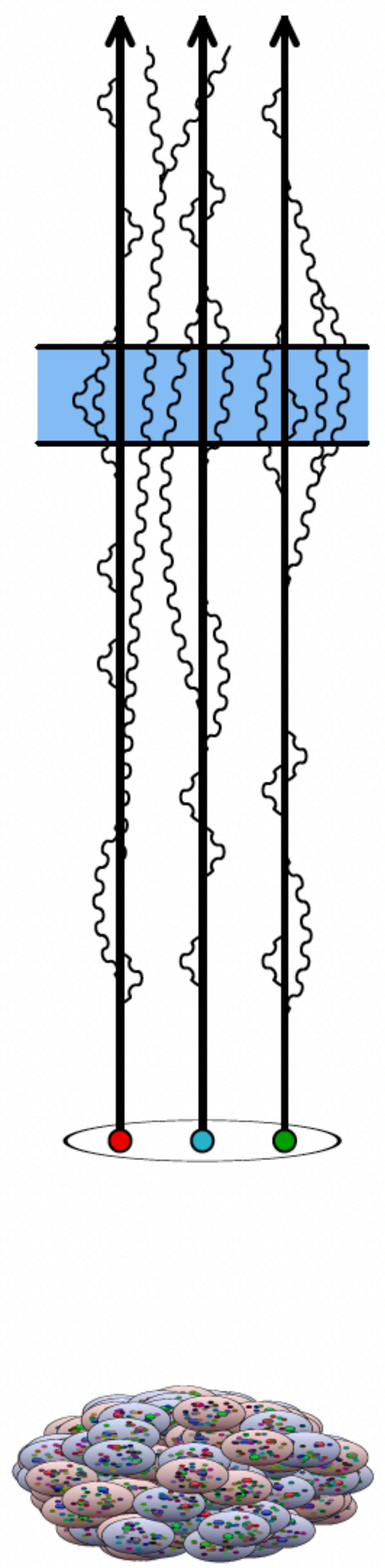
CYM Eqs

$[D_{\mu}, F^{\mu\nu}] = J^{\nu}$

THE COLOR GLASS CONDENSATE



$1/x$

x_1

x_0

Partons

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Static color sources sampled from distribution $W[x_0; \rho]$

CYM Eqs

$[D_\mu, F^{\mu\nu}] = J^\nu$

Soft Partons

Macroscopic Field

$A(x)$

SPECIAL CASE: GAUSSIAN ICS $\langle \rho_a(\mathbf{x}_\perp)\, \rho_b(\mathbf{y}_\perp) \rangle = \mu^2(\mathbf{x}_\perp)\ \delta^{(2)}(\mathbf{x}_\perp - \mathbf{y}_\perp)$

where

$\mu^2(\mathbf{x}_\perp) \sim T(x_\perp)$ Colour charge density in the nucleus

POWER-COUNTING IN THE CGC

WHICH APPROXIMATION?

Depends on $\sqrt{s}$, A_1/A_2, centrality, rapidity range, and k_T of particles

DILUTE-DILUTE

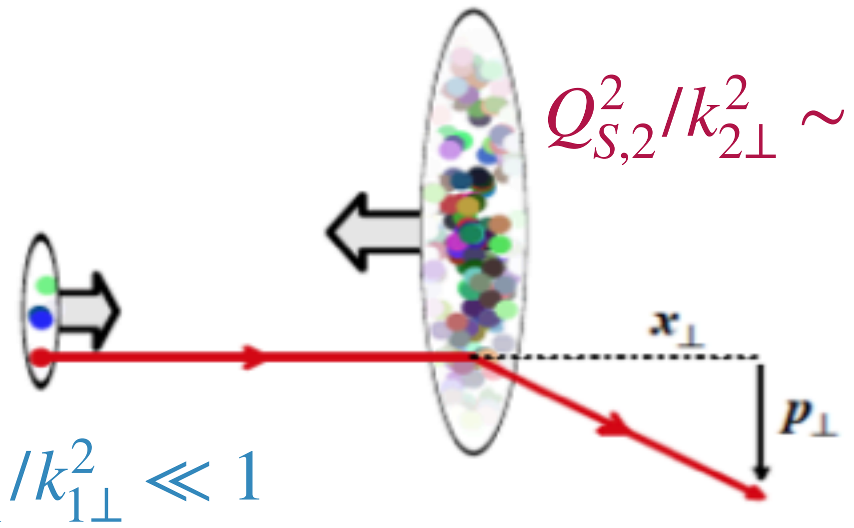
$Q^2_{S,1}/k^2_{1\perp} \ll 1$ $Q^2_{S,2}/k^2_{2\perp} \ll 1$

$gA_{1,2} \ll 1$, so that LO:

$\langle \hat{O} \rangle_x \sim \mathcal{O}(\rho_1 \rho_2)$

Match pQCD computations in the collinear limit ($k_\perp/Q^2_S \gg 1$)

DILUTE-DENSE



$Q^2_{S,2}/k^2_{2\perp} \sim 1$

$Q^2_{S,1}/k^2_{1\perp} \ll 1$

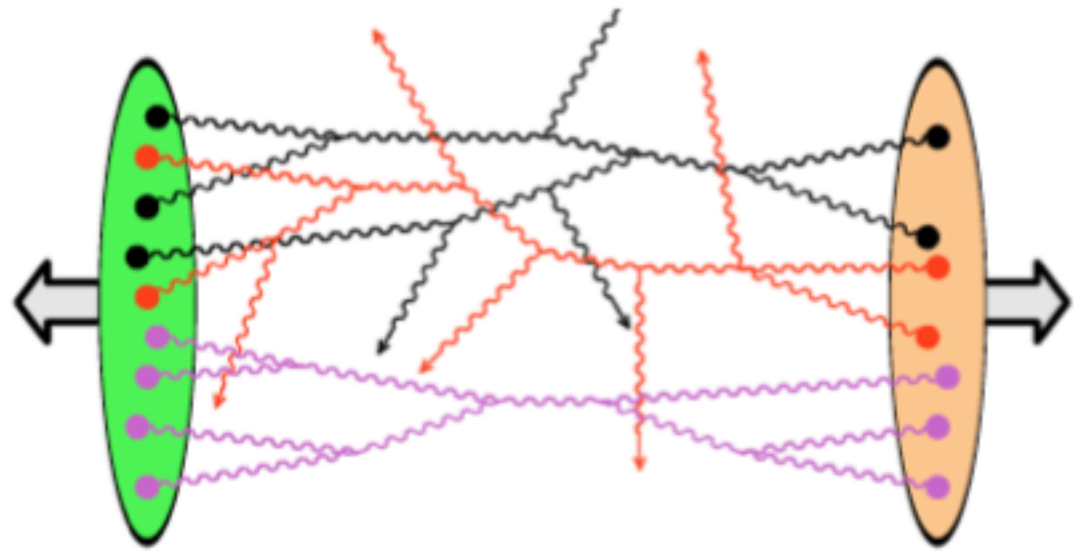
$\langle \hat{O} \rangle_x \sim \mathcal{O}(\rho_1)$ all orders in ρ_2

Forward limit: hybrid factorization.
pQCD projectile PDFs + CGC target unintegrated gluon distribution.

$\Downarrow$

Search for saturation: Probing a nuclear target with ind. partons!

DENSE-DENSE



$Q^2_{S,1}/k^2_{1\perp} \ll 1$ $Q^2_{S,2}/k^2_{2\perp} \sim 1$

$gA_{1,2} > 1 \implies \langle \hat{O} \rangle_x$
all orders in ρ_1 and ρ_2

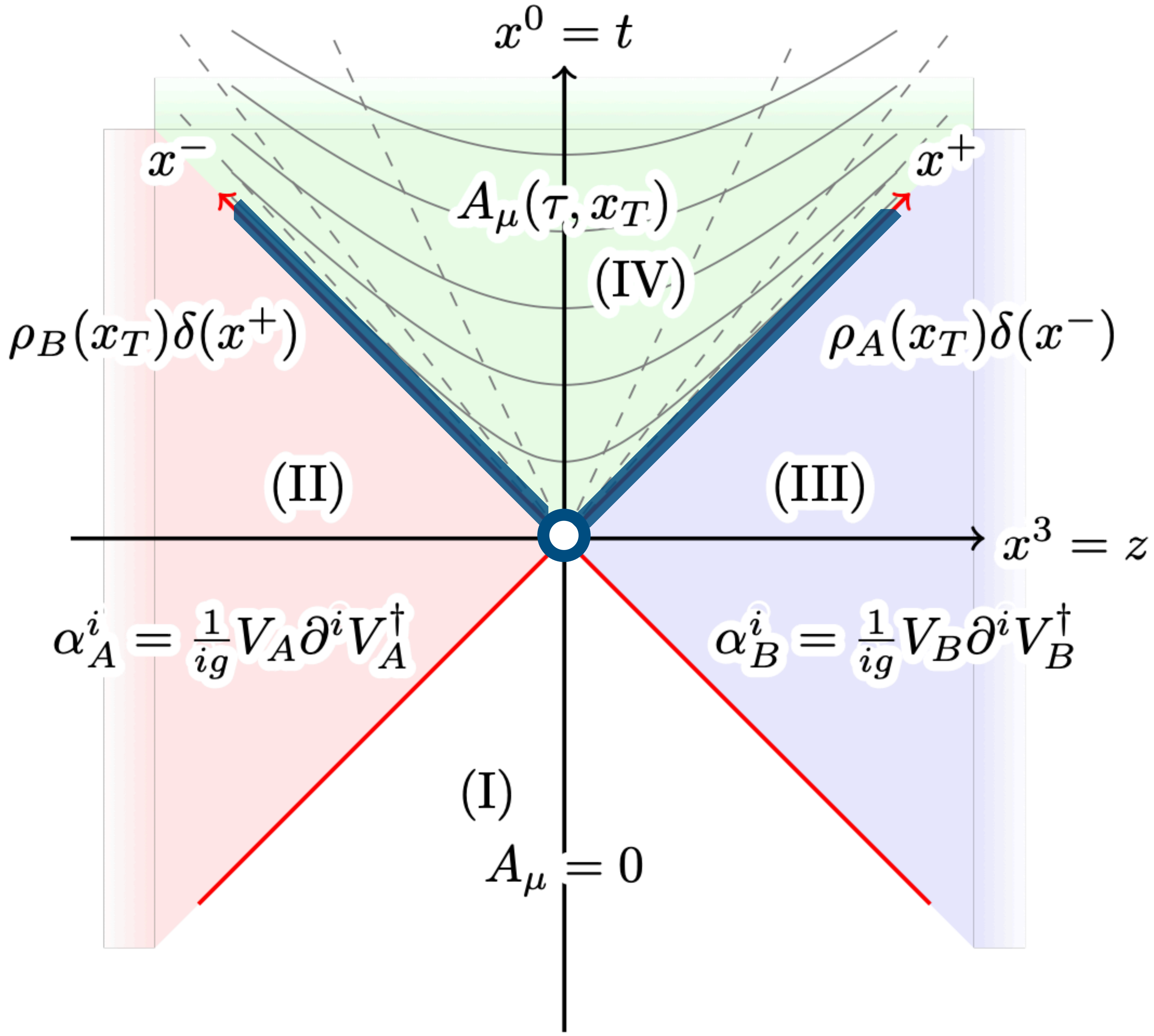
Analytically intractable:
CYM equations in 3+1 (2+1) D,
numerically solved

DENSE-DENSE: IP-GLASMA

LO approximation for the CGC evolution of a dense-dense system.

IP-GLASMA

1) Sample nucleon positions (e.g. MC-Glauber)
2) Sample color currents from those nucleons ($J_{A,B}$)
3) Solve Yang-Mills*,** in the presence of both currents, and $D_\mu J^\mu = 0$.
   - * Boundary conditions and state at $\tau = 0^+$ well known analytically
   - ** Assume boost invariance and solve numerically in τ and $\boldsymbol{x}_\perp$
4) Get energy-stress tensor, $T^{\mu\nu}$



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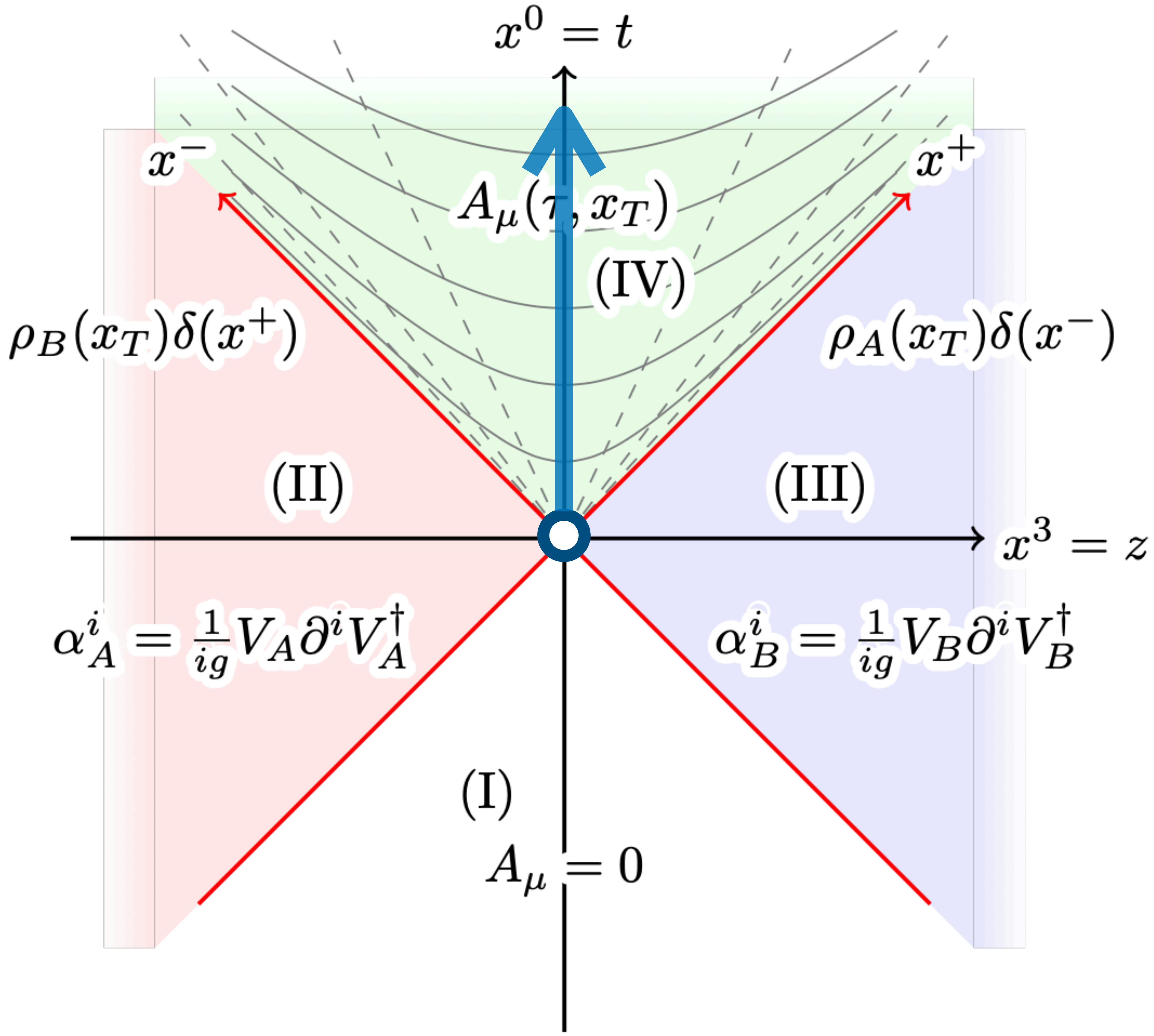

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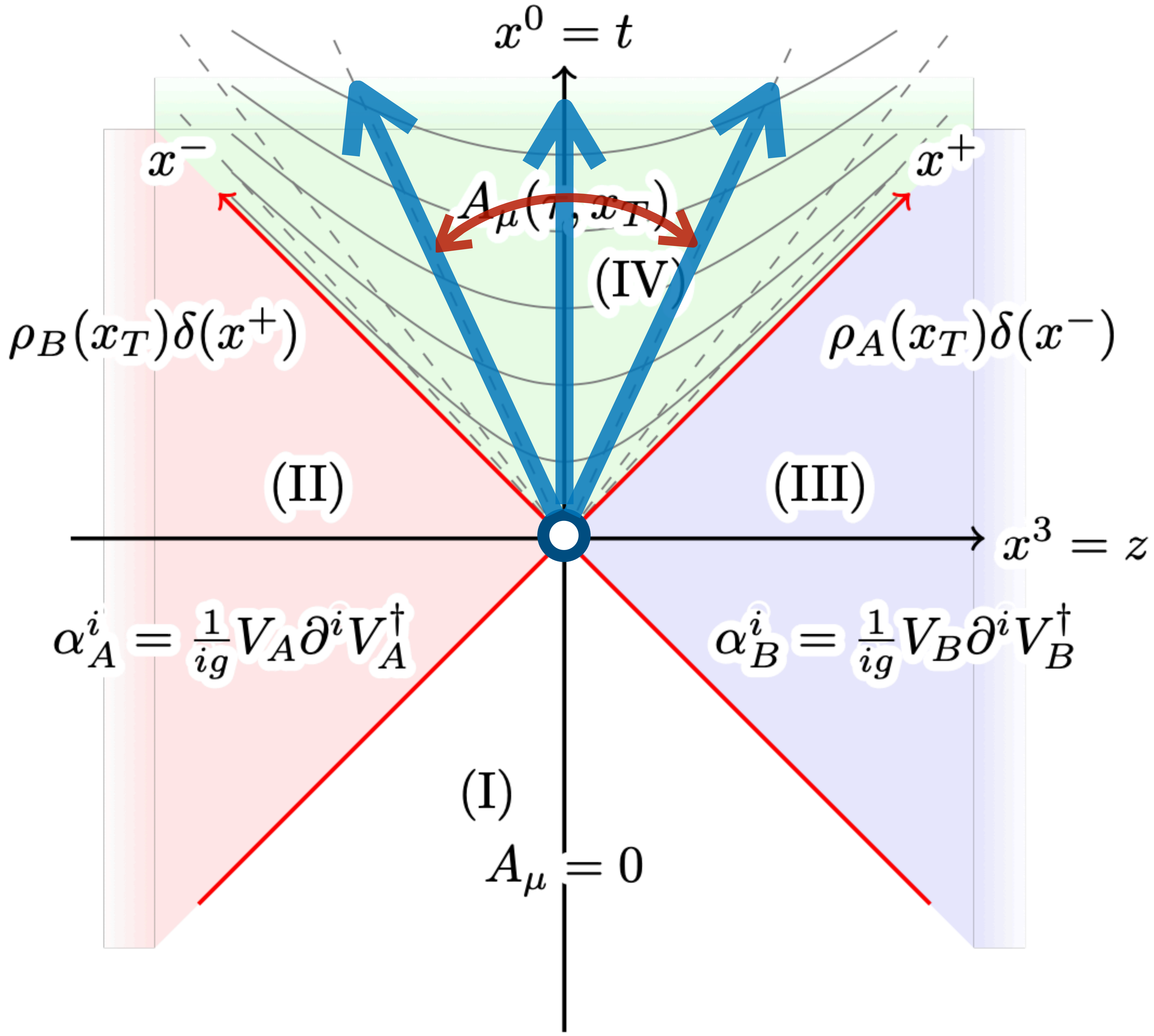
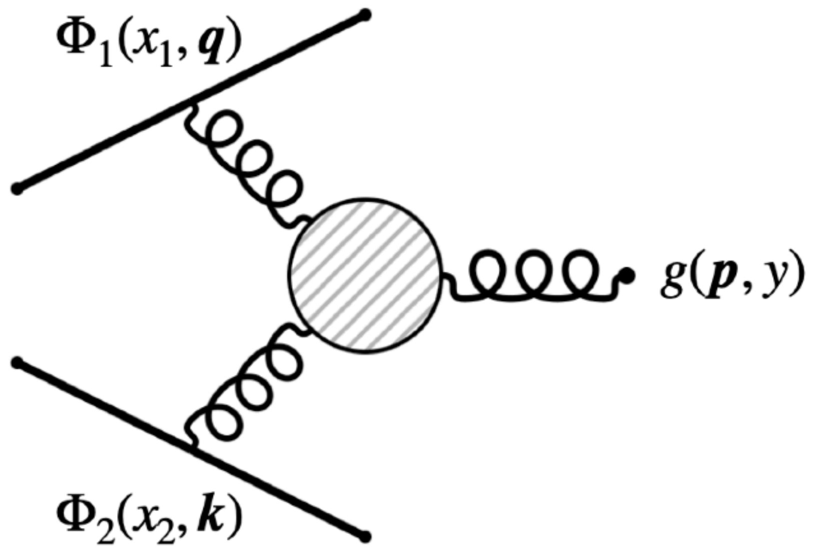
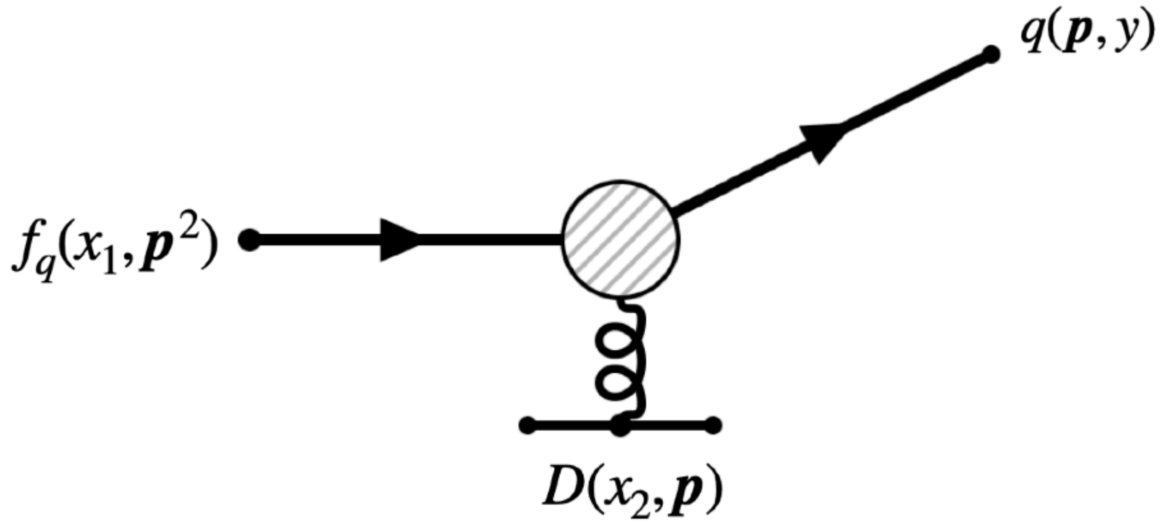
NOTE: EXTENSION TO 3D IS NOT TRIVIAL

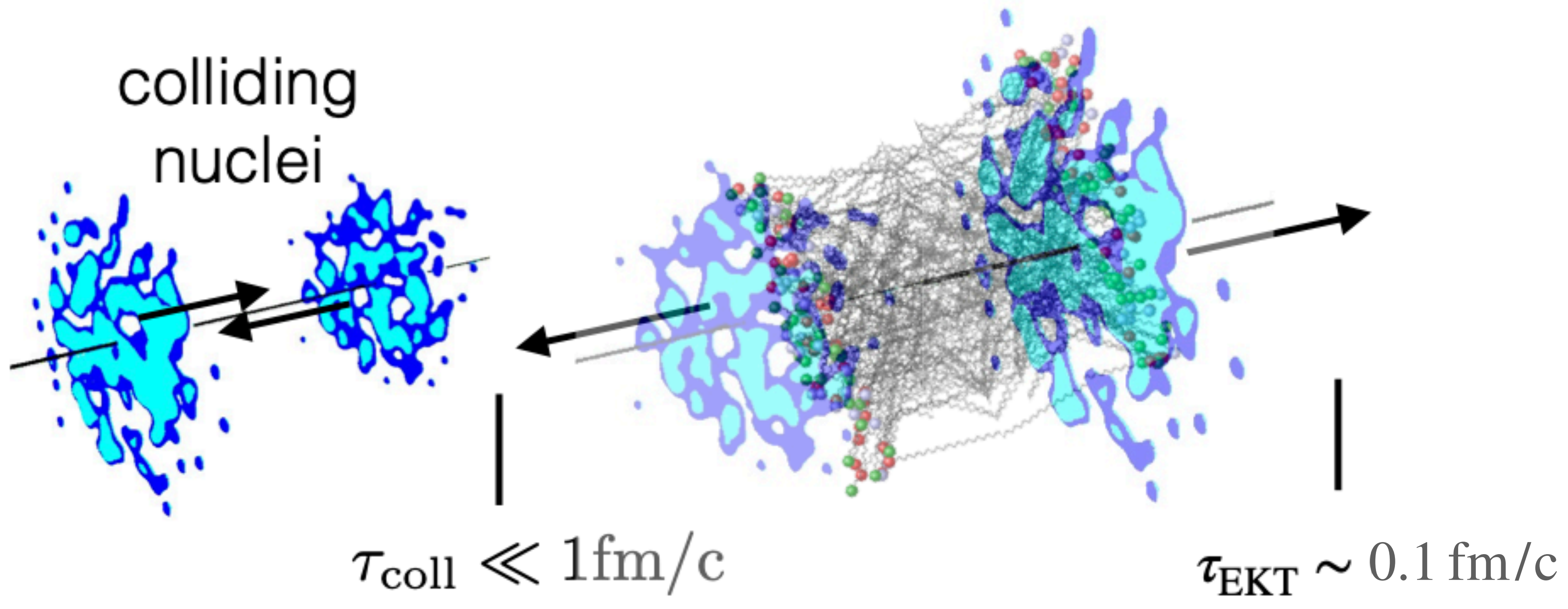


RAPIDITY RESOLUTION $\leftrightarrow$ LONG. RESOLUTION





Perturbative expansion on the sources allows simple kinematics, connection $x \leftrightarrow y$ straightforward



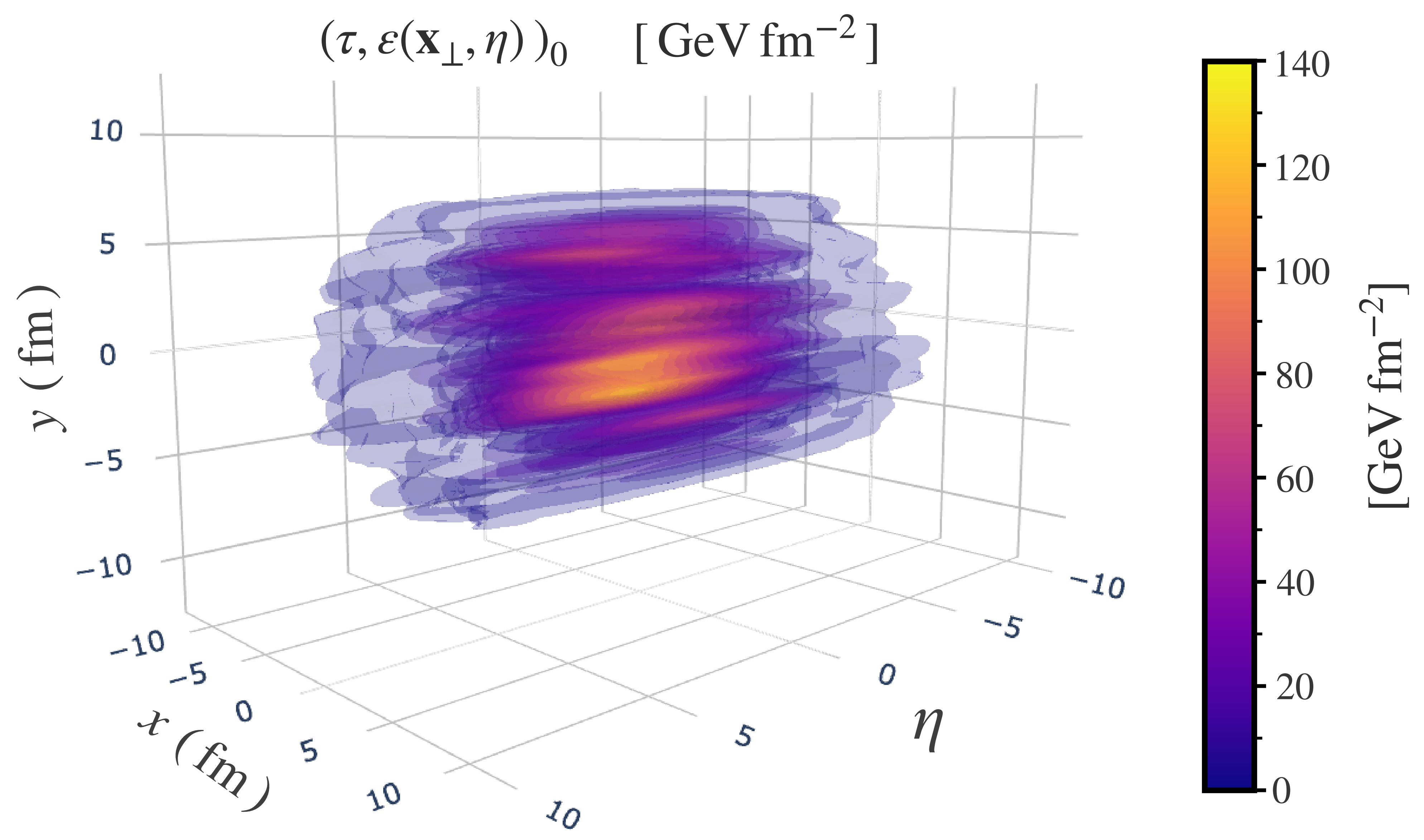
Every contribution of sources taken on account, solvable numerically, but connection $x \leftrightarrow y$ is very complex

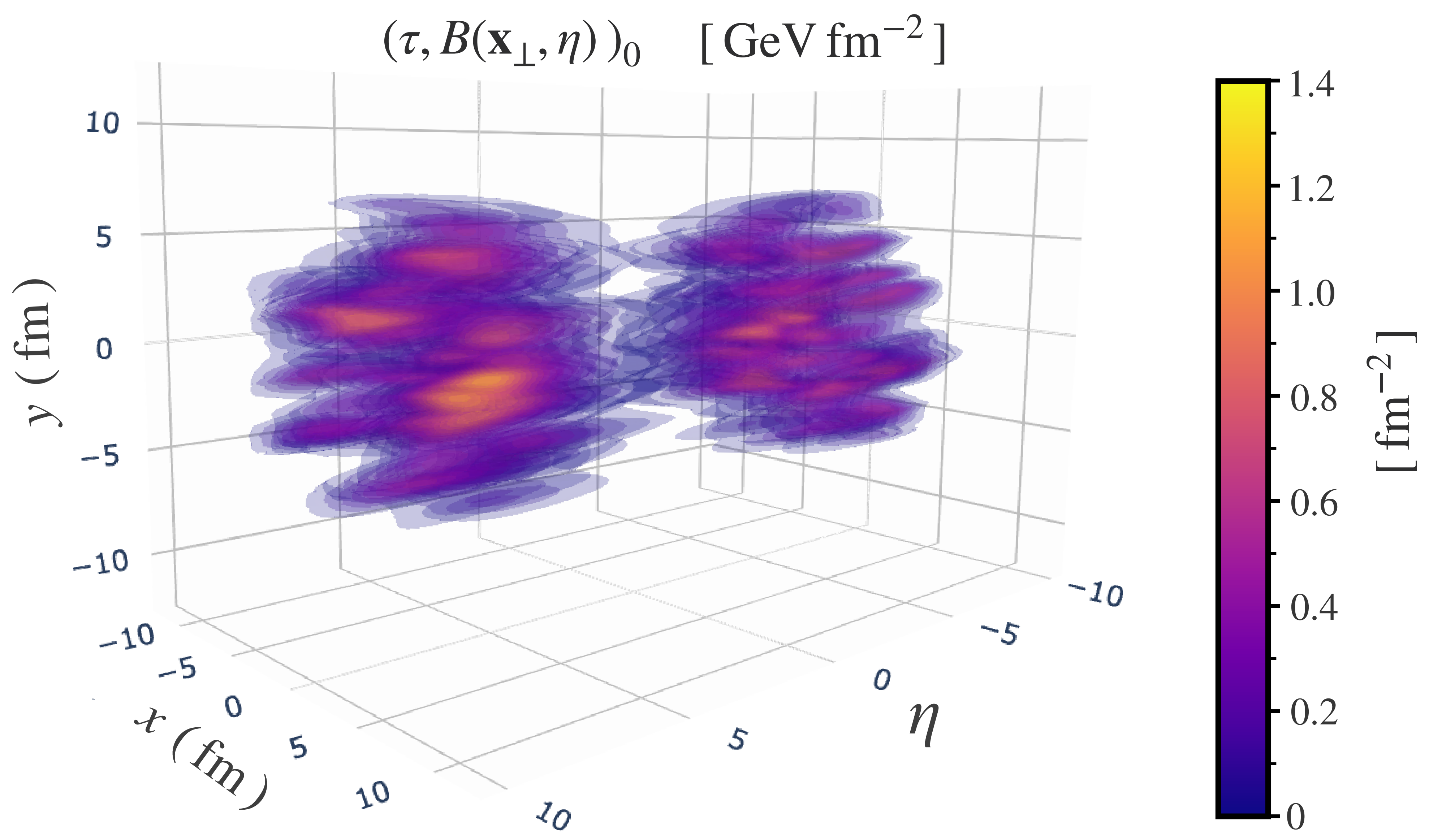
CGC IN 3D: THE McDIPPER

Monte-Carlo Dipole Parallel Event GeneRator

Framework for comparison of saturation model predictions and creation of IC for HE Heavy-Ion Collisions

Perturbative realisation of the LO glasma graph + Baryon stopping by CGC

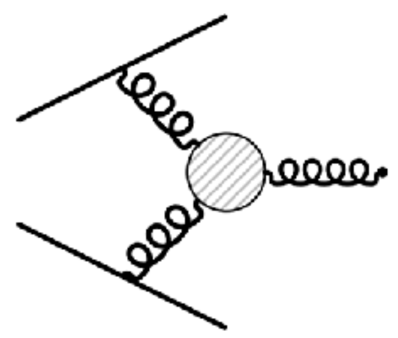




FROM MICRO TO MACRO

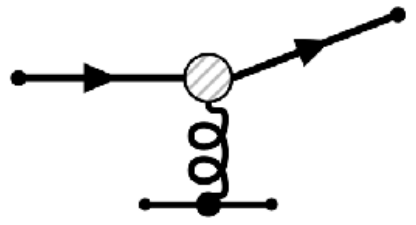
- Low-x gluons dominate the midrapidity region

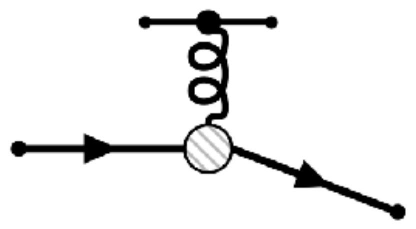
$$\frac{dN_g}{d^2\mathbf{x} d^2\boldsymbol{p} dy} = \frac{g^2}{8\pi^5 C_F\, \boldsymbol{p}^2} \int \frac{d^2\boldsymbol{q}}{(2\pi)^2} \frac{d^2\boldsymbol{k}}{(2\pi)^2} (2\pi)^2 \delta(\boldsymbol{p}+\boldsymbol{q}-\boldsymbol{p}) \times \Phi_1(x_1, \boldsymbol{x}, \boldsymbol{q})\; \Phi_2(x_2, \boldsymbol{x}, \boldsymbol{k})$$



- At forward/backward rapidities, particle production dominated by baryon stopping

$$\frac{dN_{q_f}}{d^2\mathbf{x} d^2\mathbf{p} dy} = \frac{x_1 q_f^A(x_1, \mathbf{p}^2, \mathbf{x})\; D_{\mathrm{fun}}(x_2, \mathbf{x}, \mathbf{p})}{(2\pi)^2} + \frac{x_2 q_f^A(x_2, \mathbf{p}^2, \mathbf{x})\; D_{\mathrm{fun}}(x_1, \mathbf{x}, \mathbf{p})}{(2\pi)^2}\,.$$





THE INPUT

Low-x gluons

uGDFs $\rightarrow$ $\Phi_i(x, \boldsymbol{r}, \boldsymbol{q}) \sim \boldsymbol{q}^2 D_{\mathrm{adj}}(x, \boldsymbol{r}, \boldsymbol{q})$

Dipoles $\rightarrow$ $D_{\mathrm{adj}}(x, \boldsymbol{r}, \boldsymbol{q})$, $D_{\mathrm{fun}}(x, \boldsymbol{r}, \boldsymbol{q})$

- GBW, IP-Sat, MV...

High-x partons

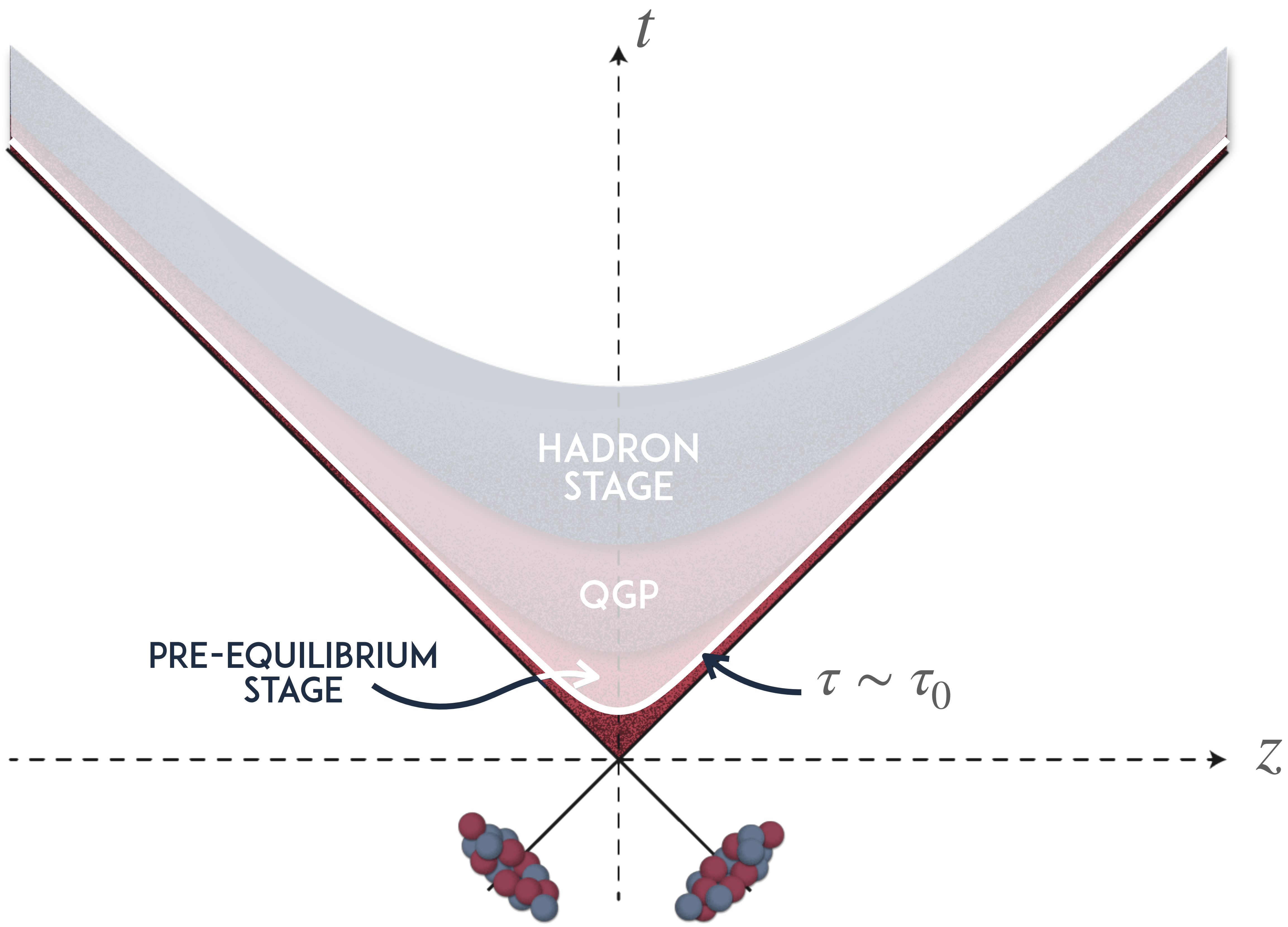
PDFs $\rightarrow$ $x_i\, q_f(x_i, \mathbf{p}^2)$

Different PDF sets*.

*Accessible in the MCDIPPER through the LHAPDF library

Systematically Improvable e.g. by including NLO gg→ $q\bar{q}$ production through gluon fusion

THE PRICE TO PAY



Knowledge of the distribution of the nuclei is still contained

As well as dynamics! Albeit in a simplified limit.

This is a story about how I am systematically enhancing both sides of this description in this limit.

FROM MICRO TO MACRO

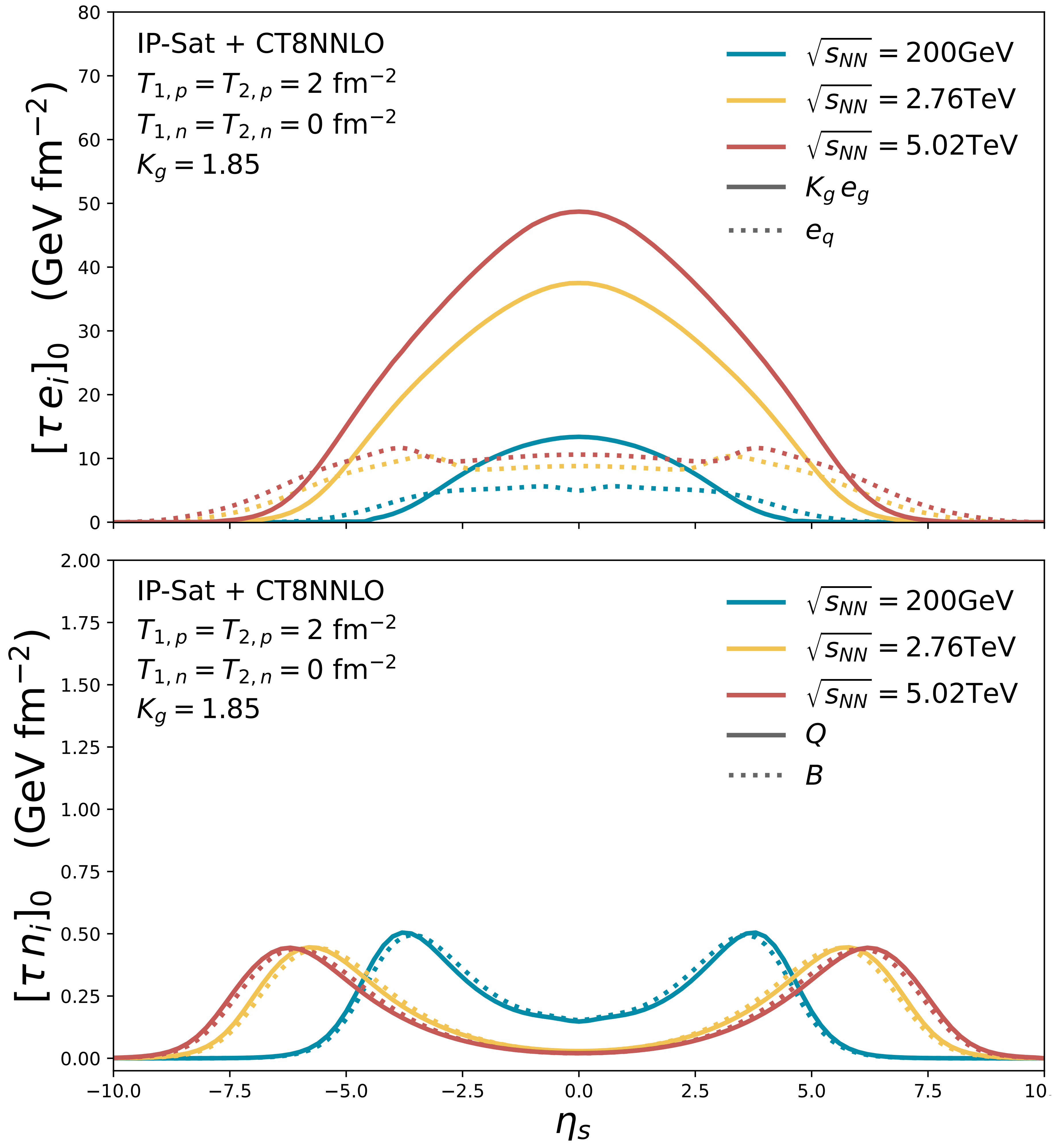
- Macroscopic quantities (energy, charges) are computed as moments of the single particle distributions
  - Total energy deposition

$$(e\tau)_0 = \int d^2\mathbf{p}\ |\mathbf{p}|\ \left[K_g \frac{dN_g}{d^2\mathbf{x}d^2\mathbf{p}dy} + \sum_{f,\bar{f}} \frac{dN_{q_f}}{d^2\mathbf{x}d^2\mathbf{p}dy} \right]_{y=\eta_s}$$

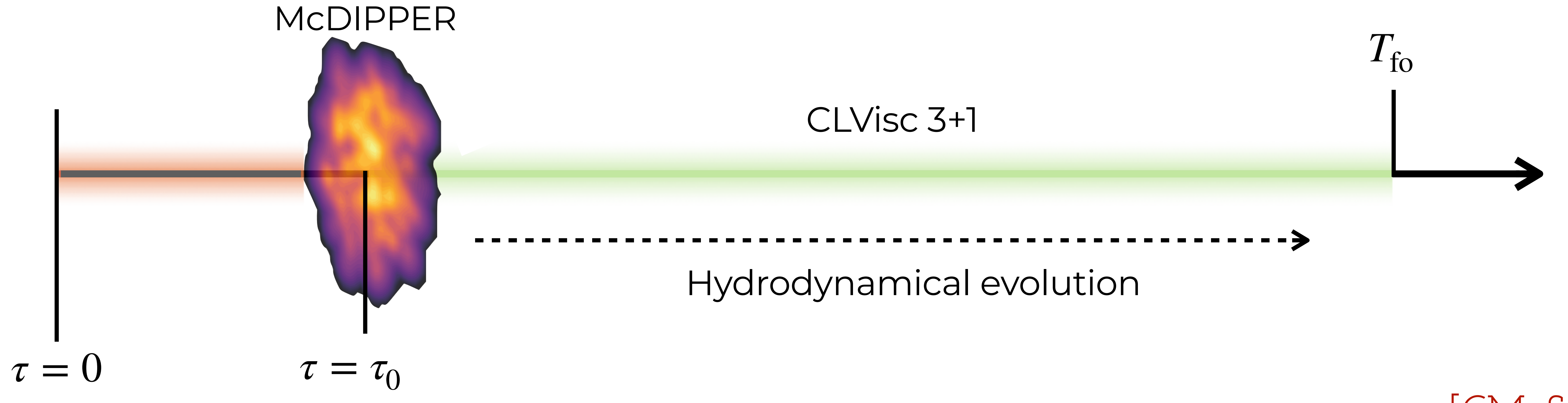
  - Charges (u,d,s) deposited can be used to compute conserved charges such as, i.e. electric charge,

$$(Q\tau)_0 = \sum_f Q_f \int d^2\mathbf{p}\ \left[\frac{dN_f}{d^2\mathbf{x}d^2\mathbf{p}dy} - \frac{dN_{\bar{f}}}{d^2\mathbf{x}d^2\mathbf{p}dy} \right]_{y=\eta_s}$$

$$(B\tau)_0 = \sum_f B_f \int d^2\mathbf{p}\ \left[\frac{dN_f}{d^2\mathbf{x}d^2\mathbf{p}dy} - \frac{dN_{\bar{f}}}{d^2\mathbf{x}d^2\mathbf{p}dy} \right]_{y=\eta_s}$$



McDIPPER+CLVisc 3+1



[GM, Schlichting, Zhu, 2501.14872]

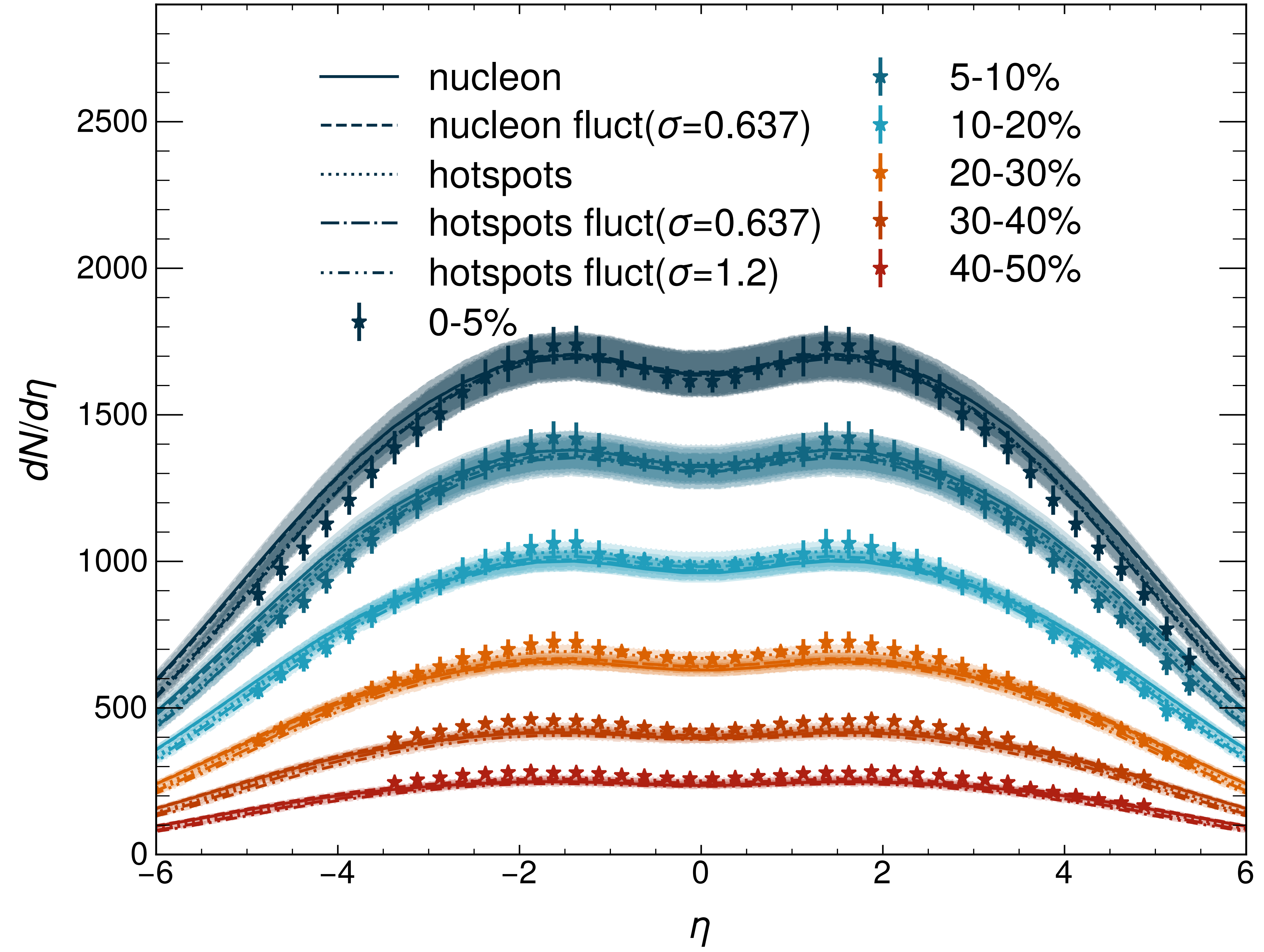
McDIPPER

- Initialisation time $\tau_0 = 0.6\,\mathrm{fm}$
- Two *tuned* parameters:
  - K_g (Fit to central PbPb data at 5.02 GeV)
  - σ (using p+p, p+Pb data to describe multiplicity fluct.)

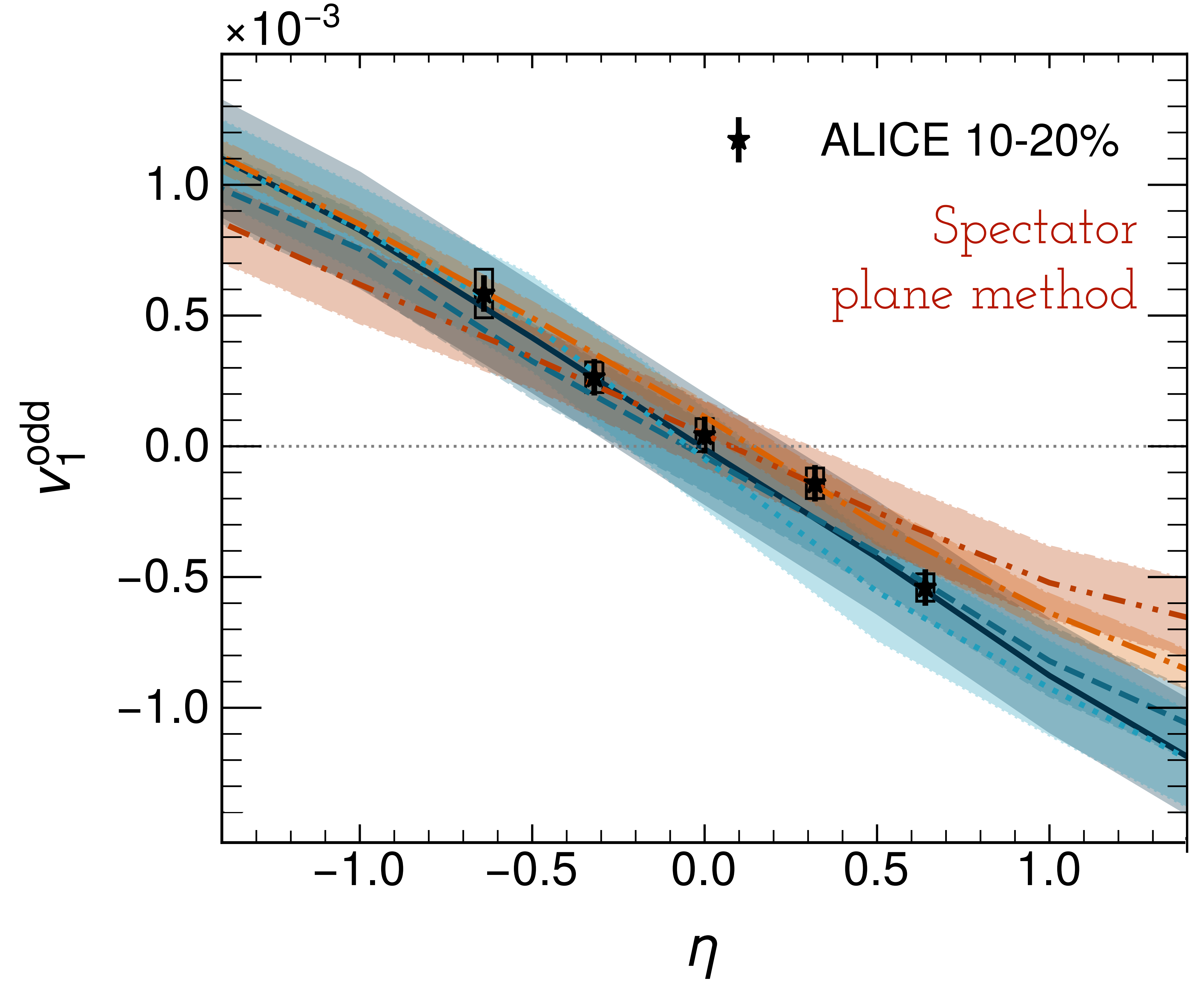
CLVisc 3+1

- Israel-Stewart-like viscous hydro evolution [PRC 98, 034916 (2018)]
- Duke transport coefficients [PRC 94, 024907 (2016)]
- Baryon diffusion included in Hydro ($C_B = 0.4$)
- **Equation of state: NEOS-B** [PRC 100, 024907 (2019)]
- Cooper-Frye particlization for cells with $e \leq 0.267\,\mathrm{GeV/fm^3}$, including viscous corrections

SOME SOFT RESULTS IN LARGE SYSTEMS



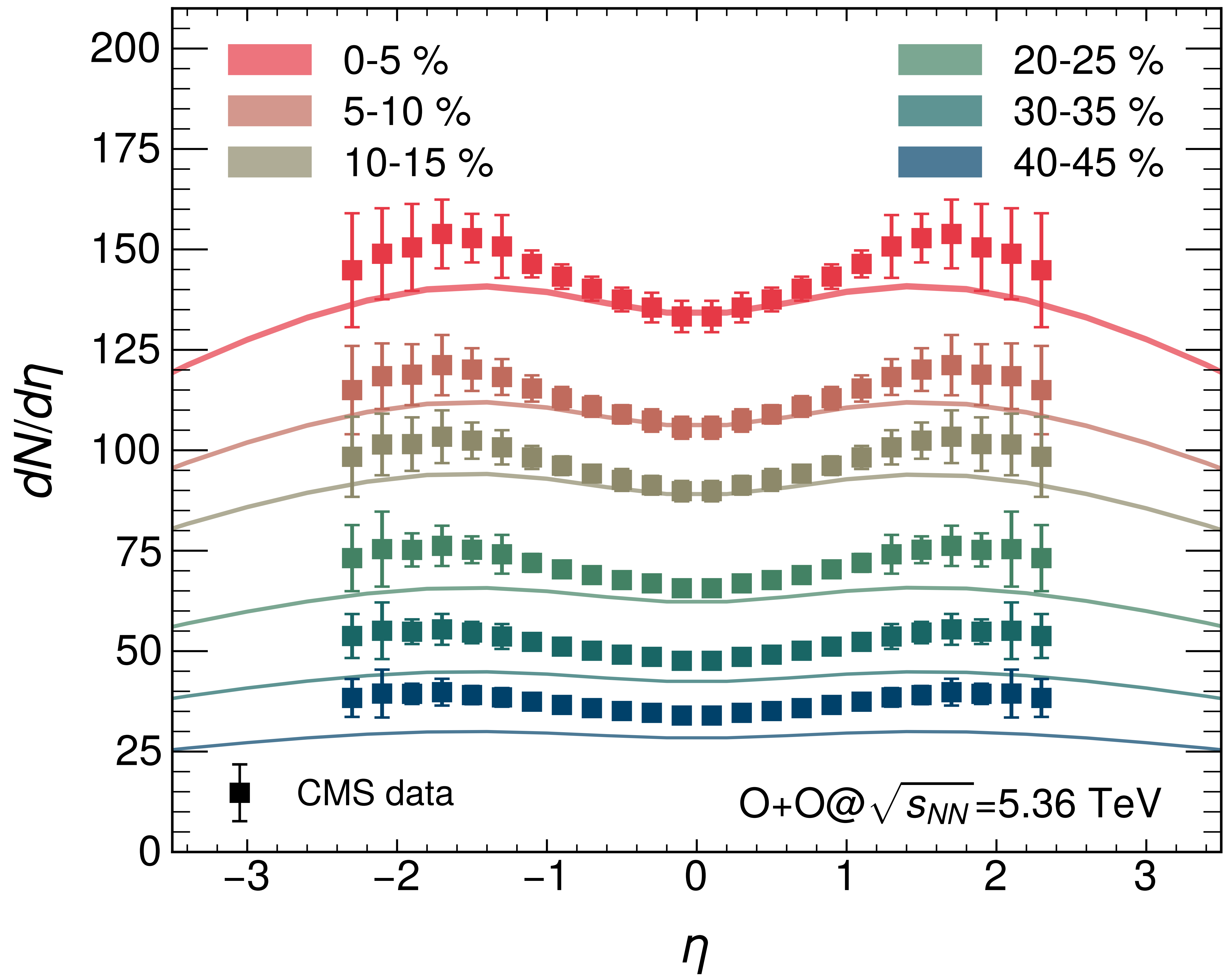
Minimal IC tuning. Shape of $\mathrm{d}N_{ch}/\mathrm{d}\eta$ given by IP-Sat



IC-information tunes the sensitive longitudinal structures, and describes data quantitatively

[GM, Schlichting, Zhu, 2501.14872]

SOME SOFT RESULTS IN SMALL SYSTEMS



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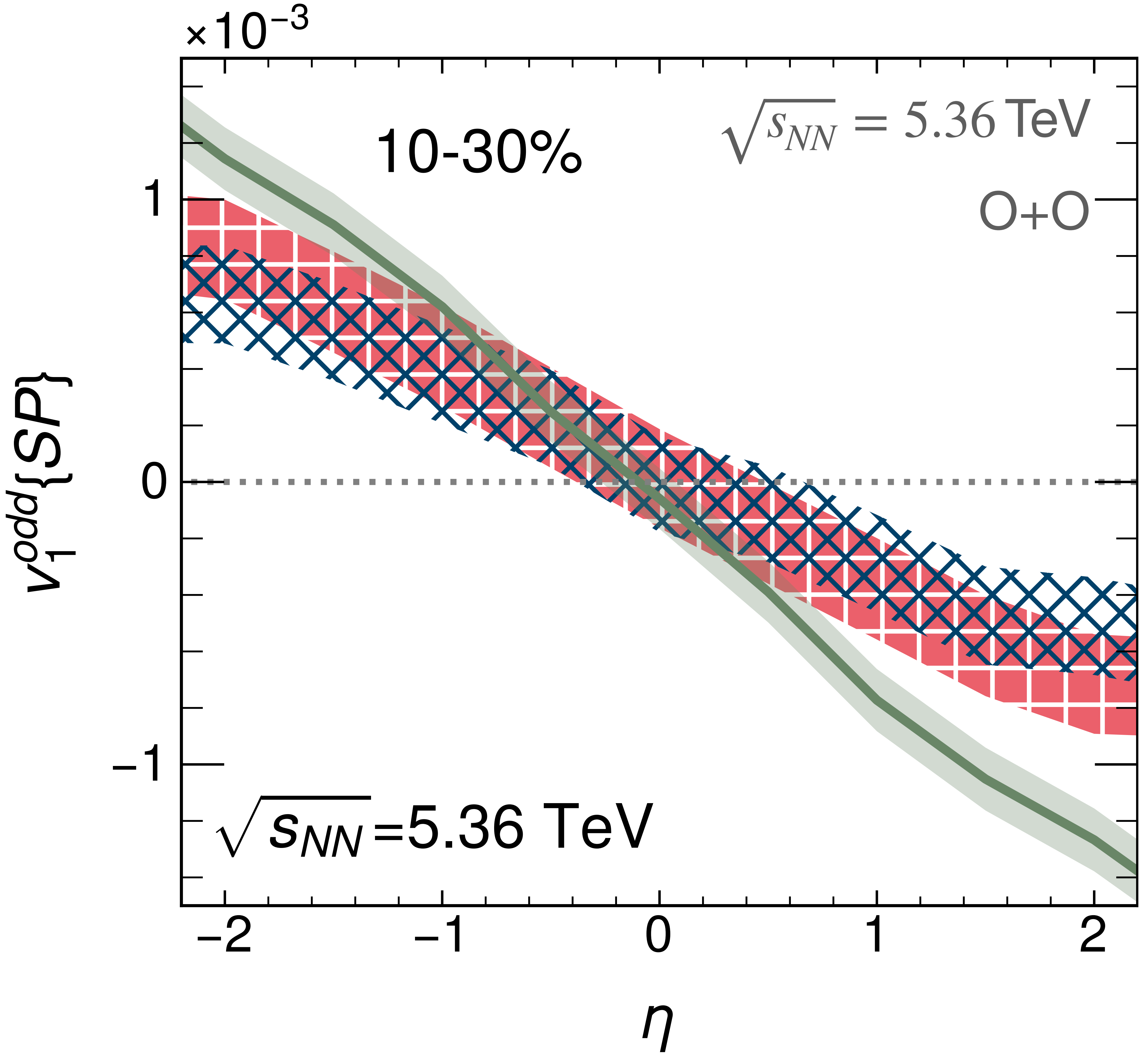


IC-information tunes the sensitive longitudinal structures, and describes data quantitatively

[GM, Zhu, 2608.XXXX]

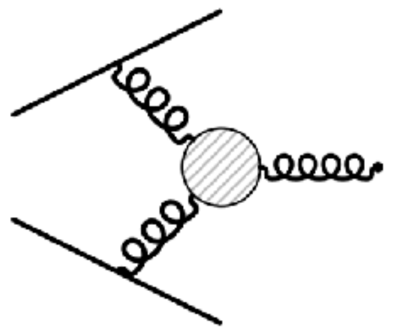
WHY DON'T WE GET $T^{xy} \neq 0$ IN THE KT-FACT FORMULA?

THE QUESTION

- Let's take a look at the particle production formula:

$$\frac{dN_g}{d^2\boldsymbol{x}d^2\boldsymbol{p}dy} = \frac{g^2}{8\pi^5 C_F \boldsymbol{p}^2}\int \frac{d^2\boldsymbol{q}}{(2\pi)^2}\frac{d^2\boldsymbol{k}}{(2\pi)^2}(2\pi)^2\delta(\boldsymbol{p}+\boldsymbol{q}-\boldsymbol{p})$$

$$\times\, \Phi_1(x_1,\boldsymbol{x},\boldsymbol{q})\ \Phi_2(x_2,\boldsymbol{x},\boldsymbol{k})$$



- Assuming a Cooper-Frye type surface for constant-τ

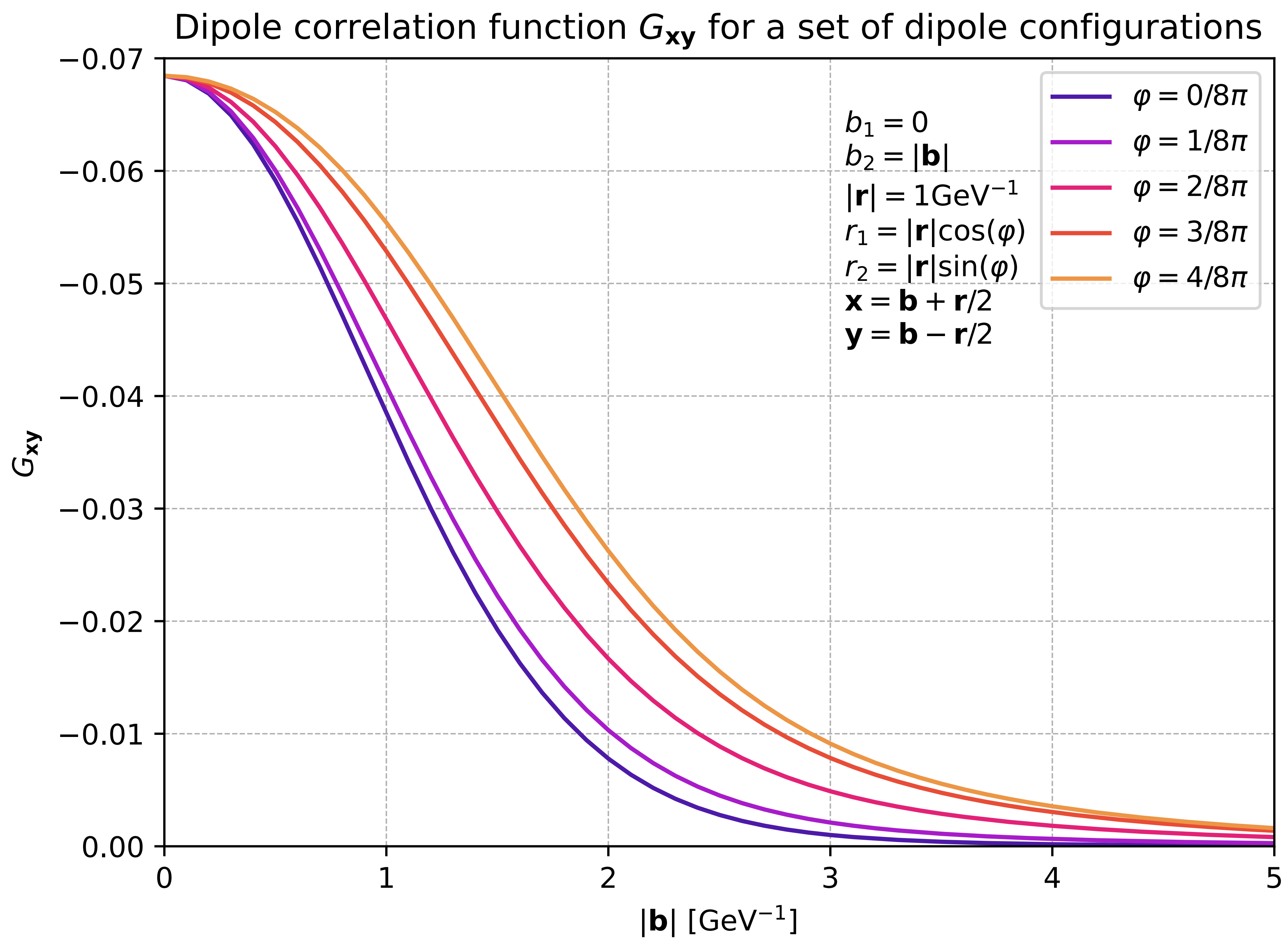
$$f(x,p) = \frac{(2\pi)^3}{\nu}\frac{1}{\tau p^\tau}\frac{dN}{d^2\mathbf{x}d^2\mathbf{p}dy}\delta(\eta-y) \implies \tau T^{\mu\nu} = \int \mathrm{d}^2\mathbf{p}\mathrm{d}y\,\frac{p^\mu p^\nu}{p^\tau}\frac{dN}{d^2\mathbf{x}d^2\mathbf{p}dy}\delta(\eta-y)$$

- Easy to see that if gluon distributions are independent functions of $\mathbf{x}$ and $\mathbf{k}$: e.g. $\Phi_i(\mathbf{x},\mathbf{k}) = \mathbf{k}^2 e^{-\mathbf{k}^2 F[\mathbf{x}]}$, then it is not possible to generate non-trivial cross-terms.

- BUT! This is not the case for a general uGDF!

In [2511.22763] we found that for a gaussian colour dist.

$$\Phi_i(\mathbf{x},\mathbf{r}) = e^{-F[|\mathbf{x}|,|\mathbf{k}|,\mathbf{x}\cdot\mathbf{r}]}$$



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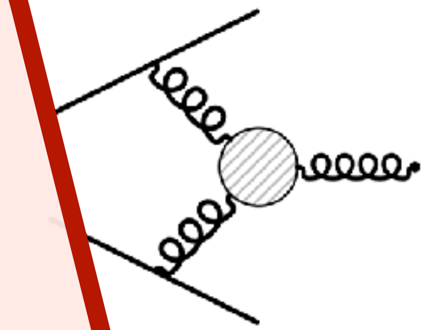

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$$f(x,p)=\frac{(2\pi)^3}{\nu}\frac{1}{\tau p^\tau}\frac{dN}{d^2\mathbf{x}d^2\mathbf{p}dy}\delta(\eta-y) \Longrightarrow T^{\mu\nu}=\int d^2\mathbf{p}dy\,\frac{p^\mu p^\nu}{p^\tau}\frac{dN}{d^2\mathbf{x}d^2\mathbf{p}dy}\delta(\eta-y)$$

- Easy to see that if gluon distributions are independent functions of $\mathbf{x}$ and $\mathbf{k}$, e.g. $\Phi_i(\mathbf{x},\mathbf{k})=\mathbf{k}^2e^{-\mathbf{k}^2F[\mathbf{x}]}$, then it is not possible to generate non-trivial cross-terms.

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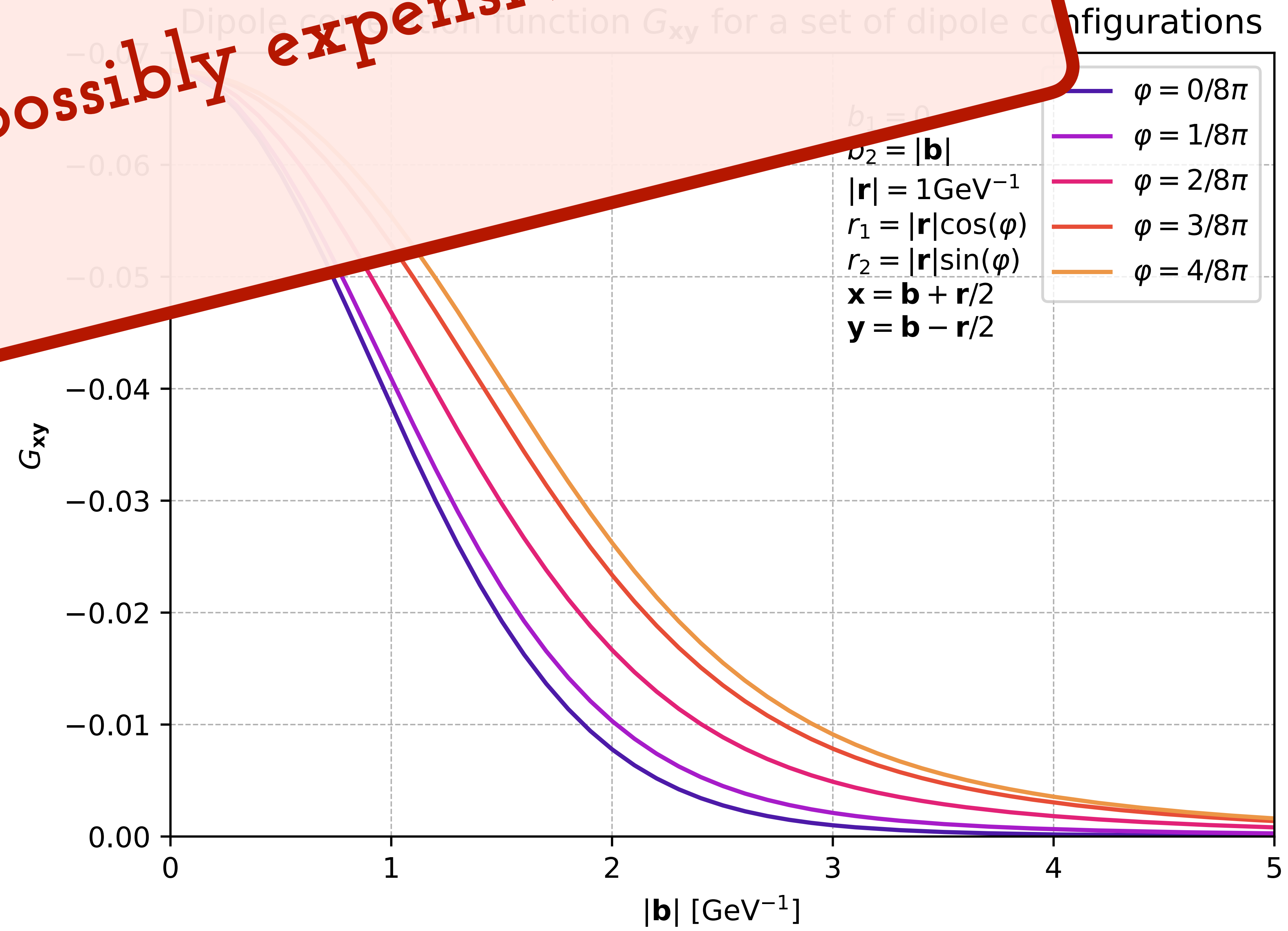
In [2511.22765] we found that for a gaussian colour dist.

$$\Phi_i(\mathbf{x},\mathbf{r})=e^{-F[|\mathbf{x}|,|\mathbf{k}|,\mathbf{x}\cdot\mathbf{r}]}$$



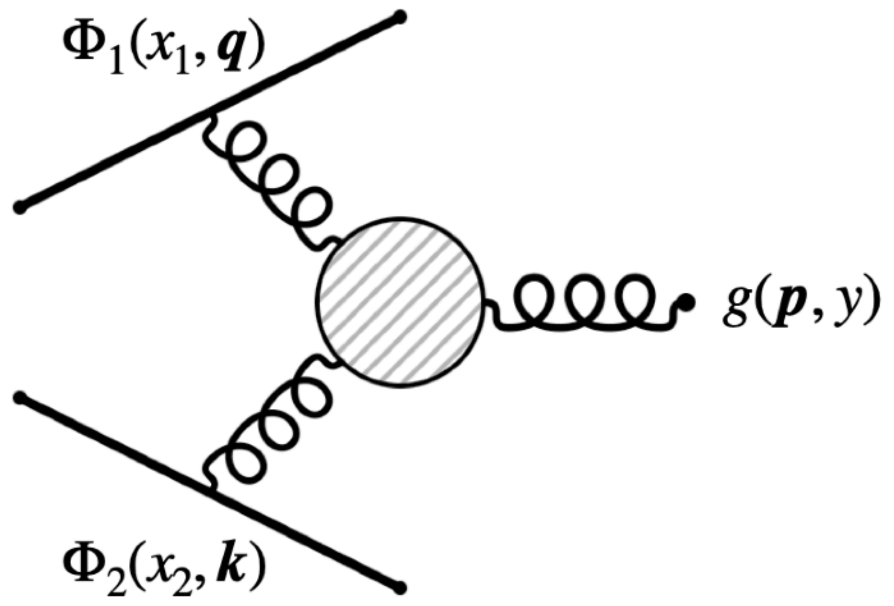
But we are interested in the full 3D-event, which means that we need the small-x evolution of the uGDFs for each event, and each possible configuration.

Doing that event by event is impossibly expensive.

A SIMPLE CASE: ANISOTROPIES WITHIN THE NUCLEI

- GBW model (gaussian gluon distributions) $D(x, \mathbf{k}_\perp, \mathbf{x}_\perp) \sim \exp\left(-\frac{\mathbf{k}_\perp^2}{Q_A^2(x, \mathbf{x}_\perp)}\right)$ $Q_A^2(x, \mathbf{x}_\perp) \sim x^{-\lambda} T(\mathbf{x}_\perp)$

- Simple kinematics
  Fixed by "measured" gluon $x_1 = \frac{p_\perp e^y}{\sqrt{s_{NN}}}$ $x_2 = \frac{p_\perp e^{-y}}{\sqrt{s_{NN}}}$



- Gluon distribution dominated by saturation scale $x_{1,2} = \frac{Q_{1,2}^2(x, T)\, e^{\pm y}}{\sqrt{s_{NN}}}$

- Analytical solution to energy $e\tau_0 \sim \frac{Q_A^2 Q_B^2}{(Q_A^2 + Q_B^2)^{5/2}}(2Q_A^4 + 2Q_B^4 + 7Q_A^2 Q_B^2)$ Complex function $e\tau_0[T_1, T_2]$

- Something like TRENTO

$$e\tau_0 \sim \left[\frac{T_A^p + T_B^p}{2}\right]^{1/p}$$

WHY IS THIS IMPORTANT?

Particle correlations take $\langle \cdot \rangle$ of powers of thermo. quantities.
Even when subleading, extra-powers may induce error.

A SIMPLE CASE

- Let's start from the beginning. $\phi_i(x, \mathbf{x}, \mathbf{k}) \sim k^2 D(x, \mathbf{x}, \mathbf{k})$ $\xleftrightarrow{\text{FT}}$ $D(\boldsymbol{r}, \boldsymbol{b}) = \frac{1}{N_c}\mathrm{Tr}\left[V(\boldsymbol{x})V^\dagger(\boldsymbol{y})\right] \equiv e^{\Gamma(\boldsymbol{x},\boldsymbol{y})}$

  Where in the MV model

  $$\Gamma(\boldsymbol{x},\boldsymbol{y}) = \lambda(\boldsymbol{x},\boldsymbol{y}) - \frac{1}{2}\lambda(\boldsymbol{x},\boldsymbol{x}) - \frac{1}{2}\lambda(\boldsymbol{y},\boldsymbol{y})$$

  and

  $$\lambda(\boldsymbol{x},\boldsymbol{y}) = g^2 C_F \int_{\boldsymbol{z}} G(\boldsymbol{x}-\boldsymbol{z})G(\boldsymbol{y}-\boldsymbol{z})\mu^2(\boldsymbol{z})$$

  Color charge dens.

- Assuming soft dependence of impact parameter and expanding

  $$\mu^2(\boldsymbol{b}-\boldsymbol{w}) = \mu^2(\boldsymbol{b}) - w^i\frac{\partial\mu^2(\boldsymbol{b})}{\partial b^i} + \frac{1}{2}w^i w^j\frac{\partial^2\mu^2(\boldsymbol{b})}{\partial b^i\partial b^j} + \mathcal{O}(|\boldsymbol{w}|^3)$$

- Then we have

  $$\Gamma(\boldsymbol{x},\boldsymbol{y}) = \Gamma_0(\boldsymbol{x},\boldsymbol{y}) + \Gamma_1(\boldsymbol{x},\boldsymbol{y}) + \Gamma_2(\boldsymbol{x},\boldsymbol{y})$$

  with $\Gamma_1(\mathbf{x},\mathbf{y}) = 0$ (parity)

A SIMPLE CASE

- After some massaging of the equations

$$\Gamma_0(\mathbf{x},\mathbf{y}) = \frac{g^2}{4\pi}\frac{\mu^2(\mathbf{b})}{m^2}C_F\left(1 - rmK_1(rm)\right) \qquad \Gamma_2(\mathbf{x},\mathbf{y}) = \frac{\alpha_S C_F}{24m^2}\nabla_i\nabla_j\mu^2(\mathbf{b})\left[xK_1(x)r^ir^j - \frac{2\delta^{ij}}{m^2}\left(2 - x^2K_2(x)\right)\right]$$

- Small-r limit

$$\Gamma_0(\mathbf{x},\mathbf{y}) = \approx -\frac{g^2 C_F}{2\pi}\mu^2(\mathbf{b})\frac{r^2}{4}\log\left[\frac{1}{rm}\right] \qquad \Gamma_2(\mathbf{x},\mathbf{y}) = \frac{\alpha_S C_F}{24m^2}\nabla_i\nabla_j\mu^2(\mathbf{b})\left(r^ir^j - \delta^{ij}r^2\right)$$

- Performing the classical method of conveniently forgetting about the log,

$$D(\mathbf{r},\mathbf{b}) = e^{-\frac{1}{4}\mathbf{r}\cdot\Sigma^{-1}\cdot\mathbf{r}} \quad \text{with} \quad \Sigma^{-1} = \begin{pmatrix} Q^2(\mathbf{b}) + \frac{1}{6m^2}\nabla_y^2 Q^2(\mathbf{b}) & -\frac{1}{6m^2}\nabla_x\nabla_y Q^2(\mathbf{b}) \\ -\frac{1}{6m^2}\nabla_x\nabla_y Q^2(\mathbf{b}) & Q^2 + \frac{1}{6m^2}\nabla_x^2 Q^2(\mathbf{b}) \end{pmatrix}$$

[O. G-M, 2608.XXXX]

IS THIS... OK?

- Having $D(\mathbf{r}, \mathbf{b}) = e^{-\frac{1}{4}\mathbf{r}\cdot\Sigma^{-1}\cdot\mathbf{r}}$ everything boils to whether Σ^{-1} is positive definite

- *Sylvester Criterion*: all of the *leading* principal minors must be positive.

$$\Longrightarrow \Sigma^{-1}_{11} > 0 \quad \text{and} \quad \det|\Sigma^{-1}| > 0$$

- *Simple Example*: Gaussian $Q^2(\mathbf{b}) = \dfrac{Q_0^2}{2\pi R^2}\exp\left(-\dfrac{\mathbf{b}^2}{2R^2}\right)$

$$\Sigma^{-1}_{11} = Q^2(\mathbf{b})\left(1 - \frac{1}{6m^2R^2}\right) + \frac{Q^2(\mathbf{b})}{6m^2R^4}\mathbf{b}^2$$

Which means that: $R > \dfrac{1}{\sqrt{6m}}$ *From which* $\det|\Sigma_{11}| > 0$ *automatically follows*

All to say that gradients have to be smaller than the regulator's size

COMPUTATION

- Now, Fourier Transforming we get

$$\frac{dN}{d^2\mathbf{x}d^2\mathbf{p}dy} = \frac{C_F}{8\pi^4\alpha_s}\frac{4\pi}{\sqrt{\det\Sigma^{-1}(x_1,\boldsymbol{x})}}\frac{4\pi}{\sqrt{\det\Sigma^{-1}(x_2,\boldsymbol{x})}}\int\frac{d^2\mathbf{q}}{(2\pi)^2}\frac{d^2\mathbf{k}}{(2\pi)^2}\frac{\mathbf{q}^2\mathbf{k}^2}{\mathbf{p}^2}$$
$$\times\; e^{-\boldsymbol{q}\cdot\Sigma(x_1,\boldsymbol{x})\cdot\boldsymbol{q}}e^{-\boldsymbol{k}\cdot\Sigma(x_2,\boldsymbol{x})\cdot\boldsymbol{k}}(2\pi)^2\delta(\mathbf{q}+\mathbf{k}-\mathbf{p}).$$

- *From which we get this absolute unit of a formula*

$$\frac{dN}{d^2\mathbf{x}d^2\mathbf{p}dy} = \frac{2C_F}{\pi^2\alpha_s\boldsymbol{p}^2}\frac{1}{\sqrt{|\Sigma_1^{-1}||\Sigma_2^{-1}|}}e^{-\frac{1}{4}\boldsymbol{p}\cdot\bar{\Sigma}\cdot\boldsymbol{p}}\frac{1}{4\pi\sqrt{|\Sigma_+|}}\Big\{H_0(\boldsymbol{p}) - 2\boldsymbol{p}\cdot\beta_-\Sigma_+^{-1}\cdot\beta_+\boldsymbol{p}$$
$$+\frac{1}{2}tr(\Sigma_+^{-1})\left[(\beta_-\cdot\boldsymbol{p})^2+(\beta_+\cdot\boldsymbol{p})^2\right]+\frac{1}{4}\left[tr[\Sigma_+^{-1}]^2+2tr[\Sigma_+^{-1}\Sigma_+^{-1}]\right]\Big\}$$

with $\Sigma_\pm = \Sigma_1 \pm \Sigma_2$ *and* $\bar{\Sigma} = \Sigma_+ - \Sigma_-\Sigma_+^{-1}\Sigma_-$

[O. G-M, 2608.XXXX]

GETTING $T^{\mu\nu}$

- We will use $\tau T^{\mu\nu} = \int \mathrm{d}^2\boldsymbol{p}\mathrm{d}y \, \frac{p^\mu p^\nu}{p^\tau} \frac{dN}{d^2\mathbf{x}d^2\mathbf{p}dy}\delta(\eta - y)$ from which $\tau T^{\tau\eta} = \tau T^{i\eta} = \tau T^{\eta\eta} = 0$

since $\delta(\eta - y)p^\eta = \delta(\eta - y)\,|\,\mathbf{p}\,|\sinh(\eta - y) = 0$

- Example: energy density $\tau T^{\tau\tau} = \frac{C_F}{2\pi^3\alpha_s} \frac{1}{\sqrt{|\Sigma_1^{-1}|\,|\Sigma_+|\,|\Sigma_2^{-1}|}} \int_{-\pi}^{\pi} \mathrm{d}\theta \int_0^\infty \mathrm{d}p J(p,\theta) e^{-\frac{1}{4}Ap^2}$

- Expanding on small gradients (a condition for positivity) we get at first order correction

$$\tau T^{\tau\tau} = \frac{C_F}{2\pi\sqrt{\pi}\alpha_s} \frac{1}{96m^2\left(\bar{Q}^2\right)^{7/2}} \Big[Q_2^2 (\nabla_x^2 Q_1^2 + \nabla_y^2 Q_1^2)(2Q_1^6 + 5Q_1^4Q_2^2 + 22Q_1^2Q_2^4 + 4Q_2^6)$$
$$+ Q_1^2 (\nabla_x^2 Q_2^2 + \nabla_y^2 Q_2^2)(4Q_1^6 + 22Q_1^4Q_2^2 + 5Q_1^2Q_2^4 + 2Q_2^6) \Big]$$

[O. G-M, 2606.XXXX]

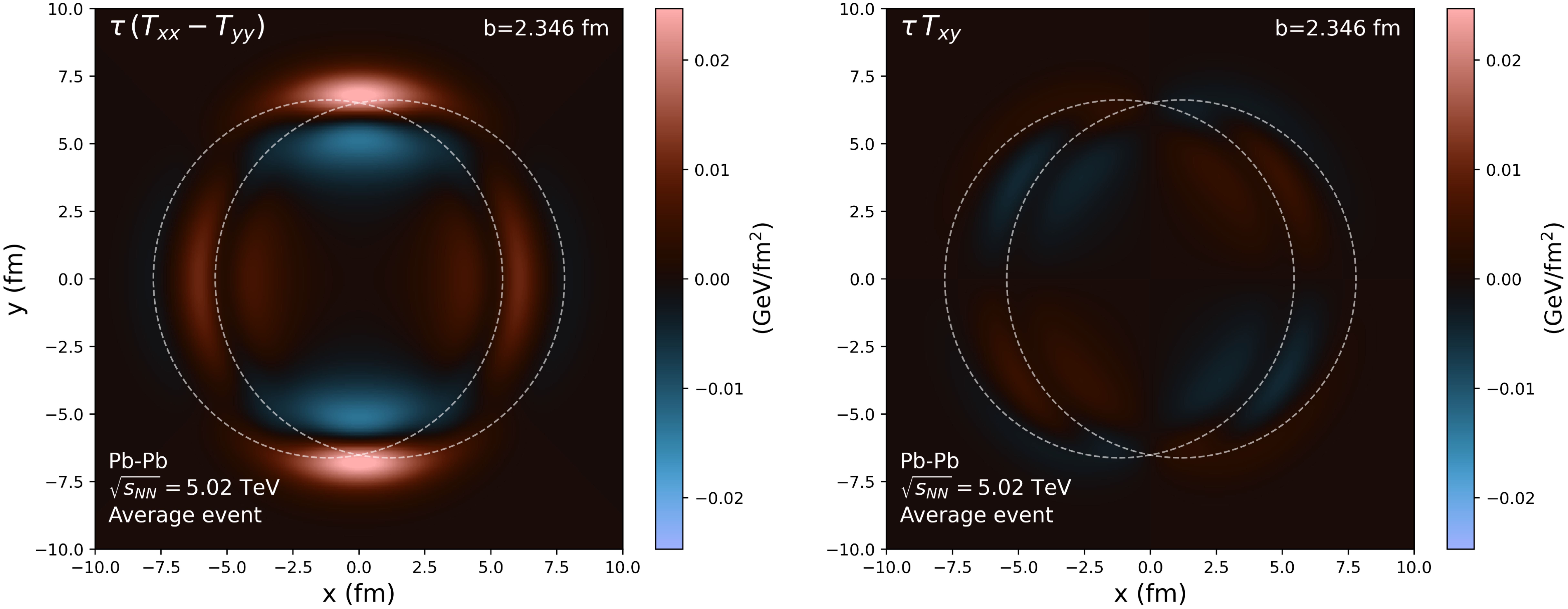
GETTING $T^{\mu\nu}$

- We also get this simple and beautiful looking transverse pressures.

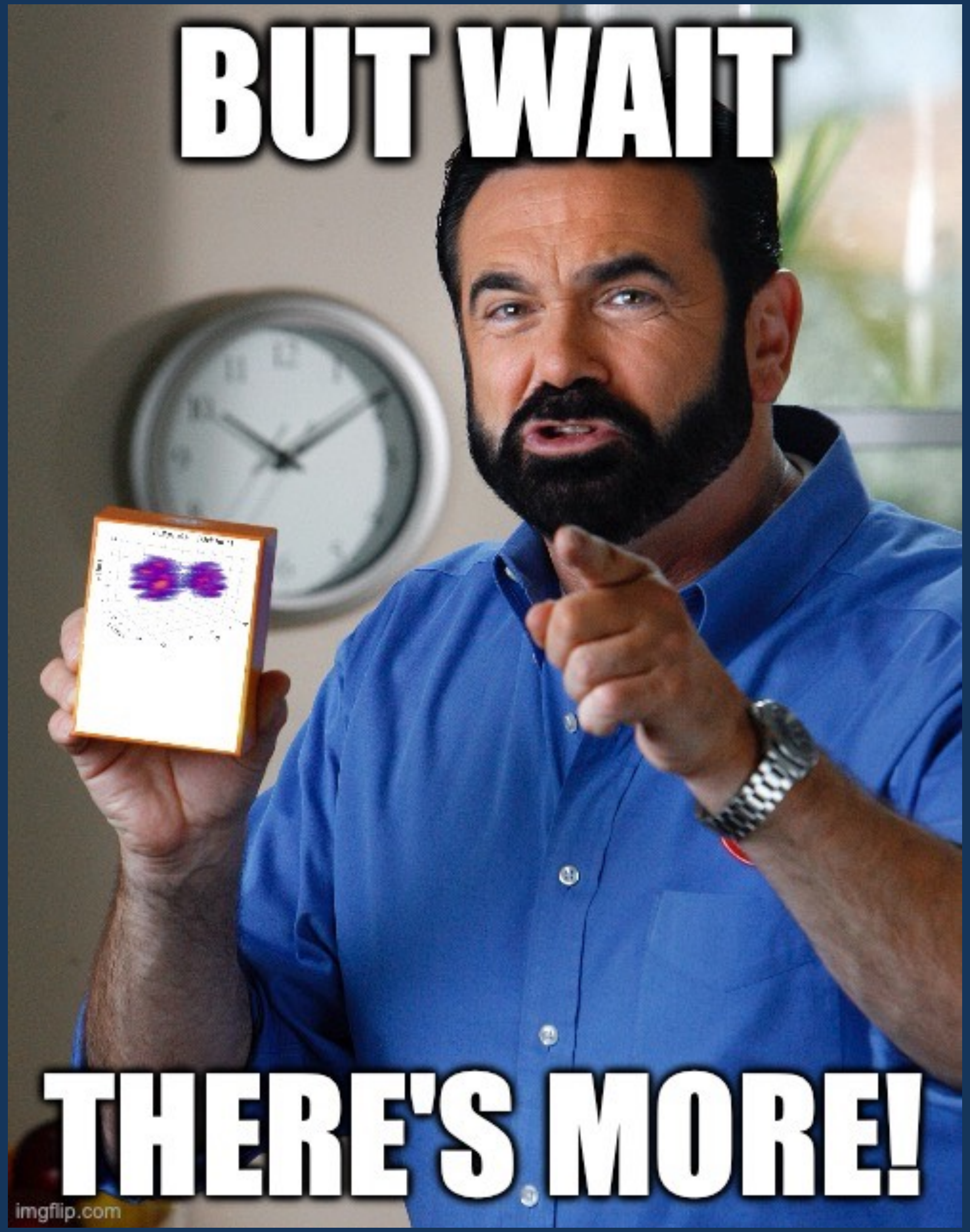
$$\tau T^{xx} = \frac{C_F}{2\pi\sqrt{\pi}\alpha_s}\frac{1}{384m^2\left(\bar{Q}^2\right)^{7/2}}\Big[Q_2^2\left(\nabla_x^2 Q_1^2\left(-2Q_1^6 - 5Q_1^4Q_2^2 + 50Q_1^2Q_2^4 + 8Q_2^6\right) + \nabla_y^2 Q_1^2\left(10Q_1^6 + 25Q_1^4Q_2^2 + 38Q_1^2Q_2^4 + 8Q_2^6\right)\right)$$
$$+\nabla_x^2 Q_2^2\left(8Q_1^8 + 50Q_1^6Q_2^2 - 5Q_1^4Q_2^4 - 2Q_1^2Q_2^6\right) + \nabla_y^2 Q_2^2\left(8Q_1^8 + 38Q_1^6Q_2^2 + 25Q_1^4Q_2^4 + 10Q_1^2Q_2^6\right)\Big]$$
$$\tau T^{yy} = \frac{C_F}{2\pi\sqrt{\pi}\alpha_s}\frac{1}{384m^2\left(\bar{Q}^2\right)^{7/2}}\Big[Q_2^2\left(\nabla_x^2 Q_1^2\left(10Q_1^6 + 25Q_1^4Q_2^2 + 38Q_1^2Q_2^4 + 8Q_2^6\right) + \nabla_y^2 Q_1^2\left(-2Q_1^6 - 5Q_1^4Q_2^2 + 50Q_1^2Q_2^4 + 8Q_2^6\right)\right)$$
$$+\nabla_x^2 Q_2^2\left(8Q_1^8 + 38Q_1^6Q_2^2 + 25Q_1^4Q_2^4 + 10Q_1^2Q_2^6\right) + \nabla_y^2 Q_2^2\left(8Q_1^8 + 50Q_1^6Q_2^2 - 5Q_1^4Q_2^4 - 2Q_1^2Q_2^6\right)\Big]$$

- And the cross terms

$$\tau T^{xy} = -\frac{C_F}{2\pi^{3/2}\alpha_s}\frac{Q_1^2Q_2^2}{64m^2\left(\bar{Q}^2\right)^{7/2}}\left(\nabla_x\nabla_y Q_1^2\left(2Q_1^4 + 5Q_1^2Q_2^2 - 2Q_2^4\right) + \nabla_x\nabla_y Q_2^2\left(-2Q_1^4 + 5Q_1^2Q_2^2 + 2Q_2^4\right)\right)$$



$\tau\,(T_{xx} - T_{yy})$
b=2.346 fm
Pb-Pb
$\sqrt{s_{NN}} = 5.02$ TeV
Average event
y (fm)
x (fm)
(GeV/fm^2)
$\tau\,T_{xy}$
b=2.346 fm
Pb-Pb
$\sqrt{s_{NN}} = 5.02$ TeV
Average event
x (fm)
(GeV/fm^2)



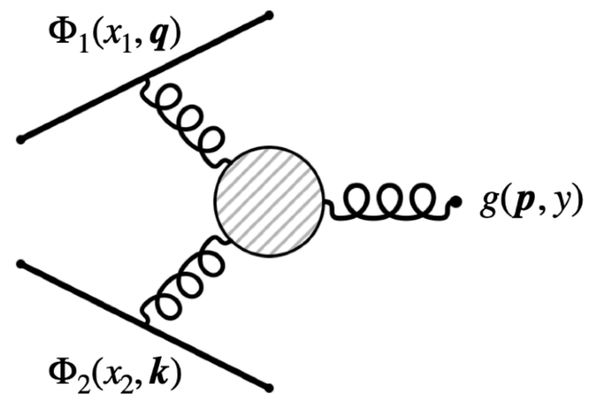
WHY IS THE KT-FACTORISED FORMULA ALREADY SO... ISOTROPIC?

SECOND SOURCE: ANISOTROPIES FROM GLUON RESOLUTION

In the dilute-dilute limit of the CGC, single gluon production is given by $E_q \dfrac{\mathrm{d}N_g}{\mathrm{d}^3p} = \dfrac{g^2}{2(2\pi)^3} |\mathscr{M}_g|^2$

$$E_q \frac{\mathrm{d}N_g}{\mathrm{d}^3p} = \frac{g^2}{2(2\pi)^3} \int_{\boldsymbol{k}_1 \bar{\boldsymbol{k}}_1} H\left(\boldsymbol{k}_1, \boldsymbol{k}_2, \bar{\boldsymbol{k}}_1, \bar{\boldsymbol{k}}_2\right) \left\langle \rho^\dagger_{A,a}(\boldsymbol{k}_1)\, \rho_{A,b}(\bar{\boldsymbol{k}}_1) \right\rangle \left\langle \rho^\dagger_{B,a}(\boldsymbol{k}_2)\, \rho_{B,b}(\bar{\boldsymbol{k}}_2) \right\rangle, \qquad \text{where} \qquad \boldsymbol{k}_2 = \boldsymbol{p} - \boldsymbol{k}_1$$

With Hard factor: $H(\boldsymbol{k}_1, \bar{\boldsymbol{k}}_1, \boldsymbol{k}_2, \bar{\boldsymbol{k}}_2) = \dfrac{p_\perp^2}{4} \left[\dfrac{4}{p_\perp^2} \left(\delta^{ij}\delta^{\kappa\lambda} + \epsilon^{ij}\epsilon^{\kappa\lambda} \right) k_1^i\, k_2^j\, \bar{k}_1^\kappa\, \bar{k}_2^\lambda \right] \dfrac{1}{\boldsymbol{k}_1^2\, \bar{\boldsymbol{k}}_1^2\, \boldsymbol{k}_2^2\, \bar{\boldsymbol{k}}_2^2}$



- Let's switch to uGDFs language:

$$g^2 \left\langle \rho_a^\dagger(\boldsymbol{k}_\perp)\, \rho_{a'}(\boldsymbol{k}'_\perp) \right\rangle = \frac{\delta^{aa'}}{\pi d_A} \left[\frac{\boldsymbol{k}_\perp + \boldsymbol{k}'_\perp}{2} \right]^2 \int_{\boldsymbol{x}_\perp} e^{\, i(\boldsymbol{k}_\perp - \boldsymbol{k}'_\perp)\cdot \boldsymbol{x}_\perp} \varphi\left(\frac{\boldsymbol{k}_\perp + \boldsymbol{k}'_\perp}{2}, \boldsymbol{x}_\perp \right).$$

- What is the underlying physics?

$$E_q \frac{\mathrm{d}N_g}{\mathrm{d}^3p} = \frac{4g^2}{(2\pi)^5 C_F \boldsymbol{p}^2} \int_{\boldsymbol{k}_1 \bar{\boldsymbol{k}}_1 \boldsymbol{k}_2 \bar{\boldsymbol{k}}_2} \int_{\boldsymbol{xy}} H\left(\boldsymbol{k}_1, \boldsymbol{k}_2, \bar{\boldsymbol{k}}_1, \bar{\boldsymbol{k}}_2\right) \left[\frac{\boldsymbol{k}_1 + \bar{\boldsymbol{k}}_1}{2} \right]^2 \left[\frac{\boldsymbol{k}_2 + \bar{\boldsymbol{k}}_2}{2} \right]^2 \mathrm{e}^{\mathrm{i}\boldsymbol{x}(\boldsymbol{k}_1 - \bar{\boldsymbol{k}}_1)} \mathrm{e}^{\mathrm{i}\boldsymbol{y}(\boldsymbol{k}_2 - \bar{\boldsymbol{k}}_2)} \varphi_A\left(\boldsymbol{x}, \frac{\boldsymbol{k}_1 + \bar{\boldsymbol{k}}_1}{2} \right) \varphi_B\left(\boldsymbol{y}, \frac{\boldsymbol{k}_2 + \bar{\boldsymbol{k}}_2}{2} \right)$$

FT of color distribution resolution

Gluon momenta sampled

- If we Wigner-transform to pair coordinates $\dfrac{\boldsymbol{k}_1+\bar{\boldsymbol{k}}_1}{2}=\boldsymbol{q}_\perp,\qquad \dfrac{\boldsymbol{k}_2+\bar{\boldsymbol{k}}_2}{2}=\boldsymbol{k}_\perp,\qquad \tilde{\boldsymbol{q}}_\perp\equiv\boldsymbol{k}_1-\bar{\boldsymbol{k}}_1,\qquad \tilde{\boldsymbol{k}}_\perp\equiv\boldsymbol{k}_2-\bar{\boldsymbol{k}}_2$

  And expand the hard factor around the relative coordinate $H=\dfrac{h}{\boldsymbol{k}_1^2\,\bar{\boldsymbol{k}}_1^2\,\boldsymbol{k}_2^2\,\bar{\boldsymbol{k}}_2^2}=\dfrac{1}{\boldsymbol{q}_\perp^4\,\boldsymbol{k}_\perp^4}\left[h_0+h_1+h_2+\cdots\right]$

- To get the ladder of contributions, a gradient expansion

$$E_p\frac{\mathrm{d}N_g}{\mathrm{d}^3p}=\sum_{i=0}^{\infty}\frac{\alpha_S}{2\pi^4p_\perp^2C_F}\int_{\boldsymbol{q}\tilde{\boldsymbol{q}}\boldsymbol{k}\tilde{\boldsymbol{k}}}\int_{xy}\Pi_2\,\frac{h_i(\boldsymbol{q},\boldsymbol{k},\tilde{\boldsymbol{q}},\tilde{\boldsymbol{k}})}{\boldsymbol{q}_\perp^2\,\boldsymbol{k}_\perp^2}\,\mathrm{e}^{\,\mathrm{i}\tilde{\boldsymbol{q}}\cdot x}\,\mathrm{e}^{\,\mathrm{i}\tilde{\boldsymbol{k}}\cdot y}\,\varphi_A(\boldsymbol{q},\boldsymbol{x})\,\varphi_B(\boldsymbol{k},\boldsymbol{y})\;\equiv\;\sum_{i=0}^{\infty}E_p\frac{\mathrm{d}N^{(i)}}{\mathrm{d}^3p}$$

$$\ni(2\pi)^2\delta(\boldsymbol{p}-\boldsymbol{q}-\tilde{\boldsymbol{q}}/2-\boldsymbol{k}-\tilde{\boldsymbol{k}}/2)$$

- We get then easily the zeroth order contribution, which is the $k_\perp$-factorised formula

$$E_p\frac{dN^{(1)}}{d^3p\,d^2x}=\frac{\alpha_s}{2\pi^4\,p_\perp^2\,C_F}\int_{\boldsymbol{q}\boldsymbol{k}}(2\pi)^2\,\delta(\boldsymbol{p}-\boldsymbol{q}-\boldsymbol{k})\,\varphi_A(\boldsymbol{q},\boldsymbol{x})\,\varphi_B(\boldsymbol{k},\boldsymbol{x})$$

- We will also get **up to the first** non-trivial correction, to be consistent ($\mathcal{O}(\partial^2)$)

$$\boxed{\begin{aligned} E_p \frac{dN^{(1)}}{d^3p\, d^2x} = \frac{\alpha_s}{2\pi^4\, p_\perp^2\, C_F} \int_{\boldsymbol{qk}} (2\pi)^2\, \delta(\boldsymbol{p}-\boldsymbol{q}-\boldsymbol{k}) \Bigg[\left(q^i k^j - (\boldsymbol{q}\cdot\boldsymbol{k})\,\delta^{ij}\right) \partial_y^i \partial_x^j \\ - \frac{q_\perp^2}{k_\perp^2}\left(k^i k^j - \tfrac{3}{4} k_\perp^2\, \delta^{ij}\right) \partial_y^i \partial_y^j - \frac{k_\perp^2}{q_\perp^2}\left(q^i q^j - \tfrac{3}{4} q_\perp^2\, \delta^{ij}\right) \partial_x^i \partial_x^j \Bigg] \frac{\varphi_A(\boldsymbol{q},\boldsymbol{x})}{q_\perp^2} \frac{\varphi_B(\boldsymbol{k},\boldsymbol{y})}{k_\perp^2} \Bigg|_{x=y} \end{aligned}}$$

Anisotropies are included due to coupling of gluon momenta and gradients of the unintegrated Gluon Distribution Functions!

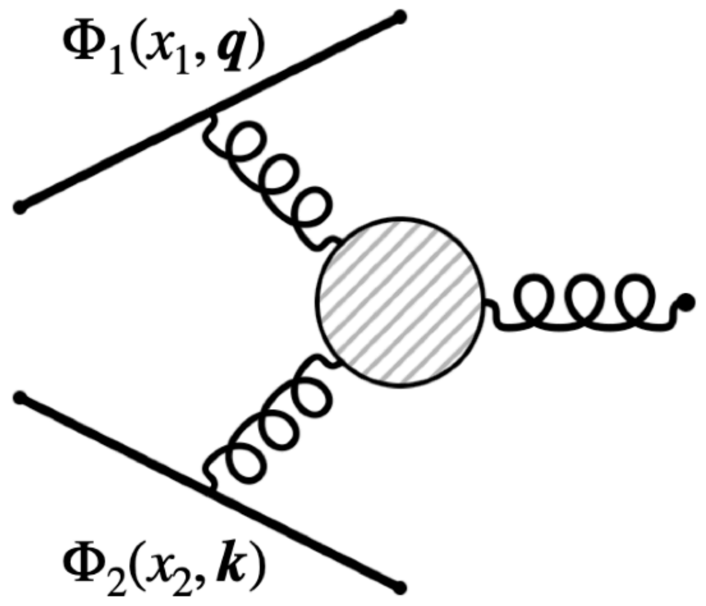
- From this, we get also an expansion on gradients for the stress-energy tensor

$$\tau T^{\mu\nu} = \underbrace{\tau T_0^{\mu\nu}}_{\mathcal{O}(\partial^0)} + \underbrace{\tau T_1^{\mu\nu}}_{\mathcal{O}(\partial^2)} + \underbrace{\tau T_2^{\mu\nu}}_{\mathcal{O}(\partial^4)} + \dots$$

A SIMPLE CASE: AGAIN

- GBW model (gaussian gluon distributions) $D(x, \mathbf{k}_\perp, \mathbf{x}_\perp) \sim \exp\left(-\frac{\mathbf{k}_\perp^2}{Q_A^2(x, \mathbf{x}_\perp)}\right)$ $Q_A^2(x, \mathbf{x}_\perp) \sim x^{-\lambda} T(\mathbf{x}_\perp)$

- Simple kinematics
  Fixed by "measured" gluon $x_1 = \frac{p_\perp e^y}{\sqrt{s_{NN}}}$ $x_2 = \frac{p_\perp e^{-y}}{\sqrt{s_{NN}}}$



- Gluon distribution dominated by saturation scale $x_{1,2} = \frac{Q_{1,2}^2(x, T)\, e^{\pm y}}{\sqrt{s_{NN}}}$

- Analytical solution to energy

$$\tau T^{\mu\nu} \sim \hat{f}^{\mu\nu} \frac{Q_A^2 Q_B^2}{(Q_A^2 + Q_B^2)^{5/2}} (2Q_A^4 + 2Q_B^4 + 7Q_A^2 Q_B^2)$$

With $\hat{f}^{\mu\nu} = \mathrm{diag}(1,1/2,1/2,0)$

SUMMARY, SINCE COMPUTATION IS LONG

- The resulting first order

$$\tau T_1^{\mu\nu} = \gamma\left[\bar{f}^{\mu\nu}\,\mathcal{T}_{\rm iso} + \mathcal{T}^{\mu\nu}_{\rm aniso}\right],$$

$$\mathcal{T}_{\rm iso} = H^{(1)}\left(\nabla Q_1^2 \cdot \nabla Q_2^2\right) + \frac{1}{2}\left(H_{01}^{2,\rm iso}\,\nabla^2 Q_2^2 + H_{02}^{2,\rm iso}\,|\nabla Q_2^2|^2\right) + \frac{1}{2}\left(H_{10}^{3,\rm iso}\,\nabla^2 Q_1^2 + H_{20}^{3,\rm iso}\,|\nabla Q_1^2|^2\right)$$

With functions such as

$$H^{(1)} = -\frac{2Q_1^4 - 11\,Q_1^2 Q_2^2 + 2Q_2^4}{4\,(Q_1^2 + Q_2^2)^{7/2}}$$

$$\mathcal{T}^{xx}_{\rm aniso} = \frac{1}{4}\Big[\left(\nabla_x Q_1^2\,\nabla_x Q_2^2 - \nabla_y Q_1^2\,\nabla_y Q_2^2\right) H^{(1)} - \left(\nabla_x^2 Q_2^2 - \nabla_y^2 Q_2^2\right) H_{01}^{2,\rm a} - \left(|\nabla_x Q_2^2|^2 - |\nabla_y Q_2^2|^2\right) H_{02}^{2,\rm a}$$
$$-\left(\nabla_x^2 Q_1^2 - \nabla_y^2 Q_1^2\right) H_{10}^{3,\rm a} - \left(|\nabla_x Q_1^2|^2 - |\nabla_y Q_1^2|^2\right) H_{20}^{3,\rm a}\Big]$$

$$\mathcal{T}^{xy}_{\rm aniso} = \frac{1}{4}\Big[\left(\nabla_x Q_1^2\,\nabla_y Q_2^2 + \nabla_y Q_1^2\,\nabla_x Q_2^2\right) H^{(1)} - 2\,\nabla_x \nabla_y Q_2^2\,H_{01}^{2,\rm a} - 2\,\nabla_x Q_2^2\,\nabla_y Q_2^2\,H_{02}^{2,\rm a} - 2\,\nabla_x \nabla_y Q_1^2\,H_{10}^{3,\rm a}$$
$$-2\,\nabla_x Q_1^2\,\nabla_y Q_1^2\,H_{20}^{3,\rm a}\Big]$$

$$\mathcal{T}^{yy}_{\rm aniso} = -\,\mathcal{T}^{xx}_{\rm aniso}$$

The rest vanish

A case for intuition

TRANSVERSE PRESSURES

P+P

Gaussian

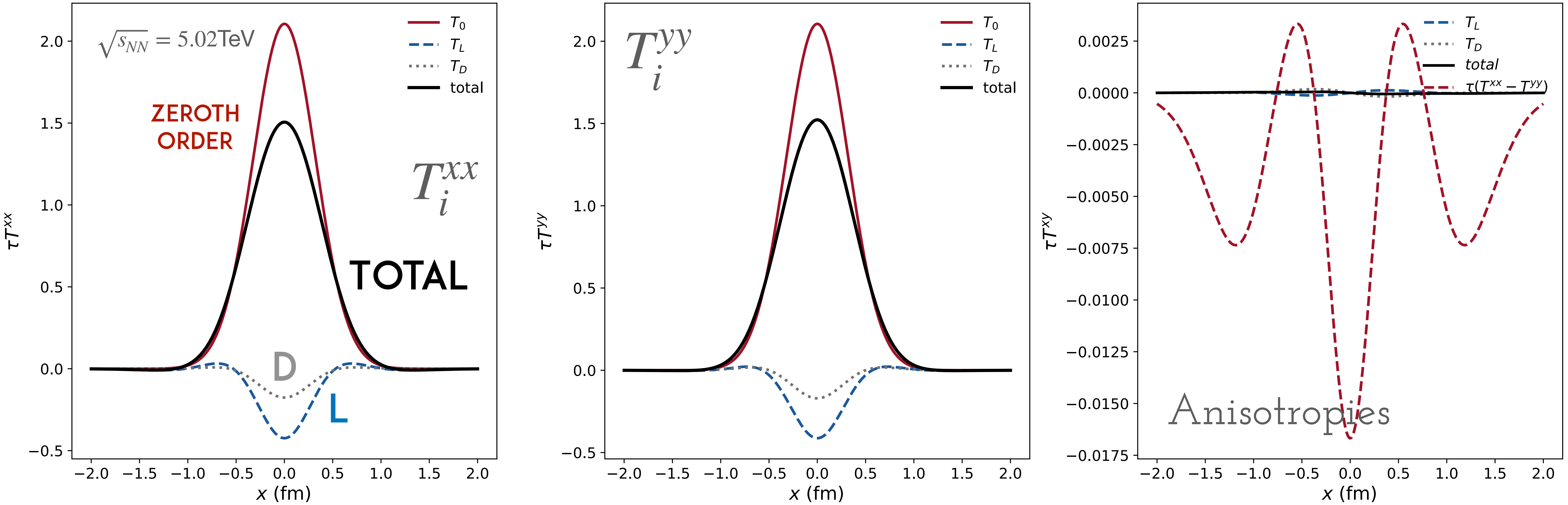
Head on $b = 0.3$ fm

Smooth

Subscript Legend

L: Correction to formula

D: Correction to gluon dist.



A case for intuition

P+P

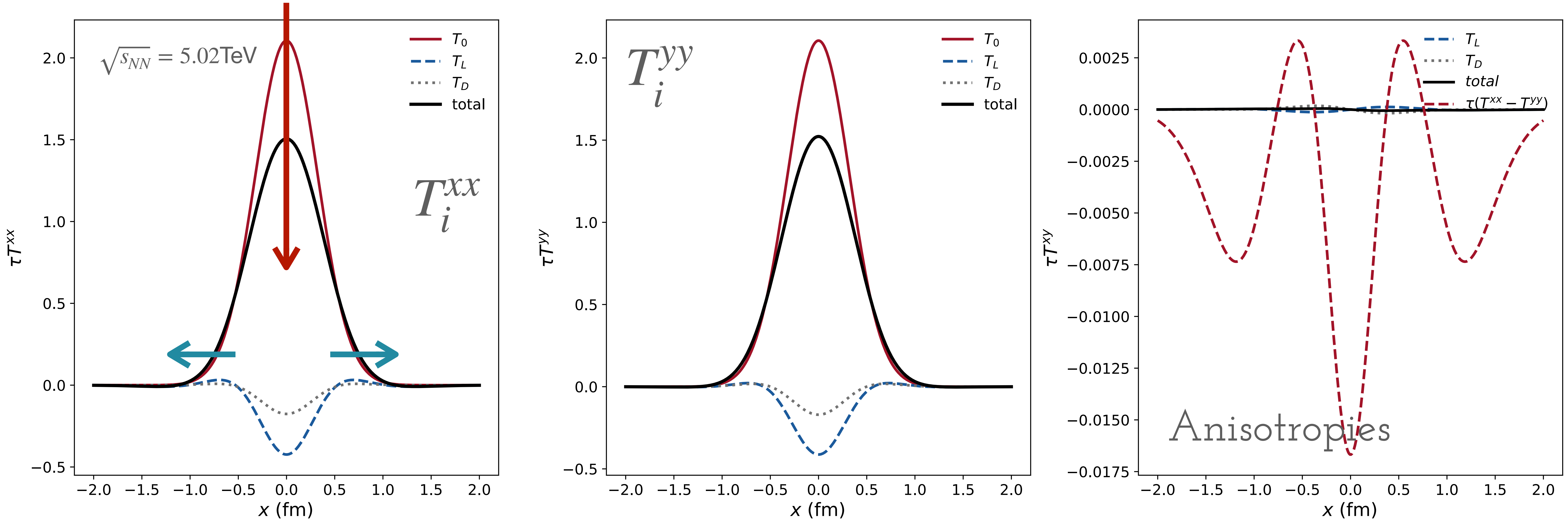
Gaussian

Head on $b = 0.3$ fm

Smooth

$\sqrt{s_{NN}} = 5.02$TeV

TRANSVERSE PRESSURES



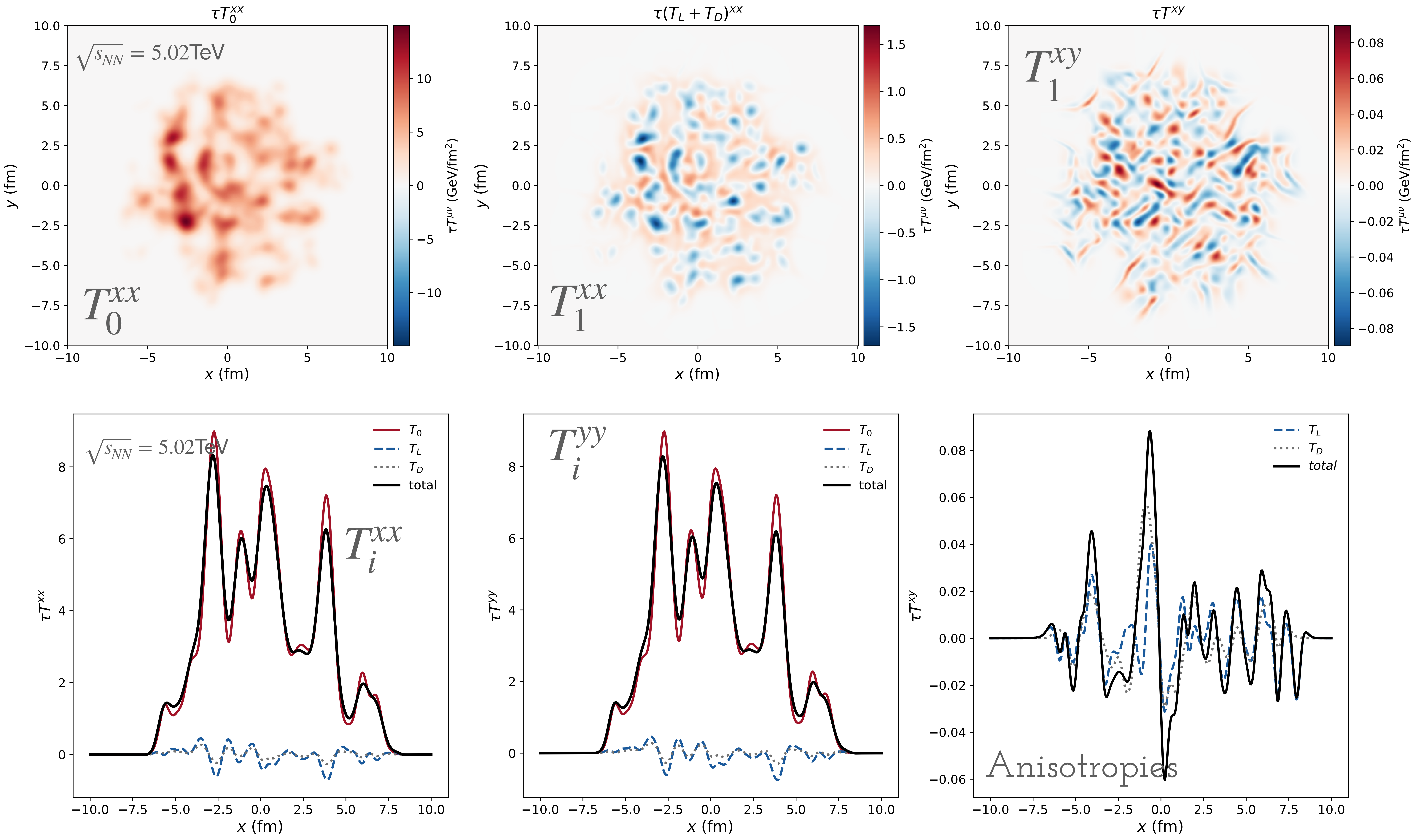
A case for intuition

PB+PB

Single event

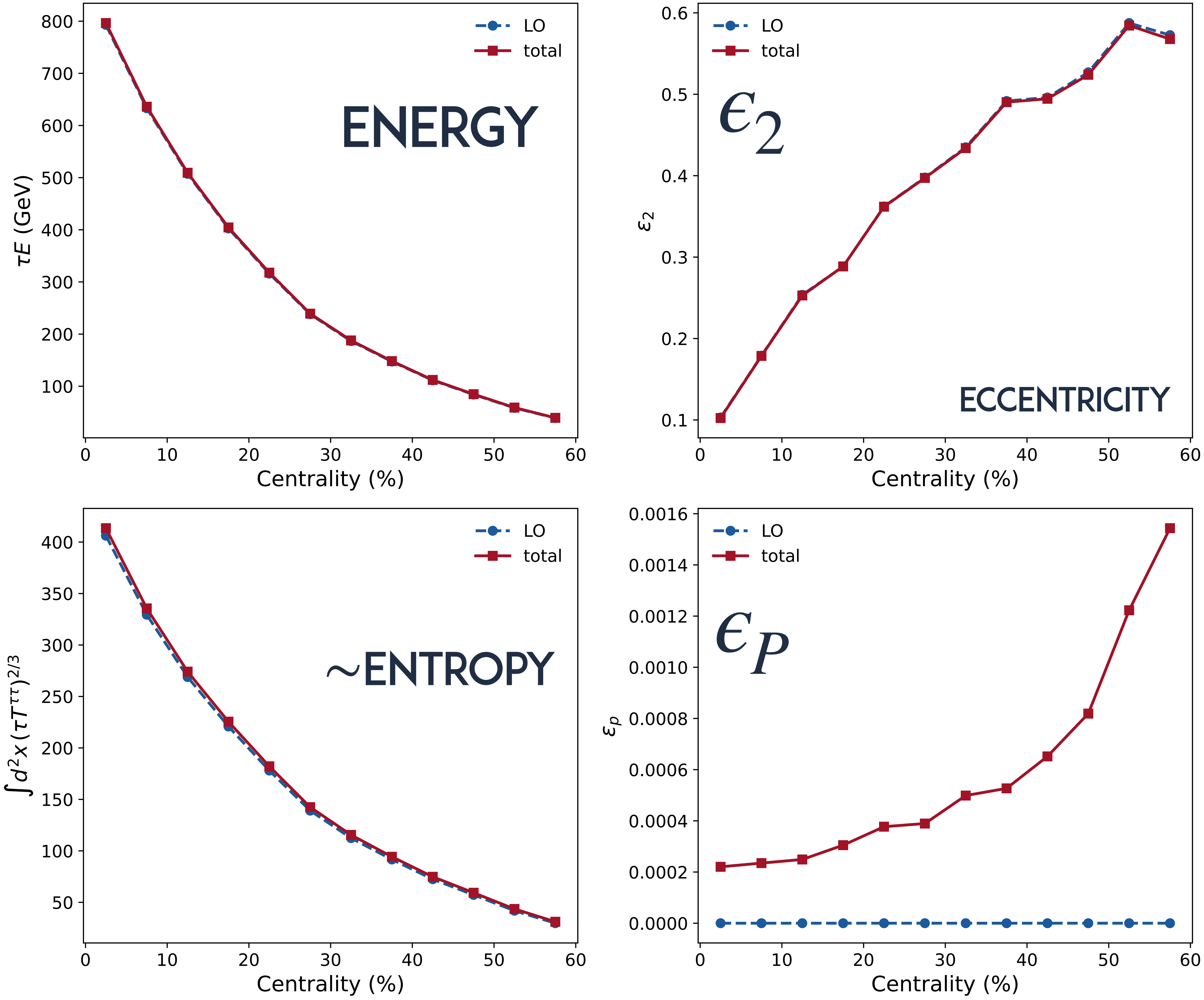
Head $b \approx 0\,\text{fm}$

TRANSVERSE PRESSURES



PB+PB

Integrated effect is negligible for simple initial state "observables"



PB+PB

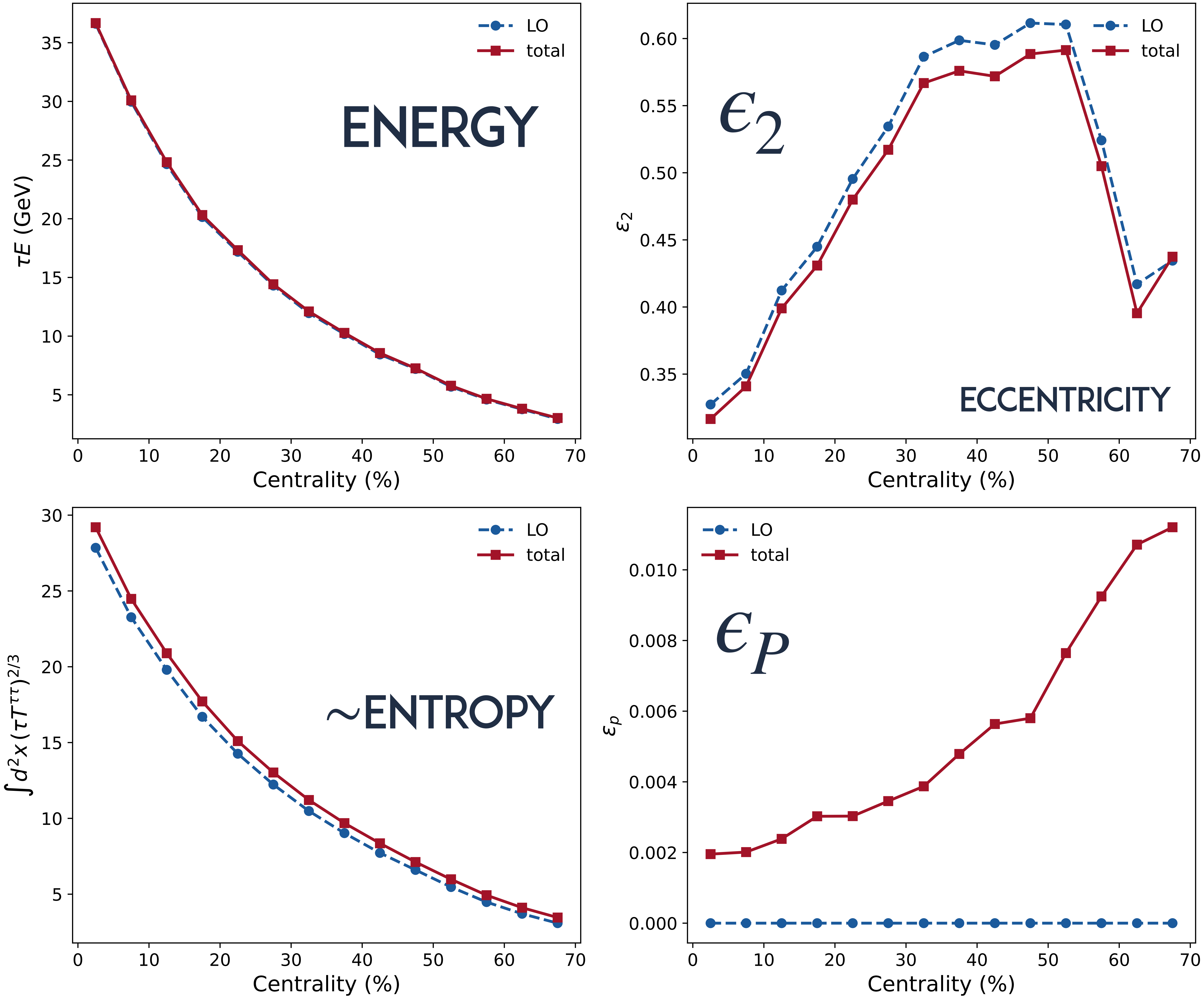
Integrated effect is negligible for simple initial state "observables"

O+O

Integrated effect is negligible for energy density but affects eccentricities and entropy, affecting multiplicity!

WHY TO CARE?

Seemingly a small effect globally, but locally is important!



SUMMARY AND CONCLUSIONS

Gradient expansion in the uGDFs and in the gluon production amplitude yields a hierarchy of a **new scalar and tensor structures**, built from local dipole amplitudes and their transverse derivatives,

Worked an **extended $k_\perp$-factorized ansatz** at first non-trivial gradient order: keeps the compact factorized form while presenting new positional dependence.

The effect should be most pronounced in fluctuation-dominated systems — O+O, Ne+Ne, as the gradients $|\nabla_\perp Q_s^2|$ are more important.

Local fluctuations and anisotropies carried through the evolution as correlations, can we probe them with jets?

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If you are interested in the initial state,

Week of Sept. 20th 2027

ECT* Trento, Italy

FONDAZIONE BRUNO KESSLER

ECT*

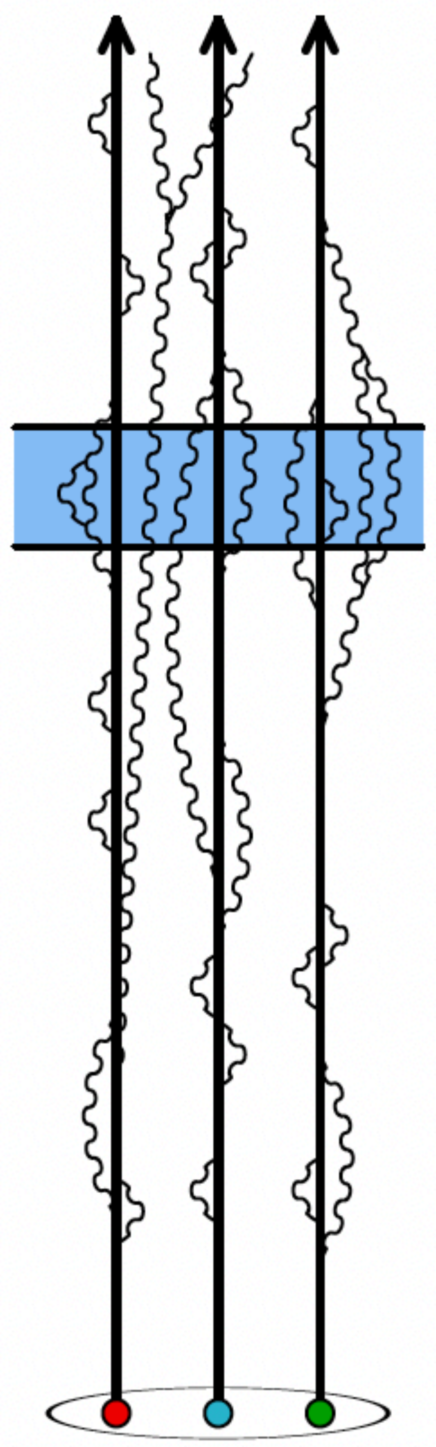
EUROPEAN CENTRE FOR THEORETICAL STUDIES IN NUCLEAR PHYSICS AND RELATED AREAS

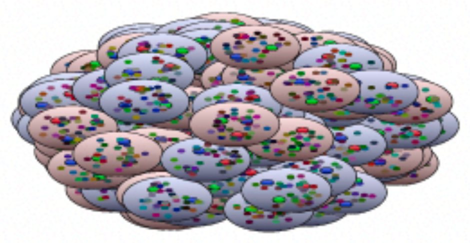
Organizers:
Oscar Garcia Montero
Pablo Guerrero
Carlota Andrés
Jacky Norohnha-Hostler
Dana Avramescu

BACK-UP

MO' QUESTIONS*

THE COLOR GLASS CONDENSATE





$1/x$

x_1

Partons

x_0

Hard Partons $J^-(x) \sim \rho(x^+, \boldsymbol{x}_\perp)$

Static color sources sampled
from distribution $W[x_0; \rho]$

Soft Partons
Macroscopic Field
$A(x)$

CYM Eqs
$[D_\mu, F^{\mu\nu}] = J^\nu$

WHAT IF WE WANT TO CHANGE x_0?

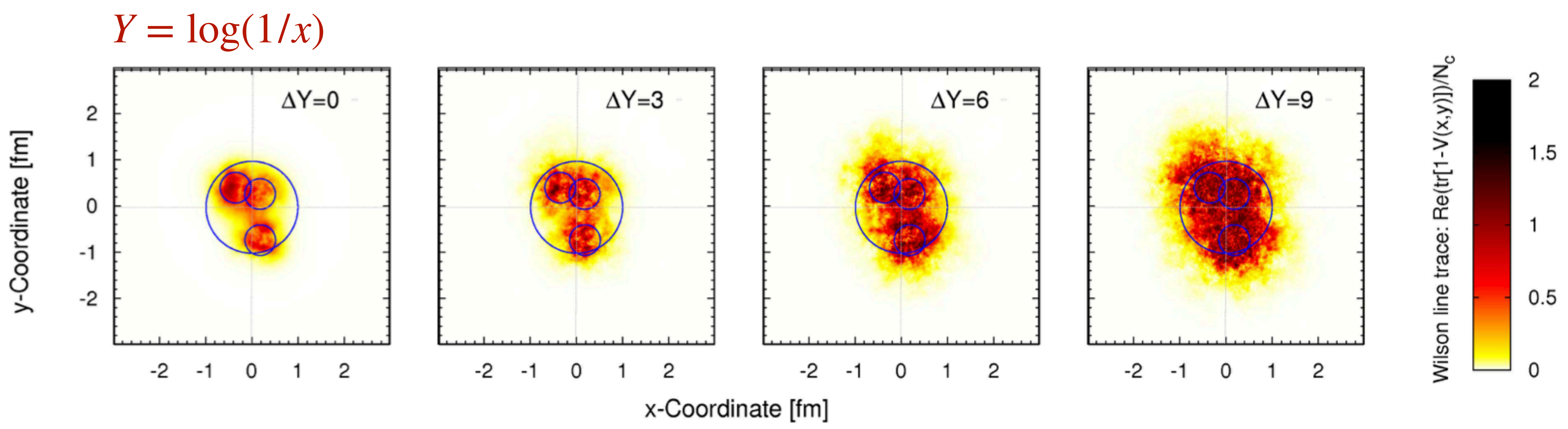
Cutoff change
changes the effective action

$W[x_0; \rho] \to W[x_1; \rho]$

RG-FLOW IN THE CGC!
RG Eqs: JIMWLK

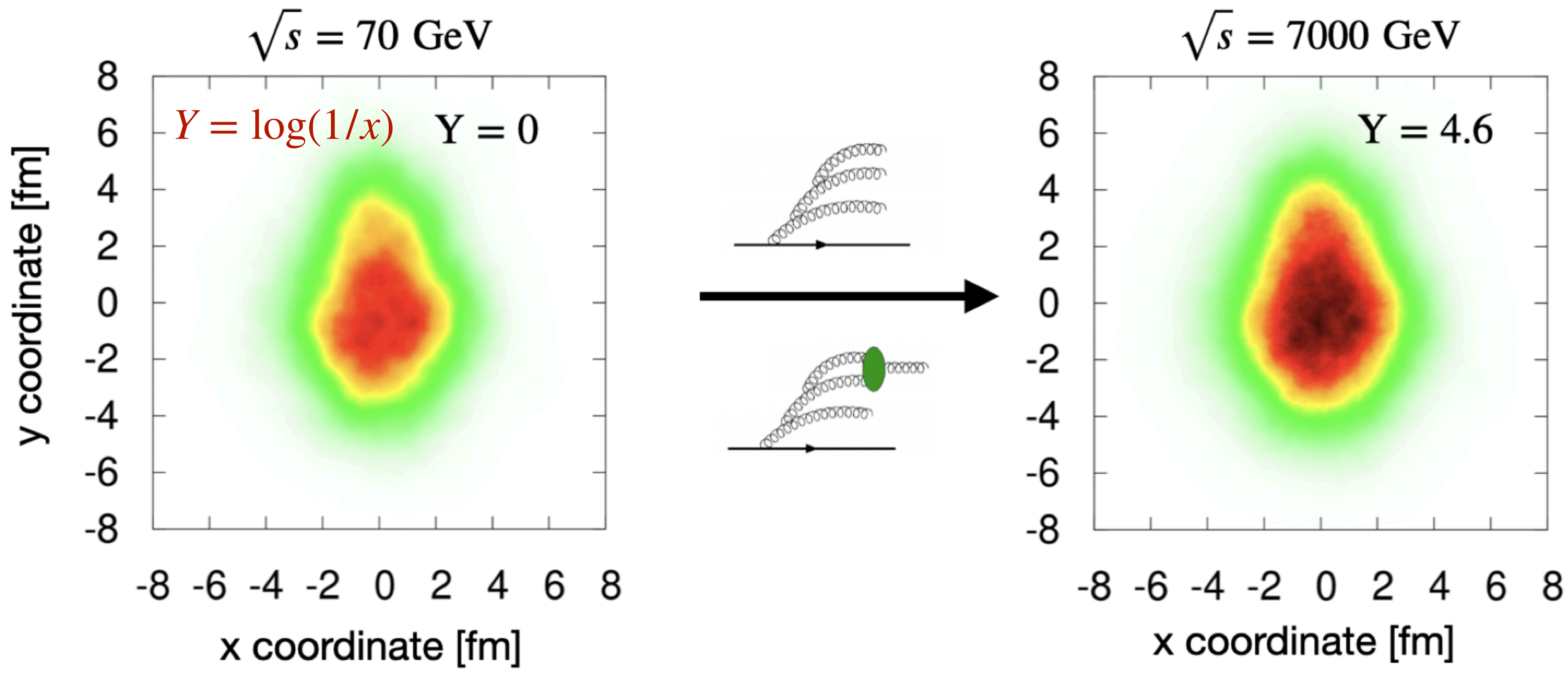
$$\frac{dW_x[\rho]}{d\log(1/x)} = \hat{H}_{eff} W_x[\rho]$$

THE EFFECT OF EVOLUTION



PROTON

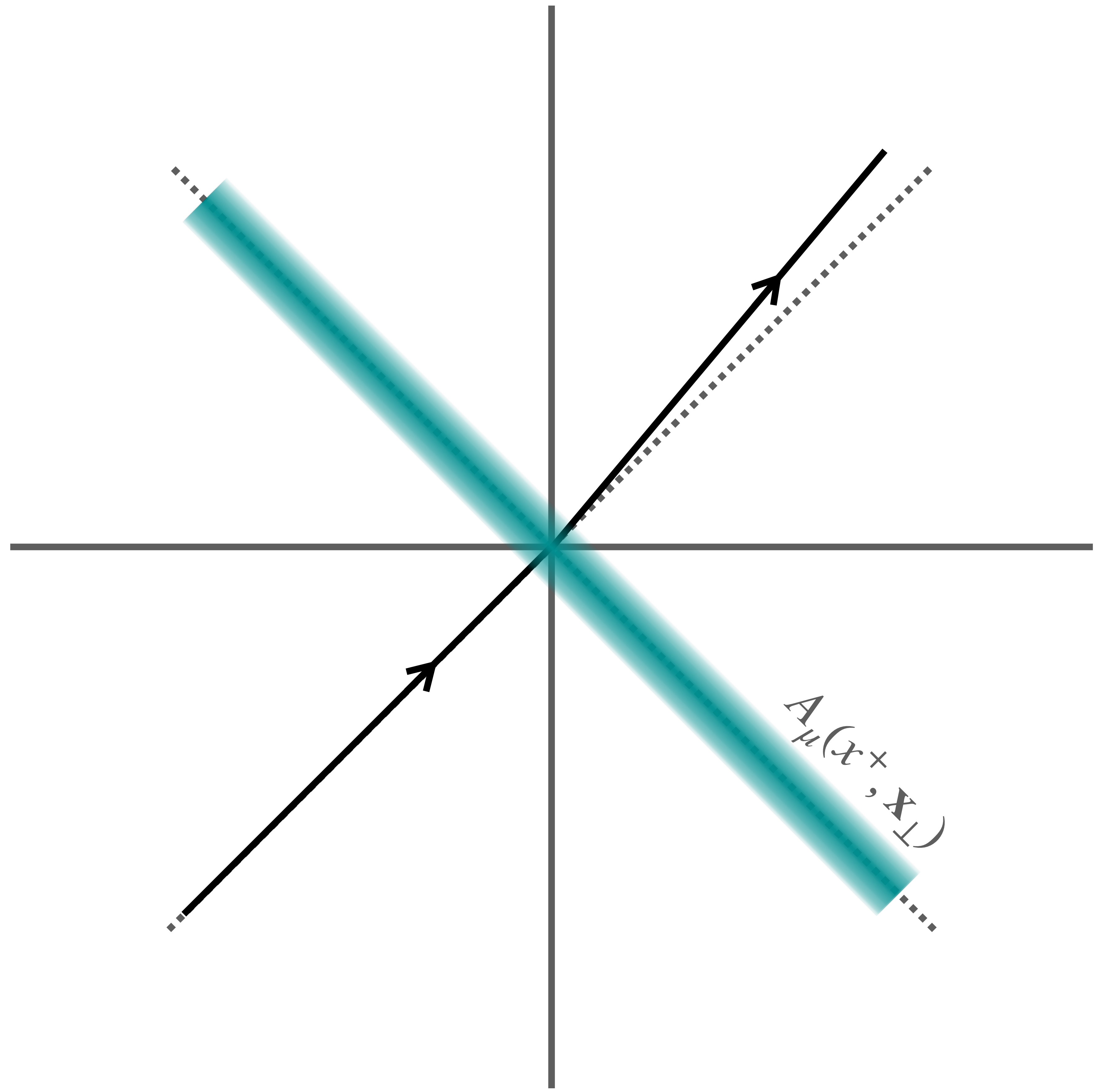
NEON

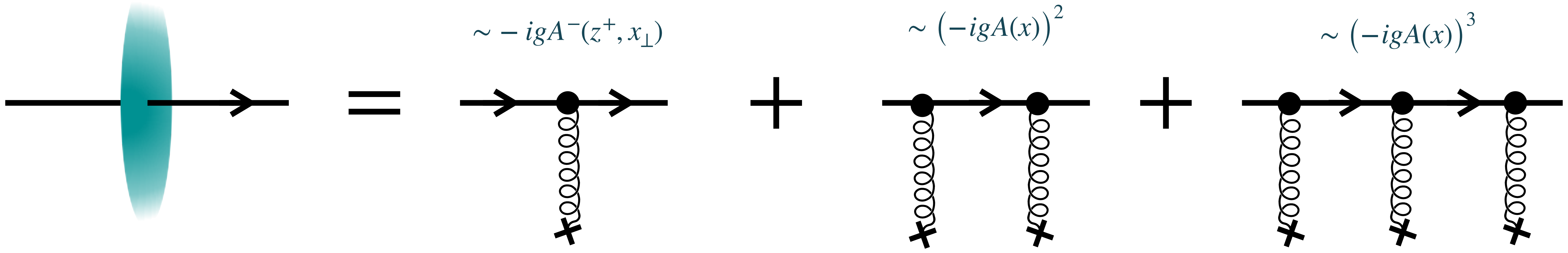


[From *EPJ Web Conf.* 296 (2024) 10005]

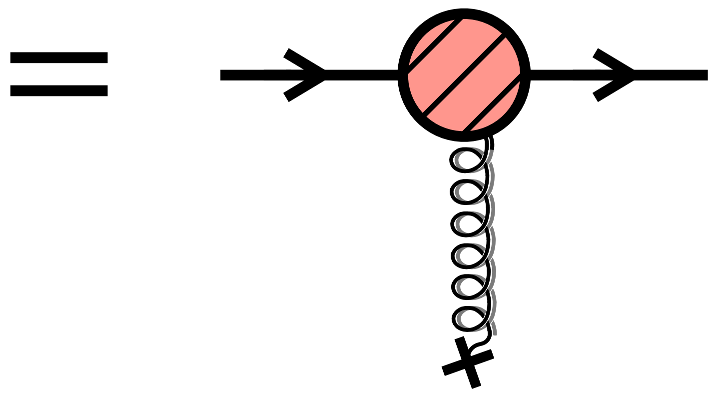
THE COLOR GLASS CONDENSATE

Simplest setting: Single charge meeting a wall of gluons





+ symmetrization



"Reggeized gluon"

$$V_{x_\perp} = e^{-ig\int dz^+ A^-(z^+, x_\perp)}$$

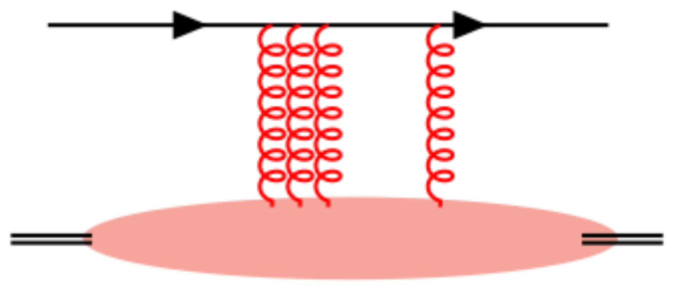
a.k.a. Wilson Line

All possible "kicks" by the gluons within the target

Test particle's transverse momentum (initially 0) is broadened dynamically (with mean around $Q_s(x)$!)

TAKE HOME MESSAGE*

CGC → Strong classical fields:
Multiple scattering, broadening



→ A theory of Wilson lines $V_{x_\perp} = e^{-ig\int dz^+ A^-(z^+, x_\perp)}$, re-sums $\alpha_s \log 1/x$ terms

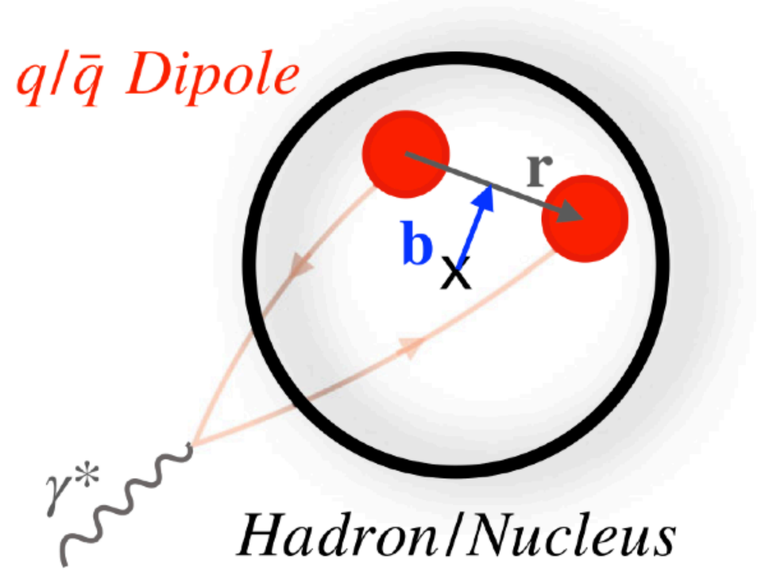
→ Observables are a combination of hard-factors and colour averaged source functionals

Dynamics of obs.

$$\langle \hat{O} \rangle_x = \int_{\{k_i\}} \Theta(\{k_i\}) \otimes \langle F[\{\rho_i\}] \rangle_x$$

Parton distributions +
Low-x evolution

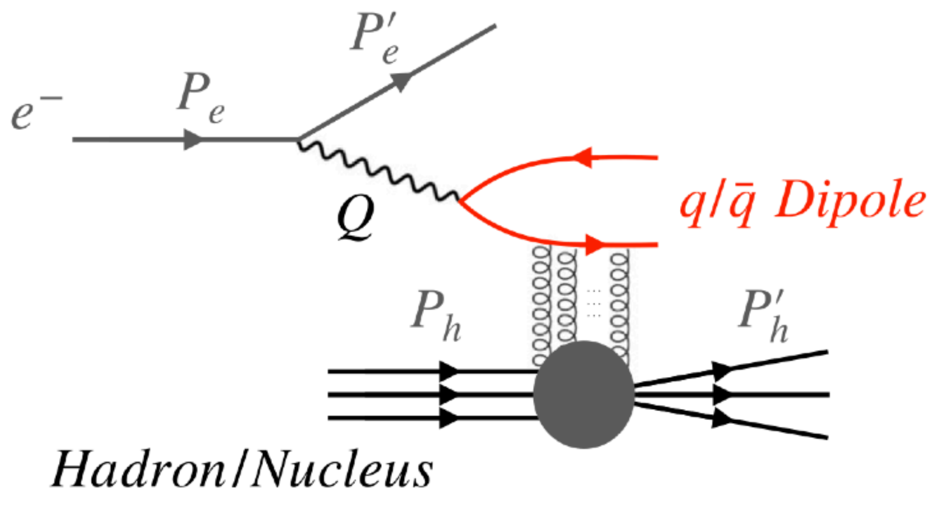
$$\langle F[\rho] \rangle_x = \int \mathscr{D}\rho \, W[x;\rho] F[\rho]$$



→ Classic example: dipole cross section at LO in α_S

$$\sigma_{L,T}^{\gamma^* p} = 2 \sum_f \int_{\mathbf{b}_\perp, \mathbf{r}_\perp} \int_z \left| \psi_{L,T}^{\gamma^* \to q\bar{q}}(r, z, Q^2) \right|^2 D(x, b, r)$$

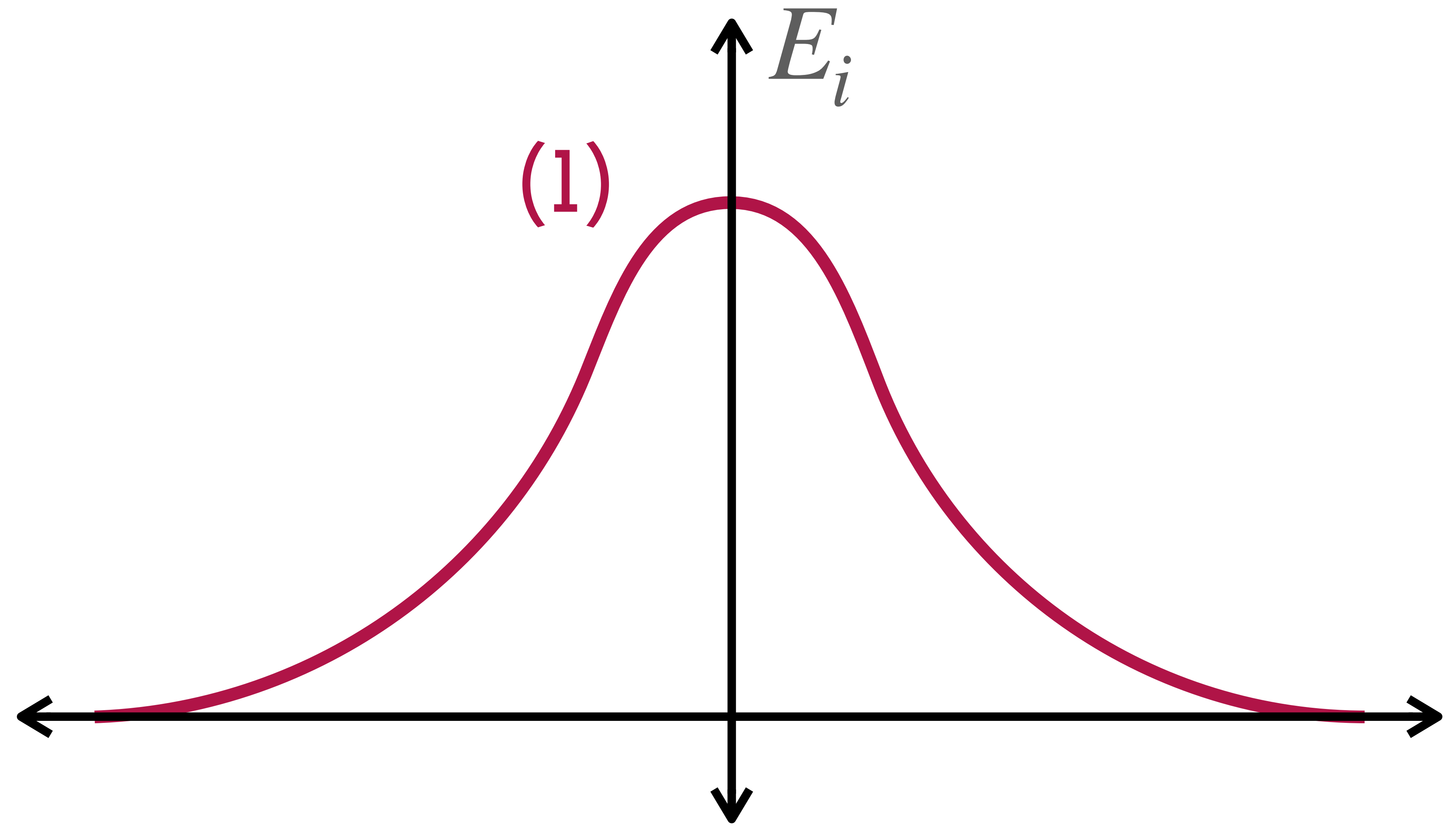
where $D(x,b,r) = \frac{1}{N_c} \mathrm{Tr_c} \left\langle 1 - V\left(r + \frac{b}{2}\right) V^\dagger \left(r - \frac{b}{2}\right) \right\rangle_x$



THE INITIAL STATE OF A HIC … IN 3D.

What do we expect to have?

- Nature of DoFs depends model-by-model
- (1) Fireball energy deposition: C.o.M of collision favours midrapidity.
  - High density of gluons, string breaking, etc.



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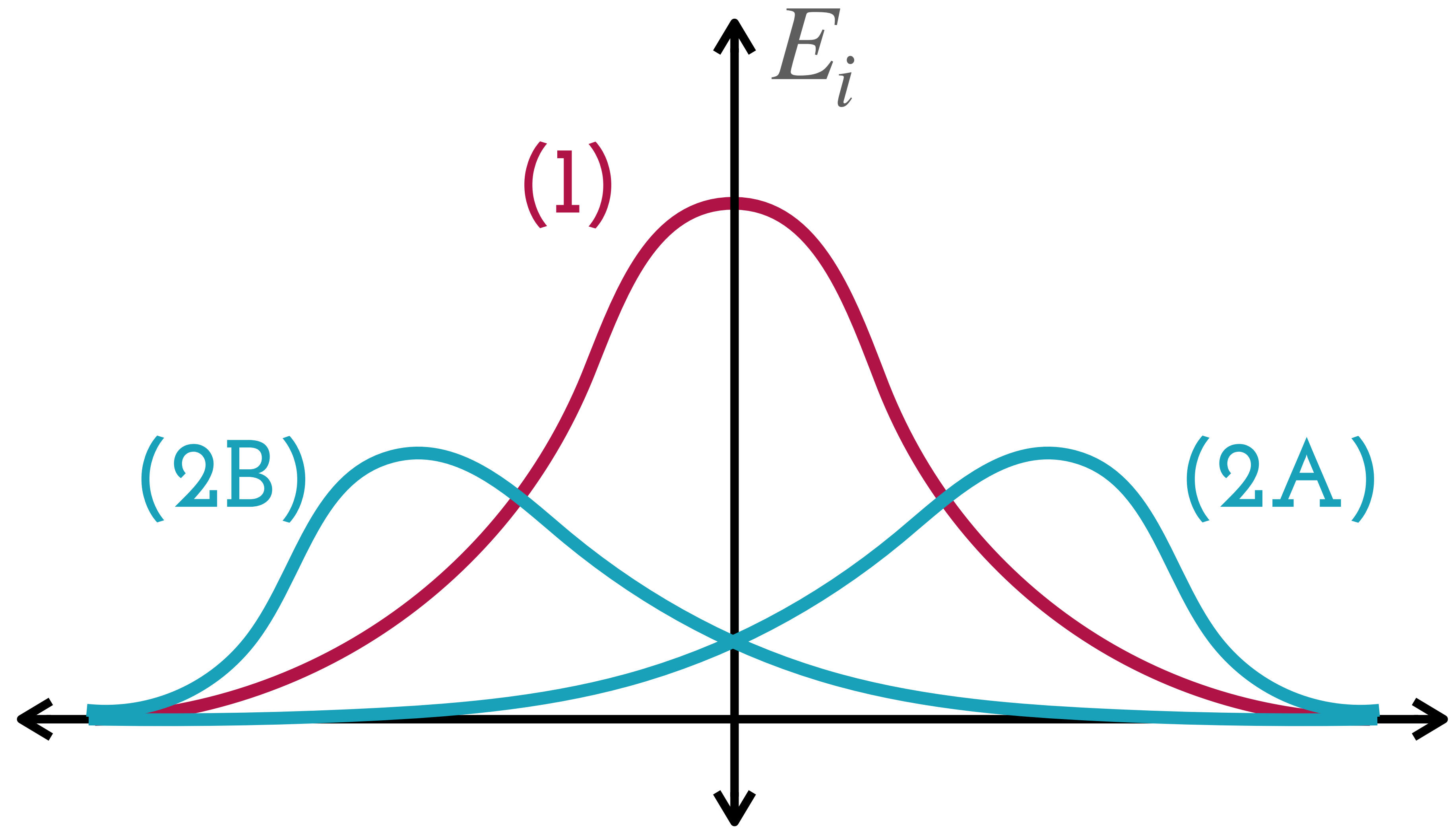
  - Quark scattering, baryon junction, hadrons?



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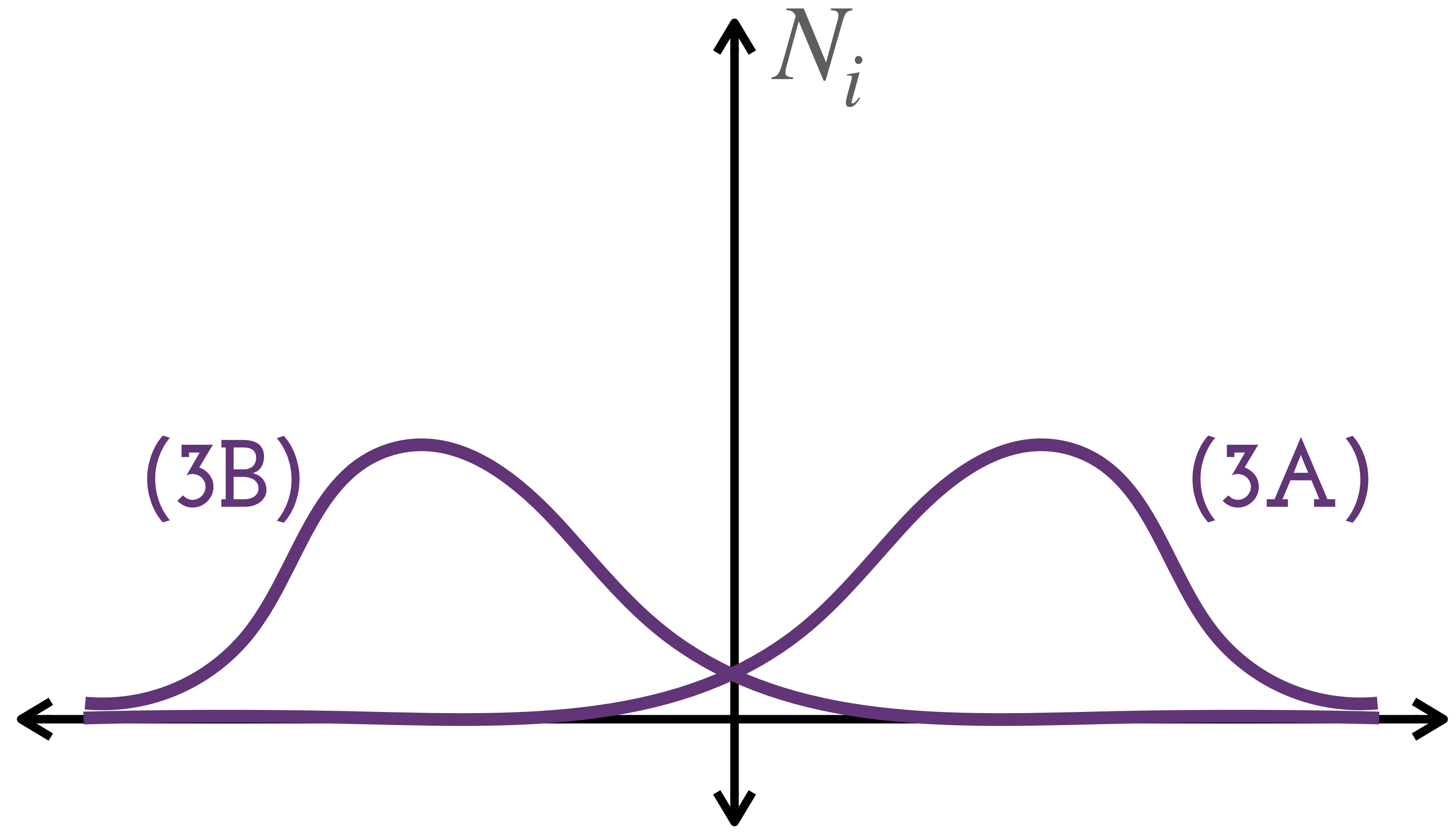
- (2) Fragmentation region charge deposition: Q,B and S (in fluctuations)



METHODS: STATE OF THE ART

DoFs/motivation behind the energy and charge deposition

LARGE-X

Collinear fact.
Described by PDFs

GEOMETRICAL

Effective description
Often parametrical

LOW-X

Overoccupied
Color fields

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CGC

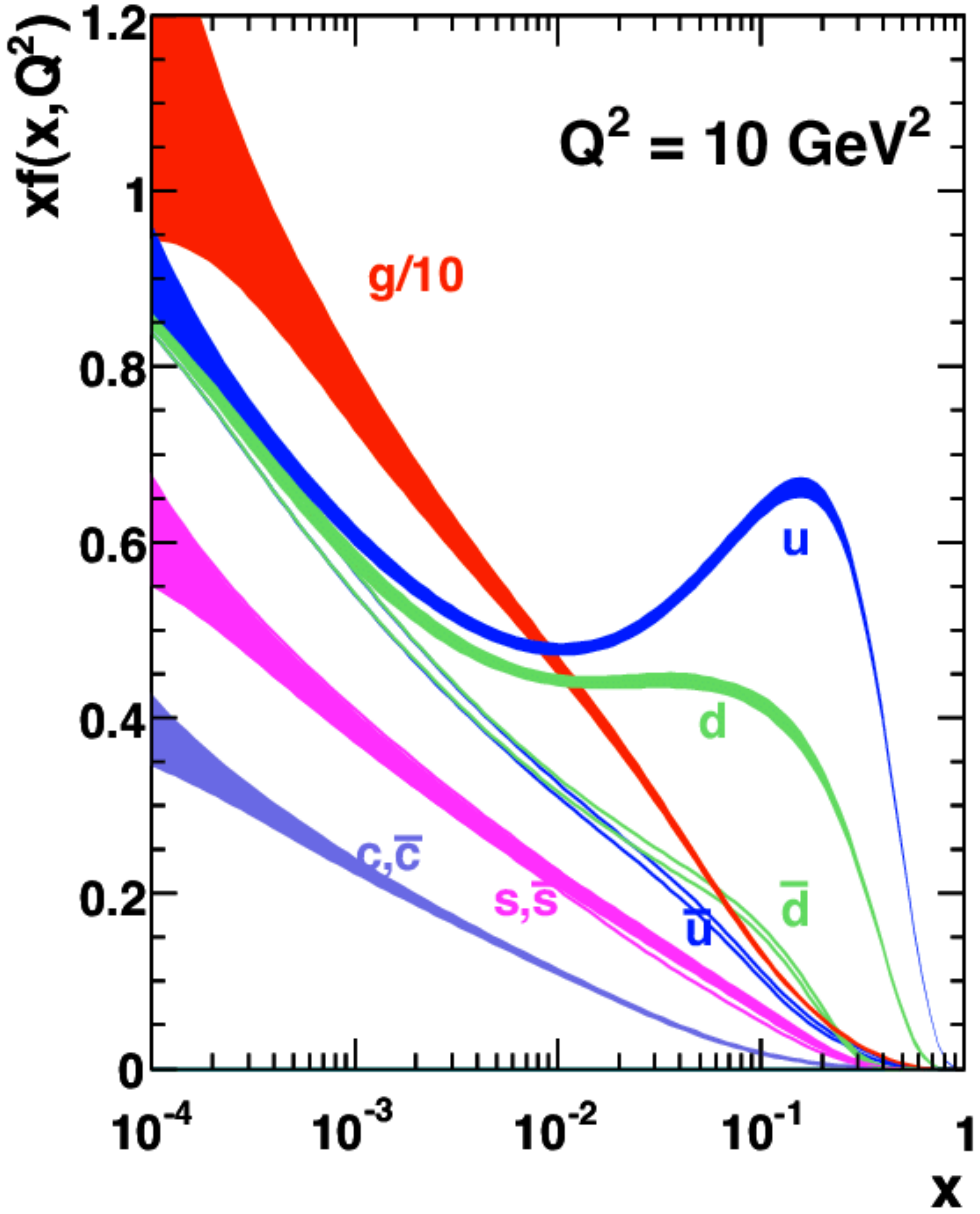
SATURATION*

I BEFORE COLLISION: SATURATION

- PDFs from fit to Experiments (DIS)

  x ~ energy/momentum fraction carried by parton

  Q^2 ~ resolution scale



① BEFORE COLLISION: SATURATION

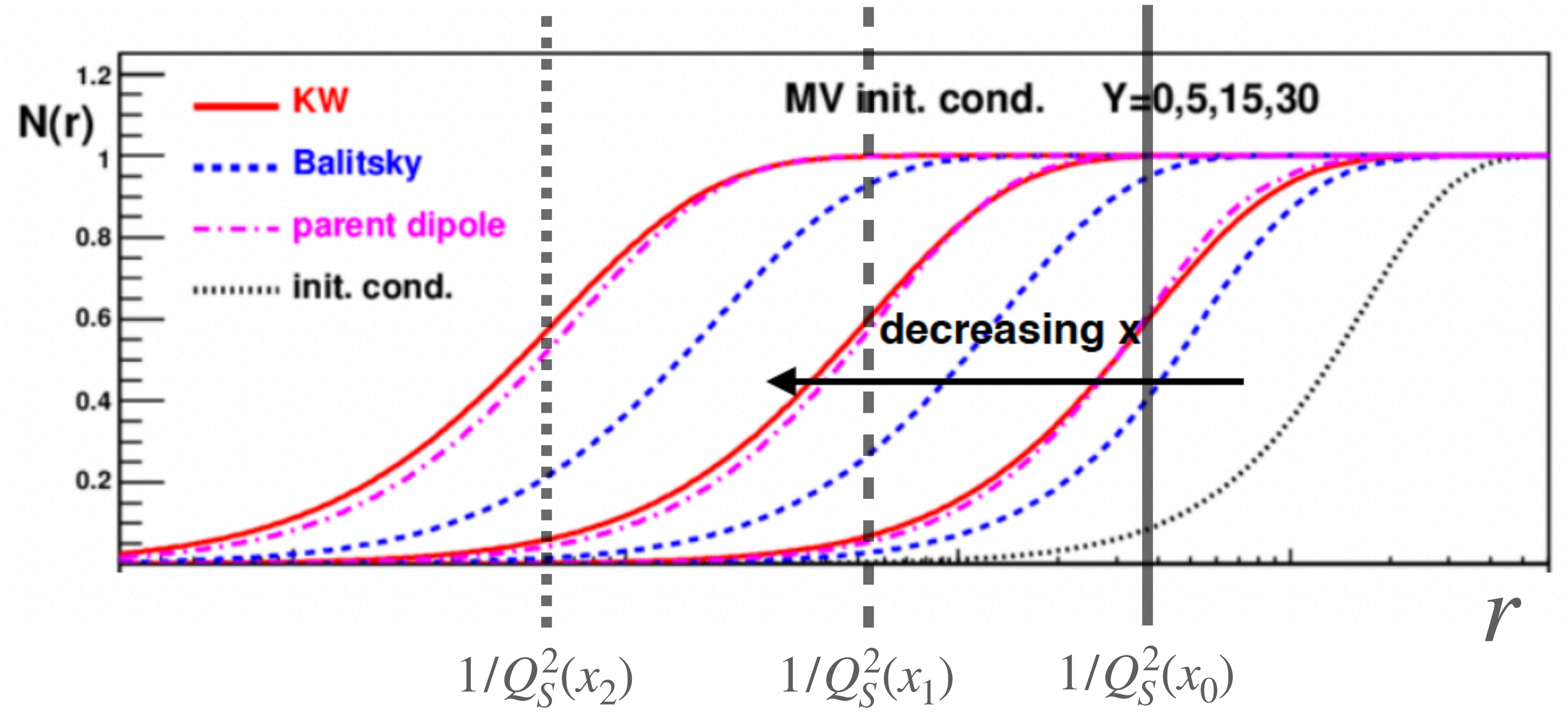
- PDFs from fit to Experiments (DIS)

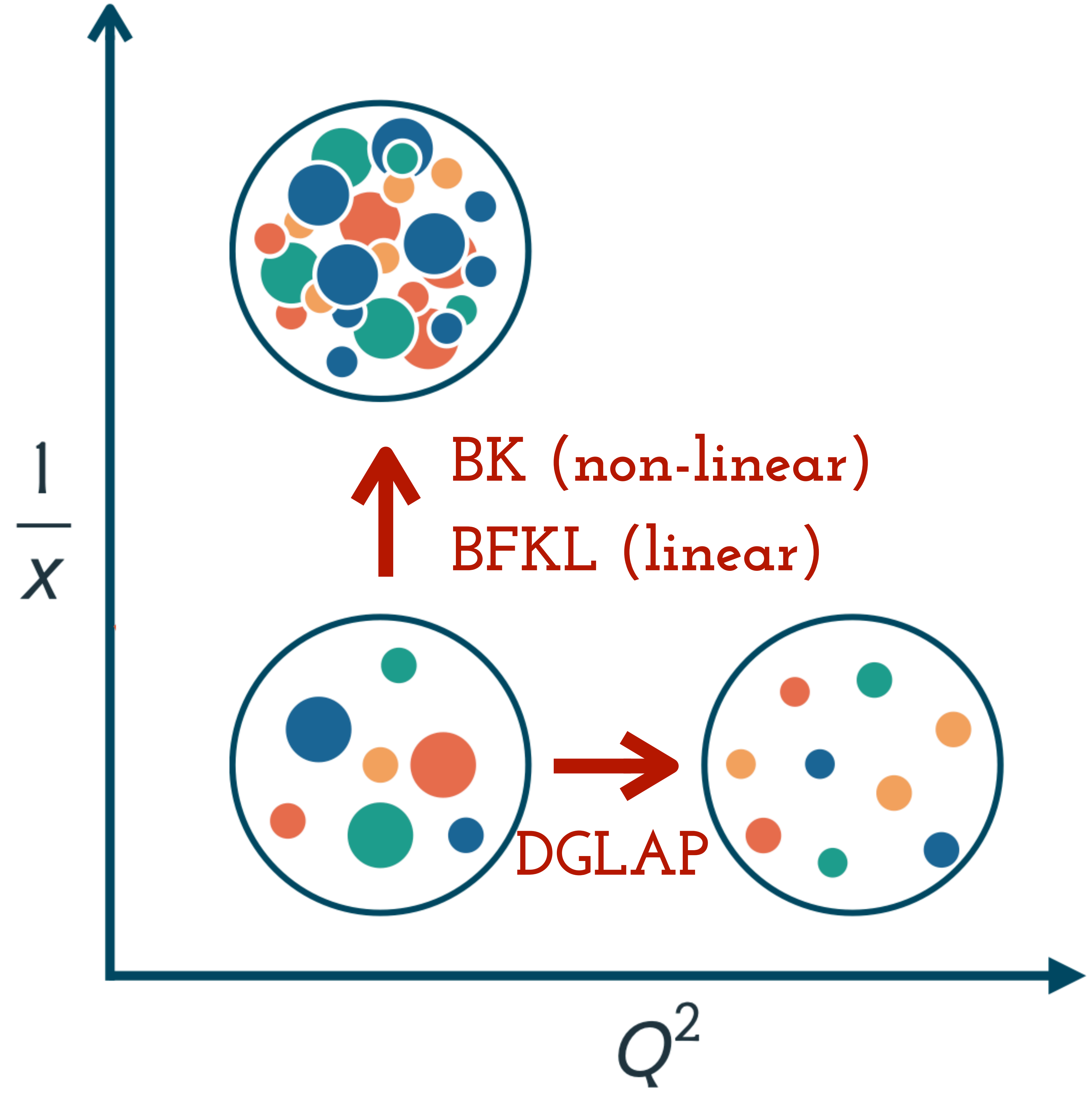
  $x \sim$ energy/momentum fraction carried by parton

  $Q^2 \sim$ resolution scale

- Non-linearities in BK: Emergence of a ***semi-hard*** saturation scale Q_s

- Gluon distributions saturate with $k_\perp < Q_S$ ($r > Q_S^{-1}$ in pos. space)





$$Q_S^2(x) \sim x^{-\lambda} A^{1/3} \sim s^{\lambda/2} A^{1/3}$$

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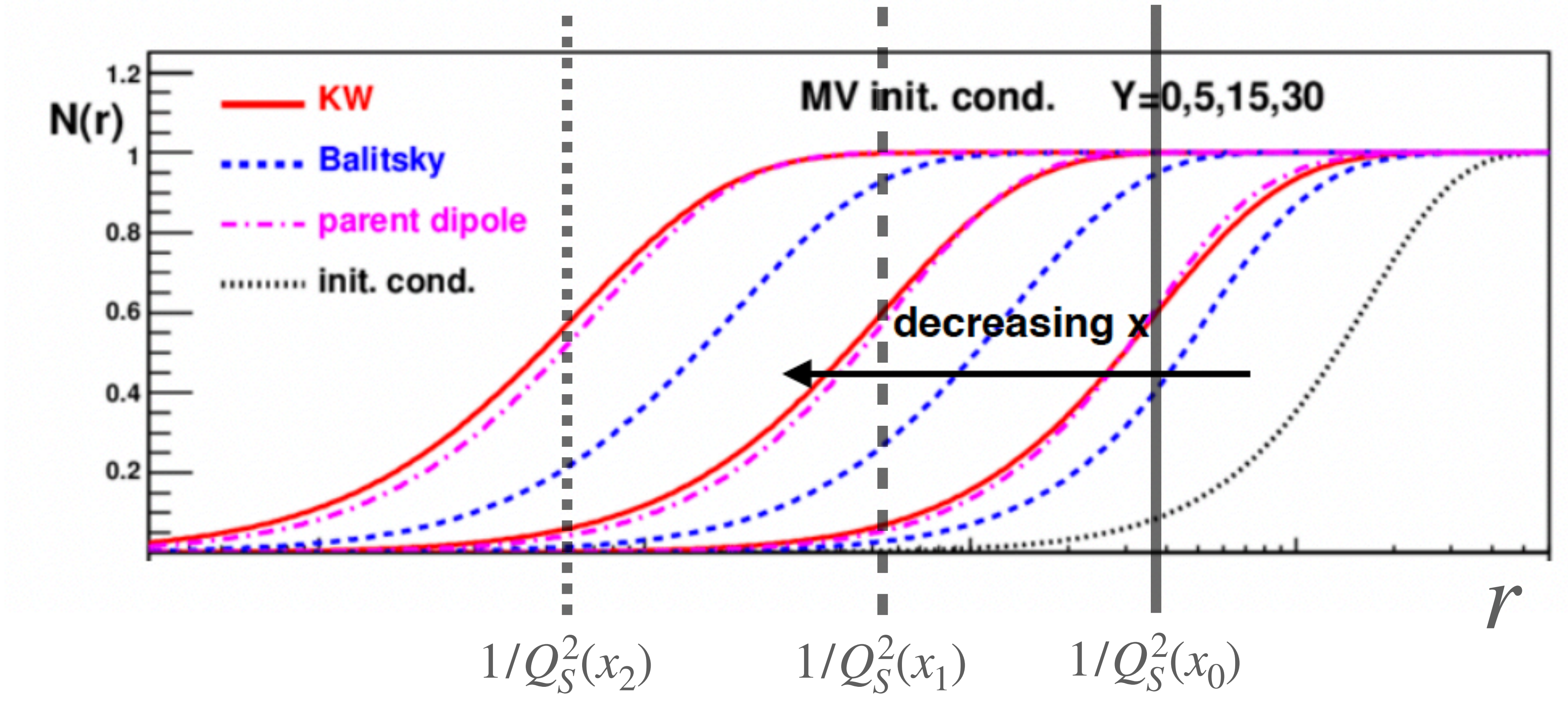

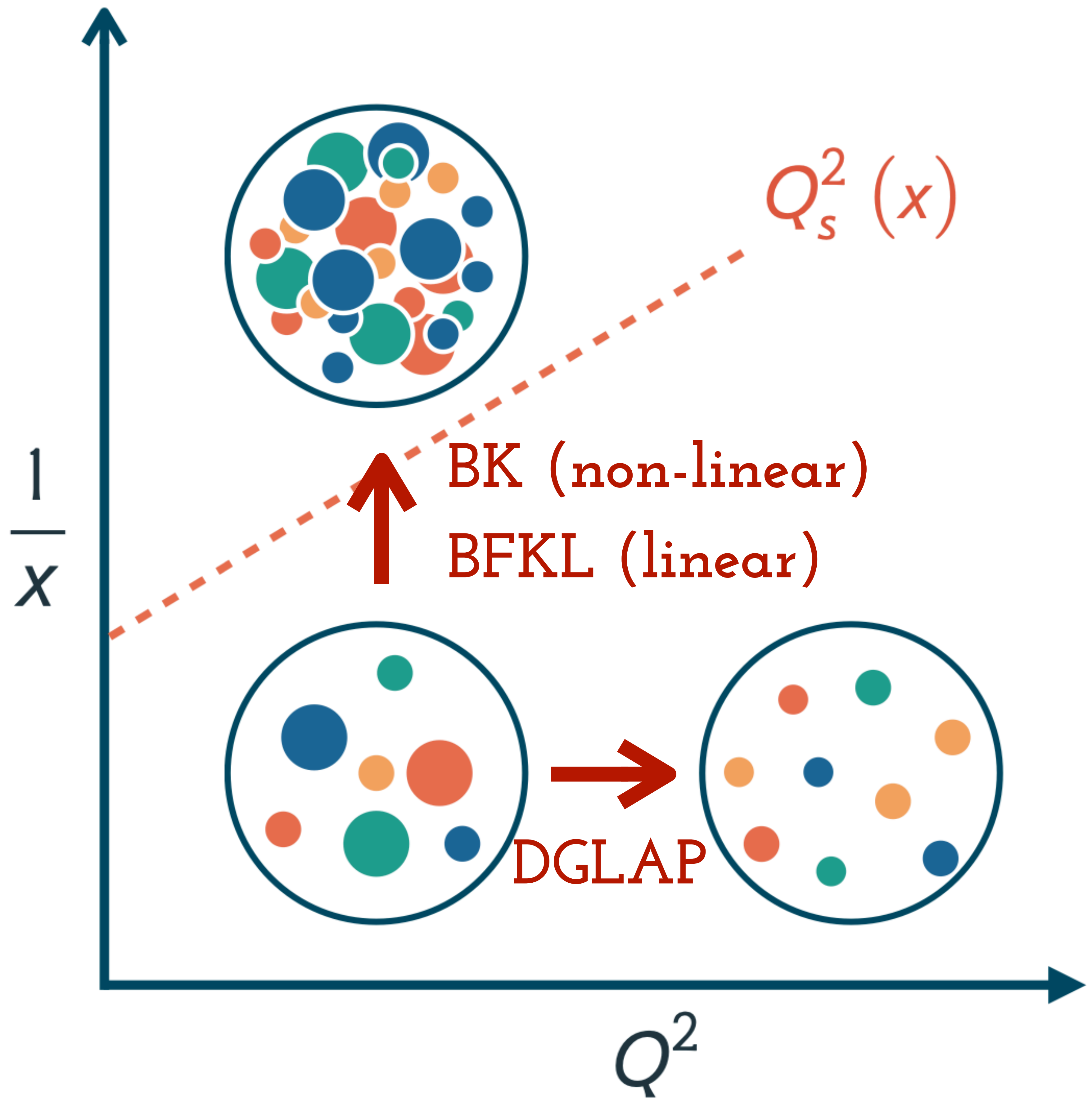

$$Q_S^2(x) \sim x^{-\lambda} A^{1/3} \sim s^{\lambda/2} A^{1/3}$$

$$H^{(1)} = -\frac{2Q_1^4 - 11\,Q_1^2Q_2^2 + 2Q_2^4}{4\,S^{7/2}}$$

$$H_{01}^{2,\mathrm{iso}} = \frac{Q_1^2\left(2Q_1^4 + 7Q_1^2Q_2^2 + Q_2^4\right)}{2\,S^{5/2}},$$

$$H_{02}^{2,\mathrm{iso}} = \frac{Q_1^2\left(4Q_1^4 - 17Q_1^2Q_2^2 - Q_2^4\right)}{4\,S^{7/2}}$$

$$H_{10}^{3,\mathrm{iso}} = \frac{Q_2^2\left(Q_1^4 + 7Q_1^2Q_2^2 + 2Q_2^4\right)}{2\,S^{5/2}},$$

$$H_{20}^{3,\mathrm{iso}} = \frac{Q_2^2\left(-Q_1^4 - 17Q_1^2Q_2^2 + 4Q_2^4\right)}{4\,S^{7/2}}$$

$$H_{01}^{2,\mathrm{a}} = \frac{Q_1^2Q_2^2\left(Q_2^2 - Q_1^2\right)}{2\,S^{5/2}},$$

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$$H_{10}^{3,\mathrm{a}} = \frac{Q_1^2Q_2^2\left(Q_1^2 - Q_2^2\right)}{2\,S^{5/2}}$$

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