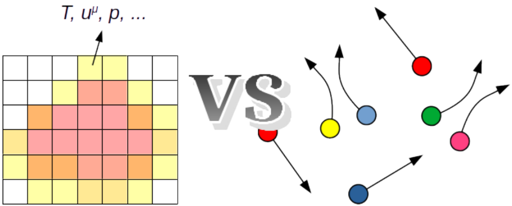
# Non-equilibrium evolution of the soft sector in hadronic collisions from kinetic theory

Clemens Werthmann
Ghent University




Funded by the European Union
European Research Council
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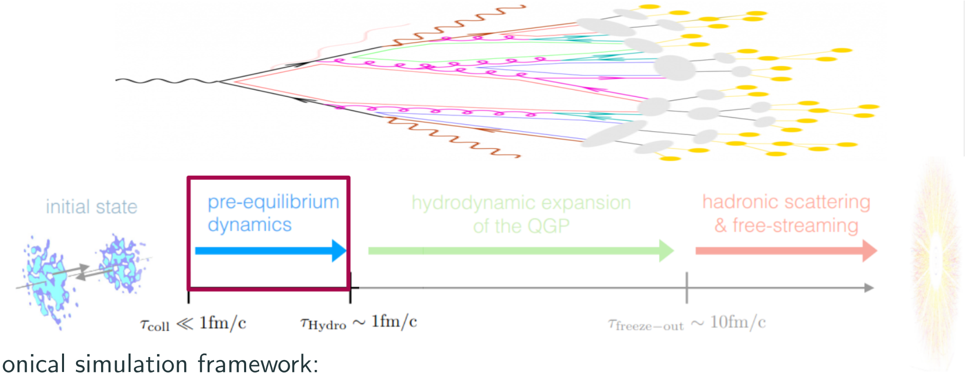
# What is this talk about?




Canonical simulation framework:

- jets modeled on top of soft background
- soft background undergoes multi-stage evolution

$\Rightarrow$ How to describe this soft background at early times?

## Cons:

Effective field theory of $T^{\mu\nu}$ for long wavelength dynamics close to equilibrium

- gradient expansion: Knudsen number $\mathrm{Kn} \sim \ell_{\mathrm{mfp}}/L$
- expansion around equilibrium: inverse Reynolds number $\mathrm{Re}^{-1} \sim |\pi_{\mu\nu}|/P$

Denicol, Niemi, Molnar, Rischke, PRD 85, 114047

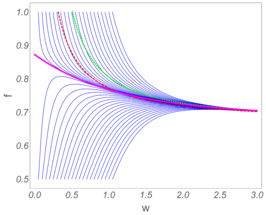
Transport coefficients computed from perturbations to equilibrium (“Green-Kubo relations”)

$$\eta = \frac{1}{T} \int_0^\infty \mathrm{d}t \int \mathrm{d}^3x \, \langle \pi^{xy}(x,t) \pi^{xy}(0,t) \rangle_{\mathrm{eq}}$$

## Pros:

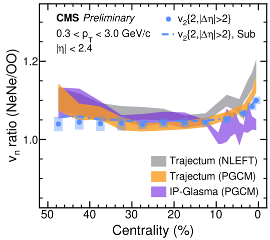
Early universal dynamics through “hydrodynamic attractor”

Heller, Spaliński, PRL 115, 072501




Accurate predictions for light ions

Giacalone et al., PRL 135, 012302
012302CMS-PAS-HIN-25-009



# Caveats of phenomenological description

## Transport coefficients:

**Expansion driven memory loss brings universal dynamics.**

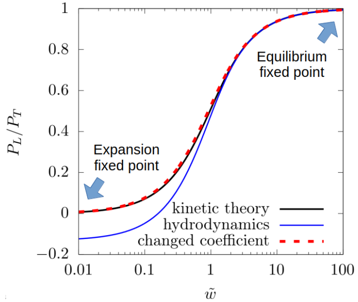
**But: attractor does not speed up equilibration!**

- early time part of attractor adjusted by changing only one second order coefficient Jean-Paul Blaizot, Li Yan, PRC 104, 055201
- transport coefficients can differ in non-equilibrium:

  $\eta/s < 1/4\pi$ in holography M.F. Wondrak, M. Kaminski, M. Bleicher, PLB 811, 135973

$\Rightarrow$ Optimizing transport coefficients to describe data:

**pheno model, not consistent effective field theory!**



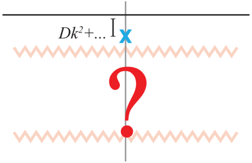

## Initial state:

$$v_2 = \kappa \epsilon_2 \,, \qquad \frac{\mathrm{d}N_{\mathrm{ch}}}{\mathrm{d}\eta} \approx \# \cdot \int_{\mathbf{x}_\perp} (e\tau^\alpha)_0{}^\gamma$$

When adjusting initial state parameters to match final state observables, may cover inaccuracies in modeling of dynamical response ($\mathrm{d}N_{\mathrm{ch}}/\mathrm{d}\eta$-estimate: Giuliano Giacalone et al., PRL 123, 262301)

# What is hydrodynamization?

Decay of non-hydrodynamic modes

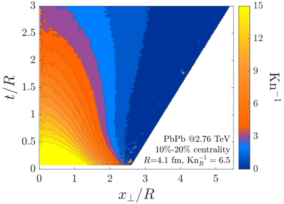



Aleksi Kurkela, Urs A. Wiedemann, Bin Wu, EPJC 79 (2019) 11, 965

Match with constitutive relations

$$T^{\mu\nu} \stackrel{?}{=} (e+P)u^\mu u^\nu - Pg^{\mu\nu}$$
$$(-\eta\sigma^{\mu\nu} + \ldots)$$

Small $\mathrm{Kn}$, $\mathrm{Re}^{-1}$




V. Nugara, V. Greco, S. Plumari, EPJC 85 (2025) 3, 311

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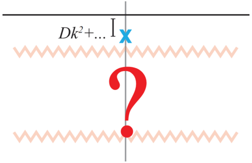


Aleksi Kurkela, Urs A. Wiedemann, Bin Wu, EPJC 79 (2019) 11, 965

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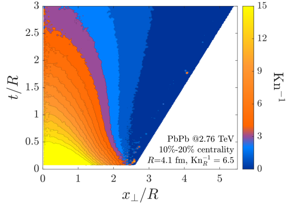



V. Nugara, V. Greco, S. Plumari, EPJC 85 (2025) 3, 311

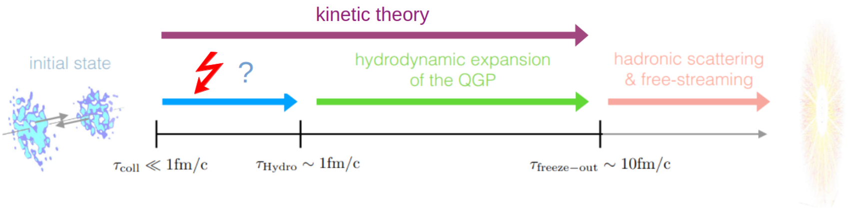
Common ground:

- hydrodynamization is a smooth transition
- applicability of hydro $\widehat{=}$ microscopic details have become irrelevant

# Assessing applicability of hydro by comparing flow

Philosophy: Won't get precise answer anyways, so let's find a practical one

$\Rightarrow$ **When does hydro accurately reproduce flow of kinetic theory?**




- need formulation of kinetic theory that is applicable in dense limit
- how to match the two descriptions?

# Dynamical descriptions

kinetic theory in conformal relaxation time approximation

$$p^\mu \partial_\mu f = -\frac{f - f_{\mathrm{eq}}}{\tau_R}\,, \quad \tau_R = 5T^{-1}\frac{\eta}{s}$$

- some microscopic info "integrated out"
  $\Rightarrow$ no diluteness requirement
- dynamics depends only on opacity $\hat{\gamma}$
  Aleksi Kurkela, Urs A. Wiedemann, Bin Wu, EPJC 79 (2019) 11, 965

$$\hat{\gamma} = \left(5\frac{\eta}{s}\right)^{-1}\left(\frac{1}{a\pi} R \frac{\mathrm{d}E_\perp^{(0)}}{\mathrm{d}\eta}\right)^{1/4}$$

total interactions $\hat{\gamma} \sim \mathrm{Kn}^{-1}$: depends on shear viscosity, transverse size and energy scale

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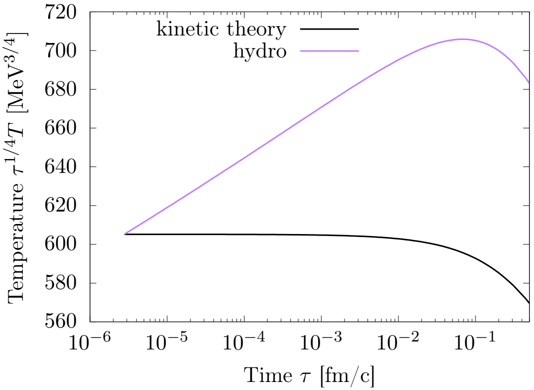
viscous hydrodynamics:

- conformal equation of state $\mathcal{E} = 3P = aT^4$
- transport coefficients from RTA
- elliptic flow of energy $\varepsilon_p$: extracted directly from hydro, no need for particlization

$$\varepsilon_p = \frac{\int_{\mathbf{x}_\perp} T^{xx} - T^{yy} + 2iT^{xy}}{\int_{\mathbf{x}_\perp} T^{xx} + T^{yy}}$$

# Temperature in pre-equilibrium hydro




$\leftarrow \eta/s = \frac{2}{4\pi}$

- smaller $P_L \Rightarrow$ hydrodynamics overestimates temperature at early times (jet quenching: off-equilibriumness more problematic)
- kinetic theory has more information on the system's state
  but: very cost intensive to run event-by-event
  $\Rightarrow$ possible solution later!

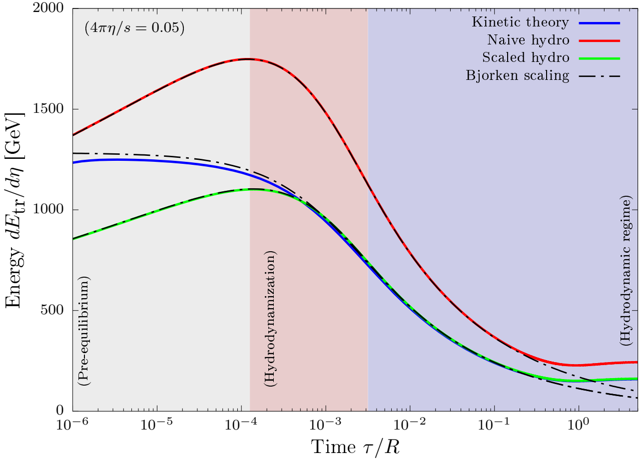
# Local Bjorken flow and "scaled hydro"

$\tau \ll \langle \nabla_\perp \rangle^{-1}$: system undergoes local Bjorken flow at each point in transverse plane

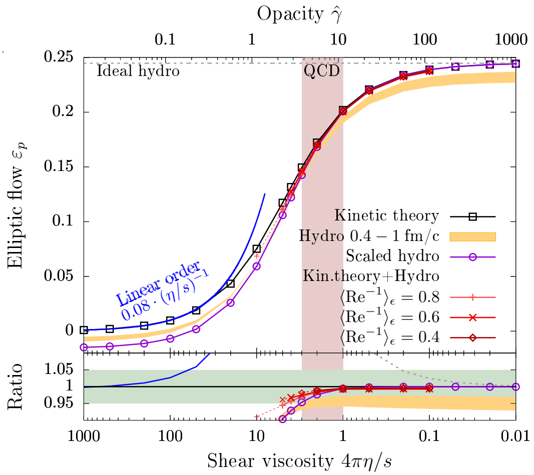
$\Rightarrow$ follows attractor behaviour: can predict evolution!

$\Rightarrow$ change initial state of hydrodynamics

Ambruș, Schlichting, CW, PRD 107 (2023) 094013



# Comparing hydro to kinetic theory (for now: average profile)




Victor E. Ambruș, Sören Schlichting, CW PRD 107 (2023) 094013

scaled hydro

- assumes equilibration before transverse expansion
- works for $\hat{\gamma} \gtrsim 4$

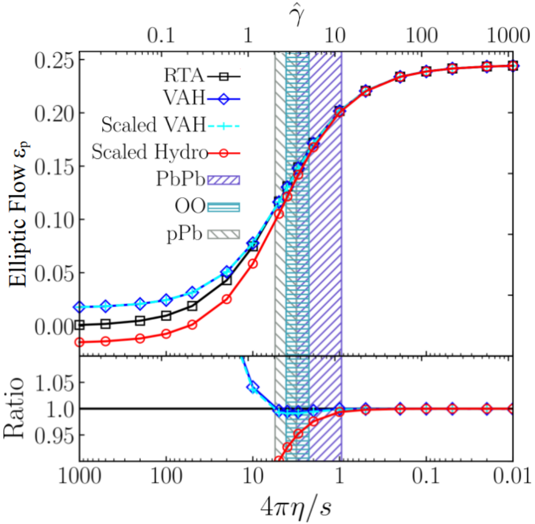
hybrid simulations

- later switch gives better agreement
- switching at fixed $\mathrm{Re}^{-1}$ gives good handle on accuracy

# Applicability of viscous anisotropic hydrodynamics

- Anisotropic hydro constructed around evolving non-equilibrium reference state $\Rightarrow$ large pressure anisotropy is no problem, can initialize very early!
- accurate beyond traditional hydro: $\hat{\gamma} \gtrsim 1$; covers high mult. pPb

Word of warning: based on flow of soft sector!

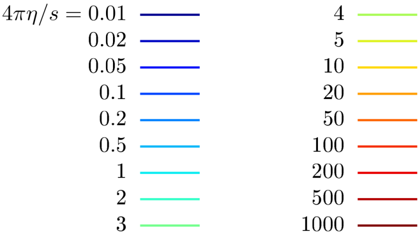


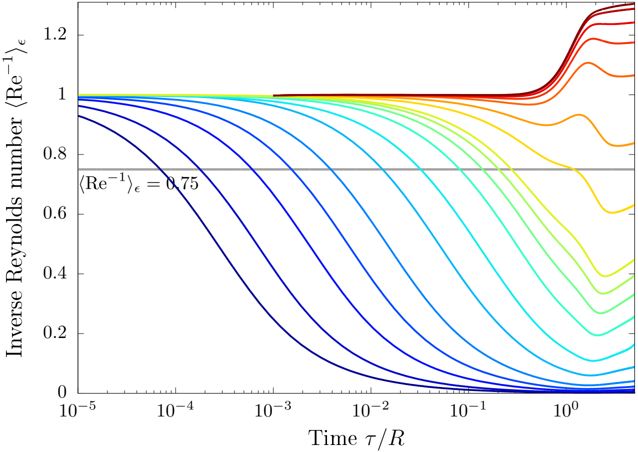

Yiyang Peng, Victor E. Ambruș, CW, Sören Schlichting, Ulrich W. Heinz, Huichao Song, PRC 113, 024918

# Equilibration of different size systems

hydrodynamization criterion $\langle \mathrm{Re}^{-1} \rangle_\epsilon \leq 0.75$: ($\mathrm{Re}^{-1} = \frac{|\pi^{\mu\nu}|}{\sqrt{6}e}$)

- takes longer for smaller systems
- some systems never equilibrate enough



# Regime of applicability of hydrodynamics

timescale of hydrodynamization:

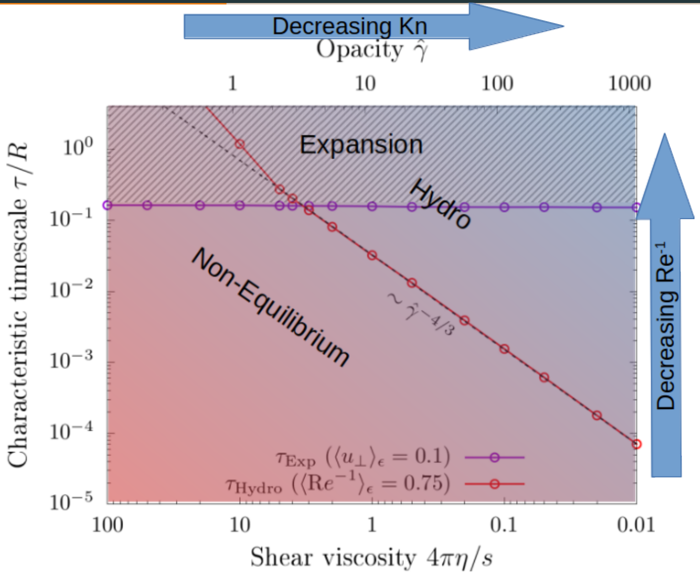
$\tau_{\mathrm{Hydro}}/R \approx 0.84\hat{\gamma}^{-4/3}$

onset of transverse expansion:

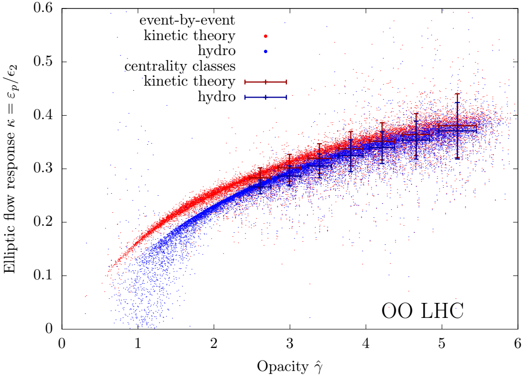
$\tau_{\mathrm{Exp}}/R \approx 0.16$

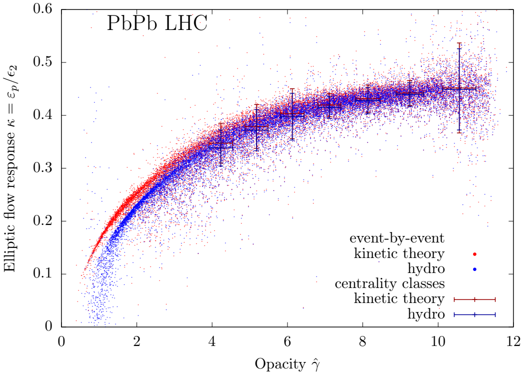
$\Rightarrow$ for $\hat{\gamma} \lesssim 3$ (central OO?), need non-equilibrium description of transverse expansion

Victor E. Ambruș, Sören Schlichting, CW PRL 130 (2023) 152301



# Event-by-event Flow Responses



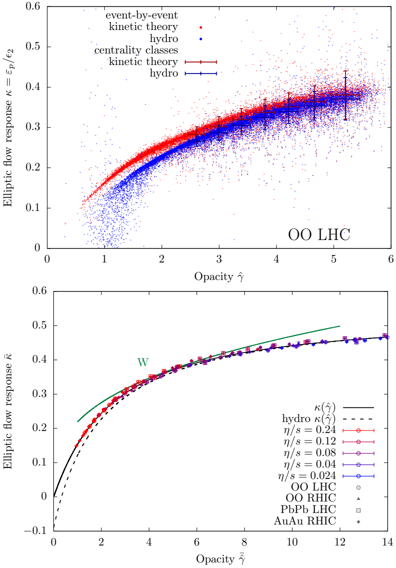




- still hydro $\rightarrow$ kinetic theory as $\hat{\gamma} \rightarrow \infty$
- some event-by-event spread, but mainly following universal curves

Victor E. Ambruș, Sören Schlichting, CW, PRD 111 (2025) 054024

# Universal flow response curve $v_2/\epsilon_2 = \kappa(\hat{\gamma})$

Hadronic collision simulations



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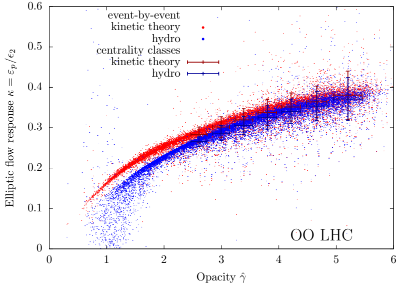
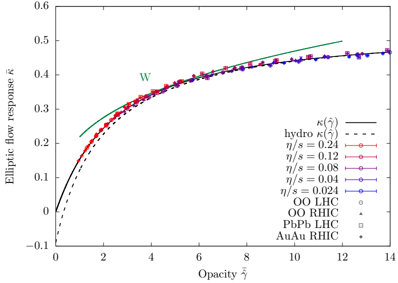
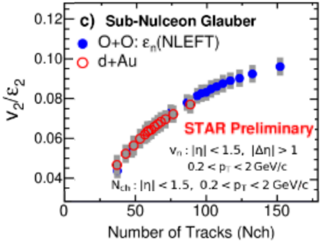
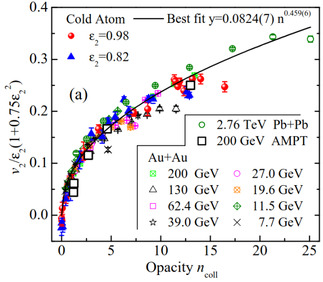



Experiment

Hadronic collisions

STAR, arXiv:2510.19645

Cold atoms

Kei Li, Hong-Fang Song, Hao-Jie Xu, Yu-Liang Sun, Fuqiang Wang, arXiv:2405.02847

# Hydrodynamization Observable: Definition

$$\mathrm{W} = \frac{2}{k}\frac{\Delta \log(c_2\{2k\})}{\Delta \log(\mathrm{d}E_\perp/\mathrm{d}y)} \approx \frac{\mathrm{d}\log\kappa}{\mathrm{d}\log\hat{\gamma}} \xrightarrow{\hat{\gamma}\to 0} 1 \quad \xrightarrow{\hat{\gamma}\to\infty} 0$$

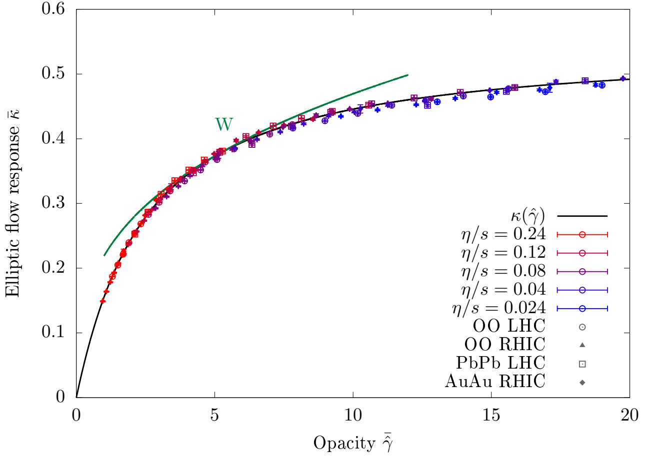
measuring hydrodynamization

cancel geometry factors between similar systems:

$$\frac{\bar{\kappa}^{2k}_{\mathrm{RHIC}}}{\bar{\kappa}^{2k}_{\mathrm{LHC}}} \approx \frac{c_2^{\mathrm{RHIC}}\{2k\}}{c_2^{\mathrm{LHC}}\{2k\}}$$

$$\frac{\hat{\gamma}_{\mathrm{RHIC}}}{\hat{\gamma}_{\mathrm{LHC}}} \approx \left(\frac{\frac{\mathrm{d}E_\perp}{\mathrm{d}\eta}_{\mathrm{RHIC}}}{\frac{\mathrm{d}E_\perp}{\mathrm{d}\eta}_{\mathrm{LHC}}}\right)^{1/4}$$

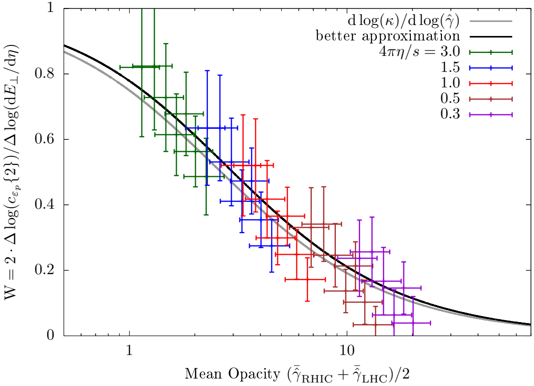
to combine the two: $\log$ turns ratios into differences



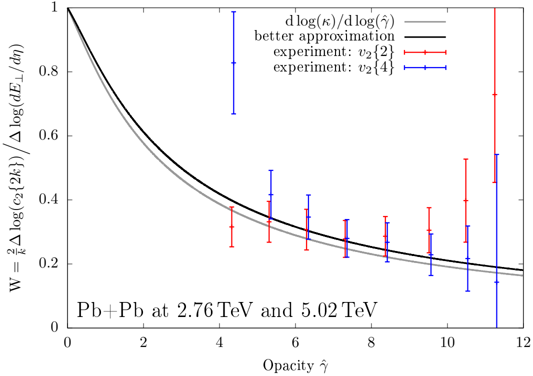
# Hydrodynamization Observable: Proof of principle

crosscheck of W-observable:

with simulations




with experimental data




previous hydrodynamization criterion: $\hat{\gamma} \sim 3$ corresponds to $\mathrm{W} \sim 0.5$

# Jet quenching with the attractor

Reminder: $\tau \ll \langle \nabla_\perp \rangle^{-1} \Rightarrow$ Bjorken flow at each transverse point

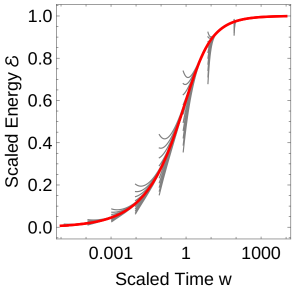
attractor behaviour also in $\mathcal{E}(w) = \frac{e\tau^{4/3}}{\lim\limits_{\tau\to\infty} e\tau^{4/3}}$

G.Giacalone, A.Mazeliauskas, S.Schlichting, PRL 123, 262301

knowing attractor curve, determine $\lim\limits_{\tau\to\infty} e\tau^{4/3}$ and map

$\tau \longleftrightarrow w \propto \tau T$: can extrapolate $e(\tau)$ to early times

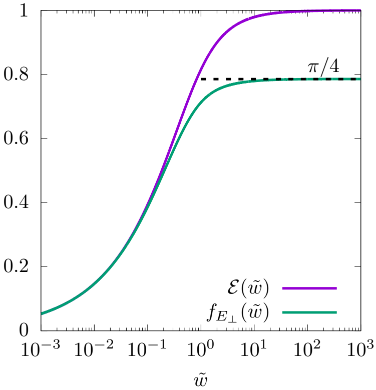
$\Rightarrow$ estimate early time jet quenching Daniel Pablos, Adam Takacs, PRC 113, 044913

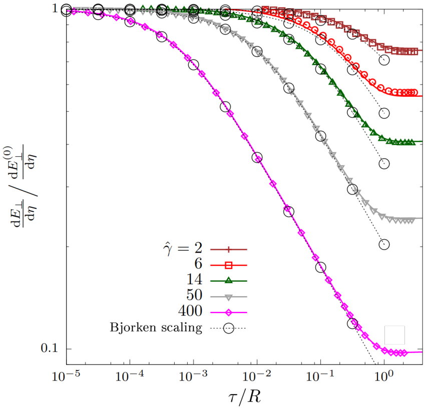



Similarly, we could also

- set different early time pressure anisotropies to test if they are excluded by data
- reconstruct any other (azimuthally isotropic) moment of the phase space distribution

# Predicting transverse energy




- Transverse energy $\frac{dE_\perp}{d^2x_\perp d\eta} = \int \frac{d^3p}{(2\pi)^3} p_\perp f$ has attractor in $f_{E_\perp} = \frac{\frac{dE_\perp}{d^2x_\perp d\eta}\tau^{1/3}}{\lim\limits_{\tau\to\infty} e\tau^{4/3}}$

  $\Rightarrow$ can also be predicted
- Comparing to 2+1D evolution: prediction accurate until onset of transverse expansion

# Predicting transverse flow

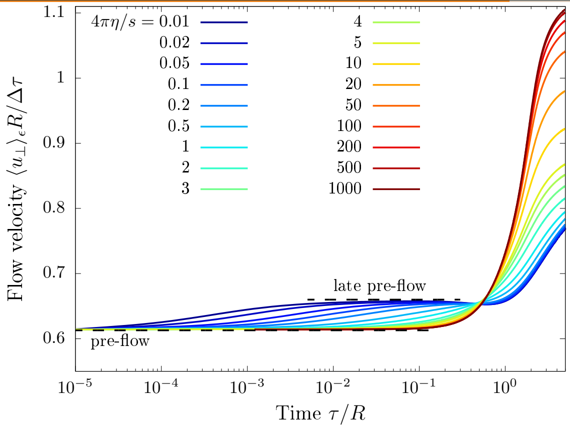
Flow buildup in pre-equilibrium can be estimated assuming

- initially diagonal $T^{\mu\nu}$
- linearized time evolution

For constant $P_L/e$:

$$u^i = -\frac{\tau}{3 - P_L/e}\frac{\partial_i e_0}{e_0}$$

Joshua Vredevoogd, Scott Pratt, PRC 79, 044915




knowledge of the attractor curve can supplement this with dynamical $P_L/e$:

$$u^i = -\frac{\tau^{1/3}}{3-(P_L/e)_0}\frac{\partial_i e_0}{e_0}\frac{2}{\mathcal{E}(1-\frac{3}{8}f_\pi)}\int_{\tau_0}^{\tau} d\bar{\tau}\,\frac{\frac{1}{3}-\frac{1}{2}f_\pi - \frac{\bar{\tilde{w}} f_\pi'}{8}}{1-\frac{\bar{\tilde{w}}}{4\mathcal{E}}\mathcal{E}'}\bar{\tau}^{-1/3}\mathcal{E}$$

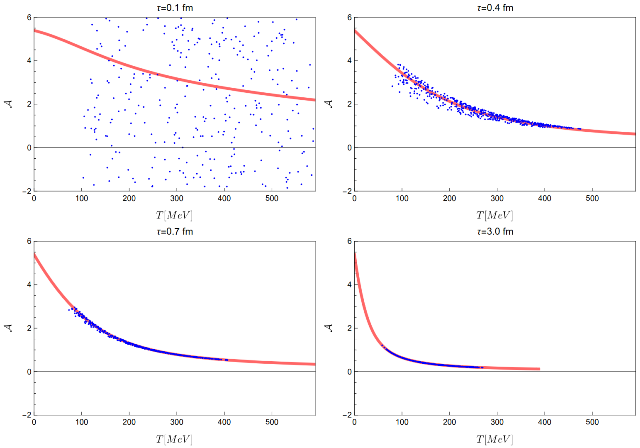
Victor Ambruș, Sören Schlichting, CW, PRD 107, 094013

# Non-conformal case

Advantage of conformality: power law relations of important quantities
$\tau_R^{-4} \propto T^4 \propto e \propto P$

MIS hydro with non-conformal eos $\rightarrow$ $(\mathcal{A} = \frac{P_T - P_L}{3e})$

- attractor behaviour survives!
- attractor curve becomes attractor surface in $(\mathcal{A}, T, \tau)$-space




$\Rightarrow$ no longer able to fully predict evolution, but it is still constrained
Michał Spaliński, arXiv:2508.21039

# Summary

- hydrodynamization time $\tau_{\mathrm{Hydro}}/R \approx 0.84\hat{\gamma}^{-4/3}$
  - "phenomenological" hydro more accurate than theoretically consistent hydro: non-equilibrium transport coefficients
  - anisotropic hydro accurate at early times

- universal flow response curve allows to measure degree of hydrodynamization independent of theory modeling

$$\mathrm{W} = \frac{2}{k}\frac{\Delta\log(c_2\{2k\})}{\Delta\log(\mathrm{d}E_\perp/\mathrm{d}y)} \approx \frac{\mathrm{d}\log\kappa}{\mathrm{d}\log\hat{\gamma}}$$

- predict early time dynamics via local Bjorken flow
  - evolution according to attractor before onset of transverse expansion