Azimuthal momentum isotropization in the early stages

Sergio Barrera Cabodevila

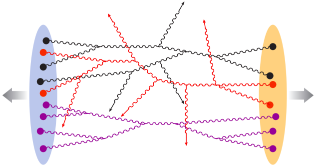
in collaboration with

Xiaojian Du, Carlos A. Salgado and Bin Wu

based on

(July 2026). SBC, Du, Salgado, and Wu [arXiv:2607.07796]

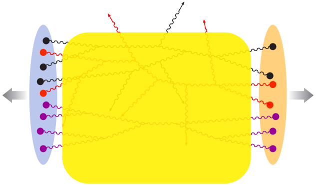
CGC



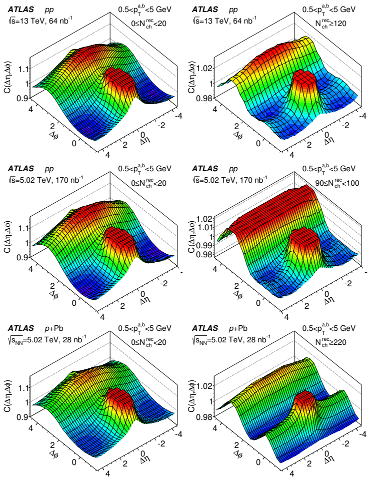
Ann. Rev. Nucl. Part. Sci. 60 (2010). Gelis, Iancu, Jalilian-Marian, and Venugopalan

Kinetic theory →

Hydro (QGP)



0.1 fm/c — 1 fm/c — 10 fm/c

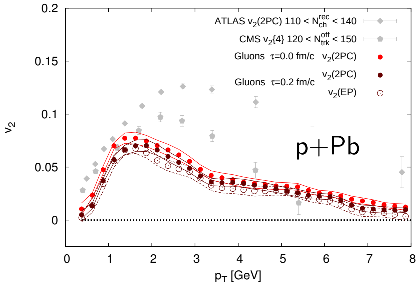


- A similar ridge to that in heavy-ion collision has been observed in small systems

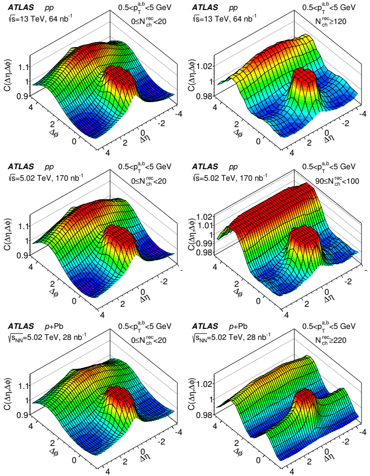
Phys. Rev. C 96.2 (2017). Aaboud et al.

- Hydrodynamics might not be the cause
- Initial state mechanisms have been proposed to explain it

Eur. Phys. J. A 56.8 (2020). Altinoluk and Armesto



Phys. Lett. B 747 (2015). Schenke, Schlichting, and Venugopalan



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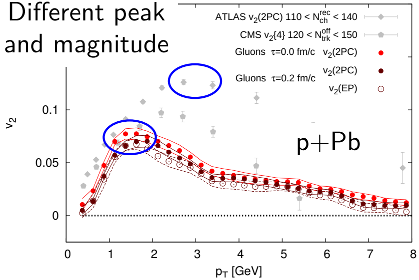
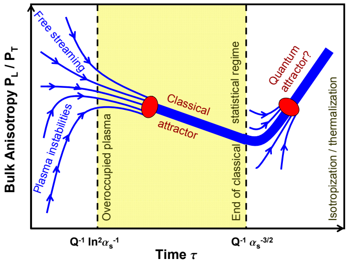
Different peak and magnitude



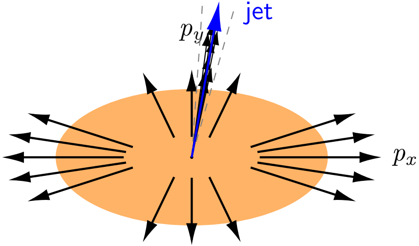
Phys. Lett. B 747 (2015). Schenke, Schlichting, and Venugopalan

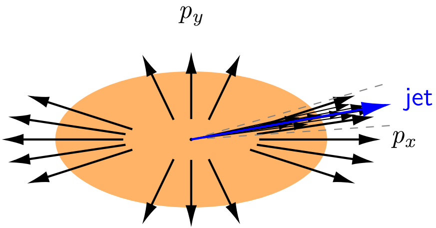
What happens to initial state anisotropies?

- We expect initial anisotropies to be washed out at hydrodynamization time in HIC
- Some questions arise:
  - Do initial azimuthal anisotropies disappear before hydrodynamization in small systems?
  - How do final-state interactions relax these anisotropies?
  - Can a hard probe (such as a jet) be sensitive to these initial anisotropies?



Phys. Rev. D 89.7 (2014). Berges, Boguslavski, Schlichting, and Venugopalan





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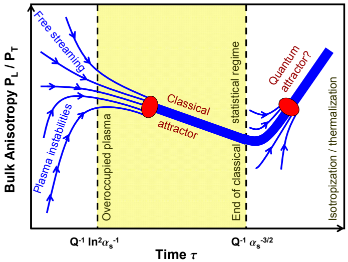

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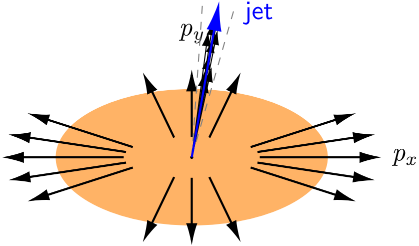
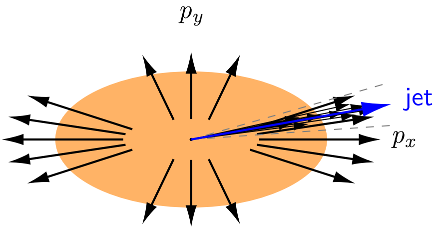
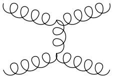
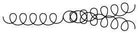




- The QCD Boltzmann equation at leading order

$$\Bigg(\partial_t + \underbrace{\frac{\mathbf{p}}{p}\cdot\nabla_{\mathbf{x}}}_{\text{Expansion}}\Bigg) f^a(\mathbf{x};\mathbf{p};t) = C^a_{2\leftrightarrow 2}[f] + C^a_{1\leftrightarrow 2}[f] \quad , \quad a = \{g, q, \bar{q}\}$$





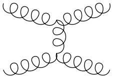
- Distribution function is related with particle spectra

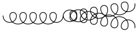
$$\frac{1}{\tau}\frac{d^3N}{d^2\mathbf{p}_T\,dy\,d\eta}\Bigg|_\tau \propto \int d^2\mathbf{x}_\perp f(\tau;\eta;\mathbf{p}_T, p_T\sinh(y-\eta))$$

QCD Kinetic theory I

- The QCD Boltzmann equation at leading order

$$\Bigg(\partial_t + \underbrace{\frac{\mathbf{p}}{p}\cdot\nabla_{\mathbf{x}}}_{\text{Expansion}}\Bigg) f^a(\mathbf{x};\mathbf{p};t) = C^a_{2\leftrightarrow 2}[f] + C^a_{1\leftrightarrow 2}[f] \quad , \quad a=\{g,q,\bar{q}\}$$





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$$\frac{1}{\tau}\frac{d^3N}{d^2\mathbf{p}_T\,dy\,d\eta}\bigg|_\tau \propto \int d^2\mathbf{x}_\perp f(\tau;\eta;\mathbf{p}_T,p_T\sinh(y-\eta))$$

- Longitudinal-boost invariant expansion + homogeneity in transverse plane

$$\left(\partial_\tau - \frac{p_z}{\tau}\partial_{p_z}\right) f^a = C^a_{2\leftrightarrow 2}[f] + C^a_{1\leftrightarrow 2}[f] \quad , \quad a=\{g,q,\bar{q}\}$$

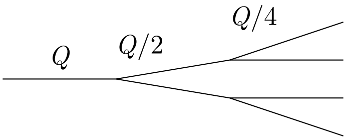
QCD Kinetic Theory II. Bottom-up thermalization

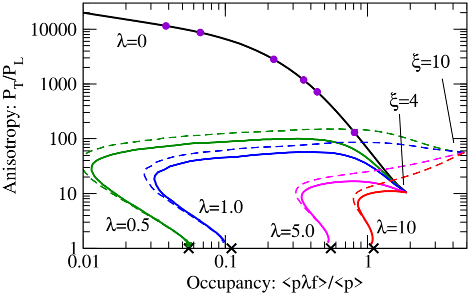
- In the weak-coupling limit → Bottom-up thermalization

Phys. Lett. B 502 (2001). Baier, Mueller, Schiff, and Son

- There are three different stages for thermalization when considering boost-invariant expansion

1. System dominated by expansion
2. Collision and expansion contribute similarly
3. Democratic cascade speeds up the thermalization





Phys. Rev. Lett. 115.18 (2015). Kurkela and Zhu

- Effective Kinetic Theory (EKT) has been widely used in this context

JHEP 01 (2003). Arnold, Moore, and Yaffe

Phys. Rev. Lett. 115.18 (2015). Kurkela and Zhu

Phys. Rev. Lett. 127.12 (2021). Du and Schlichting

- Boltzmann Equation in Diffusion Approximation (BEDA) keeps most of the relevant physics.

JHEP 06 (2024). SBC, Salgado, and Wu

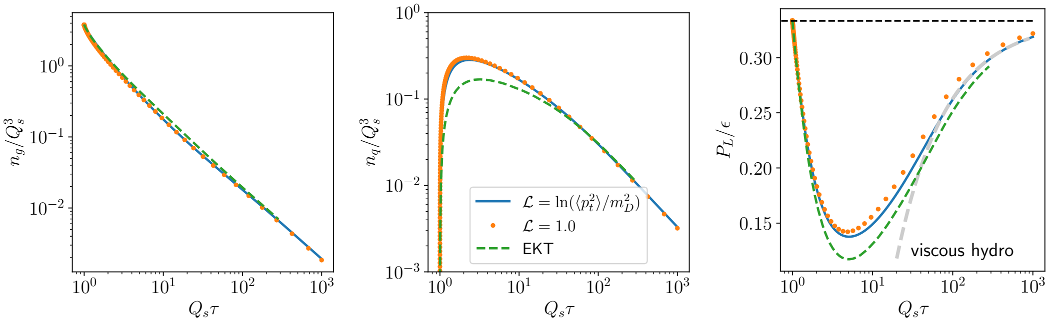
Phys. Lett. B 871 (2025). SBC, Du, Salgado, and Wu

	BEDA	EKT
$C_{2\leftrightarrow 2}$	Only small angle scatterings $\Rightarrow$ Fokker-Planck + source term	Full $2 \leftrightarrow 2$ QCD dynamics @LO
$C_{1\leftrightarrow 2}$	Deep LPM regime	Both LPM and Bethe-Heitler

- BEDA qualitatively agrees with EKT results for the same setups

Phys. Lett. B 871 (2025). SBC, Du, Salgado, and Wu

- Quantitative differences explained by differences in the collision kernels



- Evolve from CGC-like initial conditions

Phys. Rev. Lett. 115.18 (2015). Kurkela and Zhu

Phys. Rev. D 99.5 (2019). Kurkela and Mazeliauskas

$$f_0 = \frac{A}{4\pi N_c \alpha_s} \frac{e^{-\frac{2}{3}\left((p_z\xi)^2 + p_\perp^2\right)/Q_0^2}}{\sqrt{\left((p_z\xi)^2 + p_\perp^2\right)/Q_0^2}}, \qquad F_0 = 0,$$

- $\xi \rightarrow$ Quantifies anisotropy (taken as 0 for this comparison)
- $Q_0, A \rightarrow$ Phenomenological parameters fitting CGC
- Initial time $\tau_0 = Q_s^{-1}$
- Add some initial anisotropies on top

$$f_0(\mathbf{p}) = f_0(\mathbf{p}_T)\left(1 + \sum_n v_n^0(\mathbf{p}_T)\cos(n\phi)\right)$$

$$\frac{d^6N}{d^3\mathbf{x}d^3\mathbf{p}} \sim f(\mathbf{x};\mathbf{p};t_f) \Rightarrow \frac{d^3N}{d^3\mathbf{p}} \sim \int d^3\mathbf{x} f(\mathbf{x};\mathbf{p};t_f) = f(\mathbf{p};t_f)$$

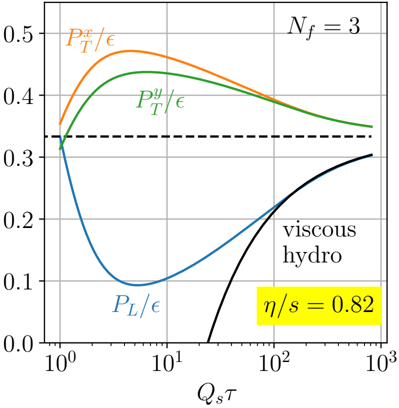
- Harmonic coefficients computed in the Event Plane

$$v_n(p_T;t) = \frac{\int dp_z d\phi \cos(n(\phi-\psi_n)) f(\mathbf{p};t)}{\int dp_z d\phi f(\mathbf{p};t)}$$

- Consider the anisotropy of the full system

$$f \equiv \nu_g f^g + 2N_f \nu_q f^q$$

- Azimuthal isotropization time is comparable to hydrodynamization time



- Start initializing with constant anisotropy and explore fully integrated v_n

$$v_n^0(\mathbf{p}_T) = v_n^0 \qquad v_n(t) \equiv \frac{\int d^3\mathbf{p}\cos(n(\phi-\psi_n))f(\mathbf{p};t)}{\int d^3\mathbf{p} f(\mathbf{p};t)}$$

- Exploring the variation due to the $C_{2\leftrightarrow 2}$ kernel

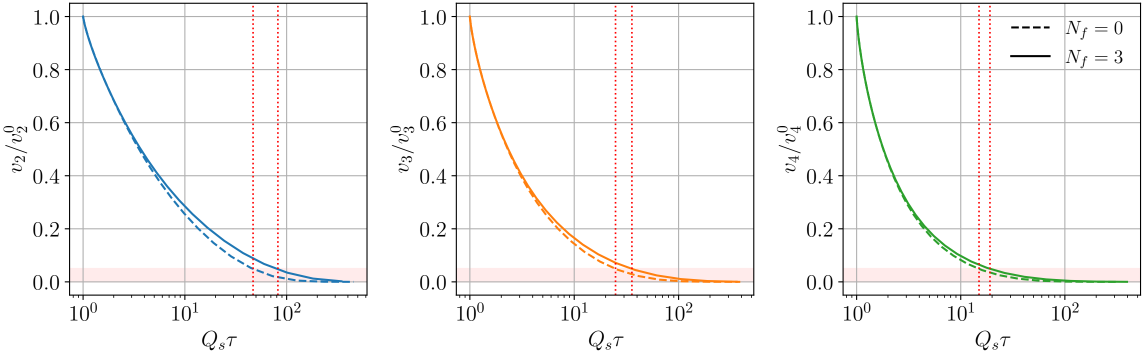
$$\partial_\tau v_n(\tau) = \frac{\hat{q}}{4n_g}\int \frac{dp\ d(\cos\theta)}{(2\pi)^2} p^2 \Bigg[-\frac{n^2 v_n^2 \tilde{f}/2}{p^2(1-\cos^2\theta)} \qquad \text{Higher } n \text{ relaxes faster}$$

$$+\frac{1}{T_*}\left(\frac{2\tilde{f}}{p}+\partial_p\tilde{f}\right) v_n^2$$

$$+\frac{1}{T_*}\left(\frac{2\tilde{f}^2}{p}+\partial_p\tilde{f}^2\right)\left(v_n^2+\sum_{k,l} v_k v_l \int \frac{d\phi}{2\pi}\cos(n\phi)\cos(k\phi)\cos(l\phi)\right)\Bigg] \qquad \text{Produces higher } n \text{ anisotropies}$$

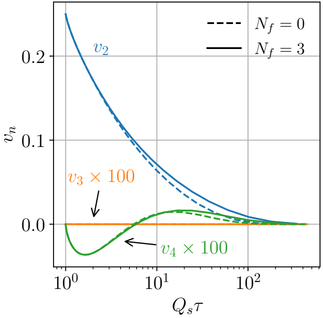
- For initial v_n, only v_{kn} with $k = 1, 2, 3\ldots$ are produced

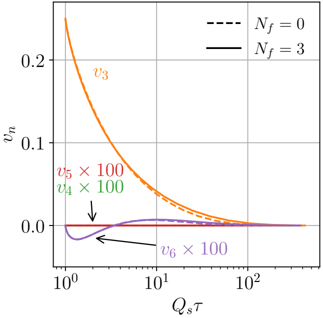
- Higher n harmonics isotropize faster
- Presence of quarks delays isotropization



Evolution of fully integrated coefficients II. Numerical results II

- For initial v_n, only v_{kn} with $k = 1, 2, 3\dots$ are produced
- They have a non-trivial evolution
- They are very small



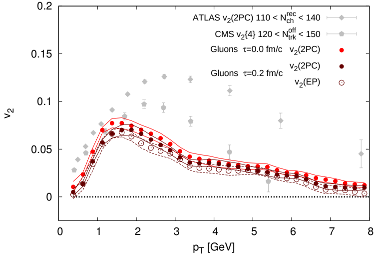


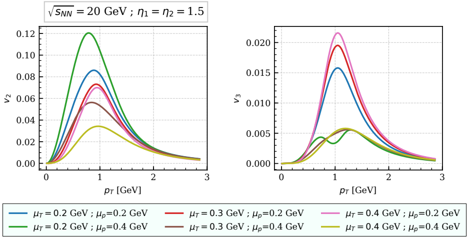
- Based on different results from initial state interactions we model

Eur. Phys. J. A 56.8 (2020). Altinoluk and Armesto

$$f_0(\mathbf{p}) = f_0(\mathbf{p}_T)\left(1 + \sum_n v_n^0(\mathbf{p}_T)\cos(n\phi)\right) \quad , \qquad v_n^0(p_T) = \tilde{v}_n \frac{p_T}{Q} e^{-p_T/Q}$$

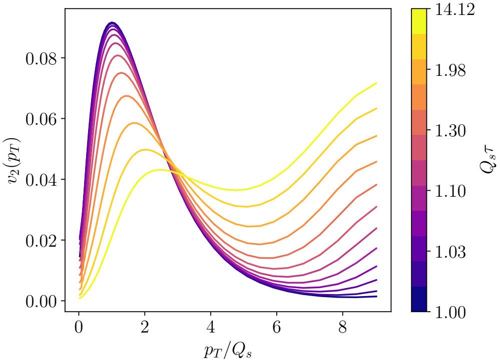
- In the following results we take $v_2 = 0.25$ and $v_{n\neq 2} = 0$





Phys. Lett. B 747 (2015). Schenke, Schlichting, and Venugopalan

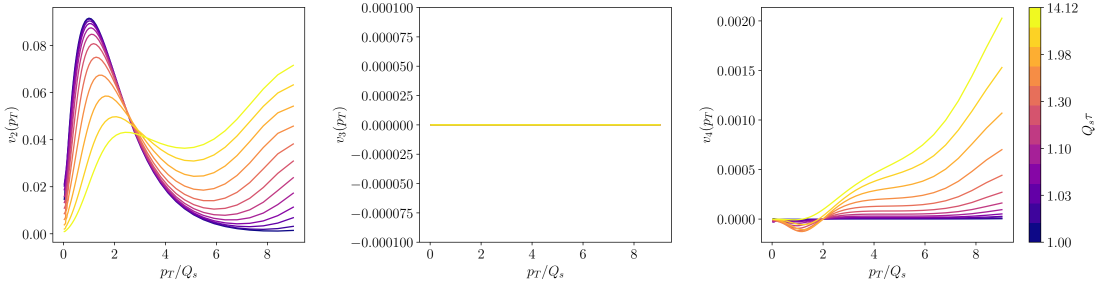
Eur. Phys. J. C 79.9 (2019). Agostini, Altinoluk, and Armesto



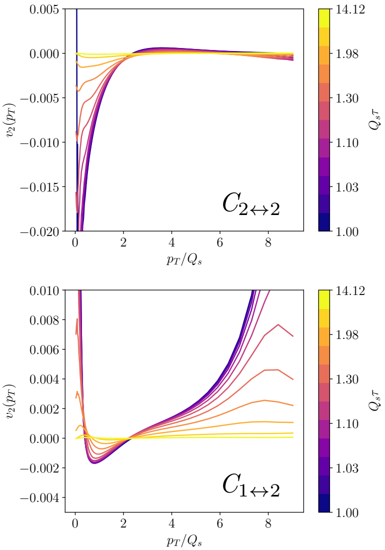
$$v_n(p_T;t) = \frac{\int dp_z d\phi \cos(n(\phi - \psi_n)) f(\mathbf{p};t)}{\int dp_z d\phi f(\mathbf{p};t)}$$

- The magnitude of the anisotropy decreases with time
- The position of the maximum is shifted to higher momenta
- Anisotropy at large p_T is produced easily because of the low occupancy in this region

- Initial $v_2 \Rightarrow$ only even harmonics are generated



p_T-dependent anisotropies III. Final state interaction effects



- Infrared:
  - $C_{1\leftrightarrow 2} \to$ Increases v_2 due to **collinear radiation** of partons from the more anisotropic regions of the phase space
  - $C_{2\leftrightarrow 2} \to$ Decreases the anisotropy more efficiently than $C_{1\leftrightarrow 2}$
- Ultraviolet:
  - $C_{1\leftrightarrow 2} \to$ Increases v_2 due to **collinear merging** of partons in the more anisotropy regions of the phase space
  - $C_{2\leftrightarrow 2} \to$ Modifies v_2 less efficiently

- Same initial condition as before

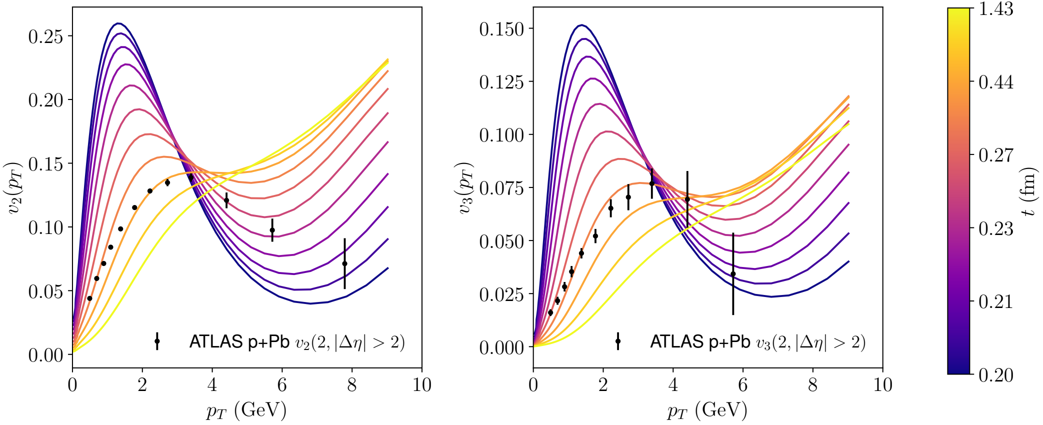
$$f_0(\mathbf{p}) = f_0(\mathbf{p}_T)\left(1 + \sum_n v_n^0(\mathbf{p}_T)\cos(n\phi)\right) \;, \qquad v_n^0(p_T) = \tilde{v}_n \frac{p}{Q} e^{-|p|/Q}$$

- Parametrization to reproduce energy density from $p + Pb$ collisions at LHC

JHEP 11 (2021). Kurkela, Mazeliauskas, and Törnkvist

- Initial time for the kinetic stage set to $t_0 = 0.2$ fm
- Initialize anisotropy with $\tilde{v}_2 = 0.75$, $\tilde{v}_3 = 0.45$ and $\tilde{v}_{n\neq 2,3} = 0$
- Position of the maximum set to $Q \sim Q_s \approx 1.22$ GeV

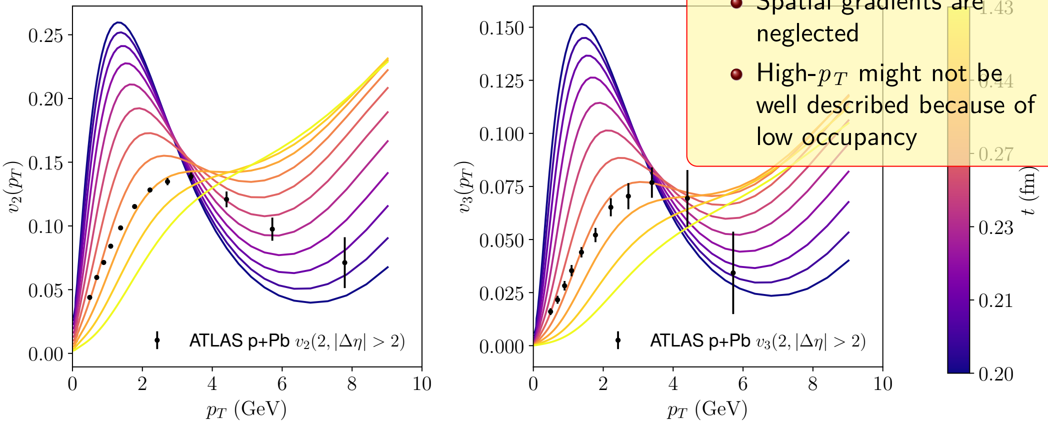
- Experimental data mimicked at a time $t \sim 0.5$ fm



- Experimental data mimicked at a time $t \sim 0.5$ fm

Disclaimer

- Spatial gradients are neglected
- High-p_T might not be well described because of low occupancy

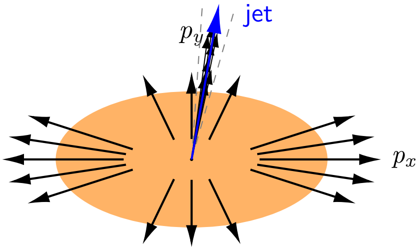


Conclusions

- Initial anisotropies may not be fully washed out at hydrodynamization time
- Harmonic coefficients with higher n isotropize faster
- Small higher-order Fourier coefficients are generated for initial $v_n \neq 0$
- Final state interactions introduce a mechanism which shifts the maximum of the azimuthal anisotropy to higher p_T, as experimental data suggest

Outlook

- Including finite size in the transverse plane enables a more complete study of the bulk's collective behavior
- We have a setup to study the influence of the initial anisotropies in hard probes, such as jets



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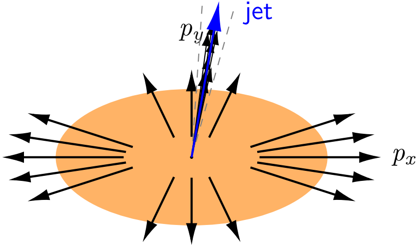
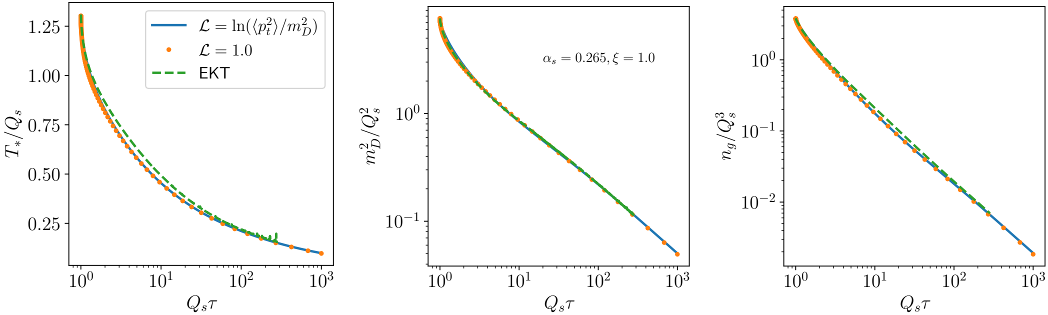


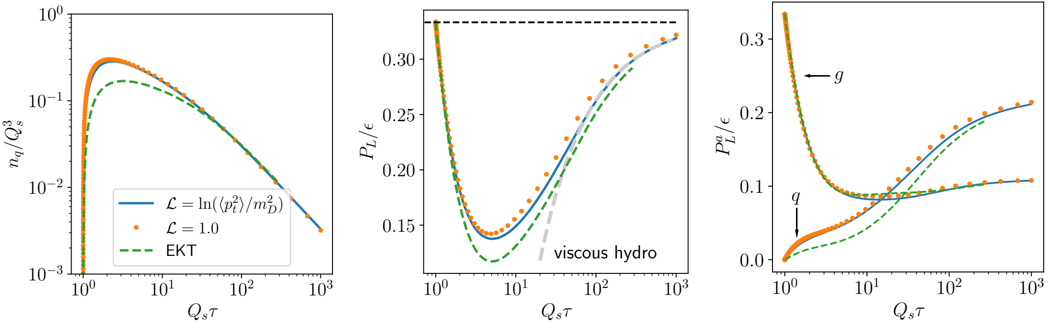
Thank you for your attention!

Back-up



- Nice quantitative agreement during whole evolution
- Gluon number density dominated by expansion

Comparison with EKT II. $\alpha_s = 0.265$



- Quantitative diferences in n_q due to differences in $g \to q\bar{q}$ splitting rates
- Higher n_q systematically leads to higher P_L/ϵ